# Computer algebra independent integration tests

Summer 2022 edition

## 4-Trig-functions/4.1-Sine/73-4.1.2.1-a+b-sin-^m-c+d-sin-^n

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 5:09am

# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 837 ]. This is test number [ 73 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 837 ) | 0.00 ( 0 ) |
| Mathematica | 97.97 ( 820 ) | 2.03 ( 17 ) |
| Maple | 76.46 ( 640 ) | 23.54 ( 197 ) |
| Fricas | 69.30 ( 580 ) | 30.70 ( 257 ) |
| Giac | 57.59 ( 482 ) | 42.41 ( 355 ) |
| Mupad | 41.10 ( 344 ) | 58.90 ( 493 ) |
| Maxima | 26.05 ( 218 ) | 73.95 ( 619 ) |
| Sympy | 19.47 ( 163 ) | 80.53 ( 674 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

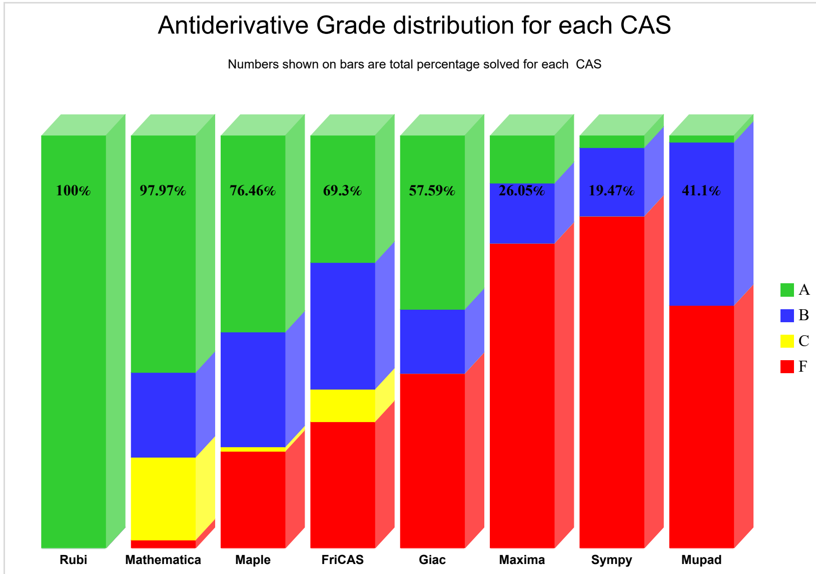
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

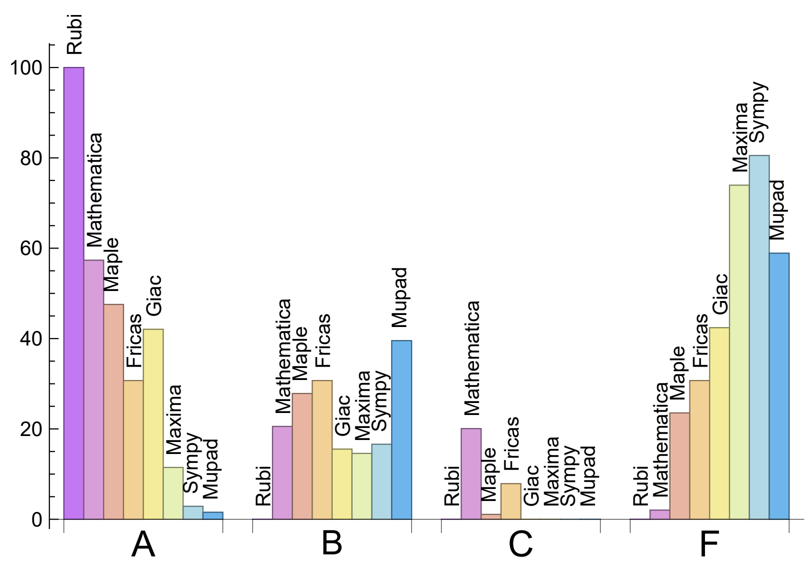
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 57.35 | 20.55 | 20.07 | 2.03 |
| Maple | 47.55 | 27.84 | 1.08 | 23.54 |
| Giac | 42.05 | 15.53 | 0.00 | 42.41 |
| Fricas | 30.70 | 30.70 | 7.89 | 30.70 |
| Maxima | 11.47 | 14.58 | 0.00 | 73.95 |
| Sympy | 2.87 | 16.61 | 0.00 | 80.53 |
| Mupad | N/A | 39.55 | 0.00 | 58.90 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 17 | 88.24 % | 11.76 % | 0.00 % |
| Maple | 197 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 257 | 83.66 % | 14.40 % | 1.95 % |
| Giac | 355 | 88.45 % | 9.01 % | 2.54 % |
| Maxima | 619 | 83.68 % | 1.94 % | 14.38 % |
| Sympy | 674 | 55.34 % | 33.23 % | 11.42 % |
| Mupad | 493 | 99.19 % | 0.81 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

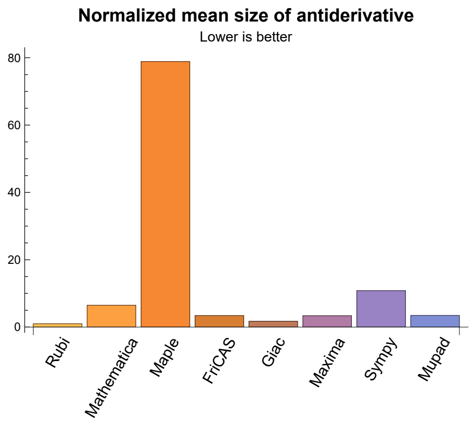
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

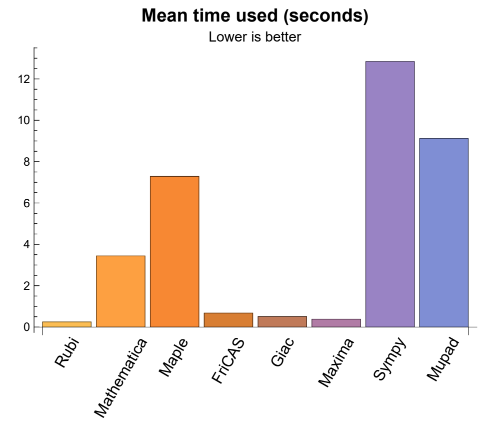
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.25 | 170.11 | 0.99 | 122.00 | 1.00 |
| Mathematica | 3.44 | 1708.25 | 6.49 | 173.00 | 1.45 |
| Maple | 7.29 | 53873.47 | 78.86 | 177.00 | 1.52 |
| Maxima | 0.38 | 329.67 | 3.37 | 182.50 | 2.31 |
| Fricas | 0.68 | 582.59 | 3.41 | 260.50 | 2.43 |
| Sympy | 12.84 | 1380.21 | 10.82 | 459.00 | 5.47 |
| Giac | 0.51 | 232.53 | 1.74 | 143.50 | 1.45 |
| Mupad | 9.12 | 483.90 | 3.44 | 149.50 | 2.01 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
80
60
40
20
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

## 1.4 list of integrals that has no closed form antiderivative

$\{221, 226, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831\}$

## 1.5 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {97, 102, 103, 109, 115, 116, 117, 119, 120, 124, 125, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 145, 146, 211, 216, 217, 218, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 473, 482, 483, 484, 485, 486, 487, 488, 542, 563, 564, 565, 566, 571, 579, 580, 581, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 631, 632, 646, 652, 653, 654, 665, 666, 667, 668, 669, 670, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 814, 819, 828, 829, 830, 835, 836, 837}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

# 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.9 Important notes about some of the results

## 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

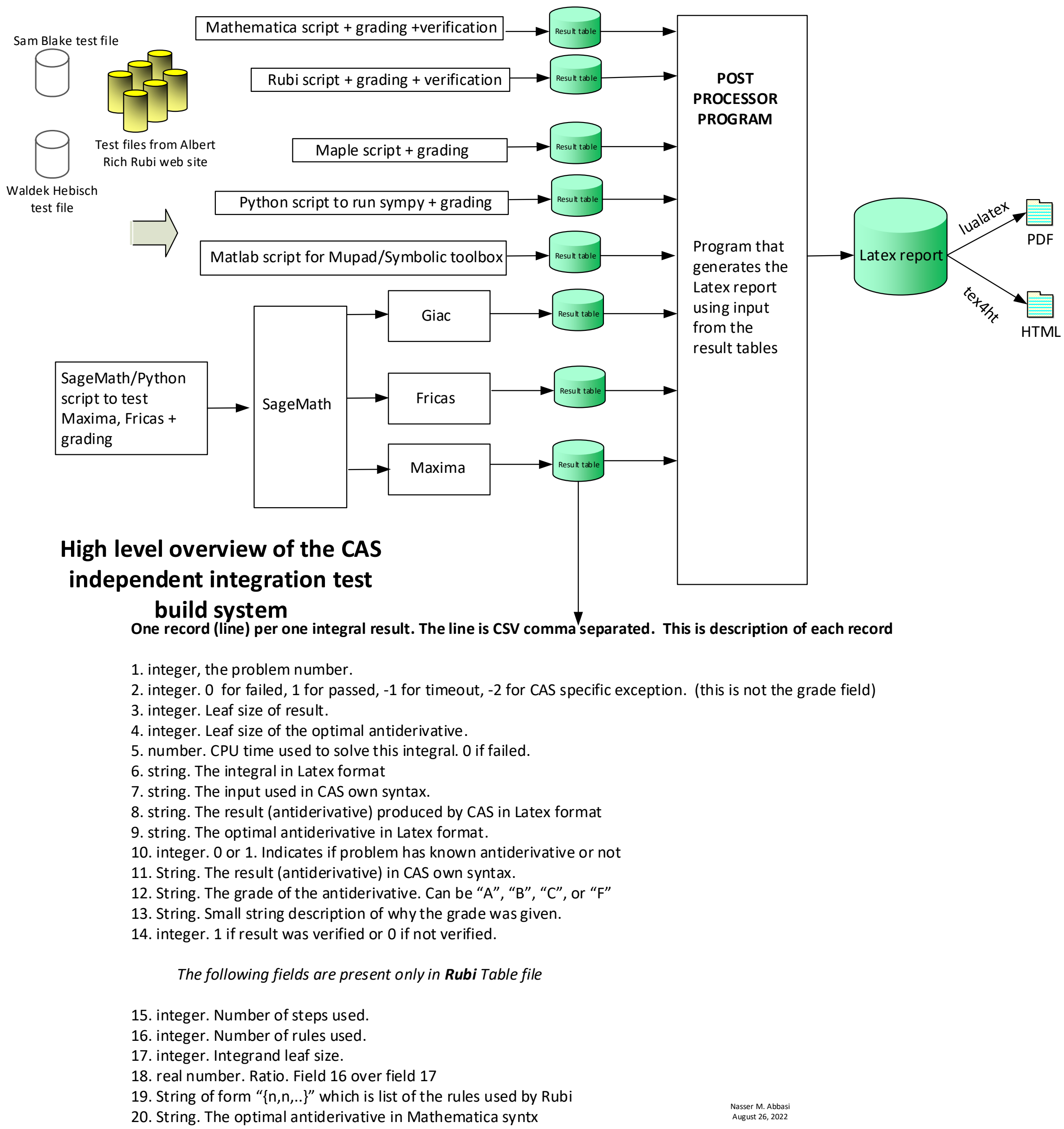
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 10, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 35, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 91, 92, 93, 94, 95, 104, 105, 106, 107, 110, 111, 112, 113, 144, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 164, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 212, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 265, 266, 267, 268, 269, 270, 272, 274, 275, 276, 277, 278, 279, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 307, 308, 309, 317, 318, 319, 320, 324, 325, 326, 327, 332, 333, 334, 335, 340, 341, 342, 343, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 401, 402, 403, 411, 412, 413, 419, 420, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 462, 463, 465, 466, 467, 468, 469, 474, 475, 476, 477, 478, 480, 481, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 529, 530, 531, 532, 535, 536, 537, 538, 539, 542, 568, 569, 570, 571, 572, 573, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 599, 604, 610, 621, 622, 623, 624, 625, 626, 627, 632, 636, 640, 643, 645, 646, 647, 648, 649, 651, 654, 658, 662, 663, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 768, 769, 771, 777, 778, 785, 786, 791, 801, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 815, 820, 821, 823, 824, 825, 826, 827, 831, 832, 833, 834 }

B grade: { 4, 6, 7, 8, 9, 11, 17, 18, 19, 20, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 49, 50, 58, 60, 118, 119, 122, 123, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 153, 161, 162, 163, 165, 173, 174, 216, 217, 218, 231, 232, 241, 243, 255, 256, 263, 264, 271, 273, 280, 281, 282, 293, 298, 301, 310, 328, 336, 345, 346, 347, 354, 365, 378, 392, 399, 400, 414, 417, 418, 421, 456, 457, 461, 464, 471, 472, 479, 525, 533, 534, 540, 541, 574, 575, 591, 592, 593, 596, 597, 598, 601, 602, 603, 605, 606, 607, 608, 611, 612, 613, 614, 615, 616, 617, 618, 619, 628, 629, 633, 634, 635, 637, 638, 639, 644, 650, 665, 666, 667, 668, 669, 670, 692, 715, 716, 722, 765, 766, 770, 772, 773, 775, 776, 779, 780, 781, 782, 783, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 819, 828, 829, 830, 835, 836, 837 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 103, 109, 115, 116, 117, 120, 121, 124, 125, 128, 129, 141, 142, 143, 145, 146, 206, 210, 211, 302, 303, 304, 305, 306, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 344, 385, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 430, 431, 432, 433, 470, 473, 482, 483, 484, 485, 486, 487, 488, 526, 527, 528, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 588, 589, 590, 594, 595, 600, 609, 620, 630, 631, 641, 642, 652, 653, 714, 745, 746, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 767, 774, 784, 814, 816, 822 }

F grade: { 96, 108, 114, 140, 222, 223, 405, 655, 656, 657, 659, 660, 661, 664, 802, 817, 818 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 88, 90, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 205, 206, 207, 208, 209, 210, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 348, 349, 350, 351, 355, 358, 359, 360, 367, 369, 370, 371, 372, 381, 382, 387, 392, 393, 394, 395, 402, 403, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 493, 506, 507, 513, 519, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 543, 544, 545, 546, 547, 553, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 686, 687, 688, 689, 690, 691, 694, 695, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 726, 746, 747, 748, 755, 763, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 37, 41, 42, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 197, 198, 202, 203, 204, 212, 238, 243, 248, 249, 251, 256, 257, 273, 282, 287, 340, 341, 344, 346, 347, 352, 353, 354, 356, 357, 361, 362, 363, 364, 365, 366, 368, 373, 374, 375, 376, 377, 378, 379, 380, 383, 384, 385, 386, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 432, 433, 442, 443, 444, 445, 451, 452, 482, 483, 484, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 514, 515, 516, 517, 518, 520, 521, 534, 535, 541, 542, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 567, 568, 569, 570, 575, 576, 577, 578, 584, 585, 586, 587, 590, 591, 592, 593, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 677, 684, 685, 692, 693, 696, 705, 714, 715, 716, 717, 718, 719, 720, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 762, 764, 769, 770, 771, 774, 775, 776, 777, 778, 780, 781, 782, 783, 784, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800 }

C grade: { 211, 765, 766, 767, 768, 772, 773, 779, 785 }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 564, 565, 566, 571, 572, 573, 574, 579, 580, 581, 582, 583, 588, 589, 594, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

### 2.1.4 Maxima

A grade: { 1, 2, 6, 7, 8, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 221, 226, 227, 228, 229, 230, 236, 237, 239, 240, 247, 250, 252, 265, 275, 286, 344, 353, 364, 377, 385, 391, 398, 411, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 446, 447, 457, 622, 633, 650, 671, 672, 673, 674, 678, 679, 680, 681, 686, 687, 688, 689, 697, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 85, 231, 232, 233, 234, 235, 238, 241, 242, 243, 244, 245, 246, 248, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 412, 413, 444, 445, 453, 454, 455, 456, 461, 462, 463, 464, 465, 466, 470, 471, 472, 473, 474, 475, 476, 480, 481, 568, 569, 570, 576, 577, 578, 585, 586, 587 }

C grade: { }

F grade: { 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 430, 431, 432, 433, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 458, 459, 460, 467, 468, 469, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 579, 580, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 675, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 21, 22, 26, 32, 33, 34, 35, 37, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 61, 64, 87, 88, 89, 90, 149, 150, 151, 152, 157, 158, 159, 160, 161, 164, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 188, 189, 221, 226, 227, 228, 229, 230, 231, 236, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 253, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 346, 348, 349, 350, 351, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 386, 387, 388, 392, 393, 394, 395, 400, 401, 402, 403, 411, 412, 413, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 449, 453, 454, 456, 457, 465, 466, 475, 476, 522, 523, 524, 526, 529, 530, 531, 532, 536, 537, 538, 539, 546, 591, 622, 628, 633, 639, 644, 650, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 686, 687, 688, 689, 690, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 709, 710, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 8, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 36, 38, 39, 40, 48, 49, 50, 51, 56, 57, 58, 59, 60, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 153, 154, 155, 156, 162, 163, 165, 172, 182, 183, 184, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 242, 243, 244, 245, 246, 254, 255, 256, 257, 258, 259, 260, 271, 272, 273, 280, 281, 282, 283, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 329, 340, 341, 347, 354, 361, 365, 373, 378, 379, 399, 432, 433, 441, 442, 443, 450, 451, 452, 455, 458, 459, 460, 461, 462, 463, 464, 467, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 480, 481, 525, 527, 528, 533, 534, 535, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 677, 683, 684, 685, 691, 692, 693, 704, 706, 707, 708, 711, 714, 715, 716, 717, 718, 719 }

C grade: { 203, 204, 207, 208, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744 }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 344, 352, 353, 362, 363, 364, 374, 375, 376, 377, 383, 384, 385, 389, 390, 391, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 634, 635, 636, 637, 638, 640, 641, 642, 643, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 705, 712, 713, 720, 721, 722, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

### 2.1.6 Sympy

A grade: { 7, 150, 152, 159, 160, 167, 169, 175, 221, 226, 429, 438, 457, 674, 680, 681, 689, 695, 697, 810, 811, 812, 813, 831 }

B grade: { 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 149, 151, 153, 157, 158, 166, 168, 178, 179, 180, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 425, 426, 427, 428, 430, 434, 435, 436, 437, 439, 444, 445, 446, 447, 453, 454, 455, 456, 458, 461, 462, 463, 464, 465, 466, 470, 471, 472, 473, 474, 475, 476, 480, 481, 671, 672, 673, 675, 678, 679, 682, 686, 687, 688, 694, 700, 701, 702 }

C grade: { }

F grade: { 8, 9, 10, 11, 17, 18, 19, 20, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 154, 155, 156, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 431, 432, 433, 440, 441, 442, 443, 448, 449, 450, 451, 452, 459, 460, 467, 468, 469, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 676, 677, 683, 684, 685, 690, 691, 692, 693, 696, 698, 699, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 164, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 298, 299, 306, 307, 319, 321, 325, 327, 329, 334, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 384, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 400, 401, 402, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 453, 454, 456, 457, 458, 462, 463, 464, 465, 466, 467, 468, 473, 474, 475, 476, 477, 478, 480, 481, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 543, 544, 547, 560, 622, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 686, 687, 688, 689, 690, 694, 695, 697, 699, 700, 701, 702, 703, 704, 706, 708, 709, 710, 711, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 9, 37, 38, 41, 42, 43, 49, 64, 65, 66, 71, 72, 74, 86, 89, 90, 154, 162, 163, 165, 171, 193, 197, 198, 201, 202, 256, 257, 293, 295, 296, 297, 300, 301, 302, 303, 304, 305, 308, 309, 310, 311, 312, 313, 315, 317, 318, 320, 322, 323, 324, 326, 328, 330, 331, 332, 333, 335, 336, 337, 338, 339, 354, 365, 378, 379, 385, 399, 432, 433, 441, 442, 443, 449, 450, 451, 452, 455, 459, 460, 461, 469, 470, 471, 472, 479, 541, 542, 545, 546, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 561, 562, 563, 568, 569, 570, 633, 650, 677, 684, 685, 691, 692, 693, 696, 698, 705, 707, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722 }

C grade: { }

F grade: { 85, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 314, 316, 386, 394, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 555, 564, 565, 566, 567, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

### 2.1.8 Mupad

A grade: { 221, 226, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 63, 64, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 207, 208, 220, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 319, 320, 326, 327, 328, 333, 334, 335, 336, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 360, 361, 366, 367, 368, 369, 370, 371, 372, 373, 379, 380, 381, 382, 392, 399, 400, 411, 412, 413, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 525, 545, 546, 568, 569, 570, 576, 577, 578, 585, 586, 587, 622, 628, 633, 639, 644, 650, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 726 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 321, 322, 323, 324, 325, 329, 330, 331, 332, 337, 338, 339, 344, 345, 352, 353, 354, 362, 363, 364, 365, 374, 375, 376, 377, 378, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 579, 580, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 634, 635, 636, 637, 638, 640, 641, 642, 643, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 105 | 96 | 103 | 89 | 221 | 94 | 225 |
| N.S. | 1 | 1.00 | 1.03 | 0.94 | 1.01 | 0.87 | 2.17 | 0.92 | 2.21 |
| time (sec) | N/A | 0.069 | 0.292 | 0.286 | 0.364 | 0.370 | 0.306 | 0.562 | 10.282 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 76 | 143 | 155 | 103 | 379 | 112 | 294 |
| N.S. | 1 | 1.00 | 0.59 | 1.11 | 1.20 | 0.80 | 2.94 | 0.87 | 2.28 |
| time (sec) | N/A | 0.105 | 0.256 | 0.354 | 0.290 | 0.372 | 0.547 | 0.556 | 10.326 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 101 | 66 | 180 | 70 | 1221 | 67 | 78 |
| N.S. | 1 | 1.00 | 1.91 | 1.25 | 3.40 | 1.32 | 23.04 | 1.26 | 1.47 |
| time (sec) | N/A | 0.050 | 0.081 | 0.100 | 0.538 | 0.349 | 2.392 | 0.544 | 6.818 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 87 | 58 | 128 | 53 | 665 | 56 | 59 |
| N.S. | 1 | 1.00 | 2.07 | 1.38 | 3.05 | 1.26 | 15.83 | 1.33 | 1.40 |
| time (sec) | N/A | 0.034 | 0.053 | 0.105 | 0.582 | 0.375 | 1.110 | 0.547 | 6.911 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 48 | 36 | 78 | 35 | 221 | 44 | 46 |
| N.S. | 1 | 1.00 | 1.78 | 1.33 | 2.89 | 1.30 | 8.19 | 1.63 | 1.70 |
| time (sec) | N/A | 0.048 | 0.044 | 0.092 | 0.523 | 0.335 | 0.541 | 0.560 | 6.770 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 42 | 24 | 32 | 28 | 34 | 19 | 19 |
| N.S. | 1 | 1.00 | 2.47 | 1.41 | 1.88 | 1.65 | 2.00 | 1.12 | 1.12 |
| time (sec) | N/A | 0.022 | 0.025 | 0.081 | 0.608 | 0.368 | 0.229 | 0.494 | 6.539 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 29 | 14 | 16 | 22 | 10 | 13 | 13 |
| N.S. | 1 | 1.00 | 2.42 | 1.17 | 1.33 | 1.83 | 0.83 | 1.08 | 1.08 |
| time (sec) | N/A | 0.007 | 0.021 | 0.059 | 0.338 | 0.408 | 0.103 | 0.464 | 0.021 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 74 | 21 | 31 | 53 | 0 | 24 | 23 |
| N.S. | 1 | 1.00 | 3.70 | 1.05 | 1.55 | 2.65 | 0.00 | 1.20 | 1.15 |
| time (sec) | N/A | 0.028 | 0.038 | 0.111 | 0.394 | 0.397 | 0.000 | 0.484 | 6.423 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 63 | 36 | 68 | 91 | 0 | 53 | 49 |
| N.S. | 1 | 1.00 | 2.42 | 1.38 | 2.62 | 3.50 | 0.00 | 2.04 | 1.88 |
| time (sec) | N/A | 0.046 | 0.114 | 0.096 | 0.451 | 0.374 | 0.000 | 0.490 | 6.714 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 83 | 54 | 97 | 134 | 0 | 73 | 69 |
| N.S. | 1 | 1.00 | 1.98 | 1.29 | 2.31 | 3.19 | 0.00 | 1.74 | 1.64 |
| time (sec) | N/A | 0.051 | 0.236 | 0.141 | 0.310 | 0.406 | 0.000 | 0.471 | 6.598 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 113 | 68 | 120 | 168 | 0 | 96 | 89 |
| N.S. | 1 | 1.00 | 2.05 | 1.24 | 2.18 | 3.05 | 0.00 | 1.75 | 1.62 |
| time (sec) | N/A | 0.054 | 0.557 | 0.127 | 0.396 | 0.399 | 0.000 | 0.478 | 6.497 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 100 | 80 | 198 | 105 | 1423 | 72 | 77 |
| N.S. | 1 | 1.00 | 1.52 | 1.21 | 3.00 | 1.59 | 21.56 | 1.09 | 1.17 |
| time (sec) | N/A | 0.093 | 0.179 | 0.152 | 0.733 | 0.361 | 5.048 | 0.584 | 6.813 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 84 | 56 | 144 | 95 | 779 | 51 | 62 |
| N.S. | 1 | 1.00 | 1.79 | 1.19 | 3.06 | 2.02 | 16.57 | 1.09 | 1.32 |
| time (sec) | N/A | 0.104 | 0.158 | 0.151 | 0.547 | 0.336 | 2.605 | 0.509 | 6.465 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 69 | 44 | 90 | 82 | 321 | 35 | 34 |
| N.S. | 1 | 1.00 | 1.97 | 1.26 | 2.57 | 2.34 | 9.17 | 1.00 | 0.97 |
| time (sec) | N/A | 0.052 | 0.090 | 0.100 | 0.513 | 0.342 | 1.347 | 0.501 | 6.482 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 29 | 27 | 62 | 60 | 87 | 21 | 21 |
| N.S. | 1 | 1.00 | 0.88 | 0.82 | 1.88 | 1.82 | 2.64 | 0.64 | 0.64 |
| time (sec) | N/A | 0.022 | 0.030 | 0.107 | 0.395 | 0.347 | 0.768 | 0.478 | 6.299 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 31 | 35 | 74 | 58 | 134 | 29 | 29 |
| N.S. | 1 | 1.00 | 0.94 | 1.06 | 2.24 | 1.76 | 4.06 | 0.88 | 0.88 |
| time (sec) | N/A | 0.016 | 0.023 | 0.079 | 0.433 | 0.369 | 0.272 | 0.563 | 6.306 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 129 | 41 | 89 | 117 | 0 | 40 | 38 |
| N.S. | 1 | 1.00 | 3.39 | 1.08 | 2.34 | 3.08 | 0.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.063 | 0.100 | 0.155 | 0.303 | 0.356 | 0.000 | 0.490 | 6.482 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 166 | 56 | 126 | 168 | 0 | 69 | 91 |
| N.S. | 1 | 1.00 | 3.69 | 1.24 | 2.80 | 3.73 | 0.00 | 1.53 | 2.02 |
| time (sec) | N/A | 0.095 | 0.261 | 0.142 | 0.321 | 0.361 | 0.000 | 0.558 | 6.540 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 203 | 74 | 155 | 220 | 0 | 93 | 111 |
| N.S. | 1 | 1.00 | 3.17 | 1.16 | 2.42 | 3.44 | 0.00 | 1.45 | 1.73 |
| time (sec) | N/A | 0.106 | 0.457 | 0.211 | 0.287 | 0.362 | 0.000 | 0.543 | 6.399 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 71 | 238 | 90 | 178 | 266 | 0 | 114 | 101 |
| N.S. | 1 | 1.09 | 3.66 | 1.38 | 2.74 | 4.09 | 0.00 | 1.75 | 1.55 |
| time (sec) | N/A | 0.108 | 2.502 | 0.180 | 0.394 | 0.398 | 0.000 | 0.581 | 6.396 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 191 | 108 | 306 | 158 | 3288 | 99 | 110 |
| N.S. | 1 | 1.00 | 1.89 | 1.07 | 3.03 | 1.56 | 32.55 | 0.98 | 1.09 |
| time (sec) | N/A | 0.158 | 0.074 | 0.238 | 0.622 | 0.349 | 24.864 | 0.470 | 7.022 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 170 | 100 | 252 | 145 | 2259 | 88 | 93 |
| N.S. | 1 | 1.00 | 1.89 | 1.11 | 2.80 | 1.61 | 25.10 | 0.98 | 1.03 |
| time (sec) | N/A | 0.148 | 0.056 | 0.217 | 0.568 | 0.343 | 15.526 | 0.558 | 6.670 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 140 | 66 | 198 | 132 | 1425 | 67 | 78 |
| N.S. | 1 | 1.00 | 1.97 | 0.93 | 2.79 | 1.86 | 20.07 | 0.94 | 1.10 |
| time (sec) | N/A | 0.159 | 0.057 | 0.201 | 0.534 | 0.336 | 8.840 | 0.489 | 6.891 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 112 | 64 | 144 | 119 | 777 | 51 | 50 |
| N.S. | 1 | 1.00 | 1.90 | 1.08 | 2.44 | 2.02 | 13.17 | 0.86 | 0.85 |
| time (sec) | N/A | 0.108 | 0.132 | 0.178 | 0.508 | 0.347 | 4.749 | 0.499 | 6.740 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 47 | 37 | 104 | 90 | 206 | 29 | 29 |
| N.S. | 1 | 1.00 | 0.94 | 0.74 | 2.08 | 1.80 | 4.12 | 0.58 | 0.58 |
| time (sec) | N/A | 0.053 | 0.045 | 0.129 | 0.299 | 0.334 | 2.760 | 0.476 | 6.637 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 41 | 45 | 116 | 88 | 277 | 37 | 37 |
| N.S. | 1 | 1.00 | 0.82 | 0.90 | 2.32 | 1.76 | 5.54 | 0.74 | 0.74 |
| time (sec) | N/A | 0.032 | 0.029 | 0.139 | 0.295 | 0.338 | 1.523 | 0.526 | 6.845 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 45 | 57 | 128 | 92 | 348 | 45 | 45 |
| N.S. | 1 | 1.00 | 0.90 | 1.14 | 2.56 | 1.84 | 6.96 | 0.90 | 0.90 |
| time (sec) | N/A | 0.026 | 0.045 | 0.101 | 0.315 | 0.362 | 0.737 | 0.551 | 6.633 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 160 | 61 | 143 | 168 | 0 | 56 | 54 |
| N.S. | 1 | 1.00 | 2.76 | 1.05 | 2.47 | 2.90 | 0.00 | 0.97 | 0.93 |
| time (sec) | N/A | 0.110 | 0.051 | 0.184 | 0.417 | 0.343 | 0.000 | 0.617 | 6.651 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 206 | 76 | 180 | 225 | 0 | 85 | 129 |
| N.S. | 1 | 1.00 | 3.17 | 1.17 | 2.77 | 3.46 | 0.00 | 1.31 | 1.98 |
| time (sec) | N/A | 0.154 | 0.109 | 0.190 | 0.583 | 0.388 | 0.000 | 0.478 | 6.725 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 247 | 94 | 209 | 276 | 0 | 109 | 97 |
| N.S. | 1 | 1.00 | 2.87 | 1.09 | 2.43 | 3.21 | 0.00 | 1.27 | 1.13 |
| time (sec) | N/A | 0.165 | 0.322 | 0.223 | 0.515 | 0.356 | 0.000 | 0.523 | 6.390 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 299 | 110 | 232 | 333 | 0 | 128 | 117 |
| N.S. | 1 | 1.00 | 2.90 | 1.07 | 2.25 | 3.23 | 0.00 | 1.24 | 1.14 |
| time (sec) | N/A | 0.164 | 0.630 | 0.233 | 0.437 | 0.353 | 0.000 | 0.474 | 6.687 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 165 | 83 | 0 | 132 | 0 | 147 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 0.53 | 0.00 | 0.84 | 0.00 | 0.93 | -0.01 |
| time (sec) | N/A | 0.159 | 0.325 | 1.713 | 0.000 | 0.345 | 0.000 | 0.512 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 141 | 73 | 0 | 111 | 0 | 120 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 0.60 | 0.00 | 0.91 | 0.00 | 0.98 | -0.01 |
| time (sec) | N/A | 0.116 | 0.200 | 1.734 | 0.000 | 0.344 | 0.000 | 0.497 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 117 | 63 | 0 | 92 | 0 | 93 | -1 |
| N.S. | 1 | 1.00 | 1.36 | 0.73 | 0.00 | 1.07 | 0.00 | 1.08 | -0.01 |
| time (sec) | N/A | 0.074 | 0.128 | 1.700 | 0.000 | 0.426 | 0.000 | 0.570 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 81 | 51 | 0 | 67 | 0 | 65 | -1 |
| N.S. | 1 | 1.00 | 1.45 | 0.91 | 0.00 | 1.20 | 0.00 | 1.16 | -0.02 |
| time (sec) | N/A | 0.032 | 0.082 | 1.774 | 0.000 | 0.334 | 0.000 | 0.596 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 65 | 43 | 0 | 50 | 0 | 36 | 33 |
| N.S. | 1 | 1.00 | 2.50 | 1.65 | 0.00 | 1.92 | 0.00 | 1.38 | 1.27 |
| time (sec) | N/A | 0.010 | 0.022 | 1.290 | 0.000 | 0.324 | 0.000 | 0.522 | 0.211 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 94 | 68 | 0 | 219 | 0 | 67 | -1 |
| N.S. | 1 | 1.00 | 2.54 | 1.84 | 0.00 | 5.92 | 0.00 | 1.81 | -0.03 |
| time (sec) | N/A | 0.036 | 0.068 | 1.392 | 0.000 | 0.387 | 0.000 | 0.490 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 178 | 104 | 0 | 258 | 0 | 122 | -1 |
| N.S. | 1 | 1.00 | 2.78 | 1.62 | 0.00 | 4.03 | 0.00 | 1.91 | -0.02 |
| time (sec) | N/A | 0.070 | 0.477 | 1.766 | 0.000 | 0.350 | 0.000 | 0.502 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 249 | 132 | 0 | 319 | 0 | 155 | -1 |
| N.S. | 1 | 1.00 | 2.44 | 1.29 | 0.00 | 3.13 | 0.00 | 1.52 | -0.01 |
| time (sec) | N/A | 0.105 | 0.520 | 2.125 | 0.000 | 0.455 | 0.000 | 0.505 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 285 | 158 | 0 | 361 | 0 | 184 | -1 |
| N.S. | 1 | 1.00 | 2.07 | 1.14 | 0.00 | 2.62 | 0.00 | 1.33 | -0.01 |
| time (sec) | N/A | 0.143 | 0.900 | 2.636 | 0.000 | 0.349 | 0.000 | 0.560 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 97 | 67 | 0 | 223 | 0 | 104 | -1 |
| N.S. | 1 | 1.00 | 2.55 | 1.76 | 0.00 | 5.87 | 0.00 | 2.74 | -0.03 |
| time (sec) | N/A | 0.034 | 0.069 | 1.293 | 0.000 | 0.370 | 0.000 | 0.592 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 96 | 70 | 0 | 223 | 0 | 106 | -1 |
| N.S. | 1 | 1.00 | 2.46 | 1.79 | 0.00 | 5.72 | 0.00 | 2.72 | -0.03 |
| time (sec) | N/A | 0.034 | 0.054 | 1.246 | 0.000 | 0.370 | 0.000 | 0.617 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 95 | 69 | 0 | 221 | 0 | 69 | -1 |
| N.S. | 1 | 1.00 | 2.38 | 1.72 | 0.00 | 5.52 | 0.00 | 1.72 | -0.02 |
| time (sec) | N/A | 0.035 | 0.055 | 1.364 | 0.000 | 0.355 | 0.000 | 0.545 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 165 | 85 | 0 | 145 | 0 | 152 | -1 |
| N.S. | 1 | 1.00 | 1.02 | 0.52 | 0.00 | 0.90 | 0.00 | 0.94 | -0.01 |
| time (sec) | N/A | 0.169 | 0.345 | 2.237 | 0.000 | 0.334 | 0.000 | 0.516 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 141 | 75 | 0 | 122 | 0 | 124 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.65 | 0.00 | 1.05 | 0.00 | 1.07 | -0.01 |
| time (sec) | N/A | 0.098 | 0.231 | 1.629 | 0.000 | 0.444 | 0.000 | 0.573 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 115 | 63 | 0 | 99 | 0 | 95 | -1 |
| N.S. | 1 | 1.00 | 1.34 | 0.73 | 0.00 | 1.15 | 0.00 | 1.10 | -0.01 |
| time (sec) | N/A | 0.045 | 0.113 | 1.843 | 0.000 | 0.326 | 0.000 | 0.673 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 89 | 53 | 0 | 76 | 0 | 67 | -1 |
| N.S. | 1 | 1.00 | 1.51 | 0.90 | 0.00 | 1.29 | 0.00 | 1.14 | -0.02 |
| time (sec) | N/A | 0.020 | 0.093 | 1.477 | 0.000 | 0.327 | 0.000 | 0.494 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 118 | 84 | 0 | 239 | 0 | 104 | -1 |
| N.S. | 1 | 1.00 | 1.79 | 1.27 | 0.00 | 3.62 | 0.00 | 1.58 | -0.02 |
| time (sec) | N/A | 0.072 | 0.105 | 1.875 | 0.000 | 0.359 | 0.000 | 0.525 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 180 | 103 | 0 | 268 | 0 | 125 | -1 |
| N.S. | 1 | 1.00 | 2.73 | 1.56 | 0.00 | 4.06 | 0.00 | 1.89 | -0.02 |
| time (sec) | N/A | 0.078 | 0.411 | 1.725 | 0.000 | 0.342 | 0.000 | 0.493 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 250 | 126 | 0 | 337 | 0 | 158 | -1 |
| N.S. | 1 | 1.00 | 2.36 | 1.19 | 0.00 | 3.18 | 0.00 | 1.49 | -0.01 |
| time (sec) | N/A | 0.114 | 0.417 | 2.254 | 0.000 | 0.438 | 0.000 | 0.517 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 286 | 144 | 0 | 380 | 0 | 188 | -1 |
| N.S. | 1 | 1.00 | 1.99 | 1.00 | 0.00 | 2.64 | 0.00 | 1.31 | -0.01 |
| time (sec) | N/A | 0.160 | 0.630 | 2.049 | 0.000 | 0.344 | 0.000 | 0.500 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 189 | 95 | 0 | 192 | 0 | 192 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 0.47 | 0.00 | 0.95 | 0.00 | 0.95 | -0.00 |
| time (sec) | N/A | 0.241 | 0.801 | 1.859 | 0.000 | 0.333 | 0.000 | 0.501 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 165 | 85 | 0 | 167 | 0 | 162 | -1 |
| N.S. | 1 | 1.00 | 1.13 | 0.58 | 0.00 | 1.14 | 0.00 | 1.11 | -0.01 |
| time (sec) | N/A | 0.112 | 0.660 | 1.770 | 0.000 | 0.380 | 0.000 | 0.616 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 141 | 75 | 0 | 140 | 0 | 132 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.65 | 0.00 | 1.21 | 0.00 | 1.14 | -0.01 |
| time (sec) | N/A | 0.061 | 0.412 | 1.786 | 0.000 | 0.329 | 0.000 | 0.720 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 117 | 65 | 0 | 115 | 0 | 102 | -1 |
| N.S. | 1 | 1.00 | 1.31 | 0.73 | 0.00 | 1.29 | 0.00 | 1.15 | -0.01 |
| time (sec) | N/A | 0.037 | 0.207 | 1.878 | 0.000 | 0.400 | 0.000 | 0.514 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 143 | 103 | 0 | 279 | 0 | 141 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 1.05 | 0.00 | 2.85 | 0.00 | 1.44 | -0.01 |
| time (sec) | N/A | 0.133 | 0.253 | 2.286 | 0.000 | 0.343 | 0.000 | 0.528 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 182 | 123 | 0 | 308 | 0 | 159 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 1.31 | 0.00 | 3.28 | 0.00 | 1.69 | -0.01 |
| time (sec) | N/A | 0.133 | 0.530 | 2.260 | 0.000 | 0.358 | 0.000 | 0.613 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 252 | 126 | 0 | 359 | 0 | 164 | -1 |
| N.S. | 1 | 1.00 | 2.38 | 1.19 | 0.00 | 3.39 | 0.00 | 1.55 | -0.01 |
| time (sec) | N/A | 0.155 | 0.513 | 2.049 | 0.000 | 0.341 | 0.000 | 0.501 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 288 | 144 | 0 | 408 | 0 | 196 | -1 |
| N.S. | 1 | 1.00 | 2.00 | 1.00 | 0.00 | 2.83 | 0.00 | 1.36 | -0.01 |
| time (sec) | N/A | 0.187 | 0.766 | 2.125 | 0.000 | 0.348 | 0.000 | 0.551 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 182 | 370 | 162 | 0 | 473 | 0 | 228 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 0.89 | 0.00 | 2.60 | 0.00 | 1.25 | -0.01 |
| time (sec) | N/A | 0.227 | 1.085 | 2.455 | 0.000 | 0.379 | 0.000 | 0.520 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 150 | 130 | 0 | 234 | 0 | 163 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 0.94 | 0.00 | 1.68 | 0.00 | 1.17 | -0.01 |
| time (sec) | N/A | 0.164 | 0.156 | 2.198 | 0.000 | 0.345 | 0.000 | 0.528 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 105 | 96 | 0 | 209 | 0 | 121 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.00 | 1.99 | 0.00 | 1.15 | -0.01 |
| time (sec) | N/A | 0.086 | 0.142 | 2.317 | 0.000 | 0.394 | 0.000 | 0.536 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 98 | 96 | 0 | 191 | 0 | 118 | 99 |
| N.S. | 1 | 1.00 | 1.36 | 1.33 | 0.00 | 2.65 | 0.00 | 1.64 | 1.38 |
| time (sec) | N/A | 0.035 | 0.071 | 2.769 | 0.000 | 0.336 | 0.000 | 0.493 | 0.745 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 73 | 75 | 0 | 167 | 0 | 111 | 49 |
| N.S. | 1 | 1.00 | 1.55 | 1.60 | 0.00 | 3.55 | 0.00 | 2.36 | 1.04 |
| time (sec) | N/A | 0.015 | 0.040 | 0.006 | 0.000 | 0.353 | 0.000 | 0.520 | 6.436 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 128 | 96 | 0 | 290 | 0 | 142 | -1 |
| N.S. | 1 | 1.00 | 1.52 | 1.14 | 0.00 | 3.45 | 0.00 | 1.69 | -0.01 |
| time (sec) | N/A | 0.075 | 0.066 | 1.772 | 0.000 | 0.350 | 0.000 | 0.593 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 168 | 133 | 0 | 412 | 0 | 217 | -1 |
| N.S. | 1 | 1.00 | 1.54 | 1.22 | 0.00 | 3.78 | 0.00 | 1.99 | -0.01 |
| time (sec) | N/A | 0.147 | 0.866 | 2.217 | 0.000 | 0.393 | 0.000 | 0.576 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 307 | 162 | 0 | 492 | 0 | 222 | -1 |
| N.S. | 1 | 1.00 | 2.10 | 1.11 | 0.00 | 3.37 | 0.00 | 1.52 | -0.01 |
| time (sec) | N/A | 0.231 | 2.179 | 2.563 | 0.000 | 0.359 | 0.000 | 0.546 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 178 | 183 | 0 | 314 | 0 | 197 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 0.00 | 1.72 | 0.00 | 1.08 | -0.01 |
| time (sec) | N/A | 0.259 | 0.282 | 1.760 | 0.000 | 0.381 | 0.000 | 0.543 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 156 | 183 | 0 | 295 | 0 | 197 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 1.26 | 0.00 | 2.03 | 0.00 | 1.36 | -0.01 |
| time (sec) | N/A | 0.169 | 0.181 | 1.855 | 0.000 | 0.349 | 0.000 | 0.513 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 134 | 143 | 0 | 274 | 0 | 172 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 1.36 | 0.00 | 2.61 | 0.00 | 1.64 | -0.01 |
| time (sec) | N/A | 0.087 | 0.174 | 1.486 | 0.000 | 0.377 | 0.000 | 0.802 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 108 | 123 | 0 | 253 | 0 | 137 | -1 |
| N.S. | 1 | 1.00 | 1.40 | 1.60 | 0.00 | 3.29 | 0.00 | 1.78 | -0.01 |
| time (sec) | N/A | 0.039 | 0.135 | 1.877 | 0.000 | 0.346 | 0.000 | 0.504 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 108 | 125 | 0 | 252 | 0 | 133 | -1 |
| N.S. | 1 | 1.00 | 1.40 | 1.62 | 0.00 | 3.27 | 0.00 | 1.73 | -0.01 |
| time (sec) | N/A | 0.027 | 0.112 | 1.714 | 0.000 | 0.364 | 0.000 | 0.550 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 223 | 172 | 0 | 453 | 0 | 138 | -1 |
| N.S. | 1 | 1.00 | 1.96 | 1.51 | 0.00 | 3.97 | 0.00 | 1.21 | -0.01 |
| time (sec) | N/A | 0.148 | 0.136 | 2.076 | 0.000 | 0.368 | 0.000 | 0.524 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 449 | 219 | 0 | 539 | 0 | 260 | -1 |
| N.S. | 1 | 1.00 | 3.12 | 1.52 | 0.00 | 3.74 | 0.00 | 1.81 | -0.01 |
| time (sec) | N/A | 0.236 | 0.437 | 2.237 | 0.000 | 0.370 | 0.000 | 0.540 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 620 | 299 | 0 | 626 | 0 | 205 | -1 |
| N.S. | 1 | 1.00 | 3.33 | 1.61 | 0.00 | 3.37 | 0.00 | 1.10 | -0.01 |
| time (sec) | N/A | 0.319 | 3.010 | 2.591 | 0.000 | 0.385 | 0.000 | 0.623 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 221 | 323 | 0 | 381 | 0 | 241 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 1.46 | 0.00 | 1.72 | 0.00 | 1.09 | -0.00 |
| time (sec) | N/A | 0.351 | 0.368 | 2.479 | 0.000 | 0.364 | 0.000 | 0.542 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 197 | 269 | 0 | 360 | 0 | 143 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 1.47 | 0.00 | 1.97 | 0.00 | 0.78 | -0.01 |
| time (sec) | N/A | 0.265 | 0.312 | 2.266 | 0.000 | 0.356 | 0.000 | 0.463 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 173 | 233 | 0 | 341 | 0 | 191 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 1.61 | 0.00 | 2.35 | 0.00 | 1.32 | -0.01 |
| time (sec) | N/A | 0.185 | 0.218 | 2.678 | 0.000 | 0.353 | 0.000 | 0.491 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 196 | 193 | 0 | 320 | 0 | 162 | -1 |
| N.S. | 1 | 1.00 | 1.83 | 1.80 | 0.00 | 2.99 | 0.00 | 1.51 | -0.01 |
| time (sec) | N/A | 0.098 | 0.138 | 2.586 | 0.000 | 0.347 | 0.000 | 0.458 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 196 | 193 | 0 | 318 | 0 | 81 | -1 |
| N.S. | 1 | 1.00 | 1.83 | 1.80 | 0.00 | 2.97 | 0.00 | 0.76 | -0.01 |
| time (sec) | N/A | 0.059 | 0.129 | 2.288 | 0.000 | 0.353 | 0.000 | 0.455 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 196 | 195 | 0 | 320 | 0 | 153 | -1 |
| N.S. | 1 | 1.00 | 1.83 | 1.82 | 0.00 | 2.99 | 0.00 | 1.43 | -0.01 |
| time (sec) | N/A | 0.042 | 0.113 | 2.418 | 0.000 | 0.354 | 0.000 | 0.449 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 296 | 261 | 0 | 539 | 0 | 163 | -1 |
| N.S. | 1 | 1.00 | 2.06 | 1.81 | 0.00 | 3.74 | 0.00 | 1.13 | -0.01 |
| time (sec) | N/A | 0.231 | 0.185 | 2.948 | 0.000 | 0.369 | 0.000 | 0.732 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 509 | 356 | 0 | 631 | 0 | 205 | -1 |
| N.S. | 1 | 1.00 | 2.93 | 2.05 | 0.00 | 3.63 | 0.00 | 1.18 | -0.01 |
| time (sec) | N/A | 0.344 | 0.422 | 2.875 | 0.000 | 0.381 | 0.000 | 0.529 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 680 | 404 | 0 | 715 | 0 | 298 | -1 |
| N.S. | 1 | 1.00 | 3.04 | 1.80 | 0.00 | 3.19 | 0.00 | 1.33 | -0.00 |
| time (sec) | N/A | 0.442 | 0.784 | 3.309 | 0.000 | 0.389 | 0.000 | 0.577 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 164 | 320 | 224 | 360 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.43 | 8.65 | 6.05 | 9.73 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.039 | 0.360 | 16.055 | 0.548 | 0.425 | 0.000 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 119 | 271 | 0 | 371 | 0 | 99 | -1 |
| N.S. | 1 | 1.00 | 3.13 | 7.13 | 0.00 | 9.76 | 0.00 | 2.61 | -0.03 |
| time (sec) | N/A | 0.048 | 0.310 | 16.375 | 0.000 | 0.420 | 0.000 | 0.756 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 123 | 52 | 0 | 28 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 7.24 | 3.06 | 0.00 | 1.65 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.029 | 1.345 | 0.372 | 0.000 | 0.326 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 125 | 54 | 0 | 163 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.98 | 1.29 | 0.00 | 3.88 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.041 | 0.061 | 0.341 | 0.000 | 0.371 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 125 | 52 | 0 | 31 | 0 | 146 | -1 |
| N.S. | 1 | 1.00 | 4.03 | 1.68 | 0.00 | 1.00 | 0.00 | 4.71 | -0.03 |
| time (sec) | N/A | 0.033 | 1.293 | 0.360 | 0.000 | 0.362 | 0.000 | 0.868 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 128 | 53 | 0 | 168 | 0 | 149 | -1 |
| N.S. | 1 | 1.00 | 3.05 | 1.26 | 0.00 | 4.00 | 0.00 | 3.55 | -0.02 |
| time (sec) | N/A | 0.045 | 0.069 | 0.331 | 0.000 | 0.378 | 0.000 | 1.449 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 121 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.150 | 0.279 | 0.494 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 160 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.203 | 0.467 | 0.328 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 151 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.109 | 0.296 | 0.345 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 138 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.052 | 0.154 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 124 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.025 | 0.135 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.081 | 1.904 | 0.117 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 143 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 8.788 | 0.113 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 373 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.207 | 9.176 | 0.351 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 363 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.108 | 9.151 | 0.347 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 351 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.053 | 8.754 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 341 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.025 | 1.156 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 2791 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 35.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.082 | 8.165 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 2800 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 35.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 8.804 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 110 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.185 | 0.310 | 0.530 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 95 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.100 | 0.186 | 0.280 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 84 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 0.100 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 70 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.025 | 0.067 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.076 | 1.842 | 0.114 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 184 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.090 | 4.491 | 0.126 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 116 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.202 | 0.330 | 0.543 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 108 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.106 | 0.211 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 130 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.057 | 0.189 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 130 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.028 | 0.133 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.087 | 6.220 | 0.133 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 230 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.099 | 6.664 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 5109 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 53.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.083 | 21.962 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 186 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.035 | 0.454 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 225 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.050 | 1.370 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 263 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.053 | 2.191 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 5111 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 48.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.105 | 6.235 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 264 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.045 | 2.849 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 234 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.091 | 1.282 | 0.103 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 274 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.097 | 1.892 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 5129 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 39.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.101 | 6.213 | 0.106 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 215 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.040 | 0.240 | 0.086 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 227 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.086 | 0.284 | 0.079 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 265 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.092 | 0.623 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 5131 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 39.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.117 | 6.231 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 266 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.052 | 0.310 | 0.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 242 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 0.483 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 276 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.140 | 0.881 | 0.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 991 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 13.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.047 | 11.890 | 0.070 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 300 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.045 | 1.535 | 0.101 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 993 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 10.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.075 | 2.676 | 0.099 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 300 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.080 | 0.441 | 0.110 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 993 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 11.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.072 | 2.584 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 301 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.083 | 0.225 | 0.147 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 995 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 9.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.112 | 2.689 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 301 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.115 | 0.243 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F(-1) | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 294 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.351 | 180.087 | 0.926 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 59941 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 278.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.213 | 120.763 | 0.856 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 28439 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 182.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.108 | 49.871 | 0.393 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 178 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.045 | 0.419 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 90 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.023 | 0.118 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 2203 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 25.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.077 | 12.467 | 0.135 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 4206 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 49.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.078 | 15.845 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 88 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.011 | 0.092 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 90 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.014 | 0.072 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 76 | 60 | 62 | 65 | 144 | 66 | 111 |
| N.S. | 1 | 1.00 | 0.99 | 0.78 | 0.81 | 0.84 | 1.87 | 0.86 | 1.44 |
| time (sec) | N/A | 0.045 | 0.115 | 0.230 | 0.271 | 0.501 | 0.178 | 0.460 | 10.285 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 60 | 49 | 52 | 50 | 92 | 50 | 68 |
| N.S. | 1 | 1.00 | 1.09 | 0.89 | 0.95 | 0.91 | 1.67 | 0.91 | 1.24 |
| time (sec) | N/A | 0.035 | 0.046 | 0.167 | 0.301 | 0.340 | 0.113 | 0.483 | 8.575 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 35 | 39 | 39 | 37 | 66 | 34 | 68 |
| N.S. | 1 | 1.00 | 0.90 | 1.00 | 1.00 | 0.95 | 1.69 | 0.87 | 1.74 |
| time (sec) | N/A | 0.011 | 0.066 | 0.109 | 0.282 | 0.438 | 0.100 | 0.425 | 7.191 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 27 | 17 | 17 | 19 | 19 | 17 | 25 |
| N.S. | 1 | 1.00 | 1.69 | 1.06 | 1.06 | 1.19 | 1.19 | 1.06 | 1.56 |
| time (sec) | N/A | 0.007 | 0.006 | 0.054 | 0.289 | 0.343 | 0.044 | 0.460 | 6.511 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 43 | 31 | 32 | 40 | 51 | 27 | 85 |
| N.S. | 1 | 1.00 | 2.53 | 1.82 | 1.88 | 2.35 | 3.00 | 1.59 | 5.00 |
| time (sec) | N/A | 0.017 | 0.012 | 0.177 | 0.270 | 0.352 | 2.809 | 0.461 | 6.797 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 52 | 33 | 43 | 68 | 0 | 62 | 28 |
| N.S. | 1 | 1.00 | 2.00 | 1.27 | 1.65 | 2.62 | 0.00 | 2.38 | 1.08 |
| time (sec) | N/A | 0.030 | 0.022 | 0.169 | 0.285 | 0.391 | 0.000 | 0.445 | 6.759 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 91 | 50 | 65 | 104 | 0 | 92 | 81 |
| N.S. | 1 | 1.00 | 1.90 | 1.04 | 1.35 | 2.17 | 0.00 | 1.92 | 1.69 |
| time (sec) | N/A | 0.038 | 0.027 | 0.231 | 0.294 | 0.362 | 0.000 | 0.461 | 6.723 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 115 | 61 | 79 | 140 | 0 | 122 | 111 |
| N.S. | 1 | 1.00 | 1.80 | 0.95 | 1.23 | 2.19 | 0.00 | 1.91 | 1.73 |
| time (sec) | N/A | 0.041 | 0.024 | 0.252 | 0.283 | 0.342 | 0.000 | 0.488 | 6.741 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 91 | 95 | 102 | 96 | 221 | 128 | 157 |
| N.S. | 1 | 1.00 | 0.81 | 0.85 | 0.91 | 0.86 | 1.97 | 1.14 | 1.40 |
| time (sec) | N/A | 0.081 | 0.227 | 0.299 | 0.275 | 0.369 | 0.296 | 0.462 | 10.366 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 117 | 89 | 91 | 89 | 211 | 86 | 85 |
| N.S. | 1 | 1.00 | 1.16 | 0.88 | 0.90 | 0.88 | 2.09 | 0.85 | 0.84 |
| time (sec) | N/A | 0.070 | 0.113 | 0.237 | 0.296 | 0.368 | 0.201 | 0.520 | 6.927 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 59 | 64 | 67 | 59 | 107 | 76 | 103 |
| N.S. | 1 | 1.00 | 0.83 | 0.90 | 0.94 | 0.83 | 1.51 | 1.07 | 1.45 |
| time (sec) | N/A | 0.035 | 0.143 | 0.186 | 0.272 | 0.352 | 0.122 | 0.471 | 8.993 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 46 | 51 | 49 | 48 | 78 | 45 | 44 |
| N.S. | 1 | 1.00 | 0.92 | 1.02 | 0.98 | 0.96 | 1.56 | 0.90 | 0.88 |
| time (sec) | N/A | 0.012 | 0.073 | 0.122 | 0.295 | 0.339 | 0.082 | 0.491 | 6.790 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 76 | 46 | 48 | 57 | 0 | 52 | 125 |
| N.S. | 1 | 1.00 | 2.17 | 1.31 | 1.37 | 1.63 | 0.00 | 1.49 | 3.57 |
| time (sec) | N/A | 0.044 | 0.018 | 0.196 | 0.281 | 0.393 | 0.000 | 0.459 | 6.480 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 76 | 46 | 56 | 84 | 0 | 79 | 105 |
| N.S. | 1 | 1.00 | 2.24 | 1.35 | 1.65 | 2.47 | 0.00 | 2.32 | 3.09 |
| time (sec) | N/A | 0.049 | 0.166 | 0.231 | 0.279 | 0.410 | 0.000 | 0.455 | 6.834 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 133 | 73 | 96 | 137 | 0 | 125 | 92 |
| N.S. | 1 | 1.00 | 2.25 | 1.24 | 1.63 | 2.32 | 0.00 | 2.12 | 1.56 |
| time (sec) | N/A | 0.057 | 0.305 | 0.340 | 0.279 | 0.362 | 0.000 | 0.448 | 6.494 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 132 | 76 | 96 | 161 | 0 | 166 | 136 |
| N.S. | 1 | 1.00 | 1.61 | 0.93 | 1.17 | 1.96 | 0.00 | 2.02 | 1.66 |
| time (sec) | N/A | 0.065 | 0.035 | 0.355 | 0.277 | 0.528 | 0.000 | 0.433 | 6.783 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 255 | 114 | 159 | 242 | 0 | 230 | 178 |
| N.S. | 1 | 1.00 | 2.32 | 1.04 | 1.45 | 2.20 | 0.00 | 2.09 | 1.62 |
| time (sec) | N/A | 0.074 | 0.039 | 0.371 | 0.278 | 0.358 | 0.000 | 0.468 | 6.876 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 193 | 147 | 145 | 157 | 145 | 393 | 180 | 417 |
| N.S. | 1 | 1.13 | 0.86 | 0.85 | 0.92 | 0.85 | 2.30 | 1.05 | 2.44 |
| time (sec) | N/A | 0.155 | 0.490 | 0.410 | 0.279 | 0.340 | 0.460 | 0.460 | 8.401 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 180 | 117 | 124 | 131 | 124 | 284 | 129 | 328 |
| N.S. | 1 | 1.12 | 0.73 | 0.78 | 0.82 | 0.78 | 1.78 | 0.81 | 2.05 |
| time (sec) | N/A | 0.155 | 0.451 | 0.331 | 0.276 | 0.449 | 0.372 | 0.455 | 8.171 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 100 | 104 | 105 | 98 | 233 | 116 | 313 |
| N.S. | 1 | 1.00 | 0.83 | 0.86 | 0.87 | 0.81 | 1.93 | 0.96 | 2.59 |
| time (sec) | N/A | 0.084 | 0.239 | 0.272 | 0.281 | 0.355 | 0.205 | 0.436 | 8.085 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 71 | 76 | 79 | 75 | 128 | 75 | 127 |
| N.S. | 1 | 1.00 | 0.79 | 0.84 | 0.88 | 0.83 | 1.42 | 0.83 | 1.41 |
| time (sec) | N/A | 0.047 | 0.118 | 0.203 | 0.275 | 0.415 | 0.133 | 0.454 | 6.738 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 81 | 75 | 77 | 84 | 0 | 114 | 259 |
| N.S. | 1 | 1.00 | 1.09 | 1.01 | 1.04 | 1.14 | 0.00 | 1.54 | 3.50 |
| time (sec) | N/A | 0.084 | 0.114 | 0.265 | 0.277 | 0.395 | 0.000 | 0.463 | 6.791 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 87 | 61 | 73 | 107 | 0 | 146 | 194 |
| N.S. | 1 | 1.00 | 1.28 | 0.90 | 1.07 | 1.57 | 0.00 | 2.15 | 2.85 |
| time (sec) | N/A | 0.088 | 0.379 | 0.287 | 0.290 | 0.363 | 0.000 | 0.440 | 6.720 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 152 | 86 | 110 | 164 | 0 | 142 | 234 |
| N.S. | 1 | 1.00 | 1.92 | 1.09 | 1.39 | 2.08 | 0.00 | 1.80 | 2.96 |
| time (sec) | N/A | 0.097 | 0.448 | 0.375 | 0.285 | 0.518 | 0.000 | 0.462 | 6.963 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 525 | 99 | 127 | 203 | 0 | 201 | 150 |
| N.S. | 1 | 1.00 | 4.82 | 0.91 | 1.17 | 1.86 | 0.00 | 1.84 | 1.38 |
| time (sec) | N/A | 0.136 | 6.143 | 0.463 | 0.277 | 0.398 | 0.000 | 0.450 | 6.785 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 322 | 129 | 175 | 251 | 0 | 269 | 203 |
| N.S. | 1 | 1.00 | 2.40 | 0.96 | 1.31 | 1.87 | 0.00 | 2.01 | 1.51 |
| time (sec) | N/A | 0.151 | 6.142 | 0.509 | 0.286 | 0.355 | 0.000 | 0.476 | 6.858 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 106 | 116 | 121 | 111 | 240 | 112 | 114 |
| N.S. | 1 | 1.00 | 0.77 | 0.85 | 0.88 | 0.81 | 1.75 | 0.82 | 0.83 |
| time (sec) | N/A | 0.105 | 0.243 | 0.267 | 0.282 | 0.530 | 0.204 | 0.427 | 6.952 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 98 | 143 | 0 | 333 | 0 | 149 | 1075 |
| N.S. | 1 | 1.00 | 0.89 | 1.30 | 0.00 | 3.03 | 0.00 | 1.35 | 9.77 |
| time (sec) | N/A | 0.203 | 0.178 | 0.187 | 0.000 | 0.382 | 0.000 | 0.437 | 7.221 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 78 | 112 | 0 | 291 | 0 | 112 | 1004 |
| N.S. | 1 | 1.00 | 0.95 | 1.37 | 0.00 | 3.55 | 0.00 | 1.37 | 12.24 |
| time (sec) | N/A | 0.117 | 0.076 | 0.148 | 0.000 | 0.368 | 0.000 | 0.446 | 7.080 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 56 | 71 | 0 | 231 | 1192 | 77 | 623 |
| N.S. | 1 | 1.00 | 0.92 | 1.16 | 0.00 | 3.79 | 19.54 | 1.26 | 10.21 |
| time (sec) | N/A | 0.081 | 0.063 | 0.143 | 0.000 | 0.463 | 146.752 | 0.446 | 6.911 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 47 | 54 | 0 | 192 | 236 | 58 | 101 |
| N.S. | 1 | 1.00 | 0.94 | 1.08 | 0.00 | 3.84 | 4.72 | 1.16 | 2.02 |
| time (sec) | N/A | 0.044 | 0.028 | 0.113 | 0.000 | 0.371 | 32.282 | 0.444 | 6.704 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 39 | 0 | 148 | 141 | 48 | 45 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 0.00 | 3.70 | 3.52 | 1.20 | 1.12 |
| time (sec) | N/A | 0.025 | 0.017 | 0.088 | 0.000 | 0.354 | 3.816 | 0.407 | 6.883 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 62 | 53 | 0 | 239 | 0 | 63 | 122 |
| N.S. | 1 | 1.00 | 1.17 | 1.00 | 0.00 | 4.51 | 0.00 | 1.19 | 2.30 |
| time (sec) | N/A | 0.048 | 0.039 | 0.181 | 0.000 | 0.624 | 0.000 | 0.451 | 6.857 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 91 | 77 | 0 | 302 | 0 | 98 | 179 |
| N.S. | 1 | 1.00 | 1.47 | 1.24 | 0.00 | 4.87 | 0.00 | 1.58 | 2.89 |
| time (sec) | N/A | 0.085 | 0.176 | 0.181 | 0.000 | 0.402 | 0.000 | 0.446 | 7.006 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 144 | 112 | 0 | 490 | 0 | 141 | 531 |
| N.S. | 1 | 1.00 | 1.71 | 1.33 | 0.00 | 5.83 | 0.00 | 1.68 | 6.32 |
| time (sec) | N/A | 0.198 | 0.349 | 0.258 | 0.000 | 0.470 | 0.000 | 0.426 | 7.388 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 125 | 156 | 0 | 577 | 0 | 194 | 586 |
| N.S. | 1 | 1.00 | 1.12 | 1.39 | 0.00 | 5.15 | 0.00 | 1.73 | 5.23 |
| time (sec) | N/A | 0.317 | 1.142 | 0.292 | 0.000 | 0.514 | 0.000 | 0.427 | 7.517 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 115 | 183 | 0 | 580 | 0 | 184 | 2500 |
| N.S. | 1 | 1.00 | 0.68 | 1.08 | 0.00 | 3.43 | 0.00 | 1.09 | 14.79 |
| time (sec) | N/A | 0.280 | 0.394 | 0.297 | 0.000 | 0.456 | 0.000 | 0.451 | 11.883 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 94 | 142 | 0 | 483 | 0 | 204 | 2578 |
| N.S. | 1 | 1.00 | 0.76 | 1.15 | 0.00 | 3.90 | 0.00 | 1.65 | 20.79 |
| time (sec) | N/A | 0.163 | 0.292 | 0.281 | 0.000 | 0.397 | 0.000 | 0.451 | 9.983 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 83 | 121 | 0 | 403 | 0 | 124 | 2562 |
| N.S. | 1 | 1.00 | 0.95 | 1.39 | 0.00 | 4.63 | 0.00 | 1.43 | 29.45 |
| time (sec) | N/A | 0.096 | 0.160 | 0.234 | 0.000 | 0.444 | 0.000 | 0.463 | 9.953 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 67 | 99 | 0 | 266 | 0 | 90 | 123 |
| N.S. | 1 | 1.00 | 1.02 | 1.50 | 0.00 | 4.03 | 0.00 | 1.36 | 1.86 |
| time (sec) | N/A | 0.048 | 0.077 | 0.178 | 0.000 | 0.383 | 0.000 | 0.434 | 6.494 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 66 | 98 | 0 | 268 | 0 | 95 | 148 |
| N.S. | 1 | 1.00 | 1.02 | 1.51 | 0.00 | 4.12 | 0.00 | 1.46 | 2.28 |
| time (sec) | N/A | 0.038 | 0.072 | 0.135 | 0.000 | 0.350 | 0.000 | 0.436 | 6.821 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 99 | 123 | 0 | 511 | 0 | 134 | 1356 |
| N.S. | 1 | 1.00 | 1.06 | 1.32 | 0.00 | 5.49 | 0.00 | 1.44 | 14.58 |
| time (sec) | N/A | 0.137 | 0.184 | 0.309 | 0.000 | 0.692 | 0.000 | 0.447 | 7.819 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 127 | 144 | 0 | 784 | 0 | 234 | 1471 |
| N.S. | 1 | 1.00 | 1.03 | 1.17 | 0.00 | 6.37 | 0.00 | 1.90 | 11.96 |
| time (sec) | N/A | 0.236 | 0.489 | 0.332 | 0.000 | 0.678 | 0.000 | 0.430 | 7.634 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 171 | 181 | 0 | 1174 | 0 | 215 | 1576 |
| N.S. | 1 | 1.00 | 1.02 | 1.08 | 0.00 | 6.99 | 0.00 | 1.28 | 9.38 |
| time (sec) | N/A | 0.406 | 0.631 | 0.360 | 0.000 | 0.775 | 0.000 | 0.440 | 7.669 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 164 | 330 | 0 | 1090 | 0 | 516 | 2500 |
| N.S. | 1 | 1.00 | 0.67 | 1.36 | 0.00 | 4.49 | 0.00 | 2.12 | 10.29 |
| time (sec) | N/A | 0.458 | 0.651 | 0.523 | 0.000 | 0.439 | 0.000 | 0.439 | 15.806 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 144 | 286 | 0 | 945 | 0 | 256 | 2500 |
| N.S. | 1 | 1.00 | 0.80 | 1.60 | 0.00 | 5.28 | 0.00 | 1.43 | 13.97 |
| time (sec) | N/A | 0.286 | 0.546 | 0.462 | 0.000 | 0.452 | 0.000 | 0.442 | 14.874 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 136 | 269 | 0 | 819 | 0 | 234 | 2500 |
| N.S. | 1 | 1.00 | 0.94 | 1.87 | 0.00 | 5.69 | 0.00 | 1.62 | 17.36 |
| time (sec) | N/A | 0.171 | 0.376 | 0.375 | 0.000 | 0.504 | 0.000 | 0.459 | 14.903 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 94 | 213 | 0 | 516 | 0 | 182 | 318 |
| N.S. | 1 | 1.00 | 0.80 | 1.81 | 0.00 | 4.37 | 0.00 | 1.54 | 2.69 |
| time (sec) | N/A | 0.105 | 0.253 | 0.319 | 0.000 | 0.437 | 0.000 | 0.435 | 7.233 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 94 | 221 | 0 | 490 | 0 | 189 | 310 |
| N.S. | 1 | 1.00 | 0.91 | 2.15 | 0.00 | 4.76 | 0.00 | 1.83 | 3.01 |
| time (sec) | N/A | 0.074 | 0.200 | 0.277 | 0.000 | 0.387 | 0.000 | 0.432 | 7.509 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 93 | 248 | 0 | 516 | 0 | 215 | 349 |
| N.S. | 1 | 1.00 | 0.91 | 2.43 | 0.00 | 5.06 | 0.00 | 2.11 | 3.42 |
| time (sec) | N/A | 0.070 | 0.138 | 0.247 | 0.000 | 0.501 | 0.000 | 0.427 | 7.323 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 140 | 270 | 0 | 1027 | 0 | 246 | 2191 |
| N.S. | 1 | 1.00 | 0.97 | 1.86 | 0.00 | 7.08 | 0.00 | 1.70 | 15.11 |
| time (sec) | N/A | 0.253 | 0.574 | 0.447 | 0.000 | 0.858 | 0.000 | 0.443 | 11.385 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 174 | 294 | 0 | 1436 | 0 | 280 | 2295 |
| N.S. | 1 | 1.00 | 0.93 | 1.57 | 0.00 | 7.68 | 0.00 | 1.50 | 12.27 |
| time (sec) | N/A | 0.417 | 0.880 | 0.497 | 0.000 | 0.870 | 0.000 | 0.445 | 9.016 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 220 | 329 | 0 | 2005 | 0 | 514 | 2405 |
| N.S. | 1 | 1.00 | 0.91 | 1.37 | 0.00 | 8.32 | 0.00 | 2.13 | 9.98 |
| time (sec) | N/A | 0.592 | 1.262 | 0.542 | 0.000 | 1.462 | 0.000 | 0.489 | 9.277 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 182 | 157 | 510 | 0 | 965 | 0 | 510 | 708 |
| N.S. | 1 | 1.00 | 0.86 | 2.80 | 0.00 | 5.30 | 0.00 | 2.80 | 3.89 |
| time (sec) | N/A | 0.159 | 0.650 | 0.533 | 0.000 | 0.428 | 0.000 | 0.487 | 10.384 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 143 | 460 | 0 | 420 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 2.67 | 0.00 | 2.44 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.126 | 2.674 | 3.418 | 0.000 | 0.111 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 61 | 239 | 0 | 371 | 0 | 0 | 55 |
| N.S. | 1 | 1.00 | 0.98 | 3.85 | 0.00 | 5.98 | 0.00 | 0.00 | 0.89 |
| time (sec) | N/A | 0.025 | 0.053 | 3.317 | 0.000 | 0.114 | 0.000 | 0.000 | 6.871 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 89 | 169 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.70 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.160 | 13.750 | 3.215 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 312 | 456 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.334 | 16.070 | 3.121 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 94 | 202 | 0 | 373 | 0 | 0 | 118 |
| N.S. | 1 | 1.00 | 0.71 | 1.53 | 0.00 | 2.83 | 0.00 | 0.00 | 0.89 |
| time (sec) | N/A | 0.080 | 2.271 | 2.990 | 0.000 | 0.112 | 0.000 | 0.000 | 7.207 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 61 | 126 | 0 | 152 | 0 | 0 | 55 |
| N.S. | 1 | 1.00 | 0.98 | 2.03 | 0.00 | 2.45 | 0.00 | 0.00 | 0.89 |
| time (sec) | N/A | 0.026 | 0.042 | 1.981 | 0.000 | 0.106 | 0.000 | 0.000 | 6.913 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 135 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.98 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.092 | 0.060 | 2.216 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 315 | 412 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.42 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.351 | 16.104 | 6.774 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 371 | 10847 | 9513 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 29.24 | 25.64 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.392 | 27.106 | 14.150 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 172 | 310 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.58 | 2.84 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.046 | 4.208 | 13.288 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 199 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.273 | 0.517 | 1.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 144 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.094 | 0.223 | 1.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 111 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.051 | 0.107 | 0.142 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 1590 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.171 | 15.608 | 0.133 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 1790 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.290 | 16.475 | 1.208 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 2298 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.387 | 15.685 | 1.266 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 188 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.085 | 0.208 | 0.938 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 73 | 0 | 0 | 0 | 0 | 0 | 127 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.74 |
| time (sec) | N/A | 0.051 | 0.073 | 0.417 | 0.000 | 0.000 | 0.000 | 0.000 | 7.473 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.026 | 1.526 | 0.117 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.334 | 2.657 | 0.490 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.201 | 3.327 | 0.371 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 193 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.130 | 0.328 | 0.085 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 120 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.043 | 0.164 | 0.006 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.021 | 1.102 | 0.120 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 64 | 149 | 158 | 82 | 314 | 100 | 292 |
| N.S. | 1 | 1.00 | 0.55 | 1.28 | 1.36 | 0.71 | 2.71 | 0.86 | 2.52 |
| time (sec) | N/A | 0.110 | 0.365 | 0.319 | 0.297 | 0.331 | 0.336 | 0.499 | 8.859 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 54 | 89 | 93 | 67 | 196 | 81 | 250 |
| N.S. | 1 | 1.00 | 0.65 | 1.07 | 1.12 | 0.81 | 2.36 | 0.98 | 3.01 |
| time (sec) | N/A | 0.083 | 0.259 | 0.236 | 0.309 | 0.329 | 0.212 | 0.439 | 9.004 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 42 | 77 | 83 | 49 | 133 | 62 | 125 |
| N.S. | 1 | 1.00 | 0.81 | 1.48 | 1.60 | 0.94 | 2.56 | 1.19 | 2.40 |
| time (sec) | N/A | 0.047 | 0.198 | 0.164 | 0.284 | 0.336 | 0.132 | 0.445 | 8.963 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 25 | 40 | 40 | 28 | 70 | 23 | 54 |
| N.S. | 1 | 1.00 | 0.86 | 1.38 | 1.38 | 0.97 | 2.41 | 0.79 | 1.86 |
| time (sec) | N/A | 0.012 | 0.019 | 0.098 | 0.282 | 0.320 | 0.080 | 0.408 | 7.154 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 83 | 38 | 88 | 70 | 88 | 37 | 46 |
| N.S. | 1 | 1.00 | 2.52 | 1.15 | 2.67 | 2.12 | 2.67 | 1.12 | 1.39 |
| time (sec) | N/A | 0.032 | 0.132 | 0.237 | 0.494 | 0.325 | 0.657 | 0.419 | 6.809 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 74 | 56 | 235 | 112 | 158 | 39 | 56 |
| N.S. | 1 | 1.00 | 2.47 | 1.87 | 7.83 | 3.73 | 5.27 | 1.30 | 1.87 |
| time (sec) | N/A | 0.052 | 0.195 | 0.277 | 0.294 | 0.316 | 1.305 | 0.422 | 6.726 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 96 | 86 | 423 | 166 | 571 | 84 | 136 |
| N.S. | 1 | 1.00 | 1.60 | 1.43 | 7.05 | 2.77 | 9.52 | 1.40 | 2.27 |
| time (sec) | N/A | 0.083 | 0.229 | 0.317 | 0.311 | 0.326 | 2.900 | 0.432 | 7.104 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 109 | 116 | 611 | 222 | 1061 | 114 | 97 |
| N.S. | 1 | 1.00 | 1.18 | 1.26 | 6.64 | 2.41 | 11.53 | 1.24 | 1.05 |
| time (sec) | N/A | 0.116 | 0.319 | 0.340 | 0.298 | 0.318 | 6.510 | 0.514 | 7.353 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 124 | 146 | 799 | 276 | 1700 | 144 | 119 |
| N.S. | 1 | 1.00 | 0.98 | 1.16 | 6.34 | 2.19 | 13.49 | 1.14 | 0.94 |
| time (sec) | N/A | 0.153 | 0.402 | 0.390 | 0.319 | 0.332 | 13.106 | 0.429 | 8.769 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 89 | 255 | 276 | 109 | 629 | 154 | 452 |
| N.S. | 1 | 1.00 | 0.59 | 1.68 | 1.82 | 0.72 | 4.14 | 1.01 | 2.97 |
| time (sec) | N/A | 0.138 | 0.722 | 0.461 | 0.287 | 0.334 | 0.773 | 0.501 | 9.229 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 79 | 211 | 225 | 92 | 530 | 133 | 284 |
| N.S. | 1 | 1.00 | 0.67 | 1.79 | 1.91 | 0.78 | 4.49 | 1.13 | 2.41 |
| time (sec) | N/A | 0.104 | 0.493 | 0.399 | 0.304 | 0.349 | 0.492 | 0.447 | 8.901 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 69 | 159 | 170 | 75 | 340 | 112 | 220 |
| N.S. | 1 | 1.00 | 0.81 | 1.87 | 2.00 | 0.88 | 4.00 | 1.32 | 2.59 |
| time (sec) | N/A | 0.072 | 1.020 | 0.313 | 0.276 | 0.327 | 0.328 | 0.466 | 10.004 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 39 | 88 | 87 | 57 | 206 | 52 | 36 |
| N.S. | 1 | 1.00 | 0.61 | 1.38 | 1.36 | 0.89 | 3.22 | 0.81 | 0.56 |
| time (sec) | N/A | 0.049 | 0.035 | 0.213 | 0.276 | 0.330 | 0.203 | 0.433 | 6.742 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 43 | 78 | 83 | 49 | 133 | 62 | 125 |
| N.S. | 1 | 1.00 | 0.83 | 1.50 | 1.60 | 0.94 | 2.56 | 1.19 | 2.40 |
| time (sec) | N/A | 0.044 | 0.232 | 0.182 | 0.288 | 0.338 | 0.130 | 0.412 | 9.017 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 130 | 55 | 236 | 111 | 454 | 103 | 118 |
| N.S. | 1 | 1.00 | 2.28 | 0.96 | 4.14 | 1.95 | 7.96 | 1.81 | 2.07 |
| time (sec) | N/A | 0.099 | 0.266 | 0.291 | 0.528 | 0.319 | 1.165 | 0.451 | 6.905 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 121 | 53 | 394 | 166 | 473 | 60 | 90 |
| N.S. | 1 | 1.00 | 1.68 | 0.74 | 5.47 | 2.31 | 6.57 | 0.83 | 1.25 |
| time (sec) | N/A | 0.095 | 0.428 | 0.332 | 0.548 | 0.318 | 2.565 | 0.421 | 6.873 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 81 | 88 | 605 | 180 | 354 | 60 | 92 |
| N.S. | 1 | 1.00 | 2.38 | 2.59 | 17.79 | 5.29 | 10.41 | 1.76 | 2.71 |
| time (sec) | N/A | 0.066 | 0.284 | 0.376 | 0.400 | 0.328 | 5.582 | 0.479 | 6.993 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 117 | 118 | 888 | 238 | 1074 | 128 | 99 |
| N.S. | 1 | 1.00 | 1.75 | 1.76 | 13.25 | 3.55 | 16.03 | 1.91 | 1.48 |
| time (sec) | N/A | 0.094 | 0.428 | 0.391 | 0.316 | 0.327 | 10.857 | 0.445 | 7.296 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 121 | 148 | 1169 | 298 | 1717 | 162 | 121 |
| N.S. | 1 | 1.00 | 1.23 | 1.51 | 11.93 | 3.04 | 17.52 | 1.65 | 1.23 |
| time (sec) | N/A | 0.129 | 0.401 | 0.433 | 0.396 | 0.338 | 20.915 | 0.455 | 8.810 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 133 | 178 | 1452 | 356 | 2509 | 196 | 143 |
| N.S. | 1 | 1.00 | 1.01 | 1.35 | 11.00 | 2.70 | 19.01 | 1.48 | 1.08 |
| time (sec) | N/A | 0.169 | 0.488 | 0.500 | 0.350 | 0.323 | 39.368 | 0.488 | 9.357 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 109 | 297 | 325 | 126 | 838 | 196 | 403 |
| N.S. | 1 | 1.00 | 0.61 | 1.65 | 1.81 | 0.70 | 4.66 | 1.09 | 2.24 |
| time (sec) | N/A | 0.155 | 1.371 | 0.599 | 0.298 | 0.347 | 1.578 | 0.474 | 9.307 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 89 | 276 | 304 | 109 | 740 | 154 | 372 |
| N.S. | 1 | 1.00 | 0.61 | 1.90 | 2.10 | 0.75 | 5.10 | 1.06 | 2.57 |
| time (sec) | N/A | 0.119 | 0.795 | 0.567 | 0.283 | 0.344 | 1.059 | 0.463 | 9.123 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 89 | 255 | 276 | 92 | 631 | 154 | 301 |
| N.S. | 1 | 1.00 | 0.79 | 2.28 | 2.46 | 0.82 | 5.63 | 1.38 | 2.69 |
| time (sec) | N/A | 0.083 | 0.684 | 0.470 | 0.289 | 0.332 | 0.731 | 0.432 | 10.454 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 49 | 140 | 142 | 74 | 398 | 73 | 143 |
| N.S. | 1 | 1.00 | 0.54 | 1.54 | 1.56 | 0.81 | 4.37 | 0.80 | 1.57 |
| time (sec) | N/A | 0.060 | 0.035 | 0.256 | 0.275 | 0.325 | 0.567 | 0.452 | 10.139 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 69 | 160 | 170 | 75 | 340 | 112 | 220 |
| N.S. | 1 | 1.00 | 0.81 | 1.88 | 2.00 | 0.88 | 4.00 | 1.32 | 2.59 |
| time (sec) | N/A | 0.068 | 1.024 | 0.292 | 0.320 | 0.336 | 0.365 | 0.431 | 10.318 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 54 | 89 | 93 | 67 | 196 | 81 | 250 |
| N.S. | 1 | 1.00 | 0.66 | 1.09 | 1.13 | 0.82 | 2.39 | 0.99 | 3.05 |
| time (sec) | N/A | 0.073 | 0.267 | 0.218 | 0.312 | 0.329 | 0.207 | 0.445 | 8.812 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 153 | 96 | 471 | 137 | 1168 | 117 | 219 |
| N.S. | 1 | 1.00 | 1.63 | 1.02 | 5.01 | 1.46 | 12.43 | 1.24 | 2.33 |
| time (sec) | N/A | 0.129 | 0.361 | 0.278 | 0.531 | 0.317 | 2.469 | 0.476 | 9.123 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 149 | 87 | 644 | 194 | 1282 | 101 | 218 |
| N.S. | 1 | 1.00 | 1.62 | 0.95 | 7.00 | 2.11 | 13.93 | 1.10 | 2.37 |
| time (sec) | N/A | 0.128 | 0.652 | 0.320 | 0.536 | 0.336 | 4.774 | 0.448 | 9.752 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 249 | 100 | 853 | 247 | 1282 | 111 | 203 |
| N.S. | 1 | 1.00 | 2.35 | 0.94 | 8.05 | 2.33 | 12.09 | 1.05 | 1.92 |
| time (sec) | N/A | 0.139 | 0.316 | 0.358 | 0.549 | 0.341 | 9.526 | 0.449 | 8.480 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 93 | 118 | 1137 | 238 | 619 | 77 | 116 |
| N.S. | 1 | 1.00 | 2.74 | 3.47 | 33.44 | 7.00 | 18.21 | 2.26 | 3.41 |
| time (sec) | N/A | 0.064 | 0.515 | 0.423 | 0.353 | 0.324 | 19.960 | 0.480 | 7.010 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 135 | 148 | 1513 | 296 | 1717 | 162 | 121 |
| N.S. | 1 | 1.00 | 1.96 | 2.14 | 21.93 | 4.29 | 24.88 | 2.35 | 1.75 |
| time (sec) | N/A | 0.098 | 0.498 | 0.523 | 0.372 | 0.321 | 35.317 | 0.498 | 8.578 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 145 | 178 | 1890 | 356 | 2509 | 196 | 143 |
| N.S. | 1 | 1.00 | 1.44 | 1.76 | 18.71 | 3.52 | 24.84 | 1.94 | 1.42 |
| time (sec) | N/A | 0.126 | 0.589 | 0.545 | 0.388 | 0.327 | 54.886 | 0.476 | 9.339 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 157 | 208 | 2266 | 414 | 3451 | 230 | 165 |
| N.S. | 1 | 1.00 | 1.19 | 1.58 | 17.17 | 3.14 | 26.14 | 1.74 | 1.25 |
| time (sec) | N/A | 0.166 | 1.197 | 0.378 | 0.444 | 0.332 | 93.293 | 0.486 | 10.171 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 209 | 238 | 2642 | 472 | 4542 | 264 | 187 |
| N.S. | 1 | 1.00 | 1.26 | 1.43 | 15.92 | 2.84 | 27.36 | 1.59 | 1.13 |
| time (sec) | N/A | 0.203 | 1.205 | 0.394 | 0.470 | 0.330 | 148.303 | 0.498 | 11.198 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 175 | 109 | 786 | 166 | 2108 | 135 | 290 |
| N.S. | 1 | 1.00 | 1.48 | 0.92 | 6.66 | 1.41 | 17.86 | 1.14 | 2.46 |
| time (sec) | N/A | 0.140 | 0.907 | 0.321 | 0.568 | 0.330 | 4.464 | 0.431 | 10.530 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 155 | 96 | 462 | 136 | 1170 | 117 | 216 |
| N.S. | 1 | 1.00 | 1.68 | 1.04 | 5.02 | 1.48 | 12.72 | 1.27 | 2.35 |
| time (sec) | N/A | 0.127 | 0.344 | 0.306 | 0.537 | 0.324 | 2.302 | 0.465 | 8.709 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 129 | 57 | 228 | 107 | 456 | 100 | 118 |
| N.S. | 1 | 1.00 | 2.30 | 1.02 | 4.07 | 1.91 | 8.14 | 1.79 | 2.11 |
| time (sec) | N/A | 0.092 | 0.253 | 0.267 | 0.550 | 0.323 | 1.172 | 0.431 | 6.994 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 79 | 38 | 83 | 68 | 90 | 37 | 45 |
| N.S. | 1 | 1.00 | 2.47 | 1.19 | 2.59 | 2.12 | 2.81 | 1.16 | 1.41 |
| time (sec) | N/A | 0.031 | 0.126 | 0.214 | 0.529 | 0.317 | 0.638 | 0.416 | 6.645 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 17 | 17 | 26 | 49 | 17 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.06 | 1.62 | 3.06 | 1.06 | 2.19 |
| time (sec) | N/A | 0.046 | 0.009 | 0.274 | 0.323 | 0.308 | 0.595 | 0.432 | 6.849 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 87 | 73 | 154 | 61 | 328 | 77 | 74 |
| N.S. | 1 | 1.00 | 1.64 | 1.38 | 2.91 | 1.15 | 6.19 | 1.45 | 1.40 |
| time (sec) | N/A | 0.077 | 0.304 | 0.253 | 0.320 | 0.321 | 1.371 | 0.438 | 7.012 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 111 | 103 | 229 | 90 | 614 | 105 | 89 |
| N.S. | 1 | 1.00 | 1.31 | 1.21 | 2.69 | 1.06 | 7.22 | 1.24 | 1.05 |
| time (sec) | N/A | 0.115 | 0.472 | 0.285 | 0.417 | 0.313 | 2.805 | 0.444 | 7.158 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 131 | 133 | 347 | 120 | 1307 | 133 | 96 |
| N.S. | 1 | 1.00 | 1.11 | 1.13 | 2.94 | 1.02 | 11.08 | 1.13 | 0.81 |
| time (sec) | N/A | 0.151 | 0.510 | 0.310 | 0.338 | 0.317 | 6.245 | 0.446 | 7.273 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 276 | 138 | 1418 | 252 | 3641 | 203 | 372 |
| N.S. | 1 | 1.00 | 1.86 | 0.93 | 9.58 | 1.70 | 24.60 | 1.37 | 2.51 |
| time (sec) | N/A | 0.170 | 0.492 | 0.407 | 0.567 | 0.329 | 15.578 | 0.461 | 10.855 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 243 | 125 | 981 | 222 | 2312 | 152 | 291 |
| N.S. | 1 | 1.00 | 1.80 | 0.93 | 7.27 | 1.64 | 17.13 | 1.13 | 2.16 |
| time (sec) | N/A | 0.163 | 0.339 | 0.398 | 0.553 | 0.323 | 8.823 | 0.464 | 10.091 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 210 | 85 | 640 | 195 | 1282 | 101 | 217 |
| N.S. | 1 | 1.00 | 2.33 | 0.94 | 7.11 | 2.17 | 14.24 | 1.12 | 2.41 |
| time (sec) | N/A | 0.124 | 0.249 | 0.345 | 0.548 | 0.324 | 4.942 | 0.446 | 9.623 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 119 | 53 | 391 | 166 | 473 | 58 | 89 |
| N.S. | 1 | 1.00 | 1.70 | 0.76 | 5.59 | 2.37 | 6.76 | 0.83 | 1.27 |
| time (sec) | N/A | 0.094 | 0.402 | 0.289 | 0.524 | 0.322 | 2.397 | 0.419 | 7.067 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 70 | 56 | 233 | 112 | 158 | 39 | 54 |
| N.S. | 1 | 1.00 | 2.41 | 1.93 | 8.03 | 3.86 | 5.45 | 1.34 | 1.86 |
| time (sec) | N/A | 0.047 | 0.188 | 0.230 | 0.322 | 0.310 | 1.288 | 0.448 | 7.029 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 87 | 73 | 154 | 60 | 328 | 77 | 74 |
| N.S. | 1 | 1.00 | 1.67 | 1.40 | 2.96 | 1.15 | 6.31 | 1.48 | 1.42 |
| time (sec) | N/A | 0.074 | 0.328 | 0.254 | 0.322 | 0.307 | 1.350 | 0.430 | 6.994 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 29 | 30 | 30 | 40 | 286 | 30 | 63 |
| N.S. | 1 | 1.00 | 0.76 | 0.79 | 0.79 | 1.05 | 7.53 | 0.79 | 1.66 |
| time (sec) | N/A | 0.047 | 0.038 | 0.322 | 0.301 | 0.321 | 1.505 | 0.436 | 6.839 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 131 | 133 | 363 | 93 | 1418 | 133 | 128 |
| N.S. | 1 | 1.00 | 1.72 | 1.75 | 4.78 | 1.22 | 18.66 | 1.75 | 1.68 |
| time (sec) | N/A | 0.083 | 0.583 | 0.307 | 0.342 | 0.313 | 6.172 | 0.430 | 7.827 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 151 | 163 | 463 | 122 | 2213 | 161 | 119 |
| N.S. | 1 | 1.00 | 1.36 | 1.47 | 4.17 | 1.10 | 19.94 | 1.45 | 1.07 |
| time (sec) | N/A | 0.115 | 0.639 | 0.382 | 0.334 | 0.314 | 12.962 | 0.466 | 6.979 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 193 | 193 | 563 | 154 | 3186 | 189 | 180 |
| N.S. | 1 | 1.00 | 1.34 | 1.34 | 3.91 | 1.07 | 22.12 | 1.31 | 1.25 |
| time (sec) | N/A | 0.155 | 0.776 | 0.437 | 0.343 | 0.316 | 28.047 | 0.480 | 9.290 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 303 | 156 | 1628 | 301 | 3643 | 186 | 364 |
| N.S. | 1 | 1.00 | 1.88 | 0.97 | 10.11 | 1.87 | 22.63 | 1.16 | 2.26 |
| time (sec) | N/A | 0.196 | 0.571 | 0.435 | 0.570 | 0.331 | 29.234 | 0.482 | 11.145 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 270 | 102 | 1192 | 270 | 2314 | 135 | 290 |
| N.S. | 1 | 1.00 | 2.18 | 0.82 | 9.61 | 2.18 | 18.66 | 1.09 | 2.34 |
| time (sec) | N/A | 0.159 | 0.407 | 0.421 | 0.582 | 0.329 | 17.260 | 0.454 | 10.904 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 239 | 100 | 849 | 245 | 1284 | 111 | 200 |
| N.S. | 1 | 1.00 | 2.32 | 0.97 | 8.24 | 2.38 | 12.47 | 1.08 | 1.94 |
| time (sec) | N/A | 0.128 | 0.291 | 0.382 | 0.549 | 0.319 | 9.384 | 0.496 | 8.939 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 81 | 88 | 602 | 180 | 354 | 60 | 90 |
| N.S. | 1 | 1.00 | 2.45 | 2.67 | 18.24 | 5.45 | 10.73 | 1.82 | 2.73 |
| time (sec) | N/A | 0.060 | 0.267 | 0.313 | 0.324 | 0.309 | 5.311 | 0.440 | 7.215 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 92 | 86 | 421 | 166 | 573 | 84 | 134 |
| N.S. | 1 | 1.00 | 1.59 | 1.48 | 7.26 | 2.86 | 9.88 | 1.45 | 2.31 |
| time (sec) | N/A | 0.073 | 0.230 | 0.257 | 0.321 | 0.311 | 2.889 | 0.439 | 7.238 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 111 | 101 | 229 | 89 | 614 | 105 | 89 |
| N.S. | 1 | 1.00 | 1.34 | 1.22 | 2.76 | 1.07 | 7.40 | 1.27 | 1.07 |
| time (sec) | N/A | 0.106 | 0.404 | 0.305 | 0.322 | 0.309 | 2.800 | 0.447 | 7.376 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 131 | 133 | 363 | 92 | 1418 | 133 | 128 |
| N.S. | 1 | 1.00 | 1.75 | 1.77 | 4.84 | 1.23 | 18.91 | 1.77 | 1.71 |
| time (sec) | N/A | 0.077 | 0.518 | 0.344 | 0.329 | 0.323 | 6.549 | 0.443 | 8.425 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 41 | 40 | 43 | 51 | 687 | 43 | 89 |
| N.S. | 1 | 1.00 | 0.69 | 0.68 | 0.73 | 0.86 | 11.64 | 0.73 | 1.51 |
| time (sec) | N/A | 0.052 | 0.093 | 0.361 | 0.329 | 0.308 | 5.119 | 0.453 | 8.343 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 193 | 193 | 563 | 115 | 3186 | 189 | 180 |
| N.S. | 1 | 1.00 | 1.99 | 1.99 | 5.80 | 1.19 | 32.85 | 1.95 | 1.86 |
| time (sec) | N/A | 0.087 | 0.738 | 0.409 | 0.341 | 0.318 | 27.872 | 0.451 | 9.416 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 213 | 223 | 662 | 144 | 4335 | 217 | 190 |
| N.S. | 1 | 1.00 | 1.63 | 1.70 | 5.05 | 1.10 | 33.09 | 1.66 | 1.45 |
| time (sec) | N/A | 0.126 | 0.891 | 0.539 | 0.330 | 0.348 | 50.441 | 0.471 | 8.198 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 233 | 253 | 763 | 176 | 5661 | 245 | 185 |
| N.S. | 1 | 1.00 | 1.40 | 1.51 | 4.57 | 1.05 | 33.90 | 1.47 | 1.11 |
| time (sec) | N/A | 0.159 | 1.045 | 0.593 | 0.350 | 0.360 | 90.012 | 0.473 | 8.616 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 104 | 79 | 0 | 192 | 0 | 144 | -1 |
| N.S. | 1 | 1.00 | 0.76 | 0.58 | 0.00 | 1.40 | 0.00 | 1.05 | -0.01 |
| time (sec) | N/A | 0.204 | 0.520 | 2.003 | 0.000 | 0.358 | 0.000 | 0.529 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 94 | 69 | 0 | 163 | 0 | 144 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 0.67 | 0.00 | 1.58 | 0.00 | 1.40 | -0.01 |
| time (sec) | N/A | 0.154 | 0.328 | 2.155 | 0.000 | 0.347 | 0.000 | 0.520 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 82 | 59 | 0 | 118 | 0 | 105 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 0.86 | 0.00 | 1.71 | 0.00 | 1.52 | -0.01 |
| time (sec) | N/A | 0.102 | 0.202 | 1.950 | 0.000 | 0.328 | 0.000 | 0.527 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 71 | 47 | 0 | 86 | 0 | 71 | -1 |
| N.S. | 1 | 1.00 | 2.09 | 1.38 | 0.00 | 2.53 | 0.00 | 2.09 | -0.03 |
| time (sec) | N/A | 0.064 | 0.086 | 1.334 | 0.000 | 0.345 | 0.000 | 0.493 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 135 | 94 | 0 | 214 | 0 | 126 | -1 |
| N.S. | 1 | 1.00 | 1.75 | 1.22 | 0.00 | 2.78 | 0.00 | 1.64 | -0.01 |
| time (sec) | N/A | 0.099 | 0.391 | 2.953 | 0.000 | 0.351 | 0.000 | 0.489 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 107 | 120 | 0 | 277 | 0 | 222 | -1 |
| N.S. | 1 | 1.00 | 1.41 | 1.58 | 0.00 | 3.64 | 0.00 | 2.92 | -0.01 |
| time (sec) | N/A | 0.098 | 0.390 | 1.727 | 0.000 | 0.353 | 0.000 | 0.530 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 176 | 189 | 0 | 364 | 0 | 229 | -1 |
| N.S. | 1 | 1.00 | 1.56 | 1.67 | 0.00 | 3.22 | 0.00 | 2.03 | -0.01 |
| time (sec) | N/A | 0.114 | 0.561 | 2.369 | 0.000 | 0.355 | 0.000 | 0.551 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 189 | 243 | 0 | 442 | 0 | 391 | -1 |
| N.S. | 1 | 1.00 | 1.30 | 1.68 | 0.00 | 3.05 | 0.00 | 2.70 | -0.01 |
| time (sec) | N/A | 0.135 | 0.702 | 2.567 | 0.000 | 0.354 | 0.000 | 0.608 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 1105 | 81 | 0 | 249 | 0 | 222 | -1 |
| N.S. | 1 | 1.00 | 7.62 | 0.56 | 0.00 | 1.72 | 0.00 | 1.53 | -0.01 |
| time (sec) | N/A | 0.225 | 6.268 | 1.769 | 0.000 | 0.336 | 0.000 | 0.621 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 96 | 71 | 0 | 214 | 0 | 187 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.65 | 0.00 | 1.96 | 0.00 | 1.72 | -0.01 |
| time (sec) | N/A | 0.177 | 3.348 | 1.728 | 0.000 | 0.342 | 0.000 | 0.572 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 84 | 61 | 0 | 163 | 0 | 144 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.84 | 0.00 | 2.23 | 0.00 | 1.97 | -0.01 |
| time (sec) | N/A | 0.133 | 0.853 | 1.938 | 0.000 | 0.348 | 0.000 | 0.567 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 73 | 49 | 0 | 123 | 0 | 107 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 1.36 | 0.00 | 3.42 | 0.00 | 2.97 | -0.03 |
| time (sec) | N/A | 0.087 | 0.155 | 1.321 | 0.000 | 0.339 | 0.000 | 0.515 | 0.000 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 130 | 112 | 0 | 258 | 0 | 220 | -1 |
| N.S. | 1 | 1.00 | 1.13 | 0.97 | 0.00 | 2.24 | 0.00 | 1.91 | -0.01 |
| time (sec) | N/A | 0.165 | 0.290 | 2.289 | 0.000 | 0.341 | 0.000 | 0.508 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 149 | 146 | 0 | 323 | 0 | 312 | -1 |
| N.S. | 1 | 1.00 | 1.30 | 1.27 | 0.00 | 2.81 | 0.00 | 2.71 | -0.01 |
| time (sec) | N/A | 0.169 | 0.420 | 2.073 | 0.000 | 0.348 | 0.000 | 0.524 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 163 | 191 | 0 | 390 | 0 | 336 | -1 |
| N.S. | 1 | 1.00 | 1.34 | 1.57 | 0.00 | 3.20 | 0.00 | 2.75 | -0.01 |
| time (sec) | N/A | 0.165 | 0.615 | 2.193 | 0.000 | 0.354 | 0.000 | 0.586 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 307 | 245 | 0 | 474 | 0 | 405 | -1 |
| N.S. | 1 | 1.00 | 1.97 | 1.57 | 0.00 | 3.04 | 0.00 | 2.60 | -0.01 |
| time (sec) | N/A | 0.191 | 0.606 | 2.473 | 0.000 | 0.365 | 0.000 | 0.638 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 371 | 299 | 0 | 563 | 0 | 343 | -1 |
| N.S. | 1 | 1.00 | 1.95 | 1.57 | 0.00 | 2.96 | 0.00 | 1.81 | -0.01 |
| time (sec) | N/A | 0.211 | 0.933 | 2.675 | 0.000 | 0.353 | 0.000 | 0.643 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 112 | 81 | 0 | 282 | 0 | 257 | -1 |
| N.S. | 1 | 1.00 | 0.77 | 0.56 | 0.00 | 1.94 | 0.00 | 1.77 | -0.01 |
| time (sec) | N/A | 0.225 | 5.396 | 2.325 | 0.000 | 0.343 | 0.000 | 0.602 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 96 | 71 | 0 | 249 | 0 | 222 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.65 | 0.00 | 2.28 | 0.00 | 2.04 | -0.01 |
| time (sec) | N/A | 0.178 | 6.021 | 2.246 | 0.000 | 0.343 | 0.000 | 0.622 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 84 | 61 | 0 | 192 | 0 | 144 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.84 | 0.00 | 2.63 | 0.00 | 1.97 | -0.01 |
| time (sec) | N/A | 0.136 | 1.874 | 2.543 | 0.000 | 0.358 | 0.000 | 0.584 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 73 | 49 | 0 | 152 | 0 | 139 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 1.36 | 0.00 | 4.22 | 0.00 | 3.86 | -0.03 |
| time (sec) | N/A | 0.088 | 0.250 | 1.687 | 0.000 | 0.334 | 0.000 | 0.558 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 156 | 129 | 0 | 287 | 0 | 304 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 0.85 | 0.00 | 1.90 | 0.00 | 2.01 | -0.01 |
| time (sec) | N/A | 0.220 | 0.463 | 2.697 | 0.000 | 0.368 | 0.000 | 0.494 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 173 | 189 | 0 | 352 | 0 | 376 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 1.26 | 0.00 | 2.35 | 0.00 | 2.51 | -0.01 |
| time (sec) | N/A | 0.221 | 0.499 | 1.880 | 0.000 | 0.365 | 0.000 | 0.571 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 187 | 239 | 0 | 418 | 0 | 400 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 1.52 | 0.00 | 2.66 | 0.00 | 2.55 | -0.01 |
| time (sec) | N/A | 0.216 | 0.628 | 2.977 | 0.000 | 0.352 | 0.000 | 0.641 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 307 | 245 | 0 | 474 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.96 | 1.56 | 0.00 | 3.02 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.224 | 0.997 | 2.619 | 0.000 | 0.373 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 371 | 299 | 0 | 563 | 0 | 539 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 1.57 | 0.00 | 2.95 | 0.00 | 2.82 | -0.01 |
| time (sec) | N/A | 0.243 | 1.612 | 3.562 | 0.000 | 0.384 | 0.000 | 0.656 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-2) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 435 | 353 | 0 | 646 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.93 | 1.57 | 0.00 | 2.87 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.263 | 2.676 | 3.273 | 0.000 | 0.377 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 112 | 69 | 258 | 79 | 0 | 343 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 0.52 | 1.95 | 0.60 | 0.00 | 2.60 | -0.01 |
| time (sec) | N/A | 0.247 | 1.297 | 1.625 | 0.604 | 0.343 | 0.000 | 0.574 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 102 | 59 | 208 | 62 | 0 | 237 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 0.60 | 2.12 | 0.63 | 0.00 | 2.42 | -0.01 |
| time (sec) | N/A | 0.184 | 0.438 | 1.501 | 0.597 | 0.355 | 0.000 | 0.565 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 88 | 49 | 158 | 44 | 0 | 67 | 90 |
| N.S. | 1 | 1.00 | 1.47 | 0.82 | 2.63 | 0.73 | 0.00 | 1.12 | 1.50 |
| time (sec) | N/A | 0.141 | 0.185 | 1.760 | 0.512 | 0.373 | 0.000 | 0.498 | 7.290 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 39 | 82 | 31 | 0 | 65 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 1.34 | 2.83 | 1.07 | 0.00 | 2.24 | 1.38 |
| time (sec) | N/A | 0.088 | 0.071 | 1.236 | 0.487 | 0.341 | 0.000 | 0.460 | 0.197 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 97 | 85 | 0 | 168 | 0 | 131 | -1 |
| N.S. | 1 | 1.00 | 1.17 | 1.02 | 0.00 | 2.02 | 0.00 | 1.58 | -0.01 |
| time (sec) | N/A | 0.113 | 0.208 | 2.098 | 0.000 | 0.346 | 0.000 | 0.510 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 125 | 134 | 0 | 226 | 0 | 304 | -1 |
| N.S. | 1 | 1.00 | 1.07 | 1.15 | 0.00 | 1.93 | 0.00 | 2.60 | -0.01 |
| time (sec) | N/A | 0.133 | 0.377 | 2.139 | 0.000 | 0.342 | 0.000 | 0.525 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 162 | 210 | 0 | 263 | 0 | 394 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 1.35 | 0.00 | 1.69 | 0.00 | 2.53 | -0.01 |
| time (sec) | N/A | 0.179 | 0.498 | 2.767 | 0.000 | 0.368 | 0.000 | 0.576 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 124 | 91 | 412 | 112 | 0 | 450 | -1 |
| N.S. | 1 | 1.00 | 0.70 | 0.52 | 2.34 | 0.64 | 0.00 | 2.56 | -0.01 |
| time (sec) | N/A | 0.284 | 1.914 | 2.235 | 0.507 | 0.382 | 0.000 | 0.608 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 112 | 79 | 362 | 98 | 0 | 125 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 0.58 | 2.66 | 0.72 | 0.00 | 0.92 | -0.01 |
| time (sec) | N/A | 0.238 | 0.746 | 2.288 | 0.502 | 0.343 | 0.000 | 0.576 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 104 | 71 | 312 | 82 | 0 | 237 | 360 |
| N.S. | 1 | 1.00 | 1.04 | 0.71 | 3.12 | 0.82 | 0.00 | 2.37 | 3.60 |
| time (sec) | N/A | 0.182 | 0.486 | 2.265 | 0.505 | 0.336 | 0.000 | 0.567 | 11.683 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 92 | 61 | 259 | 62 | 0 | 117 | 120 |
| N.S. | 1 | 1.00 | 1.35 | 0.90 | 3.81 | 0.91 | 0.00 | 1.72 | 1.76 |
| time (sec) | N/A | 0.142 | 0.217 | 2.001 | 0.508 | 0.320 | 0.000 | 0.566 | 10.323 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 73 | 49 | 161 | 50 | 0 | 116 | 227 |
| N.S. | 1 | 1.00 | 2.03 | 1.36 | 4.47 | 1.39 | 0.00 | 3.22 | 6.31 |
| time (sec) | N/A | 0.087 | 0.090 | 1.460 | 0.519 | 0.337 | 0.000 | 0.504 | 9.347 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 109 | 109 | 0 | 223 | 0 | 211 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.88 | 0.00 | 1.80 | 0.00 | 1.70 | -0.01 |
| time (sec) | N/A | 0.162 | 0.326 | 2.433 | 0.000 | 0.349 | 0.000 | 0.522 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 164 | 157 | 0 | 202 | 0 | 368 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 1.01 | 0.00 | 1.30 | 0.00 | 2.37 | -0.01 |
| time (sec) | N/A | 0.171 | 0.495 | 2.187 | 0.000 | 0.376 | 0.000 | 0.543 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 156 | 233 | 0 | 262 | 0 | 480 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 1.21 | 0.00 | 1.36 | 0.00 | 2.50 | -0.01 |
| time (sec) | N/A | 0.230 | 0.726 | 3.055 | 0.000 | 0.348 | 0.000 | 0.542 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 124 | 91 | 512 | 128 | 0 | 450 | -1 |
| N.S. | 1 | 1.00 | 0.71 | 0.52 | 2.94 | 0.74 | 0.00 | 2.59 | -0.01 |
| time (sec) | N/A | 0.284 | 1.896 | 2.109 | 0.522 | 0.363 | 0.000 | 0.612 | 0.000 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 114 | 81 | 462 | 114 | 0 | 343 | 542 |
| N.S. | 1 | 1.00 | 0.85 | 0.60 | 3.45 | 0.85 | 0.00 | 2.56 | 4.04 |
| time (sec) | N/A | 0.236 | 0.787 | 2.682 | 0.517 | 0.347 | 0.000 | 0.635 | 13.874 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 104 | 71 | 412 | 98 | 0 | 174 | 453 |
| N.S. | 1 | 1.00 | 1.00 | 0.68 | 3.96 | 0.94 | 0.00 | 1.67 | 4.36 |
| time (sec) | N/A | 0.181 | 0.531 | 2.117 | 0.507 | 0.337 | 0.000 | 0.546 | 11.949 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 92 | 61 | 359 | 80 | 0 | 219 | 355 |
| N.S. | 1 | 1.00 | 1.26 | 0.84 | 4.92 | 1.10 | 0.00 | 3.00 | 4.86 |
| time (sec) | N/A | 0.135 | 0.232 | 1.860 | 0.529 | 0.474 | 0.000 | 0.516 | 10.912 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 73 | 49 | 236 | 66 | 0 | 166 | 90 |
| N.S. | 1 | 1.00 | 2.03 | 1.36 | 6.56 | 1.83 | 0.00 | 4.61 | 2.50 |
| time (sec) | N/A | 0.086 | 0.104 | 1.964 | 0.615 | 0.346 | 0.000 | 0.474 | 9.882 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 189 | 122 | 0 | 263 | 0 | 292 | -1 |
| N.S. | 1 | 1.00 | 1.18 | 0.76 | 0.00 | 1.64 | 0.00 | 1.82 | -0.01 |
| time (sec) | N/A | 0.209 | 0.402 | 2.284 | 0.000 | 0.356 | 0.000 | 0.567 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-1) | A | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 174 | 170 | 0 | 259 | 0 | 446 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 0.89 | 0.00 | 1.36 | 0.00 | 2.34 | -0.01 |
| time (sec) | N/A | 0.227 | 0.738 | 2.357 | 0.000 | 0.343 | 0.000 | 0.547 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-1) | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 443 | 246 | 0 | 226 | 0 | 558 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 1.08 | 0.00 | 0.99 | 0.00 | 2.45 | -0.00 |
| time (sec) | N/A | 0.276 | 0.956 | 3.018 | 0.000 | 0.346 | 0.000 | 0.639 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 83 | 103 | 0 | 102 | 0 | 54 | 99 |
| N.S. | 1 | 1.00 | 1.93 | 2.40 | 0.00 | 2.37 | 0.00 | 1.26 | 2.30 |
| time (sec) | N/A | 0.055 | 0.252 | 16.276 | 0.000 | 0.324 | 0.000 | 0.488 | 8.250 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 74 | 78 | 0 | 89 | 0 | 54 | 88 |
| N.S. | 1 | 1.00 | 1.72 | 1.81 | 0.00 | 2.07 | 0.00 | 1.26 | 2.05 |
| time (sec) | N/A | 0.055 | 0.184 | 16.478 | 0.000 | 0.337 | 0.000 | 0.485 | 7.707 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 60 | 61 | 0 | 66 | 0 | 54 | 71 |
| N.S. | 1 | 1.00 | 1.40 | 1.42 | 0.00 | 1.53 | 0.00 | 1.26 | 1.65 |
| time (sec) | N/A | 0.056 | 0.135 | 15.674 | 0.000 | 0.335 | 0.000 | 0.481 | 0.845 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 39 | 44 | 0 | 47 | 0 | 54 | 47 |
| N.S. | 1 | 1.00 | 0.95 | 1.07 | 0.00 | 1.15 | 0.00 | 1.32 | 1.15 |
| time (sec) | N/A | 0.051 | 0.064 | 16.086 | 0.000 | 0.349 | 0.000 | 0.482 | 7.010 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 119 | 105 | 67 | 0 | 0 | 56 | -1 |
| N.S. | 1 | 1.00 | 2.29 | 2.02 | 1.29 | 0.00 | 0.00 | 1.08 | -0.02 |
| time (sec) | N/A | 0.072 | 0.633 | 8.930 | 0.503 | 0.000 | 0.000 | 0.494 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 84 | 69 | 0 | 64 | 0 | 56 | -1 |
| N.S. | 1 | 1.00 | 2.10 | 1.72 | 0.00 | 1.60 | 0.00 | 1.40 | -0.02 |
| time (sec) | N/A | 0.063 | 0.124 | 9.185 | 0.000 | 0.349 | 0.000 | 0.517 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 87 | 95 | 0 | 79 | 0 | 56 | 142 |
| N.S. | 1 | 1.00 | 2.02 | 2.21 | 0.00 | 1.84 | 0.00 | 1.30 | 3.30 |
| time (sec) | N/A | 0.062 | 0.126 | 8.980 | 0.000 | 0.343 | 0.000 | 0.475 | 8.895 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 87 | 120 | 0 | 97 | 0 | 56 | 190 |
| N.S. | 1 | 1.00 | 2.02 | 2.79 | 0.00 | 2.26 | 0.00 | 1.30 | 4.42 |
| time (sec) | N/A | 0.060 | 0.165 | 8.430 | 0.000 | 0.325 | 0.000 | 0.485 | 10.982 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 146 | 106 | 0 | 108 | 0 | 110 | 111 |
| N.S. | 1 | 1.00 | 1.64 | 1.19 | 0.00 | 1.21 | 0.00 | 1.24 | 1.25 |
| time (sec) | N/A | 0.127 | 0.642 | 15.477 | 0.000 | 0.342 | 0.000 | 0.528 | 8.917 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 137 | 90 | 0 | 93 | 0 | 159 | 100 |
| N.S. | 1 | 1.00 | 1.54 | 1.01 | 0.00 | 1.04 | 0.00 | 1.79 | 1.12 |
| time (sec) | N/A | 0.123 | 0.398 | 15.809 | 0.000 | 0.345 | 0.000 | 0.492 | 1.750 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 70 | 55 | 0 | 65 | 0 | 106 | 66 |
| N.S. | 1 | 1.00 | 0.79 | 0.62 | 0.00 | 0.73 | 0.00 | 1.19 | 0.74 |
| time (sec) | N/A | 0.121 | 0.261 | 15.545 | 0.000 | 0.336 | 0.000 | 0.507 | 0.889 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 60 | 63 | 0 | 66 | 0 | 54 | 71 |
| N.S. | 1 | 1.00 | 1.40 | 1.47 | 0.00 | 1.53 | 0.00 | 1.26 | 1.65 |
| time (sec) | N/A | 0.057 | 0.147 | 16.771 | 0.000 | 0.339 | 0.000 | 0.498 | 7.348 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 113 | 250 | 0 | 0 | 0 | 101 | -1 |
| N.S. | 1 | 1.00 | 1.18 | 2.60 | 0.00 | 0.00 | 0.00 | 1.05 | -0.01 |
| time (sec) | N/A | 0.134 | 0.200 | 9.322 | 0.000 | 0.000 | 0.000 | 0.468 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 153 | 375 | 147 | 0 | 0 | 91 | -1 |
| N.S. | 1 | 1.00 | 1.58 | 3.87 | 1.52 | 0.00 | 0.00 | 0.94 | -0.01 |
| time (sec) | N/A | 0.141 | 0.288 | 9.217 | 0.529 | 0.000 | 0.000 | 0.480 | 0.000 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 99 | 90 | 0 | 87 | 0 | 98 | -1 |
| N.S. | 1 | 1.00 | 2.36 | 2.14 | 0.00 | 2.07 | 0.00 | 2.33 | -0.02 |
| time (sec) | N/A | 0.068 | 0.294 | 9.809 | 0.000 | 0.329 | 0.000 | 0.503 | 0.000 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 106 | 141 | 0 | 109 | 0 | 92 | 124 |
| N.S. | 1 | 1.00 | 1.20 | 1.60 | 0.00 | 1.24 | 0.00 | 1.05 | 1.41 |
| time (sec) | N/A | 0.133 | 0.357 | 8.923 | 0.000 | 0.332 | 0.000 | 0.514 | 10.650 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 106 | 168 | 0 | 123 | 0 | 98 | 195 |
| N.S. | 1 | 1.00 | 1.15 | 1.83 | 0.00 | 1.34 | 0.00 | 1.07 | 2.12 |
| time (sec) | N/A | 0.122 | 0.656 | 9.299 | 0.000 | 0.341 | 0.000 | 0.474 | 11.814 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 106 | 196 | 0 | 141 | 0 | 92 | 225 |
| N.S. | 1 | 1.00 | 1.15 | 2.13 | 0.00 | 1.53 | 0.00 | 1.00 | 2.45 |
| time (sec) | N/A | 0.123 | 0.946 | 17.522 | 0.000 | 0.341 | 0.000 | 0.492 | 12.589 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 156 | 116 | 0 | 119 | 0 | 216 | 124 |
| N.S. | 1 | 1.00 | 1.16 | 0.87 | 0.00 | 0.89 | 0.00 | 1.61 | 0.93 |
| time (sec) | N/A | 0.181 | 0.859 | 16.493 | 0.000 | 0.356 | 0.000 | 0.510 | 9.943 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 77 | 67 | 0 | 91 | 0 | 165 | 83 |
| N.S. | 1 | 1.00 | 0.57 | 0.50 | 0.00 | 0.68 | 0.00 | 1.23 | 0.62 |
| time (sec) | N/A | 0.182 | 0.322 | 16.935 | 0.000 | 0.347 | 0.000 | 0.513 | 1.501 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 133 | 91 | 0 | 93 | 0 | 110 | 100 |
| N.S. | 1 | 1.00 | 1.49 | 1.02 | 0.00 | 1.04 | 0.00 | 1.24 | 1.12 |
| time (sec) | N/A | 0.119 | 0.409 | 17.408 | 0.000 | 0.347 | 0.000 | 0.486 | 8.271 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 72 | 79 | 0 | 88 | 0 | 54 | 86 |
| N.S. | 1 | 1.00 | 1.67 | 1.84 | 0.00 | 2.05 | 0.00 | 1.26 | 2.00 |
| time (sec) | N/A | 0.061 | 0.180 | 17.674 | 0.000 | 0.348 | 0.000 | 0.457 | 7.747 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 127 | 315 | 0 | 0 | 0 | 135 | -1 |
| N.S. | 1 | 1.00 | 0.90 | 2.23 | 0.00 | 0.00 | 0.00 | 0.96 | -0.01 |
| time (sec) | N/A | 0.198 | 0.353 | 16.858 | 0.000 | 0.000 | 0.000 | 0.481 | 0.000 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 169 | 439 | 0 | 0 | 0 | 142 | -1 |
| N.S. | 1 | 1.00 | 1.17 | 3.05 | 0.00 | 0.00 | 0.00 | 0.99 | -0.01 |
| time (sec) | N/A | 0.200 | 0.506 | 15.754 | 0.000 | 0.000 | 0.000 | 0.519 | 0.000 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 190 | 553 | 198 | 0 | 0 | 132 | -1 |
| N.S. | 1 | 1.00 | 1.29 | 3.76 | 1.35 | 0.00 | 0.00 | 0.90 | -0.01 |
| time (sec) | N/A | 0.206 | 0.713 | 16.655 | 0.493 | 0.000 | 0.000 | 0.516 | 0.000 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 110 | 129 | 0 | 117 | 0 | 138 | -1 |
| N.S. | 1 | 1.00 | 2.62 | 3.07 | 0.00 | 2.79 | 0.00 | 3.29 | -0.02 |
| time (sec) | N/A | 0.067 | 0.597 | 17.671 | 0.000 | 0.372 | 0.000 | 0.502 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 118 | 197 | 0 | 143 | 0 | 130 | 242 |
| N.S. | 1 | 1.00 | 1.34 | 2.24 | 0.00 | 1.62 | 0.00 | 1.48 | 2.75 |
| time (sec) | N/A | 0.130 | 1.376 | 17.246 | 0.000 | 0.349 | 0.000 | 0.490 | 11.575 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 118 | 225 | 0 | 159 | 0 | 139 | 273 |
| N.S. | 1 | 1.00 | 0.89 | 1.69 | 0.00 | 1.20 | 0.00 | 1.05 | 2.05 |
| time (sec) | N/A | 0.196 | 2.118 | 17.881 | 0.000 | 0.349 | 0.000 | 0.494 | 12.076 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 118 | 251 | 0 | 174 | 0 | 130 | 287 |
| N.S. | 1 | 1.00 | 0.84 | 1.79 | 0.00 | 1.24 | 0.00 | 0.93 | 2.05 |
| time (sec) | N/A | 0.192 | 3.002 | 17.428 | 0.000 | 0.352 | 0.000 | 0.515 | 12.240 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 127 | 143 | 0 | 136 | 0 | 267 | 376 |
| N.S. | 1 | 1.00 | 0.71 | 0.80 | 0.00 | 0.76 | 0.00 | 1.49 | 2.10 |
| time (sec) | N/A | 0.253 | 3.335 | 16.283 | 0.000 | 0.361 | 0.000 | 0.562 | 11.301 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 87 | 77 | 0 | 108 | 0 | 216 | 179 |
| N.S. | 1 | 1.00 | 0.49 | 0.43 | 0.00 | 0.60 | 0.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.259 | 0.613 | 17.500 | 0.000 | 0.345 | 0.000 | 0.496 | 10.648 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 107 | 117 | 0 | 119 | 0 | 165 | 124 |
| N.S. | 1 | 1.00 | 0.80 | 0.87 | 0.00 | 0.89 | 0.00 | 1.23 | 0.93 |
| time (sec) | N/A | 0.185 | 0.772 | 18.652 | 0.000 | 0.348 | 0.000 | 0.512 | 10.287 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 93 | 107 | 0 | 108 | 0 | 110 | 109 |
| N.S. | 1 | 1.00 | 1.04 | 1.20 | 0.00 | 1.21 | 0.00 | 1.24 | 1.22 |
| time (sec) | N/A | 0.121 | 0.613 | 16.937 | 0.000 | 0.339 | 0.000 | 0.497 | 9.187 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 82 | 105 | 0 | 102 | 0 | 54 | 99 |
| N.S. | 1 | 1.00 | 1.91 | 2.44 | 0.00 | 2.37 | 0.00 | 1.26 | 2.30 |
| time (sec) | N/A | 0.058 | 0.228 | 18.069 | 0.000 | 0.336 | 0.000 | 0.488 | 8.294 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 150 | 366 | 0 | 0 | 0 | 148 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 1.99 | 0.00 | 0.00 | 0.00 | 0.80 | -0.01 |
| time (sec) | N/A | 0.263 | 0.647 | 16.553 | 0.000 | 0.000 | 0.000 | 0.543 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 179 | 490 | 0 | 0 | 0 | 179 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 2.55 | 0.00 | 0.00 | 0.00 | 0.93 | -0.01 |
| time (sec) | N/A | 0.268 | 1.021 | 17.418 | 0.000 | 0.000 | 0.000 | 0.519 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 207 | 618 | 0 | 0 | 0 | 161 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 3.17 | 0.00 | 0.00 | 0.00 | 0.83 | -0.01 |
| time (sec) | N/A | 0.275 | 1.213 | 18.148 | 0.000 | 0.000 | 0.000 | 0.514 | 0.000 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 232 | 748 | 362 | 0 | 0 | 152 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 3.88 | 1.88 | 0.00 | 0.00 | 0.79 | -0.01 |
| time (sec) | N/A | 0.270 | 1.469 | 18.715 | 0.520 | 0.000 | 0.000 | 0.483 | 0.000 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 115 | 154 | 0 | 137 | 0 | 164 | -1 |
| N.S. | 1 | 1.00 | 2.74 | 3.67 | 0.00 | 3.26 | 0.00 | 3.90 | -0.02 |
| time (sec) | N/A | 0.064 | 2.739 | 18.539 | 0.000 | 0.344 | 0.000 | 0.531 | 0.000 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 128 | 247 | 0 | 175 | 0 | 176 | 317 |
| N.S. | 1 | 1.00 | 1.45 | 2.81 | 0.00 | 1.99 | 0.00 | 2.00 | 3.60 |
| time (sec) | N/A | 0.127 | 4.257 | 17.346 | 0.000 | 0.361 | 0.000 | 0.484 | 12.452 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 128 | 276 | 0 | 191 | 0 | 164 | 330 |
| N.S. | 1 | 1.00 | 0.96 | 2.08 | 0.00 | 1.44 | 0.00 | 1.23 | 2.48 |
| time (sec) | N/A | 0.193 | 6.178 | 17.085 | 0.000 | 0.353 | 0.000 | 0.518 | 12.057 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 333 | 302 | 0 | 206 | 0 | 176 | 647 |
| N.S. | 1 | 1.00 | 1.87 | 1.70 | 0.00 | 1.16 | 0.00 | 0.99 | 3.63 |
| time (sec) | N/A | 0.270 | 6.425 | 18.586 | 0.000 | 0.437 | 0.000 | 0.493 | 13.324 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 329 | 328 | 0 | 219 | 0 | 164 | 673 |
| N.S. | 1 | 1.00 | 1.75 | 1.74 | 0.00 | 1.16 | 0.00 | 0.87 | 3.58 |
| time (sec) | N/A | 0.260 | 6.459 | 19.441 | 0.000 | 0.383 | 0.000 | 0.546 | 13.981 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 136 | 319 | 0 | 0 | 0 | 135 | -1 |
| N.S. | 1 | 1.00 | 0.98 | 2.29 | 0.00 | 0.00 | 0.00 | 0.97 | -0.01 |
| time (sec) | N/A | 0.195 | 0.339 | 18.585 | 0.000 | 0.000 | 0.000 | 0.535 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 119 | 259 | 0 | 0 | 0 | 101 | -1 |
| N.S. | 1 | 1.00 | 1.28 | 2.78 | 0.00 | 0.00 | 0.00 | 1.09 | -0.01 |
| time (sec) | N/A | 0.138 | 0.180 | 19.591 | 0.000 | 0.000 | 0.000 | 0.514 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 118 | 105 | 68 | 0 | 0 | 231 | -1 |
| N.S. | 1 | 1.00 | 2.41 | 2.14 | 1.39 | 0.00 | 0.00 | 4.71 | -0.02 |
| time (sec) | N/A | 0.075 | 0.671 | 19.696 | 0.524 | 0.000 | 0.000 | 0.583 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 89 | 92 | 0 | 170 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.93 | 2.00 | 0.00 | 3.70 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.061 | 0.143 | 10.665 | 0.000 | 0.394 | 0.000 | 0.000 | 0.000 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 161 | 165 | 0 | 337 | 0 | 177 | -1 |
| N.S. | 1 | 1.00 | 1.69 | 1.74 | 0.00 | 3.55 | 0.00 | 1.86 | -0.01 |
| time (sec) | N/A | 0.125 | 0.253 | 18.800 | 0.000 | 0.393 | 0.000 | 0.517 | 0.000 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 224 | 249 | 0 | 408 | 0 | 197 | -1 |
| N.S. | 1 | 1.00 | 1.60 | 1.78 | 0.00 | 2.91 | 0.00 | 1.41 | -0.01 |
| time (sec) | N/A | 0.192 | 0.398 | 10.465 | 0.000 | 0.400 | 0.000 | 0.528 | 0.000 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 162 | 500 | 0 | 0 | 0 | 179 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 2.62 | 0.00 | 0.00 | 0.00 | 0.94 | -0.01 |
| time (sec) | N/A | 0.264 | 1.016 | 17.255 | 0.000 | 0.000 | 0.000 | 0.499 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 153 | 448 | 0 | 0 | 0 | 138 | -1 |
| N.S. | 1 | 1.00 | 1.07 | 3.13 | 0.00 | 0.00 | 0.00 | 0.97 | -0.01 |
| time (sec) | N/A | 0.201 | 0.480 | 19.306 | 0.000 | 0.000 | 0.000 | 0.517 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 134 | 388 | 146 | 0 | 0 | 115 | -1 |
| N.S. | 1 | 1.00 | 1.38 | 4.00 | 1.51 | 0.00 | 0.00 | 1.19 | -0.01 |
| time (sec) | N/A | 0.138 | 0.274 | 10.062 | 0.496 | 0.000 | 0.000 | 0.549 | 0.000 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 85 | 69 | 0 | 63 | 0 | 56 | 52 |
| N.S. | 1 | 1.00 | 2.07 | 1.68 | 0.00 | 1.54 | 0.00 | 1.37 | 1.27 |
| time (sec) | N/A | 0.059 | 0.132 | 18.188 | 0.000 | 0.369 | 0.000 | 0.451 | 7.519 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 148 | 165 | 0 | 331 | 0 | 180 | -1 |
| N.S. | 1 | 1.00 | 1.56 | 1.74 | 0.00 | 3.48 | 0.00 | 1.89 | -0.01 |
| time (sec) | N/A | 0.119 | 0.260 | 10.822 | 0.000 | 0.390 | 0.000 | 0.528 | 0.000 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 170 | 114 | 0 | 282 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 0.80 | 0.00 | 1.97 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.189 | 0.398 | 11.614 | 0.000 | 0.398 | 0.000 | 0.000 | 0.000 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 287 | 227 | 0 | 407 | 0 | 236 | -1 |
| N.S. | 1 | 1.00 | 1.50 | 1.19 | 0.00 | 2.13 | 0.00 | 1.24 | -0.01 |
| time (sec) | N/A | 0.258 | 0.483 | 11.030 | 0.000 | 0.405 | 0.000 | 0.559 | 0.000 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 202 | 684 | 0 | 0 | 0 | 198 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 2.89 | 0.00 | 0.00 | 0.00 | 0.84 | -0.00 |
| time (sec) | N/A | 0.346 | 3.236 | 17.450 | 0.000 | 0.000 | 0.000 | 0.527 | 0.000 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 187 | 631 | 0 | 0 | 0 | 162 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 3.27 | 0.00 | 0.00 | 0.00 | 0.84 | -0.01 |
| time (sec) | N/A | 0.268 | 1.285 | 18.733 | 0.000 | 0.000 | 0.000 | 0.503 | 0.000 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 172 | 566 | 197 | 0 | 0 | 135 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 3.96 | 1.38 | 0.00 | 0.00 | 0.94 | -0.01 |
| time (sec) | N/A | 0.207 | 0.746 | 18.649 | 0.509 | 0.000 | 0.000 | 0.473 | 0.000 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 86 | 93 | 0 | 87 | 0 | 98 | 118 |
| N.S. | 1 | 1.00 | 2.05 | 2.21 | 0.00 | 2.07 | 0.00 | 2.33 | 2.81 |
| time (sec) | N/A | 0.064 | 0.279 | 16.620 | 0.000 | 0.338 | 0.000 | 0.526 | 8.214 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 87 | 92 | 0 | 79 | 0 | 56 | 103 |
| N.S. | 1 | 1.00 | 2.02 | 2.14 | 0.00 | 1.84 | 0.00 | 1.30 | 2.40 |
| time (sec) | N/A | 0.059 | 0.150 | 19.157 | 0.000 | 0.339 | 0.000 | 0.498 | 7.696 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 211 | 252 | 0 | 408 | 0 | 197 | -1 |
| N.S. | 1 | 1.00 | 1.51 | 1.80 | 0.00 | 2.91 | 0.00 | 1.41 | -0.01 |
| time (sec) | N/A | 0.189 | 0.418 | 10.943 | 0.000 | 0.389 | 0.000 | 0.528 | 0.000 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 287 | 227 | 0 | 401 | 0 | 233 | -1 |
| N.S. | 1 | 1.00 | 1.53 | 1.21 | 0.00 | 2.13 | 0.00 | 1.24 | -0.01 |
| time (sec) | N/A | 0.262 | 0.460 | 11.043 | 0.000 | 0.390 | 0.000 | 0.573 | 0.000 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 237 | 134 | 0 | 310 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.57 | 0.00 | 1.31 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.327 | 0.628 | 10.941 | 0.000 | 0.410 | 0.000 | 0.000 | 0.000 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 365 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.116 | 1.971 | 0.187 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F(-1) | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.107 | 180.012 | 1.332 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 88512 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1029.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 139.294 | 0.991 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 285 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.082 | 1.076 | 0.214 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 3844 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 50.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 6.214 | 0.245 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 5391 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 62.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.097 | 6.252 | 1.339 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 6906 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 80.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 20.393 | 1.579 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F(-2) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 149 | 0 | 306 | 277 | 0 | 0 | 163 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 1.91 | 1.73 | 0.00 | 0.00 | 1.02 |
| time (sec) | N/A | 0.176 | 1.580 | 0.208 | 0.507 | 0.366 | 0.000 | 0.000 | 10.048 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | A | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 110 | 0 | 205 | 153 | 0 | 0 | 94 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 2.05 | 1.53 | 0.00 | 0.00 | 0.94 |
| time (sec) | N/A | 0.102 | 0.305 | 0.184 | 0.504 | 0.358 | 0.000 | 0.000 | 1.188 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | A | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 85 | 0 | 124 | 82 | 0 | 0 | 53 |
| N.S. | 1 | 1.00 | 1.85 | 0.00 | 2.70 | 1.78 | 0.00 | 0.00 | 1.15 |
| time (sec) | N/A | 0.048 | 0.119 | 0.176 | 0.496 | 0.375 | 0.000 | 0.000 | 0.451 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 157 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.098 | 0.982 | 0.125 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 3006 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 40.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.112 | 15.358 | 0.138 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 5136 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 69.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.113 | 19.730 | 0.173 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 157 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.092 | 0.266 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 157 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.093 | 0.236 | 0.204 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 174 | 0 | 0 | 107 | 0 | 0 | 149 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.65 | 0.00 | 0.00 | 0.91 |
| time (sec) | N/A | 0.161 | 7.289 | 1.276 | 0.000 | 0.358 | 0.000 | 0.000 | 8.249 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 136 | 0 | 0 | 77 | 0 | 0 | 111 |
| N.S. | 1 | 1.00 | 1.35 | 0.00 | 0.00 | 0.76 | 0.00 | 0.00 | 1.10 |
| time (sec) | N/A | 0.099 | 1.764 | 0.977 | 0.000 | 0.365 | 0.000 | 0.000 | 0.880 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 107 | 0 | 0 | 47 | 0 | 0 | 58 |
| N.S. | 1 | 1.00 | 2.33 | 0.00 | 0.00 | 1.02 | 0.00 | 0.00 | 1.26 |
| time (sec) | N/A | 0.046 | 0.877 | 0.343 | 0.000 | 0.348 | 0.000 | 0.000 | 0.409 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 388 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.114 | 1.858 | 0.161 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 602 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.121 | 3.926 | 0.383 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 1201 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 10.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 9.038 | 1.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 207 | 259 | 270 | 210 | 580 | 272 | 559 |
| N.S. | 1 | 1.00 | 0.91 | 1.14 | 1.19 | 0.93 | 2.56 | 1.20 | 2.46 |
| time (sec) | N/A | 0.196 | 0.865 | 0.378 | 0.294 | 0.364 | 0.387 | 0.501 | 9.941 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 124 | 182 | 189 | 150 | 386 | 191 | 460 |
| N.S. | 1 | 1.00 | 0.77 | 1.12 | 1.17 | 0.93 | 2.38 | 1.18 | 2.84 |
| time (sec) | N/A | 0.135 | 0.526 | 0.331 | 0.279 | 0.361 | 0.270 | 0.491 | 8.175 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 89 | 115 | 121 | 94 | 199 | 117 | 108 |
| N.S. | 1 | 1.00 | 0.90 | 1.16 | 1.22 | 0.95 | 2.01 | 1.18 | 1.09 |
| time (sec) | N/A | 0.065 | 0.265 | 0.238 | 0.280 | 0.349 | 0.146 | 0.588 | 6.987 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 45 | 59 | 62 | 51 | 94 | 55 | 100 |
| N.S. | 1 | 1.00 | 0.94 | 1.23 | 1.29 | 1.06 | 1.96 | 1.15 | 2.08 |
| time (sec) | N/A | 0.017 | 0.076 | 0.175 | 0.288 | 0.354 | 0.090 | 0.562 | 6.994 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 27 | 17 | 17 | 19 | 19 | 17 | 25 |
| N.S. | 1 | 1.00 | 1.69 | 1.06 | 1.06 | 1.19 | 1.19 | 1.06 | 1.56 |
| time (sec) | N/A | 0.006 | 0.006 | 0.052 | 0.291 | 0.348 | 0.045 | 0.463 | 6.718 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 182 | 72 | 0 | 237 | 537 | 86 | 449 |
| N.S. | 1 | 1.00 | 2.89 | 1.14 | 0.00 | 3.76 | 8.52 | 1.37 | 7.13 |
| time (sec) | N/A | 0.070 | 0.215 | 0.257 | 0.000 | 0.371 | 40.184 | 0.481 | 7.392 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 220 | 113 | 0 | 377 | 0 | 129 | 140 |
| N.S. | 1 | 1.00 | 2.65 | 1.36 | 0.00 | 4.54 | 0.00 | 1.55 | 1.69 |
| time (sec) | N/A | 0.072 | 0.398 | 0.360 | 0.000 | 0.361 | 0.000 | 0.461 | 6.946 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 242 | 325 | 0 | 826 | 0 | 384 | 445 |
| N.S. | 1 | 1.00 | 1.81 | 2.43 | 0.00 | 6.16 | 0.00 | 2.87 | 3.32 |
| time (sec) | N/A | 0.138 | 0.786 | 0.541 | 0.000 | 0.380 | 0.000 | 0.526 | 8.967 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 428 | 641 | 0 | 1373 | 0 | 808 | 877 |
| N.S. | 1 | 1.00 | 2.23 | 3.34 | 0.00 | 7.15 | 0.00 | 4.21 | 4.57 |
| time (sec) | N/A | 0.235 | 1.788 | 0.739 | 0.000 | 0.401 | 0.000 | 0.497 | 9.706 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 262 | 462 | 485 | 306 | 1136 | 458 | 865 |
| N.S. | 1 | 1.00 | 0.82 | 1.45 | 1.53 | 0.96 | 3.57 | 1.44 | 2.72 |
| time (sec) | N/A | 0.317 | 0.883 | 0.520 | 0.287 | 0.376 | 0.667 | 0.470 | 9.924 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 188 | 329 | 342 | 223 | 729 | 324 | 611 |
| N.S. | 1 | 1.00 | 0.81 | 1.41 | 1.47 | 0.96 | 3.13 | 1.39 | 2.62 |
| time (sec) | N/A | 0.219 | 0.578 | 0.470 | 0.294 | 0.370 | 0.419 | 0.467 | 8.420 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 148 | 219 | 227 | 150 | 459 | 208 | 440 |
| N.S. | 1 | 1.00 | 0.95 | 1.40 | 1.46 | 0.96 | 2.94 | 1.33 | 2.82 |
| time (sec) | N/A | 0.135 | 0.356 | 0.267 | 0.286 | 0.347 | 0.256 | 0.482 | 8.340 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 106 | 117 | 123 | 86 | 199 | 109 | 91 |
| N.S. | 1 | 1.00 | 1.13 | 1.24 | 1.31 | 0.91 | 2.12 | 1.16 | 0.97 |
| time (sec) | N/A | 0.042 | 0.215 | 0.233 | 0.318 | 0.359 | 0.149 | 0.440 | 6.923 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 34 | 52 | 50 | 44 | 78 | 40 | 123 |
| N.S. | 1 | 1.00 | 0.76 | 1.16 | 1.11 | 0.98 | 1.73 | 0.89 | 2.73 |
| time (sec) | N/A | 0.010 | 0.115 | 0.125 | 0.275 | 0.353 | 0.091 | 0.457 | 6.871 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 130 | 105 | 0 | 307 | 3709 | 136 | 940 |
| N.S. | 1 | 1.00 | 1.41 | 1.14 | 0.00 | 3.34 | 40.32 | 1.48 | 10.22 |
| time (sec) | N/A | 0.136 | 0.274 | 0.329 | 0.000 | 0.380 | 180.783 | 0.465 | 7.678 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 115 | 139 | 160 | 0 | 493 | 0 | 205 | 2836 |
| N.S. | 1 | 1.03 | 1.24 | 1.43 | 0.00 | 4.40 | 0.00 | 1.83 | 25.32 |
| time (sec) | N/A | 0.129 | 0.326 | 0.471 | 0.000 | 0.366 | 0.000 | 0.453 | 11.241 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 140 | 250 | 0 | 702 | 0 | 348 | 362 |
| N.S. | 1 | 1.00 | 1.01 | 1.81 | 0.00 | 5.09 | 0.00 | 2.52 | 2.62 |
| time (sec) | N/A | 0.139 | 0.402 | 0.580 | 0.000 | 0.364 | 0.000 | 0.506 | 9.427 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 196 | 523 | 0 | 1395 | 0 | 781 | 735 |
| N.S. | 1 | 1.00 | 0.95 | 2.53 | 0.00 | 6.74 | 0.00 | 3.77 | 3.55 |
| time (sec) | N/A | 0.231 | 1.640 | 0.862 | 0.000 | 0.401 | 0.000 | 0.506 | 10.316 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 286 | 269 | 1056 | 0 | 2186 | 0 | 1555 | 1411 |
| N.S. | 1 | 1.00 | 0.94 | 3.69 | 0.00 | 7.64 | 0.00 | 5.44 | 4.93 |
| time (sec) | N/A | 0.358 | 3.092 | 1.473 | 0.000 | 0.462 | 0.000 | 0.578 | 10.347 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 326 | 215 | 481 | 505 | 268 | 1176 | 373 | 773 |
| N.S. | 1 | 1.52 | 1.00 | 2.24 | 2.35 | 1.25 | 5.47 | 1.73 | 3.60 |
| time (sec) | N/A | 0.379 | 0.841 | 0.527 | 0.323 | 0.368 | 0.595 | 0.482 | 8.479 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 189 | 177 | 319 | 332 | 186 | 702 | 251 | 493 |
| N.S. | 1 | 1.15 | 1.08 | 1.95 | 2.02 | 1.13 | 4.28 | 1.53 | 3.01 |
| time (sec) | N/A | 0.184 | 0.485 | 0.400 | 0.328 | 0.371 | 0.436 | 0.466 | 8.359 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 117 | 120 | 178 | 185 | 113 | 371 | 138 | 330 |
| N.S. | 1 | 1.06 | 1.09 | 1.62 | 1.68 | 1.03 | 3.37 | 1.25 | 3.00 |
| time (sec) | N/A | 0.078 | 0.330 | 0.313 | 0.353 | 0.365 | 0.242 | 0.492 | 8.067 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 44 | 74 | 77 | 58 | 121 | 58 | 156 |
| N.S. | 1 | 1.00 | 0.70 | 1.17 | 1.22 | 0.92 | 1.92 | 0.92 | 2.48 |
| time (sec) | N/A | 0.039 | 0.224 | 0.233 | 0.349 | 0.345 | 0.150 | 0.440 | 9.120 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 162 | 184 | 0 | 419 | 0 | 239 | 2500 |
| N.S. | 1 | 1.00 | 1.13 | 1.29 | 0.00 | 2.93 | 0.00 | 1.67 | 17.48 |
| time (sec) | N/A | 0.275 | 0.430 | 0.379 | 0.000 | 0.389 | 0.000 | 0.492 | 9.220 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 162 | 209 | 0 | 664 | 0 | 395 | 2500 |
| N.S. | 1 | 1.00 | 1.01 | 1.30 | 0.00 | 4.12 | 0.00 | 2.45 | 15.53 |
| time (sec) | N/A | 0.267 | 0.466 | 0.537 | 0.000 | 0.398 | 0.000 | 0.456 | 12.955 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 196 | 353 | 0 | 1089 | 0 | 522 | 2500 |
| N.S. | 1 | 1.00 | 1.05 | 1.89 | 0.00 | 5.82 | 0.00 | 2.79 | 13.37 |
| time (sec) | N/A | 0.334 | 0.631 | 0.734 | 0.000 | 0.424 | 0.000 | 0.526 | 14.274 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 178 | 466 | 0 | 1135 | 0 | 667 | 649 |
| N.S. | 1 | 1.00 | 0.86 | 2.25 | 0.00 | 5.48 | 0.00 | 3.22 | 3.14 |
| time (sec) | N/A | 0.331 | 1.501 | 0.892 | 0.000 | 0.409 | 0.000 | 0.540 | 10.363 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 289 | 240 | 922 | 0 | 2044 | 0 | 1338 | 1231 |
| N.S. | 1 | 1.00 | 0.83 | 3.19 | 0.00 | 7.07 | 0.00 | 4.63 | 4.26 |
| time (sec) | N/A | 0.483 | 2.009 | 1.404 | 0.000 | 0.443 | 0.000 | 0.530 | 10.104 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 234 | 230 | 791 | 359 | 8605 | 306 | 451 |
| N.S. | 1 | 1.00 | 1.24 | 1.22 | 4.19 | 1.90 | 45.53 | 1.62 | 2.39 |
| time (sec) | N/A | 0.153 | 0.285 | 0.388 | 0.604 | 0.380 | 5.187 | 0.468 | 9.786 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 192 | 148 | 463 | 244 | 3602 | 172 | 282 |
| N.S. | 1 | 1.00 | 1.59 | 1.22 | 3.83 | 2.02 | 29.77 | 1.42 | 2.33 |
| time (sec) | N/A | 0.085 | 0.395 | 0.318 | 0.530 | 0.344 | 2.497 | 0.470 | 9.435 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 122 | 75 | 227 | 148 | 940 | 143 | 124 |
| N.S. | 1 | 1.00 | 1.97 | 1.21 | 3.66 | 2.39 | 15.16 | 2.31 | 2.00 |
| time (sec) | N/A | 0.096 | 0.315 | 0.307 | 0.510 | 0.348 | 1.394 | 0.407 | 7.380 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 79 | 42 | 84 | 70 | 109 | 40 | 35 |
| N.S. | 1 | 1.00 | 2.26 | 1.20 | 2.40 | 2.00 | 3.11 | 1.14 | 1.00 |
| time (sec) | N/A | 0.035 | 0.114 | 0.247 | 0.488 | 0.344 | 0.634 | 0.549 | 6.826 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 48 | 22 | 29 | 46 | 27 | 22 | 21 |
| N.S. | 1 | 1.00 | 2.09 | 0.96 | 1.26 | 2.00 | 1.17 | 0.96 | 0.91 |
| time (sec) | N/A | 0.009 | 0.031 | 0.181 | 0.295 | 0.349 | 0.388 | 0.424 | 6.975 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 114 | 83 | 0 | 510 | 916 | 100 | 121 |
| N.S. | 1 | 1.00 | 1.28 | 0.93 | 0.00 | 5.73 | 10.29 | 1.12 | 1.36 |
| time (sec) | N/A | 0.090 | 0.233 | 0.444 | 0.000 | 0.387 | 119.142 | 0.462 | 6.986 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 162 | 154 | 0 | 1153 | 0 | 311 | 309 |
| N.S. | 1 | 1.00 | 1.08 | 1.03 | 0.00 | 7.69 | 0.00 | 2.07 | 2.06 |
| time (sec) | N/A | 0.132 | 0.426 | 0.608 | 0.000 | 0.418 | 0.000 | 0.507 | 8.566 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 230 | 327 | 0 | 2410 | 0 | 482 | 753 |
| N.S. | 1 | 1.00 | 1.08 | 1.54 | 0.00 | 11.31 | 0.00 | 2.26 | 3.54 |
| time (sec) | N/A | 0.223 | 1.271 | 0.867 | 0.000 | 0.440 | 0.000 | 0.462 | 10.203 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 260 | 837 | 342 | 1426 | 592 | 17899 | 977 | 692 |
| N.S. | 1 | 1.00 | 3.22 | 1.32 | 5.48 | 2.28 | 68.84 | 3.76 | 2.66 |
| time (sec) | N/A | 0.335 | 1.117 | 0.496 | 0.520 | 0.353 | 17.396 | 0.455 | 9.426 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 378 | 250 | 986 | 452 | 8950 | 338 | 478 |
| N.S. | 1 | 1.00 | 1.94 | 1.28 | 5.06 | 2.32 | 45.90 | 1.73 | 2.45 |
| time (sec) | N/A | 0.242 | 1.249 | 0.461 | 0.533 | 0.383 | 9.530 | 0.442 | 9.207 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 212 | 157 | 641 | 318 | 3585 | 209 | 298 |
| N.S. | 1 | 1.00 | 1.77 | 1.31 | 5.34 | 2.65 | 29.88 | 1.74 | 2.48 |
| time (sec) | N/A | 0.257 | 0.241 | 0.407 | 0.504 | 0.375 | 4.993 | 0.458 | 8.385 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 172 | 108 | 390 | 205 | 915 | 132 | 93 |
| N.S. | 1 | 1.00 | 2.02 | 1.27 | 4.59 | 2.41 | 10.76 | 1.55 | 1.09 |
| time (sec) | N/A | 0.095 | 0.181 | 0.397 | 0.497 | 0.363 | 2.450 | 0.501 | 7.433 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 43 | 70 | 232 | 125 | 372 | 68 | 97 |
| N.S. | 1 | 1.00 | 0.66 | 1.08 | 3.57 | 1.92 | 5.72 | 1.05 | 1.49 |
| time (sec) | N/A | 0.036 | 0.043 | 0.265 | 0.299 | 0.319 | 1.409 | 0.444 | 7.213 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 54 | 53 | 127 | 103 | 221 | 50 | 76 |
| N.S. | 1 | 1.00 | 0.98 | 0.96 | 2.31 | 1.87 | 4.02 | 0.91 | 1.38 |
| time (sec) | N/A | 0.020 | 0.069 | 0.198 | 0.282 | 0.337 | 0.730 | 0.444 | 6.981 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 204 | 132 | 0 | 1022 | 0 | 195 | 250 |
| N.S. | 1 | 1.00 | 1.56 | 1.01 | 0.00 | 7.80 | 0.00 | 1.49 | 1.91 |
| time (sec) | N/A | 0.197 | 0.240 | 0.483 | 0.000 | 0.395 | 0.000 | 0.455 | 8.019 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 267 | 205 | 0 | 2342 | 0 | 320 | 625 |
| N.S. | 1 | 1.00 | 1.21 | 0.93 | 0.00 | 10.60 | 0.00 | 1.45 | 2.83 |
| time (sec) | N/A | 0.294 | 0.873 | 0.652 | 0.000 | 0.435 | 0.000 | 0.459 | 10.487 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 294 | 338 | 378 | 0 | 3597 | 0 | 614 | 1199 |
| N.S. | 1 | 1.00 | 1.15 | 1.29 | 0.00 | 12.23 | 0.00 | 2.09 | 4.08 |
| time (sec) | N/A | 0.424 | 0.781 | 0.862 | 0.000 | 0.501 | 0.000 | 0.569 | 10.813 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 560 | 468 | 2169 | 841 | 28065 | 776 | 898 |
| N.S. | 1 | 1.00 | 1.58 | 1.32 | 6.13 | 2.38 | 79.28 | 2.19 | 2.54 |
| time (sec) | N/A | 0.527 | 1.737 | 0.632 | 0.549 | 0.443 | 51.501 | 0.515 | 9.713 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 278 | 992 | 360 | 1636 | 669 | 15553 | 564 | 652 |
| N.S. | 1 | 1.00 | 3.57 | 1.29 | 5.88 | 2.41 | 55.95 | 2.03 | 2.35 |
| time (sec) | N/A | 0.409 | 6.861 | 0.585 | 0.528 | 0.367 | 31.068 | 0.455 | 9.536 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 683 | 248 | 1197 | 508 | 7373 | 395 | 440 |
| N.S. | 1 | 1.00 | 3.50 | 1.27 | 6.14 | 2.61 | 37.81 | 2.03 | 2.26 |
| time (sec) | N/A | 0.416 | 0.943 | 0.556 | 0.532 | 0.402 | 19.035 | 0.455 | 9.028 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | B | B | A | B |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 1366 | 194 | 852 | 364 | 2640 | 280 | 240 |
| N.S. | 1 | 1.00 | 9.62 | 1.37 | 6.00 | 2.56 | 18.59 | 1.97 | 1.69 |
| time (sec) | N/A | 0.229 | 7.779 | 0.474 | 0.516 | 0.389 | 10.000 | 0.461 | 9.904 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 84 | 139 | 601 | 254 | 1365 | 181 | 218 |
| N.S. | 1 | 1.00 | 0.67 | 1.11 | 4.81 | 2.03 | 10.92 | 1.45 | 1.74 |
| time (sec) | N/A | 0.120 | 0.088 | 0.397 | 0.292 | 0.371 | 5.641 | 0.475 | 7.486 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 63 | 114 | 421 | 202 | 1015 | 130 | 150 |
| N.S. | 1 | 1.00 | 0.62 | 1.12 | 4.13 | 1.98 | 9.95 | 1.27 | 1.47 |
| time (sec) | N/A | 0.051 | 0.058 | 0.351 | 0.288 | 0.337 | 3.122 | 0.441 | 7.251 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 76 | 85 | 221 | 159 | 558 | 78 | 133 |
| N.S. | 1 | 1.00 | 0.92 | 1.02 | 2.66 | 1.92 | 6.72 | 0.94 | 1.60 |
| time (sec) | N/A | 0.033 | 0.087 | 0.250 | 0.282 | 0.350 | 1.508 | 0.476 | 6.957 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 301 | 200 | 0 | 1789 | 0 | 364 | 466 |
| N.S. | 1 | 1.00 | 1.62 | 1.08 | 0.00 | 9.62 | 0.00 | 1.96 | 2.51 |
| time (sec) | N/A | 0.354 | 0.494 | 0.575 | 0.000 | 0.418 | 0.000 | 0.449 | 10.069 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 361 | 273 | 0 | 3292 | 0 | 517 | 987 |
| N.S. | 1 | 1.00 | 1.21 | 0.92 | 0.00 | 11.05 | 0.00 | 1.73 | 3.31 |
| time (sec) | N/A | 0.493 | 1.637 | 0.742 | 0.000 | 0.480 | 0.000 | 0.465 | 10.302 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 378 | 378 | 914 | 446 | 0 | 5295 | 0 | 794 | 1660 |
| N.S. | 1 | 1.00 | 2.42 | 1.18 | 0.00 | 14.01 | 0.00 | 2.10 | 4.39 |
| time (sec) | N/A | 0.642 | 4.059 | 1.083 | 0.000 | 0.525 | 0.000 | 0.609 | 11.645 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 55 | 115 | 309 | 150 | 889 | 112 | 94 |
| N.S. | 1 | 1.00 | 0.73 | 1.53 | 4.12 | 2.00 | 11.85 | 1.49 | 1.25 |
| time (sec) | N/A | 0.039 | 0.038 | 0.161 | 0.286 | 0.414 | 3.428 | 0.444 | 7.065 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 54 | 115 | 309 | 150 | 887 | 112 | 97 |
| N.S. | 1 | 1.00 | 0.67 | 1.42 | 3.81 | 1.85 | 10.95 | 1.38 | 1.20 |
| time (sec) | N/A | 0.046 | 0.047 | 0.169 | 0.285 | 0.327 | 3.360 | 0.515 | 7.059 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 290 | 290 | 3531 | 1315 | 0 | 592 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 12.18 | 4.53 | 0.00 | 2.04 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.307 | 6.539 | 4.821 | 0.000 | 0.186 | 0.000 | 0.000 | 0.000 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 2625 | 1034 | 0 | 521 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 11.36 | 4.48 | 0.00 | 2.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.216 | 6.338 | 5.104 | 0.000 | 0.141 | 0.000 | 0.000 | 0.000 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 1736 | 657 | 0 | 453 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 9.70 | 3.67 | 0.00 | 2.53 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.134 | 6.205 | 4.684 | 0.000 | 0.117 | 0.000 | 0.000 | 0.000 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | C | F | F | B |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 880 | 203 | 0 | 391 | 0 | 0 | 176 |
| N.S. | 1 | 1.00 | 6.38 | 1.47 | 0.00 | 2.83 | 0.00 | 0.00 | 1.28 |
| time (sec) | N/A | 0.083 | 6.189 | 5.339 | 0.000 | 0.115 | 0.000 | 0.000 | 7.619 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 938 | 246 | 0 | 543 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.55 | 1.46 | 0.00 | 3.21 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.146 | 6.288 | 3.909 | 0.000 | 0.112 | 0.000 | 0.000 | 0.000 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 1870 | 884 | 0 | 942 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 7.89 | 3.73 | 0.00 | 3.97 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.224 | 6.564 | 16.191 | 0.000 | 0.190 | 0.000 | 0.000 | 0.000 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 2815 | 1046 | 0 | 1489 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.85 | 3.29 | 0.00 | 4.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.337 | 6.787 | 23.273 | 0.000 | 0.230 | 0.000 | 0.000 | 0.000 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 378 | 378 | 322 | 1614 | 0 | 737 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 4.27 | 0.00 | 1.95 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.443 | 1.406 | 4.958 | 0.000 | 0.197 | 0.000 | 0.000 | 0.000 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 262 | 1316 | 0 | 639 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 4.42 | 0.00 | 2.14 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.316 | 1.395 | 4.868 | 0.000 | 0.198 | 0.000 | 0.000 | 0.000 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 239 | 244 | 1035 | 0 | 554 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.02 | 4.33 | 0.00 | 2.32 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.228 | 0.937 | 5.168 | 0.000 | 0.173 | 0.000 | 0.000 | 0.000 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 193 | 758 | 0 | 475 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.02 | 4.01 | 0.00 | 2.51 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.158 | 0.731 | 5.329 | 0.000 | 0.170 | 0.000 | 0.000 | 0.000 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 175 | 463 | 0 | 594 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 2.45 | 0.00 | 3.14 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.151 | 0.609 | 4.614 | 0.000 | 0.166 | 0.000 | 0.000 | 0.000 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 207 | 1221 | 0 | 1013 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.94 | 0.00 | 4.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.238 | 1.208 | 4.797 | 0.000 | 0.190 | 0.000 | 0.000 | 0.000 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 283 | 1436 | 0 | 1631 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 4.49 | 0.00 | 5.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.367 | 1.377 | 25.842 | 0.000 | 0.237 | 0.000 | 0.000 | 0.000 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 467 | 467 | 377 | 1926 | 0 | 843 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 4.12 | 0.00 | 1.81 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.638 | 1.298 | 5.450 | 0.000 | 0.231 | 0.000 | 0.000 | 0.000 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 390 | 318 | 1613 | 0 | 735 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 4.14 | 0.00 | 1.88 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.507 | 1.504 | 4.927 | 0.000 | 0.191 | 0.000 | 0.000 | 0.000 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 266 | 1316 | 0 | 638 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.14 | 0.00 | 2.01 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.393 | 1.805 | 5.971 | 0.000 | 0.173 | 0.000 | 0.000 | 0.000 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 246 | 1035 | 0 | 556 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.95 | 4.01 | 0.00 | 2.16 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.312 | 1.100 | 5.098 | 0.000 | 0.195 | 0.000 | 0.000 | 0.000 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 234 | 1031 | 0 | 803 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 3.82 | 0.00 | 2.97 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.319 | 0.992 | 5.333 | 0.000 | 0.168 | 0.000 | 0.000 | 0.000 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 280 | 232 | 1257 | 0 | 1173 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 4.49 | 0.00 | 4.19 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.379 | 1.054 | 20.666 | 0.000 | 0.217 | 0.000 | 0.000 | 0.000 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 336 | 336 | 298 | 1589 | 0 | 1622 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 4.73 | 0.00 | 4.83 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.483 | 1.408 | 27.169 | 0.000 | 0.242 | 0.000 | 0.000 | 0.000 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 419 | 419 | 351 | 2079 | 0 | 2483 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.96 | 0.00 | 5.93 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.610 | 2.425 | 38.301 | 0.000 | 0.344 | 0.000 | 0.000 | 0.000 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 246 | 298 | 1372 | 0 | 843 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.21 | 5.58 | 0.00 | 3.43 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.254 | 0.953 | 6.174 | 0.000 | 0.177 | 0.000 | 0.000 | 0.000 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 223 | 925 | 0 | 695 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 4.97 | 0.00 | 3.74 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.173 | 1.067 | 5.681 | 0.000 | 0.163 | 0.000 | 0.000 | 0.000 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 201 | 382 | 0 | 576 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.18 | 2.25 | 0.00 | 3.39 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.134 | 0.760 | 5.807 | 0.000 | 0.138 | 0.000 | 0.000 | 0.000 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 210 | 443 | 0 | 603 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 2.45 | 0.00 | 3.33 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.140 | 0.782 | 11.987 | 0.000 | 0.144 | 0.000 | 0.000 | 0.000 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 244 | 264 | 926 | 0 | 1205 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 3.80 | 0.00 | 4.94 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.219 | 1.396 | 6.777 | 0.000 | 0.180 | 0.000 | 0.000 | 0.000 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 333 | 333 | 367 | 1291 | 0 | 2243 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 3.88 | 0.00 | 6.74 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.329 | 2.955 | 23.999 | 0.000 | 0.219 | 0.000 | 0.000 | 0.000 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 256 | 310 | 1372 | 0 | 1185 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.21 | 5.36 | 0.00 | 4.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.360 | 1.737 | 21.121 | 0.000 | 0.188 | 0.000 | 0.000 | 0.000 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 283 | 1049 | 0 | 969 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.19 | 4.43 | 0.00 | 4.09 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.357 | 1.709 | 18.482 | 0.000 | 0.170 | 0.000 | 0.000 | 0.000 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 256 | 906 | 0 | 913 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 3.89 | 0.00 | 3.92 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.274 | 1.690 | 19.835 | 0.000 | 0.162 | 0.000 | 0.000 | 0.000 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 290 | 507 | 0 | 1076 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.13 | 1.97 | 0.00 | 4.19 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.294 | 1.691 | 14.206 | 0.000 | 0.172 | 0.000 | 0.000 | 0.000 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 405 | 1299 | 0 | 2093 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.24 | 3.98 | 0.00 | 6.42 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.429 | 3.278 | 23.010 | 0.000 | 0.228 | 0.000 | 0.000 | 0.000 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 405 | 405 | 674 | 1758 | 0 | 3328 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.66 | 4.34 | 0.00 | 8.22 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.560 | 6.538 | 33.346 | 0.000 | 0.336 | 0.000 | 0.000 | 0.000 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 385 | 1615 | 0 | 1558 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 5.02 | 0.00 | 4.84 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.569 | 3.877 | 28.402 | 0.000 | 0.194 | 0.000 | 0.000 | 0.000 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 441 | 1462 | 0 | 1634 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.37 | 4.53 | 0.00 | 5.06 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.584 | 4.111 | 28.632 | 0.000 | 0.172 | 0.000 | 0.000 | 0.000 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 334 | 334 | 449 | 1056 | 0 | 1703 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.34 | 3.16 | 0.00 | 5.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.511 | 3.890 | 24.839 | 0.000 | 0.200 | 0.000 | 0.000 | 0.000 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 344 | 344 | 445 | 593 | 0 | 1789 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.29 | 1.72 | 0.00 | 5.20 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.500 | 4.192 | 17.368 | 0.000 | 0.192 | 0.000 | 0.000 | 0.000 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 423 | 423 | 745 | 1851 | 0 | 3124 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.76 | 4.38 | 0.00 | 7.39 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.659 | 6.394 | 32.720 | 0.000 | 0.316 | 0.000 | 0.000 | 0.000 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 518 | 518 | 828 | 2311 | 0 | 4857 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.60 | 4.46 | 0.00 | 9.38 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.830 | 6.730 | 45.301 | 0.000 | 0.467 | 0.000 | 0.000 | 0.000 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 146 | 141 | 0 | 251 | 0 | 266 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 0.88 | 0.00 | 1.56 | 0.00 | 1.65 | -0.01 |
| time (sec) | N/A | 0.177 | 0.342 | 3.205 | 0.000 | 0.354 | 0.000 | 0.604 | 0.000 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 111 | 92 | 0 | 166 | 0 | 167 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 0.82 | 0.00 | 1.48 | 0.00 | 1.49 | -0.01 |
| time (sec) | N/A | 0.108 | 0.198 | 2.859 | 0.000 | 0.322 | 0.000 | 0.544 | 0.000 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 82 | 58 | 0 | 92 | 0 | 90 | -1 |
| N.S. | 1 | 1.00 | 1.32 | 0.94 | 0.00 | 1.48 | 0.00 | 1.45 | -0.02 |
| time (sec) | N/A | 0.039 | 0.087 | 2.593 | 0.000 | 0.332 | 0.000 | 0.516 | 0.000 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 65 | 43 | 0 | 55 | 0 | 38 | 33 |
| N.S. | 1 | 1.00 | 2.50 | 1.65 | 0.00 | 2.12 | 0.00 | 1.46 | 1.27 |
| time (sec) | N/A | 0.010 | 0.022 | 1.802 | 0.000 | 0.327 | 0.000 | 0.459 | 7.416 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 657 | 80 | 0 | 484 | 0 | 65 | -1 |
| N.S. | 1 | 1.00 | 10.77 | 1.31 | 0.00 | 7.93 | 0.00 | 1.07 | -0.02 |
| time (sec) | N/A | 0.077 | 3.836 | 2.800 | 0.000 | 0.429 | 0.000 | 0.472 | 0.000 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 871 | 155 | 0 | 828 | 0 | 139 | -1 |
| N.S. | 1 | 1.00 | 8.30 | 1.48 | 0.00 | 7.89 | 0.00 | 1.32 | -0.01 |
| time (sec) | N/A | 0.132 | 4.185 | 4.816 | 0.000 | 0.428 | 0.000 | 0.563 | 0.000 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 920 | 254 | 0 | 1304 | 0 | 222 | -1 |
| N.S. | 1 | 1.00 | 5.97 | 1.65 | 0.00 | 8.47 | 0.00 | 1.44 | -0.01 |
| time (sec) | N/A | 0.191 | 5.231 | 4.879 | 0.000 | 0.519 | 0.000 | 0.513 | 0.000 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 203 | 195 | 0 | 352 | 0 | 374 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 0.84 | 0.00 | 1.52 | 0.00 | 1.62 | -0.00 |
| time (sec) | N/A | 0.260 | 1.082 | 2.567 | 0.000 | 0.356 | 0.000 | 0.643 | 0.000 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 136 | 130 | 0 | 240 | 0 | 250 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 0.83 | 0.00 | 1.53 | 0.00 | 1.59 | -0.01 |
| time (sec) | N/A | 0.158 | 0.570 | 2.935 | 0.000 | 0.377 | 0.000 | 0.550 | 0.000 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 101 | 77 | 0 | 145 | 0 | 147 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.00 | 1.44 | 0.00 | 1.46 | -0.01 |
| time (sec) | N/A | 0.059 | 0.273 | 2.938 | 0.000 | 0.331 | 0.000 | 0.567 | 0.000 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 89 | 53 | 0 | 83 | 0 | 71 | -1 |
| N.S. | 1 | 1.00 | 1.51 | 0.90 | 0.00 | 1.41 | 0.00 | 1.20 | -0.02 |
| time (sec) | N/A | 0.022 | 0.100 | 2.119 | 0.000 | 0.460 | 0.000 | 0.508 | 0.000 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 233 | 137 | 0 | 685 | 0 | 130 | -1 |
| N.S. | 1 | 1.00 | 2.38 | 1.40 | 0.00 | 6.99 | 0.00 | 1.33 | -0.01 |
| time (sec) | N/A | 0.140 | 1.644 | 4.382 | 0.000 | 0.435 | 0.000 | 0.504 | 0.000 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 268 | 233 | 0 | 1012 | 0 | 204 | -1 |
| N.S. | 1 | 1.00 | 2.25 | 1.96 | 0.00 | 8.50 | 0.00 | 1.71 | -0.01 |
| time (sec) | N/A | 0.145 | 1.652 | 4.530 | 0.000 | 0.449 | 0.000 | 0.497 | 0.000 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 313 | 429 | 0 | 1612 | 0 | 325 | -1 |
| N.S. | 1 | 1.00 | 1.75 | 2.40 | 0.00 | 9.01 | 0.00 | 1.82 | -0.01 |
| time (sec) | N/A | 0.196 | 2.717 | 5.719 | 0.000 | 0.517 | 0.000 | 0.585 | 0.000 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 328 | 246 | 249 | 0 | 508 | 0 | 510 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 0.76 | 0.00 | 1.55 | 0.00 | 1.55 | -0.00 |
| time (sec) | N/A | 0.443 | 3.995 | 2.580 | 0.000 | 0.351 | 0.000 | 0.690 | 0.000 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 202 | 180 | 168 | 0 | 352 | 0 | 348 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.83 | 0.00 | 1.74 | 0.00 | 1.72 | -0.00 |
| time (sec) | N/A | 0.191 | 2.109 | 3.094 | 0.000 | 0.339 | 0.000 | 0.630 | 0.000 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 119 | 99 | 0 | 217 | 0 | 213 | -1 |
| N.S. | 1 | 1.00 | 0.86 | 0.72 | 0.00 | 1.57 | 0.00 | 1.54 | -0.01 |
| time (sec) | N/A | 0.076 | 1.006 | 3.661 | 0.000 | 0.369 | 0.000 | 0.524 | 0.000 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 117 | 65 | 0 | 124 | 0 | 108 | -1 |
| N.S. | 1 | 1.00 | 1.31 | 0.73 | 0.00 | 1.39 | 0.00 | 1.21 | -0.01 |
| time (sec) | N/A | 0.035 | 0.230 | 2.714 | 0.000 | 0.332 | 0.000 | 0.482 | 0.000 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 330 | 229 | 0 | 906 | 0 | 239 | -1 |
| N.S. | 1 | 1.00 | 2.32 | 1.61 | 0.00 | 6.38 | 0.00 | 1.68 | -0.01 |
| time (sec) | N/A | 0.273 | 2.510 | 4.448 | 0.000 | 0.463 | 0.000 | 0.566 | 0.000 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 350 | 393 | 0 | 1368 | 0 | 314 | -1 |
| N.S. | 1 | 1.00 | 2.11 | 2.37 | 0.00 | 8.24 | 0.00 | 1.89 | -0.01 |
| time (sec) | N/A | 0.266 | 2.827 | 5.428 | 0.000 | 0.454 | 0.000 | 0.558 | 0.000 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 379 | 567 | 0 | 2048 | 0 | 448 | -1 |
| N.S. | 1 | 1.00 | 1.95 | 2.92 | 0.00 | 10.56 | 0.00 | 2.31 | -0.01 |
| time (sec) | N/A | 0.302 | 3.199 | 5.639 | 0.000 | 0.563 | 0.000 | 0.563 | 0.000 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 155 | 285 | 0 | 409 | 0 | 294 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 1.60 | 0.00 | 2.30 | 0.00 | 1.65 | -0.01 |
| time (sec) | N/A | 0.304 | 0.419 | 3.925 | 0.000 | 0.380 | 0.000 | 0.540 | 0.000 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 125 | 185 | 0 | 306 | 0 | 199 | -1 |
| N.S. | 1 | 1.00 | 1.02 | 1.50 | 0.00 | 2.49 | 0.00 | 1.62 | -0.01 |
| time (sec) | N/A | 0.135 | 0.269 | 3.482 | 0.000 | 0.350 | 0.000 | 0.581 | 0.000 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 106 | 128 | 0 | 232 | 0 | 149 | 151 |
| N.S. | 1 | 1.00 | 1.34 | 1.62 | 0.00 | 2.94 | 0.00 | 1.89 | 1.91 |
| time (sec) | N/A | 0.050 | 0.148 | 4.682 | 0.000 | 0.350 | 0.000 | 0.605 | 8.962 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 73 | 75 | 0 | 180 | 0 | 117 | 49 |
| N.S. | 1 | 1.00 | 1.55 | 1.60 | 0.00 | 3.83 | 0.00 | 2.49 | 1.04 |
| time (sec) | N/A | 0.015 | 0.038 | 2.224 | 0.000 | 0.381 | 0.000 | 0.525 | 7.718 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 215 | 131 | 0 | 727 | 0 | 206 | -1 |
| N.S. | 1 | 1.00 | 1.75 | 1.07 | 0.00 | 5.91 | 0.00 | 1.67 | -0.01 |
| time (sec) | N/A | 0.143 | 1.296 | 5.179 | 0.000 | 0.428 | 0.000 | 0.511 | 0.000 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 324 | 453 | 0 | 1566 | 0 | 380 | -1 |
| N.S. | 1 | 1.00 | 1.85 | 2.59 | 0.00 | 8.95 | 0.00 | 2.17 | -0.01 |
| time (sec) | N/A | 0.291 | 2.484 | 5.645 | 0.000 | 0.587 | 0.000 | 0.652 | 0.000 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 414 | 1065 | 0 | 2991 | 0 | 585 | -1 |
| N.S. | 1 | 1.00 | 1.68 | 4.31 | 0.00 | 12.11 | 0.00 | 2.37 | -0.00 |
| time (sec) | N/A | 0.505 | 3.411 | 7.176 | 0.000 | 0.785 | 0.000 | 0.674 | 0.000 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 328 | 490 | 0 | 520 | 0 | 376 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 2.55 | 0.00 | 2.71 | 0.00 | 1.96 | -0.01 |
| time (sec) | N/A | 0.313 | 0.385 | 3.321 | 0.000 | 0.383 | 0.000 | 0.573 | 0.000 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 239 | 316 | 0 | 404 | 0 | 277 | -1 |
| N.S. | 1 | 1.00 | 1.73 | 2.29 | 0.00 | 2.93 | 0.00 | 2.01 | -0.01 |
| time (sec) | N/A | 0.144 | 0.230 | 3.508 | 0.000 | 0.346 | 0.000 | 0.531 | 0.000 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 150 | 176 | 0 | 315 | 0 | 192 | -1 |
| N.S. | 1 | 1.00 | 1.72 | 2.02 | 0.00 | 3.62 | 0.00 | 2.21 | -0.01 |
| time (sec) | N/A | 0.051 | 0.145 | 3.083 | 0.000 | 0.340 | 0.000 | 0.545 | 0.000 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 108 | 125 | 0 | 274 | 0 | 140 | -1 |
| N.S. | 1 | 1.00 | 1.40 | 1.62 | 0.00 | 3.56 | 0.00 | 1.82 | -0.01 |
| time (sec) | N/A | 0.031 | 0.128 | 2.405 | 0.000 | 0.350 | 0.000 | 0.541 | 0.000 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 385 | 338 | 0 | 1371 | 0 | 419 | -1 |
| N.S. | 1 | 1.00 | 2.35 | 2.06 | 0.00 | 8.36 | 0.00 | 2.55 | -0.01 |
| time (sec) | N/A | 0.299 | 1.291 | 4.432 | 0.000 | 0.547 | 0.000 | 0.563 | 0.000 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | B | F(-1) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 491 | 978 | 0 | 2603 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.02 | 4.02 | 0.00 | 10.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.497 | 3.186 | 6.056 | 0.000 | 0.820 | 0.000 | 0.000 | 0.000 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 570 | 2222 | 0 | 4237 | 0 | 1036 | -1 |
| N.S. | 1 | 1.00 | 1.79 | 6.99 | 0.00 | 13.32 | 0.00 | 3.26 | -0.00 |
| time (sec) | N/A | 0.767 | 4.489 | 7.872 | 0.000 | 1.327 | 0.000 | 0.808 | 0.000 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 400 | 688 | 0 | 638 | 0 | 420 | -1 |
| N.S. | 1 | 1.00 | 2.06 | 3.55 | 0.00 | 3.29 | 0.00 | 2.16 | -0.01 |
| time (sec) | N/A | 0.317 | 0.502 | 4.737 | 0.000 | 0.356 | 0.000 | 0.622 | 0.000 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 252 | 378 | 0 | 520 | 0 | 309 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 2.57 | 0.00 | 3.54 | 0.00 | 2.10 | -0.01 |
| time (sec) | N/A | 0.158 | 0.354 | 4.721 | 0.000 | 0.352 | 0.000 | 0.532 | 0.000 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 227 | 278 | 0 | 420 | 0 | 246 | -1 |
| N.S. | 1 | 1.00 | 1.80 | 2.21 | 0.00 | 3.33 | 0.00 | 1.95 | -0.01 |
| time (sec) | N/A | 0.071 | 0.249 | 3.870 | 0.000 | 0.366 | 0.000 | 0.605 | 0.000 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 196 | 195 | 0 | 348 | 0 | 161 | -1 |
| N.S. | 1 | 1.00 | 1.83 | 1.82 | 0.00 | 3.25 | 0.00 | 1.50 | -0.01 |
| time (sec) | N/A | 0.043 | 0.116 | 3.868 | 0.000 | 0.362 | 0.000 | 0.539 | 0.000 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 218 | 218 | 501 | 732 | 0 | 2103 | 0 | 619 | -1 |
| N.S. | 1 | 1.00 | 2.30 | 3.36 | 0.00 | 9.65 | 0.00 | 2.84 | -0.00 |
| time (sec) | N/A | 0.505 | 2.261 | 5.592 | 0.000 | 0.710 | 0.000 | 0.666 | 0.000 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 313 | 570 | 1972 | 0 | 3823 | 0 | 936 | -1 |
| N.S. | 1 | 1.00 | 1.82 | 6.30 | 0.00 | 12.21 | 0.00 | 2.99 | -0.00 |
| time (sec) | N/A | 0.750 | 3.901 | 7.968 | 0.000 | 1.237 | 0.000 | 1.140 | 0.000 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | B | F(-1) | B | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 400 | 683 | 3535 | 0 | 6119 | 0 | 1453 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 8.84 | 0.00 | 15.30 | 0.00 | 3.63 | -0.00 |
| time (sec) | N/A | 1.037 | 7.604 | 11.197 | 0.000 | 2.079 | 0.000 | 1.050 | 0.000 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 391 | 0 | 0 | 1311 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.93 | 0.00 | 0.00 | 6.46 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.262 | 2.492 | 180.000 | 0.000 | 0.740 | 0.000 | 0.000 | 0.000 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 365 | 0 | 0 | 1119 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.34 | 0.00 | 0.00 | 7.17 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.186 | 1.271 | 180.000 | 0.000 | 0.638 | 0.000 | 0.000 | 0.000 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 350 | 0 | 0 | 991 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.33 | 0.00 | 0.00 | 9.44 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.116 | 0.901 | 180.000 | 0.000 | 0.639 | 0.000 | 0.000 | 0.000 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 305 | 2707 | 0 | 807 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.00 | 44.38 | 0.00 | 13.23 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.062 | 0.795 | 11.343 | 0.000 | 0.601 | 0.000 | 0.000 | 0.000 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 84 | 99 | 193 | 137 | 0 | 157 | 145 |
| N.S. | 1 | 1.00 | 1.87 | 2.20 | 4.29 | 3.04 | 0.00 | 3.49 | 3.22 |
| time (sec) | N/A | 0.064 | 0.138 | 0.869 | 0.563 | 0.355 | 0.000 | 0.659 | 9.004 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 100 | 222 | 360 | 312 | 0 | 479 | 353 |
| N.S. | 1 | 1.00 | 1.05 | 2.34 | 3.79 | 3.28 | 0.00 | 5.04 | 3.72 |
| time (sec) | N/A | 0.130 | 0.196 | 0.182 | 0.611 | 0.361 | 0.000 | 0.861 | 13.902 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 128 | 430 | 570 | 572 | 0 | 1060 | 501 |
| N.S. | 1 | 1.00 | 0.90 | 3.03 | 4.01 | 4.03 | 0.00 | 7.46 | 3.53 |
| time (sec) | N/A | 0.198 | 0.277 | 0.241 | 0.611 | 0.381 | 0.000 | 1.270 | 16.352 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 318 | 0 | 0 | 1637 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.12 | 0.00 | 0.00 | 5.74 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.378 | 0.949 | 180.000 | 0.000 | 1.108 | 0.000 | 0.000 | 0.000 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 281 | 0 | 0 | 1391 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.23 | 0.00 | 0.00 | 6.10 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.292 | 0.647 | 180.000 | 0.000 | 0.761 | 0.000 | 0.000 | 0.000 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 247 | 0 | 0 | 1177 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.44 | 0.00 | 0.00 | 6.88 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.208 | 0.421 | 180.000 | 0.000 | 0.677 | 0.000 | 0.000 | 0.000 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 301 | 0 | 0 | 1035 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.71 | 0.00 | 0.00 | 9.32 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.140 | 0.408 | 0.295 | 0.000 | 0.716 | 0.000 | 0.000 | 0.000 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 377 | 6627 | 0 | 1351 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.22 | 56.64 | 0.00 | 11.55 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.141 | 5.954 | 9.230 | 0.000 | 0.678 | 0.000 | 0.000 | 0.000 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 104 | 345 | 325 | 335 | 0 | 0 | 387 |
| N.S. | 1 | 1.00 | 0.90 | 3.00 | 2.83 | 2.91 | 0.00 | 0.00 | 3.37 |
| time (sec) | N/A | 0.139 | 0.384 | 0.498 | 0.578 | 0.346 | 0.000 | 0.000 | 14.401 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-2) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 140 | 625 | 529 | 614 | 0 | 0 | 541 |
| N.S. | 1 | 1.00 | 0.81 | 3.63 | 3.08 | 3.57 | 0.00 | 0.00 | 3.15 |
| time (sec) | N/A | 0.222 | 0.606 | 0.276 | 0.601 | 0.382 | 0.000 | 0.000 | 17.483 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 193 | 979 | 780 | 957 | 0 | 0 | 807 |
| N.S. | 1 | 1.00 | 0.84 | 4.28 | 3.41 | 4.18 | 0.00 | 0.00 | 3.52 |
| time (sec) | N/A | 0.292 | 1.019 | 2.229 | 0.643 | 0.396 | 0.000 | 0.000 | 20.213 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 377 | 377 | 395 | 0 | 0 | 2145 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 5.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.570 | 1.894 | 180.000 | 0.000 | 1.329 | 0.000 | 0.000 | 0.000 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 327 | 0 | 0 | 1809 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 5.80 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.475 | 1.169 | 180.000 | 0.000 | 1.116 | 0.000 | 0.000 | 0.000 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 285 | 0 | 0 | 1509 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.18 | 0.00 | 0.00 | 6.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.372 | 0.638 | 180.000 | 0.000 | 0.822 | 0.000 | 0.000 | 0.000 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 256 | 0 | 0 | 1269 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.44 | 0.00 | 0.00 | 7.13 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.287 | 0.533 | 180.000 | 0.000 | 0.698 | 0.000 | 0.000 | 0.000 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 263 | 0 | 0 | 1723 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 0.00 | 0.00 | 9.57 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.291 | 0.711 | 180.000 | 0.000 | 0.723 | 0.000 | 0.000 | 0.000 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 261 | 16223 | 0 | 2363 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.43 | 88.65 | 0.00 | 12.91 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.294 | 6.231 | 0.398 | 0.000 | 0.724 | 0.000 | 0.000 | 0.000 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-2) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 152 | 793 | 486 | 670 | 0 | 0 | 590 |
| N.S. | 1 | 1.00 | 0.80 | 4.20 | 2.57 | 3.54 | 0.00 | 0.00 | 3.12 |
| time (sec) | N/A | 0.324 | 1.558 | 7.179 | 0.670 | 0.394 | 0.000 | 0.000 | 17.457 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 216 | 1223 | 731 | 1053 | 0 | 0 | 862 |
| N.S. | 1 | 1.00 | 0.85 | 4.81 | 2.88 | 4.15 | 0.00 | 0.00 | 3.39 |
| time (sec) | N/A | 0.414 | 2.729 | 7.286 | 0.735 | 0.409 | 0.000 | 0.000 | 20.609 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 304 | 1730 | 1018 | 1516 | 0 | 0 | 1155 |
| N.S. | 1 | 1.00 | 0.96 | 5.46 | 3.21 | 4.78 | 0.00 | 0.00 | 3.64 |
| time (sec) | N/A | 0.510 | 5.208 | 7.506 | 0.837 | 0.429 | 0.000 | 0.000 | 26.142 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 249 | 1893 | 0 | 0 | 3063 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 7.60 | 0.00 | 0.00 | 12.30 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.627 | 15.618 | 180.000 | 0.000 | 0.804 | 0.000 | 0.000 | 0.000 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 1639 | 0 | 0 | 2655 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.72 | 0.00 | 0.00 | 14.12 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.402 | 15.451 | 0.250 | 0.000 | 0.733 | 0.000 | 0.000 | 0.000 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 1251 | 3353 | 0 | 2034 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.87 | 23.78 | 0.00 | 14.43 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.200 | 14.108 | 0.205 | 0.000 | 0.674 | 0.000 | 0.000 | 0.000 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | A | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 283 | 191 | 0 | 499 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.58 | 2.42 | 0.00 | 6.32 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.073 | 2.703 | 10.039 | 0.000 | 0.516 | 0.000 | 0.000 | 0.000 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 306 | 874 | 0 | 1064 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.34 | 6.67 | 0.00 | 8.12 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.168 | 6.680 | 9.531 | 0.000 | 0.587 | 0.000 | 0.000 | 0.000 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 387 | 2572 | 0 | 1915 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 13.47 | 0.00 | 10.03 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.323 | 6.707 | 10.589 | 0.000 | 0.631 | 0.000 | 0.000 | 0.000 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 1844 | 0 | 0 | 3582 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 7.35 | 0.00 | 0.00 | 14.27 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.604 | 17.388 | 0.296 | 0.000 | 0.856 | 0.000 | 0.000 | 0.000 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 1625 | 6675 | 0 | 3037 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.38 | 34.41 | 0.00 | 15.65 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.371 | 17.422 | 0.224 | 0.000 | 0.851 | 0.000 | 0.000 | 0.000 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 372 | 1373 | 0 | 944 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.95 | 10.90 | 0.00 | 7.49 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.146 | 5.275 | 11.169 | 0.000 | 0.532 | 0.000 | 0.000 | 0.000 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 381 | 1268 | 0 | 1056 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.82 | 9.39 | 0.00 | 7.82 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.158 | 5.768 | 9.464 | 0.000 | 0.575 | 0.000 | 0.000 | 0.000 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 401 | 2246 | 0 | 2014 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.04 | 11.40 | 0.00 | 10.22 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.333 | 6.186 | 11.908 | 0.000 | 0.708 | 0.000 | 0.000 | 0.000 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 271 | 478 | 5040 | 0 | 3254 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.76 | 18.60 | 0.00 | 12.01 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.574 | 9.711 | 11.350 | 0.000 | 0.898 | 0.000 | 0.000 | 0.000 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 260 | 1845 | 10732 | 0 | 4041 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 7.10 | 41.28 | 0.00 | 15.54 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.572 | 17.756 | 0.302 | 0.000 | 0.935 | 0.000 | 0.000 | 0.000 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 396 | 3050 | 0 | 1364 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.15 | 16.58 | 0.00 | 7.41 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.346 | 6.944 | 11.342 | 0.000 | 0.583 | 0.000 | 0.000 | 0.000 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 412 | 3050 | 0 | 1534 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.16 | 15.97 | 0.00 | 8.03 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.320 | 6.678 | 11.130 | 0.000 | 0.628 | 0.000 | 0.000 | 0.000 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 411 | 2805 | 0 | 1704 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.04 | 13.96 | 0.00 | 8.48 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.336 | 7.354 | 11.336 | 0.000 | 0.781 | 0.000 | 0.000 | 0.000 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 462 | 4262 | 0 | 3056 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.71 | 15.79 | 0.00 | 11.32 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.587 | 8.943 | 11.621 | 0.000 | 1.024 | 0.000 | 0.000 | 0.000 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-2) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 355 | 717 | 8035 | 0 | 4942 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.02 | 22.63 | 0.00 | 13.92 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.844 | 10.194 | 11.404 | 0.000 | 2.015 | 0.000 | 0.000 | 0.000 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 373 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.122 | 1.574 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 3599 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 11.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.458 | 63.060 | 0.428 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 1774 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 9.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.189 | 71.864 | 0.381 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 275 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.054 | 2.102 | 0.113 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 90 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.022 | 0.168 | 0.028 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 363 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.090 | 1.221 | 0.125 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 363 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.088 | 1.354 | 0.406 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 363 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.088 | 1.500 | 0.488 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 365 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.133 | 1.912 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 365 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.124 | 1.365 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 365 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.115 | 1.173 | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 373 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.117 | 1.232 | 0.063 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 373 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.123 | 1.421 | 0.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 373 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.124 | 1.644 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 111 | 238 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.79 | 3.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.071 | 1.478 | 0.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 122 | 88 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.91 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.079 | 0.525 | 0.136 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 45 | 0 | 37 | 58 | 0 | 45 | 41 |
| N.S. | 1 | 1.00 | 1.61 | 0.00 | 1.32 | 2.07 | 0.00 | 1.61 | 1.46 |
| time (sec) | N/A | 0.014 | 0.045 | 0.128 | 0.511 | 0.335 | 0.000 | 0.644 | 0.432 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 131 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.080 | 0.610 | 0.123 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 167 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 0.614 | 0.121 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 95 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.016 | 0.146 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 182 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.065 | 1.110 | 0.157 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 177 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 0.919 | 0.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 97 | 0 | 0 | 44 | 0 | 0 | 43 |
| N.S. | 1 | 1.00 | 2.26 | 0.00 | 0.00 | 1.02 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.038 | 0.509 | 0.170 | 0.000 | 0.374 | 0.000 | 0.000 | 7.849 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 113 | 176 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.36 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 0.909 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 111 | 246 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.42 | 3.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.064 | 1.608 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 115 | 240 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.42 | 2.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.073 | 0.606 | 0.127 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 118 | 90 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.42 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.073 | 0.531 | 0.153 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 104 | 0 | 40 | 47 | 0 | 815 | 52 |
| N.S. | 1 | 1.00 | 2.67 | 0.00 | 1.03 | 1.21 | 0.00 | 20.90 | 1.33 |
| time (sec) | N/A | 0.013 | 5.218 | 0.138 | 0.509 | 0.347 | 0.000 | 0.723 | 0.426 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 118 | 179 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.42 | 2.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 0.614 | 0.139 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 117 | 166 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.44 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 0.687 | 0.119 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 97 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.028 | 0.175 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 118 | 184 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.64 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.086 | 0.848 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 116 | 179 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.51 | 2.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.070 | 0.667 | 0.160 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 99 | 0 | 0 | 46 | 0 | 0 | 45 |
| N.S. | 1 | 1.00 | 2.20 | 0.00 | 0.00 | 1.02 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.041 | 0.601 | 0.168 | 0.000 | 0.356 | 0.000 | 0.000 | 0.371 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 178 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.070 | 0.676 | 0.144 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 248 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 0.602 | 0.148 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 113 | 247 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.57 | 3.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.067 | 1.646 | 0.130 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 116 | 154 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.51 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.080 | 0.664 | 0.132 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 110 | 0 | 0 | 46 | 0 | 0 | 45 |
| N.S. | 1 | 1.00 | 2.44 | 0.00 | 0.00 | 1.02 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.040 | 0.601 | 0.156 | 0.000 | 0.349 | 0.000 | 0.000 | 7.829 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 155 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.075 | 0.679 | 0.145 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 155 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.067 | 0.767 | 0.132 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 97 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.035 | 0.168 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 131 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 0.948 | 0.157 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 186 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.071 | 1.227 | 0.161 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 106 | 0 | 47 | 47 | 0 | 815 | 52 |
| N.S. | 1 | 1.00 | 2.72 | 0.00 | 1.21 | 1.21 | 0.00 | 20.90 | 1.33 |
| time (sec) | N/A | 0.013 | 0.497 | 0.153 | 0.563 | 0.348 | 0.000 | 0.760 | 0.394 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 187 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 1.237 | 0.174 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 241 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.072 | 1.597 | 0.175 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 194 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.135 | 1.611 | 0.124 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 187 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.112 | 1.473 | 0.135 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.084 | 28.063 | 0.375 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.070 | 12.469 | 0.311 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.056 | 5.702 | 0.140 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 120 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.043 | 0.303 | 0.024 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 2.190 | 0.114 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 5.060 | 0.415 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 8.812 | 0.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 190 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.330 | 28.209 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 133 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.156 | 4.294 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.065 | 4.337 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 129 | 236 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.30 | 2.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.118 | 3.709 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 130 | 319 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.25 | 3.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.118 | 4.592 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 137 | 414 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.32 | 3.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.124 | 7.531 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 1736 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 16.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 6.445 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 886 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.066 | 6.292 | 0.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 942 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.070 | 6.486 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 143 | 182 | 189 | 150 | 386 | 152 | 183 |
| N.S. | 1 | 1.00 | 0.84 | 1.06 | 1.11 | 0.88 | 2.26 | 0.89 | 1.07 |
| time (sec) | N/A | 0.162 | 0.804 | 0.318 | 0.308 | 0.365 | 0.256 | 0.480 | 8.080 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 90 | 115 | 121 | 94 | 199 | 96 | 108 |
| N.S. | 1 | 1.00 | 0.85 | 1.08 | 1.14 | 0.89 | 1.88 | 0.91 | 1.02 |
| time (sec) | N/A | 0.074 | 0.361 | 0.154 | 0.298 | 0.365 | 0.178 | 0.464 | 7.894 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 52 | 59 | 62 | 51 | 94 | 48 | 52 |
| N.S. | 1 | 1.00 | 0.98 | 1.11 | 1.17 | 0.96 | 1.77 | 0.91 | 0.98 |
| time (sec) | N/A | 0.017 | 0.121 | 0.089 | 0.315 | 0.355 | 0.095 | 0.446 | 7.720 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 27 | 17 | 17 | 19 | 19 | 17 | 25 |
| N.S. | 1 | 1.00 | 1.69 | 1.06 | 1.06 | 1.19 | 1.19 | 1.06 | 1.56 |
| time (sec) | N/A | 0.007 | 0.009 | 0.043 | 0.286 | 0.365 | 0.046 | 0.436 | 7.636 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 67 | 76 | 0 | 264 | 537 | 86 | 342 |
| N.S. | 1 | 1.00 | 1.03 | 1.17 | 0.00 | 4.06 | 8.26 | 1.32 | 5.26 |
| time (sec) | N/A | 0.067 | 0.162 | 0.164 | 0.000 | 0.381 | 41.058 | 0.445 | 9.683 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 96 | 142 | 0 | 409 | 0 | 158 | 214 |
| N.S. | 1 | 1.00 | 0.98 | 1.45 | 0.00 | 4.17 | 0.00 | 1.61 | 2.18 |
| time (sec) | N/A | 0.082 | 0.444 | 0.219 | 0.000 | 0.357 | 0.000 | 0.494 | 7.920 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 157 | 351 | 0 | 816 | 0 | 428 | 477 |
| N.S. | 1 | 1.00 | 0.96 | 2.14 | 0.00 | 4.98 | 0.00 | 2.61 | 2.91 |
| time (sec) | N/A | 0.161 | 0.716 | 0.372 | 0.000 | 0.403 | 0.000 | 0.454 | 9.608 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 314 | 249 | 325 | 338 | 253 | 729 | 274 | 358 |
| N.S. | 1 | 1.00 | 0.79 | 1.04 | 1.08 | 0.81 | 2.32 | 0.87 | 1.14 |
| time (sec) | N/A | 0.398 | 1.463 | 0.428 | 0.309 | 0.369 | 0.477 | 0.485 | 8.444 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 217 | 160 | 216 | 224 | 168 | 459 | 176 | 221 |
| N.S. | 1 | 1.00 | 0.74 | 1.00 | 1.03 | 0.77 | 2.12 | 0.81 | 1.02 |
| time (sec) | N/A | 0.209 | 0.832 | 0.275 | 0.290 | 0.348 | 0.259 | 0.525 | 8.140 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 90 | 115 | 121 | 93 | 199 | 96 | 108 |
| N.S. | 1 | 1.00 | 0.84 | 1.07 | 1.13 | 0.87 | 1.86 | 0.90 | 1.01 |
| time (sec) | N/A | 0.069 | 0.338 | 0.139 | 0.294 | 0.354 | 0.146 | 0.434 | 7.765 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 46 | 51 | 49 | 48 | 78 | 45 | 44 |
| N.S. | 1 | 1.00 | 0.92 | 1.02 | 0.98 | 0.96 | 1.56 | 0.90 | 0.88 |
| time (sec) | N/A | 0.014 | 0.116 | 0.091 | 0.286 | 0.352 | 0.084 | 0.459 | 7.611 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 89 | 119 | 0 | 386 | 4032 | 134 | 2629 |
| N.S. | 1 | 1.00 | 0.96 | 1.28 | 0.00 | 4.15 | 43.35 | 1.44 | 28.27 |
| time (sec) | N/A | 0.146 | 0.254 | 0.212 | 0.000 | 0.379 | 178.525 | 0.565 | 12.432 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 134 | 218 | 0 | 694 | 0 | 249 | 2500 |
| N.S. | 1 | 1.00 | 1.04 | 1.69 | 0.00 | 5.38 | 0.00 | 1.93 | 19.38 |
| time (sec) | N/A | 0.187 | 0.646 | 0.306 | 0.000 | 0.404 | 0.000 | 0.504 | 15.469 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 202 | 467 | 0 | 1050 | 0 | 609 | 641 |
| N.S. | 1 | 1.00 | 1.03 | 2.38 | 0.00 | 5.36 | 0.00 | 3.11 | 3.27 |
| time (sec) | N/A | 0.223 | 0.999 | 0.506 | 0.000 | 0.410 | 0.000 | 0.491 | 10.265 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 346 | 933 | 0 | 1753 | 0 | 1316 | 1220 |
| N.S. | 1 | 1.00 | 1.13 | 3.06 | 0.00 | 5.75 | 0.00 | 4.31 | 4.00 |
| time (sec) | N/A | 0.407 | 1.485 | 0.906 | 0.000 | 0.436 | 0.000 | 0.583 | 11.155 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 493 | 552 | 489 | 513 | 382 | 1217 | 416 | 574 |
| N.S. | 1 | 1.23 | 1.38 | 1.22 | 1.28 | 0.96 | 3.04 | 1.04 | 1.44 |
| time (sec) | N/A | 0.660 | 1.203 | 0.700 | 0.318 | 0.387 | 0.630 | 0.717 | 8.916 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 315 | 315 | 246 | 325 | 338 | 259 | 729 | 274 | 358 |
| N.S. | 1 | 1.00 | 0.78 | 1.03 | 1.07 | 0.82 | 2.31 | 0.87 | 1.14 |
| time (sec) | N/A | 0.334 | 1.603 | 0.421 | 0.308 | 0.365 | 0.460 | 0.477 | 8.521 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 142 | 182 | 189 | 153 | 386 | 152 | 183 |
| N.S. | 1 | 1.00 | 0.83 | 1.06 | 1.11 | 0.89 | 2.26 | 0.89 | 1.07 |
| time (sec) | N/A | 0.150 | 0.724 | 0.299 | 0.295 | 0.380 | 0.246 | 0.455 | 7.953 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 71 | 76 | 79 | 75 | 128 | 75 | 127 |
| N.S. | 1 | 1.00 | 0.79 | 0.84 | 0.88 | 0.83 | 1.42 | 0.83 | 1.41 |
| time (sec) | N/A | 0.052 | 0.198 | 0.136 | 0.327 | 0.356 | 0.127 | 0.437 | 7.686 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 137 | 229 | 0 | 593 | 0 | 252 | 2500 |
| N.S. | 1 | 1.00 | 0.88 | 1.47 | 0.00 | 3.80 | 0.00 | 1.62 | 16.03 |
| time (sec) | N/A | 0.281 | 0.386 | 0.259 | 0.000 | 0.407 | 0.000 | 0.437 | 14.561 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 152 | 302 | 0 | 1081 | 0 | 585 | 2500 |
| N.S. | 1 | 1.00 | 0.73 | 1.45 | 0.00 | 5.20 | 0.00 | 2.81 | 12.02 |
| time (sec) | N/A | 0.362 | 1.158 | 0.421 | 0.000 | 0.418 | 0.000 | 0.487 | 17.637 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 521 | 670 | 0 | 1732 | 0 | 889 | 2500 |
| N.S. | 1 | 1.00 | 2.04 | 2.63 | 0.00 | 6.79 | 0.00 | 3.49 | 9.80 |
| time (sec) | N/A | 0.460 | 2.504 | 0.761 | 0.000 | 0.480 | 0.000 | 0.544 | 20.458 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 345 | 1154 | 0 | 2165 | 0 | 1677 | 1423 |
| N.S. | 1 | 1.00 | 1.06 | 3.55 | 0.00 | 6.66 | 0.00 | 5.16 | 4.38 |
| time (sec) | N/A | 0.532 | 5.333 | 1.261 | 0.000 | 0.489 | 0.000 | 0.539 | 11.797 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 52 | 67 | 0 | 163 | 180 | 73 | 94 |
| N.S. | 1 | 1.00 | 0.96 | 1.24 | 0.00 | 3.02 | 3.33 | 1.35 | 1.74 |
| time (sec) | N/A | 0.064 | 0.084 | 0.125 | 0.000 | 0.364 | 25.276 | 0.486 | 8.125 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 6 | 6 | 6 | 7 | 0 | 6 | 3 | 6 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 1.17 | 0.00 | 1.00 | 0.50 | 1.00 | 1.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.053 | 0.000 | 0.319 | 0.140 | 0.441 | 7.701 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 36 | 0 | 12 | 0 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 3.00 | 0.00 | 1.00 | 0.00 | 2.67 | 2.00 |
| time (sec) | N/A | 0.021 | 0.039 | 0.115 | 0.000 | 0.335 | 0.000 | 0.430 | 7.866 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 28 | 28 | 36 | 27 | 42 | 51 | 36 |
| N.S. | 1 | 1.00 | 0.82 | 0.82 | 1.06 | 0.79 | 1.24 | 1.50 | 1.06 |
| time (sec) | N/A | 0.026 | 0.038 | 0.073 | 0.531 | 0.343 | 0.280 | 0.544 | 7.830 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 203 | 375 | 0 | 812 | 0 | 465 | 2500 |
| N.S. | 1 | 1.00 | 0.86 | 1.60 | 0.00 | 3.46 | 0.00 | 1.98 | 10.64 |
| time (sec) | N/A | 0.483 | 0.659 | 0.299 | 0.000 | 0.414 | 0.000 | 0.477 | 16.486 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 138 | 229 | 0 | 581 | 0 | 252 | 2500 |
| N.S. | 1 | 1.00 | 0.88 | 1.47 | 0.00 | 3.72 | 0.00 | 1.62 | 16.03 |
| time (sec) | N/A | 0.261 | 0.390 | 0.267 | 0.000 | 0.395 | 0.000 | 0.497 | 14.771 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 90 | 117 | 0 | 379 | 4032 | 134 | 2628 |
| N.S. | 1 | 1.00 | 0.97 | 1.26 | 0.00 | 4.08 | 43.35 | 1.44 | 28.26 |
| time (sec) | N/A | 0.127 | 0.183 | 0.204 | 0.000 | 0.377 | 186.310 | 0.457 | 12.623 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 67 | 76 | 0 | 262 | 537 | 86 | 343 |
| N.S. | 1 | 1.00 | 1.03 | 1.17 | 0.00 | 4.03 | 8.26 | 1.32 | 5.28 |
| time (sec) | N/A | 0.051 | 0.117 | 0.146 | 0.000 | 0.403 | 41.162 | 0.476 | 10.009 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 47 | 47 | 0 | 199 | 185 | 62 | 42 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.00 | 4.23 | 3.94 | 1.32 | 0.89 |
| time (sec) | N/A | 0.024 | 0.040 | 0.095 | 0.000 | 0.382 | 4.845 | 0.534 | 7.793 |

| Problem 703 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 104 | 114 | 0 | 1093 | 0 | 145 | 2500 |
| N.S. | 1 | 1.00 | 0.89 | 0.97 | 0.00 | 9.34 | 0.00 | 1.24 | 21.37 |
| time (sec) | N/A | 0.113 | 0.190 | 0.452 | 0.000 | 1.212 | 0.000 | 0.467 | 9.989 |

| Problem 704 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 165 | 234 | 0 | 2934 | 0 | 308 | 2500 |
| N.S. | 1 | 1.00 | 0.89 | 1.26 | 0.00 | 15.86 | 0.00 | 1.66 | 13.51 |
| time (sec) | N/A | 0.329 | 1.003 | 1.180 | 0.000 | 93.131 | 0.000 | 0.503 | 22.739 |

| Problem 705 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 284 | 263 | 625 | 0 | 0 | 0 | 786 | 2500 |
| N.S. | 1 | 1.00 | 0.93 | 2.20 | 0.00 | 0.00 | 0.00 | 2.77 | 8.80 |
| time (sec) | N/A | 0.734 | 2.384 | 3.648 | 0.000 | 0.000 | 0.000 | 0.493 | 30.300 |

| Problem 706 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 199 | 456 | 0 | 1472 | 0 | 517 | 2500 |
| N.S. | 1 | 1.00 | 0.65 | 1.49 | 0.00 | 4.81 | 0.00 | 1.69 | 8.17 |
| time (sec) | N/A | 0.664 | 2.096 | 0.518 | 0.000 | 0.443 | 0.000 | 0.499 | 20.452 |

| Problem 707 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 151 | 303 | 0 | 1025 | 0 | 579 | 2500 |
| N.S. | 1 | 1.00 | 0.74 | 1.48 | 0.00 | 5.00 | 0.00 | 2.82 | 12.20 |
| time (sec) | N/A | 0.331 | 1.161 | 0.398 | 0.000 | 0.451 | 0.000 | 0.459 | 17.831 |

| Problem 708 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 133 | 220 | 0 | 682 | 0 | 249 | 2500 |
| N.S. | 1 | 1.00 | 1.03 | 1.71 | 0.00 | 5.29 | 0.00 | 1.93 | 19.38 |
| time (sec) | N/A | 0.162 | 0.606 | 0.305 | 0.000 | 0.405 | 0.000 | 0.613 | 15.469 |

| Problem 709 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 96 | 144 | 0 | 414 | 0 | 157 | 215 |
| N.S. | 1 | 1.00 | 0.99 | 1.48 | 0.00 | 4.27 | 0.00 | 1.62 | 2.22 |
| time (sec) | N/A | 0.068 | 0.346 | 0.214 | 0.000 | 0.377 | 0.000 | 0.483 | 8.021 |

| Problem 710 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 82 | 122 | 0 | 351 | 0 | 127 | 174 |
| N.S. | 1 | 1.00 | 0.99 | 1.47 | 0.00 | 4.23 | 0.00 | 1.53 | 2.10 |
| time (sec) | N/A | 0.046 | 0.213 | 0.181 | 0.000 | 0.376 | 0.000 | 0.473 | 8.240 |

| Problem 711 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 178 | 236 | 0 | 2923 | 0 | 304 | 2500 |
| N.S. | 1 | 1.00 | 0.98 | 1.30 | 0.00 | 16.15 | 0.00 | 1.68 | 13.81 |
| time (sec) | N/A | 0.306 | 0.893 | 1.227 | 0.000 | 77.603 | 0.000 | 0.492 | 22.797 |

| Problem 712 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 290 | 290 | 227 | 331 | 0 | 0 | 0 | 1005 | 2500 |
| N.S. | 1 | 1.00 | 0.78 | 1.14 | 0.00 | 0.00 | 0.00 | 3.47 | 8.62 |
| time (sec) | N/A | 0.796 | 3.036 | 3.312 | 0.000 | 0.000 | 0.000 | 0.700 | 30.890 |

| Problem 713 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 458 | 458 | 346 | 760 | 0 | 0 | 0 | 1109 | 2500 |
| N.S. | 1 | 1.00 | 0.76 | 1.66 | 0.00 | 0.00 | 0.00 | 2.42 | 5.46 |
| time (sec) | N/A | 1.585 | 6.925 | 9.087 | 0.000 | 0.000 | 0.000 | 0.572 | 45.339 |

| Problem 714 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 341 | 1123 | 0 | 3203 | 0 | 3162 | 2500 |
| N.S. | 1 | 1.00 | 0.64 | 2.10 | 0.00 | 6.00 | 0.00 | 5.92 | 4.68 |
| time (sec) | N/A | 1.441 | 4.063 | 1.159 | 0.000 | 0.528 | 0.000 | 0.600 | 25.760 |

| Problem 715 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 894 | 854 | 0 | 2362 | 0 | 1159 | 2500 |
| N.S. | 1 | 1.00 | 2.81 | 2.69 | 0.00 | 7.43 | 0.00 | 3.64 | 7.86 |
| time (sec) | N/A | 0.656 | 4.314 | 0.967 | 0.000 | 0.480 | 0.000 | 0.508 | 21.743 |

| Problem 716 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 248 | 524 | 671 | 0 | 1656 | 0 | 887 | 2500 |
| N.S. | 1 | 1.00 | 2.11 | 2.71 | 0.00 | 6.68 | 0.00 | 3.58 | 10.08 |
| time (sec) | N/A | 0.542 | 2.519 | 0.675 | 0.000 | 0.431 | 0.000 | 0.520 | 20.932 |

| Problem 717 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 204 | 465 | 0 | 1048 | 0 | 609 | 641 |
| N.S. | 1 | 1.00 | 1.04 | 2.37 | 0.00 | 5.35 | 0.00 | 3.11 | 3.27 |
| time (sec) | N/A | 0.187 | 1.017 | 0.516 | 0.000 | 0.408 | 0.000 | 0.496 | 10.443 |

| Problem 718 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 157 | 349 | 0 | 822 | 0 | 429 | 477 |
| N.S. | 1 | 1.00 | 0.97 | 2.15 | 0.00 | 5.07 | 0.00 | 2.65 | 2.94 |
| time (sec) | N/A | 0.119 | 0.680 | 0.374 | 0.000 | 0.403 | 0.000 | 0.589 | 9.793 |

| Problem 719 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 114 | 282 | 0 | 641 | 0 | 284 | 395 |
| N.S. | 1 | 1.00 | 0.87 | 2.15 | 0.00 | 4.89 | 0.00 | 2.17 | 3.02 |
| time (sec) | N/A | 0.079 | 0.429 | 0.289 | 0.000 | 0.365 | 0.000 | 0.457 | 10.150 |

| Problem 720 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 275 | 625 | 0 | 0 | 0 | 788 | 2500 |
| N.S. | 1 | 1.00 | 0.96 | 2.19 | 0.00 | 0.00 | 0.00 | 2.76 | 8.77 |
| time (sec) | N/A | 0.706 | 2.382 | 3.660 | 0.000 | 0.000 | 0.000 | 0.527 | 30.173 |

| Problem 721 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 454 | 454 | 346 | 736 | 0 | 0 | 0 | 1111 | 2500 |
| N.S. | 1 | 1.00 | 0.76 | 1.62 | 0.00 | 0.00 | 0.00 | 2.45 | 5.51 |
| time (sec) | N/A | 1.596 | 5.904 | 9.075 | 0.000 | 0.000 | 0.000 | 0.588 | 43.673 |

| Problem 722 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 669 | 669 | 1815 | 1157 | 0 | 0 | 0 | 7128 | 2500 |
| N.S. | 1 | 1.00 | 2.71 | 1.73 | 0.00 | 0.00 | 0.00 | 10.65 | 3.74 |
| time (sec) | N/A | 2.189 | 8.655 | 20.345 | 0.000 | 0.000 | 0.000 | 4.512 | 80.303 |

| Problem 723 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 275 | 1839 | 0 | 592 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 6.17 | 0.00 | 1.99 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.329 | 1.175 | 7.565 | 0.000 | 0.217 | 0.000 | 0.000 | 0.000 |

| Problem 724 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 218 | 1449 | 0 | 521 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.93 | 6.17 | 0.00 | 2.22 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.242 | 0.813 | 7.012 | 0.000 | 0.156 | 0.000 | 0.000 | 0.000 |

| Problem 725 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 152 | 862 | 0 | 453 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.76 | 0.00 | 2.50 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.140 | 0.677 | 5.785 | 0.000 | 0.123 | 0.000 | 0.000 | 0.000 |

| Problem 726 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | C | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 101 | 243 | 0 | 391 | 0 | 0 | 176 |
| N.S. | 1 | 1.00 | 0.72 | 1.74 | 0.00 | 2.79 | 0.00 | 0.00 | 1.26 |
| time (sec) | N/A | 0.083 | 2.527 | 6.927 | 0.000 | 0.118 | 0.000 | 0.000 | 8.580 |

| Problem 727 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 159 | 567 | 0 | 629 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 2.91 | 0.00 | 3.23 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.150 | 0.586 | 15.131 | 0.000 | 0.190 | 0.000 | 0.000 | 0.000 |

| Problem 728 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 199 | 887 | 0 | 1047 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.70 | 3.11 | 0.00 | 3.67 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.261 | 1.667 | 24.473 | 0.000 | 0.226 | 0.000 | 0.000 | 0.000 |

| Problem 729 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 369 | 297 | 1049 | 0 | 1601 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.80 | 2.84 | 0.00 | 4.34 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.350 | 3.287 | 33.275 | 0.000 | 0.315 | 0.000 | 0.000 | 0.000 |

| Problem 730 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 451 | 451 | 382 | 2112 | 0 | 803 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 4.68 | 0.00 | 1.78 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.620 | 1.877 | 33.132 | 0.000 | 0.191 | 0.000 | 0.000 | 0.000 |

| Problem 731 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 347 | 347 | 292 | 1575 | 0 | 684 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.54 | 0.00 | 1.97 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.412 | 1.256 | 25.848 | 0.000 | 0.191 | 0.000 | 0.000 | 0.000 |

| Problem 732 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 214 | 1100 | 0 | 575 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 4.33 | 0.00 | 2.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.266 | 0.972 | 19.710 | 0.000 | 0.181 | 0.000 | 0.000 | 0.000 |

| Problem 733 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 173 | 695 | 0 | 485 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 3.42 | 0.00 | 2.39 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.179 | 0.944 | 13.161 | 0.000 | 0.135 | 0.000 | 0.000 | 0.000 |

| Problem 734 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 172 | 888 | 0 | 810 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 3.89 | 0.00 | 3.55 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.199 | 0.930 | 17.312 | 0.000 | 0.195 | 0.000 | 0.000 | 0.000 |

| Problem 735 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 329 | 302 | 1043 | 0 | 1422 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 3.17 | 0.00 | 4.32 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.315 | 2.664 | 26.653 | 0.000 | 0.251 | 0.000 | 0.000 | 0.000 |

| Problem 736 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 460 | 460 | 424 | 1450 | 0 | 2321 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 3.15 | 0.00 | 5.05 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.565 | 5.360 | 42.309 | 0.000 | 0.354 | 0.000 | 0.000 | 0.000 |

| Problem 737 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 642 | 642 | 545 | 2728 | 0 | 1066 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 4.25 | 0.00 | 1.66 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.918 | 2.737 | 43.207 | 0.000 | 0.250 | 0.000 | 0.000 | 0.000 |

| Problem 738 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 496 | 496 | 410 | 2112 | 0 | 895 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 4.26 | 0.00 | 1.80 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.673 | 2.512 | 32.959 | 0.000 | 0.224 | 0.000 | 0.000 | 0.000 |

| Problem 739 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 375 | 375 | 306 | 1561 | 0 | 743 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 4.16 | 0.00 | 1.98 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.436 | 1.510 | 26.806 | 0.000 | 0.198 | 0.000 | 0.000 | 0.000 |

| Problem 740 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 302 | 219 | 1085 | 0 | 605 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.73 | 3.59 | 0.00 | 2.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.310 | 1.318 | 18.388 | 0.000 | 0.185 | 0.000 | 0.000 | 0.000 |

| Problem 741 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 361 | 311 | 1398 | 0 | 1083 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.86 | 3.87 | 0.00 | 3.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.411 | 2.152 | 23.587 | 0.000 | 0.200 | 0.000 | 0.000 | 0.000 |

| Problem 742 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 391 | 391 | 357 | 1379 | 0 | 1914 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 3.53 | 0.00 | 4.90 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.492 | 3.965 | 31.486 | 0.000 | 0.329 | 0.000 | 0.000 | 0.000 |

| Problem 743 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 532 | 532 | 584 | 1621 | 0 | 3078 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 3.05 | 0.00 | 5.79 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.729 | 5.546 | 43.820 | 0.000 | 0.621 | 0.000 | 0.000 | 0.000 |

| Problem 744 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | C | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 716 | 716 | 1127 | 2111 | 0 | 4727 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.57 | 2.95 | 0.00 | 6.60 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.954 | 7.114 | 68.682 | 0.000 | 0.862 | 0.000 | 0.000 | 0.000 |

| Problem 745 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 296 | 606 | 1190 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.05 | 4.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.720 | 25.584 | 19.822 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 746 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 242 | 391 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.299 | 13.183 | 7.458 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 747 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 114 | 181 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.210 | 2.373 | 6.856 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 748 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 74 | 151 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.137 | 0.125 | 5.245 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 749 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 617 | 610 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.80 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.403 | 26.710 | 19.352 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 750 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 399 | 1079 | 1072 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.70 | 2.69 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.072 | 26.968 | 31.611 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 751 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1109 | 1886 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.08 | 3.53 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.314 | 27.670 | 32.533 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 752 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 390 | 986 | 1363 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.53 | 3.49 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.799 | 27.442 | 28.559 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 753 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 891 | 1027 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.54 | 2.93 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.669 | 26.919 | 25.028 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 754 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 307 | 846 | 872 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.76 | 2.84 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.554 | 26.736 | 24.785 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 755 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 871 | 690 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.68 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.619 | 26.888 | 17.103 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 756 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 449 | 1057 | 1266 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.35 | 2.82 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.065 | 27.787 | 30.924 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 757 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 661 | 661 | 1319 | 1731 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.00 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.836 | 28.481 | 57.441 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 758 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 816 | 816 | 1526 | 2775 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.87 | 3.40 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.049 | 28.428 | 65.430 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 759 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 605 | 605 | 1323 | 2237 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.19 | 3.70 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.421 | 28.419 | 52.882 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 760 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 549 | 549 | 1149 | 1888 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.09 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.332 | 27.779 | 47.299 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 761 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 472 | 472 | 1001 | 1718 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.12 | 3.64 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.128 | 26.943 | 45.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 762 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 487 | 487 | 1038 | 1525 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.13 | 3.13 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.017 | 26.965 | 47.593 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 763 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 503 | 1069 | 867 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.13 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.062 | 26.953 | 27.932 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 764 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 682 | 682 | 1318 | 2099 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.93 | 3.08 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.772 | 28.792 | 59.789 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 765 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 888 | 888 | 1978 | 409142 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.23 | 460.75 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.218 | 7.215 | 30.361 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 766 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 784 | 784 | 1879 | 278217 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.40 | 354.87 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.255 | 9.638 | 15.300 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 767 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 628 | 628 | 228392 | 146664 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 363.68 | 233.54 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.826 | 33.264 | 13.201 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 768 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 197 | 248426 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 1254.68 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.069 | 0.151 | 21.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 769 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 409 | 263 | 46828 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.64 | 114.49 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.301 | 5.481 | 11.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 770 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 489 | 489 | 2067 | 196467 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.23 | 401.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.553 | 6.291 | 14.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 771 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1080 | 1080 | 2091 | 576490 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 533.79 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.500 | 6.988 | 50.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 772 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 870 | 870 | 1952 | 408713 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.24 | 469.79 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.150 | 6.245 | 29.431 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 773 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 740 | 740 | 1879 | 277278 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.54 | 374.70 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.470 | 9.010 | 15.299 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 774 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 644 | 644 | 222963 | 529026 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 346.22 | 821.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.988 | 33.889 | 23.920 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 775 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 600 | 600 | 1896 | 2620757 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.16 | 4367.93 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.596 | 8.924 | 39.338 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 776 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 497 | 2012 | 190877 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.05 | 384.06 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.630 | 6.249 | 14.204 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 777 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1295 | 1295 | 2276 | 753715 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.76 | 582.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 5.420 | 7.533 | 72.555 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 778 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1071 | 1071 | 2091 | 576487 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.95 | 538.27 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.334 | 6.844 | 47.412 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 779 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 894 | 894 | 1979 | 404969 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.21 | 452.99 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.147 | 6.684 | 27.798 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 780 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 745 | 745 | 1894 | 729396 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.54 | 979.06 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.421 | 9.420 | 44.536 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 781 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 780 | 780 | 2006 | 3432645 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.57 | 4400.83 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.640 | 6.515 | 53.377 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 782 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 737 | 737 | 2169 | 4937517 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.94 | 6699.48 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.180 | 6.658 | 79.849 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 783 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 772 | 772 | 1894 | 731219 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.45 | 947.17 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.521 | 9.552 | 40.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 784 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 644 | 644 | 222963 | 544053 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 346.22 | 844.80 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.012 | 34.967 | 19.259 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 785 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 197 | 247463 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 1249.81 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 0.176 | 14.830 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 786 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 191 | 1236 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 6.44 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.072 | 0.149 | 12.994 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 787 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 405 | 405 | 90261 | 41919 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 222.87 | 103.50 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.296 | 33.727 | 11.455 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 788 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 521 | 521 | 2102 | 219146 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.03 | 420.63 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.656 | 6.343 | 14.416 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 789 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 822 | 822 | 2005 | 3899958 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.44 | 4744.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.731 | 6.550 | 58.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 790 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 600 | 600 | 1896 | 2945157 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.16 | 4908.60 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.559 | 8.947 | 40.852 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 791 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 409 | 226 | 46828 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.55 | 114.49 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.295 | 3.352 | 12.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 792 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 405 | 405 | 90261 | 40621 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 222.87 | 100.30 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.296 | 36.001 | 12.916 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 793 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 495 | 495 | 2082 | 119964 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.21 | 242.35 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.606 | 6.618 | 12.807 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 794 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 681 | 681 | 2350 | 414384 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.45 | 608.49 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.811 | 7.212 | 15.955 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 795 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 736 | 736 | 2172 | 5973124 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.95 | 8115.66 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.202 | 6.725 | 89.326 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 796 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 497 | 2012 | 194980 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.05 | 392.31 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.642 | 6.313 | 13.498 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 797 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 489 | 489 | 2067 | 197175 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.23 | 403.22 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.566 | 6.352 | 14.452 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 798 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 516 | 516 | 2102 | 242899 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 4.07 | 470.73 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.673 | 6.363 | 13.840 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 799 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 688 | 688 | 2352 | 438221 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 3.42 | 636.95 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.232 | 7.359 | 18.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 800 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 941 | 941 | 2669 | 1120519 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.84 | 1190.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.051 | 7.942 | 36.367 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 801 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.030 | 2.275 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 802 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.299 | 9.404 | 0.399 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 803 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 200 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.135 | 0.380 | 0.121 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 804 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 120 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.163 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 805 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.035 | 1.280 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 806 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.034 | 2.122 | 0.375 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 807 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.034 | 7.106 | 0.517 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 808 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 31.862 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 809 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.046 | 8.427 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 810 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.042 | 0.225 | 0.063 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 811 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.043 | 3.607 | 0.064 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 812 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.048 | 4.369 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 813 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.050 | 6.625 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 814 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 272 | 28213 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 103.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.319 | 24.926 | 0.518 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 815 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 346 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.169 | 1.293 | 0.396 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 816 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 278 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.105 | 1.205 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 817 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.163 | 1.369 | 0.148 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 818 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.301 | 1.923 | 0.549 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 819 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 2967 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 26.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.159 | 14.002 | 0.123 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 820 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 297 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.341 | 0.900 | 0.529 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 821 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 222 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.172 | 0.436 | 0.551 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 822 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 307 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.080 | 1.118 | 0.124 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 823 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 157 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.158 | 0.177 | 0.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 824 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 288 | 195 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.324 | 1.862 | 0.483 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 825 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 167 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.384 | 0.383 | 0.560 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 826 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 135 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.176 | 0.250 | 0.528 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 827 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 105 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.104 | 0.144 | 0.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 828 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 1665 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.273 | 14.992 | 0.151 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 829 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 1872 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.374 | 16.418 | 0.509 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 830 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 2406 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 5.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.475 | 16.667 | 0.665 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 831 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.073 | 1.619 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 832 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 303 | 230 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.94 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.376 | 0.689 | 0.507 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 833 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 221 | 152 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.96 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.162 | 0.231 | 0.536 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 834 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 129 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.083 | 0.163 | 0.110 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 835 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 1808 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 8.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.231 | 15.733 | 0.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 836 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 1970 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 6.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.348 | 16.890 | 0.546 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 837 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 428 | 428 | 2570 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 6.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.461 | 15.952 | 0.583 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [219] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 9 | 4 | 1.00 | 21 | 0.190 |
| 2 | A | 13 | 4 | 1.00 | 21 | 0.190 |
| 3 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 4 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 5 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 6 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 7 | A | 1 | 1 | 1.00 | 8 | 0.125 |
| 8 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 9 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 10 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 11 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 12 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 13 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 14 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 15 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 16 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 17 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 18 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 19 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 20 | A | 7 | 6 | 1.09 | 13 | 0.462 |
| 21 | A | 8 | 6 | 1.00 | 13 | 0.462 |
| 22 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 23 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 24 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 25 | A | 3 | 3 | 1.00 | 13 | 0.231 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 26 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 27 | A | 3 | 2 | 1.00 | 8 | 0.250 |
| 28 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 29 | A | 7 | 6 | 1.00 | 13 | 0.462 |
| 30 | A | 8 | 7 | 1.00 | 13 | 0.538 |
| 31 | A | 8 | 6 | 1.00 | 13 | 0.462 |
| 32 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 33 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 34 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 35 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 36 | A | 1 | 1 | 1.00 | 14 | 0.071 |
| 37 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 38 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 39 | A | 4 | 3 | 1.00 | 23 | 0.130 |
| 40 | A | 5 | 3 | 1.00 | 23 | 0.130 |
| 41 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 42 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 43 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 44 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 45 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 46 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 47 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 48 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 49 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 50 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 51 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 52 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 53 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 54 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 55 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 56 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 57 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 58 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 59 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 60 | A | 6 | 5 | 1.00 | 23 | 0.217 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 62 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 63 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 64 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 65 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 66 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 67 | A | 7 | 6 | 1.00 | 23 | 0.261 |
| 68 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 69 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 70 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 71 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 72 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 73 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 74 | A | 7 | 6 | 1.00 | 23 | 0.261 |
| 75 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 76 | A | 8 | 8 | 1.00 | 23 | 0.348 |
| 77 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 78 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 79 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 80 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 81 | A | 4 | 3 | 1.00 | 14 | 0.214 |
| 82 | A | 7 | 6 | 1.00 | 21 | 0.286 |
| 83 | A | 8 | 7 | 1.00 | 23 | 0.304 |
| 84 | A | 9 | 7 | 1.00 | 23 | 0.304 |
| 85 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 86 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 87 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 88 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 89 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 90 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 91 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 92 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 93 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 94 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 95 | A | 2 | 2 | 1.00 | 14 | 0.143 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 96 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 97 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 98 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 99 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 100 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 101 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 102 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 103 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 104 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 105 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 106 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 107 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 108 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 109 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 110 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 111 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 112 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 113 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 114 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 115 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 116 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 117 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 118 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 119 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 120 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 121 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 122 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 123 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 124 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 125 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 126 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 127 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 128 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 129 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 130 | A | 5 | 5 | 1.00 | 25 | 0.200 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 131 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 132 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 133 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 134 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 135 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 136 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 137 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 138 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 139 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 140 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 141 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 142 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 143 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 144 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 145 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 146 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 147 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 148 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 149 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 150 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 151 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 152 | A | 2 | 1 | 1.00 | 10 | 0.100 |
| 153 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 154 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 155 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 156 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 157 | A | 7 | 5 | 1.00 | 21 | 0.238 |
| 158 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 159 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 160 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 161 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 162 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 163 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 164 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 165 | A | 6 | 5 | 1.00 | 21 | 0.238 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 166 | A | 8 | 6 | 1.13 | 21 | 0.286 |
| 167 | A | 4 | 3 | 1.12 | 21 | 0.143 |
| 168 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 169 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 170 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 171 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 172 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 173 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 174 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 175 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 176 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 177 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 178 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 179 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 180 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 181 | A | 5 | 5 | 1.00 | 11 | 0.454 |
| 182 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 183 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 184 | A | 8 | 7 | 1.00 | 13 | 0.538 |
| 185 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 186 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 187 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 188 | A | 5 | 5 | 1.00 | 11 | 0.454 |
| 189 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 190 | A | 6 | 6 | 1.00 | 11 | 0.546 |
| 191 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 192 | A | 8 | 7 | 1.00 | 13 | 0.538 |
| 193 | A | 8 | 8 | 1.00 | 13 | 0.615 |
| 194 | A | 7 | 7 | 1.00 | 13 | 0.538 |
| 195 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 196 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 197 | A | 6 | 5 | 1.00 | 11 | 0.454 |
| 198 | A | 6 | 6 | 1.00 | 8 | 0.750 |
| 199 | A | 7 | 7 | 1.00 | 11 | 0.636 |
| 200 | A | 8 | 7 | 1.00 | 13 | 0.538 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 201 | A | 9 | 7 | 1.00 | 13 | 0.538 |
| 202 | A | 7 | 6 | 1.00 | 12 | 0.500 |
| 203 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 204 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 205 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 206 | A | 9 | 9 | 1.00 | 23 | 0.391 |
| 207 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 208 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 209 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 210 | A | 9 | 9 | 1.00 | 23 | 0.391 |
| 211 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 212 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 213 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 214 | A | 4 | 3 | 1.00 | 23 | 0.130 |
| 215 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 216 | A | 5 | 3 | 1.00 | 23 | 0.130 |
| 217 | A | 10 | 4 | 1.00 | 23 | 0.174 |
| 218 | A | 13 | 4 | 1.00 | 23 | 0.174 |
| 219 | A | 3 | 3 | 1.00 | 36 | 0.083 |
| 220 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 221 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 222 | A | 9 | 6 | 1.00 | 21 | 0.286 |
| 223 | A | 8 | 5 | 1.00 | 21 | 0.238 |
| 224 | A | 7 | 4 | 1.00 | 19 | 0.210 |
| 225 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 226 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 227 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 228 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 229 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 230 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 231 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 232 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 233 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 234 | A | 4 | 3 | 1.00 | 24 | 0.125 |
| 235 | A | 5 | 3 | 1.00 | 24 | 0.125 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 236 | A | 7 | 5 | 1.00 | 26 | 0.192 |
| 237 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 238 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 239 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 240 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 241 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 242 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 243 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 244 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 245 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 246 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 247 | A | 8 | 5 | 1.00 | 26 | 0.192 |
| 248 | A | 7 | 5 | 1.00 | 26 | 0.192 |
| 249 | A | 6 | 4 | 1.00 | 26 | 0.154 |
| 250 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 251 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 252 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 253 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 254 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 255 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 256 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 257 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 258 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 259 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 260 | A | 6 | 3 | 1.00 | 26 | 0.115 |
| 261 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 262 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 263 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 264 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 265 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 266 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 267 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 268 | A | 6 | 4 | 1.00 | 26 | 0.154 |
| 269 | A | 7 | 5 | 1.00 | 26 | 0.192 |
| 270 | A | 6 | 5 | 1.00 | 26 | 0.192 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 271 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 272 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 273 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 274 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 275 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 276 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 277 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 278 | A | 6 | 3 | 1.00 | 26 | 0.115 |
| 279 | A | 7 | 5 | 1.00 | 26 | 0.192 |
| 280 | A | 6 | 4 | 1.00 | 26 | 0.154 |
| 281 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 282 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 283 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 284 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 285 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 286 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 287 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 288 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 289 | A | 6 | 3 | 1.00 | 26 | 0.115 |
| 290 | A | 5 | 3 | 1.00 | 26 | 0.115 |
| 291 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 292 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 293 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 294 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 295 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 296 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 297 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 298 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 299 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 300 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 301 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 302 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 303 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 304 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 305 | A | 6 | 5 | 1.00 | 28 | 0.179 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 306 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 307 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 308 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 309 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 310 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 311 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 312 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 313 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 314 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 315 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 316 | A | 8 | 5 | 1.00 | 28 | 0.179 |
| 317 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 318 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 319 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 320 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 321 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 322 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 323 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 324 | A | 6 | 3 | 1.00 | 28 | 0.107 |
| 325 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 326 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 327 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 328 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 329 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 330 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 331 | A | 7 | 6 | 1.00 | 28 | 0.214 |
| 332 | A | 6 | 3 | 1.00 | 28 | 0.107 |
| 333 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 334 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 335 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 336 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 337 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 338 | A | 7 | 6 | 1.00 | 28 | 0.214 |
| 339 | A | 8 | 7 | 1.00 | 28 | 0.250 |
| 340 | A | 1 | 1 | 1.00 | 30 | 0.033 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 341 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 342 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 343 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 344 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 345 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 346 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 347 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 348 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 349 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 350 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 351 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 352 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 353 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 354 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 355 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 356 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 357 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 358 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 359 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 360 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 361 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 362 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 363 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 364 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 365 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 366 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 367 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 368 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 369 | A | 4 | 2 | 1.00 | 30 | 0.067 |
| 370 | A | 4 | 2 | 1.00 | 30 | 0.067 |
| 371 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 372 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 373 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 374 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 375 | A | 6 | 5 | 1.00 | 30 | 0.167 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 376 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 377 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 378 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 379 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 380 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 381 | A | 4 | 2 | 1.00 | 30 | 0.067 |
| 382 | A | 4 | 2 | 1.00 | 30 | 0.067 |
| 383 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 384 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 385 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 386 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 387 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 388 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 389 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 390 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 391 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 392 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 393 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 394 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 395 | A | 5 | 3 | 1.00 | 30 | 0.100 |
| 396 | A | 7 | 5 | 1.00 | 30 | 0.167 |
| 397 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 398 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 399 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 400 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 401 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 402 | A | 5 | 3 | 1.00 | 30 | 0.100 |
| 403 | A | 6 | 3 | 1.00 | 30 | 0.100 |
| 404 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 405 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 406 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 407 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 408 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 409 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 410 | A | 4 | 4 | 1.00 | 26 | 0.154 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 411 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 412 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 413 | A | 1 | 1 | 1.00 | 28 | 0.036 |
| 414 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 415 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 416 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 417 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 418 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 419 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 420 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 421 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 422 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 423 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 424 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 425 | A | 4 | 2 | 1.00 | 23 | 0.087 |
| 426 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 427 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 428 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 429 | A | 2 | 1 | 1.00 | 10 | 0.100 |
| 430 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 431 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 432 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 433 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 434 | A | 5 | 3 | 1.00 | 25 | 0.120 |
| 435 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 436 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 437 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 438 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 439 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 440 | A | 5 | 5 | 1.03 | 25 | 0.200 |
| 441 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 442 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 443 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 444 | A | 6 | 5 | 1.52 | 25 | 0.200 |
| 445 | A | 9 | 7 | 1.15 | 25 | 0.280 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 446 | A | 8 | 6 | 1.06 | 23 | 0.261 |
| 447 | A | 7 | 5 | 1.00 | 12 | 0.417 |
| 448 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 449 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 450 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 451 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 452 | A | 9 | 8 | 1.00 | 25 | 0.320 |
| 453 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 454 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 455 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 456 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 457 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 458 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 459 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 460 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 461 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 462 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 463 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 464 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 465 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 466 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 467 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 468 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 469 | A | 8 | 7 | 1.00 | 25 | 0.280 |
| 470 | A | 5 | 4 | 1.00 | 25 | 0.160 |
| 471 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 472 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 473 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 474 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 475 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 476 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 477 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 478 | A | 8 | 7 | 1.00 | 25 | 0.280 |
| 479 | A | 9 | 7 | 1.00 | 25 | 0.280 |
| 480 | A | 4 | 3 | 1.00 | 13 | 0.231 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 481 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 482 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 483 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 484 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 485 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 486 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 487 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 488 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 489 | A | 9 | 7 | 1.00 | 27 | 0.259 |
| 490 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 491 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 492 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 493 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 494 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 495 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 496 | A | 11 | 9 | 1.00 | 27 | 0.333 |
| 497 | A | 10 | 9 | 1.00 | 27 | 0.333 |
| 498 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 499 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 500 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 501 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 502 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 503 | A | 10 | 9 | 1.00 | 27 | 0.333 |
| 504 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 505 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 506 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 507 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 508 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 509 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 510 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 511 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 512 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 513 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 514 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 515 | A | 9 | 8 | 1.00 | 27 | 0.296 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 516 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 517 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 518 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 519 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 520 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 521 | A | 10 | 8 | 1.00 | 27 | 0.296 |
| 522 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 523 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 524 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 525 | A | 1 | 1 | 1.00 | 14 | 0.071 |
| 526 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 527 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 528 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 529 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 530 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 531 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 532 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 533 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 534 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 535 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 536 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 537 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 538 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 539 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 540 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 541 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 542 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 543 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 544 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 545 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 546 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 547 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 548 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 549 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 550 | A | 6 | 6 | 1.00 | 27 | 0.222 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 551 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 552 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 553 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 554 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 555 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 556 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 557 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 558 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 559 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 560 | A | 4 | 3 | 1.00 | 14 | 0.214 |
| 561 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 562 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 563 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 564 | A | 5 | 3 | 1.00 | 29 | 0.103 |
| 565 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 566 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 567 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 568 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 569 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 570 | A | 3 | 2 | 1.00 | 29 | 0.069 |
| 571 | A | 7 | 5 | 1.00 | 29 | 0.172 |
| 572 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 573 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 574 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 575 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 576 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 577 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 578 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 579 | A | 7 | 5 | 1.00 | 29 | 0.172 |
| 580 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 581 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 582 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 583 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 584 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 585 | A | 3 | 3 | 1.00 | 29 | 0.103 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 586 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 587 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 588 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 589 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 590 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 591 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 592 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 593 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 594 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 595 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 596 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 597 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 598 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 599 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 600 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 601 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 602 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 603 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 604 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 605 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 606 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 607 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 608 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 609 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 610 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 611 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 612 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 613 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 614 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 615 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 616 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 617 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 618 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 619 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 620 | A | 2 | 2 | 1.79 | 27 | 0.074 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 621 | A | 2 | 2 | 1.91 | 27 | 0.074 |
| 622 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 623 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 624 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 625 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 626 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 627 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 628 | A | 1 | 1 | 1.00 | 27 | 0.037 |
| 629 | A | 2 | 2 | 1.36 | 27 | 0.074 |
| 630 | A | 2 | 2 | 1.42 | 27 | 0.074 |
| 631 | A | 2 | 2 | 1.42 | 29 | 0.069 |
| 632 | A | 2 | 2 | 1.42 | 29 | 0.069 |
| 633 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 634 | A | 2 | 2 | 1.42 | 29 | 0.069 |
| 635 | A | 2 | 2 | 1.44 | 27 | 0.074 |
| 636 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 637 | A | 2 | 2 | 1.64 | 29 | 0.069 |
| 638 | A | 2 | 2 | 1.51 | 29 | 0.069 |
| 639 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 640 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 641 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 642 | A | 2 | 2 | 1.57 | 29 | 0.069 |
| 643 | A | 2 | 2 | 1.51 | 29 | 0.069 |
| 644 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 645 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 646 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 647 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 648 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 649 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 650 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 651 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 652 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 653 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 654 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 655 | A | 3 | 3 | 1.00 | 25 | 0.120 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 656 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 657 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 658 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 659 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 660 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 661 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 662 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 663 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 664 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 665 | A | 3 | 3 | 1.30 | 27 | 0.111 |
| 666 | A | 3 | 3 | 1.25 | 27 | 0.111 |
| 667 | A | 3 | 3 | 1.32 | 27 | 0.111 |
| 668 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 669 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 670 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 671 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 672 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 673 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 674 | A | 2 | 1 | 1.00 | 10 | 0.100 |
| 675 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 676 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 677 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 678 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 679 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 680 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 681 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 682 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 683 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 684 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 685 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 686 | A | 5 | 4 | 1.23 | 25 | 0.160 |
| 687 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 688 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 689 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 690 | A | 6 | 6 | 1.00 | 25 | 0.240 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 691 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 692 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 693 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 694 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 695 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 696 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 697 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 698 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 699 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 700 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 701 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 702 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 703 | A | 7 | 4 | 1.00 | 25 | 0.160 |
| 704 | A | 8 | 5 | 1.00 | 25 | 0.200 |
| 705 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 706 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 707 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 708 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 709 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 710 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 711 | A | 8 | 5 | 1.00 | 25 | 0.200 |
| 712 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 713 | A | 10 | 6 | 1.00 | 25 | 0.240 |
| 714 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 715 | A | 7 | 7 | 1.00 | 25 | 0.280 |
| 716 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 717 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 718 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 719 | A | 6 | 6 | 1.00 | 12 | 0.500 |
| 720 | A | 9 | 6 | 1.00 | 25 | 0.240 |
| 721 | A | 10 | 6 | 1.00 | 25 | 0.240 |
| 722 | A | 11 | 6 | 1.00 | 25 | 0.240 |
| 723 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 724 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 725 | A | 6 | 6 | 1.00 | 25 | 0.240 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 726 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 727 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 728 | A | 7 | 6 | 1.00 | 25 | 0.240 |
| 729 | A | 8 | 6 | 1.00 | 25 | 0.240 |
| 730 | A | 9 | 7 | 1.00 | 27 | 0.259 |
| 731 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 732 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 733 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 734 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 735 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 736 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 737 | A | 10 | 8 | 1.00 | 27 | 0.296 |
| 738 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 739 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 740 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 741 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 742 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 743 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 744 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 745 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 746 | A | 8 | 8 | 1.00 | 27 | 0.296 |
| 747 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 748 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 749 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 750 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 751 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 752 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 753 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 754 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 755 | A | 9 | 9 | 1.00 | 27 | 0.333 |
| 756 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 757 | A | 11 | 10 | 1.00 | 27 | 0.370 |
| 758 | A | 11 | 11 | 1.00 | 27 | 0.407 |
| 759 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 760 | A | 10 | 10 | 1.00 | 27 | 0.370 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 761 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 762 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 763 | A | 10 | 10 | 1.00 | 27 | 0.370 |
| 764 | A | 11 | 10 | 1.00 | 27 | 0.370 |
| 765 | A | 8 | 8 | 1.00 | 29 | 0.276 |
| 766 | A | 8 | 8 | 1.00 | 29 | 0.276 |
| 767 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 768 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 769 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 770 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 771 | A | 9 | 8 | 1.00 | 29 | 0.276 |
| 772 | A | 8 | 8 | 1.00 | 29 | 0.276 |
| 773 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 774 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 775 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 776 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 777 | A | 10 | 8 | 1.00 | 29 | 0.276 |
| 778 | A | 9 | 8 | 1.00 | 29 | 0.276 |
| 779 | A | 8 | 8 | 1.00 | 29 | 0.276 |
| 780 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 781 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 782 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 783 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 784 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 785 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 786 | A | 1 | 1 | 1.00 | 29 | 0.034 |
| 787 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 788 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 789 | A | 7 | 7 | 1.00 | 29 | 0.241 |
| 790 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 791 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 792 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 793 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 794 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 795 | A | 6 | 6 | 1.00 | 29 | 0.207 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 796 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 797 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 798 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 799 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 800 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 801 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 802 | A | 8 | 5 | 1.00 | 25 | 0.200 |
| 803 | A | 7 | 4 | 1.00 | 23 | 0.174 |
| 804 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 805 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 806 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 807 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 808 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 809 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 810 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 811 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 812 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 813 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 814 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 815 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 816 | A | 6 | 4 | 1.00 | 21 | 0.190 |
| 817 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 818 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 819 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 820 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 821 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 822 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 823 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 824 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 825 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 826 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 827 | A | 6 | 4 | 1.00 | 21 | 0.190 |
| 828 | A | 7 | 5 | 1.00 | 23 | 0.217 |
| 829 | A | 10 | 5 | 1.00 | 23 | 0.217 |
| 830 | A | 12 | 5 | 1.00 | 23 | 0.217 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 831 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 832 | A | 6 | 5 | 0.94 | 27 | 0.185 |
| 833 | A | 5 | 4 | 0.96 | 27 | 0.148 |
| 834 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 835 | A | 6 | 4 | 1.00 | 27 | 0.148 |
| 836 | A | 11 | 5 | 1.00 | 27 | 0.185 |
| 837 | A | 14 | 5 | 1.00 | 27 | 0.185 |

# Chapter 3

# Listing of integrals

## Local contents

# 3.1 $\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=102

$$\frac{3a^2x}{4}-\frac{2a^2\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}-\frac{3a^2\cos(e+fx)\sin(e+fx)}{4f}-\frac{a^2\cos(e+fx)\sin^3}{2f}$$

```
[Out] 3/4*a^2*x-2*a^2*cos(f*x+e)/f+a^2*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e)^5/f-3/4*
a^2*cos(f*x+e)*sin(f*x+e)/f-1/2*a^2*cos(f*x+e)*sin(f*x+e)^3/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2713, 2715, 8}

$$-\frac{a^2\cos^5(e+fx)}{5f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3a^2\sin(e+fx)\cos(e+fx)}{4f}+\frac{3a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (2*a^2*Cos[e + f*x])/f + (a^2*Cos[e + f*x]^3)/f - (a^2*Cos[e
+ f*x]^5)/(5*f) - (3*a^2*Cos[e + f*x]*Sin[e + f*x])/(4*f) - (a^2*Cos[e + f*
x]*Sin[e + f*x]^3)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

`Q[m, 0] && RationalQ[n]`

Rubi steps

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx = \int \left(a^2\sin^3(e+fx)+2a^2\sin^4(e+fx)+a^2\sin^5(e+fx)\right)\,dx$$
$$= a^2\int \sin^3(e+fx)\,dx + a^2\int \sin^5(e+fx)\,dx + \left(2a^2\right)\int \sin^4(e+f$$
$$= -\frac{a^2\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{2}\left(3a^2\right)\int \sin^2(e+fx)\,dx - \frac{a^2\mathrm{Su}}{}$$
$$= -\frac{2a^2\cos(e+fx)}{f} + \frac{a^2\cos^3(e+fx)}{f} - \frac{a^2\cos^5(e+fx)}{5f} - \frac{3a^2\cos(}{}$$
$$= \frac{3a^2x}{4} - \frac{2a^2\cos(e+fx)}{f} + \frac{a^2\cos^3(e+fx)}{f} - \frac{a^2\cos^5(e+fx)}{5f} - 3$$

**Mathematica** [A]
time = 0.29, size = 105, normalized size = 1.03

$$-\frac{a^2\cos(e+fx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(24+15\sin(e+fx)+12\sin^2(e+fx)+10\sin^3(e+fx)+4\sin^4(e+fx))\right)}{20f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/20*(a^2*Cos[e + f*x]*(30*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[C
os[e + f*x]^2]*(24 + 15*Sin[e + f*x] + 12*Sin[e + f*x]^2 + 10*Sin[e + f*x]^
3 + 4*Sin[e + f*x]^4)))/(f*Sqrt[Cos[e + f*x]^2])
```

**Maple** [A]
time = 0.29, size = 96, normalized size = 0.94

| method | result |
|---|---|
| risch | $\frac{3a^2x}{4}-\frac{11a^2\cos(fx+e)}{8f}-\frac{a^2\cos(5fx+5e)}{80f}+\frac{a^2\sin(4fx+4e)}{16f}+\frac{3a^2\cos(3fx+3e)}{16f}-\frac{a^2\sin(2fx+2e)}{2f}$ |
| derivativedivides | $\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2\left(2+\sin^2(fx\right.}{}}{f}$ |
| default | $\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2\left(2+\sin^2(fx\right.}{}}{f}$ |

| | |
|---|---|
| norman | $\frac{\frac{3a^2x}{4}-\frac{12a^2}{5f}-\frac{3a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}-\frac{7a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{7a^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}+\frac{15a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{15a^2}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a^2*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

**Maxima** [A]
time = 0.36, size = 103, normalized size = 1.01

$$-\frac{16\,(3\,\cos\left(fx+e\right)^5-10\,\cos\left(fx+e\right)^3+15\,\cos\left(fx+e\right))a^2-80\,(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right))a^2-15\,(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right))a^2}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2 - 8
0*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 - 15*(12*f*x + 12*e + sin(4*f*x + 4
*e) - 8*sin(2*f*x + 2*e))*a^2)/f
```

**Fricas** [A]
time = 0.37, size = 89, normalized size = 0.87

$$-\frac{4\,a^2\cos\left(fx+e\right)^5-20\,a^2\cos\left(fx+e\right)^3-15\,a^2fx+40\,a^2\cos\left(fx+e\right)-5\left(2\,a^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^2*cos(f*x + e)^5 - 20*a^2*cos(f*x + e)^3 - 15*a^2*f*x + 40*a^2*c
os(f*x + e) - 5*(2*a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(94) = 188$.
time = 0.31, size = 221, normalized size = 2.17

$$\begin{cases} \frac{3a^2x\sin^4\left(e+fx\right)}{4}+\frac{3a^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{2}+\frac{3a^2x\cos^4\left(e+fx\right)}{4}-\frac{a^2\sin^4\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{5a^2\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{4f}-\frac{4a^2\sin^2\left(e+fx\right)\cos^3\left(e+fx\right)}{3f}-\frac{a^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^2\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{4f}-\frac{8a^2\cos^5\left(e+fx\right)}{15f}-\frac{2a^2\cos^3\left(e+fx\right)}{3f} & \text{for}\: f \neq 0 \\ x\left(a\sin\left(e\right)+a\right)^2\sin^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/4 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/2 + 3*a**2*x*cos(e + f*x)**4/4 - a**2*sin(e + f*x)**4*cos(e + f*x)/f
```

```
- 5*a**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*sin(e + f*x)**2*cos(e
+ f*x)**3/(3*f) - a**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*sin(e + f*x)
*cos(e + f*x)**3/(4*f) - 8*a**2*cos(e + f*x)**5/(15*f) - 2*a**2*cos(e + f*x
)**3/(3*f), Ne(f, 0)), (x*(a*sin(e) + a)**2*sin(e)**3, True))
```

**Giac** [A]
time = 0.56, size = 94, normalized size = 0.92

$$\frac{3}{4}\,a^{2}x-\frac{a^{2}\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{3\,a^{2}\cos\left(3\,fx+3\,e\right)}{16\,f}-\frac{11\,a^{2}\cos\left(fx+e\right)}{8\,f}+\frac{a^{2}\sin\left(4\,fx+4\,e\right)}{16\,f}-\frac{a^{2}\sin\left(2\,fx+2\,e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*x - 1/80*a^2*cos(5*f*x + 5*e)/f + 3/16*a^2*cos(3*f*x + 3*e)/f - 11/
8*a^2*cos(f*x + e)/f + 1/16*a^2*sin(4*f*x + 4*e)/f - 1/2*a^2*sin(2*f*x + 2*
e)/f
```

**Mupad** [B]
time = 10.28, size = 225, normalized size = 2.21

$$\frac{3\,a^2\,x}{4}-\frac{\frac{3\,a^2\,\left(e+f\,x\right)}{4}+7\,a^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3-7\,a^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7-\frac{3\,a^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{2}-\frac{a^2\,\left(15\,e+15\,f\,x-48\right)}{20}+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{15\,a^2\,\left(e+f\,x\right)}{2}-\frac{a^2\,\left(150\,e+150\,f\,x-80\right)}{20}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{15\,a^2\,\left(e+f\,x\right)}{4}-\frac{a^2\,\left(75\,e+75\,f\,x-240\right)}{20}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{15\,a^2\,\left(e+f\,x\right)}{2}-\frac{a^2\,\left(150\,e+150\,f\,x-400\right)}{20}\right)+\frac{3\,a^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + a*sin(e + f*x))^2,x)
```

```
[Out] (3*a^2*x)/4 - ((3*a^2*(e + f*x))/4 + 7*a^2*tan(e/2 + (f*x)/2)^3 - 7*a^2*tan
(e/2 + (f*x)/2)^7 - (3*a^2*tan(e/2 + (f*x)/2)^9)/2 - (a^2*(15*e + 15*f*x -
48))/20 + tan(e/2 + (f*x)/2)^6*((15*a^2*(e + f*x))/2 - (a^2*(150*e + 150*f*
x - 80))/20) + tan(e/2 + (f*x)/2)^2*((15*a^2*(e + f*x))/4 - (a^2*(75*e + 75
*f*x - 240))/20) + tan(e/2 + (f*x)/2)^4*((15*a^2*(e + f*x))/2 - (a^2*(150*e
 + 150*f*x - 400))/20) + (3*a^2*tan(e/2 + (f*x)/2))/2)/(f*(tan(e/2 + (f*x)/
2)^2 + 1)^5)
```

# 3.2 $\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=129

$$\frac{23a^3x}{16}-\frac{4a^3\cos(e+fx)}{f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{3a^3\cos^5(e+fx)}{5f}-\frac{23a^3\cos(e+fx)\sin(e+fx)}{16f}-\frac{23a^3\cos(e+f}{2}$$

```
[Out] 23/16*a^3*x-4*a^3*cos(f*x+e)/f+7/3*a^3*cos(f*x+e)^3/f-3/5*a^3*cos(f*x+e)^5/
f-23/16*a^3*cos(f*x+e)*sin(f*x+e)/f-23/24*a^3*cos(f*x+e)*sin(f*x+e)^3/f-1/6
*a^3*cos(f*x+e)*sin(f*x+e)^5/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2713, 2715, 8}

$$-\frac{3a^3\cos^5(e+fx)}{5f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{a^3\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{23a^3\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{23a^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{23a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (23*a^3*x)/16 - (4*a^3*Cos[e + f*x])/f + (7*a^3*Cos[e + f*x]^3)/(3*f) - (3*
a^3*Cos[e + f*x]^5)/(5*f) - (23*a^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (23
*a^3*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f) - (a^3*Cos[e + f*x]*Sin[e + f*x]^5
)/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx = \int \left(a^3\sin^3(e+fx)+3a^3\sin^4(e+fx)+3a^3\sin^5(e+fx)+a^3\sin\right.$$
$$= a^3\int \sin^3(e+fx)\,dx + a^3\int \sin^6(e+fx)\,dx + \left(3a^3\right)\int \sin^4(e+f$$
$$= -\frac{3a^3\cos(e+fx)\sin^3(e+fx)}{4f} - \frac{a^3\cos(e+fx)\sin^5(e+fx)}{6f} + \frac{1}{6}$$
$$= -\frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{9a^3 c}{}$$
$$= \frac{9a^3x}{8} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} -$$
$$= \frac{23a^3x}{16} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f}$$

## Mathematica [A]

time = 0.26, size = 76, normalized size = 0.59

$$\frac{a^3(1380e+1380fx-2520\cos(e+fx)+380\cos(3(e+fx))-36\cos(5(e+fx))-945\sin(2(e+fx))+135\sin(4(e+fx))-5\sin(6(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(1380*e + 1380*f*x - 2520*Cos[e + f*x] + 380*Cos[3*(e + f*x)] - 36*Cos
[5*(e + f*x)] - 945*Sin[2*(e + f*x)] + 135*Sin[4*(e + f*x)] - 5*Sin[6*(e +
f*x)]))/(960*f)
```

## Maple [A]

time = 0.35, size = 143, normalized size = 1.11

| method | result |
|---|---|
| risch | $\frac{23a^3x}{16} - \frac{21a^3\cos(fx+e)}{8f} - \frac{a^3\sin(6fx+6e)}{192f} - \frac{3a^3\cos(5fx+5e)}{80f} + \frac{9a^3\sin(4fx+4e)}{64f} + \frac{19a^3\cos(3fx+3e)}{48f} - \frac{63}{}$ |
| derivativedivides | $\frac{a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}}{f}$ |

| | |
|---|---|
| default | $\frac{a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+}{f}$ |
| norman | $\frac{\frac{23a^3x}{16}-\frac{68a^3}{15f}-\frac{23a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{391a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{75a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{75a^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{391a^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^
3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima** [A]
time = 0.29, size = 155, normalized size = 1.20

$$-\frac{192\,(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e))a^3-320\,(\cos(fx+e)^3-3\cos(fx+e))a^3-5\,(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e))a^3-90\,(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e))a^3}{960\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*
x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3 - 90*(12*f*x + 12*
e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3)/f
```

**Fricas** [A]
time = 0.37, size = 103, normalized size = 0.80

$$-\frac{144\,a^3\cos(fx+e)^5-560\,a^3\cos(fx+e)^3-345\,a^3fx+960\,a^3\cos(fx+e)+5\,(8\,a^3\cos(fx+e)^5-62\,a^3\cos(fx+e)^3+123\,a^3\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(f*x + e)^5 - 560*a^3*cos(f*x + e)^3 - 345*a^3*f*x + 960
*a^3*cos(f*x + e) + 5*(8*a^3*cos(f*x + e)^5 - 62*a^3*cos(f*x + e)^3 + 123*a
^3*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(122) = 244$.
time = 0.55, size = 379, normalized size = 2.94

$$\begin{cases} \frac{5a^3x\sin^6(e+fx)}{16}+\frac{15a^3x\sin^4(e+fx)\cos^2(e+fx)}{16}+\frac{9a^3x\sin^4(e+fx)}{8}+\frac{15a^3x\sin^2(e+fx)\cos^4(e+fx)}{16}+\frac{9a^3x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{5a^3x\cos^6(e+fx)}{16}+\frac{9a^3x\cos^4(e+fx)}{8}-\frac{11a^3\sin^5(e+fx)\cos(e+fx)}{16f}-\frac{3a^3\sin^4(e+fx)\cos(e+fx)}{f}-\frac{5a^3\sin^3(e+fx)\cos^3(e+fx)}{6f}-\frac{15a^3\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{4a^3\sin^2(e+fx)\cos^3(e+fx)}{f}-\frac{a^3\sin^2(e+fx)\cos(e+fx)}{f}-\frac{5a^3\sin(e+fx)\cos^5(e+fx)}{16f}-\frac{9a^3\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{8a^3\cos^5(e+fx)}{5f}-\frac{2a^3\cos^3(e+fx)}{3f} & \text{for } f\neq 0 \\ x(a\sin(e)+a)^3\sin^3(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(e + f*x)**6/16 + 15*a**3*x*sin(e + f*x)**4*cos(e +
f*x)**2/16 + 9*a**3*x*sin(e + f*x)**4/8 + 15*a**3*x*sin(e + f*x)**2*cos(e +
 f*x)**4/16 + 9*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a**3*x*cos(e +
 f*x)**6/16 + 9*a**3*x*cos(e + f*x)**4/8 - 11*a**3*sin(e + f*x)**5*cos(e +
f*x)/(16*f) - 3*a**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**3*sin(e + f*x)**
3*cos(e + f*x)**3/(6*f) - 15*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*
*3*sin(e + f*x)**2*cos(e + f*x)**3/f - a**3*sin(e + f*x)**2*cos(e + f*x)/f
- 5*a**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - 8*a**3*cos(e + f*x)**5/(5*f) - 2*a**3*cos(e + f*x)**3/(3*f)
, Ne(f, 0)), (x*(a*sin(e) + a)**3*sin(e)**3, True))
```

**Giac [A]**
time = 0.56, size = 112, normalized size = 0.87

$$\frac{23}{16}\,a^3x-\frac{3\,a^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{19\,a^3\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{21\,a^3\cos\left(fx+e\right)}{8\,f}-\frac{a^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{9\,a^3\sin\left(4\,fx+4\,e\right)}{64\,f}-\frac{63\,a^3\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 23/16*a^3*x - 3/80*a^3*cos(5*f*x + 5*e)/f + 19/48*a^3*cos(3*f*x + 3*e)/f -
21/8*a^3*cos(f*x + e)/f - 1/192*a^3*sin(6*f*x + 6*e)/f + 9/64*a^3*sin(4*f*x
 + 4*e)/f - 63/64*a^3*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 10.33, size = 294, normalized size = 2.28

$$\frac{23\,a^3\,x}{16}-\frac{\frac{23\,a^3\,\left(e+f\,x\right)}{16}+\frac{391\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{24}+\frac{75\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{4}-\frac{75\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{4}-\frac{391\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{24}-\frac{23\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}}{8}-\frac{a^3\,\left(345\,e+345\,f\,x-1088\right)}{240}+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{69\,a^3\,\left(e+f\,x\right)}{8}-\frac{a^3\,\left(2070\,e+2070\,f\,x-6528\right)}{240}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{345\,a^3\,\left(e+f\,x\right)}{16}-\frac{a^3\,\left(5175\,e+5175\,f\,x-960\right)}{240}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{115\,a^3\,\left(e+f\,x\right)}{4}-\frac{a^3\,\left(6900\,e+6900\,f\,x-10880\right)}{240}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{345\,a^3\,\left(e+f\,x\right)}{16}-\frac{a^3\,\left(5175\,e+5175\,f\,x-15360\right)}{240}\right)+\frac{23\,a^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + a*sin(e + f*x))^3,x)
```

```
[Out] (23*a^3*x)/16 - ((23*a^3*(e + f*x))/16 + (391*a^3*tan(e/2 + (f*x)/2)^3)/24
+ (75*a^3*tan(e/2 + (f*x)/2)^5)/4 - (75*a^3*tan(e/2 + (f*x)/2)^7)/4 - (391*
a^3*tan(e/2 + (f*x)/2)^9)/24 - (23*a^3*tan(e/2 + (f*x)/2)^11)/8 - (a^3*(345
*e + 345*f*x - 1088))/240 + tan(e/2 + (f*x)/2)^2*((69*a^3*(e + f*x))/8 - (a
^3*(2070*e + 2070*f*x - 6528))/240) + tan(e/2 + (f*x)/2)^8*((345*a^3*(e + f
*x))/16 - (a^3*(5175*e + 5175*f*x - 960))/240) + tan(e/2 + (f*x)/2)^6*((115
*a^3*(e + f*x))/4 - (a^3*(6900*e + 6900*f*x - 10880))/240) + tan(e/2 + (f*x
)/2)^4*((345*a^3*(e + f*x))/16 - (a^3*(5175*e + 5175*f*x - 15360))/240) + (
23*a^3*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

## 3.3 $\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=53

$$-\frac{3x}{2a}-\frac{4\cos(x)}{a}+\frac{4\cos^3(x)}{3a}+\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^3(x)}{a+a\sin(x)}$$

```
[Out] -3/2*x/a-4*cos(x)/a+4/3*cos(x)^3/a+3/2*cos(x)*sin(x)/a+cos(x)*sin(x)^3/(a+a
*sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2846, 2827, 2715, 8, 2713}

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{\sin^3(x)\cos(x)}{a\sin(x)+a}+\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-3*x)/(2*a) - (4*Cos[x])/a + (4*Cos[x]^3)/(3*a) + (3*Cos[x]*Sin[x])/(2*a)
+ (Cos[x]*Sin[x]^3)/(a + a*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx &= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{\int \sin^2(x)(3a-4a\sin(x))\,dx}{a^2}\\ &= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{3\int \sin^2(x)\,dx}{a}+\frac{4\int \sin^3(x)\,dx}{a}\\ &= \frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{3\int 1\,dx}{2a}-\frac{4\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)}{a}\\ &= -\frac{3x}{2a}-\frac{4\cos(x)}{a}+\frac{4\cos^3(x)}{3a}+\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^3(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica [A]**
time = 0.08, size = 101, normalized size = 1.91

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-3(7+12x)\cos\left(\frac{x}{2}\right)-18\cos\left(\frac{3x}{2}\right)-2\cos\left(\frac{5x}{2}\right)+\cos\left(\frac{7x}{2}\right)+69\sin\left(\frac{x}{2}\right)-36x\sin\left(\frac{x}{2}\right)-18\sin\left(\frac{3x}{2}\right)+2\sin\left(\frac{5x}{2}\right)+\sin\left(\frac{7x}{2}\right)\right)}{24a(1+\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-3*(7 + 12*x)*Cos[x/2] - 18*Cos[(3*x)/2] - 2*Cos[(5
*x)/2] + Cos[(7*x)/2] + 69*Sin[x/2] - 36*x*Sin[x/2] - 18*Sin[(3*x)/2] + 2*S
in[(5*x)/2] + Sin[(7*x)/2]))/(24*a*(1 + Sin[x]))
```

**Maple [A]**
time = 0.10, size = 66, normalized size = 1.25

| method | result |
|---|---|
| risch | $-\frac{3x}{2a}-\frac{7\,\mathrm{e}^{ix}}{8a}-\frac{7\,\mathrm{e}^{-ix}}{8a}-\frac{2}{\left(\mathrm{e}^{ix}+i\right)a}+\frac{\cos(3x)}{12a}+\frac{\sin(2x)}{4a}$ |

| | |
|---|---|
| default | $\frac{-\frac{2\left(\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}+\tan^4\left(\frac{x}{2}\right)+4\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{5}{3}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-3\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{2}{\tan\left(\frac{x}{2}\right)+1}}{a}$ |
| norman | $\frac{-\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{a}-\frac{11\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}-\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{3x}{2a}-\frac{16}{3a}-\frac{3x\tan\left(\frac{x}{2}\right)}{2a}-\frac{6x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{6x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{9x(\tan}{}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4(\tan}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/a*(-1/16*(1/2*tan(1/2*x)^5+tan(1/2*x)^4+4*tan(1/2*x)^2-1/2*tan(1/2*x)+5/
3)/(tan(1/2*x)^2+1)^3-3/32*arctan(tan(1/2*x))-1/16/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(47) = 94$.
time = 0.54, size = 180, normalized size = 3.40

$$-\frac{\frac{7\sin(x)}{\cos(x)+1}+\frac{39\sin(x)^2}{(\cos(x)+1)^2}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}+\frac{24\sin(x)^4}{(\cos(x)+1)^4}+\frac{9\sin(x)^5}{(\cos(x)+1)^5}+\frac{9\sin(x)^6}{(\cos(x)+1)^6}+16}{3\left(a+\frac{a\sin(x)}{\cos(x)+1}+\frac{3a\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a\sin(x)^4}{(\cos(x)+1)^4}+\frac{3a\sin(x)^5}{(\cos(x)+1)^5}+\frac{a\sin(x)^6}{(\cos(x)+1)^6}+\frac{a\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(7*sin(x)/(cos(x) + 1) + 39*sin(x)^2/(cos(x) + 1)^2 + 24*sin(x)^3/(cos
(x) + 1)^3 + 24*sin(x)^4/(cos(x) + 1)^4 + 9*sin(x)^5/(cos(x) + 1)^5 + 9*sin
(x)^6/(cos(x) + 1)^6 + 16)/(a + a*sin(x)/(cos(x) + 1) + 3*a*sin(x)^2/(cos(x
) + 1)^2 + 3*a*sin(x)^3/(cos(x) + 1)^3 + 3*a*sin(x)^4/(cos(x) + 1)^4 + 3*a*
sin(x)^5/(cos(x) + 1)^5 + a*sin(x)^6/(cos(x) + 1)^6 + a*sin(x)^7/(cos(x) +
1)^7) - 3*arctan(sin(x)/(cos(x) + 1))/a
```

**Fricas** [A]
time = 0.35, size = 70, normalized size = 1.32

$$\frac{2\cos(x)^4-\cos(x)^3-3(3x+5)\cos(x)-12\cos(x)^2+\left(2\cos(x)^3+3\cos(x)^2-9x-9\cos(x)+6\right)\sin(x)-9x-6}{6(a\cos(x)+a\sin(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(x)^4 - cos(x)^3 - 3*(3*x + 5)*cos(x) - 12*cos(x)^2 + (2*cos(x)^3
 + 3*cos(x)^2 - 9*x - 9*cos(x) + 6)*sin(x) - 9*x - 6)/(a*cos(x) + a*sin(x)
+ a)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(49) = 98$.
time = 2.39, size = 1221, normalized size = 23.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x)),x)
```

```
[Out] -9*x*tan(x/2)**7/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18
*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
- 9*x*tan(x/2)**6/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 1
8*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
 - 27*x*tan(x/2)**5/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 +
 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*
a) - 27*x*tan(x/2)**4/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5
 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) +
6*a) - 27*x*tan(x/2)**3/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)*
*5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2)
+ 6*a) - 27*x*tan(x/2)**2/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2
)**5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2
) + 6*a) - 9*x*tan(x/2)/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)*
*5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2)
+ 6*a) - 9*x/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 18
*tan(x/2)**6/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 18
*tan(x/2)**5/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 48
*tan(x/2)**4/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 48
*tan(x/2)**3/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 78
*tan(x/2)**2/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 14
*tan(x/2)/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*tan(
x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 32/(6
*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*tan(x/2)**4 + 18
*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
```

**Giac [A]**
time = 0.54, size = 67, normalized size = 1.26

$$-\frac{3\,x}{2\,a}-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,x\right)^{5}+6\,\tan\left(\frac{1}{2}\,x\right)^{4}+24\,\tan\left(\frac{1}{2}\,x\right)^{2}-3\,\tan\left(\frac{1}{2}\,x\right)+10}{3\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 2/(a*(tan(1/2*x) + 1)) - 1/3*(3*tan(1/2*x)^5 + 6*tan(1/2*x)^4 +
24*tan(1/2*x)^2 - 3*tan(1/2*x) + 10)/((tan(1/2*x)^2 + 1)^3*a)
```

**Mupad** [B]
time = 6.82, size = 78, normalized size = 1.47

$$-\frac{3\,x}{2\,a}-\frac{3\,\tan\left(\frac{x}{2}\right)^6+3\,\tan\left(\frac{x}{2}\right)^5+8\,\tan\left(\frac{x}{2}\right)^4+8\,\tan\left(\frac{x}{2}\right)^3+13\,\tan\left(\frac{x}{2}\right)^2+\frac{7\,\tan\left(\frac{x}{2}\right)}{3}+\frac{16}{3}}{a\,{\left({\tan\left(\frac{x}{2}\right)}^2+1\right)}^3\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x)),x)
```

```
[Out] - (3*x)/(2*a) - ((7*tan(x/2))/3 + 13*tan(x/2)^2 + 8*tan(x/2)^3 + 8*tan(x/2)
^4 + 3*tan(x/2)^5 + 3*tan(x/2)^6 + 16/3)/(a*(tan(x/2)^2 + 1)^3*(tan(x/2) +
1))
```

# 3.4 $\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=42

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\sin(x)}$$

```
[Out] 3/2*x/a+2*cos(x)/a-3/2*cos(x)*sin(x)/a+cos(x)*sin(x)^2/(a+a*sin(x))
```

**Rubi** [A]

time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2846, 2813}

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}+\frac{\sin^2(x)\cos(x)}{a\sin(x)+a}-\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (3*x)/(2*a) + (2*Cos[x])/a - (3*Cos[x]*Sin[x])/(2*a) + (Cos[x]*Sin[x]^2)/(a
 + a*Sin[x])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx &= \frac{\cos(x)\sin^2(x)}{a+a\sin(x)}-\frac{\int \sin(x)(2a-3a\sin(x))\,dx}{a^2}\\
&= \frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\sin(x)}
\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.
time = 0.05, size = 87, normalized size = 2.07

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(4(1+3x)\cos\left(\frac{x}{2}\right)+3\cos\left(\frac{3x}{2}\right)+\cos\left(\frac{5x}{2}\right)-20\sin\left(\frac{x}{2}\right)+12x\sin\left(\frac{x}{2}\right)+3\sin\left(\frac{3x}{2}\right)-\sin\left(\frac{5x}{2}\right)\right)}{8a(1+\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*(1 + 3*x)*Cos[x/2] + 3*Cos[(3*x)/2] + Cos[(5*x)/2
] - 20*Sin[x/2] + 12*x*Sin[x/2] + 3*Sin[(3*x)/2] - Sin[(5*x)/2]))/(8*a*(1 +
 Sin[x]))
```

**Maple** [A]
time = 0.10, size = 58, normalized size = 1.38

| method | result |
|---|---|
| risch | $\frac{3x}{2a}+\frac{e^{ix}}{2a}+\frac{e^{-ix}}{2a}+\frac{2}{(e^{ix}+i)a}-\frac{\sin(2x)}{4a}$ |
| default | $\frac{\frac{2\left(\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\tan^2\left(\frac{x}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)}{2}+1\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+3\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{16}{8\tan\left(\frac{x}{2}\right)+8}}{a}$ |
| norman | $\frac{-\frac{\tan^5\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{3x}{2a}+\frac{8}{3a}+\frac{3x\tan\left(\frac{x}{2}\right)}{2a}+\frac{9x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}+\frac{9x\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2a}+\frac{9x\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right.\right.}{2a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*(1/8*(1/2*tan(1/2*x)^3+tan(1/2*x)^2-1/2*tan(1/2*x)+1)/(tan(1/2*x)^2+1)
^2+3/16*arctan(tan(1/2*x))+1/8/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(38) = 76.
time = 0.58, size = 128, normalized size = 3.05

$$\frac{\frac{\sin(x)}{\cos(x)+1}+\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+4}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{2a\sin(x)^2}{(\cos(x)+1)^2}+\frac{2a\sin(x)^3}{(\cos(x)+1)^3}+\frac{a\sin(x)^4}{(\cos(x)+1)^4}+\frac{a\sin(x)^5}{(\cos(x)+1)^5}}+\frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] (sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^3/(cos(x) + 1)^
3 + 3*sin(x)^4/(cos(x) + 1)^4 + 4)/(a + a*sin(x)/(cos(x) + 1) + 2*a*sin(x)^
```

```
2/(cos(x) + 1)^2 + 2*a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^5/(cos(x) + 1)^5) + 3*arctan(sin(x)/(cos(x) + 1))/a
```

**Fricas** [A]
time = 0.37, size = 53, normalized size = 1.26

$$\frac{\cos(x)^3 + 3(x+1)\cos(x) + 2\cos(x)^2 - \left(\cos(x)^2 - 3x - \cos(x) + 2\right)\sin(x) + 3x + 2}{2(a\cos(x) + a\sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 + 3*(x + 1)*cos(x) + 2*cos(x)^2 - (cos(x)^2 - 3*x - cos(x) +
2)*sin(x) + 3*x + 2)/(a*cos(x) + a*sin(x) + a)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(39) = 78$.
time = 1.11, size = 665, normalized size = 15.83

$$\frac{3x\tan^5\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{3x\tan^4\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{6x\tan^3\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{6x\tan^2\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{3x\tan\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{3x}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{6\tan^4\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{6\tan^3\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{10\tan^2\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{2\tan\left(\frac{x}{2}\right)}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a} + \frac{8}{2a\tan^5\left(\frac{x}{2}\right)+2a\tan^4\left(\frac{x}{2}\right)+4a\tan^3\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x)),x)
```

```
[Out] 3*x*tan(x/2)**5/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*
tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*tan(x/2)**4/(2*a*tan(x/2)**5 + 2*a*
tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x
*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan
(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x*tan(x/2)**2/(2*a*tan(x/2)**5 + 2*a*tan
(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*ta
n(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)*
*2 + 2*a*tan(x/2) + 2*a) + 3*x/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan
(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*tan(x/2)**4/(2*a*tan(x
/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2)
 + 2*a) + 6*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**
3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 10*tan(x/2)**2/(2*a*tan(x/2)**5
 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a
) + 2*tan(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*t
an(x/2)**2 + 2*a*tan(x/2) + 2*a) + 8/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4
*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a)
```

**Giac** [A]
time = 0.55, size = 56, normalized size = 1.33

$$\frac{3x}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^3 + 2\tan\left(\frac{1}{2}x\right)^2 - \tan\left(\frac{1}{2}x\right) + 2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^2 a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + (tan(1/2*x)^3 + 2*tan(1/2*x)^2 - tan(1/2*x) + 2)/((tan(1/2*x)^2 +
 1)^2*a) + 2/(a*(tan(1/2*x) + 1))
```

**Mupad [B]**
time = 6.91, size = 59, normalized size = 1.40

$$\frac{3\,x}{2\,a}+\frac{3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^3+5\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+4}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x)),x)
```

```
[Out] (3*x)/(2*a) + (tan(x/2) + 5*tan(x/2)^2 + 3*tan(x/2)^3 + 3*tan(x/2)^4 + 4)/(
a*(tan(x/2)^2 + 1)^2*(tan(x/2) + 1))
```

# 3.5 $\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=27

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(1+\sin(x))}$$

```
[Out] -x/a-cos(x)/a-cos(x)/a/(1+sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2825, 12, 2814, 2727}

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x]),x]
```

```
[Out] -(x/a) - Cos[x]/a - Cos[x]/(a*(1 + Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx = -\frac{\cos(x)}{a} - \frac{\int \frac{a\sin(x)}{a+a\sin(x)}\,dx}{a}$$
$$= -\frac{\cos(x)}{a} - \int \frac{\sin(x)}{a+a\sin(x)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(x)}{a} + \int \frac{1}{a+a\sin(x)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(x)}{a} - \frac{\cos(x)}{a+a\sin(x)}$$

**Mathematica [A]**
time = 0.04, size = 48, normalized size = 1.78

$$-\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)(x+\cos(x))+(-2+x+\cos(x))\sin\left(\frac{x}{2}\right)\right)}{a(1+\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x]),x]
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(x + Cos[x]) + (-2 + x + Cos[x])*Sin[x/2
]))/(a*(1 + Sin[x])))
```

**Maple [A]**
time = 0.09, size = 36, normalized size = 1.33

| method | result |
|---|---|
| default | $\frac{-\frac{2}{\tan\left(\frac{x}{2}\right)+1}-\frac{2}{\tan^2\left(\frac{x}{2}\right)+1}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ |
| risch | $-\frac{x}{a}-\frac{e^{ix}}{2a}-\frac{e^{-ix}}{2a}-\frac{2}{\left(e^{ix}+i\right)a}$ |
| norman | $\frac{-\frac{4}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\tan\left(\frac{x}{2}\right)}{a}-\frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{2x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x)),x,method=_RETURNVERBOSE)
[Out] 8/a*(-1/4/(tan(1/2*x)+1)-1/4/(tan(1/2*x)^2+1)-1/4*arctan(tan(1/2*x)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(27) = 54.
time = 0.52, size = 78, normalized size = 2.89

$$-\frac{2\left(\frac{\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 2)/(a + a*sin(x)/(cos(x
) + 1) + a*sin(x)^2/(cos(x) + 1)^2 + a*sin(x)^3/(cos(x) + 1)^3) - 2*arctan(
sin(x)/(cos(x) + 1))/a
```

**Fricas** [A]
time = 0.34, size = 35, normalized size = 1.30

$$-\frac{(x+2)\cos(x)+\cos(x)^2+(x+\cos(x)-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + cos(x)^2 + (x + cos(x) - 1)*sin(x) + x + 1)/(a*cos(x) +
a*sin(x) + a)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(19) = 38$.
time = 0.54, size = 221, normalized size = 8.19

$$-\frac{x\tan^3\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{2\tan^2\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{4}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x)),x)
```

```
[Out] -x*tan(x/2)**3/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2
)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2)/(a*tan(x
/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x/(a*tan(x/2)**3 + a*tan(x/2)**2
 + a*tan(x/2) + a) - 2*tan(x/2)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x
/2) + a) - 2*tan(x/2)/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - 4/
(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a)
```

**Giac** [A]
time = 0.56, size = 44, normalized size = 1.63

$$-\frac{x}{a}-\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+\tan\left(\frac{1}{2}\,x\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)^2+\tan\left(\frac{1}{2}\,x\right)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -x/a - 2*(tan(1/2*x)^2 + tan(1/2*x) + 2)/((tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1)*a)
```

**Mupad** [B]
time = 6.77, size = 46, normalized size = 1.70

$$-\frac{x}{a}-\frac{2\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)+4}{a\,\left(\tan\left(\frac{x}{2}\right)^2+1\right)\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x)),x)
```

```
[Out] - x/a - (2*tan(x/2) + 2*tan(x/2)^2 + 4)/(a*(tan(x/2)^2 + 1)*(tan(x/2) + 1))
```

# 3.6 $\int \frac{\sin(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=17

$$\frac{x}{a}+\frac{\cos(x)}{a+a\sin(x)}$$

```
[Out] x/a+cos(x)/(a+a*sin(x))
```

**Rubi** [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2814, 2727}

$$\frac{x}{a}+\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x]),x]
[Out] x/a + Cos[x]/(a + a*Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+a\sin(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\sin(x)}\,dx\\ &= \frac{x}{a}+\frac{\cos(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.
time = 0.03, size = 42, normalized size = 2.47

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(x\cos\left(\frac{x}{2}\right)+(-2+x)\sin\left(\frac{x}{2}\right)\right)}{a(1+\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x]),x]
[Out] ((Cos[x/2] + Sin[x/2])*(x*Cos[x/2] + (-2 + x)*Sin[x/2]))/(a*(1 + Sin[x]))
```

**Maple [A]**
time = 0.08, size = 24, normalized size = 1.41

| method | result | size |
|---|---|---|
| risch | $\frac{x}{a}+\frac{2}{\left(e^{i x}+i\right) a}$ | 22 |
| default | $\frac{\frac{4}{2 \tan\left(\frac{x}{2}\right)+2}+2 \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 24 |
| norman | $\frac{\frac{x}{a}+\frac{2}{a}+\frac{x \tan\left(\frac{x}{2}\right)}{a}+\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$ | 73 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x)),x,method=_RETURNVERBOSE)
[Out] 4/a*(1/2/(tan(1/2*x)+1)+1/2*arctan(tan(1/2*x)))
```

**Maxima [A]**
time = 0.61, size = 32, normalized size = 1.88

$$\frac{2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a \sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="maxima")
[Out] 2*arctan(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

**Fricas [A]**
time = 0.37, size = 28, normalized size = 1.65

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="fricas")
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(a*cos(x) + a*sin(x) + a)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.
time = 0.23, size = 34, normalized size = 2.00

$$\frac{x\tan\left(\frac{x}{2}\right)}{a\tan\left(\frac{x}{2}\right)+a}+\frac{x}{a\tan\left(\frac{x}{2}\right)+a}+\frac{2}{a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x)
```

```
[Out] x*tan(x/2)/(a*tan(x/2) + a) + x/(a*tan(x/2) + a) + 2/(a*tan(x/2) + a)
```

**Giac** [A]
time = 0.49, size = 19, normalized size = 1.12

$$\frac{x}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] x/a + 2/(a*(tan(1/2*x) + 1))
```

**Mupad** [B]
time = 6.54, size = 19, normalized size = 1.12

$$\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*sin(x)),x)
```

```
[Out] 2/(a*(tan(x/2) + 1)) + x/a
```

# 3.7 $\int \frac{1}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=12

$$-\frac{\cos(x)}{a+a\sin(x)}$$

[Out] -cos(x)/(a+a*sin(x))

**Rubi** [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$-\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[x])^(-1),x]

[Out] -(Cos[x]/(a + a*Sin[x]))

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(x)}\,dx = -\frac{\cos(x)}{a+a\sin(x)}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.
time = 0.02, size = 29, normalized size = 2.42

$$\frac{2\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a+a\sin(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[x])^(-1),x]

[Out] (2*Sin[x/2]*(Cos[x/2] + Sin[x/2]))/(a + a*Sin[x])

**Maple** [A]
time = 0.06, size = 14, normalized size = 1.17

| method | result | size |
|---|---|---|
| default | $-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$ | 14 |
| norman | $-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$ | 14 |
| risch | $-\frac{2}{\left(e^{ix}+i\right)a}$ | 16 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/(tan(1/2*x)+1)
```

**Maxima** [A]
time = 0.34, size = 16, normalized size = 1.33

$$-\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(a + a*sin(x)/(cos(x) + 1))
```

**Fricas** [A]
time = 0.41, size = 22, normalized size = 1.83

$$-\frac{\cos\left(x\right)-\sin\left(x\right)+1}{a\cos\left(x\right)+a\sin\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)
```

**Sympy** [A]
time = 0.10, size = 10, normalized size = 0.83

$$-\frac{2}{a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x)
```

```
[Out] -2/(a*tan(x/2) + a)
```

**Giac** [A]
time = 0.46, size = 13, normalized size = 1.08

$$-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="giac")
[Out] -2/(a*(tan(1/2*x) + 1))
```

**Mupad** [B]
time = 0.02, size = 13, normalized size = 1.08

$$-\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x)),x)
[Out] -2/(a*(tan(x/2) + 1))
```

# 3.8 $\int \frac{\csc(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=20

$$-\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cos(x)}{a+a\sin(x)}$$

```
[Out] -arctanh(cos(x))/a+cos(x)/(a+a*sin(x))
```

**Rubi** [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2826, 3855, 2727}

$$\frac{\cos(x)}{a\sin(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/a) + Cos[x]/(a + a*Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+a\sin(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\int \frac{1}{a+a\sin(x)}\,dx\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cos(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 74 vs. $2(20) = 40$.
time = 0.04, size = 74, normalized size = 3.70

$$-\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\left(2+\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin\left(\frac{x}{2}\right)\right)}{a(1+\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (2 + L
og[Cos[x/2]] - Log[Sin[x/2]])*Sin[x/2]))/(a*(1 + Sin[x])))
```

**Maple** [A]
time = 0.11, size = 21, normalized size = 1.05

| method | result | size |
|---|---|---|
| default | $\frac{\frac{2}{\tan\left(\frac{x}{2}\right)+1}+\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 21 |
| norman | $\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 24 |
| risch | $\frac{2}{\left(e^{ix}+i\right)a}-\frac{\ln\left(e^{ix}+1\right)}{a}+\frac{\ln\left(e^{ix}-1\right)}{a}$ | 42 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(2/(tan(1/2*x)+1)+ln(tan(1/2*x)))
```

**Maxima** [A]
time = 0.39, size = 31, normalized size = 1.55

$$\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 0.40, size = 53, normalized size = 2.65

$$-\frac{(\cos(x)+\sin(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+\sin(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)+2\sin(x)-2}{2\left(a\cos(x)+a\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*l
og(-1/2*cos(x) + 1/2) - 2*cos(x) + 2*sin(x) - 2)/(a*cos(x) + a*sin(x) + a)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)/(sin(x) + 1), x)/a
```

**Giac** [A]
time = 0.48, size = 24, normalized size = 1.20

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a + 2/(a*(tan(1/2*x) + 1))
```

**Mupad** [B]
time = 6.42, size = 23, normalized size = 1.15

$$\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))),x)
```

```
[Out] 2/(a*(tan(x/2) + 1)) + log(tan(x/2))/a
```

## 3.9 $\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=26

$$\frac{\tanh^{-1}(\cos(x))}{a}-\frac{2\cot(x)}{a}+\frac{\cot(x)}{a+a\sin(x)}$$

```
[Out] arctanh(cos(x))/a-2*cot(x)/a+cot(x)/(a+a*sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {2847, 2827, 3852, 8, 3855}

$$-\frac{2\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x]),x]
```

```
[Out] ArcTanh[Cos[x]]/a - (2*Cot[x])/a + Cot[x]/(a + a*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)}{a+a\sin(x)}-\frac{\int \csc^2(x)(-2a+a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)}{a+a\sin(x)}-\frac{\int \csc(x)\,dx}{a}+\frac{2\int \csc^2(x)\,dx}{a}\\ &= \frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a+a\sin(x)}-\frac{2\text{Subst}(\int 1\,dx,x,\cot(x))}{a}\\ &= \frac{\tanh^{-1}(\cos(x))}{a}-\frac{2\cot(x)}{a}+\frac{\cot(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.11, size = 63, normalized size = 2.42

$$\frac{-\cot\left(\frac{x}{2}\right)+2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{4\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\tan\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x]),x]
```

```
[Out] (-Cot[x/2] + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (4*Sin[x/2])/(Cos[x/2] + S
in[x/2]) + Tan[x/2])/(2*a)
```

**Maple** [A]
time = 0.10, size = 36, normalized size = 1.38

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{4}{\tan\left(\frac{x}{2}\right)+1}}{2a}$ | 36 |
| norman | $\frac{-\frac{3\tan\left(\frac{x}{2}\right)}{a}-\frac{1}{2a}+\frac{\tan^3\left(\frac{x}{2}\right)}{2a}}{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 53 |
| risch | $-\frac{2\left(-2+i e^{ix}+e^{2ix}\right)}{\left(e^{2ix}-1\right)\left(e^{ix}+i\right)a}-\frac{\ln\left(e^{ix}-1\right)}{a}+\frac{\ln\left(e^{ix}+1\right)}{a}$ | 66 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(tan(1/2*x)-1/tan(1/2*x)-2*ln(tan(1/2*x))-4/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 0.45, size = 68, normalized size = 2.62

$$-\frac{\frac{5\sin(x)}{\cos(x)+1}+1}{2\left(\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{\sin(x)}{2\,a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(5*sin(x)/(cos(x) + 1) + 1)/(a*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x
) + 1)^2) - log(sin(x)/(cos(x) + 1))/a + 1/2*sin(x)/(a*(cos(x) + 1))
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.
time = 0.37, size = 91, normalized size = 3.50

$$\frac{4\cos(x)^2+\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2\,(2\cos(x)+1)\sin(x)+2\cos(x)-2}{2\left(a\cos(x)^2-(a\cos(x)+a)\sin(x)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(x)^2 + (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(1/2*cos(x) + 1/2
) - (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*(2*cos(
x) + 1)*sin(x) + 2*cos(x) - 2)/(a*cos(x)^2 - (a*cos(x) + a)*sin(x) - a)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(sin(x) + 1), x)/a
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(26) = 52$.
time = 0.49, size = 53, normalized size = 2.04

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}+\frac{\tan\left(\frac{1}{2}x\right)}{2\,a}+\frac{\tan\left(\frac{1}{2}x\right)^2-4\tan\left(\frac{1}{2}x\right)-1}{2\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2*x)))/a + 1/2*tan(1/2*x)/a + 1/2*(tan(1/2*x)^2 - 4*tan(1/2*
x) - 1)/((tan(1/2*x)^2 + tan(1/2*x))*a)
```

**Mupad [B]**
time = 6.71, size = 49, normalized size = 1.88

$$\frac{\tan\left(\frac{x}{2}\right)}{2\,a}-\frac{5\tan\left(\frac{x}{2}\right)+1}{2\,a\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))),x)
```

```
[Out] tan(x/2)/(2*a) - (5*tan(x/2) + 1)/(2*a*tan(x/2) + 2*a*tan(x/2)^2) - log(tan
(x/2))/a
```

# 3.10 $\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=42

$$-\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}$$

```
[Out] -3/2*arctanh(cos(x))/a+2*cot(x)/a-3/2*cot(x)*csc(x)/a+cot(x)*csc(x)/(a+a*si
n(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(2*a) + (2*Cot[x])/a - (3*Cot[x]*Csc[x])/(2*a) + (Cot[
x]*Csc[x])/(a + a*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{\int \csc^3(x)(-3a+2a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{2\int \csc^2(x)\,dx}{a}+\frac{3\int \csc^3(x)\,dx}{a}\\ &= -\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}+\frac{3\int \csc(x)\,dx}{2a}+\frac{2\mathrm{Subst}(\int 1\,dx,x,\cot(x))}{a}\\ &= -\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica [A]**
time = 0.24, size = 83, normalized size = 1.98

$$\frac{4\cot\left(\frac{x}{2}\right)-\csc^2\left(\frac{x}{2}\right)-12\log\left(\cos\left(\frac{x}{2}\right)\right)+12\log\left(\sin\left(\frac{x}{2}\right)\right)+\sec^2\left(\frac{x}{2}\right)-\frac{16\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-4\tan\left(\frac{x}{2}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (4*Cot[x/2] - Csc[x/2]^2 - 12*Log[Cos[x/2]] + 12*Log[Sin[x/2]] + Sec[x/2]^2
 - (16*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 4*Tan[x/2])/(8*a)
```

**Maple [A]**
time = 0.14, size = 54, normalized size = 1.29

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| default | $\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-2\tan\left(\frac{x}{2}\right)-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}+\frac{2}{\tan\left(\frac{x}{2}\right)}+6\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{8}{\tan\left(\frac{x}{2}\right)+1}}{4a}$ | 54 |
| norman | $\frac{\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{1}{8a}+\frac{3\tan\left(\frac{x}{2}\right)}{8a}-\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^5\left(\frac{x}{2}\right)}{8a}}{\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$ | 75 |
| risch | $\frac{3\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+3i\mathrm{e}^{3ix}+4-i\mathrm{e}^{ix}}{\left(\mathrm{e}^{2ix}-1\right)^2\left(\mathrm{e}^{ix}+i\right)a}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)}{2a}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)}{2a}$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(1/2*tan(1/2*x)^2-2*tan(1/2*x)-1/2/tan(1/2*x)^2+2/tan(1/2*x)+6*ln(tan
(1/2*x))+8/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(38) = 76$.
time = 0.31, size = 97, normalized size = 2.31

$$-\frac{\frac{4\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a}+\frac{\frac{3\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^2}{(\cos(x)+1)^2}-1}{8\left(\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a + 1/8*(3*sin(x)/(c
os(x) + 1) + 20*sin(x)^2/(cos(x) + 1)^2 - 1)/(a*sin(x)^2/(cos(x) + 1)^2 + a
*sin(x)^3/(cos(x) + 1)^3) + 3/2*log(sin(x)/(cos(x) + 1))/a
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(38) = 76$.
time = 0.41, size = 134, normalized size = 3.19

$$\frac{8\cos(x)^3+6\cos(x)^2-3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\left(4\cos(x)^2+\cos(x)-2\right)\sin(x)-6\cos(x)-4}{4\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)+\left(a\cos(x)^2-a\right)\sin(x)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(x)^3 + 6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2 - 1)*sin(
x) - cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2
 - 1)*sin(x) - cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2*(4*cos(x)^2 + cos(x)
- 2)*sin(x) - 6*cos(x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) + (a*cos(x)
^2 - a)*sin(x) - a)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(sin(x) + 1), x)/a
```

**Giac** [A]
time = 0.47, size = 73, normalized size = 1.74

$$\frac{3\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4a\tan\left(\frac{1}{2}x\right)}{8a^2}+\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}-\frac{18\tan\left(\frac{1}{2}x\right)^2-4\tan\left(\frac{1}{2}x\right)+1}{8a\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*log(abs(tan(1/2*x)))/a + 1/8*(a*tan(1/2*x)^2 - 4*a*tan(1/2*x))/a^2 + 2/
(a*(tan(1/2*x) + 1)) - 1/8*(18*tan(1/2*x)^2 - 4*tan(1/2*x) + 1)/(a*tan(1/2*
x)^2)
```

**Mupad** [B]
time = 6.60, size = 69, normalized size = 1.64

$$\frac{10\tan\left(\frac{x}{2}\right)^2+\frac{3\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2}}{4a\tan\left(\frac{x}{2}\right)^3+4a\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{2a}+\frac{\tan\left(\frac{x}{2}\right)^2}{8a}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))),x)
```

```
[Out] ((3*tan(x/2))/2 + 10*tan(x/2)^2 - 1/2)/(4*a*tan(x/2)^2 + 4*a*tan(x/2)^3) -
tan(x/2)/(2*a) + tan(x/2)^2/(8*a) + (3*log(tan(x/2)))/(2*a)
```

# 3.11 $\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx$

**Optimal**. Leaf size=55

$$\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}$$

```
[Out] 3/2*arctanh(cos(x))/a-4*cot(x)/a-4/3*cot(x)^3/a+3/2*cot(x)*csc(x)/a+cot(x)*
csc(x)^2/(a+a*sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2847, 2827, 3852, 3853, 3855}

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (3*ArcTanh[Cos[x]])/(2*a) - (4*Cot[x])/a - (4*Cot[x]^3)/(3*a) + (3*Cot[x]*C
sc[x])/(2*a) + (Cot[x]*Csc[x]^2)/(a + a*Sin[x])
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{\int \csc^4(x)(-4a+3a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc^3(x)\,dx}{a}+\frac{4\int \csc^4(x)\,dx}{a}\\ &= \frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc(x)\,dx}{2a}-\frac{4\mathrm{Subst}\left(\int (1+x^2)\,dx,x,\cot(x)\right)}{a}\\ &= \frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.
time = 0.56, size = 113, normalized size = 2.05

$$\frac{-20\cot\left(\frac{x}{2}\right)+3\csc^2\left(\frac{x}{2}\right)+36\log\left(\cos\left(\frac{x}{2}\right)\right)-36\log\left(\sin\left(\frac{x}{2}\right)\right)-3\sec^2\left(\frac{x}{2}\right)+8\csc^3(x)\sin^4\left(\frac{x}{2}\right)+\frac{48\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)\sin(x)+20\tan\left(\frac{x}{2}\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-20*Cot[x/2] + 3*Csc[x/2]^2 + 36*Log[Cos[x/2]] - 36*Log[Sin[x/2]] - 3*Sec[
x/2]^2 + 8*Csc[x]^3*Sin[x/2]^4 + (48*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (Csc
[x/2]^4*Sin[x])/2 + 20*Tan[x/2])/(24*a)
```

**Maple [A]**
time = 0.13, size = 68, normalized size = 1.24

| method | result | size |
|---|---|---|
| default | $\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{x}{2}\right)\right)+7\tan\left(\frac{x}{2}\right)-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}+\frac{1}{\tan\left(\frac{x}{2}\right)^2}-\frac{7}{\tan\left(\frac{x}{2}\right)}-12\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{16}{\tan\left(\frac{x}{2}\right)+1}}{8a}$ | 68 |

| | | |
|---|---|---|
| norman | $\frac{-\frac{1}{24a}+\frac{\tan\left(\frac{x}{2}\right)}{12a}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a}-\frac{\tan^6\left(\frac{x}{2}\right)}{12a}+\frac{\tan^7\left(\frac{x}{2}\right)}{24a}-\frac{15\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a}}{\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$ | 97 |
| risch | $-\frac{9i\,e^{5ix}-24\,e^{4ix}+9\,e^{6ix}-24i\,e^{3ix}+39\,e^{2ix}+7i\,e^{ix}-16}{3(e^{2ix}-1)^3(e^{ix}+i)a}+\frac{3\ln(e^{ix}+1)}{2a}-\frac{3\ln(e^{ix}-1)}{2a}$ | 99 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(1/3*tan(1/2*x)^3-tan(1/2*x)^2+7*tan(1/2*x)-1/3/tan(1/2*x)^3+1/tan(1/
2*x)^2-7/tan(1/2*x)-12*ln(tan(1/2*x))-16/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(49) = 98$.

time = 0.40, size = 120, normalized size = 2.18

$$\frac{\frac{21\,\sin(x)}{\cos(x)+1}-\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{24\,a}+\frac{\frac{2\,\sin(x)}{\cos(x)+1}-\frac{18\,\sin(x)^2}{(\cos(x)+1)^2}-\frac{69\,\sin(x)^3}{(\cos(x)+1)^3}-1}{24\left(\frac{a\sin(x)^3}{(\cos(x)+1)^3}+\frac{a\sin(x)^4}{(\cos(x)+1)^4}\right)}-\frac{3\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(21*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x)
 + 1)^3)/a + 1/24*(2*sin(x)/(cos(x) + 1) - 18*sin(x)^2/(cos(x) + 1)^2 - 69*
sin(x)^3/(cos(x) + 1)^3 - 1)/(a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x
) + 1)^4) - 3/2*log(sin(x)/(cos(x) + 1))/a
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(49) = 98$.

time = 0.40, size = 168, normalized size = 3.05

$$\frac{32\,\cos(x)^4+14\,\cos(x)^3-48\,\cos(x)^2+9\left(\cos(x)^4-2\,\cos(x)^2-\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\sin(x)+1\right)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)-9\left(\cos(x)^4-2\,\cos(x)^2-\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\sin(x)+1\right)\log\left(-\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)+2\left(16\,\cos(x)^3+9\,\cos(x)^2-15\,\cos(x)-6\right)\sin(x)-18\,\cos(x)+12}{12\left(a\cos(x)^4-2\,a\cos(x)^2-\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*cos(x)^4 + 14*cos(x)^3 - 48*cos(x)^2 + 9*(cos(x)^4 - 2*cos(x)^2 -
(cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(1/2*cos(x) + 1/2) - 9*(c
os(x)^4 - 2*cos(x)^2 - (cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(-
1/2*cos(x) + 1/2) + 2*(16*cos(x)^3 + 9*cos(x)^2 - 15*cos(x) - 6)*sin(x) - 1
8*cos(x) + 12)/(a*cos(x)^4 - 2*a*cos(x)^2 - (a*cos(x)^3 + a*cos(x)^2 - a*co
s(x) - a)*sin(x) + a)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(sin(x) + 1), x)/a
```

**Giac [A]**
time = 0.48, size = 96, normalized size = 1.75

$$-\frac{3 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-3a^2\tan\left(\frac{1}{2}x\right)^2+21a^2\tan\left(\frac{1}{2}x\right)}{24a^3}-\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}+\frac{66\tan\left(\frac{1}{2}x\right)^3-21\tan\left(\frac{1}{2}x\right)^2+3\tan\left(\frac{1}{2}x\right)-1}{24a\tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(tan(1/2*x)))/a + 1/24*(a^2*tan(1/2*x)^3 - 3*a^2*tan(1/2*x)^2 +
 21*a^2*tan(1/2*x))/a^3 - 2/(a*(tan(1/2*x) + 1)) + 1/24*(66*tan(1/2*x)^3 -
21*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/(a*tan(1/2*x)^3)
```

**Mupad [B]**
time = 6.50, size = 89, normalized size = 1.62

$$\frac{7\tan\left(\frac{x}{2}\right)}{8a}-\frac{23\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-\frac{2\tan\left(\frac{x}{2}\right)}{3}+\frac{1}{3}}{8a\tan\left(\frac{x}{2}\right)^4+8a\tan\left(\frac{x}{2}\right)^3}-\frac{\tan\left(\frac{x}{2}\right)^2}{8a}+\frac{\tan\left(\frac{x}{2}\right)^3}{24a}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))),x)
```

```
[Out] (7*tan(x/2))/(8*a) - (6*tan(x/2)^2 - (2*tan(x/2))/3 + 23*tan(x/2)^3 + 1/3)/
(8*a*tan(x/2)^3 + 8*a*tan(x/2)^4) - tan(x/2)^2/(8*a) + tan(x/2)^3/(24*a) -
(3*log(tan(x/2)))/(2*a)
```

## 3.12 $\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=66

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}-\frac{7\cos(x)\sin(x)}{2a^2}+\frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}$$

```
[Out] 7/2*x/a^2+16/3*cos(x)/a^2-7/2*cos(x)*sin(x)/a^2+8/3*cos(x)*sin(x)^2/a^2/(1+
sin(x))+1/3*cos(x)*sin(x)^3/(a+a*sin(x))^2
```

**Rubi** [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2844, 3056, 2813}

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}+\frac{8\sin^2(x)\cos(x)}{3a^2(\sin(x)+1)}-\frac{7\sin(x)\cos(x)}{2a^2}+\frac{\sin^3(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (7*x)/(2*a^2) + (16*Cos[x])/(3*a^2) - (7*Cos[x]*Sin[x])/(2*a^2) + (8*Cos[x]
*Sin[x]^2)/(3*a^2*(1 + Sin[x])) + (Cos[x]*Sin[x]^3)/(3*(a + a*Sin[x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{\sin^2(x)(3a-5a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}-\frac{\int \sin(x)\left(16a^2-21a^2\sin(x)\right)\,dx}{3a^4}\\ &= \frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}-\frac{7\cos(x)\sin(x)}{2a^2}+\frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}\end{aligned}$$

**Mathematica [A]**
time = 0.18, size = 100, normalized size = 1.52

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(21(-7+12x)\cos\left(\frac{x}{2}\right)+(239-84x)\cos\left(\frac{3x}{2}\right)+3\left(-5\cos\left(\frac{5x}{2}\right)+\cos\left(\frac{7x}{2}\right)+2(-50+56x+(27+28x)\cos(x)+6\cos(2x)+\cos(3x))\sin\left(\frac{x}{2}\right)\right)\right)}{48a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(21*(-7 + 12*x)*Cos[x/2] + (239 - 84*x)*Cos[(3*x)/2]
 + 3*(-5*Cos[(5*x)/2] + Cos[(7*x)/2] + 2*(-50 + 56*x + (27 + 28*x)*Cos[x] +
 6*Cos[2*x] + Cos[3*x])*Sin[x/2])))/(48*a^2*(1 + Sin[x])^2)
```

**Maple [A]**
time = 0.15, size = 80, normalized size = 1.21

| method | result |
|---|---|
| default | $\frac{-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{6}{\tan\left(\frac{x}{2}\right)+1}+\frac{2\left(\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+2\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{\tan\left(\frac{x}{2}\right)}{2}+2\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+7\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$ |
| risch | $\frac{7x}{2a^2}+\frac{i e^{2ix}}{8a^2}+\frac{e^{ix}}{a^2}+\frac{e^{-ix}}{a^2}-\frac{i e^{-2ix}}{8a^2}+\frac{8e^{2ix}-\frac{22}{3}+14i e^{ix}}{\left(e^{ix}+i\right)^3 a^2}$ |
| norman | $\frac{\frac{7\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{a}+\frac{25\tan\left(\frac{x}{2}\right)}{a}+\frac{92\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{7x}{2a}+\frac{32}{3a}+\frac{21x\tan\left(\frac{x}{2}\right)}{2a}+\frac{49x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}+\frac{91x\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a}+\frac{63x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{77x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+77$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)

[Out] 32/a^2*(-1/24/(tan(1/2*x)+1)^3+1/16/(tan(1/2*x)+1)^2+3/16/(tan(1/2*x)+1)+1/16*(1/2*tan(1/2*x)^3+2*tan(1/2*x)^2-1/2*tan(1/2*x)+2)/(tan(1/2*x)^2+1)^2+7/32*arctan(tan(1/2*x)))

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(56) = 112$.
time = 0.73, size = 198, normalized size = 3.00

$$\frac{\frac{75\sin(x)}{\cos(x)+1}+\frac{97\sin(x)^2}{(\cos(x)+1)^2}+\frac{126\sin(x)^3}{(\cos(x)+1)^3}+\frac{98\sin(x)^4}{(\cos(x)+1)^4}+\frac{63\sin(x)^5}{(\cos(x)+1)^5}+\frac{21\sin(x)^6}{(\cos(x)+1)^6}+32}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{5a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{7a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{7a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{5a^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{3a^2\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^2\sin(x)^7}{(\cos(x)+1)^7}\right)}+\frac{7\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")

[Out] 1/3*(75*sin(x)/(cos(x) + 1) + 97*sin(x)^2/(cos(x) + 1)^2 + 126*sin(x)^3/(cos(x) + 1)^3 + 98*sin(x)^4/(cos(x) + 1)^4 + 63*sin(x)^5/(cos(x) + 1)^5 + 21*sin(x)^6/(cos(x) + 1)^6 + 32)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 5*a^2*sin(x)^2/(cos(x) + 1)^2 + 7*a^2*sin(x)^3/(cos(x) + 1)^3 + 7*a^2*sin(x)^4/(cos(x) + 1)^4 + 5*a^2*sin(x)^5/(cos(x) + 1)^5 + 3*a^2*sin(x)^6/(cos(x) + 1)^6 + a^2*sin(x)^7/(cos(x) + 1)^7) + 7*arctan(sin(x)/(cos(x) + 1))/a^2

**Fricas** [A]
time = 0.36, size = 105, normalized size = 1.59

$$-\frac{3\cos(x)^4-(21x-31)\cos(x)^2-6\cos(x)^3+(21x+38)\cos(x)+\left(3\cos(x)^3+(21x+40)\cos(x)+9\cos(x)^2+42x+2\right)\sin(x)+42x-2}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")

[Out] -1/6*(3*cos(x)^4 - (21*x - 31)*cos(x)^2 - 6*cos(x)^3 + (21*x + 38)*cos(x) + (3*cos(x)^3 + (21*x + 40)*cos(x) + 9*cos(x)^2 + 42*x + 2)*sin(x) + 42*x - 2)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1423 vs. $2(70) = 140$.
time = 5.05, size = 1423, normalized size = 21.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] 21*x*tan(x/2)**7/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/
2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 1
8*a**2*tan(x/2) + 6*a**2) + 63*x*tan(x/2)**6/(6*a**2*tan(x/2)**7 + 18*a**2*
tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)*
*3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 105*x*tan(x/2)**5/(
6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*ta
n(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) +
6*a**2) + 147*x*tan(x/2)**4/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*
a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(
x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 147*x*tan(x/2)**3/(6*a**2*tan(x/2)**
7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a*
*2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 105*x*t
an(x/2)**2/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5
+ 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2
*tan(x/2) + 6*a**2) + 63*x*tan(x/2)/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)*
*6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a
**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 21*x/(6*a**2*tan(x/2)**7 + 1
8*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*ta
n(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 42*tan(x/2)*
*6/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**
2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2
) + 6*a**2) + 126*tan(x/2)**5/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 3
0*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*ta
n(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 196*tan(x/2)**4/(6*a**2*tan(x/2)**
7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a*
*2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 252*tan
(x/2)**3/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 +
42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*t
an(x/2) + 6*a**2) + 194*tan(x/2)**2/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)*
*6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a
**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 150*tan(x/2)/(6*a**2*tan(x/2
)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42
*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 64/(
6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*ta
n(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) +
6*a**2)
```

**Giac [A]**
time = 0.58, size = 72, normalized size = 1.09

$$\frac{7\,x}{2\,a^2}+\frac{\tan\left(\frac{1}{2}\,x\right)^3+4\,\tan\left(\frac{1}{2}\,x\right)^2-\tan\left(\frac{1}{2}\,x\right)+4}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^2a^2}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,x\right)^2+21\,\tan\left(\frac{1}{2}\,x\right)+10\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*x/a^2 + (tan(1/2*x)^3 + 4*tan(1/2*x)^2 - tan(1/2*x) + 4)/((tan(1/2*x)^2
 + 1)^2*a^2) + 2/3*(9*tan(1/2*x)^2 + 21*tan(1/2*x) + 10)/(a^2*(tan(1/2*x) +
 1)^3)
```

**Mupad [B]**
time = 6.81, size = 77, normalized size = 1.17

$$\frac{7\,x}{2\,a^2}+\frac{7\tan\left(\frac{x}{2}\right)^6+21\tan\left(\frac{x}{2}\right)^5+\frac{98\tan\left(\frac{x}{2}\right)^4}{3}+42\tan\left(\frac{x}{2}\right)^3+\frac{97\tan\left(\frac{x}{2}\right)^2}{3}+25\tan\left(\frac{x}{2}\right)+\frac{32}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x))^2,x)
```

```
[Out] (7*x)/(2*a^2) + (25*tan(x/2) + (97*tan(x/2)^2)/3 + 42*tan(x/2)^3 + (98*tan(
x/2)^4)/3 + 21*tan(x/2)^5 + 7*tan(x/2)^6 + 32/3)/(a^2*(tan(x/2)^2 + 1)^2*(t
an(x/2) + 1)^3)
```

# 3.13 $\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=47

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(1+\sin(x))}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}$$

```
[Out] -2*x/a^2-4/3*cos(x)/a^2-2*cos(x)/a^2/(1+sin(x))+1/3*cos(x)*sin(x)^2/(a+a*si
n(x))^2
```

**Rubi** [A]
time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2844, 3047, 3102, 12, 2814, 2727}

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(\sin(x)+1)}+\frac{\sin^2(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] (-2*x)/a^2 - (4*Cos[x])/(3*a^2) - (2*Cos[x])/(a^2*(1 + Sin[x])) + (Cos[x]*S
in[x]^2)/(3*(a + a*Sin[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{\sin(x)(2a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{2a\sin(x)-4a\sin^2(x)}{a+a\sin(x)}\,dx}{3a^2}\\
&= -\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{6a^2\sin(x)}{a+a\sin(x)}\,dx}{3a^3}\\
&= -\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{2\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{2\cos(x)}{a^2+a^2\sin(x)}
\end{aligned}
$$

**Mathematica [A]**
time = 0.16, size = 84, normalized size = 1.79

$$
-\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(6(-5+6x)\cos\left(\frac{x}{2}\right)+(41-12x)\cos\left(\frac{3x}{2}\right)-3\cos\left(\frac{5x}{2}\right)+6(-9+8x+4(1+x)\cos(x)+\cos(2x))\sin\left(\frac{x}{2}\right)\right)}{12a^2(1+\sin(x))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] -1/12*((Cos[x/2] + Sin[x/2])*(6*(-5 + 6*x)*Cos[x/2] + (41 - 12*x)*Cos[(3*x)
/2] - 3*Cos[(5*x)/2] + 6*(-9 + 8*x + 4*(1 + x)*Cos[x] + Cos[2*x])*Sin[x/2])
)/(a^2*(1 + Sin[x])^2)
```

**Maple [A]**
time = 0.15, size = 56, normalized size = 1.19

| method | result |
|---|---|
| default | $\frac{\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{4}{\tan\left(\frac{x}{2}\right)+1}-\frac{2}{\tan^{2}\left(\frac{x}{2}\right)+1}-4\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$ |
| risch | $-\frac{2x}{a^{2}}-\frac{e^{ix}}{2a^{2}}-\frac{e^{-ix}}{2a^{2}}-\frac{2\left(15i e^{ix}+9\,e^{2ix}-8\right)}{3a^{2}\left(e^{ix}+i\right)^{3}}$ |
| norman | $\frac{-\frac{16\tan\left(\frac{x}{2}\right)}{a}-\frac{4\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{a}-\frac{2x}{a}-\frac{20}{3a}-\frac{6x\tan\left(\frac{x}{2}\right)}{a}-\frac{12x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}-\frac{20x\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}-\frac{24x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}-\frac{24x\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a}-\frac{20x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a}-1}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/a^2*(1/12/(tan(1/2*x)+1)^3-1/8/(tan(1/2*x)+1)^2-1/4/(tan(1/2*x)+1)-1/8/(
tan(1/2*x)^2+1)-1/4*arctan(tan(1/2*x)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(43) = 86$.
time = 0.55, size = 144, normalized size = 3.06

$$-\frac{4\left(\frac{12\sin(x)}{\cos(x)+1}+\frac{11\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{9\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3\sin(x)^{4}}{(\cos(x)+1)^{4}}+5\right)}{3\left(a^{2}+\frac{3a^{2}\sin(x)}{\cos(x)+1}+\frac{4a^{2}\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{4a^{2}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3a^{2}\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{a^{2}\sin(x)^{5}}{(\cos(x)+1)^{5}}\right)}-\frac{4\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -4/3*(12*sin(x)/(cos(x) + 1) + 11*sin(x)^2/(cos(x) + 1)^2 + 9*sin(x)^3/(cos
(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 5)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1
) + 4*a^2*sin(x)^2/(cos(x) + 1)^2 + 4*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*s
in(x)^4/(cos(x) + 1)^4 + a^2*sin(x)^5/(cos(x) + 1)^5) - 4*arctan(sin(x)/(co
s(x) + 1))/a^2
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(43) = 86$.
time = 0.34, size = 95, normalized size = 2.02

$$-\frac{(6\,x-11)\cos\left(x\right)^{2}+3\,\cos\left(x\right)^{3}-(6\,x+13)\cos\left(x\right)-\left(2\,(3\,x+7)\cos\left(x\right)+3\,\cos\left(x\right)^{2}+12\,x+1\right)\sin\left(x\right)-12\,x+1}{3\,\left(a^{2}\cos\left(x\right)^{2}-a^{2}\cos\left(x\right)-2\,a^{2}-\left(a^{2}\cos\left(x\right)+2\,a^{2}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*((6*x - 11)*cos(x)^2 + 3*cos(x)^3 - (6*x + 13)*cos(x) - (2*(3*x + 7)*c
os(x) + 3*cos(x)^2 + 12*x + 1)*sin(x) - 12*x + 1)/(a^2*cos(x)^2 - a^2*cos(x
) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(48) = 96$.
time = 2.61, size = 779, normalized size = 16.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] -6*x*tan(x/2)**5/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 18*x*tan(x/2)**4/(
3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan
(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 24*x*tan(x/2)**3/(3*a**2*tan(x/2)**5
 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*
tan(x/2) + 3*a**2) - 24*x*tan(x/2)**2/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)
**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
 - 18*x*tan(x/2)/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 6*x/(3*a**2*tan(x/
2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*
a**2*tan(x/2) + 3*a**2) - 12*tan(x/2)**4/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x
/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a*
*2) - 36*tan(x/2)**3/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan
(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 44*tan(x/2)**2
/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*t
an(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 48*tan(x/2)/(3*a**2*tan(x/2)**5 +
9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan
(x/2) + 3*a**2) - 20/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan
(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
```

**Giac** [A]
time = 0.51, size = 51, normalized size = 1.09

$$-\frac{2\,x}{a^2}-\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)a^2}-\frac{2\left(6\,\tan\left(\frac{1}{2}\,x\right)^2+15\,\tan\left(\frac{1}{2}\,x\right)+7\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*x/a^2 - 2/((tan(1/2*x)^2 + 1)*a^2) - 2/3*(6*tan(1/2*x)^2 + 15*tan(1/2*x)
 + 7)/(a^2*(tan(1/2*x) + 1)^3)
```

**Mupad [B]**
time = 6.46, size = 62, normalized size = 1.32

$$-\frac{2\,x}{a^2}-\frac{4\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4+12\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3+\frac{44\,{\mathrm{tan}\left(\frac{x}{2}\right)}^2}{3}+16\,\mathrm{tan}\left(\frac{x}{2}\right)+\frac{20}{3}}{a^2\,\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+1\right)\,{\left(\mathrm{tan}\left(\frac{x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x))^2,x)
```

```
[Out] - (2*x)/a^2 - (16*tan(x/2) + (44*tan(x/2)^2)/3 + 12*tan(x/2)^3 + 4*tan(x/2)
^4 + 20/3)/(a^2*(tan(x/2)^2 + 1)*(tan(x/2) + 1)^3)
```

# 3.14 $\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=35

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(1+\sin(x))}-\frac{\cos(x)}{3(a+a\sin(x))^2}$$

```
[Out] x/a^2+5/3*cos(x)/a^2/(1+sin(x))-1/3*cos(x)/(a+a*sin(x))^2
```

**Rubi** [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2837, 2814, 2727}

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] x/a^2 + (5*Cos[x])/(3*a^2*(1 + Sin[x])) - Cos[x]/(3*(a + a*Sin[x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx = -\frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{\int \frac{-2a+3a\sin(x)}{a+a\sin(x)}\,dx}{3a^2}$$
$$= \frac{x}{a^2} - \frac{\cos(x)}{3(a+a\sin(x))^2} - \frac{5\int \frac{1}{a+a\sin(x)}\,dx}{3a}$$
$$= \frac{x}{a^2} - \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{5\cos(x)}{3\left(a^2+a^2\sin(x)\right)}$$

**Mathematica** [A]
time = 0.09, size = 69, normalized size = 1.97

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(3(-4+3x)\cos\left(\frac{x}{2}\right)+(10-3x)\cos\left(\frac{3x}{2}\right)+6(-3+2x+x\cos(x))\sin\left(\frac{x}{2}\right)\right)}{6a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(3*(-4 + 3*x)*Cos[x/2] + (10 - 3*x)*Cos[(3*x)/2] + 6
*(-3 + 2*x + x*Cos[x])*Sin[x/2]))/(6*a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.10, size = 44, normalized size = 1.26

| method | result |
|---|---|
| risch | $\frac{x}{a^2}+\frac{4\,\mathrm{e}^{2ix}-\frac{10}{3}+6i\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+i\right)^3 a^2}$ |
| default | $\frac{-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{8}{4\tan\left(\frac{x}{2}\right)+4}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$ |
| norman | $\frac{\frac{x}{a}+\frac{x\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}+\frac{6\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{12\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{8}{3a}+\frac{3x\tan\left(\frac{x}{2}\right)}{a}+\frac{5x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{7x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{7x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{5x\left(\tan^5\left(\right.\right.}{a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2 a\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/a^2*(-1/6/(tan(1/2*x)+1)^3+1/4/(tan(1/2*x)+1)^2+1/4/(tan(1/2*x)+1)+1/4*ar
ctan(tan(1/2*x)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(31) = 62.
time = 0.51, size = 90, normalized size = 2.57

$$\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+4\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 4)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
1)^3) + 2*arctan(sin(x)/(cos(x) + 1))/a^2
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(31) = 62$.
time = 0.34, size = 82, normalized size = 2.34

$$\frac{(3\,x-5)\cos\left(x\right)^{2}-(3\,x+4)\cos\left(x\right)-((3\,x+5)\cos\left(x\right)+6\,x+1)\sin\left(x\right)-6\,x+1}{3\left(a^{2}\cos\left(x\right)^{2}-a^{2}\cos\left(x\right)-2\,a^{2}-(a^{2}\cos\left(x\right)+2\,a^{2})\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 5)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 5)*cos(x) + 6*x + 1)*s
in(x) - 6*x + 1)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*
sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(32) = 64$.
time = 1.35, size = 321, normalized size = 9.17

$$\frac{3x\tan^{3}\left(\frac{x}{2}\right)}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{9x\tan^{2}\left(\frac{x}{2}\right)}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{9x\tan\left(\frac{x}{2}\right)}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{3x}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{6\tan^{2}\left(\frac{x}{2}\right)}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{18\tan\left(\frac{x}{2}\right)}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}+\frac{8}{3a^{2}\tan^{3}\left(\frac{x}{2}\right)+9a^{2}\tan^{2}\left(\frac{x}{2}\right)+9a^{2}\tan\left(\frac{x}{2}\right)+3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] 3*x*tan(x/2)**3/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2)
+ 3*a**2) + 9*x*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a*
*2*tan(x/2) + 3*a**2) + 9*x*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)*
*2 + 9*a**2*tan(x/2) + 3*a**2) + 3*x/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)*
*2 + 9*a**2*tan(x/2) + 3*a**2) + 6*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2
*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 18*tan(x/2)/(3*a**2*tan(x/2)**3
+ 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 8/(3*a**2*tan(x/2)**3 +
9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
```

**Giac** [A]
time = 0.50, size = 35, normalized size = 1.00

$$\frac{x}{a^{2}}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,x\right)^{2}+9\,\tan\left(\frac{1}{2}\,x\right)+4\right)}{3\,a^{2}\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] x/a^2 + 2/3*(3*tan(1/2*x)^2 + 9*tan(1/2*x) + 4)/(a^2*(tan(1/2*x) + 1)^3)
```

**Mupad** [B]
time = 6.48, size = 34, normalized size = 0.97

$$\frac{x}{a^2}+\frac{2\tan\left(\frac{x}{2}\right)^2+6\tan\left(\frac{x}{2}\right)+\frac{8}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x))^2,x)
```

```
[Out] x/a^2 + (6*tan(x/2) + 2*tan(x/2)^2 + 8/3)/(a^2*(tan(x/2) + 1)^3)
```

## 3.15 $\int \frac{\sin(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=33

$$\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{2\cos(x)}{3\left(a^2+a^2\sin(x)\right)}$$

```
[Out] 1/3*cos(x)/(a+a*sin(x))^2-2/3*cos(x)/(a^2+a^2*sin(x))
```

**Rubi** [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2829, 2727}

$$\frac{\cos(x)}{3(a\sin(x)+a)^2}-\frac{2\cos(x)}{3\left(a^2\sin(x)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x])^2,x]
```

```
[Out] Cos[x]/(3*(a + a*Sin[x])^2) - (2*Cos[x])/(3*(a^2 + a^2*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= \frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{2\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 29, normalized size = 0.88

$$-\frac{-3+\cos(x)+\cos(2x)-4\sin(x)+\sin(2x)}{3a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^2,x]
[Out] -1/3*(-3 + Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.11, size = 27, normalized size = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}}{a^{2}}$ | 27 |
| risch | $-\frac{2\left(3i\,e^{ix}+3\,e^{2ix}-2\right)}{3\left(e^{ix}+i\right)^{3}a^{2}}$ | 33 |
| norman | $\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2}{3a}-\frac{2\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3a}}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}$ | 60 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 4/a^2*(1/3/(tan(1/2*x)+1)^3-1/2/(tan(1/2*x)+1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.
time = 0.40, size = 62, normalized size = 1.88

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="maxima")
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 1)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 3*a^2*s
in(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) + 1)^3)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.
time = 0.35, size = 60, normalized size = 1.82

$$\frac{2\cos(x)^2+(2\cos(x)+1)\sin(x)+\cos(x)-1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="fricas")

[Out] 1/3*(2*cos(x)^2 + (2*cos(x) + 1)*sin(x) + cos(x) - 1)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.
time = 0.77, size = 87, normalized size = 2.64

$$-\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{2}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+a*sin(x))**2,x)

[Out] -6*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)

**Giac** [A]
time = 0.48, size = 21, normalized size = 0.64

$$-\frac{2\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="giac")

[Out] -2/3*(3*tan(1/2*x) + 1)/(a^2*(tan(1/2*x) + 1)^3)

**Mupad** [B]
time = 6.30, size = 21, normalized size = 0.64

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)+1\right)}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a + a*sin(x))^2,x)

[Out] -(2*(3*tan(x/2) + 1))/(3*a^2*(tan(x/2) + 1)^3)

# 3.16 $\int \frac{1}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=33

$$-\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{\cos(x)}{3\left(a^2+a^2\sin(x)\right)}$$

```
[Out] -1/3*cos(x)/(a+a*sin(x))^2-1/3*cos(x)/(a^2+a^2*sin(x))
```

**Rubi** [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2729, 2727}

$$-\frac{\cos(x)}{3\left(a^2\sin(x)+a^2\right)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-2),x]
```

```
[Out] -1/3*Cos[x]/(a + a*Sin[x])^2 - Cos[x]/(3*(a^2 + a^2*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= -\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

**Mathematica** [A]
time = 0.02, size = 31, normalized size = 0.94

$$-\frac{-3+4\cos(x)+\cos(2x)-4\sin(x)+\sin(2x)}{6a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-2),x]
[Out] -1/6*(-3 + 4*Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.08, size = 35, normalized size = 1.06

| method | result | size |
|---|---|---|
| risch | $-\frac{2i(i+3\,\mathrm{e}^{ix})}{3(\mathrm{e}^{ix}+i)^3a^2}$ | 27 |
| default | $\frac{-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{2}{\tan\left(\frac{x}{2}\right)+1}}{a^2}$ | 35 |
| norman | $\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{4}{3a}}{a\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$ | 39 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 2/a^2*(-2/3/(tan(1/2*x)+1)^3+1/(tan(1/2*x)+1)^2-1/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(29) = 58.
time = 0.43, size = 74, normalized size = 2.24

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="maxima")
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 2)/(a^2 + 3*a^2*s
in(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
 1)^3)
```

**Fricas** [A]
time = 0.37, size = 58, normalized size = 1.76

$$\frac{\cos(x)^2+(\cos(x)-1)\sin(x)+2\cos(x)+1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + (cos(x) - 1)*sin(x) + 2*cos(x) + 1)/(a^2*cos(x)^2 - a^2*cos
(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(31) = 62$.
time = 0.27, size = 134, normalized size = 4.06

$$-\frac{6\tan^2\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{4}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**2,x)
```

```
[Out] -6*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) +
 3*a**2) - 6*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan
(x/2) + 3*a**2) - 4/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x
/2) + 3*a**2)
```

**Giac** [A]
time = 0.56, size = 29, normalized size = 0.88

$$-\frac{2\left(3\,\tan\left(\frac{1}{2}\,x\right)^2+3\,\tan\left(\frac{1}{2}\,x\right)+2\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*x)^2 + 3*tan(1/2*x) + 2)/(a^2*(tan(1/2*x) + 1)^3)
```

**Mupad** [B]
time = 6.31, size = 29, normalized size = 0.88

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)^2+3\tan\left(\frac{x}{2}\right)+2\right)}{3\,a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x))^2,x)
```

```
[Out] -(2*(3*tan(x/2) + 3*tan(x/2)^2 + 2))/(3*a^2*(tan(x/2) + 1)^3)
```

# 3.17 $\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=38

$$-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}$$

```
[Out] -arctanh(cos(x))/a^2+4/3*cos(x)/a^2/(1+sin(x))+1/3*cos(x)/(a+a*sin(x))^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2845, 3057, 12, 3855}

$$\frac{4\cos(x)}{3a^2(\sin(x)+1)}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (4*Cos[x])/(3*a^2*(1 + Sin[x])) + Cos[x]/(3*(a + a
*Sin[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc(x)(3a-a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int 3a^2\csc(x)\,dx}{3a^4}\\ &= \frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \csc(x)\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(38) = 76.
time = 0.10, size = 129, normalized size = 3.39

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{3x}{2}\right)\left(8+3\log\left(\cos\left(\frac{x}{2}\right)\right)-3\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\cos\left(\frac{x}{2}\right)\left(-6-9\log\left(\cos\left(\frac{x}{2}\right)\right)+9\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\left(3+2\log\left(\cos\left(\frac{x}{2}\right)\right)+\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin\left(\frac{x}{2}\right)\right)}{6a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(Cos[(3*x)/2]*(8 + 3*Log[Cos[x/2]] - 3*Log[Sin[x/2]]
) + Cos[x/2]*(-6 - 9*Log[Cos[x/2]] + 9*Log[Sin[x/2]]) - 6*(3 + 2*Log[Cos[x/
2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) - 2*Log[Sin[x/2]])*Sin[x/2]))/
(6*a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.16, size = 41, normalized size = 1.08

| method | result | size |
|---|---|---|
| default | $\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{4}{\tan\left(\frac{x}{2}\right)+1}}{a^{2}}$ | 41 |

| | | |
|---|---|---|
| norman | $\frac{\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{10}{3a}+\frac{6\tan\left(\frac{x}{2}\right)}{a}}{a\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$ | 49 |
| risch | $\frac{6i\mathrm{e}^{ix}+2\,\mathrm{e}^{2ix}-\frac{8}{3}}{\left(\mathrm{e}^{ix}+i\right)^3a^2}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{a^2}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{a^2}$ | 59 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(ln(tan(1/2*x))+4/3/(tan(1/2*x)+1)^3-2/(tan(1/2*x)+1)^2+4/(tan(1/2*x)
+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.
time = 0.30, size = 89, normalized size = 2.34

$$\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{6\sin(x)^2}{(\cos(x)+1)^2}+5\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 5)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
1)^3) + log(sin(x)/(cos(x) + 1))/a^2
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(34) = 68$.
time = 0.36, size = 117, normalized size = 3.08

$$-\frac{8\cos(x)^2+3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2\,(4\cos(x)-1)\sin(x)+10\cos(x)+2}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*cos(x)^2 + 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(1/2*
cos(x) + 1/2) - 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(-1/2*co
s(x) + 1/2) + 2*(4*cos(x) - 1)*sin(x) + 10*cos(x) + 2)/(a^2*cos(x)^2 - a^2*
cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

**Giac [A]**
time = 0.49, size = 40, normalized size = 1.05

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2}+\frac{2\left(6\,\tan\left(\frac{1}{2}\,x\right)^2+9\,\tan\left(\frac{1}{2}\,x\right)+5\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^2 + 2/3*(6*tan(1/2*x)^2 + 9*tan(1/2*x) + 5)/(a^2*(ta
n(1/2*x) + 1)^3)
```

**Mupad [B]**
time = 6.48, size = 38, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{4\tan\left(\frac{x}{2}\right)^2+6\tan\left(\frac{x}{2}\right)+\frac{10}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))^2),x)
```

```
[Out] log(tan(x/2))/a^2 + (6*tan(x/2) + 4*tan(x/2)^2 + 10/3)/(a^2*(tan(x/2) + 1)^
3)
```

# 3.18 $\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=45

$$\frac{2\tanh^{-1}(\cos(x))}{a^2}-\frac{10\cot(x)}{3a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}$$

```
[Out] 2*arctanh(cos(x))/a^2-10/3*cot(x)/a^2+2*cot(x)/a^2/(1+sin(x))+1/3*cot(x)/(a
+a*sin(x))^2
```

**Rubi** [A]
time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$,
Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$-\frac{10\cot(x)}{3a^2}+\frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(\sin(x)+1)}+\frac{\cot(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[x]])/a^2 - (10*Cot[x])/(3*a^2) + (2*Cot[x])/(a^2*(1 + Sin[x]
)) + Cot[x]/(3*(a + a*Sin[x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^2(x)(4a-2a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^2(x)\left(10a^2-6a^2\sin(x)\right)\,dx}{3a^4}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{2\int \csc(x)\,dx}{a^2}+\frac{10\int \csc^2(x)\,dx}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{10\mathrm{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}-\frac{10\cot(x)}{3a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}
\end{aligned}
$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(45) = 90.
time = 0.26, size = 166, normalized size = 3.69

$$
\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)-2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+28\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2-3\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+12\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-12\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+3\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3\tan\left(\frac{x}{2}\right)\right)}{6(a+a\sin(x))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 28*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 3*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 12*Log[Cos[x
/2]]*(Cos[x/2] + Sin[x/2])^3 - 12*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 3
*(Cos[x/2] + Sin[x/2])^3*Tan[x/2]))/(6*(a + a*Sin[x])^2)
```

**Maple** [A]
time = 0.14, size = 56, normalized size = 1.24

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)-\frac{8}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{12}{\tan\left(\frac{x}{2}\right)+1}-\frac{1}{\tan\left(\frac{x}{2}\right)}-4\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^{2}}$ | 56 |
| norman | $\frac{-\frac{1}{2a}+\frac{\tan^{5}\left(\frac{x}{2}\right)}{2a}-\frac{25\tan\left(\frac{x}{2}\right)}{3a}-\frac{31\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2a}-\frac{19\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2a}}{a\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$ | 78 |
| risch | $-\frac{4\left(-11\,\mathrm{e}^{2ix}+9i\mathrm{e}^{3ix}+5-12i\mathrm{e}^{ix}+3\,\mathrm{e}^{4ix}\right)}{3\left(\mathrm{e}^{2ix}-1\right)\left(\mathrm{e}^{ix}+i\right)^{3}a^{2}}-\frac{2\ln\left(\mathrm{e}^{ix}-1\right)}{a^{2}}+\frac{2\ln\left(\mathrm{e}^{ix}+1\right)}{a^{2}}$ | 84 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(tan(1/2*x)-8/3/(tan(1/2*x)+1)^3+4/(tan(1/2*x)+1)^2-12/(tan(1/2*x)+
1)-1/tan(1/2*x)-4*ln(tan(1/2*x)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(41) = 82$.
time = 0.32, size = 126, normalized size = 2.80

$$-\frac{\frac{41\sin(x)}{\cos(x)+1}+\frac{69\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{39\sin(x)^{3}}{(\cos(x)+1)^{3}}+3}{6\left(\frac{a^{2}\sin(x)}{\cos(x)+1}+\frac{3a^{2}\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{3a^{2}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{a^{2}\sin(x)^{4}}{(\cos(x)+1)^{4}}\right)}-\frac{2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^{2}}+\frac{\sin(x)}{2a^{2}(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(41*sin(x)/(cos(x) + 1) + 69*sin(x)^2/(cos(x) + 1)^2 + 39*sin(x)^3/(co
s(x) + 1)^3 + 3)/(a^2*sin(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 +
 3*a^2*sin(x)^3/(cos(x) + 1)^3 + a^2*sin(x)^4/(cos(x) + 1)^4) - 2*log(sin(x
)/(cos(x) + 1))/a^2 + 1/2*sin(x)/(a^2*(cos(x) + 1))
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(41) = 82$.
time = 0.36, size = 168, normalized size = 3.73

$$-\frac{10\cos(x)^{3}-4\cos(x)^{2}-3\left(\cos(x)^{3}+2\cos(x)^{2}+\left(\cos(x)^{2}-\cos(x)-2\right)\sin(x)-\cos(x)-2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^{3}+2\cos(x)^{2}+\left(\cos(x)^{2}-\cos(x)-2\right)\sin(x)-\cos(x)-2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(10\cos(x)^{2}+14\cos(x)+1\right)\sin(x)-13\cos(x)+1}{3\left(a^{2}\cos(x)^{3}+2a^{2}\cos(x)^{2}-a^{2}\cos(x)-2a^{2}+\left(a^{2}\cos(x)^{2}-a^{2}\cos(x)-2a^{2}\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*cos(x)^3 - 4*cos(x)^2 - 3*(cos(x)^3 + 2*cos(x)^2 + (cos(x)^2 - cos
(x) - 2)*sin(x) - cos(x) - 2)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + 2*cos(x
)^2 + (cos(x)^2 - cos(x) - 2)*sin(x) - cos(x) - 2)*log(-1/2*cos(x) + 1/2) -
 (10*cos(x)^2 + 14*cos(x) + 1)*sin(x) - 13*cos(x) + 1)/(a^2*cos(x)^3 + 2*a^
2*cos(x)^2 - a^2*cos(x) - 2*a^2 + (a^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x
))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

**Giac [A]**
time = 0.56, size = 69, normalized size = 1.53

$$-\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^2}+\frac{4\,\tan\left(\frac{1}{2}\,x\right)-1}{2\,a^2\tan\left(\frac{1}{2}\,x\right)}-\frac{2\left(9\,\tan\left(\frac{1}{2}\,x\right)^2+15\,\tan\left(\frac{1}{2}\,x\right)+8\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*x)/a^2 + 1/2*(4*tan(1/2*x) - 1)/(
a^2*tan(1/2*x)) - 2/3*(9*tan(1/2*x)^2 + 15*tan(1/2*x) + 8)/(a^2*(tan(1/2*x)
 + 1)^3)
```

**Mupad [B]**
time = 6.54, size = 91, normalized size = 2.02

$$\frac{\tan\left(\frac{x}{2}\right)}{2\,a^2}-\frac{13\tan\left(\frac{x}{2}\right)^3+23\tan\left(\frac{x}{2}\right)^2+\frac{41\tan\left(\frac{x}{2}\right)}{3}+1}{2\,a^2\tan\left(\frac{x}{2}\right)^4+6\,a^2\tan\left(\frac{x}{2}\right)^3+6\,a^2\tan\left(\frac{x}{2}\right)^2+2\,a^2\tan\left(\frac{x}{2}\right)}-\frac{2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - ((41*tan(x/2))/3 + 23*tan(x/2)^2 + 13*tan(x/2)^3 + 1)/(2
*a^2*tan(x/2) + 6*a^2*tan(x/2)^2 + 6*a^2*tan(x/2)^3 + 2*a^2*tan(x/2)^4) - (
2*log(tan(x/2)))/a^2
```

## 3.19 $\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=64

$$-\frac{7\tanh^{-1}(\cos(x))}{2a^2}+\frac{16\cot(x)}{3a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}$$

```
[Out] -7/2*arctanh(cos(x))/a^2+16/3*cot(x)/a^2-7/2*cot(x)*csc(x)/a^2+8/3*cot(x)*c
sc(x)/a^2/(1+sin(x))+1/3*cot(x)*csc(x)/(a+a*sin(x))^2
```

**Rubi** [A]
time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\frac{16\cot(x)}{3a^2}-\frac{7\tanh^{-1}(\cos(x))}{2a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[x]])/(2*a^2) + (16*Cot[x])/(3*a^2) - (7*Cot[x]*Csc[x])/(2*a
^2) + (8*Cot[x]*Csc[x])/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x])/(3*(a + a*Si
n[x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^3(x)(5a-3a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^3(x)\left(21a^2-16a^2\sin(x)\right)\,dx}{3a^4}\\
&= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}-\frac{16\int \csc^2(x)\,dx}{3a^2}+\frac{7\int \csc^3(x)\,dx}{a^2}\\
&= -\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{7\int \csc(x)\,dx}{2a^2}+\frac{16\text{Subst}(\\
&= -\frac{7\tanh^{-1}(\cos(x))}{2a^2}+\frac{16\cot(x)}{3a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(\\
\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 203 vs. 2(64) = 128.
time = 0.46, size = 203, normalized size = 3.17

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-16\sin\left(\frac{x}{2}\right)-3\left(1+\cot\left(\frac{x}{2}\right)\right)^3\sin\left(\frac{x}{2}\right)+8\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-160\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2+24\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-84\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+84\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-24\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3\tan\left(\frac{x}{2}\right)+3\cos\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^3\right)}{24a^2(1+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-16*Sin[x/2] - 3*(1 + Cot[x/2])^3*Sin[x/2] + 8*(Cos
[x/2] + Sin[x/2]) - 160*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 24*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^3 - 84*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 84*Log[Sin
[x/2]]*(Cos[x/2] + Sin[x/2])^3 - 24*(Cos[x/2] + Sin[x/2])^3*Tan[x/2] + 3*Co
s[x/2]*(1 + Tan[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.21, size = 74, normalized size = 1.16

| method | result | size |
|---|---|---|
| default | $\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-4\tan\left(\frac{x}{2}\right)-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}+\frac{4}{\tan\left(\frac{x}{2}\right)}+14\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{16}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{8}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{32}{\tan\left(\frac{x}{2}\right)+1}}{4a^2}$ | 74 |
| risch | $\frac{63i e^{5ix}+21 e^{6ix}-126i e^{3ix}-98 e^{4ix}+75i e^{ix}+97 e^{2ix}-32}{3(e^{2ix}-1)^2(e^{ix}+i)^3a^2}-\frac{7\ln(e^{ix}+1)}{2a^2}+\frac{7\ln(e^{ix}-1)}{2a^2}$ | 99 |
| norman | $\frac{\frac{14\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{1}{8a}+\frac{5\tan\left(\frac{x}{2}\right)}{8a}-\frac{5\left(\tan^6\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^7\left(\frac{x}{2}\right)}{8a}+\frac{95\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a}+\frac{151\left(\tan^2\left(\frac{x}{2}\right)\right)}{12a}}{\tan\left(\frac{x}{2}\right)^2a\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{7\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2}$ | 100 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/2*tan(1/2*x)^2-4*tan(1/2*x)-1/2/tan(1/2*x)^2+4/tan(1/2*x)+14*ln(
tan(1/2*x))+16/3/(tan(1/2*x)+1)^3-8/(tan(1/2*x)+1)^2+32/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(54) = 108.
time = 0.29, size = 155, normalized size = 2.42

$$\frac{\frac{15\sin(x)}{\cos(x)+1}+\frac{239\sin(x)^2}{(\cos(x)+1)^2}+\frac{405\sin(x)^3}{(\cos(x)+1)^3}+\frac{216\sin(x)^4}{(\cos(x)+1)^4}-3}{24\left(\frac{a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^2\sin(x)^5}{(\cos(x)+1)^5}\right)}-\frac{\frac{8\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a^2}+\frac{7\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(15*sin(x)/(cos(x) + 1) + 239*sin(x)^2/(cos(x) + 1)^2 + 405*sin(x)^3/(
cos(x) + 1)^3 + 216*sin(x)^4/(cos(x) + 1)^4 - 3)/(a^2*sin(x)^2/(cos(x) + 1)
```

```
^2 + 3*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + a^2*si
n(x)^5/(cos(x) + 1)^5) - 1/8*(8*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)
^2)/a^2 + 7/2*log(sin(x)/(cos(x) + 1))/a^2
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(54) = 108$.
time = 0.36, size = 220, normalized size = 3.44

$$-\frac{64 \cos(x)^4 + 86 \cos(x)^3 - 54 \cos(x)^2 + 21 \left(\cos(x)^4 - \cos(x)^3 - 3 \cos(x)^2 - \left(\cos(x)^3 + 2 \cos(x)^2 - \cos(x) - 2\right) \sin(x) + \cos(x) + 2\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 21 \left(\cos(x)^4 - \cos(x)^3 - 3 \cos(x)^2 - \left(\cos(x)^3 + 2 \cos(x)^2 - \cos(x) - 2\right) \sin(x) + \cos(x) + 2\right) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2 \left(32 \cos(x)^3 - 11 \cos(x)^2 - 38 \cos(x) + 2\right) \sin(x) - 80 \cos(x) - 4}{12 \left(a^2 \cos(x)^4 - a^2 \cos(x)^3 - 3 a^2 \cos(x)^2 + a^2 \cos(x) + 2 a^2 - \left(a^2 \cos(x)^3 + 2 a^2 \cos(x)^2 - a^2 \cos(x) - 2 a^2\right) \sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(64*cos(x)^4 + 86*cos(x)^3 - 54*cos(x)^2 + 21*(cos(x)^4 - cos(x)^3 -
3*cos(x)^2 - (cos(x)^3 + 2*cos(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(
1/2*cos(x) + 1/2) - 21*(cos(x)^4 - cos(x)^3 - 3*cos(x)^2 - (cos(x)^3 + 2*co
s(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) + 2*(32*co
s(x)^3 - 11*cos(x)^2 - 38*cos(x) + 2)*sin(x) - 80*cos(x) - 4)/(a^2*cos(x)^4
 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2 - (a^2*cos(x)^3 + 2*a
^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

**Giac** [A]
time = 0.54, size = 93, normalized size = 1.45

$$\frac{7 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2 a^2} + \frac{a^2 \tan\left(\frac{1}{2}x\right)^2 - 8 a^2 \tan\left(\frac{1}{2}x\right)}{8 a^4} - \frac{42 \tan\left(\frac{1}{2}x\right)^2 - 8 \tan\left(\frac{1}{2}x\right) + 1}{8 a^2 \tan\left(\frac{1}{2}x\right)^2} + \frac{2 \left(12 \tan\left(\frac{1}{2}x\right)^2 + 21 \tan\left(\frac{1}{2}x\right) + 11\right)}{3 a^2 \left(\tan\left(\frac{1}{2}x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*log(abs(tan(1/2*x)))/a^2 + 1/8*(a^2*tan(1/2*x)^2 - 8*a^2*tan(1/2*x))/a^
4 - 1/8*(42*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(a^2*tan(1/2*x)^2) + 2/3*(12*t
an(1/2*x)^2 + 21*tan(1/2*x) + 11)/(a^2*(tan(1/2*x) + 1)^3)
```

**Mupad** [B]
time = 6.40, size = 111, normalized size = 1.73

$$\frac{36\tan\left(\frac{x}{2}\right)^4+\frac{135\tan\left(\frac{x}{2}\right)^3}{2}+\frac{239\tan\left(\frac{x}{2}\right)^2}{6}+\frac{5\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2}}{4\,a^2\tan\left(\frac{x}{2}\right)^5+12\,a^2\tan\left(\frac{x}{2}\right)^4+12\,a^2\tan\left(\frac{x}{2}\right)^3+4\,a^2\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{a^2}+\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^2}+\frac{7\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))^2),x)
```

```
[Out] ((5*tan(x/2))/2 + (239*tan(x/2)^2)/6 + (135*tan(x/2)^3)/2 + 36*tan(x/2)^4 -
 1/2)/(4*a^2*tan(x/2)^2 + 12*a^2*tan(x/2)^3 + 12*a^2*tan(x/2)^4 + 4*a^2*tan
(x/2)^5) - tan(x/2)/a^2 + tan(x/2)^2/(8*a^2) + (7*log(tan(x/2)))/(2*a^2)
```

# 3.20 $\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=65

$$\frac{5\tanh^{-1}(\cos(x))}{a^2}-\frac{4\cot(x)}{a^2}-\frac{\cot^3(x)}{3a^2}+\frac{\cot(x)\csc(x)}{a^2}-\frac{\cos(x)}{3a^2(1+\sin(x))^2}-\frac{13\cos(x)}{3a^2(1+\sin(x))}$$

```
[Out] 5*arctanh(cos(x))/a^2-4*cot(x)/a^2-1/3*cot(x)^3/a^2+cot(x)*csc(x)/a^2-1/3*c
os(x)/a^2/(1+sin(x))^2-13/3*cos(x)/a^2/(1+sin(x))
```

**Rubi** [A]
time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2845, 3057, 2827, 3852, 3853, 3855}

$$-\frac{4\cot^3(x)}{a^2}-\frac{12\cot(x)}{a^2}+\frac{5\tanh^{-1}(\cos(x))}{a^2}+\frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc^2(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[x]])/a^2 - (12*Cot[x])/a^2 - (4*Cot[x]^3)/a^2 + (5*Cot[x]*Cs
c[x])/a^2 + (10*Cot[x]*Csc[x]^2)/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x]^2)/(
3*(a + a*Sin[x])^2)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^4(x)(6a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^4(x)\left(36a^2-30a^2\sin(x)\right)\,dx}{3a^4}\\ &= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}-\frac{10\int \csc^3(x)\,dx}{a^2}+\frac{12\int \csc^4(x)\,dx}{a^2}\\ &= \frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}-\frac{5\int \csc(x)\,dx}{a^2}-\frac{12\text{Subst}(\int\end{aligned}$$

$$= \frac{5\tanh^{-1}(\cos(x))}{a^2}-\frac{12\cot(x)}{a^2}-\frac{4\cot^3(x)}{a^2}+\frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{c}{3(}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 238 vs. $2(65) = 130$.

time = 2.50, size = 238, normalized size = 3.66

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-\cos\left(\frac{x}{2}\right)\left(1+\cot\left(\frac{x}{2}\right)\right)^{3}+16\sin\left(\frac{x}{2}\right)+6\left(1+\cot\left(\frac{x}{2}\right)\right)^{3}\sin\left(\frac{x}{2}\right)-8\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+208\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^{2}-44\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^{3}+120\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^{3}-120\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^{3}+44\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^{3}\tan\left(\frac{x}{2}\right)-6\cos\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^{3}+\sin\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^{3}\right)}{24a^{2}(1+\sin(x))^{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-(Cos[x/2]*(1 + Cot[x/2])^3) + 16*Sin[x/2] + 6*(1 +
 Cot[x/2])^3*Sin[x/2] - 8*(Cos[x/2] + Sin[x/2]) + 208*Sin[x/2]*(Cos[x/2] +
Sin[x/2])^2 - 44*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 120*Log[Cos[x/2]]*(Cos[
x/2] + Sin[x/2])^3 - 120*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 44*(Cos[x/
2] + Sin[x/2])^3*Tan[x/2] - 6*Cos[x/2]*(1 + Tan[x/2])^3 + Sin[x/2]*(1 + Tan
[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

**Maple** [A]
time = 0.18, size = 90, normalized size = 1.38

| method | result |
|---|---|
| default | $\frac{\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}-2\left(\tan^{2}\left(\frac{x}{2}\right)\right)+15\tan\left(\frac{x}{2}\right)-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{x}{2}\right)^{2}}-\frac{15}{\tan\left(\frac{x}{2}\right)}-40\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{32}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{16}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{80}{\tan\left(\frac{x}{2}\right)+1}}{8a^{2}}$ |
| risch | $-\frac{2\left(-85\,e^{6ix}+45ie^{7ix}+153\,e^{4ix}-135ie^{5ix}+15\,e^{8ix}-99\,e^{2ix}+155ie^{3ix}+24-57ie^{ix}\right)}{3\left(e^{2ix}-1\right)^{3}\left(e^{ix}+i\right)^{3}a^{2}}+\frac{5\ln\left(e^{ix}+1\right)}{a^{2}}-\frac{5\ln\left(e^{ix}-1\right)}{a^{2}}$ |
| norman | $\frac{-\frac{1}{24a}+\frac{\tan\left(\frac{x}{2}\right)}{8a}-\frac{5\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{4a}+\frac{5\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4a}-\frac{\tan^{8}\left(\frac{x}{2}\right)}{8a}+\frac{\tan^{9}\left(\frac{x}{2}\right)}{24a}-\frac{115\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{6a}-\frac{145\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4a}-\frac{85\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4a}}{\tan\left(\frac{x}{2}\right)^{3}a\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^2*(1/3*tan(1/2*x)^3-2*tan(1/2*x)^2+15*tan(1/2*x)-1/3/tan(1/2*x)^3+2/t
an(1/2*x)^2-15/tan(1/2*x)-40*ln(tan(1/2*x))-32/3/(tan(1/2*x)+1)^3+16/(tan(1
/2*x)+1)^2-80/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(59) = 118$.
time = 0.39, size = 178, normalized size = 2.74

$$\frac{\frac{3\sin(x)}{\cos(x)+1}-\frac{30\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{342\sin(x)^{3}}{(\cos(x)+1)^{3}}-\frac{561\sin(x)^{4}}{(\cos(x)+1)^{4}}-\frac{285\sin(x)^{5}}{(\cos(x)+1)^{5}}-1}{24\left(\frac{a^{2}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3a^{2}\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{3a^{2}\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{a^{2}\sin(x)^{6}}{(\cos(x)+1)^{6}}\right)}+\frac{\frac{45\sin(x)}{\cos(x)+1}-\frac{6\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}}{24\,a^{2}}-\frac{5\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*sin(x)/(cos(x) + 1) - 30*sin(x)^2/(cos(x) + 1)^2 - 342*sin(x)^3/(co
s(x) + 1)^3 - 561*sin(x)^4/(cos(x) + 1)^4 - 285*sin(x)^5/(cos(x) + 1)^5 - 1
```

```
)/(a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + 3*a^2*sin(
x)^5/(cos(x) + 1)^5 + a^2*sin(x)^6/(cos(x) + 1)^6) + 1/24*(45*sin(x)/(cos(x
) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^2 - 5*log(s
in(x)/(cos(x) + 1))/a^2
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(59) = 118$.
time = 0.40, size = 266, normalized size = 4.09

$$-\frac{48 \cos(x)^5 - 18 \cos(x)^4 - 108 \cos(x)^3 + 22 \cos(x)^2 - 15\left(\cos(x)^5 + 2\cos(x)^4 - 2\cos(x)^3 - 4\cos(x)^2 + \left(\cos(x)^4 - \cos(x)^3 - 3\cos(x)^2 + \cos(x) + 2\right)\sin(x) + \cos(x) + 2\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 15\left(\cos(x)^5 + 2\cos(x)^4 - 2\cos(x)^3 - 4\cos(x)^2 + \left(\cos(x)^4 - \cos(x)^3 - 3\cos(x)^2 + \cos(x) + 2\right)\sin(x) + \cos(x) + 2\right)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 2\left(24\cos(x)^4 + 33\cos(x)^3 - 21\cos(x)^2 - 32\cos(x) - 1\right)\sin(x) + 62\cos(x) - 2}{6\left(a^2\cos(x)^5 + 2a^2\cos(x)^4 - 2a^2\cos(x)^3 - 4a^2\cos(x)^2 + a^2\cos(x) + 2a^2 + \left(a^2\cos(x)^4 - a^2\cos(x)^3 - 3a^2\cos(x)^2 + a^2\cos(x) + 2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(48*cos(x)^5 - 18*cos(x)^4 - 108*cos(x)^3 + 22*cos(x)^2 - 15*(cos(x)^5
 + 2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2
 + cos(x) + 2)*sin(x) + cos(x) + 2)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^5 +
2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2 +
cos(x) + 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) - 2*(24*cos(x)^4 +
33*cos(x)^3 - 21*cos(x)^2 - 32*cos(x) - 1)*sin(x) + 62*cos(x) - 2)/(a^2*cos
(x)^5 + 2*a^2*cos(x)^4 - 2*a^2*cos(x)^3 - 4*a^2*cos(x)^2 + a^2*cos(x) + 2*a
^2 + (a^2*cos(x)^4 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2)*si
n(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin^2(x)+2\sin(x)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

**Giac** [A]
time = 0.58, size = 114, normalized size = 1.75

$$-\frac{5\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2} + \frac{110\tan\left(\frac{1}{2}x\right)^6 + 45\tan\left(\frac{1}{2}x\right)^5 - 231\tan\left(\frac{1}{2}x\right)^4 - 232\tan\left(\frac{1}{2}x\right)^3 - 30\tan\left(\frac{1}{2}x\right)^2 + 3\tan\left(\frac{1}{2}x\right) - 1}{24\left(\tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right)\right)^3 a^2} + \frac{a^4\tan\left(\frac{1}{2}x\right)^3 - 6a^4\tan\left(\frac{1}{2}x\right)^2 + 45a^4\tan\left(\frac{1}{2}x\right)}{24a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -5*log(abs(tan(1/2*x)))/a^2 + 1/24*(110*tan(1/2*x)^6 + 45*tan(1/2*x)^5 - 23
1*tan(1/2*x)^4 - 232*tan(1/2*x)^3 - 30*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/((t
```

```
an(1/2*x)^2 + tan(1/2*x))^3*a^2) + 1/24*(a^4*tan(1/2*x)^3 - 6*a^4*tan(1/2*x
)^2 + 45*a^4*tan(1/2*x))/a^6
```

**Mupad [B]**
time = 6.40, size = 101, normalized size = 1.55

$$\frac{15\tan\left(\frac{x}{2}\right)}{8\,a^2}-\frac{\tan\left(\frac{x}{2}\right)^2}{4\,a^2}+\frac{\tan\left(\frac{x}{2}\right)^3}{24\,a^2}-\frac{5\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\frac{95\tan\left(\frac{x}{2}\right)^5}{8}+\frac{187\tan\left(\frac{x}{2}\right)^4}{8}+\frac{57\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\tan\left(\frac{x}{2}\right)^2}{4}-\frac{\tan\left(\frac{x}{2}\right)}{8}+\frac{1}{24}}{a^2\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))^2),x)
```

```
[Out] (15*tan(x/2))/(8*a^2) - tan(x/2)^2/(4*a^2) + tan(x/2)^3/(24*a^2) - (5*log(t
an(x/2)))/a^2 - ((5*tan(x/2)^2)/4 - tan(x/2)/8 + (57*tan(x/2)^3)/4 + (187*t
an(x/2)^4)/8 + (95*tan(x/2)^5)/8 + 1/24)/(a^2*tan(x/2)^3*(tan(x/2) + 1)^3)
```

## 3.21 $\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=101

$$-\frac{23x}{2a^3}-\frac{136\cos(x)}{5a^3}+\frac{136\cos^3(x)}{15a^3}+\frac{23\cos(x)\sin(x)}{2a^3}+\frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}+\frac{23\cos(x)\sin^3(x)}{3\,(a^3+a^3\sin(x))}$$

```
[Out] -23/2*x/a^3-136/5*cos(x)/a^3+136/15*cos(x)^3/a^3+23/2*cos(x)*sin(x)/a^3+1/5
*cos(x)*sin(x)^5/(a+a*sin(x))^3+13/15*cos(x)*sin(x)^4/a/(a+a*sin(x))^2+23/3
*cos(x)*sin(x)^3/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2844, 3056, 2827, 2715, 8, 2713}

$$-\frac{23x}{2a^3}+\frac{136\cos^3(x)}{15a^3}-\frac{136\cos(x)}{5a^3}+\frac{23\sin^3(x)\cos(x)}{3\,(a^3\sin(x)+a^3)}+\frac{23\sin(x)\cos(x)}{2a^3}+\frac{\sin^5(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{13\sin^4(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] (-23*x)/(2*a^3) - (136*Cos[x])/(5*a^3) + (136*Cos[x]^3)/(15*a^3) + (23*Cos[
x]*Sin[x])/(2*a^3) + (Cos[x]*Sin[x]^5)/(5*(a + a*Sin[x])^3) + (13*Cos[x]*Si
n[x]^4)/(15*a*(a + a*Sin[x])^2) + (23*Cos[x]*Sin[x]^3)/(3*(a^3 + a^3*Sin[x]
))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}-\frac{\int \frac{\sin^4(x)(5a-8a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}-\frac{\int \frac{\sin^3(x)\left(52a^2-63a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}+\frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)}-\frac{\int \sin^2(x)\left(345a^3-40\right.}{15a^6}\\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}+\frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)}-\frac{23\int \sin^2(x)\,dx}{a^3}+\frac{136}{}\\
&= \frac{23\cos(x)\sin(x)}{2a^3}+\frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}+\frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)}-\frac{23}{}\\
&= -\frac{23x}{2a^3}-\frac{136\cos(x)}{5a^3}+\frac{136\cos^3(x)}{15a^3}+\frac{23\cos(x)\sin(x)}{2a^3}+\frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3}+\frac{13\cos(}{15a(a+}
\end{aligned}$$

**Mathematica** [A]
time = 0.07, size = 191, normalized size = 1.89

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(24\sin\left(\frac{x}{2}\right)-12\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-224\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2+112\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+1576\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4-690x\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-405\cos(x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+5\cos(3x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+45\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\sin(2x)\right)}{60(a+a\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(24*Sin[x/2] - 12*(Cos[x/2] + Sin[x/2]) - 224*Sin[x/
2]*(Cos[x/2] + Sin[x/2])^2 + 112*(Cos[x/2] + Sin[x/2])^3 + 1576*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 - 690*x*(Cos[x/2] + Sin[x/2])^5 - 405*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 + 5*Cos[3*x]*(Cos[x/2] + Sin[x/2])^5 + 45*(Cos[x/2] + Sin[x
/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^3)
```

**Maple** [A]
time = 0.24, size = 108, normalized size = 1.07

| method | result |
|---|---|
| default | $\frac{-\frac{4\left(\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+3\left(\tan^4\left(\frac{x}{2}\right)\right)+7\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{3\tan\left(\frac{x}{2}\right)}{4}+\frac{10}{3}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-23\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}+\frac{8}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\cdots}{a^3}$ |
| risch | $-\frac{23x}{2a^3}+\frac{e^{3ix}}{24a^3}-\frac{3i e^{2ix}}{8a^3}-\frac{27\,e^{ix}}{8a^3}-\frac{27\,e^{-ix}}{8a^3}+\frac{3i e^{-2ix}}{8a^3}+\frac{e^{-3ix}}{24a^3}-\frac{2\left(810i e^{3ix}+225\,e^{4ix}-1160\,e^{2ix}-760i e^{ix}+197\right)}{15\left(e^{ix}+i\right)^5 a^3}$ |
| norman | $\frac{-\frac{115x\left(\tan^{16}\left(\frac{x}{2}\right)\right)}{2a}-\frac{4409\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{460x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{15025\left(\tan^9\left(\frac{x}{2}\right)\right)}{3a}-\frac{2944x\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}-\frac{2300x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{3335x\left(\tan^8\left(\frac{x}{2}\right)\right)}{a}-\frac{1564x(\cdots}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 128/a^3*(-1/32*(3/4*tan(1/2*x)^5+3*tan(1/2*x)^4+7*tan(1/2*x)^2-3/4*tan(1/2*
x)+10/3)/(tan(1/2*x)^2+1)^3-23/128*arctan(tan(1/2*x))-1/80/(tan(1/2*x)+1)^5
+1/32/(tan(1/2*x)+1)^4+1/48/(tan(1/2*x)+1)^3-1/16/(tan(1/2*x)+1)^2-5/32/(ta
n(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(87) = 174.
time = 0.62, size = 306, normalized size = 3.03

$$-\frac{\frac{2375\sin(x)}{\cos(x)+1}+\frac{5347\sin(x)^2}{(\cos(x)+1)^2}+\frac{9230\sin(x)^3}{(\cos(x)+1)^3}+\frac{12622\sin(x)^4}{(\cos(x)+1)^4}+\frac{13340\sin(x)^5}{(\cos(x)+1)^5}+\frac{11684\sin(x)^6}{(\cos(x)+1)^6}+\frac{8050\sin(x)^7}{(\cos(x)+1)^7}+\frac{4370\sin(x)^8}{(\cos(x)+1)^8}+\frac{1725\sin(x)^9}{(\cos(x)+1)^9}+\frac{345\sin(x)^{10}}{(\cos(x)+1)^{10}}+544}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{13a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{25a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{38a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{46a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{46a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{38a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{25a^3\sin(x)^8}{(\cos(x)+1)^8}+\frac{13a^3\sin(x)^9}{(\cos(x)+1)^9}+\frac{5a^3\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{a^3\sin(x)^{11}}{(\cos(x)+1)^{11}}\right)}-\frac{23\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(2375*sin(x)/(cos(x) + 1) + 5347*sin(x)^2/(cos(x) + 1)^2 + 9230*sin(x
)^3/(cos(x) + 1)^3 + 12622*sin(x)^4/(cos(x) + 1)^4 + 13340*sin(x)^5/(cos(x)
```

```
 + 1)^5 + 11684*sin(x)^6/(cos(x) + 1)^6 + 8050*sin(x)^7/(cos(x) + 1)^7 + 43
70*sin(x)^8/(cos(x) + 1)^8 + 1725*sin(x)^9/(cos(x) + 1)^9 + 345*sin(x)^10/(
cos(x) + 1)^10 + 544)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 13*a^3*sin(x)^2/(c
os(x) + 1)^2 + 25*a^3*sin(x)^3/(cos(x) + 1)^3 + 38*a^3*sin(x)^4/(cos(x) + 1
)^4 + 46*a^3*sin(x)^5/(cos(x) + 1)^5 + 46*a^3*sin(x)^6/(cos(x) + 1)^6 + 38*
a^3*sin(x)^7/(cos(x) + 1)^7 + 25*a^3*sin(x)^8/(cos(x) + 1)^8 + 13*a^3*sin(x
)^9/(cos(x) + 1)^9 + 5*a^3*sin(x)^10/(cos(x) + 1)^10 + a^3*sin(x)^11/(cos(x
) + 1)^11) - 23*arctan(sin(x)/(cos(x) + 1))/a^3
```

**Fricas** [**A**]
time = 0.35, size = 158, normalized size = 1.56

$$\frac{10 \cos(x)^6 - 15 \cos(x)^5 - (345 x + 839) \cos(x)^3 - 140 \cos(x)^4 - (1035 x - 668) \cos(x)^2 + 6 (115 x + 233) \cos(x) + \left(10 \cos(x)^5 + 25 \cos(x)^4 - (345 x - 724) \cos(x)^2 - 115 \cos(x)^3 + 6 (115 x + 232) \cos(x) + 1380 x - 6\right) \sin(x) + 1380 x + 6}{30 \left(a^3 \cos(x)^3 + 3 a^3 \cos(x)^2 - 2 a^3 \cos(x) - 4 a^3 + \left(a^3 \cos(x)^2 - 2 a^3 \cos(x) - 4 a^3\right) \sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*cos(x)^6 - 15*cos(x)^5 - (345*x + 839)*cos(x)^3 - 140*cos(x)^4 - (
1035*x - 668)*cos(x)^2 + 6*(115*x + 233)*cos(x) + (10*cos(x)^5 + 25*cos(x)^
4 - (345*x - 724)*cos(x)^2 - 115*cos(x)^3 + 6*(115*x + 232)*cos(x) + 1380*x
 - 6)*sin(x) + 1380*x + 6)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3288 vs. $2(107) = 214$.
time = 24.86, size = 3288, normalized size = 32.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a+a*sin(x))**3,x)
```

```
[Out] -345*x*tan(x/2)**11/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**
3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*ta
n(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/
2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1725*x*tan(x/
2)**10/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9
 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1
380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a
**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 4485*x*tan(x/2)**9/(30*a**
3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*ta
n(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x
/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**
2 + 150*a**3*tan(x/2) + 30*a**3) - 8625*x*tan(x/2)**8/(30*a**3*tan(x/2)**11
 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 11
40*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*
```

a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*t
an(x/2) + 30*a**3) - 13110*x*tan(x/2)**7/(30*a**3*tan(x/2)**11 + 150*a**3*t
an(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x
/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)
**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*
a**3) - 15870*x*tan(x/2)**6/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 +
 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380
*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**
3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 15870
*x*tan(x/2)**5/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan
(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2
)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3
 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 13110*x*tan(x/2)**
4/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 75
0*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a
**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 8625*x*tan(x/2)**3/(30*a**3*tan
(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2
)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**
5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 1
50*a**3*tan(x/2) + 30*a**3) - 4485*x*tan(x/2)**2/(30*a**3*tan(x/2)**11 + 15
0*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a*
*3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*
tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) - 1725*x*tan(x/2)/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**
10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 +
1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750
*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 3
45*x/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 +
 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 138
0*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**
3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 690*tan(x/2)**10/(30*a**3*ta
n(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/
2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)*
*5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 +
150*a**3*tan(x/2) + 30*a**3) - 3450*tan(x/2)**9/(30*a**3*tan(x/2)**11 + 150
*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**
3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*t
an(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2
) + 30*a**3) - 8740*tan(x/2)**8/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**
10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 +
1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750
*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1
6100*tan(x/2)**7/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*t
an(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x

```
/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)*
*3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 23368*tan(x/2)**
6/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 75
0*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a
**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 26680*tan(x/2)**5/(30*a**3*tan(
x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)
**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2...
```

**Giac** [A]
time = 0.47, size = 99, normalized size = 0.98

$$-\frac{23\,x}{2\,a^3}-\frac{9\,\tan\left(\frac{1}{2}\,x\right)^5+36\,\tan\left(\frac{1}{2}\,x\right)^4+84\,\tan\left(\frac{1}{2}\,x\right)^2-9\,\tan\left(\frac{1}{2}\,x\right)+40}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3a^3}-\frac{4\left(75\,\tan\left(\frac{1}{2}\,x\right)^4+330\,\tan\left(\frac{1}{2}\,x\right)^3+530\,\tan\left(\frac{1}{2}\,x\right)^2+355\,\tan\left(\frac{1}{2}\,x\right)+86\right)}{15\,a^3\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*x/a^3 - 1/3*(9*tan(1/2*x)^5 + 36*tan(1/2*x)^4 + 84*tan(1/2*x)^2 - 9*t
an(1/2*x) + 40)/((tan(1/2*x)^2 + 1)^3*a^3) - 4/15*(75*tan(1/2*x)^4 + 330*ta
n(1/2*x)^3 + 530*tan(1/2*x)^2 + 355*tan(1/2*x) + 86)/(a^3*(tan(1/2*x) + 1)^
5)
```

**Mupad** [B]
time = 7.02, size = 110, normalized size = 1.09

$$-\frac{23\,x}{2\,a^3}-\frac{23\,\tan\left(\frac{x}{2}\right)^{10}+115\,\tan\left(\frac{x}{2}\right)^9+\frac{874\,\tan\left(\frac{x}{2}\right)^8}{3}+\frac{1610\,\tan\left(\frac{x}{2}\right)^7}{3}+\frac{11684\,\tan\left(\frac{x}{2}\right)^6}{15}+\frac{2668\,\tan\left(\frac{x}{2}\right)^5}{3}+\frac{12622\,\tan\left(\frac{x}{2}\right)^4}{15}+\frac{1846\,\tan\left(\frac{x}{2}\right)^3}{3}+\frac{5347\,\tan\left(\frac{x}{2}\right)^2}{15}+\frac{475\,\tan\left(\frac{x}{2}\right)}{3}+\frac{544}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a + a*sin(x))^3,x)
```

```
[Out] - (23*x)/(2*a^3) - ((475*tan(x/2))/3 + (5347*tan(x/2)^2)/15 + (1846*tan(x/2
)^3)/3 + (12622*tan(x/2)^4)/15 + (2668*tan(x/2)^5)/3 + (11684*tan(x/2)^6)/1
5 + (1610*tan(x/2)^7)/3 + (874*tan(x/2)^8)/3 + 115*tan(x/2)^9 + 23*tan(x/2)
^10 + 544/15)/(a^3*(tan(x/2)^2 + 1)^3*(tan(x/2) + 1)^5)
```

# 3.22 $\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=90

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}-\frac{13\cos(x)\sin(x)}{2a^3}+\frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3}+\frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2}+\frac{76\cos(x)\sin^2(x)}{15\left(a^3+a^3\sin(x)\right)}$$

```
[Out] 13/2*x/a^3+152/15*cos(x)/a^3-13/2*cos(x)*sin(x)/a^3+1/5*cos(x)*sin(x)^4/(a+
a*sin(x))^3+11/15*cos(x)*sin(x)^3/a/(a+a*sin(x))^2+76/15*cos(x)*sin(x)^2/(a
^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.15, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2844, 3056, 2813}

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}+\frac{76\sin^2(x)\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{13\sin(x)\cos(x)}{2a^3}+\frac{\sin^4(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{11\sin^3(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] (13*x)/(2*a^3) + (152*Cos[x])/(15*a^3) - (13*Cos[x]*Sin[x])/(2*a^3) + (Cos[
x]*Sin[x]^4)/(5*(a + a*Sin[x])^3) + (11*Cos[x]*Sin[x]^3)/(15*a*(a + a*Sin[x
])^2) + (76*Cos[x]*Sin[x]^2)/(15*(a^3 + a^3*Sin[x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx = \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^3(x)(4a-7a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} - \frac{\int \frac{\sin^2(x)(33a^2-43a^2\sin(x))}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{76\cos(x)\sin^2(x)}{15(a^3+a^3\sin(x))} - \frac{\int \sin(x)\,(152a^3-19}{15a^6}$$

$$= \frac{13x}{2a^3} + \frac{152\cos(x)}{15a^3} - \frac{13\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{7}{15}$$

**Mathematica** [A]
time = 0.06, size = 170, normalized size = 1.89

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-24\sin\left(\frac{x}{2}\right)+12\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+184\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2-92\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-1016\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4+390x\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+180\cos(x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-15\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\sin(2x)\right)}{60(a+a\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-24*Sin[x/2] + 12*(Cos[x/2] + Sin[x/2]) + 184*Sin[x
/2]*(Cos[x/2] + Sin[x/2])^2 - 92*(Cos[x/2] + Sin[x/2])^3 - 1016*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 + 390*x*(Cos[x/2] + Sin[x/2])^5 + 180*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 - 15*(Cos[x/2] + Sin[x/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^
3)
```

**Maple** [A]
time = 0.22, size = 100, normalized size = 1.11

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{13x}{2a^3}+\frac{ie^{2ix}}{8a^3}+\frac{3\,e^{ix}}{2a^3}+\frac{3\,e^{-ix}}{2a^3}-\frac{ie^{-2ix}}{8a^3}+\frac{70ie^{3ix}+20\,e^{4ix}-\frac{194ie^{ix}}{3}-\frac{298\,e^{2ix}}{3}+\frac{254}{15}}{(e^{ix}+i)^5a^3}$ |
| default | $\frac{\frac{4\left(\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{x}{2}\right)}{4}+\frac{3}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+13\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{6}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{}{\tan(}}{a^3}$ |
| norman | $\frac{\frac{4717\left(\tan^6\left(\frac{x}{2}\right)\right)}{3a}+\frac{455x\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a}+\frac{3491\left(\tan^9\left(\frac{x}{2}\right)\right)}{3a}+\frac{2015x\left(\tan^7\left(\frac{x}{2}\right)\right)}{2a}+\frac{1755x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2a}+\frac{2015x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}+\frac{1313x\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a}+\frac{845x\left(\tan^4\right.}{2a}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64/a^3*(1/16*(1/4*tan(1/2*x)^3+3/2*tan(1/2*x)^2-1/4*tan(1/2*x)+3/2)/(tan(1/
2*x)^2+1)^2+13/64*arctan(tan(1/2*x))+1/40/(tan(1/2*x)+1)^5-1/16/(tan(1/2*x)
+1)^4-1/48/(tan(1/2*x)+1)^3+3/32/(tan(1/2*x)+1)^2+3/16/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(78) = 156$.
time = 0.57, size = 252, normalized size = 2.80

$$\frac{\frac{1325\sin(x)}{\cos(x)+1}+\frac{2673\sin(x)^2}{(\cos(x)+1)^2}+\frac{3805\sin(x)^3}{(\cos(x)+1)^3}+\frac{4329\sin(x)^4}{(\cos(x)+1)^4}+\frac{3575\sin(x)^5}{(\cos(x)+1)^5}+\frac{2275\sin(x)^6}{(\cos(x)+1)^6}+\frac{975\sin(x)^7}{(\cos(x)+1)^7}+\frac{195\sin(x)^8}{(\cos(x)+1)^8}+304}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{12a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{20a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{26a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{26a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{20a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{12a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{5a^3\sin(x)^8}{(\cos(x)+1)^8}+\frac{a^3\sin(x)^9}{(\cos(x)+1)^9}\right)}+\frac{13\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(1325*sin(x)/(cos(x) + 1) + 2673*sin(x)^2/(cos(x) + 1)^2 + 3805*sin(x)
^3/(cos(x) + 1)^3 + 4329*sin(x)^4/(cos(x) + 1)^4 + 3575*sin(x)^5/(cos(x) +
1)^5 + 2275*sin(x)^6/(cos(x) + 1)^6 + 975*sin(x)^7/(cos(x) + 1)^7 + 195*sin
(x)^8/(cos(x) + 1)^8 + 304)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 12*a^3*sin(x
)^2/(cos(x) + 1)^2 + 20*a^3*sin(x)^3/(cos(x) + 1)^3 + 26*a^3*sin(x)^4/(cos(
x) + 1)^4 + 26*a^3*sin(x)^5/(cos(x) + 1)^5 + 20*a^3*sin(x)^6/(cos(x) + 1)^6
 + 12*a^3*sin(x)^7/(cos(x) + 1)^7 + 5*a^3*sin(x)^8/(cos(x) + 1)^8 + a^3*sin
(x)^9/(cos(x) + 1)^9) + 13*arctan(sin(x)/(cos(x) + 1))/a^3
```

**Fricas** [A]
time = 0.34, size = 145, normalized size = 1.61

$$\frac{15\cos(x)^5+(195x+449)\cos(x)^3+60\cos(x)^4+(585x-358)\cos(x)^2-6(65x+128)\cos(x)-\left(15\cos(x)^4-(195x-404)\cos(x)^2-45\cos(x)^3+6(65x+127)\cos(x)+780x-6\right)\sin(x)-780x-6}{30\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(x)^5 + (195*x + 449)*cos(x)^3 + 60*cos(x)^4 + (585*x - 358)*co
s(x)^2 - 6*(65*x + 128)*cos(x) - (15*cos(x)^4 - (195*x - 404)*cos(x)^2 - 45
```

```
*cos(x)^3 + 6*(65*x + 127)*cos(x) + 780*x - 6)*sin(x) - 780*x - 6)/(a^3*cos
(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x
) - 4*a^3)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2259 vs. $2(95) = 190$.
time = 15.53, size = 2259, normalized size = 25.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+a*sin(x))**3,x)
```

```
[Out] 195*x*tan(x/2)**9/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*ta
n(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)
**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*
a**3) + 975*x*tan(x/2)**8/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360
*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*
tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) + 2340*x*tan(x/2)**7/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)
**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 +
780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a*
*3*tan(x/2) + 30*a**3) + 3900*x*tan(x/2)**6/(30*a**3*tan(x/2)**9 + 150*a**3
*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x
/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2
 + 150*a**3*tan(x/2) + 30*a**3) + 5070*x*tan(x/2)**5/(30*a**3*tan(x/2)**9 +
 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a
**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*ta
n(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 5070*x*tan(x/2)**4/(30*a**3*tan(
x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**
6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 36
0*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 3900*x*tan(x/2)**3/(30*
a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*t
an(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2
)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 2340*x*tan(x/2
)**2/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 6
00*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**
3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 975*x
*tan(x/2)/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**
7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 60
0*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) +
195*x/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 +
600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a*
*3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 390*
tan(x/2)**8/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)
```

```
**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 +
600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3)
+ 1950*tan(x/2)**7/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*t
an(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2
)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30
*a**3) + 4550*tan(x/2)**6/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360
*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*
tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) + 7150*tan(x/2)**5/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**
8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 78
0*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3
*tan(x/2) + 30*a**3) + 8658*tan(x/2)**4/(30*a**3*tan(x/2)**9 + 150*a**3*tan
(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)*
*5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 1
50*a**3*tan(x/2) + 30*a**3) + 7610*tan(x/2)**3/(30*a**3*tan(x/2)**9 + 150*a
**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*ta
n(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)
**2 + 150*a**3*tan(x/2) + 30*a**3) + 5346*tan(x/2)**2/(30*a**3*tan(x/2)**9
+ 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*
a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 2650*tan(x/2)/(30*a**3*tan(x/2)
**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 +
780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a*
*3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 608/(30*a**3*tan(x/2)**9 +
150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a*
*3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan
(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3)
```

**Giac** [A]
time = 0.56, size = 88, normalized size = 0.98

$$\frac{13\,x}{2\,a^3}+\frac{\tan\left(\frac{1}{2}\,x\right)^3+6\,\tan\left(\frac{1}{2}\,x\right)^2-\tan\left(\frac{1}{2}\,x\right)+6}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^2a^3}+\frac{2\left(90\,\tan\left(\frac{1}{2}\,x\right)^4+405\,\tan\left(\frac{1}{2}\,x\right)^3+665\,\tan\left(\frac{1}{2}\,x\right)^2+445\,\tan\left(\frac{1}{2}\,x\right)+107\right)}{15\,a^3\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] 13/2*x/a^3 + (tan(1/2*x)^3 + 6*tan(1/2*x)^2 - tan(1/2*x) + 6)/((tan(1/2*x)^
2 + 1)^2*a^3) + 2/15*(90*tan(1/2*x)^4 + 405*tan(1/2*x)^3 + 665*tan(1/2*x)^2
 + 445*tan(1/2*x) + 107)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad** [B]
time = 6.67, size = 93, normalized size = 1.03

$$\frac{13\,x}{2\,a^3}+\frac{13\tan\left(\frac{x}{2}\right)^8+65\tan\left(\frac{x}{2}\right)^7+\frac{455\tan\left(\frac{x}{2}\right)^6}{3}+\frac{715\tan\left(\frac{x}{2}\right)^5}{3}+\frac{1443\tan\left(\frac{x}{2}\right)^4}{5}+\frac{761\tan\left(\frac{x}{2}\right)^3}{3}+\frac{891\tan\left(\frac{x}{2}\right)^2}{5}+\frac{265\tan\left(\frac{x}{2}\right)}{3}+\frac{304}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + a*sin(x))^3,x)
```

```
[Out] (13*x)/(2*a^3) + ((265*tan(x/2))/3 + (891*tan(x/2)^2)/5 + (761*tan(x/2)^3)/
3 + (1443*tan(x/2)^4)/5 + (715*tan(x/2)^5)/3 + (455*tan(x/2)^6)/3 + 65*tan(
x/2)^7 + 13*tan(x/2)^8 + 304/15)/(a^3*(tan(x/2)^2 + 1)^2*(tan(x/2) + 1)^5)
```

# 3.23 $\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=71

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}+\frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{3\cos(x)}{a^3+a^3\sin(x)}$$

```
[Out] -3*x/a^3-9/5*cos(x)/a^3+1/5*cos(x)*sin(x)^3/(a+a*sin(x))^3+3/5*cos(x)*sin(x
)^2/a/(a+a*sin(x))^2-3*cos(x)/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2844, 3056, 3047, 3102, 12, 2814, 2727}

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}-\frac{3\cos(x)}{a^3\sin(x)+a^3}+\frac{\sin^3(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{3\sin^2(x)\cos(x)}{5a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (-3*x)/a^3 - (9*Cos[x])/(5*a^3) + (Cos[x]*Sin[x]^3)/(5*(a + a*Sin[x])^3) +
(3*Cos[x]*Sin[x]^2)/(5*a*(a + a*Sin[x])^2) - (3*Cos[x])/(a^3 + a^3*Sin[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^2(x)(3a-6a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{\sin(x)(18a^2-27a^2\sin(x))}{a+a\sin(x)}\,dx}{15a^4} \\
&= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{18a^2\sin(x)-27a^2\sin^2(x)}{a+a\sin(x)}\,dx}{15a^4} \\
&= -\frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{45a^3\sin(x)}{a+a\sin(x)}\,dx}{15a^5} \\
&= -\frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{3\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a^2} \\
&= -\frac{3x}{a^3} - \frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} + \frac{3\int \frac{1}{a+a\sin(x)}\,dx}{a^2} \\
&= -\frac{3x}{a^3} - \frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{3\cos(x)}{a^3+a^3\sin(x)}
\end{aligned}$$

**Mathematica [A]**
time = 0.06, size = 140, normalized size = 1.97

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)-12\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2+6\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+48\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4-15x\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-5\cos(x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\right)}{5(a+a\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-Cos[x/2] + Sin[x/2] - 12*Sin[x/2]*(Cos[x/2] + Sin[
x/2])^2 + 6*(Cos[x/2] + Sin[x/2])^3 + 48*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 -
 15*x*(Cos[x/2] + Sin[x/2])^5 - 5*Cos[x]*(Cos[x/2] + Sin[x/2])^5))/(5*(a +
a*Sin[x])^3)
```

**Maple [A]**
time = 0.20, size = 66, normalized size = 0.93

| method | result |
|---|---|
| default | $\frac{-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{6}{\tan\left(\frac{x}{2}\right)+1}-\frac{2}{\tan^2\left(\frac{x}{2}\right)+1}-6\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$ |
| risch | $-\frac{3x}{a^3}-\frac{e^{ix}}{2a^3}-\frac{e^{-ix}}{2a^3}-\frac{4\left(-70\,e^{2ix}+50i e^{3ix}+15\,e^{4ix}-45i e^{ix}+12\right)}{5\left(e^{ix}+i\right)^5 a^3}$ |

| | |
|---|---|
| norman | $\frac{-\frac{2172\left(\tan^6\left(\frac{x}{2}\right)\right)}{5a}-\frac{90x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{170\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}-\frac{252x\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}-\frac{252x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{213x\left(\tan^8\left(\frac{x}{2}\right)\right)}{a}-\frac{213x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{153x\left(\tan^4\left(\frac{x}{2}\right)\right.}{a}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 32/a^3*(-1/20/(tan(1/2*x)+1)^5+1/8/(tan(1/2*x)+1)^4-1/8/(tan(1/2*x)+1)^2-3/
16/(tan(1/2*x)+1)-1/16/(tan(1/2*x)^2+1)-3/16*arctan(tan(1/2*x)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(65) = 130$.
time = 0.53, size = 198, normalized size = 2.79

$$-\frac{2\left(\frac{105\sin(x)}{\cos(x)+1}+\frac{189\sin(x)^2}{(\cos(x)+1)^2}+\frac{200\sin(x)^3}{(\cos(x)+1)^3}+\frac{160\sin(x)^4}{(\cos(x)+1)^4}+\frac{75\sin(x)^5}{(\cos(x)+1)^5}+\frac{15\sin(x)^6}{(\cos(x)+1)^6}+24\right)}{5\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{11a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{15a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{15a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{11a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{6\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(105*sin(x)/(cos(x) + 1) + 189*sin(x)^2/(cos(x) + 1)^2 + 200*sin(x)^3/
(cos(x) + 1)^3 + 160*sin(x)^4/(cos(x) + 1)^4 + 75*sin(x)^5/(cos(x) + 1)^5 +
 15*sin(x)^6/(cos(x) + 1)^6 + 24)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 11*a^3
*sin(x)^2/(cos(x) + 1)^2 + 15*a^3*sin(x)^3/(cos(x) + 1)^3 + 15*a^3*sin(x)^4
/(cos(x) + 1)^4 + 11*a^3*sin(x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) +
 1)^6 + a^3*sin(x)^7/(cos(x) + 1)^7) - 6*arctan(sin(x)/(cos(x) + 1))/a^3
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(65) = 130$.
time = 0.34, size = 132, normalized size = 1.86

$$-\frac{3(5x+13)\cos(x)^3+5\cos(x)^4+(45x-28)\cos(x)^2-3(10x+21)\cos(x)+((15x-34)\cos(x)^2+5\cos(x)^3-2(15x+31)\cos(x)-60x+1)\sin(x)-60x-1}{5\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(3*(5*x + 13)*cos(x)^3 + 5*cos(x)^4 + (45*x - 28)*cos(x)^2 - 3*(10*x +
 21)*cos(x) + ((15*x - 34)*cos(x)^2 + 5*cos(x)^3 - 2*(15*x + 31)*cos(x) - 6
0*x + 1)*sin(x) - 60*x - 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(71) = 142$.
time = 8.84, size = 1425, normalized size = 20.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] -15*x*tan(x/2)**7/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x
/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 +
25*a**3*tan(x/2) + 5*a**3) - 75*x*tan(x/2)**6/(5*a**3*tan(x/2)**7 + 25*a**3
*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)
**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 165*x*tan(x/2)**5/
(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) +
 5*a**3) - 225*x*tan(x/2)**4/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55
*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan
(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 225*x*tan(x/2)**3/(5*a**3*tan(x/2)*
*7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a
**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 165*x*
tan(x/2)**2/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5
 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**
3*tan(x/2) + 5*a**3) - 75*x*tan(x/2)/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)
**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*
a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 15*x/(5*a**3*tan(x/2)**7 +
25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*t
an(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 30*tan(x/2)
**6/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a*
*3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/
2) + 5*a**3) - 150*tan(x/2)**5/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 +
55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*t
an(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 320*tan(x/2)**4/(5*a**3*tan(x/2)*
*7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a
**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 400*ta
n(x/2)**3/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 +
 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*
tan(x/2) + 5*a**3) - 378*tan(x/2)**2/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)
**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*
a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 210*tan(x/2)/(5*a**3*tan(x/
2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 7
5*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 48/
(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) +
 5*a**3)
```

**Giac** [A]
time = 0.49, size = 67, normalized size = 0.94

$$-\frac{3\,x}{a^3}-\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)a^3}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,x\right)^4+70\,\tan\left(\frac{1}{2}\,x\right)^3+120\,\tan\left(\frac{1}{2}\,x\right)^2+80\,\tan\left(\frac{1}{2}\,x\right)+19\right)}{5\,a^3\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*x/a^3 - 2/((tan(1/2*x)^2 + 1)*a^3) - 2/5*(15*tan(1/2*x)^4 + 70*tan(1/2*x
)^3 + 120*tan(1/2*x)^2 + 80*tan(1/2*x) + 19)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad [B]**
time = 6.89, size = 78, normalized size = 1.10

$$-\frac{3\,x}{a^3}-\frac{6\,\tan\left(\frac{x}{2}\right)^6+30\,\tan\left(\frac{x}{2}\right)^5+64\,\tan\left(\frac{x}{2}\right)^4+80\,\tan\left(\frac{x}{2}\right)^3+\frac{378\,\tan\left(\frac{x}{2}\right)^2}{5}+42\,\tan\left(\frac{x}{2}\right)+\frac{48}{5}}{a^3\,\left({\tan\left(\frac{x}{2}\right)}^2+1\right)\,{\left(\tan\left(\frac{x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x))^3,x)
```

```
[Out] - (3*x)/a^3 - (42*tan(x/2) + (378*tan(x/2)^2)/5 + 80*tan(x/2)^3 + 64*tan(x/
2)^4 + 30*tan(x/2)^5 + 6*tan(x/2)^6 + 48/5)/(a^3*(tan(x/2)^2 + 1)*(tan(x/2)
 + 1)^5)
```

## 3.24 $\int \frac{\sin^3(x)}{(a+a\sin(x))^3} dx$

**Optimal**. Leaf size=59

$$\frac{x}{a^3}+\frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{29\cos(x)}{15\,(a^3+a^3\sin(x))}$$

```
[Out] x/a^3+1/5*cos(x)*sin(x)^2/(a+a*sin(x))^3-7/15*cos(x)/a/(a+a*sin(x))^2+29/15
*cos(x)/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2844, 3047, 3098, 2814, 2727}

$$\frac{x}{a^3}+\frac{29\cos(x)}{15\,(a^3\sin(x)+a^3)}+\frac{\sin^2(x)\cos(x)}{5(a\sin(x)+a)^3}-\frac{7\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] x/a^3 + (Cos[x]*Sin[x]^2)/(5*(a + a*Sin[x])^3) - (7*Cos[x])/(15*a*(a + a*Si
n[x])^2) + (29*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin(x)(2a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{2a\sin(x)-5a\sin^2(x)}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{-14a^2+15a^2\sin(x)}{a+a\sin(x)}\,dx}{15a^4} \\
&= \frac{x}{a^3} + \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} - \frac{29\int \frac{1}{a+a\sin(x)}\,dx}{15a^2} \\
&= \frac{x}{a^3} + \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} + \frac{29\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

**Mathematica [A]**
time = 0.13, size = 112, normalized size = 1.90

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(30(-9+5x)\cos\left(\frac{x}{2}\right)+(230-75x)\cos\left(\frac{3x}{2}\right)-15x\cos\left(\frac{5x}{2}\right)-370\sin\left(\frac{x}{2}\right)+150x\sin\left(\frac{x}{2}\right)-90\sin\left(\frac{3x}{2}\right)+75x\sin\left(\frac{3x}{2}\right)+64\sin\left(\frac{5x}{2}\right)-15x\sin\left(\frac{5x}{2}\right)\right)}{60a^3(1+\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^3,x]
[Out] ((Cos[x/2] + Sin[x/2])*(30*(-9 + 5*x)*Cos[x/2] + (230 - 75*x)*Cos[(3*x)/2]
- 15*x*Cos[(5*x)/2] - 370*Sin[x/2] + 150*x*Sin[x/2] - 90*Sin[(3*x)/2] + 75*
x*Sin[(3*x)/2] + 64*Sin[(5*x)/2] - 15*x*Sin[(5*x)/2]))/(60*a^3*(1 + Sin[x])
^3)
```

**Maple [A]**
time = 0.18, size = 64, normalized size = 1.08

| method | result |
|---|---|
| risch | $\frac{x}{a^{3}}+\frac{18 i \mathrm{e}^{3 i x}+6 \,\mathrm{e}^{4 i x}-\frac{46 i \mathrm{e}^{i x}}{3}-\frac{74 \,\mathrm{e}^{2 i x}}{3}+\frac{64}{15}}{a^{3} \left(\mathrm{e}^{i x}+i\right)^{5}}$ |
| default | $\frac{-\frac{4}{\left(\tan \left(\frac{x}{2}\right)+1\right)^{4}}+\frac{8}{5 \left(\tan \left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{3 \left(\tan \left(\frac{x}{2}\right)+1\right)^{3}}+\frac{2}{\left(\tan \left(\frac{x}{2}\right)+1\right)^{2}}+\frac{16}{8 \tan \left(\frac{x}{2}\right)+8}+2 \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{a^{3}}$ |
| norman | $\frac{\frac{x}{a}+\frac{x \left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}+\frac{2 \left(\tan^{10}\left(\frac{x}{2}\right)\right)}{a}+\frac{10 \left(\tan^{9}\left(\frac{x}{2}\right)\right)}{a}+\frac{48 \left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{44}{15 a}+\frac{5 x \tan \left(\frac{x}{2}\right)}{a}+\frac{13 x \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}+\frac{25 x \left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{38 x \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}+46x}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/a^3*(-1/4/(tan(1/2*x)+1)^4+1/10/(tan(1/2*x)+1)^5+1/12/(tan(1/2*x)+1)^3+1
/8/(tan(1/2*x)+1)^2+1/8/(tan(1/2*x)+1)+1/8*arctan(tan(1/2*x)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(53) = 106.
time = 0.51, size = 144, normalized size = 2.44

$$\frac{2 \left(\frac{95 \sin(x)}{\cos(x)+1} + \frac{145 \sin(x)^2}{(\cos(x)+1)^2} + \frac{75 \sin(x)^3}{(\cos(x)+1)^3} + \frac{15 \sin(x)^4}{(\cos(x)+1)^4} + 22\right)}{15 \left(a^3 + \frac{5 a^3 \sin(x)}{\cos(x)+1} + \frac{10 a^3 \sin(x)^2}{(\cos(x)+1)^2} + \frac{10 a^3 \sin(x)^3}{(\cos(x)+1)^3} + \frac{5 a^3 \sin(x)^4}{(\cos(x)+1)^4} + \frac{a^3 \sin(x)^5}{(\cos(x)+1)^5}\right)} + \frac{2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(95*sin(x)/(cos(x) + 1) + 145*sin(x)^2/(cos(x) + 1)^2 + 75*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 22)/(a^3 + 5*a^3*sin(x)/(cos(x)
 + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5
*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + 2*arctan(sin(
x)/(cos(x) + 1))/a^3
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.
time = 0.35, size = 119, normalized size = 2.02

$$\frac{(15\,x+32)\cos(x)^3 + (45\,x-19)\cos(x)^2 - 6\,(5\,x+9)\cos(x) + \left((15\,x-32)\cos(x)^2 - 3\,(10\,x+17)\cos(x) - 60\,x+3\right)\sin(x) - 60\,x - 3}{15\left(a^3\cos(x)^3 + 3\,a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3 + \left(a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*x + 32)*cos(x)^3 + (45*x - 19)*cos(x)^2 - 6*(5*x + 9)*cos(x) + ((
15*x - 32)*cos(x)^2 - 3*(10*x + 17)*cos(x) - 60*x + 3)*sin(x) - 60*x - 3)/(
```

```
a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^
3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(58) = 116$.
time = 4.75, size = 777, normalized size = 13.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**3,x)
```

```
[Out] 15*x*tan(x/2)**5/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(
x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 75*x*tan(x/2
)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 15
0*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*x*tan(x/2)**3/(15*a*
*3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(
x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*x*tan(x/2)**2/(15*a**3*tan(x/2)
**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 7
5*a**3*tan(x/2) + 15*a**3) + 75*x*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2)
 + 15*a**3) + 15*x/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*ta
n(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 30*tan(x/2
)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 15
0*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*tan(x/2)**3/(15*a**3
*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/
2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 290*tan(x/2)**2/(15*a**3*tan(x/2)**5
+ 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a*
*3*tan(x/2) + 15*a**3) + 190*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/
2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15
*a**3) + 44/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)*
*3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3)
```

**Giac** [A]
time = 0.50, size = 51, normalized size = 0.86

$$\frac{x}{a^3} + \frac{2\left(15\tan\left(\frac{1}{2}x\right)^4 + 75\tan\left(\frac{1}{2}x\right)^3 + 145\tan\left(\frac{1}{2}x\right)^2 + 95\tan\left(\frac{1}{2}x\right) + 22\right)}{15\,a^3\left(\tan\left(\frac{1}{2}x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] x/a^3 + 2/15*(15*tan(1/2*x)^4 + 75*tan(1/2*x)^3 + 145*tan(1/2*x)^2 + 95*tan
(1/2*x) + 22)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad [B]**
time = 6.74, size = 50, normalized size = 0.85

$$\frac{x}{a^3}+\frac{2\tan\left(\frac{x}{2}\right)^4+10\tan\left(\frac{x}{2}\right)^3+\frac{58\tan\left(\frac{x}{2}\right)^2}{3}+\frac{38\tan\left(\frac{x}{2}\right)}{3}+\frac{44}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x))^3,x)
```

```
[Out] x/a^3 + ((38*tan(x/2))/3 + (58*tan(x/2)^2)/3 + 10*tan(x/2)^3 + 2*tan(x/2)^4
 + 44/15)/(a^3*(tan(x/2) + 1)^5)
```

# 3.25 $\int \frac{\sin^2(x)}{(a+a\sin(x))^3} dx$

**Optimal**. Leaf size=50

$$-\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{8\cos(x)}{15a(a+a\sin(x))^2}-\frac{7\cos(x)}{15\left(a^3+a^3\sin(x)\right)}$$

```
[Out] -1/5*cos(x)/(a+a*sin(x))^3+8/15*cos(x)/a/(a+a*sin(x))^2-7/15*cos(x)/(a^3+a^
3*sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2837, 2829, 2727}

$$-\frac{7\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{8\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] -1/5*Cos[x]/(a + a*Sin[x])^3 + (8*Cos[x])/(15*a*(a + a*Sin[x])^2) - (7*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{(a+a\sin(x))^3}\,dx &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{-3a+5a\sin(x)}{(a+a\sin(x))^2}\,dx}{5a^2}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{8\cos(x)}{15a(a+a\sin(x))^2}+\frac{7\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{8\cos(x)}{15a(a+a\sin(x))^2}-\frac{7\cos(x)}{15\left(a^3+a^3\sin(x)\right)}\end{aligned}$$

**Mathematica** [A]
time = 0.05, size = 47, normalized size = 0.94

$$\frac{70-15\cos(x)-42\cos(2x)+7\cos(3x)+105\sin(x)-12\sin(2x)-7\sin(3x)}{60a^3(1+\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] (70 - 15*Cos[x] - 42*Cos[2*x] + 7*Cos[3*x] + 105*Sin[x] - 12*Sin[2*x] - 7*S
in[3*x])/(60*a^3*(1 + Sin[x])^3)
```

**Maple** [A]
time = 0.13, size = 37, normalized size = 0.74

| method | result | size |
|---|---|---|
| default | $\frac{-\frac{8}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}}{a^3}$ | 37 |
| risch | $-\frac{2\left(30i\,e^{3ix}+15\,e^{4ix}-40\,e^{2ix}-20i\,e^{ix}+7\right)}{15\left(e^{ix}+i\right)^5a^3}$ | 48 |
| norman | $\frac{-\frac{4}{15a}-\frac{4\tan\left(\frac{x}{2}\right)}{3a}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{5a}-\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{8\left(\tan^6\left(\frac{x}{2}\right)\right)}{3a}-\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3a}-\frac{28\left(\tan^4\left(\frac{x}{2}\right)\right)}{5a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$ | 93 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/a^3*(-1/3/(tan(1/2*x)+1)^3+1/2/(tan(1/2*x)+1)^4-1/5/(tan(1/2*x)+1)^5)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(44) = 88.
time = 0.30, size = 104, normalized size = 2.08

$$-\frac{4\left(\frac{5\sin(x)}{\cos(x)+1}+\frac{10\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)/(cos(x) + 1) + 10*sin(x)^2/(cos(x) + 1)^2 + 1)/(a^3 + 5*a^3
*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(co
s(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(44) = 88$.
time = 0.33, size = 90, normalized size = 1.80

$$-\frac{7\cos(x)^3+\cos(x)^2-\left(7\cos(x)^2+6\cos(x)-3\right)\sin(x)-9\cos(x)-3}{15\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(7*cos(x)^3 + cos(x)^2 - (7*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos(x
) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^
2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(48) = 96$.
time = 2.76, size = 206, normalized size = 4.12

$$-\frac{40\tan^2\left(\frac{x}{2}\right)}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}-\frac{20\tan\left(\frac{x}{2}\right)}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}-\frac{4}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] -40*tan(x/2)**2/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x
/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 20*tan(x/2)/(
15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3
*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 4/(15*a**3*tan(x/2)**5 + 75*a*
*3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(
x/2) + 15*a**3)
```

**Giac** [A]
time = 0.48, size = 29, normalized size = 0.58

$$-\frac{4\left(10\tan\left(\frac{1}{2}x\right)^2+5\tan\left(\frac{1}{2}x\right)+1\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad** [B]
time = 6.64, size = 29, normalized size = 0.58

$$-\frac{4\left(10\tan\left(\frac{x}{2}\right)^2+5\tan\left(\frac{x}{2}\right)+1\right)}{15\,a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x))^3,x)
[Out] -(4*(5*tan(x/2) + 10*tan(x/2)^2 + 1))/(15*a^3*(tan(x/2) + 1)^5)
```

# 3.26 $\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=50

$$\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}-\frac{\cos(x)}{5\left(a^3+a^3\sin(x)\right)}$$

```
[Out] 1/5*cos(x)/(a+a*sin(x))^3-1/5*cos(x)/a/(a+a*sin(x))^2-1/5*cos(x)/(a^3+a^3*s
in(x))
```

**Rubi** [A]

time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2829, 2729, 2727}

$$-\frac{\cos(x)}{5\left(a^3\sin(x)+a^3\right)}-\frac{\cos(x)}{5a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] Cos[x]/(5*(a + a*Sin[x])^3) - Cos[x]/(5*a*(a + a*Sin[x])^2) - Cos[x]/(5*(a^
3 + a^3*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{3\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}\\ &= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{5a^2}\\ &= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}-\frac{\cos(x)}{5\left(a^3+a^3\sin(x)\right)}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 41, normalized size = 0.82

$$\frac{\sin^2\left(\frac{x}{2}\right)\left(7+4\cos(x)-\cos(2x)+8\sin(x)+\sin(2x)\right)}{5a^3(1+\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] (Sin[x/2]^2*(7 + 4*Cos[x] - Cos[2*x] + 8*Sin[x] + Sin[2*x]))/(5*a^3*(1 + Si
n[x])^3)
```

**Maple** [A]
time = 0.14, size = 45, normalized size = 0.90

| method | result | size |
|---|---|---|
| risch | $-\frac{2i\left(5i\,e^{2ix}+5\,e^{3ix}-i-5\,e^{ix}\right)}{5\left(e^{ix}+i\right)^5 a^3}$ | 42 |
| default | $\frac{\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^3}}{a^3}$ | 45 |
| norman | $\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{2}{5a}-\frac{12\left(\tan^2\left(\frac{x}{2}\right)\right)}{5a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$ | 82 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^3*(2/5/(tan(1/2*x)+1)^5-1/(tan(1/2*x)+1)^4-1/2/(tan(1/2*x)+1)^2+1/(tan(
1/2*x)+1)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(44) = 88.
time = 0.29, size = 116, normalized size = 2.32

$$-\frac{2\left(\frac{5\sin(x)}{\cos(x)+1}+\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\sin(x)^3}{(\cos(x)+1)^3}+1\right)}{5\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(5*sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^3/(cos(x
) + 1)^3 + 1)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) +
1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3
*sin(x)^5/(cos(x) + 1)^5)
```

**Fricas [A]**
time = 0.34, size = 88, normalized size = 1.76

$$-\frac{\cos\left(x\right)^{3}-2\,\cos\left(x\right)^{2}-\left(\cos\left(x\right)^{2}+3\,\cos\left(x\right)+1\right)\sin\left(x\right)-2\,\cos\left(x\right)+1}{5\,\left(a^{3}\cos\left(x\right)^{3}+3\,a^{3}\cos\left(x\right)^{2}-2\,a^{3}\cos\left(x\right)-4\,a^{3}+\left(a^{3}\cos\left(x\right)^{2}-2\,a^{3}\cos\left(x\right)-4\,a^{3}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(cos(x)^3 - 2*cos(x)^2 - (cos(x)^2 + 3*cos(x) + 1)*sin(x) - 2*cos(x) +
 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 -
 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(44) = 88$.
time = 1.52, size = 277, normalized size = 5.54

$$-\frac{10\tan^{3}\left(\frac{x}{2}\right)}{5a^{3}\tan^{5}\left(\frac{x}{2}\right)+25a^{3}\tan^{4}\left(\frac{x}{2}\right)+50a^{3}\tan^{3}\left(\frac{x}{2}\right)+50a^{3}\tan^{2}\left(\frac{x}{2}\right)+25a^{3}\tan\left(\frac{x}{2}\right)+5a^{3}}-\frac{10\tan^{2}\left(\frac{x}{2}\right)}{5a^{3}\tan^{5}\left(\frac{x}{2}\right)+25a^{3}\tan^{4}\left(\frac{x}{2}\right)+50a^{3}\tan^{3}\left(\frac{x}{2}\right)+50a^{3}\tan^{2}\left(\frac{x}{2}\right)+25a^{3}\tan\left(\frac{x}{2}\right)+5a^{3}}-\frac{10\tan\left(\frac{x}{2}\right)}{5a^{3}\tan^{5}\left(\frac{x}{2}\right)+25a^{3}\tan^{4}\left(\frac{x}{2}\right)+50a^{3}\tan^{3}\left(\frac{x}{2}\right)+50a^{3}\tan^{2}\left(\frac{x}{2}\right)+25a^{3}\tan\left(\frac{x}{2}\right)+5a^{3}}-\frac{2}{5a^{3}\tan^{5}\left(\frac{x}{2}\right)+25a^{3}\tan^{4}\left(\frac{x}{2}\right)+50a^{3}\tan^{3}\left(\frac{x}{2}\right)+50a^{3}\tan^{2}\left(\frac{x}{2}\right)+25a^{3}\tan\left(\frac{x}{2}\right)+5a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))**3,x)
```

```
[Out] -10*tan(x/2)**3/(5*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*tan(x/2
)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 10*tan(x/2)**2/(5
*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*tan(x/2)**3 + 50*a**3*tan
(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 10*tan(x/2)/(5*a**3*tan(x/2)**5 + 2
5*a**3*tan(x/2)**4 + 50*a**3*tan(x/2)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*ta
n(x/2) + 5*a**3) - 2/(5*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*ta
n(x/2)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3)
```

**Giac [A]**
time = 0.53, size = 37, normalized size = 0.74

$$-\frac{2\,{\left(5\,\tan\left(\frac{1}{2}\,x\right)^{3}+5\,\tan\left(\frac{1}{2}\,x\right)^{2}+5\,\tan\left(\frac{1}{2}\,x\right)+1\right)}}{5\,a^{3}{\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*tan(1/2*x)^3 + 5*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x)
+ 1)^5)
```

**Mupad** [B]
time = 6.85, size = 37, normalized size = 0.74

$$-\frac{2\left(5\tan\left(\frac{x}{2}\right)^3+5\tan\left(\frac{x}{2}\right)^2+5\tan\left(\frac{x}{2}\right)+1\right)}{5\,a^3\,\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*sin(x))^3,x)
```

```
[Out] -(2*(5*tan(x/2) + 5*tan(x/2)^2 + 5*tan(x/2)^3 + 1))/(5*a^3*(tan(x/2) + 1)^5
)
```

# 3.27 $\int \frac{1}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=50

$$-\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{2\cos(x)}{15a(a+a\sin(x))^2}-\frac{2\cos(x)}{15\left(a^3+a^3\sin(x)\right)}$$

```
[Out] -1/5*cos(x)/(a+a*sin(x))^3-2/15*cos(x)/a/(a+a*sin(x))^2-2/15*cos(x)/(a^3+a^
3*sin(x))
```

**Rubi** [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2729, 2727}

$$-\frac{2\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{2\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-3),x]
```

```
[Out] -1/5*Cos[x]/(a + a*Sin[x])^3 - (2*Cos[x])/(15*a*(a + a*Sin[x])^2) - (2*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(x))^3}\,dx &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{2\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}\\
&= -\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{2\cos(x)}{15a(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\
&= -\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{2\cos(x)}{15a(a+a\sin(x))^2}-\frac{2\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

**Mathematica [A]**
time = 0.04, size = 45, normalized size = 0.90

$$-\frac{5\cos\left(\frac{3x}{2}\right)-10\sin\left(\frac{x}{2}\right)+\sin\left(\frac{5x}{2}\right)}{15a^3\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-3),x]
```

```
[Out] -1/15*(5*Cos[(3*x)/2] - 10*Sin[x/2] + Sin[(5*x)/2])/(a^3*(Cos[x/2] + Sin[x/
2])^5)
```

**Maple [A]**
time = 0.10, size = 57, normalized size = 1.14

| method | result | size |
|---|---|---|
| risch | $\frac{-\frac{4}{15}+\frac{8\,{\rm e}^{2ix}}{3}+\frac{4i{\rm e}^{ix}}{3}}{\left({\rm e}^{ix}+i\right)^5 a^3}$ | 33 |
| default | $\frac{-\frac{2}{\tan\left(\frac{x}{2}\right)+1}-\frac{16}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}}{a^3}$ | 57 |
| norman | $\frac{-\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{14}{15a}-\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{8\tan\left(\frac{x}{2}\right)}{3a}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{3a}}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*(-1/(tan(1/2*x)+1)-8/3/(tan(1/2*x)+1)^3+2/(tan(1/2*x)+1)^2-4/5/(tan(1
/2*x)+1)^5+2/(tan(1/2*x)+1)^4)
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(44) = 88.
time = 0.31, size = 128, normalized size = 2.56

$$-\frac{2\left(\frac{20\sin(x)}{\cos(x)+1}+\frac{40\sin(x)^2}{(\cos(x)+1)^2}+\frac{30\sin(x)^3}{(\cos(x)+1)^3}+\frac{15\sin(x)^4}{(\cos(x)+1)^4}+7\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(x)/(cos(x) + 1) + 40*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 7)/(a^3 + 5*a^3*sin(x)/(cos(x)
+ 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*
a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 0.36, size = 92, normalized size = 1.84

$$-\frac{2\,\cos\left(x\right)^{3}-4\,\cos\left(x\right)^{2}-\left(2\,\cos\left(x\right)^{2}+6\,\cos\left(x\right)-3\right)\sin\left(x\right)-9\,\cos\left(x\right)-3}{15\,\left(a^{3}\cos\left(x\right)^{3}+3\,a^{3}\cos\left(x\right)^{2}-2\,a^{3}\cos\left(x\right)-4\,a^{3}+\left(a^{3}\cos\left(x\right)^{2}-2\,a^{3}\cos\left(x\right)-4\,a^{3}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^3 - 4*cos(x)^2 - (2*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos
(x) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x
)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(49) = 98$.
time = 0.74, size = 348, normalized size = 6.96

$$-\frac{30\tan^{4}\left(\frac{x}{2}\right)}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}-\frac{60\tan^{3}\left(\frac{x}{2}\right)}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}-\frac{80\tan^{2}\left(\frac{x}{2}\right)}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}-\frac{40\tan\left(\frac{x}{2}\right)}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}-\frac{14}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**3,x)
```

```
[Out] -30*tan(x/2)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x
/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 60*tan(x/2)**
3/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a
**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 80*tan(x/2)**2/(15*a**3*tan
(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**
2 + 75*a**3*tan(x/2) + 15*a**3) - 40*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**
3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x
/2) + 15*a**3) - 14/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*t
an(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3)
```

**Giac** [A]
time = 0.55, size = 45, normalized size = 0.90

$$-\frac{2\,\left(15\,\tan\left(\frac{1}{2}\,x\right)^{4}+30\,\tan\left(\frac{1}{2}\,x\right)^{3}+40\,\tan\left(\frac{1}{2}\,x\right)^{2}+20\,\tan\left(\frac{1}{2}\,x\right)+7\right)}{15\,a^{3}\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*x)^4 + 30*tan(1/2*x)^3 + 40*tan(1/2*x)^2 + 20*tan(1/2*x)
+ 7)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad [B]**
time = 6.63, size = 45, normalized size = 0.90

$$-\frac{2\left(15\tan\left(\frac{x}{2}\right)^4+30\tan\left(\frac{x}{2}\right)^3+40\tan\left(\frac{x}{2}\right)^2+20\tan\left(\frac{x}{2}\right)+7\right)}{15\,a^3\,\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x))^3,x)
```

```
[Out] -(2*(20*tan(x/2) + 40*tan(x/2)^2 + 30*tan(x/2)^3 + 15*tan(x/2)^4 + 7))/(15*
a^3*(tan(x/2) + 1)^5)
```

## 3.28 $\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=58

$$-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\,(a^3+a^3\sin(x))}$$

```
[Out] -arctanh(cos(x))/a^3+1/5*cos(x)/(a+a*sin(x))^3+7/15*cos(x)/a/(a+a*sin(x))^2
+22/15*cos(x)/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2845, 3057, 12, 3855}

$$\frac{22\cos(x)}{15\,(a^3\sin(x)+a^3)}-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{7\cos(x)}{15a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^3) + Cos[x]/(5*(a + a*Sin[x])^3) + (7*Cos[x])/(15*a*(a
+ a*Sin[x])^2) + (22*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc(x)(5a-2a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{\csc(x)\left(15a^2-7a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int 15a^3\csc(x)\,dx}{15a^6}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int \csc(x)\,dx}{a^3}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}
$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(58) = 116.
time = 0.05, size = 160, normalized size = 2.76

$$
\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-6\sin\left(\frac{x}{2}\right)+3\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-14\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2+7\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-44\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4-15\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+15\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\right)}{15(a+a\sin(x))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-6*Sin[x/2] + 3*(Cos[x/2] + Sin[x/2]) - 14*Sin[x/2]
*(Cos[x/2] + Sin[x/2])^2 + 7*(Cos[x/2] + Sin[x/2])^3 - 44*Sin[x/2]*(Cos[x/2
] + Sin[x/2])^4 - 15*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 15*Log[Sin[x/2
]]*(Cos[x/2] + Sin[x/2])^5))/(15*(a + a*Sin[x])^3)
```

**Maple** [**A**]
time = 0.18, size = 61, normalized size = 1.05

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\tan(\frac{x}{2}))+\frac{8}{5(\tan(\frac{x}{2})+1)^5}-\frac{4}{(\tan(\frac{x}{2})+1)^4}+\frac{20}{3(\tan(\frac{x}{2})+1)^3}-\frac{6}{(\tan(\frac{x}{2})+1)^2}+\frac{6}{\tan(\frac{x}{2})+1}}{a^3}$ | 61 |
| norman | $\frac{\frac{6(\tan^4(\frac{x}{2}))}{a}+\frac{64}{15a}+\frac{18(\tan^3(\frac{x}{2}))}{a}+\frac{74(\tan^2(\frac{x}{2}))}{3a}+\frac{46\tan(\frac{x}{2})}{3a}}{a^2(\tan(\frac{x}{2})+1)^5}+\frac{\ln(\tan(\frac{x}{2}))}{a^3}$ | 71 |
| risch | $\frac{10i e^{3ix}+2e^{4ix}-\frac{38i e^{ix}}{3}-\frac{58e^{2ix}}{3}+\frac{44}{15}}{(e^{ix}+i)^5a^3}-\frac{\ln(e^{ix}+1)}{a^3}+\frac{\ln(e^{ix}-1)}{a^3}$ | 74 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(ln(tan(1/2*x))+8/5/(tan(1/2*x)+1)^5-4/(tan(1/2*x)+1)^4+20/3/(tan(1/2
*x)+1)^3-6/(tan(1/2*x)+1)^2+6/(tan(1/2*x)+1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(52) = 104$.

time = 0.42, size = 143, normalized size = 2.47

$$\frac{2\left(\frac{115\sin(x)}{\cos(x)+1}+\frac{185\sin(x)^2}{(\cos(x)+1)^2}+\frac{135\sin(x)^3}{(\cos(x)+1)^3}+\frac{45\sin(x)^4}{(\cos(x)+1)^4}+32\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(115*sin(x)/(cos(x) + 1) + 185*sin(x)^2/(cos(x) + 1)^2 + 135*sin(x)^3/
(cos(x) + 1)^3 + 45*sin(x)^4/(cos(x) + 1)^4 + 32)/(a^3 + 5*a^3*sin(x)/(cos(
x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 +
 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + log(sin(x)/
(cos(x) + 1))/a^3
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.

time = 0.34, size = 168, normalized size = 2.90

$$\frac{44\cos(x)^3-58\cos(x)^2-15\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+15\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\left(22\cos(x)^2+51\cos(x)-3\right)\sin(x)-108\cos(x)-6}{30\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(44*cos(x)^3 - 58*cos(x)^2 - 15*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2
*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^3 +
3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(
```

```
x) + 1/2) - 2*(22*cos(x)^2 + 51*cos(x) - 3)*sin(x) - 108*cos(x) - 6)/(a^3*c
os(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos
(x) - 4*a^3)*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

**Giac** [A]
time = 0.62, size = 56, normalized size = 0.97

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3} + \frac{2\left(45\tan\left(\frac{1}{2}x\right)^4 + 135\tan\left(\frac{1}{2}x\right)^3 + 185\tan\left(\frac{1}{2}x\right)^2 + 115\tan\left(\frac{1}{2}x\right) + 32\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^3 + 2/15*(45*tan(1/2*x)^4 + 135*tan(1/2*x)^3 + 185*t
an(1/2*x)^2 + 115*tan(1/2*x) + 32)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad** [B]
time = 6.65, size = 54, normalized size = 0.93

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3} + \frac{6\tan\left(\frac{x}{2}\right)^4 + 18\tan\left(\frac{x}{2}\right)^3 + \frac{74\tan\left(\frac{x}{2}\right)^2}{3} + \frac{46\tan\left(\frac{x}{2}\right)}{3} + \frac{64}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))^3),x)
```

```
[Out] log(tan(x/2))/a^3 + ((46*tan(x/2))/3 + (74*tan(x/2)^2)/3 + 18*tan(x/2)^3 +
6*tan(x/2)^4 + 64/15)/(a^3*(tan(x/2) + 1)^5)
```

# 3.29 $\int \frac{\csc^2(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=65

$$\frac{3\tanh^{-1}(\cos(x))}{a^3}-\frac{24\cot(x)}{5a^3}+\frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}$$

```
[Out] 3*arctanh(cos(x))/a^3-24/5*cot(x)/a^3+1/5*cot(x)/(a+a*sin(x))^3+3/5*cot(x)/
a/(a+a*sin(x))^2+3*cot(x)/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.15, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$-\frac{24\cot(x)}{5a^3}+\frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{3\cot(x)}{a^3\sin(x)+a^3}+\frac{3\cot(x)}{5a(a\sin(x)+a)^2}+\frac{\cot(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[x]])/a^3 - (24*Cot[x])/(5*a^3) + Cot[x]/(5*(a + a*Sin[x])^3)
 + (3*Cot[x])/(5*a*(a + a*Sin[x])^2) + (3*Cot[x])/(a^3 + a^3*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc^2(x)(6a-3a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{\int \frac{\csc^2(x)\left(27a^2-18a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}+\frac{\int \csc^2(x)\left(72a^3-45a^3\sin(x\right.}{15a^6}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}-\frac{3\int \csc(x)\,dx}{a^3}+\frac{24\int \csc^2(x}{5a^3}\\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}-\frac{24\text{Subst}}{}\\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3}-\frac{24\cot(x)}{5a^3}+\frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x}
\end{aligned}
$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(65) = 130.

time = 0.11, size = 206, normalized size = 3.17

$$
\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)-2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+16\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2-8\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+76\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4-5\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+30\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-30\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+5\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\tan\left(\frac{x}{2}\right)\right)}{10(a+a\sin(x))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 16*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 8*(Cos[x/2] + Sin[x/2])^3 + 76*Sin[x/2]*(Cos[x/2]
 + Sin[x/2])^4 - 5*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 30*Log[Cos[x/2]]*(Cos
[x/2] + Sin[x/2])^5 - 30*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 5*(Cos[x/2
] + Sin[x/2])^5*Tan[x/2]))/(10*(a + a*Sin[x])^3)
```

**Maple** [A]
time = 0.19, size = 76, normalized size = 1.17

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)-\frac{16}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{8}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{16}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{16}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{24}{\tan\left(\frac{x}{2}\right)+1}-\frac{1}{\tan\left(\frac{x}{2}\right)}-6\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^{3}}$ | 76 |
| risch | $-\frac{2\left(-160\,\mathrm{e}^{4ix}+75i\mathrm{e}^{5ix}+189\,\mathrm{e}^{2ix}-200i\mathrm{e}^{3ix}-24+105i\mathrm{e}^{ix}+15\,\mathrm{e}^{6ix}\right)}{5(\mathrm{e}^{2ix}-1)(\mathrm{e}^{ix}+i)^{5}a^{3}}-\frac{3\ln\left(\mathrm{e}^{ix}-1\right)}{a^{3}}+\frac{3\ln\left(\mathrm{e}^{ix}+1\right)}{a^{3}}$ | 99 |
| norman | $\frac{-\frac{53\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}-\frac{1}{2a}+\frac{\tan^{7}\left(\frac{x}{2}\right)}{2a}-\frac{20\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a}-\frac{125\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2a}-\frac{73\tan\left(\frac{x}{2}\right)}{5a}-\frac{167\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2a}}{\tan\left(\frac{x}{2}\right)a^{2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{3}}$ | 100 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*(tan(1/2*x)-16/5/(tan(1/2*x)+1)^5+8/(tan(1/2*x)+1)^4-16/(tan(1/2*x)
+1)^3+16/(tan(1/2*x)+1)^2-24/(tan(1/2*x)+1)-1/tan(1/2*x)-6*ln(tan(1/2*x)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.
time = 0.58, size = 180, normalized size = 2.77

$$-\frac{\frac{121\,\sin(x)}{\cos(x)+1}+\frac{410\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{610\,\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{425\,\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{125\,\sin(x)^{5}}{(\cos(x)+1)^{5}}+5}{10\left(\frac{a^{3}\sin(x)}{\cos(x)+1}+\frac{5\,a^{3}\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{10\,a^{3}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{10\,a^{3}\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{5\,a^{3}\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{a^{3}\sin(x)^{6}}{(\cos(x)+1)^{6}}\right)}-\frac{3\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^{3}}+\frac{\sin(x)}{2\,a^{3}(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/10*(121*sin(x)/(cos(x) + 1) + 410*sin(x)^2/(cos(x) + 1)^2 + 610*sin(x)^3
/(cos(x) + 1)^3 + 425*sin(x)^4/(cos(x) + 1)^4 + 125*sin(x)^5/(cos(x) + 1)^5
 + 5)/(a^3*sin(x)/(cos(x) + 1) + 5*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin
(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 5*a^3*sin(x)^5/(cos
(x) + 1)^5 + a^3*sin(x)^6/(cos(x) + 1)^6) - 3*log(sin(x)/(cos(x) + 1))/a^3
+ 1/2*sin(x)/(a^3*(cos(x) + 1))
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(59) = 118$.

time = 0.39, size = 225, normalized size = 3.46

$$\frac{48 \cos(x)^4 + 114 \cos(x)^3 - 60 \cos(x)^2 + 15 \left(\cos(x)^4 - 2 \cos(x)^3 - 5 \cos(x)^2 - \left(\cos(x)^3 + 3 \cos(x)^2 - 2 \cos(x) - 4\right) \sin(x) + 2 \cos(x) + 4\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 15 \left(\cos(x)^4 - 2 \cos(x)^3 - 5 \cos(x)^2 - \left(\cos(x)^3 + 3 \cos(x)^2 - 2 \cos(x) - 4\right) \sin(x) + 2 \cos(x) + 4\right) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2 \left(24 \cos(x)^3 - 33 \cos(x)^2 - 63 \cos(x) - 1\right) \sin(x) - 124 \cos(x) + 2}{10 \left(a^3 \cos(x)^4 - 2 a^3 \cos(x)^3 - 5 a^3 \cos(x)^2 + 2 a^3 \cos(x) + 4 a^3 - \left(a^3 \cos(x)^3 + 3 a^3 \cos(x)^2 - 2 a^3 \cos(x) - 4 a^3\right) \sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(48*cos(x)^4 + 114*cos(x)^3 - 60*cos(x)^2 + 15*(cos(x)^4 - 2*cos(x)^3
- 5*cos(x)^2 - (cos(x)^3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4
)*log(1/2*cos(x) + 1/2) - 15*(cos(x)^4 - 2*cos(x)^3 - 5*cos(x)^2 - (cos(x)^
3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2
) + 2*(24*cos(x)^3 - 33*cos(x)^2 - 63*cos(x) - 1)*sin(x) - 124*cos(x) + 2)/
(a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3 - (a
^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

**Giac** [A]
time = 0.48, size = 85, normalized size = 1.31

$$-\frac{3 \log\left(\left|\tan\left(\frac{1}{2} x\right)\right|\right)}{a^3} + \frac{\tan\left(\frac{1}{2} x\right)}{2 a^3} + \frac{6 \tan\left(\frac{1}{2} x\right) - 1}{2 a^3 \tan\left(\frac{1}{2} x\right)} - \frac{4 \left(15 \tan\left(\frac{1}{2} x\right)^4 + 50 \tan\left(\frac{1}{2} x\right)^3 + 70 \tan\left(\frac{1}{2} x\right)^2 + 45 \tan\left(\frac{1}{2} x\right) + 12\right)}{5 a^3 \left(\tan\left(\frac{1}{2} x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*tan(1/2*x) - 1)/(
a^3*tan(1/2*x)) - 4/5*(15*tan(1/2*x)^4 + 50*tan(1/2*x)^3 + 70*tan(1/2*x)^2
+ 45*tan(1/2*x) + 12)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad** [B]
time = 6.72, size = 129, normalized size = 1.98

$$\frac{\tan\left(\frac{x}{2}\right)}{2 a^3} - \frac{25 \tan\left(\frac{x}{2}\right)^5 + 85 \tan\left(\frac{x}{2}\right)^4 + 122 \tan\left(\frac{x}{2}\right)^3 + 82 \tan\left(\frac{x}{2}\right)^2 + \frac{121 \tan\left(\frac{x}{2}\right)}{5} + 1}{2 a^3 \tan\left(\frac{x}{2}\right)^6 + 10 a^3 \tan\left(\frac{x}{2}\right)^5 + 20 a^3 \tan\left(\frac{x}{2}\right)^4 + 20 a^3 \tan\left(\frac{x}{2}\right)^3 + 10 a^3 \tan\left(\frac{x}{2}\right)^2 + 2 a^3 \tan\left(\frac{x}{2}\right)} - \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))^3),x)
```

```
[Out] tan(x/2)/(2*a^3) - ((121*tan(x/2))/5 + 82*tan(x/2)^2 + 122*tan(x/2)^3 + 85*
tan(x/2)^4 + 25*tan(x/2)^5 + 1)/(2*a^3*tan(x/2) + 10*a^3*tan(x/2)^2 + 20*a^
3*tan(x/2)^3 + 20*a^3*tan(x/2)^4 + 10*a^3*tan(x/2)^5 + 2*a^3*tan(x/2)^6) -
(3*log(tan(x/2)))/a^3
```

# 3.30 $\int \frac{\csc^3(x)}{(a+a\sin(x))^3}\,dx$

**Optimal**. Leaf size=86

$$-\frac{13\tanh^{-1}(\cos(x))}{2a^3}+\frac{152\cot(x)}{15a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)}$$

```
[Out] -13/2*arctanh(cos(x))/a^3+152/15*cot(x)/a^3-13/2*cot(x)*csc(x)/a^3+1/5*cot(
x)*csc(x)/(a+a*sin(x))^3+11/15*cot(x)*csc(x)/a/(a+a*sin(x))^2+76/15*cot(x)*
csc(x)/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\frac{152\cot(x)}{15a^3}-\frac{13\tanh^{-1}(\cos(x))}{2a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{76\cot(x)\csc(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{11\cot(x)\csc(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] (-13*ArcTanh[Cos[x]])/(2*a^3) + (152*Cot[x])/(15*a^3) - (13*Cot[x]*Csc[x])/
(2*a^3) + (Cot[x]*Csc[x])/(5*(a + a*Sin[x])^3) + (11*Cot[x]*Csc[x])/(15*a*(
a + a*Sin[x])^2) + (76*Cot[x]*Csc[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{\csc^3(x)(7a-4a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{\csc^3(x)(43a^2-33a^2\sin(x))}{a+a\sin(x)}\,dx}{15a^4} \\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} + \frac{\int \csc^3(x)\left(195a^3-15\right.}{15a^6} \\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} - \frac{152\int \csc^2(x)\,dx}{15a^3} + \frac{13}{} \\
&= -\frac{13\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} + \frac{1}{} \\
&= -\frac{13\tanh^{-1}(\cos(x))}{2a^3} + \frac{152\cot(x)}{15a^3} - \frac{13\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x}{15a(a+a}
\end{aligned}
$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 247 vs. $2(86) = 172$.

time = 0.32, size = 247, normalized size = 2.87

$$
\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(-48\sin\left(\frac{x}{2}\right)-15\left(1+\cot\left(\frac{x}{2}\right)\right)^5\sin^3\left(\frac{x}{2}\right)+24\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-272\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2+136\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3-1712\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4+180\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-780\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+780\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-180\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\tan\left(\frac{x}{2}\right)+15\cos^3\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^5\right)}{120a^3(1+\sin(x))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-48*Sin[x/2] - 15*(1 + Cot[x/2])^5*Sin[x/2]^3 + 24*
(Cos[x/2] + Sin[x/2]) - 272*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 136*(Cos[x/2
] + Sin[x/2])^3 - 1712*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 + 180*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^5 - 780*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 780*Log[S
in[x/2]]*(Cos[x/2] + Sin[x/2])^5 - 180*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] + 1
5*Cos[x/2]^3*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

**Maple [A]**
time = 0.22, size = 94, normalized size = 1.09

| method | result |
|---|---|
| default | $\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-6\tan\left(\frac{x}{2}\right)-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}+\frac{6}{\tan\left(\frac{x}{2}\right)}+26\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{32}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{16}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}+\frac{112}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{40}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{80}{\tan\left(\frac{x}{2}\right)}}{4a^3}$ |
| risch | $\frac{975ie^{7ix}+195\,e^{8ix}-3575ie^{5ix}-2275\,e^{6ix}+3805ie^{3ix}+4329\,e^{4ix}-1325ie^{ix}-2673\,e^{2ix}+304}{15\left(e^{2ix}-1\right)^2\left(e^{ix}+i\right)^5a^3}+\frac{13\ln\left(e^{ix}-1\right)}{2a^3}-\frac{13\ln\left(e^{ix}+1\right)}{2a^3}$ |

| norman | $\frac{\frac{39(\tan^6(\frac{x}{2}))}{a}-\frac{1}{8a}+\frac{7\tan(\frac{x}{2})}{8a}-\frac{7(\tan^8(\frac{x}{2}))}{8a}+\frac{\tan^9(\frac{x}{2})}{8a}+\frac{251(\tan^5(\frac{x}{2}))}{2a}+\frac{649(\tan^3(\frac{x}{2}))}{6a}+\frac{883(\tan^2(\frac{x}{2}))}{30a}+\frac{1013(\tan^4(\frac{x}{2}))}{6a}}{\tan(\frac{x}{2})^2 a^2 (\tan(\frac{x}{2})+1)^5} + \frac{13\ln(\tan}{2a^3}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(1/2*tan(1/2*x)^2-6*tan(1/2*x)-1/2/tan(1/2*x)^2+6/tan(1/2*x)+26*ln(
tan(1/2*x))+32/5/(tan(1/2*x)+1)^5-16/(tan(1/2*x)+1)^4+112/3/(tan(1/2*x)+1)^
3-40/(tan(1/2*x)+1)^2+80/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(74) = 148$.
time = 0.52, size = 209, normalized size = 2.43

$$\frac{\frac{105\sin(x)}{\cos(x)+1}+\frac{2782\sin(x)^2}{(\cos(x)+1)^2}+\frac{9410\sin(x)^3}{(\cos(x)+1)^3}+\frac{13645\sin(x)^4}{(\cos(x)+1)^4}+\frac{9285\sin(x)^5}{(\cos(x)+1)^5}+\frac{2580\sin(x)^6}{(\cos(x)+1)^6}-15}{120\left(\frac{a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{5a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{10a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{10a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{\frac{12\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a^3}+\frac{13\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*sin(x)/(cos(x) + 1) + 2782*sin(x)^2/(cos(x) + 1)^2 + 9410*sin(x)
^3/(cos(x) + 1)^3 + 13645*sin(x)^4/(cos(x) + 1)^4 + 9285*sin(x)^5/(cos(x) +
 1)^5 + 2580*sin(x)^6/(cos(x) + 1)^6 - 15)/(a^3*sin(x)^2/(cos(x) + 1)^2 + 5
*a^3*sin(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(
x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) + 1)^6 + a^3*sin(x)^7/(cos(x)
+ 1)^7) - 1/8*(12*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a^3 + 13/2
*log(sin(x)/(cos(x) + 1))/a^3
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(74) = 148$.
time = 0.36, size = 276, normalized size = 3.21

$$\frac{608\cos(x)^5-826\cos(x)^4-2174\cos(x)^3+784\cos(x)^2-195\left(\cos(x)^5+3\cos(x)^4-3\cos(x)^3-7\cos(x)^2+\left(\cos(x)^4-2\cos(x)^3-5\cos(x)^2+2\cos(x)+4\right)\sin(x)+2\cos(x)+4\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+195\left(\cos(x)^5+3\cos(x)^4-3\cos(x)^3-7\cos(x)^2+\left(\cos(x)^4-2\cos(x)^3-5\cos(x)^2+2\cos(x)+4\right)\sin(x)+2\cos(x)+4\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\left(304\cos(x)^4+717\cos(x)^3-370\cos(x)^2-762\cos(x)+6\right)\sin(x)+1536\cos(x)+12}{60\left(a^3\cos(x)^5+3a^3\cos(x)^4-3a^3\cos(x)^3-7a^3\cos(x)^2+2a^3\cos(x)+4a^3+\left(a^3\cos(x)^4-2a^3\cos(x)^3-5a^3\cos(x)^2+2a^3\cos(x)+4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(608*cos(x)^5 - 826*cos(x)^4 - 2174*cos(x)^3 + 784*cos(x)^2 - 195*(cos
(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 - 5*c
os(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(1/2*cos(x) + 1/2) + 195*
(cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 -
 5*cos(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2) -
 2*(304*cos(x)^4 + 717*cos(x)^3 - 370*cos(x)^2 - 762*cos(x) + 6)*sin(x) + 1
536*cos(x) + 12)/(a^3*cos(x)^5 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*co
```

```
s(x)^2 + 2*a^3*cos(x) + 4*a^3 + (a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(
x)^2 + 2*a^3*cos(x) + 4*a^3)*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**3,x)
[Out] Integral(csc(x)**3/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

**Giac** [A]
time = 0.52, size = 109, normalized size = 1.27

$$\frac{13\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^3}-\frac{78\tan\left(\frac{1}{2}x\right)^2-12\tan\left(\frac{1}{2}x\right)+1}{8a^3\tan\left(\frac{1}{2}x\right)^2}+\frac{a^3\tan\left(\frac{1}{2}x\right)^2-12a^3\tan\left(\frac{1}{2}x\right)}{8a^6}+\frac{2\left(150\tan\left(\frac{1}{2}x\right)^4+525\tan\left(\frac{1}{2}x\right)^3+745\tan\left(\frac{1}{2}x\right)^2+485\tan\left(\frac{1}{2}x\right)+127\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
[Out] 13/2*log(abs(tan(1/2*x)))/a^3 - 1/8*(78*tan(1/2*x)^2 - 12*tan(1/2*x) + 1)/(
a^3*tan(1/2*x)^2) + 1/8*(a^3*tan(1/2*x)^2 - 12*a^3*tan(1/2*x))/a^6 + 2/15*(
150*tan(1/2*x)^4 + 525*tan(1/2*x)^3 + 745*tan(1/2*x)^2 + 485*tan(1/2*x) + 1
27)/(a^3*(tan(1/2*x) + 1)^5)
```

**Mupad** [B]
time = 6.39, size = 97, normalized size = 1.13

$$\frac{\tan\left(\frac{x}{2}\right)^2}{8a^3}-\frac{3\tan\left(\frac{x}{2}\right)}{2a^3}+\frac{13\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^3}+\frac{\frac{43\tan\left(\frac{x}{2}\right)^6}{2}+\frac{619\tan\left(\frac{x}{2}\right)^5}{8}+\frac{2729\tan\left(\frac{x}{2}\right)^4}{24}+\frac{941\tan\left(\frac{x}{2}\right)^3}{12}+\frac{1391\tan\left(\frac{x}{2}\right)^2}{60}+\frac{7\tan\left(\frac{x}{2}\right)}{8}-\frac{1}{8}}{a^3\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))^3),x)
[Out] tan(x/2)^2/(8*a^3) - (3*tan(x/2))/(2*a^3) + (13*log(tan(x/2)))/(2*a^3) + ((
7*tan(x/2))/8 + (1391*tan(x/2)^2)/60 + (941*tan(x/2)^3)/12 + (2729*tan(x/2)
^4)/24 + (619*tan(x/2)^5)/8 + (43*tan(x/2)^6)/2 - 1/8)/(a^3*tan(x/2)^2*(tan
(x/2) + 1)^5)
```

# 3.31 $\int \frac{\csc^4(x)}{(a+a\sin(x))^3} dx$

**Optimal**. Leaf size=103

$$\frac{23\tanh^{-1}(\cos(x))}{2a^3}-\frac{136\cot(x)}{5a^3}-\frac{136\cot^3(x)}{15a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2}+\frac{23\,\mathrm{c}}{3\,(a}$$

```
[Out] 23/2*arctanh(cos(x))/a^3-136/5*cot(x)/a^3-136/15*cot(x)^3/a^3+23/2*cot(x)*c
sc(x)/a^3+1/5*cot(x)*csc(x)^2/(a+a*sin(x))^3+13/15*cot(x)*csc(x)^2/a/(a+a*s
in(x))^2+23/3*cot(x)*csc(x)^2/(a^3+a^3*sin(x))
```

**Rubi** [A]
time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2845, 3057, 2827, 3852, 3853, 3855}

$$-\frac{136\cot^3(x)}{15a^3}-\frac{136\cot(x)}{5a^3}+\frac{23\tanh^{-1}(\cos(x))}{2a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{13\cot(x)\csc^2(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc^2(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (23*ArcTanh[Cos[x]])/(2*a^3) - (136*Cot[x])/(5*a^3) - (136*Cot[x]^3)/(15*a^
3) + (23*Cot[x]*Csc[x])/(2*a^3) + (Cot[x]*Csc[x]^2)/(5*(a + a*Sin[x])^3) +
(13*Cot[x]*Csc[x]^2)/(15*a*(a + a*Sin[x])^2) + (23*Cot[x]*Csc[x]^2)/(3*(a^3
 + a^3*Sin[x]))
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc^4(x)(8a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{\csc^4(x)\left(63a^2-52a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)}+\frac{\int \csc^4(x)\left(408a^3-345a\right.}{15a^6}\\
&= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)}-\frac{23\int \csc^3(x)\,dx}{a^3}+\frac{136\int}{}\\
&= \frac{23\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}+\frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)}-\frac{23\int}{}\\
&= \frac{23\tanh^{-1}(\cos(x))}{2a^3}-\frac{136\cot(x)}{5a^3}-\frac{136\cot^3(x)}{15a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3}
\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 299 vs. 2(103) = 206.
time = 0.63, size = 299, normalized size = 2.90

$$\frac{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(48\sin\left(\frac{x}{2}\right)-5\cos\left(\frac{x}{2}\right)\left(1+\cot\left(\frac{x}{2}\right)\right)^5\sin^2\left(\frac{x}{2}\right)+45\left(1+\cot\left(\frac{x}{2}\right)\right)^5\sin^3\left(\frac{x}{2}\right)-24\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+352\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2-176\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3+2752\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4-400\cot\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+1380\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5-1380\log\left(\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5+400\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^5\tan\left(\frac{x}{2}\right)-45\cos^3\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^5+5\cos^2\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)\right)^5\right)}{120a^3(1+\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(48*Sin[x/2] - 5*Cos[x/2]*(1 + Cot[x/2])^5*Sin[x/2]^
2 + 45*(1 + Cot[x/2])^5*Sin[x/2]^3 - 24*(Cos[x/2] + Sin[x/2]) + 352*Sin[x/2
]*(Cos[x/2] + Sin[x/2])^2 - 176*(Cos[x/2] + Sin[x/2])^3 + 2752*Sin[x/2]*(Co
s[x/2] + Sin[x/2])^4 - 400*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 1380*Log[Cos[
x/2]]*(Cos[x/2] + Sin[x/2])^5 - 1380*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5
+ 400*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] - 45*Cos[x/2]^3*(1 + Tan[x/2])^5 + 5
*Cos[x/2]^2*Sin[x/2]*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

**Maple** [A]
time = 0.23, size = 110, normalized size = 1.07

| method | result |
|---|---|
| default | $\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{x}{2}\right)\right)+27\tan\left(\frac{x}{2}\right)-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}+\frac{3}{\tan\left(\frac{x}{2}\right)^2}-\frac{27}{\tan\left(\frac{x}{2}\right)}-92\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{64}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{32}{\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{256}{3\left(\tan\left(\frac{x}{2}\right)+1\right.}}{8a^3}$ |
| risch | $-\frac{11684\,e^{6ix}-12622\,e^{4ix}-544-4370\,e^{8ix}+1725ie^{9ix}-8050ie^{7ix}+13340ie^{5ix}+5347\,e^{2ix}-9230ie^{3ix}+2375ie^{ix}+345\,e^{10ix}}{15\left(e^{2ix}-1\right)^3\left(e^{ix}+i\right)^5a^3}+\frac{23\ln(e}{2e}$ |
| norman | $\frac{-\frac{1}{24a}+\frac{\tan\left(\frac{x}{2}\right)}{6a}-\frac{23\left(\tan^2\left(\frac{x}{2}\right)\right)}{12a}+\frac{23\left(\tan^9\left(\frac{x}{2}\right)\right)}{12a}-\frac{\tan^{10}\left(\frac{x}{2}\right)}{6a}+\frac{\tan^{11}\left(\frac{x}{2}\right)}{24a}-\frac{228\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{1067\left(\tan^3\left(\frac{x}{2}\right)\right)}{20a}-\frac{611\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a}-\frac{1567\left(\tan^4\left(\frac{x}{2}\right.}{8a}}{\tan\left(\frac{x}{2}\right)^3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^3*(1/3*tan(1/2*x)^3-3*tan(1/2*x)^2+27*tan(1/2*x)-1/3/tan(1/2*x)^3+3/t
an(1/2*x)^2-27/tan(1/2*x)-92*ln(tan(1/2*x))-64/5/(tan(1/2*x)+1)^5+32/(tan(1
/2*x)+1)^4-256/3/(tan(1/2*x)+1)^3+96/(tan(1/2*x)+1)^2-240/(tan(1/2*x)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(89) = 178.
time = 0.44, size = 232, normalized size = 2.25

$$\frac{\frac{20\sin(x)}{\cos(x)+1}-\frac{230\sin(x)^2}{(\cos(x)+1)^2}-\frac{4777\sin(x)^3}{(\cos(x)+1)^3}-\frac{15785\sin(x)^4}{(\cos(x)+1)^4}-\frac{22390\sin(x)^5}{(\cos(x)+1)^5}-\frac{14940\sin(x)^6}{(\cos(x)+1)^6}-\frac{4005\sin(x)^7}{(\cos(x)+1)^7}-5}{120\left(\frac{a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{10a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{10a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{5a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{a^3\sin(x)^8}{(\cos(x)+1)^8}\right)}+\frac{\frac{81\sin(x)}{\cos(x)+1}-\frac{9\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{24a^3}-\frac{23\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(20*sin(x)/(cos(x) + 1) - 230*sin(x)^2/(cos(x) + 1)^2 - 4777*sin(x)^3
/(cos(x) + 1)^3 - 15785*sin(x)^4/(cos(x) + 1)^4 - 22390*sin(x)^5/(cos(x) +
1)^5 - 14940*sin(x)^6/(cos(x) + 1)^6 - 4005*sin(x)^7/(cos(x) + 1)^7 - 5)/(a
^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(x)^
5/(cos(x) + 1)^5 + 10*a^3*sin(x)^6/(cos(x) + 1)^6 + 5*a^3*sin(x)^7/(cos(x)
+ 1)^7 + a^3*sin(x)^8/(cos(x) + 1)^8) + 1/24*(81*sin(x)/(cos(x) + 1) - 9*si
n(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^3 - 23/2*log(sin(x)/(cos
(x) + 1))/a^3
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(89) = 178$.
time = 0.35, size = 333, normalized size = 3.23

$$\frac{1088 \cos(x)^6 + 2574 \cos(x)^5 - 2428 \cos(x)^4 - 5338 \cos(x)^3 + 1372 \cos(x)^2 + 345 \left(\cos(x)^6 - 2 \cos(x)^5 - 6 \cos(x)^4 + 4 \cos(x)^3 + 9 \cos(x)^2 - \left(\cos(x)^5 + 3 \cos(x)^4 - 3 \cos(x)^3 - 7 \cos(x)^2 + 2 \cos(x) + 4\right) \sin(x) - 2 \cos(x) - 4\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 345 \left(\cos(x)^6 - 2 \cos(x)^5 - 6 \cos(x)^4 + 4 \cos(x)^3 + 9 \cos(x)^2 - \left(\cos(x)^5 + 3 \cos(x)^4 - 3 \cos(x)^3 - 7 \cos(x)^2 + 2 \cos(x) + 4\right) \sin(x) - 2 \cos(x) - 4\right) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2 \left(544 \cos(x)^5 - 743 \cos(x)^4 - 1957 \cos(x)^3 + 712 \cos(x)^2 + 1398 \cos(x) + 6\right) \sin(x) + 2784 \cos(x) - 12}{60 \left(a^3 \cos(x)^6 - 2 a^3 \cos(x)^5 - 6 a^3 \cos(x)^4 + 4 a^3 \cos(x)^3 + 9 a^3 \cos(x)^2 - 2 a^3 \cos(x) - 4 a^3 - \left(a^3 \cos(x)^5 + 3 a^3 \cos(x)^4 - 3 a^3 \cos(x)^3 - 7 a^3 \cos(x)^2 + 2 a^3 \cos(x) + 4 a^3\right) \sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(1088*cos(x)^6 + 2574*cos(x)^5 - 2428*cos(x)^4 - 5338*cos(x)^3 + 1372*
cos(x)^2 + 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos(x)^4 + 4*cos(x)^3 + 9*cos(x)^
2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + 2*cos(x) + 4)*sin(x)
 - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) - 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos
(x)^4 + 4*cos(x)^3 + 9*cos(x)^2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*c
os(x)^2 + 2*cos(x) + 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(x) + 1/2) + 2*(
544*cos(x)^5 - 743*cos(x)^4 - 1957*cos(x)^3 + 712*cos(x)^2 + 1398*cos(x) +
6)*sin(x) + 2784*cos(x) - 12)/(a^3*cos(x)^6 - 2*a^3*cos(x)^5 - 6*a^3*cos(x)
^4 + 4*a^3*cos(x)^3 + 9*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 - (a^3*cos(x)^5
 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3)
*sin(x))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

**Giac** [A]
time = 0.47, size = 128, normalized size = 1.24

$$-\frac{23 \log\left(\left|\tan\left(\frac{1}{2} x\right)\right|\right)}{2 a^3} + \frac{506 \tan\left(\frac{1}{2} x\right)^3 - 81 \tan\left(\frac{1}{2} x\right)^2 + 9 \tan\left(\frac{1}{2} x\right) - 1}{24 a^3 \tan\left(\frac{1}{2} x\right)^3} - \frac{2 \left(225 \tan\left(\frac{1}{2} x\right)^4 + 810 \tan\left(\frac{1}{2} x\right)^3 + 1160 \tan\left(\frac{1}{2} x\right)^2 + 760 \tan\left(\frac{1}{2} x\right) + 197\right)}{15 a^3 \left(\tan\left(\frac{1}{2} x\right) + 1\right)^5} + \frac{a^6 \tan\left(\frac{1}{2} x\right)^3 - 9 a^6 \tan\left(\frac{1}{2} x\right)^2 + 81 a^6 \tan\left(\frac{1}{2} x\right)}{24 a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*log(abs(tan(1/2*x)))/a^3 + 1/24*(506*tan(1/2*x)^3 - 81*tan(1/2*x)^2 +
 9*tan(1/2*x) - 1)/(a^3*tan(1/2*x)^3) - 2/15*(225*tan(1/2*x)^4 + 810*tan(1/
2*x)^3 + 1160*tan(1/2*x)^2 + 760*tan(1/2*x) + 197)/(a^3*(tan(1/2*x) + 1)^5)
 + 1/24*(a^6*tan(1/2*x)^3 - 9*a^6*tan(1/2*x)^2 + 81*a^6*tan(1/2*x))/a^9
```

**Mupad** [B]
time = 6.69, size = 117, normalized size = 1.14

$$\frac{27\tan\left(\frac{x}{2}\right)}{8\,a^3}-\frac{3\tan\left(\frac{x}{2}\right)^2}{8\,a^3}+\frac{\tan\left(\frac{x}{2}\right)^3}{24\,a^3}-\frac{23\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a^3}-\frac{\frac{267\tan\left(\frac{x}{2}\right)^7}{8}+\frac{249\tan\left(\frac{x}{2}\right)^6}{2}+\frac{2239\tan\left(\frac{x}{2}\right)^5}{12}+\frac{3157\tan\left(\frac{x}{2}\right)^4}{24}+\frac{4777\tan\left(\frac{x}{2}\right)^3}{120}+\frac{23\tan\left(\frac{x}{2}\right)^2}{12}-\frac{\tan\left(\frac{x}{2}\right)}{6}+\frac{1}{24}}{a^3\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))^3),x)
```

```
[Out] (27*tan(x/2))/(8*a^3) - (3*tan(x/2)^2)/(8*a^3) + tan(x/2)^3/(24*a^3) - (23*
log(tan(x/2)))/(2*a^3) - ((23*tan(x/2)^2)/12 - tan(x/2)/6 + (4777*tan(x/2)^
3)/120 + (3157*tan(x/2)^4)/24 + (2239*tan(x/2)^5)/12 + (249*tan(x/2)^6)/2 +
 (267*tan(x/2)^7)/8 + 1/24)/(a^3*tan(x/2)^3*(tan(x/2) + 1)^5)
```

# 3.32 $\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=158

$$-\frac{32a\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{64\cos(c+dx)\sqrt{a+}}{315d}$$

```
[Out] -32/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-32/45*a*cos(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)-16/63*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/9
*a*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+64/315*cos(d*x+c)*(a+a*
sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2849, 2838, 2830, 2725}

$$-\frac{2a\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{16a\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{32\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105ad}+\frac{64\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{32a\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-32*a*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a*Cos[c + d*x]*S
in[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c +
d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (64*Cos[c + d*x]*Sqrt[a + a*Sin[c
+ d*x]])/(315*d) - (32*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
```

```
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{8}{9}\int \sin^3(c+dx)\sqrt{a+a\sin(c+$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{16}{21}$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}-\frac{32}{}$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{64}{}$$
$$= -\frac{32a\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos}{9d}$$

**Mathematica [A]**
time = 0.33, size = 165, normalized size = 1.04

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(-1890\cos\left(\frac{1}{2}(c+dx)\right)-420\cos\left(\frac{3}{2}(c+dx)\right)+252\cos\left(\frac{5}{2}(c+dx)\right)+45\cos\left(\frac{7}{2}(c+dx)\right)-35\cos\left(\frac{9}{2}(c+dx)\right)+1890\sin\left(\frac{1}{2}(c+dx)\right)-420\sin\left(\frac{3}{2}(c+dx)\right)-252\sin\left(\frac{5}{2}(c+dx)\right)+45\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-1890*Cos[(c + d*x)/2] - 420*Cos[(3*(c + d*x))
/2] + 252*Cos[(5*(c + d*x))/2] + 45*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d
*x))/2] + 1890*Sin[(c + d*x)/2] - 420*Sin[(3*(c + d*x))/2] - 252*Sin[(5*(c
+ d*x))/2] + 45*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**
time = 1.71, size = 83, normalized size = 0.53

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)\left(35\left(\sin^{4}(dx+c)\right)+40\left(\sin^{3}(dx+c)\right)+48\left(\sin^{2}(dx+c)\right)+64\sin(dx+c)+128\right)}{315\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+40*sin(d*x+c)^3+48*s
in(d*x+c)^2+64*sin(d*x+c)+128)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^4, x)
```

**Fricas [A]**
time = 0.34, size = 132, normalized size = 0.84

$$-\frac{2\left(35\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{4}-118\cos\left(dx+c\right)^{3}+26\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}+40\cos\left(dx+c\right)^{3}-78\cos\left(dx+c\right)^{2}-104\cos\left(dx+c\right)+107\right)\sin\left(dx+c\right)+211\cos\left(dx+c\right)+107\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 118*cos(d*x + c)^3 + 26*cos(
d*x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 - 78*cos(d*x + c)^2 - 1
04*cos(d*x + c) + 107)*sin(d*x + c) + 211*cos(d*x + c) + 107)*sqrt(a*sin(d*
x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**4, x)
```

**Giac [A]**
time = 0.51, size = 147, normalized size = 0.93

$$\frac{\sqrt{2}\left(1890\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+420\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+252\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+45\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+35\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(1890*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c) + 420*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*
x + 3/2*c) + 252*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x
+ 5/2*c) + 45*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/2*d*x + 7
/2*c) + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9/4*pi + 9/2*d*x + 9/2*
c))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2), x)
```

# 3.33 $\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=122

$$-\frac{4a\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d}-\frac{12\cos(c+dx)(a}{3}$$

```
[Out] -12/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-4/5*a*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-2/7*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+8/35*cos
(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2849, 2838, 2830, 2725}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{4a\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[
c + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin
[c + d*x]])/(35*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
```

```
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{6}{7}\int \sin^2(c+dx)\sqrt{a+a\sin(c+a$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}-\frac{12\cos(c+dx)(a+a\sin(c+dx))^3}{35ad}$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d}$$

$$= -\frac{4a\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c$$

**Mathematica [A]**
time = 0.20, size = 141, normalized size = 1.16

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(-105\cos\left(\frac{1}{2}(c+dx)\right)-35\cos\left(\frac{3}{2}(c+dx)\right)+7\cos\left(\frac{5}{2}(c+dx)\right)+5\cos\left(\frac{7}{2}(c+dx)\right)+105\sin\left(\frac{1}{2}(c+dx)\right)-35\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)\right)}{140d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-105*Cos[(c + d*x)/2] - 35*Cos[(3*(c + d*x))/2
] + 7*Cos[(5*(c + d*x))/2] + 5*Cos[(7*(c + d*x))/2] + 105*Sin[(c + d*x)/2]
- 35*Sin[(3*(c + d*x))/2] - 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]
))/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**
time = 1.73, size = 73, normalized size = 0.60

| method | result | size |
|---|---|---|

| default | $\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)\left(5\left(\sin^3(dx+c)\right)+6\left(\sin^2(dx+c)\right)+8\sin(dx+c)+16\right)}{35\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ | 73 |
|---|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(5*sin(d*x+c)^3+6*sin(d*x+c)^2+8*sin(d
*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^3, x)
```

**Fricas** [**A**]
time = 0.34, size = 111, normalized size = 0.91

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{3}-12\cos\left(dx+c\right)^{2}+\left(5\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-13\cos\left(dx+c\right)+9\right)\sin\left(dx+c\right)-22\cos\left(dx+c\right)-9\right)\sqrt{a\sin\left(dx+c\right)+a}}{35\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 12*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 - cos(d*x + c)^2 - 13*cos(d*x + c) + 9)*sin(d*x + c) - 22*cos(d*x +
c) - 9)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.50, size = 120, normalized size = 0.98

$$\frac{\sqrt{2}\left(105\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+35\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)+7\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)+5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)\right)\sqrt{a}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*(105*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c) + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x +
 3/2*c) + 7*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x + 5/2
*c) + 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/2*d*x + 7/2*c))
*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

# 3.34 $\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=86

$$-\frac{14a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}+\frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad}$$

```
[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-14/15*a*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+4/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2838, 2830, 2725}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{14a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-14*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(15*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/
(5*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad} + \frac{2\int\left(\frac{3a}{2}-a\sin(c+dx)\right)\sqrt{}}{5a}$$

$$= \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2\cos(c+dx)(a+a\sin(c+d}{5ad}$$

$$= -\frac{14a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} -$$

**Mathematica** [A]
time = 0.13, size = 117, normalized size = 1.36

$$-\frac{\sqrt{a(1+\sin(c+dx))}\left(30\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)-3\cos\left(\frac{5}{2}(c+dx)\right)-30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/30*(Sqrt[a*(1 + Sin[c + d*x])]*(30*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*x)
)/2] - 3*Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2
] + 3*Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple** [A]
time = 1.70, size = 63, normalized size = 0.73

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)\left(3\left(\sin^2(dx+c)\right)+4\sin(dx+c)+8\right)}{15\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+4*sin(d*x+c)+8)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^2, x)
```

**Fricas [A]**
time = 0.43, size = 92, normalized size = 1.07

$$\frac{2\left(3\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-\left(3\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)-7\right)\sin\left(dx+c\right)-11\cos\left(dx+c\right)-7\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x + c
) - 7)*sin(d*x + c) - 11*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a)/(d*cos(
d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2, x)
```

**Giac [A]**
time = 0.57, size = 93, normalized size = 1.08

$$\frac{\sqrt{2}\left(30\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)+3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c) + 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x + 3/
2*c) + 3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x + 5/2*c)
)*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int{\sin\left(c+d\,x\right)}^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

# 3.35 $\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=56

$$-\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

```
[Out] -2/3*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2830, 2725}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}\int \sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

**Mathematica [A]**
time = 0.08, size = 81, normalized size = 1.45

$$-\frac{\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-4\sin^3\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/3*((3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 4*Sin[(c + d*x)/2]^3)*Sq
rt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**
time = 1.77, size = 51, normalized size = 0.91

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)(\sin(dx+c)+2)}{3\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$ | 51 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c), x)
```

**Fricas [A]**
time = 0.33, size = 67, normalized size = 1.20

$$-\frac{2\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)+1\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x), x)
```

**Giac** [A]
time = 0.60, size = 65, normalized size = 1.16

$$\frac{\sqrt{2}\left(3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
1/2*c) + sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x + 3/2*c)
)*sqrt(a)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

# 3.36 $\int \sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=26

$$-\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.02, size = 65, normalized size = 2.50

$$\frac{2\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**
time = 1.29, size = 43, normalized size = 1.65

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)}{\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ | 43 |
| risch | $-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)}{\left(\mathrm{e}^{2i(dx+c)}+2i\,\mathrm{e}^{i(dx+c)}-1\right)d}$ | 74 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(d*x+c))*a*(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.
time = 0.32, size = 50, normalized size = 1.92

$$-\frac{2\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/(d*cos(d*x +
c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(c + d*x) + a), x)
```

**Giac** [A]
time = 0.52, size = 36, normalized size = 1.38

$$\frac{2\sqrt{2}\,\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)/d
```

**Mupad** [B]
time = 0.21, size = 33, normalized size = 1.27

$$-\frac{2\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -(2*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(d*(sin(c + d*x) + 1))
```

# 3.37 $\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=37

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2852, 212}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.
time = 0.07, size = 94, normalized size = 2.54

$$\frac{\left(-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(31) = 62$.
time = 1.39, size = 68, normalized size = 1.84

| method | result | size |
|---|---|---|
| default | $-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)}{\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ | 68 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctanh((-a*(sin(d*x+c)
-1))^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c), x)
```

**Fricas** [A]
time = 0.39, size = 219, normalized size = 5.92

$$\left[\frac{\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2\,d},\ -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{-a}\,(\sin(dx+c)-2)}{2\,a\cos(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(-a)*arctan(1/2*sqrt(a*sin(d*x
+ c) + a)*sqrt(-a)*(sin(d*x + c) - 2)/(a*cos(d*x + c)))/d]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.
time = 0.49, size = 67, normalized size = 1.81

$$-\frac{\sqrt{a}\,\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt(
2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

# 3.38 $\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=64

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-a*cot(d*x+c)/
d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2851, 2852, 212}

$$-\frac{a\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) - (
a*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{1}{2}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 178 vs. 2(64) = 128.
time = 0.48, size = 178, normalized size = 2.78

$$-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(2\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)+\left(\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*Si
n[(c + d*x)/2] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4
])))
```

**Maple** [**A**]
time = 1.77, size = 104, normalized size = 1.62

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+\text{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a^2 \ldots\right.}{\sin(dx+c)a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)*a^(3/2)+arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2*sin(d*x+c))/sin(d*x+c)/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^2, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(56) = 112.
time = 0.35, size = 258, normalized size = 4.03

$$\frac{\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{4\left(d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\csc^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(56) = 112.
time = 0.50, size = 122, normalized size = 1.91

$$-\frac{\sqrt{2}\left(\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^2, x)
```

# 3.39 $\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=102

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -3/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-3/4*a*cot
(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x
+c))^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2851, 2852, 212}

$$-\frac{3a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (3*a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}+\frac{3}{4}\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}+\frac{3}{8}\int \csc(c$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}-\frac{(3a)\mathrm{Subs}}{}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(102) = 204.
time = 0.52, size = 249, normalized size = 2.44

$$\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-2\cos\left(\frac{1}{2}(c+dx)\right)-6\cos\left(\frac{3}{2}(c+dx)\right)-3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)-6\sin\left(\frac{3}{2}(c+dx)\right)\right)}{4d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-2*Cos[(c + d*x)/2] - 6*Cos
[(3*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[
2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2]))/(4*d
*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

**Maple** [A]
time = 2.12, size = 132, normalized size = 1.29

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{3}{2}}\sin(dx+c)+3\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx \cdots}{\sqrt{a \cdots}}\right.\right.}{4\sin(dx+c)^{2}a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)} \cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*
a^(3/2)*sin(d*x+c)+3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^
2*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/
(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^3, x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(86) = 172.
time = 0.45, size = 319, normalized size = 3.13

$$\frac{3\left(\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\,\cos(dx+c)^{2}+(3\,\cos(dx+c)+1)\sin(dx+c)+2\,\cos(dx+c)-1\right)\sqrt{a\sin(dx+c)+a}}{16\left(d\cos(dx+c)^{3}+d\cos(dx+c)^{2}-d\cos(dx+c)+\left(d\cos(dx+c)^{2}-d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(\sin(c+dx)+1)}\,\csc^{3}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**3, x)
```

**Giac [A]**
time = 0.50, size = 155, normalized size = 1.52

$$-\frac{\sqrt{2}\left(3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-5\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) + 4*(6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^3 - 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^3, x)
```

# 3.40 $\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=138

$$-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)}{3d\sqrt{a+a\,\mathrm{s}}}$$

```
[Out] -5/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-5/8*a*cot
(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-5/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-1/3*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2851, 2852, 212}

$$-\frac{5a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (5*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (5*a*Cot[c + d*x]*Cs
c[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5}{6}\int \csc^3(c+dx)\sqrt{a+a\sin(c+d\ldots}\\
&= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5}{8}\int \ldots\\
&= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c\ldots}{3d\sqrt{a\ldots}}\\
&= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c\ldots}{3d\sqrt{a\ldots}}\\
&= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}\ldots
\end{aligned}
$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(138) = 276.
time = 0.90, size = 285, normalized size = 2.07

$$
\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-84\cos\left(\frac{1}{2}(c+dx)\right)-10\cos\left(\frac{3}{2}(c+dx)\right)+30\cos\left(\frac{5}{2}(c+dx)\right)+84\sin\left(\frac{1}{2}(c+dx)\right)-45\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+45\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-10\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)+15\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-15\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-84*Cos[(c + d*x)/2] - 10*
Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] + 84*Sin[(c + d*x)/2] - 45*L
og[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 45*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 10*Sin[(3*(c + d*x))/2] - 30
*Sin[(5*(c + d*x))/2] + 15*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 15*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3)
```

**Maple [A]**
time = 2.64, size = 158, normalized size = 1.14

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(15\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{3}{2}}\left(\sin^{2}(dx+c)\right)+15\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\right.}}{}\right.\right.}{24\sin(dx+c)^{3}a^{\frac{3}{2}}\cos(dx+c)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c)-1))^(1/2
)*a^(3/2)*sin(d*x+c)^2+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^3*a^2+10*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)+8*(-a*(sin(d*x+c
)-1))^(1/2)*a^(3/2))/sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)
/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^4, x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(118) = 236$.
time = 0.35, size = 361, normalized size = 2.62

$$\frac{15\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(15\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)^{2}-\left(15\cos\left(dx+c\right)^{2}+10\cos\left(dx+c\right)-13\right)\sin\left(dx+c\right)-23\cos\left(dx+c\right)-13\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(d\cos\left(dx+c\right)^{4}-2d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(15*cos(d*x + c)^3 + 5*cos(d*x + c)^2 - (15*cos(d*x + c)^2 + 10*cos(d*x +
 c) - 13)*sin(d*x + c) - 23*cos(d*x + c) - 13)*sqrt(a*sin(d*x + c) + a))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2
 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\csc^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**4, x)
```

**Giac [A]**
time = 0.56, size = 184, normalized size = 1.33

$$-\frac{\sqrt{2}\left(15\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(60\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-80\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+33\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/
2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) + 4*(60*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^5 - 80*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^3 + 33*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^4, x)
```

# 3.41 $\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=38

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a-a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2852, 212}

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a - a*Sin[c + d*x]]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx &= \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(38) = 76.

time = 0.07, size = 97, normalized size = 2.55

$$\frac{\left(\log \left(1-\cos \left(\frac{1}{2}(c+dx)\right)-\sin \left(\frac{1}{2}(c+dx)\right)\right)-\log \left(1+\cos \left(\frac{1}{2}(c+dx)\right)+\sin \left(\frac{1}{2}(c+dx)\right)\right)\right)\sqrt{a-a\sin(c+dx)}}{d\left(\cos \left(\frac{1}{2}(c+dx)\right)-\sin \left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a - a*Sin[c + d*x]])/(d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

time = 1.29, size = 67, normalized size = 1.76

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(dx+c)-1)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}\,\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}{\sqrt{a}}\right)}{\cos(dx+c)\sqrt{a-a\sin\left(dx+c\right)}\,d}$ | 67 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctanh((a*(1+sin(d*x+c))
)^(1/2)/a^(1/2))/cos(d*x+c)/(a-a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(d*x + c) + a)*csc(d*x + c), x)
```

**Fricas** [A]

time = 0.37, size = 223, normalized size = 5.87

$$\left[\frac{\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2-(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{-a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)-\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2\,d}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a\sin\left(dx+c\right)+a}\,\sqrt{-a}\,(\sin(dx+c)+2)}{2\,a\cos(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(-a)*arctan(1/2*sqrt(-a*sin(d*x
 + c) + a)*sqrt(-a)*(sin(d*x + c) + 2)/(a*cos(d*x + c)))/d]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(c+dx\right)-1\right)}\,\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(32) = 64.
time = 0.59, size = 104, normalized size = 2.74

$$\frac{\sqrt{a}\,\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*log(abs(-4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(cos(-1
/4*pi + 1/2*d*x + 1/2*c) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x
 + 1/2*c) - 1)/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6))*sgn(sin(-1/4*pi +
 1/2*d*x + 1/2*c))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a-a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
```

```
[Out] int((a - a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

# 3.42 $\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=39

$$\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}$$

```
[Out] 2*arctan(cos(d*x+c)*a^(1/2)/(-a+a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2852, 210}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a + a*Sin[c + d*x]]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(39) = 78$.
time = 0.05, size = 96, normalized size = 2.46

$$\frac{\left(\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sqrt{a(-1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a*(-1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.
time = 1.25, size = 70, normalized size = 1.79

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(dx+c)-1)\sqrt{-a\left(1+\sin\left(dx+c\right)\right)}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{-a\left(1+\sin\left(dx+c\right)\right)}}{\sqrt{a}}\right)}{\cos(dx+c)\sqrt{a\sin\left(dx+c\right)-a}\,d}$ | 70 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a*sin(d*x+c)-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(sin(d*x+c)-1)*(-a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctan((-a*(1+sin(d*x+c)
))^(1/2)/a^(1/2))/cos(d*x+c)/(a*sin(d*x+c)-a)^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) - a)*csc(d*x + c), x)
```

**Fricas** [A]
time = 0.37, size = 223, normalized size = 5.72

$$\left[\frac{\sqrt{-a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2-(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)-a}\,\sqrt{-a}-9a\cos(dx+c)-\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2d},\ -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a\sin\left(dx+c\right)-a}\,(\sin(dx+c)+2)}{2\sqrt{a}\cos(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) - a)*sqrt(-a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(a)*arctan(1/2*sqrt(a*sin(d*x
 + c) - a)*(sin(d*x + c) + 2)/(sqrt(a)*cos(d*x + c)))/d]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)-1\right)}\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(33) = 66.
time = 0.62, size = 106, normalized size = 2.72

$$\frac{\sqrt{-a}\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-a)*log(abs(-4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(cos(-
1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*
x + 1/2*c) - 1)/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6))*sgn(sin(-1/4*pi
+ 1/2*d*x + 1/2*c))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,\sin\left(c+d\,x\right)-a}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) - a)^(1/2)/sin(c + d*x),x)
```

```
[Out] int((a*sin(c + d*x) - a)^(1/2)/sin(c + d*x), x)
```

# 3.43 $\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}$$

```
[Out] 2*arctan(cos(d*x+c)*a^(1/2)/(-a-a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2852, 210}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a(-\sin(c+dx))-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a - a*Sin[c + d*x]]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx &= \frac{(2a)\text{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(40) = 80.
time = 0.06, size = 95, normalized size = 2.38

$$\frac{\left(-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sqrt{-a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[-(a*(1 + Sin[c + d*x]))])/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))
```

**Maple** [A]
time = 1.36, size = 69, normalized size = 1.72

| method | result | size |
|---|---|---|
| default | $-\frac{2(1+\sin(dx+c))\sqrt{a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)}{\cos(dx+c)\sqrt{-a-a\sin\left(dx+c\right)}\,d}$ | 69 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+sin(d*x+c))*(a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctan((a*(sin(d*x+c)-1)
)^(1/2)/a^(1/2))/cos(d*x+c)/(-a-a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(d*x + c) - a)*csc(d*x + c), x)
```

**Fricas** [A]
time = 0.36, size = 221, normalized size = 5.52

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{-a\sin\left(dx+c\right)-a}\,\sqrt{-a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2\,d},\ \frac{\sqrt{a}\,\arctan\left(\frac{\sqrt{-a\sin\left(dx+c\right)-a}\,(\sin(dx+c)-2)}{2\,\sqrt{a}\,\cos(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
 c) - a)*sqrt(-a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(a)*arctan(1/2*sqrt(-a*sin(d*
x + c) - a)*(sin(d*x + c) - 2)/(sqrt(a)*cos(d*x + c)))/d]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(c+dx\right)+1\right)}\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.
time = 0.54, size = 69, normalized size = 1.72

$$-\frac{\sqrt{-a}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt
(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-a-a\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
```

```
[Out] int((- a - a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

# 3.44 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=162

$$-\frac{68a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{136a\cos(c+dx)\cdots}{3\cdots}$$

```
[Out] -68/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-68/45*a^2*cos(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)-34/63*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2
/9*a^2*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+136/315*a*cos(d*x+c
)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2842, 21, 2849, 2838, 2830, 2725}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{68a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{68\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105d}+\frac{136a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*
x]*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Si
n[c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (136*a*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(315*d) - (68*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1
05*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{2}{9}\int \frac{\sin^3(c+dx)\left(\frac{17a^2}{2}+\frac{17}{2}a^2\,\mathrm{s}\right.}{\sqrt{a+a\sin(c+d}}$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{1}{9}(17a)\int \sin^3(c+dx)\sqrt{a+a\,\mathrm{s}}$$

$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{68a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2}{9}$$

**Mathematica** [A]
time = 0.35, size = 165, normalized size = 1.02

$$\frac{(a(1+\sin(c+dx)))^{3/2}\left(-3780\cos\left(\frac{1}{2}(c+dx)\right)-1050\cos\left(\frac{3}{2}(c+dx)\right)+378\cos\left(\frac{5}{2}(c+dx)\right)+135\cos\left(\frac{7}{2}(c+dx)\right)-35\cos\left(\frac{9}{2}(c+dx)\right)+3780\sin\left(\frac{1}{2}(c+dx)\right)-1050\sin\left(\frac{3}{2}(c+dx)\right)-378\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-3780*Cos[(c + d*x)/2] - 1050*Cos[(3*(c + d*
x))/2] + 378*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 3780*Sin[(c + d*x)/2] - 1050*Sin[(3*(c + d*x))/2] - 378*Sin[(
5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [A]
time = 2.24, size = 85, normalized size = 0.52

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)(35(\sin^4(dx+c))+85(\sin^3(dx+c))+102(\sin^2(dx+c))+136\sin(dx+c)+272)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 85 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+85*sin(d*x+c)^3+10
2*sin(d*x+c)^2+136*sin(d*x+c)+272)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^3, x)
```

**Fricas [A]**
time = 0.33, size = 145, normalized size = 0.90

$$-\frac{2\,\left(35\,a\cos\left(dx+c\right)^{5}-50\,a\cos\left(dx+c\right)^{4}-172\,a\cos\left(dx+c\right)^{3}+134\,a\cos\left(dx+c\right)^{2}+409\,a\cos\left(dx+c\right)-\left(35\,a\cos\left(dx+c\right)^{4}+85\,a\cos\left(dx+c\right)^{3}-87\,a\cos\left(dx+c\right)^{2}-221\,a\cos\left(dx+c\right)+188\,a\right)\sin\left(dx+c\right)+188\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^3 +
134*a*cos(d*x + c)^2 + 409*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos
(d*x + c)^3 - 87*a*cos(d*x + c)^2 - 221*a*cos(d*x + c) + 188*a)*sin(d*x + c
) + 188*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sin^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)**3, x)
```

**Giac [A]**
time = 0.52, size = 152, normalized size = 0.94

$$\frac{\sqrt{2}\,\left(3780\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1050\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+378\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+135\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+35\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(3780*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) + 1050*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3
/2*d*x + 3/2*c) + 378*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5
/2*d*x + 5/2*c) + 135*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7
```

```
/2*d*x + 7/2*c) + 35*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9/4*pi + 9/
2*d*x + 9/2*c))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+dx)^3 (a+a\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

# 3.45 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=116

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d}+\frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d}-\frac{2\cos\cdots}{\cdots}$$

```
[Out] 4/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d*x+c)*(a+a*sin(d*x+c))^(5
/2)/a/d-152/105*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-38/105*a*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2726, 2725}

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}-\frac{38a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-152*a^2*Cos[c + d*x])/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(35*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7ad}+\frac{2\int\left(\frac{5a}{2}-a\sin(c+dx)\right)(\ldots}{7\ldots}\\ &= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d}-\frac{2\cos(c+dx)(a+a\sin(c+\ldots}{7ad}\\ &= -\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d}+\frac{4\cos(c+dx)(a+a\sin(\ldots}{35d}\\ &= -\frac{152a^2\cos(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx\ldots}}{105d}\end{aligned}$$

**Mathematica** [A]
time = 0.23, size = 141, normalized size = 1.22

$$\frac{(a(1+\sin(c+dx)))^{3/2}\left(-735\cos\left(\frac{1}{2}(c+dx)\right)-175\cos\left(\frac{3}{2}(c+dx)\right)+63\cos\left(\frac{5}{2}(c+dx)\right)+15\cos\left(\frac{7}{2}(c+dx)\right)+735\sin\left(\frac{1}{2}(c+dx)\right)-175\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)\right)}{420d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-735*Cos[(c + d*x)/2] - 175*Cos[(3*(c + d*x)
)/2] + 63*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] + 735*Sin[(c + d*x
)/2] - 175*Sin[(3*(c + d*x))/2] - 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c +
d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [A]
time = 1.63, size = 75, normalized size = 0.65

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)\left(15\left(\sin^3(dx+c)\right)+39\left(\sin^2(dx+c)\right)+52\sin(dx+c)+104\right)}{105\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 75 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(15*sin(d*x+c)^3+39*sin(d*x+c)^2+52
*sin(d*x+c)+104)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^2, x)
```

**Fricas** [A]
time = 0.44, size = 122, normalized size = 1.05

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}-43\,a\cos\left(dx+c\right)^{2}-143\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(dx+c\right)^{2}-67\,a\cos\left(dx+c\right)+76\,a\right)\sin\left(dx+c\right)-76\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 43*a*cos(d*x + c)^2 - 14
3*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 67*a*cos(d*
x + c) + 76*a)*sin(d*x + c) - 76*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c
) + d*sin(d*x + c) + d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sin^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)**2, x)
```

**Giac** [A]
time = 0.57, size = 124, normalized size = 1.07

$$\frac{\sqrt{2}\left(735\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+175\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+63\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+15\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(735*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c) + 175*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*
```

```
d*x + 3/2*c) + 63*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d
*x + 5/2*c) + 15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/2*d*
x + 7/2*c))*sqrt(a)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+dx)^2 (a+a\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

# 3.46 $\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=86

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}$$

```
[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-8/5*a^2*cos(d*x+c)/d/(a+a*sin(d*x+
c))^(1/2)-2/5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2726, 2725}

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(5*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))
/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{3}{5}\int (a+a\sin(c+dx))^{3/2}$$
$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c+dx)(a+a\sin(c}{5d}$$
$$= -\frac{8a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-$$

**Mathematica** [A]
time = 0.11, size = 115, normalized size = 1.34

$$-\frac{(a(1+\sin(c+dx)))^{3/2}\left(20\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)-\cos\left(\frac{5}{2}(c+dx)\right)-20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{10d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/10*((a*(1 + Sin[c + d*x]))^(3/2)*(20*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*
x))/2] - Cos[(5*(c + d*x))/2] - 20*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [A]
time = 1.84, size = 63, normalized size = 0.73

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)(\sin^2(dx+c)+3\sin(dx+c)+6)}{5\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)^2+3*sin(d*x+c)+6)/cos(d*x
+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c), x)
```

**Fricas [A]**
time = 0.33, size = 99, normalized size = 1.15

$$\frac{2\left(a\cos\left(dx+c\right)^{3}-2\,a\cos\left(dx+c\right)^{2}-7\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)-4\,a\right)\sin\left(dx+c\right)-4\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 7*a*cos(d*x + c) - (a*cos(d*x
+ c)^2 + 3*a*cos(d*x + c) - 4*a)*sin(d*x + c) - 4*a)*sqrt(a*sin(d*x + c) +
a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sin\left(c+dx\right)+1)\right)^{\frac{3}{2}}\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x), x)
```

**Giac [A]**
time = 0.67, size = 95, normalized size = 1.10

$$\frac{\sqrt{2}\left(20\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\sqrt{a}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2)*(20*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c) + 5*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x
+ 3/2*c) + a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x + 5/
2*c))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(c+d\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

# 3.47 $\int (a + a\sin(c + dx))^{3/2}\, dx$

**Optimal**. Leaf size=59

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

```
[Out] -8/3*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/d
```

**Rubi** [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}(4a)\int\sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

**Mathematica** [A]
time = 0.09, size = 89, normalized size = 1.51

$$-\frac{(a(1+\sin(c+dx)))^{3/2}\left(9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[c + d*x]))^(3/2)*(9*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/
2] - 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3)
```

**Maple** [A]
time = 1.48, size = 53, normalized size = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)(\sin(dx+c)+5)}{3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 53 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)+5)/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas** [A]
time = 0.33, size = 76, normalized size = 1.29

$$-\frac{2\left(a\cos\left(dx+c\right)^2+5\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)-4\,a\right)\sin\left(dx+c\right)+4\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c) + (a*cos(d*x + c) - 4*a)*sin(d*x
+ c) + 4*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*sin(c + d*x) + a)**(3/2), x)
```

**Giac [A]**
time = 0.49, size = 67, normalized size = 1.14

$$\frac{\sqrt{2}\,\left(9\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(2)*(9*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c) + a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x + 3/
2*c))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^(3/2), x)
```

# 3.48 $\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=66

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-2*a^2*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2842, 21, 2852, 212}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
```

```
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+2\int \frac{\csc(c+dx)\left(\frac{a^2}{2}+\frac{1}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}} \\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+a\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx \\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+}{\sqrt{a+a\sin(}}\right.}{d} \\
&= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

**Mathematica** [A]
time = 0.11, size = 118, normalized size = 1.79

$$\frac{\left(-2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sin(c+dx)))^{3/2}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*(a*(1 + Sin[
c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [A]
time = 1.88, size = 84, normalized size = 1.27

| method | result |
|---|---|
| default | $-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\, a\left(\sqrt{a-a\sin\left(dx+c\right)}+\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)\right.}{\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*((a-a*sin(d*x+c))^(1/2)+a^(1/
2)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(58) = 116$.
time = 0.36, size = 239, normalized size = 3.62

$$\frac{\left(a\cos\left(dx+c\right)+a\sin\left(dx+c\right)+a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(a\cos\left(dx+c\right)-a\sin\left(dx+c\right)+a\right)\sqrt{a\sin\left(dx+c\right)+a}}{2\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\csc\left(c+dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)

[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*csc(c + d*x), x)

**Giac [A]**
time = 0.52, size = 104, normalized size = 1.58

$$-\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-4\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*(sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 4*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))*sqrt(a)/d

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x),x)

[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x), x)

# 3.49 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=66

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-a^2*cot(d*x
+c)/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 21, 2852, 212}

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{a^2\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^2*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
```

```
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-a\int \frac{\csc(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sin(c+dx\right.}{\sqrt{a+a\sin(c+dx)}}\\ &= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{1}{2}(3a)\int \csc(c+dx)\sqrt{a+a\sin(c+d}\\ &= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c}{\sqrt{a+a\sin}}\right.}{d}\\ &= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(66) = 132.
time = 0.41, size = 180, normalized size = 2.73

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(2\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)+3\left(\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2] + 3*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x
)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*
x)/4])))
```

**Maple [A]**
time = 1.72, size = 103, normalized size = 1.56

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)a\sin(dx+c)+\sqrt{a-a\sin\left(\right.}\right.}{\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(3*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*a*sin(d*x+c)+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^2, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(58) = 116.
time = 0.34, size = 268, normalized size = 4.06

$$\frac{3\left(a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(a\cos\left(dx+c\right)-a\sin\left(dx+c\right)+a\right)\sqrt{a\sin\left(dx+c\right)+a}}{4\left(d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(58) = 116.
time = 0.49, size = 125, normalized size = 1.89

$$-\frac{\sqrt{2}\left(3\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1}\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(3*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/
2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) + 4*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^2, x)
```

# 3.50 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=106

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -7/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-7/4*a^2*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 21, 2851, 2852, 212}

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{7a^2\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (7*a^2*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*
Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
```

```
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{1}{2}a\int \frac{\csc^2(c+dx)\left(-\frac{7a}{2}-\frac{7}{2}a\sin(\right.}{\sqrt{a+a\sin(c+dx)}} \\
&= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(7a)\int \csc^2(c+dx)\sqrt{a+a\sin(} \\
&= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(7a) \\
&= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{(7a^2)\,\mathrm{S}}{} \\
&= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(106) = 212.

time = 0.42, size = 250, normalized size = 2.36

$$\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(6\cos\left(\frac{1}{2}(c+dx)\right)-14\cos\left(\frac{3}{2}(c+dx)\right)-7\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+7\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-7\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\sin\left(\frac{1}{2}(c+dx)\right)-14\sin\left(\frac{3}{2}(c+dx)\right)\right)}{4d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(6*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] - 7*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Co
s[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] - 7*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 6*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2]))/(
4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

**Maple [A]**
time = 2.25, size = 126, normalized size = 1.19

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(9\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{5}{2}}-7(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}+7\operatorname{arctanh}\left(\right.\right.}{4\sin(dx+c)^2 a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(9*(-a*(sin(d*x+c)-1))^(1/2)*
a^(5/2)-7*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+7*arctanh((-a*(sin(d*x+c)-1))^(
1/2)/a^(1/2))*a^3*sin(d*x+c)^2)/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^3, x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(90) = 180$.
time = 0.44, size = 337, normalized size = 3.18

$$\frac{7\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(7a\cos(dx+c)^2+2a\cos(dx+c)+(7a\cos(dx+c)+5a)\sin(dx+c)-5a\right)\sqrt{a\sin(dx+c)+a}}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2-d\cos(dx+c)+\left(d\cos(dx+c)^2-d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
+ c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c
) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c
)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c
)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(7*a*cos(d
*x + c)^2 + 2*a*cos(d*x + c) + (7*a*cos(d*x + c) + 5*a)*sin(d*x + c) - 5*a)
*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x
 + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

**Giac [A]**
time = 0.52, size = 158, normalized size = 1.49

$$-\frac{\sqrt{2}\left(7\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(14\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(7*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)) + 4*(14*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^3 - 9*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^3, x)
```

# 3.51 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

**Optimal**. Leaf size=144

$$-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+}{3d\sqrt{a+}}$$

```
[Out] -11/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-11/8*a^2
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-11/12*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-1/3*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {2841, 21, 2851, 2852, 212}

$$-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{11a^2\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (11*a^2*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (11*a^2*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
```

```
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{1}{3}a\int \frac{\csc^3(c+dx)\left(-\frac{11a}{2} - \frac{11}{2}a\,\mathrm{s}\right.}{\sqrt{a+a\sin(c+d}}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{6}(11a)\int \csc^3(c+dx)\sqrt{a+a\,\mathrm{si}}$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\,\mathrm{co}}{3d}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\,\mathrm{co}}{3d}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx}}$$

**Mathematica** [A]
time = 0.63, size = 286, normalized size = 1.99

$$\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-108\cos\left(\frac{1}{2}(c+dx)\right)-22\cos\left(\frac{3}{2}(c+dx)\right)+66\cos\left(\frac{5}{2}(c+dx)\right)+108\sin\left(\frac{1}{2}(c+dx)\right)-99\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+99\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-22\sin\left(\frac{3}{2}(c+dx)\right)-66\sin\left(\frac{5}{2}(c+dx)\right)+33\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-33\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-108*Cos[(c + d*x)/2] -
22*Cos[(3*(c + d*x))/2] + 66*Cos[(5*(c + d*x))/2] + 108*Sin[(c + d*x)/2] -
99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 99*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 22*Sin[(3*(c + d*x))/2]
- 66*Sin[(5*(c + d*x))/2] + 33*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[3*(c + d*x)] - 33*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x
)/4]^2)^3)
```

**Maple** [A]
time = 2.05, size = 144, normalized size = 1.00

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(33(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{5}{2}}+33\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{5}\right.}{24a^{\frac{7}{2}}\sin(dx+c)^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+\right.}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(33*(-a*(sin(d*x+c)-
1))^(5/2)*a^(5/2)+33*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x
+c)^3-88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+63*(-a*(sin(d*x+c)-1))^(1/2)*a^(
9/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^4, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(124) = 248$.
time = 0.34, size = 380, normalized size = 2.64

$$\frac{33\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)+a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(33\,a\cos\left(dx+c\right)^{3}+11\,a\cos\left(dx+c\right)^{2}-41\,a\cos\left(dx+c\right)-\left(33\,a\cos\left(dx+c\right)^{2}+22\,a\cos\left(dx+c\right)-19\,a\right)\sin\left(dx+c\right)-19\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(33*a*cos(d*x + c)^3 + 11*a*cos(d*x + c)^2 - 41*a*cos(d*x + c)
- (33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) - 19*a)*sin(d*x + c) - 19*a)*sqr
t(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

**Giac [A]**
time = 0.50, size = 188, normalized size = 1.31

$$-\frac{\sqrt{2}\left(33\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(132\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-176\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+63\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(33*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) + 4*(132*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^5 - 176*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^3 + 63*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt
(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\sin\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^4,x)
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^4, x)
```

# 3.52 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=203

$$-\frac{284a^3\cos(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}-\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}}-\frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}+\frac{568a^2\cos(c+d}{}$$

```
[Out] -284/231*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-284/99*a^3*cos(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-710/693*a^3*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(
1/2)-46/99*a^3*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+568/693*a^2
*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-2/11*a^2*cos(d*x+c)*sin(d*x+c)^4*(a+a*
sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2842, 3060, 2849, 2838, 2830, 2725}

$$-\frac{46a^3\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{710a^3\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{284a^3\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{568a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{693d}-\frac{284a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{231d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-284*a^3*Cos[c + d*x])/(99*d*Sqrt[a + a*Sin[c + d*x]]) - (710*a^3*Cos[c +
d*x]*Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) - (46*a^3*Cos[c + d*x
]*Sin[c + d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (568*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(693*d) - (2*a^2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[
a + a*Sin[c + d*x]])/(11*d) - (284*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2
))/(231*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2}{11}\int \sin^3(c$$

$$= -\frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{}}{11d}$$

$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{284a^3\cos(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - 46$$

**Mathematica [A]**
time = 0.80, size = 189, normalized size = 0.93

$$-\frac{(a(1+\sin(c+dx)))^{5/2}\left(31878\cos\left(\frac{1}{2}(c+dx)\right)+8778\cos\left(\frac{3}{2}(c+dx)\right)-3465\cos\left(\frac{5}{2}(c+dx)\right)-1287\cos\left(\frac{7}{2}(c+dx)\right)+385\cos\left(\frac{9}{2}(c+dx)\right)+63\cos\left(\frac{11}{2}(c+dx)\right)-31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\sin\left(\frac{5}{2}(c+dx)\right)-1287\sin\left(\frac{7}{2}(c+dx)\right)-385\sin\left(\frac{9}{2}(c+dx)\right)+63\sin\left(\frac{11}{2}(c+dx)\right)\right)}{11088d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/11088*((a*(1 + Sin[c + d*x]))^(5/2)*(31878*Cos[(c + d*x)/2] + 8778*Cos[(
3*(c + d*x))/2] - 3465*Cos[(5*(c + d*x))/2] - 1287*Cos[(7*(c + d*x))/2] + 3
85*Cos[(9*(c + d*x))/2] + 63*Cos[(11*(c + d*x))/2] - 31878*Sin[(c + d*x)/2]
 + 8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] - 1287*Sin[(7*(c +
 d*x))/2] - 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

**Maple [A]**
time = 1.86, size = 95, normalized size = 0.47

| method | result |
|---|---|
| default | $\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)\left(63(\sin^5(dx+c))+224(\sin^4(dx+c))+355(\sin^3(dx+c))+426(\sin^2(dx+c))+568\sin(dx+c)+1136\right)}{693\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(63*sin(d*x+c)^5+224*sin(d*x+c)^4+3
55*sin(d*x+c)^3+426*sin(d*x+c)^2+568*sin(d*x+c)+1136)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^3, x)
```

**Fricas [A]**
time = 0.33, size = 192, normalized size = 0.95

$$-\frac{2\,{\left(63\,a^{2} \cos\left(d x + c\right)^{6} + 224\,a^{2} \cos\left(d x + c\right)^{5} - 320\,a^{2} \cos\left(d x + c\right)^{4} - 874\,a^{2} \cos\left(d x + c\right)^{3} + 593\,a^{2} \cos\left(d x + c\right)^{2} + 1786\,a^{2} \cos\left(d x + c\right) + 800\,a^{2} + {\left(63\,a^{2} \cos\left(d x + c\right)^{5} - 161\,a^{2} \cos\left(d x + c\right)^{4} - 481\,a^{2} \cos\left(d x + c\right)^{3} + 393\,a^{2} \cos\left(d x + c\right)^{2} + 986\,a^{2} \cos\left(d x + c\right) - 800\,a^{2}\right)} \sin\left(d x + c\right)\right)} \sqrt{a \sin\left(d x + c\right) + a}}{693\,{\left(d \cos\left(d x + c\right) + d \sin\left(d x + c\right) + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 - 320*a^2*cos(d*x +
c)^4 - 874*a^2*cos(d*x + c)^3 + 593*a^2*cos(d*x + c)^2 + 1786*a^2*cos(d*x +
 c) + 800*a^2 + (63*a^2*cos(d*x + c)^5 - 161*a^2*cos(d*x + c)^4 - 481*a^2*c
os(d*x + c)^3 + 393*a^2*cos(d*x + c)^2 + 986*a^2*cos(d*x + c) - 800*a^2)*si
n(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.50, size = 192, normalized size = 0.95

$$\frac{\sqrt{2}\,{\left(31878\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right) + 8778\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{3}{4}\,\pi + \frac{3}{2}\,d x + \frac{3}{2}\,c\right) + 3465\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{5}{4}\,\pi + \frac{5}{2}\,d x + \frac{5}{2}\,c\right) + 1287\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,d x + \frac{7}{2}\,c\right) + 385\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{9}{4}\,\pi + \frac{9}{2}\,d x + \frac{9}{2}\,c\right) + 63\,a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,d x + \frac{1}{2}\,c\right)\right) \sin\left(-\frac{11}{4}\,\pi + \frac{11}{2}\,d x + \frac{11}{2}\,c\right)\right)} \sqrt{a}}{11088\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/11088*sqrt(2)*(31878*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c) + 8778*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*
pi + 3/2*d*x + 3/2*c) + 3465*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5
/4*pi + 5/2*d*x + 5/2*c) + 1287*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-7/4*pi + 7/2*d*x + 7/2*c) + 385*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-9/4*pi + 9/2*d*x + 9/2*c) + 63*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-11/4*pi + 11/2*d*x + 11/2*c))*sqrt(a)/d
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2), x)
```

# 3.53 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=146

$$-\frac{832a^3\cos(c+dx)}{315d\sqrt{a+a\sin(c+dx)}}-\frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d}-\frac{26a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{105d}+$$

```
[Out] -26/105*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+4/63*cos(d*x+c)*(a+a*sin(d*x+
c))^(5/2)/d-2/9*cos(d*x+c)*(a+a*sin(d*x+c))^(7/2)/a/d-832/315*a^3*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-208/315*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2726, 2725}

$$-\frac{832a^3\cos(c+dx)}{315d\sqrt{a\sin(c+dx)+a}}-\frac{208a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{7/2}}{9ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{63d}-\frac{26a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-832*a^3*Cos[c + d*x])/(315*d*Sqrt[a + a*Sin[c + d*x]]) - (208*a^2*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*d) - (26*a*Cos[c + d*x]*(a + a*Sin[c +
 d*x])^(3/2))/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(63*d)
- (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2))/(9*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{7/2}}{9ad}+\frac{2\int\left(\frac{7a}{2}-a\sin(c+dx)\right)(\ldots}{9\ldots}\\
&= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{63d}-\frac{2\cos(c+dx)(a+a\sin(c+\ldots}{9ad}\\
&= -\frac{26a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{105d}+\frac{4\cos(c+dx)(a+a\sin\ldots}{63d}\\
&= -\frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d}-\frac{26a\cos(c+dx)(a+\ldots}{105d}\\
&= -\frac{832a^3\cos(c+dx)}{315d\sqrt{a+a\sin(c+dx)}}-\frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+\ldots}}{315d}
\end{aligned}
$$

**Mathematica [A]**
time = 0.66, size = 165, normalized size = 1.13

$$\frac{(a(1+\sin(c+dx)))^{5/2}\left(-8190\cos\left(\frac{1}{2}(c+dx)\right)-2100\cos\left(\frac{3}{2}(c+dx)\right)+756\cos\left(\frac{5}{2}(c+dx)\right)+225\cos\left(\frac{7}{2}(c+dx)\right)-35\cos\left(\frac{9}{2}(c+dx)\right)+8190\sin\left(\frac{1}{2}(c+dx)\right)-2100\sin\left(\frac{3}{2}(c+dx)\right)-756\sin\left(\frac{5}{2}(c+dx)\right)+225\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-8190*Cos[(c + d*x)/2] - 2100*Cos[(3*(c + d*
x))/2] + 756*Cos[(5*(c + d*x))/2] + 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 8190*Sin[(c + d*x)/2] - 2100*Sin[(3*(c + d*x))/2] - 756*Sin[(
5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

**Maple [A]**
time = 1.77, size = 85, normalized size = 0.58

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)\left(35\left(\sin^4(dx+c)\right)+130\left(\sin^3(dx+c)\right)+219\left(\sin^2(dx+c)\right)+292\sin(dx+c)+584\right)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 85 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+130*sin(d*x+c)^3+2
19*sin(d*x+c)^2+292*sin(d*x+c)+584)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^2, x)
```

**Fricas** [**A**]
time = 0.38, size = 167, normalized size = 1.14

$$-\frac{2\left(35\,a^2\cos\left(dx+c\right)^5-95\,a^2\cos\left(dx+c\right)^4-289\,a^2\cos\left(dx+c\right)^3+263\,a^2\cos\left(dx+c\right)^2+838\,a^2\cos\left(dx+c\right)+416\,a^2-\left(35\,a^2\cos\left(dx+c\right)^4+130\,a^2\cos\left(dx+c\right)^3-159\,a^2\cos\left(dx+c\right)^2-422\,a^2\cos\left(dx+c\right)+416\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 289*a^2*cos(d*x + c
)^3 + 263*a^2*cos(d*x + c)^2 + 838*a^2*cos(d*x + c) + 416*a^2 - (35*a^2*cos
(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 159*a^2*cos(d*x + c)^2 - 422*a^2*cos
(d*x + c) + 416*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sin^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(5/2)*sin(c + d*x)**2, x)
```

**Giac** [**A**]
time = 0.62, size = 162, normalized size = 1.11

$$\frac{\sqrt{2}\left(8190\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2100\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+756\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+225\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+35\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(8190*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c) + 2100*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi
 + 3/2*d*x + 3/2*c) + 756*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*
pi + 5/2*d*x + 5/2*c) + 225*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/
4*pi + 7/2*d*x + 7/2*c) + 35*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9
/4*pi + 9/2*d*x + 9/2*c))*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2), x)
```

# 3.54 $\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=116

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{7d}-\frac{2\mathrm{c}}{}$$

```
[Out] -2/7*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d*x+c)*(a+a*sin(d*x+c))^
(5/2)/d-64/21*a^3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-16/21*a^2*cos(d*x+c)*
(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2830, 2726, 2725}

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(21*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(7*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d}+\frac{5}{7}\int (a+a\sin(c+dx))^{5/2}$$
$$= -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{7d}-\frac{2\cos(c+dx)(a+a\sin(c}{7d}$$
$$= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos(c+dx)(a+a\sin}{7d}$$
$$= -\frac{64a^3\cos(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}$$

**Mathematica** [A]
time = 0.41, size = 141, normalized size = 1.22

$$\frac{(a(1+\sin(c+dx)))^{5/2}\left(-315\cos\left(\frac{1}{2}(c+dx)\right)-77\cos\left(\frac{3}{2}(c+dx)\right)+21\cos\left(\frac{5}{2}(c+dx)\right)+3\cos\left(\frac{7}{2}(c+dx)\right)+315\sin\left(\frac{1}{2}(c+dx)\right)-77\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+3\sin\left(\frac{7}{2}(c+dx)\right)\right)}{84d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-315*Cos[(c + d*x)/2] - 77*Cos[(3*(c + d*x))
/2] + 21*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 315*Sin[(c + d*x)/
2] - 77*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x)
)/2]))/(84*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

**Maple** [A]
time = 1.79, size = 75, normalized size = 0.65

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)\left(3\left(\sin^3(dx+c)\right)+12\left(\sin^2(dx+c)\right)+23\sin(dx+c)+46\right)}{21\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ | 75 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^3+12*sin(d*x+c)^2+23*s
in(d*x+c)+46)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c), x)
```

**Fricas [A]**
time = 0.33, size = 140, normalized size = 1.21

$$\frac{2\,\left(3\,a^{2}\cos\left(dx+c\right)^{4}+12\,a^{2}\cos\left(dx+c\right)^{3}-17\,a^{2}\cos\left(dx+c\right)^{2}-58\,a^{2}\cos\left(dx+c\right)-32\,a^{2}+\left(3\,a^{2}\cos\left(dx+c\right)^{3}-9\,a^{2}\cos\left(dx+c\right)^{2}-26\,a^{2}\cos\left(dx+c\right)+32\,a^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{21\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 - 17*a^2*cos(d*x + c)^2
- 58*a^2*cos(d*x + c) - 32*a^2 + (3*a^2*cos(d*x + c)^3 - 9*a^2*cos(d*x + c)
^2 - 26*a^2*cos(d*x + c) + 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(
d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sin\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(5/2)*sin(c + d*x), x)
```

**Giac [A]**
time = 0.72, size = 132, normalized size = 1.14

$$\frac{\sqrt{2}\,\left(315\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+77\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+3\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\sqrt{a}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*sqrt(2)*(315*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c) + 77*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/
2*d*x + 3/2*c) + 21*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5
/2*d*x + 5/2*c) + 3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7
/2*d*x + 7/2*c))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2),x)
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2), x)
```

# 3.55 $\int (a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=89

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}$$

```
[Out] -2/5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-64/15*a^3*cos(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)-16/15*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(15*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int (a+a\sin(c+dx))^{3/2}\,dx\\ &= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}\\ &= -\frac{64a^3\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c-}{}\end{aligned}$$

**Mathematica [A]**
time = 0.21, size = 117, normalized size = 1.31

$$-\frac{(a(1+\sin(c+dx)))^{5/2}\left(150\cos\left(\frac{1}{2}(c+dx)\right)+25\cos\left(\frac{3}{2}(c+dx)\right)-3\cos\left(\frac{5}{2}(c+dx)\right)-150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/30*((a*(1 + Sin[c + d*x]))^(5/2)*(150*Cos[(c + d*x)/2] + 25*Cos[(3*(c +
d*x))/2] - 3*Cos[(5*(c + d*x))/2] - 150*Sin[(c + d*x)/2] + 25*Sin[(3*(c + d
*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^5)
```

**Maple [A]**
time = 1.88, size = 65, normalized size = 0.73

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)(3(\sin^2(dx+c))+14\sin(dx+c)+43)}{15\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 65 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+14*sin(d*x+c)+43)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas [A]**
time = 0.40, size = 115, normalized size = 1.29

$$\frac{2\left(3a^2\cos(dx+c)^3-11a^2\cos(dx+c)^2-46a^2\cos(dx+c)-32a^2-\left(3a^2\cos(dx+c)^2+14a^2\cos(dx+c)-32a^2\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}}{15\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 46*a^2*cos(d*x + c) -
32*a^2 - (3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) - 32*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (c+dx)+a)^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(5/2), x)
```

**Giac** [A]
time = 0.51, size = 102, normalized size = 1.15

$$\frac{\sqrt{2}\,\left(150\,a^2\mathrm{sgn}\left(\cos\left(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)\right)\sin\left(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+25\,a^2\mathrm{sgn}\left(\cos\left(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)\right)\sin\left(-\tfrac{3}{4}\,\pi+\tfrac{3}{2}\,dx+\tfrac{3}{2}\,c\right)+3\,a^2\mathrm{sgn}\left(\cos\left(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)\right)\sin\left(-\tfrac{5}{4}\,\pi+\tfrac{5}{2}\,dx+\tfrac{5}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(150*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c) + 25*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/
2*d*x + 3/2*c) + 3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/
2*d*x + 5/2*c))*sqrt(a)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2), x)
```

# 3.56 $\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=98

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

```
[Out] -2*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-14/3*a^3*co
s(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)
/d
```

**Rubi** [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2842, 3060, 2852, 212}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{14a^3\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^3*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{2}{3}\int \csc(c+dx)\sqrt{a+a\sin}\\ &= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\\ &= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\\ &= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.25, size = 143, normalized size = 1.46

$$-\frac{(a(1+\sin(c+dx)))^{5/2}\left(15\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-15\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[c + d*x]))^(5/2)*(15*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))
/2] + 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d
```

```
*x)/2] + Sin[(c + d*x)/2]] - 15*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

**Maple [A]**
time = 2.29, size = 103, normalized size = 1.05

| method | result |
|---|---|
| default | $-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\, a\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}+9a\sqrt{a}\right.}{3\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*(3*a^(3/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)+9*a*(a-a*sin(d*x+c))^(1/2))/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(84) = 168$.
time = 0.34, size = 279, normalized size = 2.85

$$\frac{3\left(a^2\cos\left(dx+c\right)+a^2\sin\left(dx+c\right)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(a^2\cos\left(dx+c\right)^2+8a^2\cos\left(dx+c\right)+7a^2+\left(a^2\cos\left(dx+c\right)-7a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{6\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x
 + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
+ c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c)
 - 1)) - 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c) + 7*a^2 + (a^2*cos(d*x
+ c) - 7*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*s
in(d*x + c) + d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.53, size = 141, normalized size = 1.44

$$-\frac{\sqrt{2}\left(8\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\sqrt{2}\,a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-36\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(8*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^3 + 3*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)) - 36*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c))*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x), x)
```

# 3.57 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=94

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] -5*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-a^3*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(1/2)-a^2*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 3060, 2852, 212}

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{a^3\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^3*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - a\int \csc(c+dx)\left(-\frac{5a}{2} - \right.$$

$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}$$

$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - $$

$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - $$

**Mathematica [A]**
time = 0.53, size = 182, normalized size = 1.94

$$-\frac{a^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(2\cos\left(\frac{3}{2}(c+dx)\right)+5\left(\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)+2\sin\left(\frac{3}{2}(c+dx)\right)\right)}{d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^2*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(3*(c + d*x))/2
] + 5*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]])*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Co
t[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + S
ec[(c + d*x)/4])))
```

**Maple [A]**
time = 2.26, size = 123, normalized size = 1.31

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{3}{2}}\left(\sin(dx+c)\left(2\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{a}+5\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{\sqrt{}}\right.\right.\right.}{\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*(sin(d*x+c)*(2*(a-a*sin(d
*x+c))^(1/2)*a^(1/2)+5*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*sin(
d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^2, x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(84) = 168$.
time = 0.36, size = 308, normalized size = 3.28

$$\frac{5\left(a^2\cos\left(dx+c\right)^2-a^2-\left(a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(2a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-a^2+\left(2a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{4\left(d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 - a^2 - (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (co
s(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)
*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin
(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(
d*x + c) - cos(d*x + c) - 1)) + 4*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)
- a^2 + (2*a^2*cos(d*x + c) + a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/
(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.61, size = 159, normalized size = 1.69

$$-\frac{\sqrt{2}\left(5\sqrt{2}\,a^{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-8a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{4a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1}\right)\sqrt{a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(5*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) - 8*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c) + 4*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\sin\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^2, x)
```

# 3.58 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=106

$$-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

```
[Out] -19/4*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-9/4*a^3*
cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a^2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(
d*x+c))^(1/2)/d
```

**Rubi** [A]

time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 3059, 2852, 212}

$$-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{9a^3\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d
) - (9*a^3*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]
*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} - \frac{1}{2}a\int \csc^2(c+$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(}}{2d}$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(}}{2d}$$

$$= -\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(106) = 212.
time = 0.51, size = 252, normalized size = 2.38

$$\frac{a^2\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(14\cos\left(\frac{1}{2}(c+dx)\right)-22\cos\left(\frac{3}{2}(c+dx)\right)-19\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+19\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+19\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-19\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-14\sin\left(\frac{1}{2}(c+dx)\right)-22\sin\left(\frac{3}{2}(c+dx)\right)\right)}{4d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2] - 2
2*Cos[(3*(c + d*x))/2] - 19*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
```

```
19*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 19*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 19*Cos[2*(c + d*x)]*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 14*Sin[(c + d*x)/2] - 22*Sin[(3*(c + d*x)
)/2]))/(4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^2)
```

**Maple** [A]
time = 2.05, size = 126, normalized size = 1.19

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\left(19\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)(\sin^2(dx+c))a^2+13\sqrt{}\right.}{4\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(19*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+13*(-a*(sin(d*x+c)-1))^(1/2)*a^(
3/2)-11*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^3, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(90) = 180$.
time = 0.34, size = 359, normalized size = 3.39

$$\frac{19\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-a^2+\left(a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(11\,a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)-9\,a^2+\left(11\,a^2\cos\left(dx+c\right)+9\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
+ (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
```

```
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(11*a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) - 9*a^2 + (11*a^2*cos(d*x +
c) + 9*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*c
os(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.50, size = 164, normalized size = 1.55

$$-\frac{\sqrt{2}\left(19\,\sqrt{2}\,a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(22\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-13\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(19*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)) + 4*(22*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3 - 13*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*s
qrt(a)/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\sin\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^3, x)
```

# 3.59 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=144

$$-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+$$

```
[Out] -25/8*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-25/8*a^3
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-13/12*a^3*cot(d*x+c)*csc(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-1/3*a^2*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]
time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 3059, 2851, 2852, 212}

$$-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{25a^3\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-25*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (25*a^3*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (13*a^3*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{1}{3}a\int \csc^3(c+$$

$$= -\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+}}{3d}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(}{}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(}{}$$

$$= -\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica** [A]
time = 0.77, size = 288, normalized size = 2.00

$$\frac{a^2\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-228\cos\left(\frac{1}{2}(c+dx)\right)+14\cos\left(\frac{3}{2}(c+dx)\right)+150\cos\left(\frac{5}{2}(c+dx)\right)+228\sin\left(\frac{1}{2}(c+dx)\right)-225\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+225\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+14\sin\left(\frac{3}{2}(c+dx)\right)-150\sin\left(\frac{5}{2}(c+dx)\right)+75\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-75\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-228*Cos[(c + d*x)/2]
+ 14*Cos[(3*(c + d*x))/2] + 150*Cos[(5*(c + d*x))/2] + 228*Sin[(c + d*x)/2]
 - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 225*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 14*Sin[(3*(c + d*x)
)/2] - 150*Sin[(5*(c + d*x))/2] + 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]]*Sin[3*(c + d*x)] - 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*S
in[3*(c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c
 + d*x)/4]^2)^3)
```

**Maple** [A]
time = 2.12, size = 144, normalized size = 1.00

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(75(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+75\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\right.}{24\sin(dx+c)^3a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(75*(-a*(sin(d*x+c)-1))^(5/2
)*a^(3/2)+75*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3-18
4*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+117*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/
sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^4, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(124) = 248$.
time = 0.35, size = 408, normalized size = 2.83

$$\frac{75\left(a^2\cos(dx+c)^4-2a^2\cos(dx+c)^2+a^2-\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2-a^2\cos(dx+c)-a^2\right)\sin(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(75a^2\cos(dx+c)^3+41a^2\cos(dx+c)^2-83a^2\cos(dx+c)-49a^2-\left(75a^2\cos(dx+c)^2+34a^2\cos(dx+c)-49a^2\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}}{96\left(d\cos(dx+c)^4-2d\cos(dx+c)^2-\left(d\cos(dx+c)^3+d\cos(dx+c)^2-d\cos(dx+c)-d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2 - (a^2*cos(d*x +
c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2)*sin(d*x + c))*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(75*a^2*cos(d*x + c)^3 + 41*a^2*cos(d*x + c)^2 -
83*a^2*cos(d*x + c) - 49*a^2 - (75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
 - 49*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)
*sin(d*x + c) + d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.55, size = 196, normalized size = 1.36

$$-\frac{\sqrt{2}\left(75\,\sqrt{2}\,a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\,\left(300\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-368\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+117\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(75*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)) + 4*(300*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^5 - 368*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 117*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1
)^3)*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\sin\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^4,x)
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^4, x)
```

# 3.60 $\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

**Optimal**. Leaf size=182

$$-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d}-\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}-\frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}}-\frac{17a^3\,\mathrm{cc}}{24d}$$

```
[Out] -163/64*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-163/64
*a^3*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-163/96*a^3*cot(d*x+c)*csc(d*x+c)/d
/(a+a*sin(d*x+c))^(1/2)-17/24*a^3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c)
)^(1/2)-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

**Rubi** [A]

time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 3059, 2851, 2852, 212}

$$-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{163a^3\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-163*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64
*d) - (163*a^3*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (163*a^3*Cot
[c + d*x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{1}{4}a\int \csc^4(c+\cdots$$

$$= -\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+\cdots}}{4d}\cdots$$

$$= -\frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \cdots$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a\cdots}{2\cdots}$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a\cdots}{2\cdots}$$

$$= -\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+d\cdots}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 370 vs. $2(182) = 364$.
time = 1.08, size = 370, normalized size = 2.03

$$-\frac{a^2\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-1030\cos\left(\frac{1}{2}(c+dx)\right)+3102\cos\left(\frac{3}{2}(c+dx)\right)-326\cos\left(\frac{5}{2}(c+dx)\right)-978\cos\left(\frac{7}{2}(c+dx)\right)+1467\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-1956\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+489\cos(4(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-1467\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+1956\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-489\cos(4(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+1030\sin\left(\frac{1}{2}(c+dx)\right)+3102\sin\left(\frac{3}{2}(c+dx)\right)+326\sin\left(\frac{5}{2}(c+dx)\right)-978\sin\left(\frac{7}{2}(c+dx)\right)\right)}{192d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/192*(a^2*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(-1030*Cos[(c +
d*x)/2] + 3102*Cos[(3*(c + d*x))/2] - 326*Cos[(5*(c + d*x))/2] - 978*Cos[(7
*(c + d*x))/2] + 1467*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1956*C
os[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 489*Cos[4*(c
 + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1467*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 1956*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 489*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 1030*Sin[(c + d*x)/2] + 3102*Sin[(3*(c + d*x))/2] + 326
*Sin[(5*(c + d*x))/2] - 978*Sin[(7*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

**Maple** [A]
time = 2.46, size = 162, normalized size = 0.89

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-489(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{5}{2}}+1793(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{7}{2}}-2303(-a(\sin(d\right.}{192a^{\frac{7}{2}}\sin(dx+c)^{4}\cos(dx+c)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-489*(-a*(sin(d*x+c)-1))^(
7/2)*a^(5/2)+1793*(-a*(sin(d*x+c)-1))^(5/2)*a^(7/2)-2303*(-a*(sin(d*x+c)-1)
)^(3/2)*a^(9/2)+1047*(-a*(sin(d*x+c)-1))^(1/2)*a^(11/2)+489*arctanh((-a*(si
n(d*x+c)-1))^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4)/a^(7/2)/sin(d*x+c)^4/cos(d*x+
c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^5, x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(158) = 316$.
time = 0.38, size = 473, normalized size = 2.60

$$\frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4-2a^2\cos(dx+c)^3-2a^2\cos(dx+c)^2+a^2\cos(dx+c)+a^2+\left(a^2\cos(dx+c)^4-2a^2\cos(dx+c)^2+a^2\right)\sin(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(489a^2\cos(dx+c)^4+326a^2\cos(dx+c)^3-836a^2\cos(dx+c)^2-374a^2\cos(dx+c)+299a^2+\left(489a^2\cos(dx+c)^3+163a^2\cos(dx+c)^2-673a^2\cos(dx+c)-299a^2\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}}{768\left(d\cos(dx+c)^5+d\cos(dx+c)^4-2d\cos(dx+c)^3-2d\cos(dx+c)^2+d\cos(dx+c)+\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3
- 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + a^2 + (a^2*cos(d*x + c)^4 - 2*a
^2*cos(d*x + c)^2 + a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
489*a^2*cos(d*x + c)^4 + 326*a^2*cos(d*x + c)^3 - 836*a^2*cos(d*x + c)^2 -
374*a^2*cos(d*x + c) + 299*a^2 + (489*a^2*cos(d*x + c)^3 + 163*a^2*cos(d*x
+ c)^2 - 673*a^2*cos(d*x + c) - 299*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c)
```

```
+ a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d
*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*si
n(d*x + c) + d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.52, size = 228, normalized size = 1.25

$$-\frac{\sqrt{2}\left(489\,\sqrt{2}\,a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(3912\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7172\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4606\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1047\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}\right)\sqrt{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/768*sqrt(2)*(489*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*
x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c)) + 4*(3912*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 7172*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 4606*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 1047*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/
2*c)^2 - 1)^4)*sqrt(a)/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\sin\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^5, x)
```

# 3.61 $\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=139

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad}$$

```
[Out] arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^
(1/2)-28/15*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)^2
/d/(a+a*sin(d*x+c))^(1/2)+2/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a/d
```

**Rubi** [A]
time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2857, 3047, 3102, 2830, 2728, 212}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}-\frac{28\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (28*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[
c + d*x]*Sin[c + d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(15*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} - \frac{2\int \frac{\frac{a^2}{2}}{\sqrt{a+}}$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+}}{15ad}$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+}}{15ad}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(}{5d\sqrt{}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.16, size = 150, normalized size = 1.08

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left((-60-60i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-60\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{5}{2}(c+dx)\right)+60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((-60 - 60*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - 60*Cos[(c + d*x)/2] + 5*Cos[
(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2] + 5*Sin[(3*
(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [A]
time = 2.20, size = 130, normalized size = 0.94

| method | result |
|---|---|
| default | $\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(15a^{\frac{5}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)-6(a-a\sin(dx+c))^{\frac{5}{2}}\right.}{15a^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-6*(a-a*sin(d*x+c))^(5/2)+10*(a-a
*sin(d*x+c))^(3/2)*a-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [**A**]
time = 0.35, size = 234, normalized size = 1.68

$$\frac{\frac{15\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\left(3\cos(dx+c)^3+4\cos(dx+c)^2-\left(3\cos(dx+c)^2-\cos(dx+c)-17\right)\sin(dx+c)-16\cos(dx+c)-17\right)\sqrt{a\sin(dx+c)+a}}{30\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(3*cos
(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c) - 17)*sin
(d*x + c) - 16*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
c) + a*d*sin(d*x + c) + a*d)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.53, size = 163, normalized size = 1.17

$$-\frac{\frac{15\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{15\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sqrt{2}\left(12a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-10a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+15a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{5}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/30*(15*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*
c) + 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sqrt(2)*(12*a^(9/
2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 10*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/
2*c)^3 + 15*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^5*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^3}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/2), x)
```

# 3.62 $\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=105

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad}$$

```
[Out] -arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a
^(1/2)+4/3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/a/d
```

**Rubi** [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2728, 212}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)]])/(Sqrt[a]*d)) + (4*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos
[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a}$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \int \frac{1}{\sqrt{a+a\sin}}$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} - \frac{2\mathrm{Subst}\left(\int \frac{1}{2a-}\right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.14, size = 105, normalized size = 1.00

$$-\frac{\left((-6-6i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/3*(((-6 - 6*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c +
d*x)/4])] - 2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [A]
time = 2.32, size = 96, normalized size = 0.91

| method | result |
|---|---|
| default | $\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)+2(a-a\sin(dx+c))^{\frac{3}{2}}\right)}{3a^{2}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+2*(a-a*sin(d*x+c))^(3/2))/a^2/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(88) = 176.
time = 0.39, size = 209, normalized size = 1.99

$$\frac{\frac{3\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a\sin\left(dx+c\right)+a}}{6\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac [A]**
time = 0.54, size = 121, normalized size = 1.15

$$-\frac{\frac{8\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{3\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(8*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3/(sqrt(a)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))) - 3*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sq
rt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 3*sqrt(2)*log(-sin(-1/4*pi + 1
/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/2), x)
```

# 3.63 $\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=72

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^
(1/2)-2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2728, 212}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.07, size = 98, normalized size = 1.36

$$-\frac{2\left((1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-2*((1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])] + Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [A]
time = 2.77, size = 96, normalized size = 1.33

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)+2\sqrt{a-a\,\text{si}}\right.}{a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-a^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)
/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(61) = 122$.
time = 0.34, size = 191, normalized size = 2.65

$$\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (
cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*
x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 -
(cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin
(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [A]
time = 0.49, size = 118, normalized size = 1.64

$$-\frac{\frac{\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))) - sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)
/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sqrt(2)*sin(-1/4*pi + 1/
2*d*x + 1/2*c)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [B]**
time = 0.75, size = 99, normalized size = 1.38

$$-\frac{\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(c+d\,x\right)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(c+d\,x\right)}}{2}\right)\middle|1\right)\right)\,\sqrt{{\cos\left(c+d\,x\right)}^2}\,\sqrt{\frac{a+a\,\sin\left(c+d\,x\right)}{2\,a}}}{d\,\cos\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -((4*ellipticE(asin((2^(1/2)*(1 - sin(c + d*x))^(1/2))/2), 1) - 2*ellipticF
(asin((2^(1/2)*(1 - sin(c + d*x))^(1/2))/2), 1))*(cos(c + d*x)^2)^(1/2)*((a
 + a*sin(c + d*x))/(2*a))^(1/2))/(d*cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)
)
```

# 3.64 $\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}$$

[Out] -arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)

**Rubi** [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2728, 212}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + a*Sin[c + d*x]],x]

[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(Sqrt[a]*d))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.04, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4]
)]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [A]
time = 0.01, size = 75, normalized size = 1.60

| method | result | size |
|---|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ | 75 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*(
sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [A]
time = 0.35, size = 167, normalized size = 3.55

$$\left[\frac{\sqrt{2}\,\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{2\sqrt{a}\,d},\ \frac{\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{-\frac{1}{a}}}{\cos(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqr
t(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3
*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(
d*x + c) - 2))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(d*
x + c) + a)*sqrt(-1/a)/cos(d*x + c))/d]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(c + d*x) + a), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(38) = 76.
time = 0.52, size = 111, normalized size = 2.36

$$\frac{\frac{\sqrt{2}\,\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\,\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*d
*x + 1/2*c) + 2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - sqrt(2)*l
og(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*d*x + 1/2*c) -
2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [B]
time = 6.44, size = 49, normalized size = 1.04

$$-\frac{\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(c+d\,x\right)\right)}{a}}}{d\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -(ellipticF(pi/4 - c/2 - (d*x)/2, 1)*((2*(a + a*sin(c + d*x)))/a)^(1/2))/(d
*(a + a*sin(c + d*x))^(1/2))
```

# 3.65 $\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=84

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2
*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2859, 2728, 212, 2852}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2859

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{}{\sqrt{a -}}\right.}{d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{\sqrt{a}\,d}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.07, size = 128, normalized size = 1.52

$$-\frac{\left((2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*
(1 + Sin[c + d*x])]))
```

**Maple [A]**
time = 1.77, size = 96, normalized size = 1.14

| method | result |
|---|---|
| default | $\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)-2\,\text{arctanh}\left(\frac{\sqrt{a-a\,\text{si}}}{\sqrt{}}\right.\right.}{\sqrt{a}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*
sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-2*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)
))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(69) = 138$.
time = 0.35, size = 290, normalized size = 3.45

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c)
 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqr
t(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c
) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3
)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 +
 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 +
 (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a*d)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(69) = 138.

time = 0.59, size = 142, normalized size = 1.69

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) + log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) - log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)))/(sqrt(a)*d)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+dx)\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

# 3.66 $\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=109

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-cot(d*x+
c)/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]

time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2858, 3064, 2728, 212, 2852}

$$-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - (Sqr
t[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(S
qrt[a]*d) - Cot[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a} \\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a} + \int \frac{}{\sqrt{a+a}} \\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2\text{Su}}{} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} -
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.87, size = 168, normalized size = 1.54

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left((8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{4}(c+dx)\right)\right)}{4d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*Sqrt[a*(1 + Sin[c +
 d*x])])
```

**Maple [A]**
time = 2.22, size = 133, normalized size = 1.22

| method | result |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)a^{3}\left(\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-\mathrm{arctanh}\left(\right.\right.\right.}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(sin(d*x+c)*a^3*(2^(1/2)*
arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c)
)^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(5/2))/sin(d*x+c)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(92) = 184.
time = 0.39, size = 412, normalized size = 3.78

$$\frac{\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{2\sqrt{2}\left(a\cos(dx+c)^2-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\,\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{4\left(ad\cos(dx+c)^2-ad-(ad\cos(dx+c)+ad)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
```

```
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 2*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*
x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*
cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x
+ c) + 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c
))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.
time = 0.58, size = 217, normalized size = 1.99

$$\frac{\frac{\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))) - sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/
(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - log(abs(1/2*sqrt(2) + sin(-
1/4*pi + 1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) +
 log(abs(-1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((2*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))))/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^2\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

# 3.67 $\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=146

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{\text{co}\ldots}{2\ldots}$$

```
[Out] -7/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)+1/4
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)
```

**Rubi** [A]

time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2858, 3063, 3064, 2728, 212, 2852}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(Sqrt[a]*d) + Cot[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc^2(c+dx)(a-3a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc(c+dx)\left(-\frac{7a^2}{2}+\frac{1}{2}a^2\sin(c+\cdots\right.}{\sqrt{a+a\sin(c+dx)}}}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{7\int \csc(c+dx)\sqrt{a+a\,\mathrm{si}\cdots}}{8a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{7\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \cdots\right.}{4d}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\cdots\right.}{\sqrt{a}\,d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 2.18, size = 307, normalized size = 2.10

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-8-(64+64i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)-28\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+28\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+4\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - (64 + 64*I)*(-1)^(3/4)*ArcTanh
[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 4*Cot[(c + d*x)/4] - Csc
[(c + d*x)/4]^2 - 28*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 28*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c +
 d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - S
in[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d
*Sqrt[a*(1 + Sin[c + d*x])])
```

**Maple** [A]
time = 2.56, size = 162, normalized size = 1.11

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(7a^5\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin^2(dx+c))+(-a(\sin(dx+c)\right.}{4a^{\frac{11}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{}}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(7*a^5*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2)-4*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(d*x+c)^2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(121) = 242$.
time = 0.36, size = 492, normalized size = 3.37

$$\frac{7\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{8\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad+\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)

```
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*
x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

**Giac** [A]
time = 0.55, size = 222, normalized size = 1.52

$$-\frac{\frac{4\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{7\log\left(\frac{\left|-16\sqrt{2}-32\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|16\sqrt{2}-32\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))) - 4*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c)
+ 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 7*log(abs(-16*sqrt(2)
- 32*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(16*sqrt(2) - 32*sin(-1/4*pi + 1/2*
d*x + 1/2*c)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(2
*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^3\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

## 3.68 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=183

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}}-\frac{9\cos(c+dx)\,\mathrm{si}}{10ad\sqrt{a+a\,\mathrm{si}}}$$

```
[Out] 1/2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+15/4*arctanh(1/2*cos(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-31/5*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-9/10*cos(d*x+c)*sin(d*x+c)^2/a/d/(a+a*sin(d
*x+c))^(1/2)+13/10*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

**Rubi** [A]
time = 0.26, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2844, 3062, 3047, 3102, 2830, 2728, 212}

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{13\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{10a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{9\sin^2(c+dx)\cos(c+dx)}{10ad\sqrt{a\sin(c+dx)+a}}-\frac{31\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(2*d*(a + a*Sin[c + d*x]
)^(3/2)) - (31*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (9*Cos[c +
d*x]*Sin[c + d*x]^2)/(10*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(10*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)\left(-9a^2+\frac{39}{4}a^2\sin\ldots\right.}{\sqrt{a+a\sin(c+\ldots}}}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-9a^2\sin(c+dx)+\frac{39}{4}a^2\sin\ldots}{\sqrt{a+a\sin(c+\ldots}}}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} + \frac{13\cos(c+dx)\sqrt{a+\ldots}}{10a^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+\ldots}{10ad\sqrt{a+a\sin(c+d\ldots}}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+\ldots}{10ad\sqrt{a+a\sin(c+d\ldots}}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\,\ldots}{5ad\sqrt{a\ldots}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.28, size = 178, normalized size = 0.97

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-55\cos\left(\frac{1}{2}(c+dx)\right)-41\cos\left(\frac{3}{2}(c+dx)\right)-3\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx)\right)+55\sin\left(\frac{1}{2}(c+dx)\right)-(150+150i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)(1+\sin(c+dx))-41\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{20d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-55*Cos[(c + d*x)/2] - 41*Cos[(3*(c
 + d*x))/2] - 3*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 55*Sin[(c + d
*x)/2] - (150 + 150*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(c + d*x)/4])]*(1 + Sin[c + d*x]) - 41*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c +
 d*x))/2] + Sin[(7*(c + d*x))/2]))/(20*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 1.76, size = 183, normalized size = 1.00

| method | result |
| --- | --- |
| default | $\frac{\left(\sin(dx+c)\left(-80\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{5}{2}}-8(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{a}+75\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{20a^{\frac{9}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(sin(d*x+c)*(-80*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8*(a-a*sin(d*x+c))^(5/
2)*a^(1/2)+75*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^3)-90*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8*(a-a*sin(d*x+c))^(5/2)*a^(1/2)+75*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*
x+c)-1))^(1/2)/a^(9/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(156) = 312$.
time = 0.38, size = 314, normalized size = 1.72

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(4\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{3}-48\cos\left(dx+c\right)^{2}+\left(4\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}-40\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)-45\cos\left(dx+c\right)-5\right)\sqrt{a\sin\left(dx+c\right)+a}}{40\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2a^{2}d-\left(a^{2}d\cos\left(dx+c\right)+2a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 -
48*cos(d*x + c)^2 + (4*cos(d*x + c)^3 + 8*cos(d*x + c)^2 - 40*cos(d*x + c)
+ 5)*sin(d*x + c) - 45*cos(d*x + c) - 5)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*
d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}\left(c + dx\right)}{\left(a\left(\sin\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac** [A]
time = 0.54, size = 197, normalized size = 1.08

$$-\frac{\frac{75\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{75\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)} + \frac{10\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{32\sqrt{2}\left(2a^{\frac{17}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+5a^{\frac{17}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*(75*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) - 75*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*
c) + 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 10*sqrt(2)*sin(-1/4
*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 32*sqrt(2)*(2*a^(17/2)*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^5 + 5*a^(17/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^10*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(c+d\,x\right)}^4}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(3/2), x)
```

# 3.69 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=145

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}}-\frac{7\cos(c+dx)}{}$$

```
[Out] 1/2*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/2)-11/4*arctanh(1/2*cos(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+13/3*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-7/6*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

**Rubi** [A]
time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3047, 3102, 2830, 2728, 212}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{6a^2d}+\frac{\sin^2(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(2*d*(a + a*Sin[c + d*x
])^(3/2)) + (13*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (7*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(6*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{7}{2}a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}\ldots}{\sqrt{a+a\sin\ldots}}}{3\ldots}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\ldots}}{6a^2d}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\ldots}}{6a^2d}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\ldots}{3ad\sqrt{\ldots}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.18, size = 156, normalized size = 1.08

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(11\cos\left(\frac{1}{2}(c+dx)\right)+7\cos\left(\frac{3}{2}(c+dx)\right)+\cos\left(\frac{5}{2}(c+dx)\right)-11\sin\left(\frac{1}{2}(c+dx)\right)+(33+33i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)(1+\sin(c+dx))+7\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)\right)}{6d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(11*Cos[(c + d*x)/2] + 7*Cos[(3*(c +
 d*x))/2] + Cos[(5*(c + d*x))/2] - 11*Sin[(c + d*x)/2] + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d
*x]) + 7*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2]))/(6*d*(a*(1 + Sin[c +
 d*x]))^(3/2))
```

**Maple** [A]
time = 1.86, size = 183, normalized size = 1.26

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{\left(\sin(dx+c)\left(33\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}-24\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}-8(a-a\sin(dx+c))^{\frac{3}{2}}\right.\right.}{12a^{\frac{7}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(sin(d*x+c)*(33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))*a^2-24*(a-a*sin(d*x+c))^(1/2)*a^(3/2)-8*(a-a*sin(d*x+c))^(3/2)*a^(1/
2))+33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-30*(
a-a*sin(d*x+c))^(1/2)*a^(3/2)-8*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*(-a*(sin(d*
x+c)-1))^(1/2)/a^(7/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(122) = 244.
time = 0.35, size = 295, normalized size = 2.03

$$\frac{33\sqrt{2}\left(\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(4\cos(dx+c)^{3}+16\cos(dx+c)^{2}-\left(4\cos(dx+c)^{2}-12\cos(dx+c)+3\right)\sin(dx+c)+15\cos(dx+c)+3\right)\sqrt{a\sin(dx+c)+a}}{24\left(a^{2}d\cos(dx+c)^{2}-a^{2}d\cos(dx+c)-2a^{2}d-\left(a^{2}d\cos(dx+c)+2a^{2}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^3 + 16*cos(d*x + c)^2 -
 (4*cos(d*x + c)^2 - 12*cos(d*x + c) + 3)*sin(d*x + c) + 15*cos(d*x + c) +
3)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.51, size = 197, normalized size = 1.36

$$\frac{\frac{33\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{33\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{6\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}\left(2a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+3a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{6}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(33*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) - 33*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c
) + 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 6*sqrt(2)*sin(-1/4*p
i + 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 16*sqrt(2)*(2*a^(9/2)*sin(-1/4*pi + 1/2*d*
x + 1/2*c)^3 + 3*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^6*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(3/2), x)
```

# 3.70 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=105

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+7/4*arctanh(1/2*cos(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-2*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2837, 2830, 2728, 212}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[
c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{-\frac{3a}{2}+2a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{7\int \frac{1}{\sqrt{a+a\sin(c+a}}}{4a}$$

$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{7\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\,\text{co}}{ad\sqrt{a+}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.17, size = 134, normalized size = 1.28

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(3\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)+(7+7i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)(1+\sin(c+dx))+2\sin\left(\frac{3}{2}(c+dx)\right)\right)}{2d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(3*Cos[(c + d*x)/2] + 2*Cos[(3*
(c + d*x))/2] - 3*Sin[(c + d*x)/2] + (7 + 7*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d*x]) + 2*Sin[(3*(c + d
*x))/2]))/(d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 1.49, size = 143, normalized size = 1.36

| method | result |
|---|---|
| default | $-\frac{\left(\sin(dx+c)\left(-7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a+8\sqrt{a-a\sin(dx+c)}\,\sqrt{a}\right)-7\sqrt{2}\,\operatorname{arctanh}\left(\right.\right.}{4a^{\frac{5}{2}}\cos(dx+c)\sqrt{a+a\sin(}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(d*x+c)*(-7*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))*a+8*(a-a*sin(d*x+c))^(1/2)*a^(1/2))-7*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+10*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(
-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(88) = 176.
time = 0.38, size = 274, normalized size = 2.61

$$\frac{7\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(4\cos(dx+c)^2+(4\cos(dx+c)-1)\sin(dx+c)+5\cos(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{8\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2d-(a^2d\cos(dx+c)+2a^2d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(4*cos(d*x + c)^2 + (4*cos(d*x + c) - 1)*
sin(d*x + c) + 5*cos(d*x + c) + 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
 + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin
(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac [A]**
time = 0.80, size = 172, normalized size = 1.64

$$-\frac{\frac{7\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{7\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(7*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))) - 7*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c)
+ 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 16*sqrt(2)*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*sqrt(
2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a
^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^2}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(3/2), x)
```

# 3.71 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] 1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-3/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*
2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2829, 2728, 212}

$$\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a} \\
&= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad} \\
&= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}
\end{aligned}
$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.13, size = 108, normalized size = 1.40

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)+(3+3i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)(1+\sin(c+dx))\right)}{2d(a(1+\sin(c+dx)))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 1.88, size = 123, normalized size = 1.60

| method | result |
|---|---|
| default | $\frac{\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sin(dx+c)-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a+2\right.}{4a^{\frac{5}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(5/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a*sin(d*x+c)-3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a+2*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(62) = 124$.
time = 0.35, size = 253, normalized size = 3.29

$$\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)-4\sqrt{a\sin\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2a^{2}d-\left(a^{2}d\cos\left(dx+c\right)+2a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) - 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) -
sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (
a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(62) = 124$.
time = 0.50, size = 137, normalized size = 1.78

$$\frac{\frac{3\sqrt{2}\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}-\frac{3\sqrt{2}\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}+\frac{2\sqrt{2}\sin(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)}{\left(\sin(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)^{2}-1\right)a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) - 3*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) +
```

```
 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*sqrt(2)*sin(-1/4*pi +
 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(3/2), x)
```

## 3.72 $\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1/4*arctanh(1/2*cos(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

**Rubi** [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2729, 2728, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sq
rt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$
$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.11, size = 108, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)+(1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)(1+\sin(c+dx))\right)}{2d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-3/2),x]
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4
])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 1.71, size = 125, normalized size = 1.62

| method | result |
|---|---|
| default | $-\frac{\left(\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(dx+c)+2\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-}}{}\right.\right.}{4a^{\frac{7}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^2*sin(d*x+c)+2*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(62) = 124$.
time = 0.36, size = 252, normalized size = 3.27

$$\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3\,a\cos(dx+c)-(a\cos(dx+c)-2\,a)\sin(dx+c)+2\,a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{8\,\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2\,a^{2}d-\left(a^{2}d\cos\left(dx+c\right)+2\,a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x +
c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)
*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
+ c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*
x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^
2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-3/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.
time = 0.55, size = 133, normalized size = 1.73

$$\frac{\sqrt{2}\left(\frac{\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\mathrm{asgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\mathrm{asgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\mathrm{asgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{8\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))) - log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(-1/
```

```
4*pi + 1/2*d*x + 1/2*c))) - 2*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/(sqrt(a)*
d)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*sin(c + d*x))^(3/2), x)
```

## 3.73 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(3/2)+5/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+
a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

**Rubi** [A]
time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2845, 3064, 2728, 212, 2852}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc(c+dx)\left(2a-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\;}{\sqrt{a\cdots}}}{\cdots}$$

$$= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} + \frac{5\cdots}{\cdots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}\cdots$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.14, size = 223, normalized size = 1.96

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)-(5+5i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-2\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+2\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)}{2d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 - (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 2*Log[1 + Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 2*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/
(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 2.08, size = 172, normalized size = 1.51

| method | result |
|---|---|
| default | $\frac{\left(\sin(dx+c)a^3\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-8\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)+2\sqrt{a-a\,\mathrm{s}\cdots\right.}{4a^{\frac{9}{2}}\cos(\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(9/2)*(sin(d*x+c)*a^3*(5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))-8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))+2*(a-a*sin(d*x+c
))^(1/2)*a^(5/2)+5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^3-8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1))^
(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(93) = 186.
time = 0.37, size = 453, normalized size = 3.97

$$\frac{5\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{8\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2d-(a^2d\cos(dx+c)+2a^2d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
 - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)
^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)
^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*sqrt(a*sin(
d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^
2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (c+dx)}{(a (\sin (c+dx)+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac [A]**
time = 0.52, size = 138, normalized size = 1.21

$$\frac{\frac{4 \log \left(\left|\frac{1}{2} \sqrt{2}+\sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}-\frac{4 \log \left(\left|-\frac{1}{2} \sqrt{2}+\sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}+\frac{\sqrt{2} \sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-1\right) a^{\frac{3}{2}} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*log(abs(1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(3/2)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*log(abs(-1/2*sqrt(2) + sin(-1/4*pi + 1
/2*d*x + 1/2*c)))/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + sqrt(2)*s
in(-1/4*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^(3/
2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin (c+d\,x)\,{\left(a+a\,\sin (c+d\,x)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

# 3.74 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=144

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{3\ldots}{2ad\sqrt{\ldots}}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cot(d*x+
c)/d/(a+a*sin(d*x+c))^(3/2)-9/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a
*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-3/2*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(
1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3063, 3064, 2728, 212, 2852}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3\cot(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Cot[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (3*Cot[
c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(3a-\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-3a^2+\frac{3}{2}a^2\sin(c\right.}{\sqrt{a+a\sin(c+dx}}}{2a^3}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{3\int\csc(c+dx)\sqrt{a+a}}{2a^2}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{3\text{Subst}\left(\int\frac{1}{a-x^2}\,dx,x,-\right.}{ad}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + $$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.44, size = 449, normalized size = 3.12

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)-2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+(18+18i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-\cot\left(\frac{1}{4}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+6\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-6\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+\frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}-\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\tan\left(\frac{1}{4}(c+dx)\right)\right)}{4d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(4*Sin[(c + d*x)/2] - 2*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (18
 + 18*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - Cot[(c + d*x)/4]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 6*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/4]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4]) - (2*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c
+ d*x)/4] + Sin[(c + d*x)/4]) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan
[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 2.24, size = 219, normalized size = 1.52

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{\left(9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{2}\left(dx+c\right)\right)a+9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(d*x+c)^2*a+9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/
a^(1/2))*a*sin(d*x+c)-12*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x
+c)^2*a+6*(-a*(sin(d*x+c)-1))^(1/2)*sin(d*x+c)*a^(1/2)-12*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*a*sin(d*x+c)+4*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2))
*(-a*(sin(d*x+c)-1))^(1/2)/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(119) = 238.
time = 0.37, size = 539, normalized size = 3.74

$$\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+6\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\cos\left(dx+c\right)^{2}+\left(3\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}}{8\left(a^{2}d\cos\left(dx+c\right)^{3}+2a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2a^{2}d+\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d
*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^
2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c)
 + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 6*(cos(
d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c
```

```
)^2 + (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*
x + c) + a))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x
 + c) - 2*a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d)*sin
(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(119) = 238.
time = 0.54, size = 260, normalized size = 1.81

$$\frac{\frac{9\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{9\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{12\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{12\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(6\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(9*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) - 9*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) +
 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 12*log(abs(1/2*sqrt(2)
+ sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))) + 12*log(abs(-1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(3/2)
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(6*sqrt(a)*sin(-1/4*pi +
1/2*d*x + 1/2*c)^3 - 5*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^4 - 3*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 1)*a^2*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

# 3.75 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

**Optimal**. Leaf size=186

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{}{4\ldots}$$

```
[Out] -19/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cot(
d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+13/4*arctanh(1/2*cos(d*x+c)*a^(1
/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+7/4*cot(d*x+c)/a/d/(a
+a*sin(d*x+c))^(1/2)-cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]

time = 0.32, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3063, 3064, 2728, 212, 2852}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (13*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])
/(2*Sqrt[2]*a^(3/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*d*(a + a*Sin[c + d*
x])^(3/2)) + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(4a-\frac{5}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)(-7a^2+6a^2\sin(}{\sqrt{a+a\sin(c+d}}}{4a^3}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+d}{ad\sqrt{a+a\sin(c+dx}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+d}{ad\sqrt{a+a\sin(c+dx}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+d}{ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 3.01, size = 620, normalized size = 3.33

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-32*Sin[(c + d*x)/2] + 16*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 24*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (208 + 208*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 12*Cot[(c + d*x)/4]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 - Csc[(c + d*x)/4]^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2 - 76*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 76*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + Sec[(c + d*x)/4]^2*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) -
 (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] + Sin[(c + d
```

```
*x)/4])^2 + (24*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2*Tan[(c + d*x)/4]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

**Maple** [A]
time = 2.59, size = 299, normalized size = 1.61

| method | result |
|---|---|
| default | $\frac{\left(13\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{3}\left(dx+c\right)\right)a^{2}+13\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(13*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
d*x+c)^3*a^2+13*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(d*x+c)^2*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2-19*ar
ctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2+3*(-a*(sin(d*x+c)
-1))^(1/2)*a^(3/2)*sin(d*x+c)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c
)-19*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin
(d*x+c)-1))^(1/2)*a^(3/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))*(-a*(sin(d*x
+c)-1))^(1/2)/a^(7/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(157) = 314$.
time = 0.39, size = 626, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos
(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x +
```

```
 c) + 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 19*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(
d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) + cos(d*x + c) + 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(7*cos(d*x + c
)^3 + 4*cos(d*x + c)^2 - (7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x +
c) - 5*cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^4 -
a^2*d*cos(d*x + c)^3 - 3*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + 2*a^2*
d - (a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

**Giac** [A]
time = 0.62, size = 205, normalized size = 1.10

$$\frac{\frac{19\log\left(\frac{\left|-16\sqrt{2}-32\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|16\sqrt{2}-32\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\left(10\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(19*log(abs(-16*sqrt(2) - 32*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(16*sqr
t(2) - 32*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))) + 2*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*pi + 1/
2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*sqrt
(2)*(10*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3*sqrt(a)*sin(-1/4*pi +
1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^2*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

# 3.76 $\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=221

$$\frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}-\frac{1729}{120a^2d\sqrt{}}$$

```
[Out] 1/4*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(5/2)+21/16*cos(d*x+c)*sin(d
*x+c)^3/a/d/(a+a*sin(d*x+c))^(3/2)+283/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1729/120*cos(d*x+c)/a^2/d/(
a+a*sin(d*x+c))^(1/2)-157/80*cos(d*x+c)*sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))
^(1/2)+787/240*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

**Rubi** [A]
time = 0.35, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2844, 3056, 3062, 3047, 3102, 2830, 2728, 212}

$$\frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{787\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{240a^3d}-\frac{157\sin^2(c+dx)\cos(c+dx)}{80a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1729\cos(c+dx)}{120a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\sin^4(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}+\frac{21\sin^3(c+dx)\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (283*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*d*(a + a*Sin[c + d*
x])^(5/2)) + (21*Cos[c + d*x]*Sin[c + d*x]^3)/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) - (1729*Cos[c + d*x])/(120*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (157*Co
s[c + d*x]*Sin[c + d*x]^2)/(80*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (787*Cos[c
 + d*x]*Sqrt[a + a*Sin[c + d*x]])/(240*a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^3(c+dx)\left(4a-\frac{13}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(\frac{63a^2}{2}-\frac{157}{4}\cdots\right.}{\sqrt{a+a\sin(\cdots}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin\cdots}{80a^2d\sqrt{a+a\sin(\cdots}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin\cdots}{80a^2d\sqrt{a+a\sin(\cdots}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin\cdots}{80a^2d\sqrt{a+a\sin(\cdots}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+\cdots}{120a^2d\sqrt{a+a\sin\cdots}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+\cdots}{120a^2d\sqrt{a+a\sin\cdots}}$$

$$= \frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos\cdots}{16ad(\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.37, size = 221, normalized size = 1.00

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(2547\cos\left(\frac{1}{2}(c+dx)\right)+3603\cos\left(\frac{3}{2}(c+dx)\right)-872\cos\left(\frac{5}{2}(c+dx)\right)+52\cos\left(\frac{7}{2}(c+dx)\right)+12\cos\left(\frac{9}{2}(c+dx)\right)-2547\sin\left(\frac{1}{2}(c+dx)\right)+(8490+8490i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+3603\sin\left(\frac{3}{2}(c+dx)\right)+872\sin\left(\frac{5}{2}(c+dx)\right)+52\sin\left(\frac{7}{2}(c+dx)\right)-12\sin\left(\frac{9}{2}(c+dx)\right)\right)}{480d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/480*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2547*Cos[(c + d*x)/2] + 3603
*Cos[(3*(c + d*x))/2] - 872*Cos[(5*(c + d*x))/2] + 52*Cos[(7*(c + d*x))/2]
+ 12*Cos[(9*(c + d*x))/2] - 2547*Sin[(c + d*x)/2] + (8490 + 8490*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4 + 3603*Sin[(3*(c + d*x))/2] + 872*Sin[(5*(c + d*x)
)/2] + 52*Sin[(7*(c + d*x))/2] - 12*Sin[(9*(c + d*x))/2]))/(d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

**Maple** [A]
time = 2.48, size = 323, normalized size = 1.46

| method | result |
|---|---|
| default | $-\frac{\left(\sin(dx+c)\left(384(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{a}+640(a-a\sin(dx+c))^{\frac{3}{2}}a^{\frac{3}{2}}+7680\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{5}{2}}-8490\sqrt{2}\,\operatorname{arctanh}\left(\right.\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/a^(11/2)*(sin(d*x+c)*(384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)+640*(a-a*si
n(d*x+c))^(3/2)*a^(3/2)+7680*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*ar
ctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)+(-192*(a-a*sin(d*x+c
))^(5/2)*a^(1/2)-320*(a-a*sin(d*x+c))^(3/2)*a^(3/2)-3840*(a-a*sin(d*x+c))^(
1/2)*a^(5/2)+4245*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2
))*a^3)*cos(d*x+c)^2+384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)-470*(a-a*sin(d*x+c)
)^(3/2)*a^(3/2)+9780*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1))^(1/2)/(1
+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^5/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(190) = 380$.
time = 0.36, size = 381, normalized size = 1.72

$$\frac{4245\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(96\cos\left(dx+c\right)^{5}+256\cos\left(dx+c\right)^{4}-1760\cos\left(dx+c\right)^{3}+2475\cos\left(dx+c\right)^{2}-\left(96\cos\left(dx+c\right)^{4}-160\cos\left(dx+c\right)^{3}-1920\cos\left(dx+c\right)^{2}-4395\cos\left(dx+c\right)-60\right)\sin\left(dx+c\right)+4335\cos\left(dx+c\right)-60\right)\sqrt{a\sin\left(dx+c\right)+a}}{960\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/960*(4245*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin
(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2
*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))
+ 4*(96*cos(d*x + c)^5 + 256*cos(d*x + c)^4 - 1760*cos(d*x + c)^3 + 2475*co
s(d*x + c)^2 - (96*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 1920*cos(d*x + c)^
2 - 4395*cos(d*x + c) - 60)*sin(d*x + c) + 4335*cos(d*x + c) - 60)*sqrt(a*s
in(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a
^3*d)*sin(d*x + c))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.54, size = 241, normalized size = 1.09

$$-\frac{\frac{4245\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4245\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{30\sqrt{2}\left(37\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-35\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{256\sqrt{2}\left(6a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+5a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+30a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/960*(4245*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) - 4245*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x +
 1/2*c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 30*sqrt(2)*(37
*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 35*sqrt(a)*sin(-1/4*pi + 1/2*d*
x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))) - 256*sqrt(2)*(6*a^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c
```

```
)^5 + 5*a^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 30*a^(25/2)*sin(-1/4*pi
 + 1/2*d*x + 1/2*c))/(a^15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+dx)^5}{(a+a\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(sin(c + d*x)^5/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.77 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=183

$$-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}+\frac{19}{24a^2d\sqrt{}}$$

[Out] 1/4*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(5/2)+17/16*cos(d*x+c)*sin(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(3/2)-163/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+197/24*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-95/48*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d

**Rubi** [A]

time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2844, 3056, 3047, 3102, 2830, 2728, 212}

$$-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{95\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{48a^3d}+\frac{197\cos(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\sin^3(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}+\frac{17\sin^2(c+dx)\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]

[Out] (-163*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (17*Cos[c + d*x]*Sin[c + d*x]^2)/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) + (197*Cos[c + d*x])/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (95*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(48*a^3*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{11}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(17a^2-\frac{95}{4}a^2 \cdots\right.}{\sqrt{a+a\sin(c \cdots}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{17a^2\sin(c+dx)-\frac{95}{4}a^2 \cdots}{\sqrt{a+a\sin(c \cdots}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{95\cos(c+dx)\sqrt{a \cdots}}{48a^{3 \cdots}}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+d \cdots}{24a^2d\sqrt{a+a\sin(c \cdots}}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+d \cdots}{24a^2d\sqrt{a+a\sin(c \cdots}}$$

$$= -\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\,\mathrm{c}\cdots}{16a\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.31, size = 197, normalized size = 1.08

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(279\cos\left(\frac{1}{2}(c+dx)\right)+399\cos\left(\frac{3}{2}(c+dx)\right)-88\cos\left(\frac{5}{2}(c+dx)\right)+8\cos\left(\frac{7}{2}(c+dx)\right)-279\sin\left(\frac{1}{2}(c+dx)\right)+(978+978i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+399\sin\left(\frac{3}{2}(c+dx)\right)+88\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}(c+dx)\right)\right)}{96d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(279*Cos[(c + d*x)/2] + 399*Cos[(3*(
c + d*x))/2] - 88*Cos[(5*(c + d*x))/2] + 8*Cos[(7*(c + d*x))/2] - 279*Sin[(
c + d*x)/2] + (978 + 978*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
 Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 399*Sin[(3*(c
 + d*x))/2] + 88*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2]))/(96*d*(a*(
1 + Sin[c + d*x]))^(5/2))
```

**Maple** [A]
time = 2.27, size = 269, normalized size = 1.47

| method | result |
|---|---|
| default | $-\frac{\left(\sin(dx+c)\left(978\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}-768\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}-128(a-a\sin(dx+c\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(sin(d*x+c)*(978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a
^(1/2))*a^2-768*(a-a*sin(d*x+c))^(1/2)*a^(3/2)-128*(a-a*sin(d*x+c))^(3/2)*a
^(1/2))+(-489*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^2+384*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+64*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*co
s(d*x+c)^2+978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^2-1092*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+46*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*
(-a*(sin(d*x+c)-1))^(1/2)/a^(9/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c)
)^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(156) = 312$.
time = 0.36, size = 360, normalized size = 1.97

$$\frac{489\sqrt{2}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(32\cos(dx+c)^{4}-160\cos(dx+c)^{3}+279\cos(dx+c)^{2}+\left(32\cos(dx+c)^{3}+192\cos(dx+c)^{2}+471\cos(dx+c)+12\right)\sin(dx+c)+459\cos(dx+c)-12\right)\sqrt{a\sin(dx+c)+a}}{192\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}-2a^{3}d\cos(dx+c)-4a^{3}d+\left(a^{3}d\cos(dx+c)^{2}-2a^{3}d\cos(dx+c)-4a^{3}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2
*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) -
 4*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 + 279*cos(d*x + c)^2 + (32*cos(d
*x + c)^3 + 192*cos(d*x + c)^2 + 471*cos(d*x + c) + 12)*sin(d*x + c) + 459*
```

```
cos(d*x + c) - 12)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac [A]**
time = 0.46, size = 143, normalized size = 0.78

$$\frac{\frac{3\sqrt{2}\left(29\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-27\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{128\sqrt{2}\left(a^{\frac{13}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a^{\frac{13}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^9\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(3*sqrt(2)*(29*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 27*sqrt(a)*s
in(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^
3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 128*sqrt(2)*(a^(13/2)*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^3 + 3*a^(13/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^9*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(c+d\,x\right)}^4}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=145

$$\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}-\frac{9\cos(c}{4a^2d\sqrt{a+a}}$$

```
[Out] 1/4*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c))^(5/2)-13/16*cos(d*x+c)/a/d/(
a+a*sin(d*x+c))^(3/2)+75/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin
(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-9/4*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/
2)
```

**Rubi** [A]

time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3047, 3098, 2830, 2728, 212}

$$\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{9\cos(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\sin^2(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}-\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (75*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(4*d*(a + a*Sin[c + d*x
])^(5/2)) - (13*Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (9*Cos[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{9}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{9}{2}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13}{16ad(a-}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.22, size = 173, normalized size = 1.19

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-45\cos\left(\frac{1}{2}(c+dx)\right)-69\cos\left(\frac{3}{2}(c+dx)\right)+16\cos\left(\frac{5}{2}(c+dx)\right)+45\sin\left(\frac{1}{2}(c+dx)\right)-(150+150i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4-69\sin\left(\frac{3}{2}(c+dx)\right)-16\sin\left(\frac{5}{2}(c+dx)\right)\right)}{32d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-45*Cos[(c + d*x)/2] - 69*Cos[(3*(c
 + d*x))/2] + 16*Cos[(5*(c + d*x))/2] + 45*Sin[(c + d*x)/2] - (150 + 150*I)
*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^4 - 69*Sin[(3*(c + d*x))/2] - 16*Sin[(5*(c +
 d*x))/2]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

**Maple** [A]
time = 2.68, size = 233, normalized size = 1.61

| method | result |
|---|---|
| default | $\frac{\left(\sin(dx+c)\left(150\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2-128\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}\right)+\left(-75\sqrt{2}\,\operatorname{arctan}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(9/2)*(sin(d*x+c)*(150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2))*a^2-128*(a-a*sin(d*x+c))^(1/2)*a^(3/2))+(-75*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+64*(a-a*sin(d*x+c))^(1/2)*
a^(3/2))*cos(d*x+c)^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a^2+42*(a-a*sin(d*x+c))^(3/2)*a^(1/2)-204*(a-a*sin(d*x+c))^(1/2)
*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(122) = 244$.
time = 0.35, size = 341, normalized size = 2.35

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(32\cos\left(dx+c\right)^{3}-53\cos\left(dx+c\right)^{2}-\left(32\cos\left(dx+c\right)^{2}+85\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)-81\cos\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4
*(32*cos(d*x + c)^3 - 53*cos(d*x + c)^2 - (32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 4)*sin(d*x + c) - 81*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3
*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d
 + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.49, size = 191, normalized size = 1.32

$$-\frac{\frac{75\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{75\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{128\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\left(21\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-19\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(75*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) - 75*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*
c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 128*sqrt(2)*sin(-1/
4*pi + 1/2*d*x + 1/2*c)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*s
qrt(2)*(21*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 19*sin(-1/4*pi + 1/2*d*x + 1/
2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.79 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)+13/16*cos(d*x+c)/a/d/(a+a*sin(d*x+
c))^(3/2)-19/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/
2))*2^(1/2)/a^(5/2)/d
```

**Rubi** [A]

time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2837, 2829, 2728, 212}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (13*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2}+4a\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{19\int \frac{1}{\sqrt{a+a\sin(c}}}{32a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{19\text{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{1}{16ad(a}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.14, size = 196, normalized size = 1.83

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)-4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-26\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+13\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+(19+19i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\right)}{16d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) - 26*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + 13*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (19 + 19*I)*(-1
)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(88) = 176.
time = 2.59, size = 193, normalized size = 1.80

| method | result |
|---|---|
| default | $-\frac{\left(-19\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\left(\cos^{2}\left(dx+c\right)\right)+38\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.}{32a^{\frac{9}{2}}(}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/32/a^(9/2)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(d*x+c)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(d*x+c)+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-44*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+26*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(88) = 176.
time = 0.35, size = 320, normalized size = 2.99

$$\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(13\cos\left(dx+c\right)^{2}+\left(13\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)+9\cos\left(dx+c\right)-4\right)\sqrt{a\sin\left(dx+c\right)+a}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4*(13*cos(d*x + c)^2 + (13*cos(d*x + c) + 4)*sin(d*x + c) + 9*cos(d*x + c) - 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac** [A]
time = 0.46, size = 162, normalized size = 1.51

$$\frac{\frac{19\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{19\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\left(13\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-11\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(19*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) - 19*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c
) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*sqrt(2)*(13*sqrt(a
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 11*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))))/d
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.80 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

```
[Out] 1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-5/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c)
)^(3/2)-5/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))
*2^(1/2)/a^(5/2)/d
```

**Rubi** [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2829, 2729, 2728, 212}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{5\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (5*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{5\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{a+a\sin(c+}}}{32a^2}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{5\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{1}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5}{16ad(a}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.13, size = 196, normalized size = 1.83

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)+4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+10\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-5\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+(5+5i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\right)}{16d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - 5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (5 + 5*I)*(-1)^
(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(88) = 176.
time = 2.29, size = 193, normalized size = 1.80

| method | result |
|---|---|
| default | $-\frac{\left(-5\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^{3}(\cos^{2}(dx+c))+10\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.}{32a^{\frac{11}{2}}(1}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-5*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^3*c
os(d*x+c)^2+10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^3*sin(d*x+c)-10*(a-a*sin(d*x+c))^(3/2)*a^(3/2)+12*(a-a*sin(d*x+c))^(1/2)*
a^(5/2)+10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)
*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(88) = 176$.
time = 0.35, size = 318, normalized size = 2.97

$$\frac{5\sqrt{2}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(5\cos(dx+c)^{2}+(5\cos(dx+c)+4)\sin(dx+c)+\cos(dx+c)-4\right)\sqrt{a\sin(dx+c)+a}}{64\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}-2a^{3}d\cos(dx+c)-4a^{3}d+\left(a^{3}d\cos(dx+c)^{2}-2a^{3}d\cos(dx+c)-4a^{3}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 4)*sin(d*x + c) + cos(d*x + c) - 4)*s
qrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2
*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c
) - 4*a^3*d)*sin(d*x + c))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac** [A]
time = 0.45, size = 81, normalized size = 0.76

$$-\frac{\sqrt{2}\left(5\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{32\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{3}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*sqrt(2)*(5*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3*sqrt(a)*sin(-
1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*d*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{{(a+a\,\sin(c+d\,x))}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.81 $\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-3/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c
))^(3/2)-3/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)*2^(1/2)/a^(5/2)/d
```

**Rubi** [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{3\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (3*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{3\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}\\
&= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{a+a\sin(c+}}}{32a^2}\\
&= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}-\frac{3\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx\right.}{}\\
&= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}-\frac{3}{16ad(a}
\end{aligned}
$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.11, size = 196, normalized size = 1.83

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)-4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+(3+3i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\right)}{16d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 6*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - 3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(88) = 176.
time = 2.42, size = 195, normalized size = 1.82

| method | result |
|---|---|
| default | $-\frac{\left(\sin(dx+c)\left(6\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{32a^{\frac{9}{2}}(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^(9/2)*(sin(d*x+c)*(6*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
```

```
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(d*x+c)^2+14*(a-a*sin(d*x+c))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(88) = 176.
time = 0.35, size = 320, normalized size = 2.99

$$\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(3\cos\left(dx+c\right)^{2}+\left(3\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)+7\cos\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(3*cos(d*x + c)^2 + (3*cos(d*x + c) - 4)*sin(d*x + c) + 7*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-5/2), x)
```

**Giac [A]**
time = 0.45, size = 153, normalized size = 1.43

$$\frac{\sqrt{2}\left(\frac{3\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{3\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\left(3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{64\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(3*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))) - 3*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*(3*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3
- 5*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)
^2*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/(sqrt(a)*d)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a*sin(c + d*x))^(5/2), x)
```

# 3.82 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=144

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{16\ldots}{}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(5/2)+11/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)+43
/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/
a^(5/2)/d
```

**Rubi** [A]
time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 3064, 2728, 212, 2852}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{11\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (43*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (11*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc(c+dx)\left(4a-\frac{3}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc(c+dx)\left(8a^2-\frac{11}{4}a^2\,\mathrm{s}\right.}{\sqrt{a+a\sin(c+}}}{8a^4}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \csc(c+dx)\sqrt{a+}}{a^3}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{2\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,\right.}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.18, size = 296, normalized size = 2.06

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)+4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-22\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+11\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-(43+43i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4-16\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+16\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\right)}{16d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) - 22*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 + 11*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (43 + 43*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4 - 16*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 16*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 +
 Sin[c + d*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(119) = 238$.
time = 2.95, size = 261, normalized size = 1.81

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\left(-2\sin(dx+c)a^5\left(-43\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)+64\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)+a^5\left(-\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-2*sin(d*x+c)*a^5*(-43*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))+64*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))+a^5*(-43*2^(1/2)*
arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+64*arctanh((a-a*sin(d*x
+c))^(1/2)/a^(1/2)))*cos(d*x+c)^2-22*(a-a*sin(d*x+c))^(3/2)*a^(7/2)+52*(a-a
*sin(d*x+c))^(1/2)*a^(9/2)+86*2^(1/2)*a^5*arctanh(1/2*(a-a*sin(d*x+c))^(1/2
)*2^(1/2)/a^(1/2))-128*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^5)*(-a*(si
n(d*x+c)-1))^(1/2)/a^(15/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(119) = 238.
time = 0.37, size = 539, normalized size = 3.74

$$\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^2-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)+32\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+\left(\cos(dx+c)+3\right)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(11\cos(dx+c)^2+\left(11\cos(dx+c)-4\right)\sin(dx+c)+15\cos(dx+c)+4\right)\sqrt{a\sin(dx+c)+a}}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d+\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 3
2*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*cos(d*x + c) - 4
)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(
d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(
```

```
d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(
d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(11
*cos(d*x + c)^2 + (11*cos(d*x + c) - 4)*sin(d*x + c) + 15*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac** [**A**]
time = 0.73, size = 163, normalized size = 1.13

$$\frac{\frac{32\,\log\left(\left|\frac{1}{2}\,\sqrt{2}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{32\,\log\left(\left|-\frac{1}{2}\,\sqrt{2}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\left(11\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-13\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(32*log(abs(1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(5/2)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 32*log(abs(-1/2*sqrt(2) + sin(-1/4*pi
+ 1/2*d*x + 1/2*c)))/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + sqrt(2
)*(11*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 13*sqrt(a)*sin(-1/4*pi + 1
/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

# 3.83 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=174

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{}{16ad}$$

[Out] 5*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cot(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)+15/16*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-115/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-35/16*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)

**Rubi** [A]
time = 0.34, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2845, 3057, 3063, 3064, 2728, 212, 2852}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{35\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{15\cot(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cot(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]

[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) - (115*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + Cot[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (15*Cot[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (35*Cot[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
```

```
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^2(c+dx)\left(5a-\frac{5}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{35a^2}{2}-\frac{45}{4}a^2 \right.}{\sqrt{a+a\sin(c+}}}{8a^4}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.42, size = 509, normalized size = 2.93

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 38*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 - 19*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 8*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4 + (115 + 115*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
- 4*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 40*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
 - 40*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4 + (8*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
```

```
^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) - (8*Sin[(c + d*x)/4]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) - 4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(16*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(145) = 290$.
time = 2.88, size = 356, normalized size = 2.05

| method | result |
|---|---|
| default | $\frac{\left(-115\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{3}\left(dx+c\right)\right)a^{2}-230\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-\right.}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(9/2)*(-115*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(d*x+c)^3*a^2-230*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+160*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1
/2))*sin(d*x+c)^3*a^2+38*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)-32*(-
a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2-115*2^(1/2)*arctanh(1/2*(-a*(s
in(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)*a^2+320*arctanh((-a*(sin(d*
x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2-148*(-a*(sin(d*x+c)-1))^(1/2)*a^(3
/2)*sin(d*x+c)+160*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^2*sin(d*x+c
)-32*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)/sin(d*x+c
)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(145) = 290$.
time = 0.38, size = 631, normalized size = 3.63

$$\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-5\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)+4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)+80\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-5\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)+4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+\left(\cos(dx+c)+3\right)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(35\cos\left(dx+c\right)^{3}-20\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{2}+55\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)-51\cos\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{64\left(a^{3}d\cos\left(dx+c\right)^{4}-2a^{3}d\cos\left(dx+c\right)^{3}-5a^{3}d\cos\left(dx+c\right)^{2}+2a^{3}d\cos\left(dx+c\right)+4a^{3}d-\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (
cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*co
s(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*
cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2
)*sin(d*x + c) - cos(d*x + c) - 2)) + 80*(cos(d*x + c)^4 - 2*cos(d*x + c)^3
 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
35*cos(d*x + c)^3 - 20*cos(d*x + c)^2 - (35*cos(d*x + c)^2 + 55*cos(d*x + c
) + 4)*sin(d*x + c) - 51*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d
*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac [A]**
time = 0.53, size = 205, normalized size = 1.18

$$\frac{\frac{80\log\left(\frac{\left|-4\sqrt{2}+8\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|4\sqrt{2}+8\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{32\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(19\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(80*log(abs(-4*sqrt(2) + 8*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(4*sqrt(
2) + 8*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))) - 32*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((2*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^2 - 1)*a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - sqrt(2
)*(19*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 21*sqrt(a)*sin(-1/4*pi + 1
```

```
/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

# 3.84 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

**Optimal**. Leaf size=224

$$-\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d}+\frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{19\cdots}{16\cdots}$$

```
[Out] -39/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cot(
d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)+19/16*cot(d*x+c)*csc(d*x+c)/a/d/
(a+a*sin(d*x+c))^(3/2)+219/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*s
in(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+63/16*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))
^(1/2)-31/16*cot(d*x+c)*csc(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.44, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2845, 3057, 3063, 3064, 2728, 212, 2852}

$$-\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{63\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}-\frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+dx)\csc(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cot(c+dx)\csc(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-39*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (219*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(16*Sqrt[2]*a^(5/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(4*d*(a + a*Sin[c +
d*x])^(5/2)) + (19*Cot[c + d*x]*Csc[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) + (63*Cot[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (31*Cot[c
+ d*x]*Csc[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^3(c+dx)\left(6a-\frac{7}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(31a^2-\frac{95}{4}a^2 s\right.}{\sqrt{a+a\sin(c+}}}{8a^4}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{31\cot(c+dx)\csc(c}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+}}$$

$$= -\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.78, size = 680, normalized size = 3.04

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*Sin[(c + d*x)/2] + 8*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) - 108*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + 54*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 40*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^4 - (438 + 438*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
```

```
)^4 + 20*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - Csc[(c
+ d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 156*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 156*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4 + Sec[(c + d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (2*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4])^2 - (40*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[
(c + d*x)/4] - Sin[(c + d*x)/4]) - (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*Sin[(c + d*x)/4]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(32*d*(a*(1 + S
in[c + d*x]))^(5/2))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(189) = 378$.
time = 3.31, size = 404, normalized size = 1.80

| method | result |
|---|---|
| default | $\frac{\left(219\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{4}\left(dx+c\right)\right)a^{2}+438\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-\right.}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(219*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(d*x+c)^4*a^2+438*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(d*x+c)^3*a^2-312*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(
d*x+c)^4*a^2+219*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1
/2))*sin(d*x+c)^2*a^2-624*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^3*a^2-126*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)^2+172*(-a*(sin(
d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2-312*arctanh((-a*(sin(d*x+c)-1))^(1/2)
/a^(1/2))*sin(d*x+c)^2*a^2-144*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)
+112*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)-72*(-a*(sin(d*x+c)-1))^(3
/2)*a^(1/2)+56*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)
/a^(9/2)/sin(d*x+c)^2/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(189) = 378$.

time = 0.39, size = 715, normalized size = 3.19

$$\frac{219\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4-3\cos(dx+c)^3-7\cos(dx+c)^2+\left(\cos(dx+c)^4-2\cos(dx+c)^3-5\cos(dx+c)^2+2\cos(dx+c)+4\right)\sin(dx+c)+2\cos(dx+c)+4\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+156\left(\cos(dx+c)^5+3\cos(dx+c)^4-3\cos(dx+c)^3-7\cos(dx+c)^2+\left(\cos(dx+c)^4-2\cos(dx+c)^3-5\cos(dx+c)^2+2\cos(dx+c)+4\right)\sin(dx+c)+2\cos(dx+c)+4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(63\cos(dx+c)^4+95\cos(dx+c)^3-51\cos(dx+c)^2+\left(63\cos(dx+c)^3-32\cos(dx+c)^2-83\cos(dx+c)+4\right)\sin(dx+c)-87\cos(dx+c)-4\right)\sqrt{a\sin(dx+c)+a}}{64\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4-3a^3d\cos(dx+c)^3-7a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+4a^3d+\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^3-5a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+4a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(219*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7
*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2
*cos(d*x + c) + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) +
 156*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7*cos(d*x + c)
^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2*cos(d*x + c)
 + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(63*cos(d*x + c)^4 + 95*cos(d*x + c)^3 - 51*cos(d*x + c)^2 + (63*cos(d*x +
 c)^3 - 32*cos(d*x + c)^2 - 83*cos(d*x + c) + 4)*sin(d*x + c) - 87*cos(d*x
+ c) - 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x
 + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*
cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(5/2), x)
```

**Giac** [A]

time = 0.58, size = 298, normalized size = 1.33

$$-\frac{\frac{219\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{219\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{312\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{312\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(252\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-568\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+399\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-85\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(219*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))) - 219*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/
2*c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 312*log(abs(1/2*s
qrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))) + 312*log(abs(-1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c)))/
(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(252*sqrt(a)*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^7 - 568*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5
 + 399*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 85*sqrt(a)*sin(-1/4*pi +
1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 - 3*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^2 + 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

# 3.85 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=37

$$-\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

[Out] -2*arcsin(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f

**Rubi** [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2853, 222}

$$-\frac{2\sqrt{a}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]

[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.36, size = 164, normalized size = 4.43

$$\frac{(1+i)e^{\frac{1}{2}i(e+fx)}\sqrt{-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)-i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right)\right)\sqrt{a(1+\sin(e+fx))}}{\sqrt{2}\sqrt{-1+e^{2i(e+fx)}}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*E^((I/2)*(e + f*x))*Sqrt[((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e
 + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] - I*ArcTanh[E^(I*(e + f*x
))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sin[e + f*x])])/(Sqrt[2]*Sq
rt[-1 + E^((2*I)*(e + f*x))]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(31) = 62$.
time = 16.06, size = 320, normalized size = 8.65

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(\sqrt{\sin}\left(fx+e\right)\right)\left(\ln\left(-\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\sqrt{2}\sin(fx+e)-\cos(fx\cdots}{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\sqrt{2}\sin(fx+e)+\cos(fx\cdots}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x
+e)^(1/2)*(ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-cos(
f*x+e)+sin(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e
)+cos(f*x+e)-sin(f*x+e)-1))+4*arctan(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^
```

```
(1/2)+1)+4*arctan(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)-1)+ln(-((-(-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+cos(f*x+e)-sin(f*x+e)-1)/
((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-cos(f*x+e)+sin(f*x+
e)+1)))*2^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(33) = 66$.
time = 0.55, size = 224, normalized size = 6.05

$$-\frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}} - 3\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)\right)\right)\sqrt{a} + 6\sqrt{2}\sqrt{a}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}} - \frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{2}\sqrt{a}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*sqrt(a)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2) - 3*sqrt(2)
*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(f*x + e)/(cos(f*
x + e) + 1)))))*sqrt(a) + 6*sqrt(2)*sqrt(a)*sqrt(sin(f*x + e)/(cos(f*x + e)
 + 1)) - 2*(3*sqrt(2)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(2)*sqr
t(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(33) = 66$.
time = 0.42, size = 360, normalized size = 9.73

$$\left[\frac{\sqrt{-a}\log\left(\frac{128a\cos(fx+e)^5-128a\cos(fx+e)^4-416a\cos(fx+e)^3+128a\cos(fx+e)^2-8\left(16\cos(fx+e)^4-24\cos(fx+e)^3-66\cos(fx+e)^2+\left(16\cos(fx+e)^3+40\cos(fx+e)^2-26\cos(fx+e)-51\right)\sin(fx+e)+25\cos(fx+e)+51\right)\sqrt{a\sin(fx+e)+a}\sqrt{-a}\sqrt{\sin(fx+e)}+289a\cos(fx+e)+\left(128a\cos(fx+e)^4+256a\cos(fx+e)^3-160a\cos(fx+e)^2-288a\cos(fx+e)+a\right)\sin(fx+e)+a}{\cos(fx+e)+\sin(fx+e)+1}\right)}{4f}, \frac{\sqrt{a}\arctan\left(\frac{\left(8\cos(fx+e)^2+8\sin(fx+e)-9\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}\sqrt{\sin(fx+e)}}{4\left(2a\cos(fx+e)^3+a\cos(fx+e)\sin(fx+e)-2a\cos(fx+e)\right)}\right)}{2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 - 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
3 - 66*cos(f*x + e)^2 + (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(a*sin(f*x + e) + a)*s
qrt(-a)*sqrt(sin(f*x + e)) + 289*a*cos(f*x + e) + (128*a*cos(f*x + e)^4 + 2
56*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(f*
x + e) + a)/(cos(f*x + e) + sin(f*x + e) + 1))/f, 1/2*sqrt(a)*arctan(1/4*(8
*cos(f*x + e)^2 + 8*sin(f*x + e) - 9)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*sqrt
(sin(f*x + e))/(2*a*cos(f*x + e)^3 + a*cos(f*x + e)*sin(f*x + e) - 2*a*cos(
f*x + e)))/f]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin(e+fx)+1\right)}}{\sqrt{\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(sin(f*x + e)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/sin(e + f*x)^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/sin(e + f*x)^(1/2), x)
```

# 3.86 $\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

```
[Out] 2*arcsin(cos(f*x+e)*a^(1/2)/(a-a*sin(f*x+e))^(1/2))*a^(1/2)/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2853, 222}

$$\frac{2\sqrt{a}\,\text{ArcSin}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a - a*Sin[e + f*x]]])/f
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

$$= \frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.31, size = 119, normalized size = 3.13

$$-\frac{\sqrt{-1+e^{2i(e+fx)}}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)+i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right)\right)\sqrt{a-a\sin(e+fx)}}{\left(-i+e^{i(e+fx)}\right)f\sqrt{-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[-1 + E^((2*I)*(e + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] +
 I*ArcTanh[E^(I*(e + f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a - a*Sin[
e + f*x]])/((-I + E^(I*(e + f*x)))*f*Sqrt[-Sin[e + f*x]]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(32) = 64.
time = 16.38, size = 271, normalized size = 7.13

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)\left(\ln\left(-\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sqrt{2}\,\sin(fx+e)-\cos(fx+e)+\sin(}{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sqrt{2}\,\sin(fx+e)+\cos(fx+e)-\sin(}\right.\right.}{2f\sqrt{-\sin\left(fx+e\right)}\,(-1+\cos(fx+e)+\sin(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x
+e)*(ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-cos(f*x+e)
+sin(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+cos(
f*x+e)-sin(f*x+e)-1))-ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(
```

```
f*x+e)+cos(f*x+e)-sin(f*x+e)-1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2
)*sin(f*x+e)-cos(f*x+e)+sin(f*x+e)+1)))/(-sin(f*x+e))^(1/2)/(-1+cos(f*x+e)+
sin(f*x+e))*2^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/sqrt(-sin(f*x + e)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(34) = 68$.
time = 0.42, size = 371, normalized size = 9.76

$$\left[\frac{\sqrt{-a}\log\left(\frac{128\,a\cos(fx+e)^5-128\,a\cos(fx+e)^4-416\,a\cos(fx+e)^3+128\,a\cos(fx+e)^2+8\left(16\cos(fx+e)^4-24\cos(fx+e)^3-66\cos(fx+e)^2-\left(16\cos(fx+e)^3+40\cos(fx+e)^2-26\cos(fx+e)-51\right)\sin(fx+e)+25\cos(fx+e)+51\right)\sqrt{-a\sin(fx+e)+a}\sqrt{-a}\sqrt{-\sin(fx+e)}+289\,a\cos(fx+e)-\left(128\,a\cos(fx+e)^4+256\,a\cos(fx+e)^3-160\,a\cos(fx+e)^2-288\,a\cos(fx+e)+a\right)\sin(fx+e)+a}{\cos(fx+e)-\sin(fx+e)+1}\right)}{4f},\ -\frac{\sqrt{a}\arctan\left(\frac{\left(8\cos(fx+e)^2-8\sin(fx+e)-9\right)\sqrt{-a\sin(fx+e)+a}\sqrt{a}\sqrt{-\sin(fx+e)}}{4\left(2\,a\cos(fx+e)^3-a\cos(fx+e)\sin(fx+e)-2\,a\cos(fx+e)\right)}\right)}{2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 + 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
3 - 66*cos(f*x + e)^2 - (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(-a*sin(f*x + e) + a)*
sqrt(-a)*sqrt(-sin(f*x + e)) + 289*a*cos(f*x + e) - (128*a*cos(f*x + e)^4 +
 256*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(
f*x + e) + a)/(cos(f*x + e) - sin(f*x + e) + 1))/f, -1/2*sqrt(a)*arctan(1/4
*(8*cos(f*x + e)^2 - 8*sin(f*x + e) - 9)*sqrt(-a*sin(f*x + e) + a)*sqrt(a)*
sqrt(-sin(f*x + e))/(2*a*cos(f*x + e)^3 - a*cos(f*x + e)*sin(f*x + e) - 2*a
*cos(f*x + e)))/f]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{-\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(-sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/sqrt(-sin(e + f*x)), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(34) = 68$.
time = 0.76, size = 99, normalized size = 2.61

$$-\frac{4\sqrt{a}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2\left(2\sqrt{2}-\sqrt{-\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}+6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-1}\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-3}\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(2) - sqrt(-tan(-1/8*pi
+ 1/4*f*x + 1/4*e)^4 + 6*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 1))/(tan(-1/8*p
i + 1/4*f*x + 1/4*e)^2 - 3)))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a-a\sin\left(e+fx\right)}}{\sqrt{-\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(e + f*x))^(1/2)/(-sin(e + f*x))^(1/2),x)
```

```
[Out] int((a - a*sin(e + f*x))^(1/2)/(-sin(e + f*x))^(1/2), x)
```

# 3.87 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx$

**Optimal**. Leaf size=17

$$-\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{1+\sin(x)}\right)$$

```
[Out] -arcsin(cos(x)/(1+sin(x)))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2860, 222}

$$-\sqrt{2}\,\text{ArcSin}\left(\frac{\cos(x)}{\sin(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]
```

```
[Out] -(Sqrt[2]*ArcSin[Cos[x]/(1 + Sin[x])])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2860

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{\cos(x)}{1+\sin(x)}\right)\right)\\ &= -\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{1+\sin(x)}\right)\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 1.34, size = 123, normalized size = 7.24

$$\frac{2\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)\right)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{\sin(x)}}{\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{1+\sin(x)}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 - Sqrt[2], ArcSi
n[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]]]
, -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^2]
*Sqrt[1 + Sin[x]]*Tan[x/4]^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.
time = 0.37, size = 52, normalized size = 3.06

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)-\sin(x))\left(\sqrt{\sin}\,(x)\right)\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{1+\sin(x)}\,(-1+\cos(x))}$ | 52 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)-sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(1+sin(x))^(1/2)/(-1+cos(x))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

**Fricas** [A]
time = 0.33, size = 28, normalized size = 1.65

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)+1}\sqrt{\sin(x)}}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(x) + 1)*sqrt(sin(x))/(cos(x) + sin(x) + 1
))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x)+1}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(1+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{\sin(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(sin(x) + 1)^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(sin(x) + 1)^(1/2)), x)
```

# 3.88 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx$

**Optimal**. Leaf size=42

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)}{\sqrt{a}}$$

```
[Out] -arctan(1/2*cos(x)*a^(1/2)*2^(1/2)/sin(x)^(1/2)/(a+a*sin(x))^(1/2))*2^(1/2)
/a^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2861, 211}

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a\sin(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]
)]])/Sqrt[a])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx = -\left((2a)\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, \frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)\right)$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)}{\sqrt{a}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.06, size = 125, normalized size = 2.98

$$\frac{2\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)\right)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{\sin(x)}}{\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a(1+\sin(x))}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 - Sqrt[2], ArcSi
n[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]]]
, -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^2]
*Sqrt[a*(1 + Sin[x])]*Tan[x/4]^(3/2))
```

**Maple** [A]
time = 0.34, size = 54, normalized size = 1.29

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)-\sin(x))\left(\sqrt{\sin}\,(x)\right)\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{a\,(1+\sin\,(x))}\,(-1+\cos(x))}$ | 54 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)-sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(a*(1+sin(x)))^(1/2)/(-1+cos(x))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

**Fricas [A]**
time = 0.37, size = 163, normalized size = 3.88

$$\left[\frac{1}{4}\,\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\log\left(\frac{17\,\cos\left(x\right)^{3}-4\,\sqrt{2}\,\left(3\,\cos\left(x\right)^{2}+\left(3\,\cos\left(x\right)+4\right)\sin\left(x\right)-\cos\left(x\right)-4\right)\sqrt{a\sin\left(x\right)+a}\,\sqrt{-\frac{1}{a}}\,\sqrt{\sin\left(x\right)}+3\,\cos\left(x\right)^{2}+\left(17\,\cos\left(x\right)^{2}+14\,\cos\left(x\right)-4\right)\sin\left(x\right)-18\,\cos\left(x\right)-4}{\cos\left(x\right)^{3}+3\,\cos\left(x\right)^{2}+\left(\cos\left(x\right)^{2}-2\,\cos\left(x\right)-4\right)\sin\left(x\right)-2\,\cos\left(x\right)-4}\right),\ \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\sin\left(x\right)+a}\,\left(3\,\sin\left(x\right)-1\right)}{4\,\sqrt{a}\,\cos\left(x\right)\sqrt{\sin\left(x\right)}}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/a)*log((17*cos(x)^3 - 4*sqrt(2)*(3*cos(x)^2 + (3*cos(x
) + 4)*sin(x) - cos(x) - 4)*sqrt(a*sin(x) + a)*sqrt(-1/a)*sqrt(sin(x)) + 3*
cos(x)^2 + (17*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3
+ 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)), 1/2*sqrt(
2)*arctan(1/4*sqrt(2)*sqrt(a*sin(x) + a)*(3*sin(x) - 1)/(sqrt(a)*cos(x)*sqr
t(sin(x))))/sqrt(a)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(x\right)+1\right)}\,\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a+a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(x) + 1))*sqrt(sin(x))), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sin\left(x\right)}\,\sqrt{a+a\,\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(a + a*sin(x))^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(a + a*sin(x))^(1/2)), x)
```

# 3.89 $\int \frac{1}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\,dx$

**Optimal**. Leaf size=31

$$\sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)$$

```
[Out] arctanh(1/2*cos(x)*2^(1/2)/(1-sin(x))^(1/2)/sin(x)^(1/2))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2861, 212}

$$\sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] Sqrt[2]*ArcTanh[Cos[x]/(Sqrt[2]*Sqrt[1 - Sin[x]]*Sqrt[Sin[x]])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\cos(x)}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)\right)\\
&= \sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)
\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.29, size = 125, normalized size = 4.03

$$\frac{2\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)\right)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\sin(x)}{\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{-((-1+\sin(x))\sin(x))}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 - Sqrt[2], ArcS
in[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sin[x])/(Sqrt[1 - Cot[x/4]^2]*Sqr
t[-((-1 + Sin[x])*Sin[x])]*Tan[x/4]^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.
time = 0.36, size = 52, normalized size = 1.68

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)+\sin(x))\left(\sqrt{\sin}\,(x)\right)\operatorname{arctanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{1-\sin\left(x\right)}\,(-1+\cos(x))}$ | 52 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(1-sin(x))^(1/2)/(-1+cos(x))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-sin(x) + 1)*sqrt(sin(x))), x)
```

**Fricas** [A]
time = 0.36, size = 31, normalized size = 1.00

$$\sqrt{2}\,\log\left(\frac{\sqrt{2}\,\sqrt{-\sin\left(x\right)+1}\,\sqrt{\sin\left(x\right)}+\cos\left(x\right)}{\sin\left(x\right)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log((sqrt(2)*sqrt(-sin(x) + 1)*sqrt(sin(x)) + cos(x))/(sin(x) - 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(1 - sin(x))*sqrt(sin(x))), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(24) = 48.
time = 0.87, size = 146, normalized size = 4.71

$$\frac{\sqrt{2}\left(\log\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2-\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+1\right)-\log\left(\left|-\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+3\right|\right)-\log\left(\left|-\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+1\right|\right)\right)}{2\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(log(tan(-1/8*pi + 1/4*x)^2 - sqrt(tan(-1/8*pi + 1/4*x)^4 - 6*t
an(-1/8*pi + 1/4*x)^2 + 1) + 1) - log(abs(-tan(-1/8*pi + 1/4*x)^2 + sqrt(ta
n(-1/8*pi + 1/4*x)^4 - 6*tan(-1/8*pi + 1/4*x)^2 + 1) + 3)) - log(abs(-tan(-
1/8*pi + 1/4*x)^2 + sqrt(tan(-1/8*pi + 1/4*x)^4 - 6*tan(-1/8*pi + 1/4*x)^2
+ 1) + 1)))/sgn(sin(-1/4*pi + 1/2*x))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{1-\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(1 - sin(x))^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(1 - sin(x))^(1/2)), x)
```

## 3.90 $\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{a-a\sin(x)}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(x)}{\sqrt{2}\,\sqrt{\sin(x)}\,\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

```
[Out] arctanh(1/2*cos(x)*a^(1/2)*2^(1/2)/sin(x)^(1/2)/(a-a*sin(x))^(1/2))*2^(1/2)
/a^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2861, 214}

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(x)}{\sqrt{2}\,\sqrt{\sin(x)}\,\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]])
])/Sqrt[a]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\,dx = -\left((2a)\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, -\frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)\right)$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.07, size = 128, normalized size = 3.05

$$\frac{2\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)\right)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\sqrt{\sin(x)}}{\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a-a\sin(x)}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 - Sqrt[2], ArcS
in[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^
2]*Sqrt[a - a*Sin[x]]*Tan[x/4]^(3/2))
```

**Maple [A]**
time = 0.33, size = 53, normalized size = 1.26

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)+\sin(x))\left(\sqrt{\sin}\,(x)\right)\operatorname{arctanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{-a\,(-1+\sin(x))}\,(-1+\cos(x))}$ | 53 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(-a*(-1+sin(x)))^(1/2)/(-1+cos(x))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*sin(x) + a)*sqrt(sin(x))), x)
```

**Fricas [A]**
time = 0.38, size = 168, normalized size = 4.00

$$\left[\frac{\sqrt{2}\log\left(\frac{17\cos(x)^3+3\cos(x)^2+\frac{4\sqrt{2}\left(3\cos(x)^2-(3\cos(x)+4)\sin(x)-\cos(x)-4\right)\sqrt{-a\sin(x)+a}\sqrt{\sin(x)}}{\sqrt{a}}-\left(17\cos(x)^2+14\cos(x)-4\right)\sin(x)-18\cos(x)-4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)}{4\sqrt{a}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\sin(x)+a}\sqrt{-\frac{1}{a}}(3\sin(x)+1)}{4\cos(x)\sqrt{\sin(x)}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((17*cos(x)^3 + 3*cos(x)^2 + 4*sqrt(2)*(3*cos(x)^2 - (3*cos
(x) + 4)*sin(x) - cos(x) - 4)*sqrt(-a*sin(x) + a)*sqrt(sin(x))/sqrt(a) - (1
7*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3 + 3*cos(x)^2
- (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))/sqrt(a), -1/2*sqrt(2)*s
qrt(-1/a)*arctan(1/4*sqrt(2)*sqrt(-a*sin(x) + a)*sqrt(-1/a)*(3*sin(x) + 1)/
(cos(x)*sqrt(sin(x))))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a(\sin(x)-1)}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a-a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(sin(x) - 1))*sqrt(sin(x))), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(31) = 62.
time = 1.45, size = 149, normalized size = 3.55

$$\frac{\sqrt{2}\left(\log\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2-\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+1\right)-\log\left(\left|-\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+3\right|\right)-\log\left(\left|-\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+\sqrt{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^4-6\tan\left(-\frac{1}{8}\pi+\frac{1}{4}x\right)^2+1}+1\right|\right)\right)}{2\sqrt{a}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(log(tan(-1/8*pi + 1/4*x)^2 - sqrt(tan(-1/8*pi + 1/4*x)^4 - 6*t
an(-1/8*pi + 1/4*x)^2 + 1) + 1) - log(abs(-tan(-1/8*pi + 1/4*x)^2 + sqrt(ta
n(-1/8*pi + 1/4*x)^4 - 6*tan(-1/8*pi + 1/4*x)^2 + 1) + 3)) - log(abs(-tan(-
```

```
1/8*pi + 1/4*x)^2 + sqrt(tan(-1/8*pi + 1/4*x)^4 - 6*tan(-1/8*pi + 1/4*x)^2
+ 1) + 1)))/(sqrt(a)*sgn(sin(-1/4*pi + 1/2*x)))
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sin(x)}\ \sqrt{a-a\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(a - a*sin(x))^(1/2)),x)
[Out] int(1/(sin(x)^(1/2)*(a - a*sin(x))^(1/2)), x)
```

# 3.91 $\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

**Optimal**. Leaf size=184

$$\frac{4\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)\sqrt[3]{\sin(c+dx)}}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)\sin^{\frac{4}{3}}(c+dx)}{36a^2d\sqrt{\cos^2(c+dx)}}-$$

```
[Out] -1/9*cos(d*x+c)*sin(d*x+c)^(1/3)/a^2/d/(1+sin(d*x+c))-1/3*cos(d*x+c)*sin(d*
x+c)^(1/3)/d/(a+a*sin(d*x+c))^2+4/9*cos(d*x+c)*hypergeom([1/6, 1/2],[7/6],s
in(d*x+c)^2)*sin(d*x+c)^(1/3)/a^2/d/(cos(d*x+c)^2)^(1/2)-1/36*cos(d*x+c)*hy
pergeom([1/2, 2/3],[5/3],sin(d*x+c)^2)*sin(d*x+c)^(4/3)/a^2/d/(cos(d*x+c)^2
)^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2843, 3057, 2827, 2722}

$$\frac{4\sqrt[3]{\sin(c+dx)}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sin^{\frac{4}{3}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)}{36a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sqrt[3]{\sin(c+dx)}\cos(c+dx)}{9a^2d(\sin(c+dx)+1)}-\frac{\sqrt[3]{\sin(c+dx)}\cos(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2]*Sin[c + d*
x]^(1/3))/(9*a^2*d*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*Hypergeometric2F1[
1/2, 2/3, 5/3, Sin[c + d*x]^2]*Sin[c + d*x]^(4/3))/(36*a^2*d*Sqrt[Cos[c + d
*x]^2]) - (Cos[c + d*x]*Sin[c + d*x]^(1/3))/(9*a^2*d*(1 + Sin[c + d*x])) -
(Cos[c + d*x]*Sin[c + d*x]^(1/3))/(3*d*(a + a*Sin[c + d*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{a}{3}+\frac{2}{3}a\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)(a+a\sin(c+dx))}\,dx}{3a^2}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{4a^2}{9}-\frac{1}{9}a^2\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)}\,d}{3a^4}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} - \frac{\int \sqrt[3]{\sin(c+dx)}\,d}{27a^2}$$

$$= \frac{4\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)\sqrt[3]{\sin(c+dx)}}{9a^2d\sqrt{\cos^2(c+dx)}} - \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3}\right.}{36a^2d\sqrt{}}$$

**Mathematica [A]**
time = 0.28, size = 121, normalized size = 0.66

$$\frac{\sec^3(c+dx)\sqrt[3]{\sin(c+dx)}\left(80\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)+27\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(c+dx)\right)\sin(c+dx)+4(-25+5\cos(2(c+dx))+27\sin(c+dx))\right)}{180a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*Sin[c + d*x]^(1/3)*(80*(Cos[c + d*x]^2)^(3/2)*Hypergeometri
c2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2] + 27*(Cos[c + d*x]^2)^(3/2)*Hypergeomet
ric2F1[2/3, 5/2, 5/3, Sin[c + d*x]^2]*Sin[c + d*x] + 4*(-25 + 5*Cos[2*(c +
d*x)] + 27*Sin[c + d*x])))/(180*a^2*d)
```

**Maple [F]**
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{1}{3}}\left(dx+c\right)}{\left(a+a\sin\left(dx+c\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^(1/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt[3]{\sin\left(c+dx\right)}}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/3)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**(1/3)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**
2
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^{1/3}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/3)/(a + a*sin(c + d*x))^2,x)
[Out] int(sin(c + d*x)^(1/3)/(a + a*sin(c + d*x))^2, x)
```

# 3.92 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=161

$$-\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d}-\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d}-\frac{67\cos(c+dx)\,{}_2F_1$$

```
[Out] -63/220*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-3/11*cos(d*x+c)*sin(d*x+c)^2*(a
+a*sin(d*x+c))^(2/3)/d-67/110*cos(d*x+c)*hypergeom([-1/6, 1/2],[3/2],1/2-1/
2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c))^(7/6)-3/44*co
s(d*x+c)*(a+a*sin(d*x+c))^(5/3)/a/d
```

**Rubi** [A]
time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2862, 3047, 3102, 2830, 2731, 2730}

$$-\frac{67\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{55\ 2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{11d}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{44ad}-\frac{63\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{220d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-63*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(220*d) - (3*Cos[c + d*x]*Sin
[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3))/(11*d) - (67*Cos[c + d*x]*Hypergeom
etric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/3))/
(55*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c + d*
x])^(5/3))/(44*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx &= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d}+\frac{3\int \sin(c+d\\ &= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d}+\frac{3\int(a+a\sin\\ &= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d}-\frac{3\cos(c+dx\\ &= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d}-\frac{3\cos(c+dx)\sin^2(c+d}{11\\ &= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d}-\frac{3\cos(c+dx)\sin^2(c+d}{11\\ &= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d}-\frac{3\cos(c+dx)\sin^2(c+d}{11\end{aligned}$$

**Mathematica** [**A**]
time = 0.47, size = 160, normalized size = 0.99

$$\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sin(c+dx)))^{2/3}\left(67\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{1-\sin(c+dx)}\,(-144+25\cos(2(c+dx))-92\sin(c+dx)+10\sin(3(c+dx)))\right)}{440d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(67*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(-144 + 25*Cos[2*(c + d*x)] - 92*Sin[c + d*x] + 10*Sin[3
*(c + d*x)])))/(440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c
+ d*x]])
```

**Maple** [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(2/3),x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(2/3), x)
```

# 3.93 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=126

$$\frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{19\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{2/3}}{10\ 2^{5/6}d(1+\sin(c+dx))^{7/6}}$$

```
[Out] 9/40*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-19/20*cos(d*x+c)*hypergeom([-1/6,
1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+
c))^(7/6)-3/8*cos(d*x+c)*(a+a*sin(d*x+c))^(5/3)/a/d
```

**Rubi** [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2731, 2730}

$$-\frac{19\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{8ad}+\frac{9\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (9*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(40*d) - (19*Cos[c + d*x]*Hyper
geometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/
3))/(10*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(5/3))/(8*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad}+\frac{3\int\left(\frac{5a}{3}-a\sin(c+dx)\right)(a}{8a}$$

$$=\frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{3\cos(c+dx)(a+a\sin(c+}{8ad}$$

$$=\frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{3\cos(c+dx)(a+a\sin(c+}{8ad}$$

$$=\frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{19\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\right.}{10\ 2^{5/6}}$$

**Mathematica** [A]
time = 0.30, size = 151, normalized size = 1.20

$$\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sin(c+dx)))^{2/3}\left(19\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{1-\sin(c+dx)}\,(5\cos(2(c+dx))-14(2+\sin(c+dx)))\right)}{80d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(19*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(5*Cos[2*(c + d*x)] - 14*(2 + Sin[c + d*x]))))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

**Maple** [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\sin^2(dx+c)\right)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}}\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(2/3),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(2/3), x)
```

# 3.94 $\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=96

$$-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx}{5d(1+\sin(c+dx))^{7/6}}$$

```
[Out] -3/5*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-4/5*cos(d*x+c)*hypergeom([-1/6, 1/
2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c)
)^(7/6)
```

**Rubi** [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2731, 2730}

$$-\frac{4\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(5*d) - (4*2^(1/6)*Cos[c + d*x
]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(2/3))/(5*d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{2}{5}\int (a+a\sin(c+dx))^{2/3}\, d$$

$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{\left(2(a+a\sin(c+dx))^{2/3}\right)\int}{5(1+\sin(c+}$$

$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} - \frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\right.}{5d}$$

**Mathematica** [A]
time = 0.15, size = 138, normalized size = 1.44

$$-\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sin(c+dx)))^{2/3}\left(-\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{1-\sin(c+dx)}\,(2+\sin(c+dx))\right)}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(-(S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2]) + Sqr
t[1 - Sin[c + d*x]]*(2 + Sin[c + d*x])))/(5*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

**Maple** [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sin(dx+c)\left(a+a\sin(dx+c)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}} \sin(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(2/3),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(2/3), x)
```

# 3.95 $\int (a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=66

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}$$

```
[Out] -2*cos(d*x+c)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+
c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c))^(7/6)
```

**Rubi** [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x
])/2]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{2/3}\,dx &= \frac{(a+a\sin(c+dx))^{2/3}\int (1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}\\ &= -\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}\end{aligned}$$

**Mathematica [A]**
time = 0.14, size = 124, normalized size = 1.88

$$-\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)(a(1+\sin(c+dx)))^{2/3}}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{2-2\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-2*Hypergeometric2F1[1/6, 1/2, 7
/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]])*(a*(1 + Sin[c
+ d*x]))^(2/3))/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[2 - 2*Sin[c
 + d*x]])
```

**Maple [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sin(d*x+c))^(2/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin{(c + dx)} + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(2/3), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\sin\left(c+d\,x\right)\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3),x)
```

```
[Out] int((a + a*sin(c + d*x))^(2/3), x)
```

# 3.96 $\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=77

$$-\frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}$$

```
[Out] -2*2^(1/6)*AppellF1(1/2,1,-1/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+a*sin(d*x+c))^(2/3)/d/(1+sin(d*x+c))^(7/6)
```

**Rubi** [A]

time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2\sqrt[6]{2}\,\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 1, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$

$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\mathrm{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)\sqrt{x}}\,dx, x, 1-\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\mathrm{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{1-x^2}\,dx, x, \sqrt{\right.}}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+}{d(1+\sin(c+dx))^{7/6}}$$

**Mathematica [F]**
time = 1.90, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3), x]
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin{(c+dx)}+1))^{\frac{2}{3}}\csc{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*csc(c + d*x), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(a+a\,\sin{(c+d\,x)})}^{2/3}}{\sin{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x), x)
```

# 3.97 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

**Optimal**. Leaf size=77

$$-\frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}$$

```
[Out] -2*2^(1/6)*AppellF1(1/2,2,-1/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+a*sin(d*x+c))^(2/3)/d/(1+sin(d*x+c))^(7/6)
```

**Rubi** [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 2, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc^2(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$

$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)^2\sqrt{x}}\,dx, x, 1}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{(1-x^2)^2}\,dx, x,}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c}{d(1+\sin(c+dx))^{7/6}}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 8.79, size = 143, normalized size = 1.86

$$-\frac{2e^{i(c+dx)}\left(-i-e^{i(c+dx)}+\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(-i+e^{i(c+dx)}\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)\right)(a(1+\sin(c+dx)))^{2/3}}{d\left(-i+e^{i(c+dx)}\right)\left(i+e^{i(c+dx)}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*E^(I*(c + d*x))*(-I - E^(I*(c + d*x)) + (1 + I/E^(I*(c + d*x)))^(2/3)*(
-I + E^(I*(c + d*x)))*Hypergeometric2F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))
])*(a*(1 + Sin[c + d*x]))^(2/3))/(d*(-I + E^(I*(c + d*x)))*(I + E^(I*(c + d
*x)))^2)
```

**Maple [F]**
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin{(c+dx)}+1))^{\frac{2}{3}}\csc^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*csc(c + d*x)**2, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin{(c+d\,x)})^{2/3}}{\sin{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x)^2, x)
```

# 3.98 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=162

$$-\frac{388\ 2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{455d(1+\sin(c+dx))^{5/6}}-\frac{72\cos(c+dx)(a+a\sin(c+}{455d}$$

```
[Out] -388/455*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c
))*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)-72/455*cos(d*x+c)*(a+a*sin
(d*x+c))^(4/3)/d-3/13*cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3)/d-6/65
*cos(d*x+c)*(a+a*sin(d*x+c))^(7/3)/a/d
```

**Rubi** [A]
time = 0.21, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2862, 3047, 3102, 2830, 2731, 2730}

$$-\frac{388\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{455d(\sin(c+dx)+1)^{5/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{13d}-\frac{6\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{65ad}-\frac{72\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{455d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-388*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
 d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(455*d*(1 + Sin[c + d*x])^(5/6)) - (7
2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(455*d) - (3*Cos[c + d*x]*Sin[c
+ d*x]^2*(a + a*Sin[c + d*x])^(4/3))/(13*d) - (6*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(7/3))/(65*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int \sin(c+dx}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int (a+a\sin(}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} - \frac{6\cos(c+dx)(}{}$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx}{13c}$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx}{13c}$$

$$= -\frac{388\ 2^{5/6} a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\,\mathrm{s}}}{455d(1+\sin(c+dx))^{5/6}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 9.18, size = 373, normalized size = 2.30

$$\frac{(a(1+\sin(c+dx)))^{4/3}\left(\frac{291(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}(i+e^{i(c+dx)})\left(20e^{i(c+dx)}\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(c+dx)}\right)-2(1+ie^{-i(c+dx)})^{2/3}(1+e^{2i(c+dx)})\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(c+dx)}\right)\sqrt{2-2\sin(c+dx)}\right)}{20\sqrt{2}(1+ie^{-i(c+dx)})^{2/3}\sqrt{ie^{-i(c+dx)}\left(-i+e^{i(c+dx)}\right)^2}}-\frac{3}{40}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(-1940+790\cos(c+dx)-98\cos(3(c+dx))+278\sin(2(c+dx))-35\sin(4(c+dx)))\right)}{91d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((291*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-1940 + 790*Cos[c + d*x] - 98*Cos[3*(c + d*x)]
 + 278*Sin[2*(c + d*x)] - 35*Sin[4*(c + d*x)]))/40))/(91*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^3)
```

**Maple** [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2 - a)*si
n(d*x + c) + a)*(a*sin(d*x + c) + a)^(1/3), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(4/3),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(4/3), x)
```

# 3.99 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=127

$$-\frac{37\ 2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{35d(1+\sin(c+dx))^{5/6}}+\frac{9\cos(c+dx)(a+a\sin(c+a}{70d}$$

```
[Out] -37/35*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))
*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)+9/70*cos(d*x+c)*(a+a*sin(d*x
+c))^(4/3)/d-3/10*cos(d*x+c)*(a+a*sin(d*x+c))^(7/3)/a/d
```

**Rubi** [A]
time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2731, 2730}

$$-\frac{37\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{35d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{10ad}+\frac{9\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-37*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(35*d*(1 + Sin[c + d*x])^(5/6)) + (9*C
os[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(70*d) - (3*Cos[c + d*x]*(a + a*Sin
[c + d*x])^(7/3))/(10*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx &= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{7/3}}{10ad}+\frac{3\int\left(\frac{7a}{3}-a\sin(c+dx)\right)(a\ldots}{10a\ldots}\\
&= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d}-\frac{3\cos(c+dx)(a+a\sin(c+\ldots}{10ad}\\
&= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d}-\frac{3\cos(c+dx)(a+a\sin(c+\ldots}{10ad}\\
&= -\frac{37\ 2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin\ldots}}{35d(1+\sin(c+dx))^{5/6}}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 9.15, size = 363, normalized size = 2.86

$$\frac{(a(1+\sin(c+dx)))^{4/3}\left(\frac{111(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(20e^{i(c+dx)}\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(c+dx)}\right)-2\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(1+e^{2i(c+dx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(c+dx)}\right)\sqrt{2-2\sin(c+dx)}\right)}{20\sqrt{2}\left(1+ie^{-i(c+dx)}\right)^{2/3}\sqrt{ie^{-i(c+dx)}\left(-i+e^{i(c+dx)}\right)^2}}-\frac{3}{10}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-185+60\cos(c+dx)-7\cos(3(c+dx))+22\sin(2(c+dx))\right)\right)}{28d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((111*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-185 + 60*Cos[c + d*x] - 7*Cos[3*(c + d*x)] +
22*Sin[2*(c + d*x)]))/10))/(28*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\sin^{2}\left(dx+c\right)\right) \left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(d*x + c)^2 + (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*(a*s
in(d*x + c) + a)^(1/3), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{4}{3}}\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(4/3)*sin(c + d*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3), x)
```

# 3.100 $\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=97

$$-\frac{8\ 2^{5/6} a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{7d(1+\sin(c+dx))^{5/6}}-\frac{3\cos(c+dx)(a+a\sin(c+dx}{7d}$$

```
[Out] -8/7*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(
a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)-3/7*cos(d*x+c)*(a+a*sin(d*x+c)
)^(4/3)/d
```

**Rubi** [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2731, 2730}

$$-\frac{8\ 2^{5/6} a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{7d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-8*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(7*d*(1 + Sin[c + d*x])^(5/6)) - (3*Cos
[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(7*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d} + \frac{4}{7}\int (a+a\sin(c+dx))^{4/3}\,d$$

$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d} + \frac{\left(4a\sqrt[3]{a+a\sin(c+dx)}\right)\int}{7\sqrt[3]{1+\sin(}}$$

$$= -\frac{8\ 2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(}}{7d(1+\sin(c+dx))^{5/6}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 8.75, size = 351, normalized size = 3.62

$$\frac{(a(1+\sin(c+dx)))^{4/3}\left(\frac{3(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(20e^{i(c+dx)}\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(c+dx)}\right)-2\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(1+e^{2i(c+dx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(c+dx)}\right)\sqrt{2-2\sin(c+dx)}\right)}{2\sqrt{2}\left(1+ie^{-i(c+dx)}\right)^{2/3}\sqrt{ie^{-i(c+dx)}\left(-i+e^{i(c+dx)}\right)^2}}-\frac{3}{2}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(-10+4\cos(c+dx)+\sin(2(c+dx)))\right)}{7d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((3*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I
*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*S
in[c + d*x]]))/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(
2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(-10 + 4*Cos[c + d*x] + Sin[2*(c + d*x)]))/2))/(7*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral(-(a*cos(d*x + c)^2 - a*sin(d*x + c) - a)*(a*sin(d*x + c) + a)^(1/3
), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{4}{3}}\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(4/3)*sin(c + d*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3), x)
```

# 3.101 $\int (a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=67

$$-\frac{2\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

```
[Out] -2*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+
a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

**Rubi** [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{4/3}\,dx &= \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int (1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}\\ &= -\frac{2\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.16, size = 341, normalized size = 5.09

$$\frac{\left(-\frac{3}{2}(-5+\cos(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\left(20e^{i(c+dx)}\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(c+dx)}\right)-2\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(1+e^{2i(c+dx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(c+dx)}\right)\sqrt{2-2\sin(c+dx)}\right)}{4\sqrt{2}\left(1+ie^{-i(c+dx)}\right)^{2/3}\sqrt{ie^{-i(c+dx)}\left(-i+e^{i(c+dx)}\right)^2}}\right)(a(1+\sin(c+dx)))^{4/3}}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (((-3*(-5 + Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/2 + (3*(-1
)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*
x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I
/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 5/
6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5
/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*Sin[c + d*x]]))/(4*Sqrt[2]*E^(((3*I)/2
)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2
)/E^(I*(c + d*x))]))*(a*(1 + Sin[c + d*x]))^(4/3))/(2*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3)
```

**Maple** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(4/3), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (c+dx)+a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(4/3), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin (c+d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int((a + a*sin(c + d*x))^(4/3), x)
```

# 3.102 $\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=78

$$-\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

```
[Out] -2*2^(5/6)*a*AppellF1(1/2,1,-5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

**Rubi** [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 1, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$

$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)\sqrt{x}}\,dx, x, 1-\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{\left(2-x^2\right)^{5/6}}{1-x^2}\,dx, x, \sqrt{\right.}}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{2\ 2^{5/6}aF_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+}{d(1+\sin(c+dx))^{5/6}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 8.17, size = 2791, normalized size = 35.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
) - ((15 + 15*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3)*(1
+ Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*Sec[(c + d*x)/
2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2
- I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]) + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*
(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]))*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^3) + ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5
/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])]
```

*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(
(5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]),
(1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 +
I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + I*Appell
F1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 +
Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) - (3*Cos[(3*(c + d*x))/2]*Csc
[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c
+ d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/
(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (
1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) -
AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/
2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*(
(-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(4*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((-3*Sec[(c + d*x)
/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*
2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hyp
ergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*T
an[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/
2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 +
2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^
(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2) + ((8 + (1 + I)
*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*
Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1
/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 +
2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))
^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2 - (Tan[(c + d*x)
/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/(2*(1 + Tan[(c +
d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(1/3)) + (3*((1
+ Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-1/2*(AppellF1[2/3, 1
/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Co
t[(c + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2) + ((1 + I)*(((1 - I)*(I + Cot[(c
+ d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3,
((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*Sec[(c + d*
x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(c + d*x)/2]
)*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/2)*Csc[(c + d*x)
/2]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) +
(1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]))
/(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(2/3) - ((1/6 +
I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2
- I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/
3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I + Cot[(c + d*x)/
2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot
[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 - I)*(I + Cot[(c +

```
d*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((2 + 2*I) - (2
 - 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^
(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3,
 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c
 + d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3
, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*C
sc[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 -
 I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(I + Tan[(c + d*x
)/2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 + I) + (1 - I)*Ta
n[(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)*Sec[(c + d*x)/2
]^2)/(2 + 2*Tan[(c + d*x)/2]))*(-Hypergeometric...
```

**Maple** [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^{4/3}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x), x)
```

# 3.103 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

**Optimal**. Leaf size=78

$$-\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

```
[Out] -2*2^(5/6)*a*AppellF1(1/2,2,-5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

**Rubi** [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 2, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc^2(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$

$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^2\sqrt{x}}\,dx, x,\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x^2)^{5/6}}{(1-x^2)^2}\,dx, x,\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(}{d(1+\sin(c+dx))^{5/6}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 8.80, size = 2800, normalized size = 35.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((-1 - Cot[c + d*x])*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2) - ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2
+ I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 +
Sin[c + d*x]))^(4/3)*(1 + Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*Sec[(c + d*x)/2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 +
Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] +
Sec[(c + d*x)/2]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c
+ d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c
+ d*x)/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((10 + 10*I)*AppellF
1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 +
```

Tan[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + T
an[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3,
 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*
x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2])
, (1/2 - I/2)*(1 + Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) + (Cos[(3*(
c + d*x))/2]*Csc[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/
2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan
[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c +
d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2
])))/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((
-3*Sec[(c + d*x)/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/
3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x
)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d
*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot
[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2)
 + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*
x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c +
d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1
/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
 d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Co
t[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2
- (Tan[(c + d*x)/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/
(2*(1 + Tan[(c + d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])
^(1/3)) + (3*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-1/2*
(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/
2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*(
(-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2) + ((1 + I)*(((1
- I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F
1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/
2])]*Sec[(c + d*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I +
Cot[(c + d*x)/2])*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/
2)*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Ta
n[(c + d*x)/2]))/(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^
(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c +
 d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(
c + d*x)/2])^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I
+ Cot[(c + d*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2
 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 -

```
I)*(I + Cot[(c + d*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))
/((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Co
t[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/3
0)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 -
I/2)*(1 + Cot[(c + d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/
3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[
(c + d*x)/2])]*Csc[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3
)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(
I + Tan[(c + d*x)/2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 +
 I) + (1 - I)*Tan[(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)
*Sec[(c + d*x)/2]^2)/(2 + 2*Tan[(c + d*x)/2]))*...
```

**Maple** [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^2 \left(dx+c\right)\right) \left(a+a\sin \left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{4/3}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x)^2, x)
```

# 3.104 $\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=161

$$-\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\ 2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}+\frac{3}{}$$

```
[Out] -99/80*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)-3/8*cos(d*x+c)*sin(d*x+c)^2/d/(a
+a*sin(d*x+c))^(1/3)+37/80*cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*si
n(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)+3/40*cos(d*
x+c)*(a+a*sin(d*x+c))^(2/3)/a/d
```

**Rubi** [A]
time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2862, 3047, 3102, 2830, 2731, 2730}

$$\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\ 2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{8d\sqrt[3]{a\sin(c+dx)+a}}+\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40ad}-\frac{99\cos(c+dx)}{80d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-99*Cos[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*Sin[
c + d*x]^2)/(8*d*(a + a*Sin[c + d*x])^(1/3)) + (37*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(40*2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) + (3*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(2/3))/(40*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\int \frac{\sin(c+dx)\left(2a-\frac{1}{3}a\sin(c+dx)\right)}{\sqrt[3]{a+a\sin(c+dx)}}dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\int \frac{2a\sin(c+dx)-\frac{1}{3}a\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40ad} + \frac{9\int \frac{-\frac{2a}{9}\cdots}{\sqrt[3]{a\cdots}}}{\cdots}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\cdots}{40ad\cdots}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\cdots}{40ad\cdots}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{37\cos(c+dx)\,{}_2F_1\cdots}{40\ 2^{5/6}d\sqrt[6]{1+\sin(\cdots}}$$

**Mathematica** [A]
time = 0.31, size = 110, normalized size = 0.68

$$\frac{3\cos(c+dx)\left(-37\sqrt{2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{1-\sin(c+dx)}\,(-36+5\cos(2(c+dx))+2\sin(c+dx))\right)}{80d\sqrt{1-\sin(c+dx)}\,\sqrt[3]{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (3*Cos[c + d*x]*(-37*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2] + Sqrt[1 - Sin[c + d*x]]*(-36 + 5*Cos[2*(c + d*x)] + 2*Sin[
c + d*x])))/(80*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

**Maple** [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/3),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/3), x)
```

# 3.105 $\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=126

$$\frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\ \sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}$$

```
[Out] 9/10*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)-7/10*cos(d*x+c)*hypergeom([1/2, 5/
6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c)
)^(1/3)-3/5*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/a/d
```

**Rubi** [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2731, 2730}

$$-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\ \sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5ad}+\frac{9\cos(c+dx)}{10d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (9*Cos[c + d*x])/(10*d*(a + a*Sin[c + d*x])^(1/3)) - (7*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*d*(1 + Sin[c +
 d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*(a + a*Sin[c + d
*x])^(2/3))/(5*a*d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad} + \frac{3\int \frac{\frac{2a}{3}-a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad} + \frac{7}{10}\int \frac{}{\sqrt[3]{a+a}}$$

$$= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad} + \frac{\left(7\sqrt[3]{1+\sin(}\right.}{1}$$

$$= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\ \sqrt[3]{a+a\sin(c+dx)}} - \frac{3}{}$$

**Mathematica [A]**
time = 0.19, size = 95, normalized size = 0.75

$$-\frac{3\cos(c+dx)\left(-14\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{2-2\sin(c+dx)}\,(-1+2\sin(c+dx))\right)}{10d\sqrt{2-2\sin(c+dx)}\,\sqrt[3]{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(-14*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*
x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]*(-1 + 2*Sin[c + d*x])))/(10*d*Sqrt[2 -
2*Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

**Maple [F]**
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c + dx)}{\sqrt[3]{a(\sin(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin \left(c+d\,x\right)^2}{{\left(a+a\,\sin \left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/3), x)
```

# 3.106 $\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=93

$$-\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}+\frac{\cos(c+dx)\,_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -3/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)+1/2*cos(d*x+c)*hypergeom([1/2, 5/6
],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))
^(1/3)
```

**Rubi** [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2830, 2731, 2730}

$$\frac{\cos(c+dx)\,_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(2*d*(a + a*Sin[c + d*x])^(1/3)) + (Cos[c + d*x]*Hypergeo
metric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx\\ &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}-\frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{2\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}+\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.10, size = 84, normalized size = 0.90

$$-\frac{3\cos(c+dx)\left(2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)}{2d\sqrt{2-2\sin(c+dx)}\sqrt[3]{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)
/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]))/(2*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*(1 + S
in[c + d*x]))^(1/3))
```

**Maple** [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c + dx)}{\sqrt[3]{a(\sin(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c + d\,x)}{{(a + a\,\sin(c + d\,x))}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/3),x)
```

```
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/3), x)
```

# 3.107 $\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin
(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1/3),x]
```

```
[Out] -((2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])
/2])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica** [A]
time = 0.07, size = 70, normalized size = 1.06

$$\frac{3\sqrt{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[3]{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1/3),x]
[Out] (3*Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*
d*x)/4]^2])/(d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

**Maple** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(1/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^(-1/3), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a\sin(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-1/3), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(1/3),x)
```

```
[Out] int(1/(a + a*sin(c + d*x))^(1/3), x)
```

# 3.108 $\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -2^(1/6)*AppellF1(1/2,1,5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)
/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\sqrt[6]{2}\,\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 1, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\operatorname{Subst}\left(\int \frac{1}{(1-x)(2-x)^{5/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{5/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica [F]**
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3), x]
```

**Maple [F]**
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/3)),x)
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/3)), x)
```

# 3.109 $\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -2^(1/6)*AppellF1(1/2,2,5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)
/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]
time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 2, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\mathrm{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{5/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\mathrm{Subst}\left(\int \frac{1}{(1-x^2)^2(2-x^2)^{5/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 4.49, size = 184, normalized size = 2.39

$$\frac{2\,2^{2/3}\cos^{\frac{2}{3}}\left(\frac{1}{4}(2c-\pi+2dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(1+4\sin(c+dx)+4i\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)(1+i\cos(c+dx)+\sin(c+dx))^{2/3}\right)}{5d\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\right)^{2/3}\left(1+e^{2i(c+dx)}\right)\sqrt[3]{a(1+\sin(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (2*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(2/3)*(Cos[c + d*x] + I*Sin[c + d*x])*
(1 + 4*Sin[c + d*x] + (4*I)*Cos[c + d*x]*Hypergeometric2F1[1/3, 2/3, 4/3, (
-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])^(2/3)))/(5*d*(-(((
-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c + d*x))))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*(a*(1 + Sin[c + d*x]))^(1/3))
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/3)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/3)), x)
```

# 3.110 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

**Optimal**. Leaf size=162

$$\frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\right.}{ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{}}$$

```
[Out] 6/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-3/5*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*
sin(d*x+c))^(4/3)+6/5*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/3)-2*2^(1/6)*cos(d
*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))/a/d/(1+sin(d*x+c))^(1/
6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]

time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2862, 3047, 3098, 2830, 2731, 2730}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a\sin(c+dx)+a}}+\frac{6\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (6*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x]*Sin[c +
 d*x]^2)/(5*d*(a + a*Sin[c + d*x])^(4/3)) + (6*Cos[c + d*x])/(5*a*d*(a + a*
Sin[c + d*x])^(1/3)) - (2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6,
3/2, (1 - Sin[c + d*x])/2])/(a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*
x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{3\int \frac{\sin(c+dx)\left(2a-\frac{4}{3}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{4/3}}dx}{5a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{3\int \frac{2a\sin(c+dx)-\frac{4}{3}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}dx}{5a}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{9\int \frac{-\frac{40a^2}{9}+\frac{20}{9}a^2\sin(c+}{\sqrt[3]{a+a\sin(c+}}}{25a^3}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+}}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+}}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+}}$$

**Mathematica [A]**
time = 0.33, size = 116, normalized size = 0.72

$$\frac{3\cos(c+dx)\left(20\sqrt{2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)(1+\sin(c+dx))+\sqrt{1-\sin(c+dx)}\,(7+\cos(2(c+dx))+4\sin(c+dx))\right)}{10d\sqrt{1-\sin(c+dx)}\,(a(1+\sin(c+dx)))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x]*(20*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
+ 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + Cos[2*(c +
d*x)] + 4*Sin[c + d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]
))^(4/3))
```

**Maple [F]**
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*
cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(4/3),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(4/3), x)
```

# 3.111 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

**Optimal**. Leaf size=129

$$-\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}+\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\;2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -3/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-3/2*cos(d*x+c)/a/d/(a+a*sin(d*x+c)
)^(1/3)+13/10*2^(1/6)*cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x
+c))/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]

time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2837, 2830, 2731, 2730}

$$\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\;2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x])/(2*a*
d*(a + a*Sin[c + d*x])^(1/3)) + (13*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6
, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a +
a*Sin[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx = -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{3\int \frac{-\frac{4a}{3}+\frac{5}{3}a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a^2}$$

$$= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}} - \frac{13\int \frac{1}{\sqrt[3]{a+a\sin(c+}}$$

$$= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}} - \frac{\left(13\sqrt[3]{1+\sin(c+dx)}\right.}{10a\sqrt[3]{a+}}$$

$$= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}} + \frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2}\right.}{5\,2^{5/6}ad\sqrt[6]{1+\sin(c-}}$$

**Mathematica** [A]
time = 0.21, size = 108, normalized size = 0.84

$$-\frac{3\cos(c+dx)\left(13\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)(1+\sin(c+dx))+\sqrt{1-\sin(c+dx)}\,(7+5\sin(c+dx))\right)}{10d\sqrt{1-\sin(c+dx)}\,(a(1+\sin(c+dx)))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
[Out] (-3*Cos[c + d*x]*(13*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + 5*Sin[c +
d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(4/3))
```

**Maple** [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^
2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(4/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+dx)^2}{(a+a\sin(c+dx))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(4/3), x)
```

# 3.112 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

**Optimal**. Leaf size=99

$$\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{4\sqrt[6]{2}\,\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] 3/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-4/5*2^(1/6)*cos(d*x+c)*hypergeom([1
/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c)
)^(1/3)
```

**Rubi** [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2829, 2731, 2730}

$$\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}-\frac{4\sqrt[6]{2}\,\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (4*2^(1/6)*Cos[c + d*x]
*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*a*d*(1 + Sin[c
+ d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{4\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a}\\ &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{\left(4\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{5a\sqrt[3]{a+a\sin(c+dx)}}\\ &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.19, size = 130, normalized size = 1.31

$$\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{2-2\sin(c+dx)}+8\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)(1+\sin(c+dx))\right)}{5d\sqrt{2-2\sin(c+dx)}\,(a(1+\sin(c+dx)))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(Sqrt[2 - 2*Sin[c + d*x]] + 8*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c
 + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*
(1 + Sin[c + d*x]))^(4/3))
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{{(a+a\,\sin(c+d\,x))}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(4/3), x)
```

# 3.113 $\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx$

**Optimal**. Leaf size=69

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -1/2*cos(d*x+c)*hypergeom([1/2, 11/6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/a/d
/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]

time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2731, 2730}

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Sin[c + d*x])/2])/(2
^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.13, size = 130, normalized size = 1.88

$$-\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{2-2\sin(c+dx)}-2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)(1+\sin(c+dx))\right)}{5d\sqrt{2-2\sin(c+dx)}\,(a(1+\sin(c+dx)))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])*(Sqrt[2 - 2*Sin[c + d*x]] - 2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*
c + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a
*(1 + Sin[c + d*x]))^(4/3))
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(1/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
c) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin{(c+dx)}+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral((a*sin(c + d*x) + a)**(-4/3), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(4/3),x)
[Out] int(1/(a + a*sin(c + d*x))^(4/3), x)
```

# 3.114 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -1/2*AppellF1(1/2,1,11/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)*2^
(1/6)/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 1, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\text{Subst}\left(\int \frac{1}{(1-x)(2-x)^{11/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\text{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{11/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica [F]**
time = 6.22, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3), x]
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3)),x)
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3)), x)
```

# 3.115 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}} \, dx$

**Optimal**. Leaf size=80

$$-\frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

```
[Out] -1/2*AppellF1(1/2,2,11/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)*2^
(1/6)/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

**Rubi** [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 2, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\text{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{11/6}\sqrt{x}}\,dx,x,1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\text{Subst}\left(\int \frac{1}{(1-x^2)^2(2-x^2)^{11/6}}\,dx,x,\sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 6.66, size = 230, normalized size = 2.88

$$\frac{8\,2^{2/3}\cos^{\frac{8}{3}}\left(\frac{1}{4}(2c-\pi+2dx)\right)(\cos(2(c+dx))+i\sin(2(c+dx)))\left(6-14\cos(2(c+dx))+35\sin(c+dx)+14i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)(1+i\cos(c+dx)+\sin(c+dx))^{2/3}(2\cos(c+dx)+\sin(2(c+dx)))\right)}{55d\left(-1+ie^{i(c+dx)}\right)^3\left(-i+e^{i(c+dx)}\right)\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\right)^{2/3}(a(1+\sin(c+dx)))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (8*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(8/3)*(Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)])*(6 - 14*Cos[2*(c + d*x)] + 35*Sin[c + d*x] + (14*I)*Hypergeometric2
F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])
^(2/3)*(2*Cos[c + d*x] + Sin[2*(c + d*x)])))/(55*d*(-1 + I*E^(I*(c + d*x)))
^3*(-I + E^(I*(c + d*x)))*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c
 + d*x))))^(2/3)*(a*(1 + Sin[c + d*x]))^(4/3))
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(4/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3)), x)
```

# 3.116 $\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=96

$$-\frac{2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{1+\sin(e+fx)}}-\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(3+2*n)/(1+
sin(f*x+e))^(1/2)-2*cos(f*x+e)*sin(f*x+e)^(1+n)/f/(3+2*n)/(1+sin(f*x+e))^(1
/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2842, 21, 2855, 67}

$$-\frac{2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]
])/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]]) - (2*Cos[e + f*x]*Sin[e + f*x]^(1 +
 n))/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
```

```
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{2\int \frac{\sin^n(e+fx)\left(\frac{1}{2}(5+4n)+\frac{1}{2}(5+4n)\sin(e+\right.}{\sqrt{1+\sin(e+fx)}}}{3+2n}$$

$$= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\int \sin^n(e+fx)\sqrt{1+\sin}}{3+2n}$$

$$= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{((5+4n)\cos(e+fx))\text{Subst}\left(\int\right.}{f(3+2n)\sqrt{1-\sin(e+f}}$$

$$= -\frac{2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{1+\sin(e+fx)}} - \frac{2\cos(e+}{f(3+2n}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 21.96, size = 5109, normalized size = 53.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sin^n(fx+e)\right)\left(1+\sin(fx+e)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(e + fx) + 1)^{\frac{3}{2}} \sin^n(e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((sin(e + f*x) + 1)**(3/2)*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin \left(e+f\,x\right)^n\,{\left(\sin \left(e+f\,x\right)+1\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(3/2),x)
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(3/2), x)
```

# 3.117 $\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx$

**Optimal**. Leaf size=43

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(1+sin(f*x+e))^(1/2
)
```

**Rubi** [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2855, 67}

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sqrt
[1 + Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{x^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.45, size = 186, normalized size = 4.33

$$\frac{2^{1-n}e^{i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{1+n}\left(i(-1+2n)\,{}_2F_1\left(1,\frac{1}{4}(3+2n);\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)+e^{i(e+fx)}(1+2n)\,{}_2F_1\left(1,\frac{1}{4}(5+2n);\frac{1}{4}(5-2n);e^{2i(e+fx)}\right)\right)\sqrt{1+\sin(e+fx)}}{\left(i+e^{i(e+fx)}\right)f(-1+2n)(1+2n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (2^(1 - n)*E^(I*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x
)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4, E^
((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 + 2*
n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*Sqrt[1 + Sin[e + f*x]])/((I + E^(I
*(e + f*x)))*f*(-1 + 2*n)*(1 + 2*n))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sin^n\left(fx+e\right)\right)\sqrt{1+\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(e + fx) + 1} \sin^n(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e + f\,x\right)^n\,\sqrt{\sin\left(e + f\,x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(1/2),x)
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(1/2), x)
```

# 3.118 $\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(1+sin
(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2864, 129, 440}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx &= -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}\\ &= -\frac{(2\cos(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^n}{2-x^2}\,dx,x,\sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}\\ &= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(58) = 116.
time = 1.37, size = 225, normalized size = 3.88

$$\frac{\cos(e+fx)(-\sin(e+fx))^{-n}\sin^n(e+fx)\sqrt{1+\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(-\sin(e+fx))^n\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}-(1+2n)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^n\right)}{4f(1+2n)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 - n, -
1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[e +
f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*
x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-
Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

**Maple [F]**
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sin^n(fx+e)}{\sqrt{1+\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e + fx\right)}{\sqrt{\sin\left(e + fx\right) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/sqrt(sin(e + f*x) + 1), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\sin\left(e+f\,x\right)}^n}{\sqrt{\sin\left(e+f\,x\right)+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(1/2), x)
```

# 3.119 $\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=60

$$-\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(1
+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2864, 129, 440}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos
[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{(2\cos(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2f\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 263 vs. 2(60) = 120.
time = 2.19, size = 263, normalized size = 4.38

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))^2-\frac{4(1+\sin(e+fx))\sqrt{1-\frac{2}{1+\sin(e+fx)}}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(2(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{-1+4n^2}\right)}{8f\sqrt{1+\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2
, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e
+ f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1 + 2*
n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e
 + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 -
(1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sin^n(fx+e)}{(1+\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/(sin(f*x + e) + 1)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sin(f*x + e)^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin(f*x
 + e) - 2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e + fx\right)}{\left(\sin\left(e + fx\right) + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/(sin(e + f*x) + 1)**(3/2), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\sin\left(e+f\,x\right)}^n}{{\left(\sin\left(e+f\,x\right)+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(3/2), x)
```

# 3.120 $\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=106

$$-\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(3+2*n)
/(a+a*sin(f*x+e))^(1/2)-2*a^2*cos(f*x+e)*sin(f*x+e)^(1+n)/f/(3+2*n)/(a+a*si
n(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2842, 21, 2855, 67}

$$-\frac{2a^2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]])/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sin[e +
 f*x]^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
```

```
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\sin^n(e+fx)\left(\frac{1}{2}a^2(5+4n)+\frac{1}{2}a^2(5-}{\sqrt{a+a\sin(e+}}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(5+4n))\int \sin^n(e+fx)\sqrt{}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\right)\mathrm{Su}}{f(3+2n)\sqrt{a-a\sin}}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2}{f(3}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.24, size = 5111, normalized size = 48.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]
[Out] Result too large to show
```

**Maple [F]**
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sin^n\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+fx)^n \left(a+a\sin(e+fx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(3/2), x)
```

# 3.121 $\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=46

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}}$$

[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)

**Rubi** [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2855, 67}

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]

[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx &= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 2.85, size = 264, normalized size = 5.74

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\left(e^{ifx}(1+2n)\,{}_2F_1\left(\frac{1}{4}(1-2n),-n;\frac{1}{4}(5-2n);e^{2ifx}(\cos(e)+i\sin(e))^2\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)+(-1+2n)\,{}_2F_1\left(\frac{1}{4}(-1-2n),-n;\frac{1}{4}(3-2n);e^{2ifx}(\cos(e)+i\sin(e))^2\right)\left(i\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\right)\left(1-e^{2ifx}\cos^2(e)+e^{2ifx}\sin^2(e)-ie^{2ifx}\sin(2e)\right)^{-n}\sin^n(e+fx)\sqrt{a(1+\sin(e+fx))}}{f(-1+2n)(1+2n)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*Sin[e + f*x]^n*Sqrt[a*(1 + Sin[e + f*
x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2 + E
^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]))
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\sin^n(fx+e)\right)\sqrt{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\sin\left(e+f\,x\right)}^n\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(1/2), x)
```

# 3.122 $\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=60

$$-\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(a+a*s
in(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{(2\cos(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^n}{2-x^2}\,dx,x,\sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(60) = 120.
time = 1.28, size = 234, normalized size = 3.90

$$\frac{\cos(e+fx)\sin^{2n}(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(-\sin(e+fx))^n\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}-(1+2n)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^n\right)}{4af(1+2n)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^(2*n)*Sqrt[a*(1 + Sin[e + f*x])]*(4*AppellF1[-1/
2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-
Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Ap
pellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Si
n[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*n)*(-1 + Sin[e
+ f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

**Maple** [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.123 $\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/a/f/
(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2864, 129, 440}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos
[e + f*x])/(a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{(2\cos(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx,x,\sqrt{1-\sin(e+fx)}\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 274 vs. 2(65) = 130.
time = 1.89, size = 274, normalized size = 4.22

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(a^2F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))^2-\frac{4a(-1+\sin(e+fx))\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(2a(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{(-1+4n^2)\sqrt{1-\frac{2}{1+\sin(e+fx)}}}\right)}{8a^3f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -
1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e
 + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e + f
*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])
])
```

**Maple** [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.124 $\int (d\sin(e+fx))^n(1+\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=130

$$-\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)(3+2n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(1+sin(f*x+e))^(1/2)+(5+4*n)
*cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(d*sin(f*x+e))^(1+n)/d/f
/(1+n)/(3+2*n)/(1-sin(f*x+e))^(1/2)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2842, 21, 2855, 66}

$$\frac{(4n+5)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)(2n+3)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[1 + Sin[e +
f*x]]) + ((5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e
 + f*x]]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(3 + 2*n)*Sqrt[1 - Sin[e +
f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
```

```
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}d(5+4n)+\frac{1}{2}d(}{\sqrt{1+\sin(e+}}}{d(3+2n)}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\int(d\sin(e+fx))^n}{3+2n}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{((5+4n)\cos(e+fx))\text{Subs}}{f(3+2n)\sqrt{1-\sin(}}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{df(1+n)(3+2n)}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.21, size = 5129, normalized size = 39.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
[Out] Result too large to show
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n (\sin(e+fx)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(sin(e + f*x) + 1)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin\left(e+f\,x\right))^n\,\left(\sin\left(e+f\,x\right)+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(3/2),x)
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(3/2), x)
```

# 3.125 $\int (d\sin(e+fx))^n \sqrt{1+\sin(e+fx)}\, dx$

**Optimal**. Leaf size=72

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(d*sin(f*x+e))^(1+n)/d/f/
(1+n)/(1-sin(f*x+e))^(1/2)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2855, 66}

$$\frac{\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e + f*x]]*(d*Sin[e +
 f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\,dx &= \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.24, size = 215, normalized size = 2.99

$$\frac{(1-i)2^{-n}e^{\frac{1}{2}i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{1+n}\left(i(-1+2n)\,{}_2F_1\left(1,\frac{1}{4}(3+2n);\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)+e^{i(e+fx)}(1+2n)\,{}_2F_1\left(1,\frac{1}{4}(5+2n);\frac{1}{4}(5-2n);e^{2i(e+fx)}\right)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}}{f(-1+2n)(1+2n)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] ((1 - I)*E^((I/2)*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f
*x)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4,
E^((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 +
2*n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[
e + f*x]])/(2^n*f*(-1 + 2*n)*(1 + 2*n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[e + f*x]^n)
```

**Maple** [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^n\sqrt{1+\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n \sqrt{\sin(e+fx)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(1/2),x)
[Out] Integral((d*sin(e + f*x))**n*sqrt(sin(e + f*x) + 1), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\sin(e+fx))^n \sqrt{\sin(e+fx)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(1/2),x)
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(1/2), x)
```

# 3.126 $\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=78

$$-\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f
*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2865, 2864, 129, 440}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^n}{2-x^2}\,dx, x, \sqrt{1-\mathrm{si}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 227 vs. 2(78) = 156.
time = 0.28, size = 227, normalized size = 2.91

$$\frac{\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(-\sin(e+fx))^n\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}-(1+2n)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^n\right)}{4f(1+2n)(-1+\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 -
n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[
e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Appell
F1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e
+ f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x]
)*(-Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

**Maple [F]**
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^n}{\sqrt{1+\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/sqrt(sin(e + f*x) + 1), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d \sin(e+f x))^n}{\sqrt{\sin(e+f x)+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(1/2),x)
[Out] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(1/2), x)
```

# 3.127 $\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*s
in(f*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2865, 2864, 129, 440}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos
[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\mathrm{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\mathrm{si}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx, x, \sqrt{1-\mathrm{s}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+f}{2f\sqrt{1+\sin(e+fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(80) = 160.
time = 0.62, size = 265, normalized size = 3.31

$$\frac{\sec(e+fx)(d\sin(e+fx))^n\left(F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))^2-\frac{4(1+\sin(e+fx))\sqrt{1-\frac{2}{1+\sin(e+fx)}}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(2(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{-1+4n^2}\right)}{8f\sqrt{1+\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1
+ 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + S
in[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(
1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^n}{(1+\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/(sin(f*x + e) + 1)^(3/2), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin
(f*x + e) - 2), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin{(e+fx)})^n}{(\sin{(e+fx)}+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/(sin(e + f*x) + 1)**(3/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\sin\left(e+f\,x\right))^n}{{\left(\sin\left(e+f\,x\right)+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(3/2),x)
[Out] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(3/2), x)
```

# 3.128 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=131

$$-\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*
x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*cos(f*x+e)*(d
*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 21, 2855, 69, 67}

$$-\frac{2a^2(4n+5)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*
x]]) - (2*a^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}a^2d(5+\ldots}{\sqrt{a+a\ldots}}}{d(3\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(5+4n))\int (d\sin(e\ldots}{\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a^3(5+4n)\cos(e+f\ldots}{f(3+2n)\sqrt{a-\ldots}}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a^3(5+4n)\cos(e+f\ldots}{\ldots}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{\ldots}}{f(3+2n)\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 6.23, size = 5131, normalized size = 39.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int (d \sin (fx+e))^n (a+a \sin (fx+e))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^{\frac{3}{2}} (d \sin (e+fx))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin\left(e+f\,x\right))^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(3/2),x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(3/2), x)
```

# 3.129 $\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}\, dx$

**Optimal**. Leaf size=66

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/
(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2855, 69, 67}

$$-\frac{2a\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin
[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int\frac{x}{\sqrt{a-}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.31, size = 266, normalized size = 4.03

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\left(e^{ifx}(1+2n)\,{}_2F_1\left(\frac{1}{4}(1-2n),-n;\frac{1}{4}(5-2n);e^{2ifx}(\cos(e)+i\sin(e))^2\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)+(-1+2n)\,{}_2F_1\left(\frac{1}{4}(-1-2n),-n;\frac{1}{4}(3-2n);e^{2ifx}(\cos(e)+i\sin(e))^2\right)\left(i\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\right)\left(1-e^{2ifx}\cos^2(e)+e^{2ifx}\sin^2(e)-ie^{2ifx}\sin(2e)\right)^{-n}(d\sin(e+fx))^n\sqrt{a(1+\sin(e+fx))}}{f(-1+2n)(1+2n)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e
+ f*x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2
 + E^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]))
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^n\sqrt{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(1/2), x)
```

# 3.130 $\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f
*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2866, 2865, 2864, 129, 440}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\mathrm{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\,\mathrm{Subst}\left(\int \frac{(1-x^2)^n}{2-x^2}\,dx, x, \sqrt{1-\mathrm{si}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(80) = 160.
time = 0.48, size = 242, normalized size = 3.02

$$\frac{\cos(e+fx)\sin^n(e+fx)(d\sin(e+fx))^n\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(-\sin(e+fx))^n\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}-(1+2n)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^n\right)}{4af(1+2n)(-1+\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
 f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]
 - (1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]
]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*
n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

**Maple** [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (fx+e))^{n}}{\sqrt{a+a \sin (fx+e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (e+fx))^{n}}{\sqrt{a (\sin (e+fx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\sin\left(e+f\,x\right))^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.131 $\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=85

$$-\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{2af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*s
in(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2866, 2865, 2864, 129, 440}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos
[e + f*x]*(d*Sin[e + f*x])^n)/(a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}\\
&= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\operatorname{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\mathrm{s}\ldots\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n)\operatorname{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx, x, \sqrt{1-\ldots}\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+f\ldots}{2af\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 276 vs. 2(85) = 170.

time = 0.88, size = 276, normalized size = 3.25

$$
\frac{\sec(e+fx)(d\sin(e+fx))^n\left(a^2F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))^2-\frac{4a(-1+\sin(e+fx))\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(2a(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{(-1+4n^2)\sqrt{1-\frac{2}{1+\sin(e+fx)}}}\right)}{8a^3f\sqrt{a(1+\sin(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e +
 f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/
(-Sin[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 -
n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-
1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + S
in[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e
 + f*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f
*x])])
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (fx+e))^{n}}{(a+a \sin (fx+e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (e+fx))^{n}}{(a (\sin (e+fx)+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\sin\left(e+f\,x\right))^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.132 $\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=71

$$-\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)/f/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2864, 138}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx = -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}; -n, \frac{1}{2}-m; \frac{3}{2}; 1-\sin(e+fx), \frac{1}{2}(1-\sin(e+fx))\right)\cos(}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 991 vs. 2(71) = 142.
time = 11.89, size = 991, normalized size = 13.96

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (15*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*
e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(1 + Sin[e + f*x])^m)/(f*
(15*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(
2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[e + f*x] + 10*(n*AppellF1[3/2, 1 -
 n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]
^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/
4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2 + (9*Appel
lF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi +
2*f*x)/4]^2]*Cos[e + f*x]*Cot[(2*e + Pi + 2*f*x)/4]*(-5*n*AppellF1[3/2, 1 -
 n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]
^2] - 5*(1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x
)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + 2*(2*n*(1 + m + n)*AppellF1[5/2, 1
- n, 2 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4
]^2] + (-1 + n)*n*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (2 + m^2 + 3*n + n^2 + m*(3 + 2*n)
)*AppellF1[5/2, -n, 3 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e
- Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2)*Csc[(2*e + Pi + 2*f*x)/4]
^2)/(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[
(2*e - Pi + 2*f*x)/4]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Cot[(2
*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1
[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f
*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2) - 15*AppellF1[1/2, -n, 1 + m + n, 3
/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[e + f*x]
 + 30*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan
[(2*e - Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sin^{n}\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(e+fx\right)+1\right)^{m}\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**m,x)
```

```
[Out] Integral((sin(e + f*x) + 1)**m*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (e+f\,x)^n\,{\left (\sin (e+f\,x)+1\right )}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^m,x)
```

```
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^m, x)
```

# 3.133 $\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=68

$$\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)/f/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2864, 138}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 - Sin[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1+\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}; -n, \frac{1}{2}-m; \frac{3}{2}; 1+\sin(e+fx), \frac{1}{2}(1+\sin(e+fx))\right)\text{ co}}{f\sqrt{1-\sin(e+fx)}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(68) = 136.
time = 1.53, size = 300, normalized size = 4.41

$$-\frac{(3+2m)F_1\left(\frac{1}{2}+m; -n, 1+m+n; \frac{3}{2}+m; \cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right), -\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos(e+fx)(1-\sin(e+fx))^m(-\sin(e+fx))^n}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m; -n, 1+m+n; \frac{3}{2}+m; \cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right), -\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right) - 2\left(nF_1\left(\frac{3}{2}+m; 1-n, 1+m+n; \frac{5}{2}+m; \cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right), -\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right) + (1+m+n)F_1\left(\frac{3}{2}+m; -n, 2+m+n; \frac{5}{2}+m; \cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right), -\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(-
Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3
/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*A
ppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2
*e - Pi + 2*f*x)/4]^2)))
```

**Maple** [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int (1-\sin(fx+e))^m(-\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
```

```
[Out] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sin(e+fx))^n (1-\sin(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(-sin(f*x+e))**n,x)
[Out] Integral((-sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (-\sin(e+f\,x))^n\,(1-\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(e + f*x))^n*(1 - sin(e + f*x))^m,x)
[Out] int((-sin(e + f*x))^n*(1 - sin(e + f*x))^m, x)
```

# 3.134 $\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=91

$$-\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2865, 2864, 138}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin
[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \sin^n(e+fx)(1+\sin(e+fx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n(2\cdot}{\sqrt{}}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 993 vs. 2(91) = 182.
time = 2.68, size = 993, normalized size = 10.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (15*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*
e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m)
/(f*(15*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[e + f*x] + 10*(n*AppellF1[3/2,
 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x
)/4]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f
*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2 + (9*A
ppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - P
i + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[(2*e + Pi + 2*f*x)/4]*(-5*n*AppellF1[3/2,
 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x
)/4]^2] - 5*(1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2
*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + 2*(2*n*(1 + m + n)*AppellF1[5/2
, 1 - n, 2 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*
x)/4]^2] + (-1 + n)*n*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Cot[(2*e + Pi +
2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (2 + m^2 + 3*n + n^2 + m*(3 +
2*n))*AppellF1[5/2, -n, 3 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(
2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2)*Csc[(2*e + Pi + 2*f*x
)/4]^2)/(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -
Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Co
t[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*Appe
```

```
llF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi +
 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2) - 15*AppellF1[1/2, -n, 1 + m +
n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[e +
f*x] + 30*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2,
-Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n} \left(1+\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(e+fx\right)\right)^{n} \left(\sin\left(e+fx\right)+1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**m,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(sin(e + f*x) + 1)**m, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\sin (e+f\,x))^n\,(\sin (e+f\,x)+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^m,x)
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^m, x)
```

# 3.135 $\int (1-\sin(e+fx))^m (d\sin(e+fx))^n \, dx$

**Optimal**. Leaf size=90

$$\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(d*sin(f*x+e))^n/f/((-sin(f*x+e))^n)/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2865, 2864, 138}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sqrt[1 - Sin[e + f*x]]*(-Sin[
e + f*x])^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int (1-\sin(e+fx))^m(d\sin(e+fx))^n\,dx = \left((-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\int (1-\sin(e+fx))^m(-\sin$$

$$= \frac{\left(\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{(1-x)^n}{}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\mathrm{c}}{f\sqrt{1-\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(90) = 180.
time = 0.44, size = 300, normalized size = 3.33

$$-\frac{(3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos(e+fx)(1-\sin(e+fx))^m(d\sin(e+fx))^n}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)-2\left(nF_1\left(\frac{3}{2}+m;1-n,1+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+(1+m+n)F_1\left(\frac{3}{2}+m;-n,2+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(d
*Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*
AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -
Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(
2*e - Pi + 2*f*x)/4]^2)))
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (1-\sin(fx+e))^m\,(d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
```

```
[Out] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n (1-\sin(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(d*sin(f*x+e))**n,x)
[Out] Integral((d*sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin(e+f\,x))^n\,(1-\sin(e+f\,x))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(1 - sin(e + f*x))^m,x)
[Out] int((d*sin(e + f*x))^n*(1 - sin(e + f*x))^m, x)
```

# 3.136 $\int \sin^n(e+fx)(a+a\sin(e+fx))^m \, dx$

**Optimal**. Leaf size=87

$$-\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin}{f}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2866, 2864, 138}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])
^m)/f)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int \sin^n(e+fx)(1+\sin$$

$$= -\frac{\left(\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m\right)\text{Subst}}{f\sqrt{1-\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\text{co}}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 993 vs. 2(87) = 174.
time = 2.58, size = 993, normalized size = 11.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (15*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*
e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(a*(1 + Sin[e + f*x]))^m)
/(f*(15*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[e + f*x] + 10*(n*AppellF1[3/2,
 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x
)/4]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f
*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2 + (9*A
ppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - P
i + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[(2*e + Pi + 2*f*x)/4]*(-5*n*AppellF1[3/2,
 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x
)/4]^2] - 5*(1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2
*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + 2*(2*n*(1 + m + n)*AppellF1[5/2
, 1 - n, 2 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*
x)/4]^2] + (-1 + n)*n*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Cot[(2*e + Pi +
2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (2 + m^2 + 3*n + n^2 + m*(3 +
2*n))*AppellF1[5/2, -n, 3 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(
2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2)*Csc[(2*e + Pi + 2*f*x
)/4]^2)/(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -
Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Co
t[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*Appe
```

```
llF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi +
 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2) - 15*AppellF1[1/2, -n, 1 + m +
n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[e +
f*x] + 30*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2,
-Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int (\sin^n(fx+e))\,(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m \sin^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*sin(e + f*x)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+f\,x)^n\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^m,x)
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^m, x)
```

# 3.137 $\int (-\sin(e+fx))^n (a-a\sin(e+fx))^m \, dx$

**Optimal**. Leaf size=85

$$\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(a-a\sin(e+fx))^m}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(1-sin(f*x+e))^(-1/2-m)*(a-a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2866, 2864, 138}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(a-a\sin(e+fx))^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(a - a*Sin[e + f*x])^m
)/f
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx)$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(a-a\sin(e+fx))^m\right)\text{Sub}}{f\sqrt{1+\sin(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(85) = 170.
time = 0.22, size = 301, normalized size = 3.54

$$-\frac{(3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos(e+fx)(-\sin(e+fx))^n(a-a\sin(e+fx))^m}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)-2\left(nF_1\left(\frac{3}{2}+m;1-n,1+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+(1+m+n)F_1\left(\frac{3}{2}+m;-n,2+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(-Sin[e + f*x])^n*(a -
a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n
*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
-Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n
, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[
(2*e - Pi + 2*f*x)/4]^2)))
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(-\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

```
[Out] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int(-\sin(e+fx))^n(-a(\sin(e+fx)-1))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)
[Out] Integral((-sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int(-\sin(e+f\,x))^n\,(a-a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(e + f*x))^n*(a - a*sin(e + f*x))^m,x)
[Out] int((-sin(e + f*x))^n*(a - a*sin(e + f*x))^m, x)
```

# 3.138 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=107

$$-\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1}{f}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(sin(f*
x+e)^n)
```

**Rubi** [A]

time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2866, 2865, 2864, 138}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(
a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^n$$

$$= \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-m}(a+a\sin(\right.$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx)\right.}{f}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx)\right.}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 995 vs. 2(107) = 214.
time = 2.69, size = 995, normalized size = 9.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (15*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*
e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x])
)^m)/(f*(15*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2
, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[e + f*x] + 10*(n*AppellF1[
3/2, 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2
*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi +
 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2 +
(9*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e
 - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*Cot[(2*e + Pi + 2*f*x)/4]*(-5*n*AppellF1[
3/2, 1 - n, 1 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2
```

```
*f*x)/4]^2] - 5*(1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi
 + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + 2*(2*n*(1 + m + n)*AppellF1
[5/2, 1 - n, 2 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi +
2*f*x)/4]^2] + (-1 + n)*n*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Cot[(2*e + P
i + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (2 + m^2 + 3*n + n^2 + m*(
3 + 2*n))*AppellF1[5/2, -n, 3 + m + n, 7/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2)*Csc[(2*e + Pi + 2
*f*x)/4]^2)/(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^
2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2
, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*
AppellF1[3/2, -n, 2 + m + n, 5/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e -
Pi + 2*f*x)/4]^2])*Cot[(2*e + Pi + 2*f*x)/4]^2) - 15*AppellF1[1/2, -n, 1 +
m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[
e + f*x] + 30*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Cot[(2*e + Pi + 2*f*x)/4]
^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (d\sin(e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m,x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m, x)
```

# 3.139 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=107

$$\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^n}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(1-sin(f*x+e))^(-1/2-m)*(d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m/f/((-sin(f
*x+e))^n)
```

**Rubi** [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2866, 2865, 2864, 138}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(d*Sin[e + f*x])^n*(a
- a*Sin[e + f*x])^m)/(f*(-Sin[e + f*x])^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx$$

$$= \left((1-\sin(e+fx))^{-m}(-\sin(e+fx))^{-n}(d\sin(e+fx))^n(a-a$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(-\sin(e+fx))^{-n}(d\sin(e\right.}{f}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(107) = 214.
time = 0.24, size = 301, normalized size = 2.81

$$-\frac{(3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos(e+fx)(d\sin(e+fx))^n(a-a\sin(e+fx))^m}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m;-n,1+m+n;\frac{3}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)-2\left(nF_1\left(\frac{3}{2}+m;1-n,1+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+(1+m+n)F_1\left(\frac{3}{2}+m;-n,2+m+n;\frac{5}{2}+m;\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -((((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(a -
 a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n
, 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(
n*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
 -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m +
n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan
[(2*e - Pi + 2*f*x)/4]^2)))
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^n\,(a-a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n(-a(\sin(e+fx)-1))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (d \sin (e + f x))^n (a - a \sin (e + f x))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a - a*sin(e + f*x))^m,x)
[Out] int((d*sin(e + f*x))^n*(a - a*sin(e + f*x))^m, x)
```

# 3.140 $\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=294

$$\frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)}-\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)}-\frac{\cos(c+dx)\,\mathrm{si}}{}$$

```
[Out] (-n^2-n+9)*cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^4+10*n^3+35*n^2+50*n+24)-n*co
s(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^n/d/(3+n)/(4+n)-cos(d*x+c)*sin(d*x+c
)^3*(a+a*sin(d*x+c))^n/d/(4+n)-2^(1/2+n)*(n^4+6*n^3+17*n^2+12*n+9)*cos(d*x+
c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)
*(a+a*sin(d*x+c))^n/d/(4+n)/(n^3+6*n^2+11*n+6)-(n^2+3*n+9)*cos(d*x+c)*(a+a*
sin(d*x+c))^(1+n)/a/d/(4+n)/(n^2+5*n+6)
```

**Rubi** [A]
time = 0.35, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2862, 3062, 3047, 3102, 2830, 2731, 2730}

$$-\frac{2^{n+\frac{1}{2}}(n^4+6n^3+17n^2+12n+9)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)(n+4)}+\frac{(-n^2-n+9)\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+1)(n+2)(n+3)(n+4)}-\frac{(n^2+3n+9)\cos(c+dx)(a\sin(c+dx)+a)^{n+1}}{ad(n+2)(n+3)(n+4)}-\frac{\sin^3(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+4)}-\frac{n\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] ((9 - n - n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 +
 n)*(4 + n)) - (n*Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3
 + n)*(4 + n)) - (Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n)/(d*(4
 + n)) - (2^(1/2 + n)*(9 + 12*n + 17*n^2 + 6*n^3 + n^4)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1
/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n)*(4 + n)) - ((9 +
 3*n + n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n)*(3 + n)
*(4 + n))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^n}{d(4+n)} + \frac{\int \sin^2(c+dx)(a+$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(}{}$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(}{}$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(}{}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c}{d(3-}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c}{d(3-}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c}{d(3-}$$

**Mathematica** [**F**]
time = 180.09, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] $Aborted
```

**Maple** [**F**]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(\sin^4\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sin(d*x + c) + a)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n \sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x)**4, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^n, x)
```

# 3.141 $\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=215

$$-\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)}-\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)}-\frac{2^{\frac{1}{2}+n}n(5+3n+n^2$$

```
[Out] -(4+n)*cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^3+6*n^2+11*n+6)-cos(d*x+c)*sin(d*
x+c)^2*(a+a*sin(d*x+c))^n/d/(3+n)-2^(1/2+n)*n*(n^2+3*n+5)*cos(d*x+c)*hyperg
eom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin
(d*x+c))^n/d/(n^3+6*n^2+11*n+6)-n*cos(d*x+c)*(a+a*sin(d*x+c))^(1+n)/a/d/(n^
2+5*n+6)
```

**Rubi** [A]
time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2862, 3047, 3102, 2830, 2731, 2730}

$$-\frac{2^{n+\frac{1}{2}}n(n^2+3n+5)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)}-\frac{n\cos(c+dx)(a\sin(c+dx)+a)^{n+1}}{ad\,(n^2+5n+6)}-\frac{\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+3)}-\frac{(n+4)\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(((4 + n)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n))
) - (Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3 + n)) - (2^(
1/2 + n)*n*(5 + 3*n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/(d*(1 + n)*(2 + n)*(3 + n)) - (n*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 +
n))/(a*d*(6 + 5*n + n^2))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} + \frac{\int \sin(c+dx)(a+$$
$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} + \frac{\int (a+a\sin(c+dx$$
$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} - \frac{n\cos(c+dx)(a+}{ad\,(6+5}$$
$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx}{d(3+}$$
$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx}{d(3+}$$
$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx}{d(3+}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 120.76, size = 59941, normalized size = 278.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^n, x)
```

# 3.142 $\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=156

$$\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\,(2+3n+n^2)}-\frac{2^{\frac{1}{2}+n}(1+n+n^2)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin($$

```
[Out] cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^2+3*n+2)-2^(1/2+n)*(n^2+n+1)*cos(d*x+c)*
hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a
+a*sin(d*x+c))^n/d/(n^2+3*n+2)-cos(d*x+c)*(a+a*sin(d*x+c))^(1+n)/a/d/(2+n)
```

**Rubi** [A]
time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2838, 2830, 2731, 2730}

$$-\frac{2^{n+\frac{1}{2}}(n^2+n+1)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)}+\frac{\cos(c+dx)(a\sin(c+dx)+a)^n}{d\,(n^2+3n+2)}-\frac{\cos(c+dx)(a\sin(c+dx)+a)^{n+1}}{ad(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(2 + 3*n + n^2)) - (2^(1/2 + n)*(1
 + n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(
2 + n)) - (Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)} + \frac{\int (a(1+n)-a\sin(c+dx)}{a(2+}$$
$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\,(2+3n+n^2)} - \frac{\cos(c+dx)(a+a\sin(c+dx))^1}{ad(2+n)}$$
$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\,(2+3n+n^2)} - \frac{\cos(c+dx)(a+a\sin(c+dx))^1}{ad(2+n)}$$
$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\,(2+3n+n^2)} - \frac{2^{\frac{1}{2}+n}(1+n+n^2)\cos(c+dx)\,{}_2}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 49.87, size = 28439, normalized size = 182.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n \sin^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^n, x)
```

# 3.143 $\int \sin(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=109

$$-\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)}-\frac{2^{\frac{1}{2}+n}n\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))}{d(1+n)}$$

```
[Out] -cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(1+n)-2^(1/2+n)*n*cos(d*x+c)*hypergeom([1/
2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c)
)^n/d/(1+n)
```

**Rubi** [A]

time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2830, 2731, 2730}

$$-\frac{2^{n+\frac{1}{2}}n\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)}-\frac{\cos(c+dx)(a\sin(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n))) - (2^(1/2 + n)*n*Cos[c
 + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin
[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)}+\frac{n\int(a+a\sin(c+dx))^n\,dx}{1+n}$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)}+\frac{(n(1+\sin(c+dx))^{-n}(a+a\sin \cdots}{1 \cdots}$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)}-\frac{2^{\frac{1}{2}+n}n\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\cdots\right.}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.42, size = 178, normalized size = 1.63

$$-\frac{\sqrt[4]{-1}\,2^{-1-2n}e^{-\frac{3}{2}i(c+dx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(i+e^{i(c+dx)}\right)\right)^{1+2n}\left(e^{2i(c+dx)}(-1+n)\,{}_2F_1\left(1,n;-n;-ie^{-i(c+dx)}\right)-(1+n)\,{}_2F_1\left(1,2+n;2-n;-ie^{-i(c+dx)}\right)\right)(a(1+\sin(c+dx)))^n\sin^{-2n}\left(\frac{1}{4}(2c+\pi+2dx)\right)}{d(-1+n)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(((-1)^(1/4)*2^(-1 - 2*n)*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(
c + d*x))))^(1 + 2*n)*(E^((2*I)*(c + d*x))*(-1 + n)*Hypergeometric2F1[1, n,
 -n, (-I)/E^(I*(c + d*x))] - (1 + n)*Hypergeometric2F1[1, 2 + n, 2 - n, (-I
)/E^(I*(c + d*x))])*(a*(1 + Sin[c + d*x]))^n)/(d*E^(((3*I)/2)*(c + d*x))*(-
1 + n)*(1 + n)*Sin[(2*c + Pi + 2*d*x)/4]^(2*n)))
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^n \sin (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (c+d\,x)\,\left(a+a\,\sin (c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^n, x)
```

# 3.144 $\int (a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=74

$$-\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n}{d}$$

```
[Out] -2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+s
in(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

**Rubi** [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2731, 2730}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int (1+\sin(c+dx))^n\,dx$$
$$= -\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a}{d}$$

**Mathematica** [A]
time = 0.12, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c-\pi+2dx)\right)\right)(a(1+\sin(c+dx)))^n}{(d+2dn)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(a*(1 + Sin[c + d*x]))^n)/((d + 2*d*n)*
Sqrt[1 - Sin[c + d*x]])
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^n,x)
```

```
[Out] int((a+a*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(c+d\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n,x)
```

```
[Out] int((a + a*sin(c + d*x))^n, x)
```

# 3.145 $\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=85

$$-\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n}{d}$$

```
[Out] -2^(1/2+n)*AppellF1(1/2,1,1/2-n,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*
x+c)*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

**Rubi** [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^nF_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 1, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc(c+dx)(1+\sin(c+$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\int\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+}{d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 12.47, size = 2203, normalized size = 25.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -1/2*(Csc[c + d*x]*(a + a*Sin[c + d*x])^n*(AppellF1[2*n, n, n, 1 + 2*n, (-1
 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)
/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(
(I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n - AppellF
1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 +
Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c +
Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*
x)/2]))^n))/(d*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1/2*(n*Tan[(-c + Pi/2 - d*x
)/2]*(AppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])
, (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2]
)/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 +
Tan[(-c + Pi/2 - d*x)/2]))^n - AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Ta
```

```
n[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan
[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi
/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n))/(Sec[(-c + Pi/2 - d*x)/2]
^2)^n + ((1/4 + I/4)*Cos[(-c + Pi/2 - d*x)/2]^4*Csc[c + d*x]^2*(I*n*(1 + 2*
n)*AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1
+ I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2*(-1 + Tan[(
-c + Pi/2 - d*x)/2])^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2
 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2
]))^(-1 + n) - n*(1 + 2*n)*AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-
c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2
 - d*x)/2]^2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2*((-I + Tan[(-c + Pi/2 - d*x)
/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(-c + Pi/2 - d*x)/2
])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n + 2*n^2*(AppellF1[1 + 2*n, n, 1 + n, 2
 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2
 - d*x)/2])] - I*AppellF1[1 + 2*n, 1 + n, n, 2 + 2*n, (1 - I)/(1 + Tan[(-c
+ Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])])*Sec[(-c + Pi/2
- d*x)/2]^2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2*((-I + Tan[(-c + Pi/2 - d*x)/
2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 +
Tan[(-c + Pi/2 - d*x)/2]))^n - n*(1 + 2*n)*AppellF1[2*n, n, n, 1 + 2*n, (-1
 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)
/2])]*Sec[(-c + Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan
[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + P
i/2 - d*x)/2]))^(-1 + n)*(1 + Tan[(-c + Pi/2 - d*x)/2])^2 + I*n*(1 + 2*n)*A
ppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 +
 I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2*((-I + Tan[
(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(
-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2
 - d*x)/2])^2 + 2*n^2*((-I)*AppellF1[1 + 2*n, n, 1 + n, 2*(1 + n), (-1 - I)
/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]
 + AppellF1[1 + 2*n, 1 + n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2 - d
*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])])*Sec[(-c + Pi/2 - d*x)/2
]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I
 + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-
c + Pi/2 - d*x)/2])^2))/((1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n)))
```

**Maple** [F]

time = 0.14, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n \csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**n,x)
[Out] Integral((a*(sin(c + d*x) + 1))**n*csc(c + d*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^n}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^n/sin(c + d*x), x)
```

# 3.146 $\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=85

$$-\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n}{d}$$

```
[Out] -2^(1/2+n)*AppellF1(1/2,2,1/2-n,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*
x+c)*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

**Rubi** [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2866, 2864, 129, 440}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^nF_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 2, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc^2(c+dx)(1+\sin(c$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\Big(\int}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\Big(}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+}{d}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 15.84, size = 4206, normalized size = 49.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n
, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x
)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c
 + Pi/2 - d*x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 +
Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I +
 Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c
 + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 -
 d*x)/2])))/(d*(1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/
2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])*(-1/2*((Sec[(-c + Pi/2 - d*x)/2
]^2)^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/
2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2
```

```
- d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))
^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n) - App
ellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (
-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(
-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan
[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/((1 + 2*n)*(-1 +
 Tan[(-c + Pi/2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) - ((Sec[(-c +
Pi/2 - d*x)/2]^2)^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 +
Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Ta
n[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/
2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/
2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2
- d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2
 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/
2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/(2*
(1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2*(1 + Tan[(-c + Pi/2 - d*x)/2]))
 - (n*Tan[(-c + Pi/2 - d*x)/2]*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/
(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1
 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c
+ Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 -
d*x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c +
Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c +
 Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 -
d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2]))
)/((1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/2 - d*x)/2])
*(1 + Tan[(-c + Pi/2 - d*x)/2])) + (-1/2*(AppellF1[1 + 2*n, n, n, 2*(1 + n)
, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 -
 d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-
c + Pi/2 - d*x)/2]))^n) - (AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Ta
n[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c +
Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x
)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n)
/2 - (((1/4 - I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, n, 1 + n, 1 + 2*(1 + n), (
-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*
x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2]
)^2) + ((1/4 + I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, 1 + n, n, 1 + 2*(1 + n),
(-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d
*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2
])^2))*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*
((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan
[(-c + Pi/2 - d*x)/2]) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1
+ Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I
 + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I
+ Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c
```

```
 + Pi/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + Pi/2 - d*
x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(-I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(
-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (
-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*
x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n
*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*
(1 + Tan[(-c + Pi/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c
 + Pi/2 - d*x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(-c + Pi/2 - d*x
)/2]))/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - (-1 + Tan[(-c + Pi/2 - d*x)
/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[...
```

**Maple** [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sin\left(c+dx\right)+1)\right)^n\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(c + d\,x))^n}{\sin(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^n/sin(c + d*x)^2, x)
```

# 3.147 $\int (1+\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=58

$$-\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{1+\sin(c+dx)}}$$

```
[Out] -2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))/d/(1
+sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2730\}$

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2])/(d*Sqrt[1 + Sin[c + d*x]]))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1+\sin(c+dx))^n\,dx = -\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{1+\sin(c+dx)}}$$

**Mathematica** [A]
time = 0.09, size = 88, normalized size = 1.52

$$\frac{\sqrt{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c-\pi+2dx)\right)\right)(1+\sin(c+dx))^n}{(d+2dn)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 + Sin[c + d*x])^n)/((d + 2*d*n)*Sqrt
[1 - Sin[c + d*x]])
```

**Maple [F]**
time = 0.06, size = 0, normalized size = 0.00

$$\int (1+\sin(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(d*x+c))^n,x)
```

```
[Out] int((1+sin(d*x+c))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((sin(d*x + c) + 1)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(c+dx)+1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))**n,x)
```

```
[Out] Integral((sin(c + d*x) + 1)**n, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (\sin (c+d\,x)+1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x) + 1)^n,x)
[Out] int((sin(c + d*x) + 1)^n, x)
```

# 3.148 $\int (1-\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=57

$$\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{1-\sin(c+dx)}}$$

```
[Out] 2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2+1/2*sin(d*x+c))/d/(1-
sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2730}

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[c +
 d*x])/2])/(d*Sqrt[1 - Sin[c + d*x]])
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1-\sin(c+dx))^n\,dx = \frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{1-\sin(c+dx)}}$$

**Mathematica** [A]
time = 0.07, size = 90, normalized size = 1.58

$$\frac{\cos(c+dx)\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)^{-\frac{1}{2}-n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c-\pi+2dx)\right)\right)(1-\sin(c+dx))^n}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*(Cos[(2*c + Pi + 2*d*x)/4]^2)^(-1/2 - n)*Hypergeometric2F1[1/
2, 1/2 - n, 3/2, (Cos[c + d*x]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 - Sin[c
 + d*x])^n)/d
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (1-\sin (dx+c))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(d*x+c))^n,x)
```

```
[Out] int((1-sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((-sin(d*x + c) + 1)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (1-\sin (c+dx))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))**n,x)
```

```
[Out] Integral((1 - sin(c + d*x))**n, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (1-\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin(c + d*x))^n,x)
[Out] int((1 - sin(c + d*x))^n, x)
```

# 3.149 $\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=77

$$\frac{3bx}{8}-\frac{a\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}-\frac{3b\cos(e+fx)\sin(e+fx)}{8f}-\frac{b\cos(e+fx)\sin^3(e+fx)}{4f}$$

```
[Out] 3/8*b*x-a*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f-3/8*b*cos(f*x+e)*sin(f*x+e)/f-1
/4*b*cos(f*x+e)*sin(f*x+e)^3/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2827, 2713, 2715, 8}

$$\frac{a\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)}{f}-\frac{b\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{3b\sin(e+fx)\cos(e+fx)}{8f}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[e + f*x])/f + (a*Cos[e + f*x]^3)/(3*f) - (3*b*Cos[e + f*
x]*Sin[e + f*x])/(8*f) - (b*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx = a\int \sin^3(e+fx)\,dx + b\int \sin^4(e+fx)\,dx$$
$$= -\frac{b\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}(3b)\int \sin^2(e+fx)\,dx - \frac{a\text{Subst}(\ldots}{\ldots}$$
$$= -\frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx)}{8f} - \ldots$$
$$= \frac{3bx}{8} - \frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx\ldots}{8f}$$

**Mathematica [A]**
time = 0.11, size = 76, normalized size = 0.99

$$\frac{3b(e+fx)}{8f} - \frac{3a\cos(e+fx)}{4f} + \frac{a\cos(3(e+fx))}{12f} - \frac{b\sin(2(e+fx))}{4f} + \frac{b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*(e + f*x))/(8*f) - (3*a*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])/(12
*f) - (b*Sin[2*(e + f*x)])/(4*f) + (b*Sin[4*(e + f*x)])/(32*f)
```

**Maple [A]**
time = 0.23, size = 60, normalized size = 0.78

| method | result |
|---|---|
| derivativedivides | $\frac{b\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}}{f}$ |
| default | $\frac{b\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}}{f}$ |
| risch | $\frac{3bx}{8}-\frac{3a\cos(fx+e)}{4f}+\frac{b\sin(4fx+4e)}{32f}+\frac{a\cos(3fx+3e)}{12f}-\frac{b\sin(2fx+2e)}{4f}$ |
| norman | $\frac{\frac{3bx}{8}-\frac{4a}{3f}-\frac{3b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{11b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{11b\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{3b\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{3bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{9bx\left(\tan^4\left(\ldots\right)\right)}{4}\ldots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a*
(2+sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima** [A]
time = 0.27, size = 62, normalized size = 0.81

$$\frac{32\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)a+3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)b}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a + 3*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*b)/f
```

**Fricas** [A]
time = 0.50, size = 65, normalized size = 0.84

$$\frac{8\,a\cos\left(fx+e\right)^{3}+9\,bfx-24\,a\cos\left(fx+e\right)+3\left(2\,b\cos\left(fx+e\right)^{3}-5\,b\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(f*x + e)^3 + 9*b*f*x - 24*a*cos(f*x + e) + 3*(2*b*cos(f*x + e
)^3 - 5*b*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 0.18, size = 144, normalized size = 1.87

$$\begin{cases}-\frac{a\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a\cos^{3}\left(e+fx\right)}{3f}+\frac{3bx\sin^{4}\left(e+fx\right)}{8}+\frac{3bx\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+\frac{3bx\cos^{4}\left(e+fx\right)}{8}-\frac{5b\sin^{3}\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{3b\sin\left(e+fx\right)\cos^{3}\left(e+fx\right)}{8f} & \text{for}\ f\neq 0\\ x(a+b\sin\left(e\right))\sin^{3}\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*cos(e + f*x)**3/(3*f) +
3*b*x*sin(e + f*x)**4/8 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b*x*c
os(e + f*x)**4/8 - 5*b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**3, True))
```

**Giac** [A]
time = 0.46, size = 66, normalized size = 0.86

$$\frac{3}{8}\,bx+\frac{a\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,a\cos\left(fx+e\right)}{4\,f}+\frac{b\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{b\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*f*x + 3*e)/f - 3/4*a*cos(f*x + e)/f + 1/32*b*sin(4*f
*x + 4*e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 10.29, size = 111, normalized size = 1.44

$$\frac{3\,b\,x}{8}-\frac{-\frac{3\,b\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{4}-\frac{11\,b\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{4}+4\,a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+\frac{11\,b\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{4}+\frac{16\,a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2}{3}+\frac{3\,b\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}+\frac{4\,a}{3}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)),x)
```

```
[Out] (3*b*x)/8 - ((4*a)/3 + (3*b*tan(e/2 + (f*x)/2))/4 + (16*a*tan(e/2 + (f*x)/2
)^2)/3 + 4*a*tan(e/2 + (f*x)/2)^4 + (11*b*tan(e/2 + (f*x)/2)^3)/4 - (11*b*t
an(e/2 + (f*x)/2)^5)/4 - (3*b*tan(e/2 + (f*x)/2)^7)/4)/(f*(tan(e/2 + (f*x)/
2)^2 + 1)^4)
```

# 3.150 $\int \sin^2(e + fx)(a + b\sin(e + fx))\, dx$

**Optimal**. Leaf size=55

$$\frac{ax}{2} - \frac{b\cos(e+fx)}{f} + \frac{b\cos^3(e+fx)}{3f} - \frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*x-b*cos(f*x+e)/f+1/3*b*cos(f*x+e)^3/f-1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2827, 2715, 8, 2713}

$$-\frac{a\sin(e+fx)\cos(e+fx)}{2f} + \frac{ax}{2} + \frac{b\cos^3(e+fx)}{3f} - \frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[e + f*x])/f + (b*Cos[e + f*x]^3)/(3*f) - (a*Cos[e + f*x]*S
in[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))\,dx = a\int \sin^2(e+fx)\,dx + b\int \sin^3(e+fx)\,dx$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}a\int 1\,dx - \frac{b\text{Subst}\left(\int (1-x^2)\,dx, x\right.}{f}$$
$$= \frac{ax}{2} - \frac{b\cos(e+fx)}{f} + \frac{b\cos^3(e+fx)}{3f} - \frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica [A]**
time = 0.05, size = 60, normalized size = 1.09

$$\frac{a(e+fx)}{2f} - \frac{3b\cos(e+fx)}{4f} + \frac{b\cos(3(e+fx))}{12f} - \frac{a\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(e + f*x))/(2*f) - (3*b*Cos[e + f*x])/(4*f) + (b*Cos[3*(e + f*x)])/(12*f
) - (a*Sin[2*(e + f*x)])/(4*f)
```

**Maple [A]**
time = 0.17, size = 49, normalized size = 0.89

| method | result |
|---|---|
| risch | $\frac{ax}{2} - \frac{3b\cos(fx+e)}{4f} + \frac{b\cos(3fx+3e)}{12f} - \frac{a\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-\frac{b\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3} + a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$ |
| default | $\frac{-\frac{b\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3} + a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$ |
| norman | $\frac{\frac{a\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f} + \frac{ax}{2} - \frac{4b}{3f} - \frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f} + \frac{3ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2} + \frac{3ax\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2} + \frac{ax\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2} - \frac{4b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e))
```

**Maxima [A]**
time = 0.30, size = 52, normalized size = 0.95

$$\frac{3\,(2\,fx + 2\,e - \sin\,(2\,fx + 2\,e))a + 4\,\left(\cos\,(fx+e)^3 - 3\,\cos\,(fx+e)\right)b}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a + 4*(cos(f*x + e)^3 - 3*cos(f*x
 + e))*b)/f
```

**Fricas** [A]
time = 0.34, size = 50, normalized size = 0.91

$$\frac{2\,b\cos\left(fx+e\right)^{3}+3\,afx-3\,a\cos\left(fx+e\right)\sin\left(fx+e\right)-6\,b\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*cos(f*x + e)^3 + 3*a*f*x - 3*a*cos(f*x + e)*sin(f*x + e) - 6*b*cos
(f*x + e))/f
```

**Sympy** [A]
time = 0.11, size = 92, normalized size = 1.67

$$\begin{cases} \frac{ax\sin^{2}\left(e+fx\right)}{2}+\frac{ax\cos^{2}\left(e+fx\right)}{2}-\frac{a\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b\cos^{3}\left(e+fx\right)}{3f} & \text{for}\: f\neq 0 \\ x(a+b\sin\left(e\right))\sin^{2}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 - a*sin(e + f*x)*c
os(e + f*x)/(2*f) - b*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*cos(e + f*x)**3/
(3*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**2, True))
```

**Giac** [A]
time = 0.48, size = 50, normalized size = 0.91

$$\frac{1}{2}\,ax+\frac{b\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,b\cos\left(fx+e\right)}{4\,f}-\frac{a\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*b*cos(3*f*x + 3*e)/f - 3/4*b*cos(f*x + e)/f - 1/4*a*sin(2*f*
x + 2*e)/f
```

**Mupad** [B]
time = 8.57, size = 68, normalized size = 1.24

$$\frac{a\,x}{2}-\frac{-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}+4\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,b}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)),x)
```

```
[Out] (a*x)/2 - ((4*b)/3 + a*tan(e/2 + (f*x)/2) - a*tan(e/2 + (f*x)/2)^5 + 4*b*ta
n(e/2 + (f*x)/2)^2)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^3)
```

# 3.151 $\int \sin(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=39

$$\frac{bx}{2}-\frac{a\cos(e+fx)}{f}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*b*x-a*cos(f*x+e)/f-1/2*b*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2813\}$

$$-\frac{a\cos(e+fx)}{f}-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
[Out] (b*x)/2 - (a*Cos[e + f*x])/f - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))\,dx=\frac{bx}{2}-\frac{a\cos(e+fx)}{f}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.07, size = 35, normalized size = 0.90

$$-\frac{4a\cos(e+fx)+b(-2(e+fx)+\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
[Out] -1/4*(4*a*Cos[e + f*x] + b*(-2*(e + f*x) + Sin[2*(e + f*x)]))/f
```

**Maple [A]**
time = 0.11, size = 39, normalized size = 1.00

| method | result |
|---|---|
| risch | $\frac{bx}{2}-\frac{a\cos(fx+e)}{f}-\frac{b\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos(fx+e)a}{f}$ |
| default | $\frac{b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos(fx+e)a}{f}$ |
| norman | $\frac{\frac{b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{2a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{bx}{2}-\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{bx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{2a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(b*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-cos(f*x+e)*a)
```

**Maxima [A]**
time = 0.28, size = 39, normalized size = 1.00

$$\frac{(2\,fx+2\,e-\sin{(2\,fx+2\,e)})b-4\,a\cos{(fx+e)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] 1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*b - 4*a*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.44, size = 37, normalized size = 0.95

$$\frac{bfx-b\cos{(fx+e)}\sin{(fx+e)}-2\,a\cos{(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] 1/2*(b*f*x - b*cos(f*x + e)*sin(f*x + e) - 2*a*cos(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.
time = 0.10, size = 66, normalized size = 1.69

$$\begin{cases} -\frac{a\cos{(e+fx)}}{f}+\frac{bx\sin^2{(e+fx)}}{2}+\frac{bx\cos^2{(e+fx)}}{2}-\frac{b\sin{(e+fx)}\cos{(e+fx)}}{2f} & \text{for } f\neq 0 \\ x(a+b\sin{(e)})\sin{(e)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*cos(e + f*x)/f + b*x*sin(e + f*x)**2/2 + b*x*cos(e + f*x)**2/
2 - b*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e),
 True))
```

**Giac** [A]
time = 0.43, size = 34, normalized size = 0.87

$$\frac{1}{2}\,bx - \frac{a\cos\left(fx+e\right)}{f} - \frac{b\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*b*x - a*cos(f*x + e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 7.19, size = 68, normalized size = 1.74

$$\frac{b\,x}{2} - \frac{-b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 + 2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + 2\,a}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)),x)
```

```
[Out] (b*x)/2 - (2*a + b*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 - b*tan(e/
2 + (f*x)/2)^3)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^2)
```

# 3.152 $\int (a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=16

$$ax-\frac{b\cos(e+fx)}{f}$$

```
[Out] a*x-b*cos(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2718}

$$ax-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x],x]
[Out] a*x - (b*Cos[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax+b\int \sin(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)}{f}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 27, normalized size = 1.69

$$ax-\frac{b\cos(e)\cos(fx)}{f}+\frac{b\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x],x]
[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f
```

**Maple** [A]
time = 0.05, size = 17, normalized size = 1.06

| method | result | size |
|---|---|---|
| default | $ax-\frac{b\cos(fx+e)}{f}$ | 17 |
| risch | $ax-\frac{b\cos(fx+e)}{f}$ | 17 |
| derivativedivides | $\frac{a(fx+e)-b\cos(fx+e)}{f}$ | 22 |
| norman | $\frac{ax+ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{2b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ | 52 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-b*cos(f*x+e)/f
```

**Maxima** [A]
time = 0.29, size = 17, normalized size = 1.06

$$ax-\frac{b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.34, size = 19, normalized size = 1.19

$$\frac{afx-b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (a*f*x - b*cos(f*x + e))/f
```

**Sympy** [A]
time = 0.04, size = 19, normalized size = 1.19

$$ax+b\left(\begin{cases}-\frac{\cos(e+fx)}{f} & \text{for } f\neq 0\\ x\sin(e) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)
```

```
[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

**Giac** [A]
time = 0.46, size = 17, normalized size = 1.06

$$ax - \frac{b\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

**Mupad** [B]
time = 6.51, size = 25, normalized size = 1.56

$$a\,x - \frac{2\,b}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x),x)
```

```
[Out] a*x - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

# 3.153 $\int \csc(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=17

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] b*x-a*arctanh(cos(f*x+e))/f
```

**Rubi** [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2814, 3855}

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*ArcTanh[Cos[e + f*x]])/f
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)(a+b\sin(e+fx))\,dx&=bx+a\int \csc(e+fx)\,dx\\&=bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(17) = 34.
time = 0.01, size = 43, normalized size = 2.53

$$bx-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*Log[Cos[e/2 + (f*x)/2]])/f + (a*Log[Sin[e/2 + (f*x)/2]])/f
```

**Maple** [A]
time = 0.18, size = 31, normalized size = 1.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b(fx+e)}{f}$ | 31 |
| default | $\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b(fx+e)}{f}$ | 31 |
| risch | $bx+\frac{a\ln(\mathrm{e}^{i(fx+e)}-1)}{f}-\frac{a\ln(\mathrm{e}^{i(fx+e)}+1)}{f}$ | 40 |
| norman | $\frac{bx+bx\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$ | 51 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*ln(csc(f*x+e)-cot(f*x+e))+b*(f*x+e))
```

**Maxima** [A]
time = 0.27, size = 32, normalized size = 1.88

$$\frac{(fx+e)b-a\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b - a*log(cot(f*x + e) + csc(f*x + e)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.
time = 0.35, size = 40, normalized size = 2.35

$$\frac{2\,bfx-a\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+a\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*f*x - a*log(1/2*cos(f*x + e) + 1/2) + a*log(-1/2*cos(f*x + e) + 1/
2))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(14) = 28$.
time = 2.81, size = 51, normalized size = 3.00

$$a\left(\begin{cases} \frac{x\cot(e)\csc(e)}{\cot(e)+\csc(e)} + \frac{x\csc^2(e)}{\cot(e)+\csc(e)} & \text{for } f = 0 \\ -\frac{\log(\cot(e+fx)+\csc(e+fx))}{f} & \text{otherwise} \end{cases}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] a*Piecewise((x*cot(e)*csc(e)/(cot(e) + csc(e)) + x*csc(e)**2/(cot(e) + csc(
e)), Eq(f, 0)), (-log(cot(e + f*x) + csc(e + f*x))/f, True)) + b*x
```

**Giac** [A]
time = 0.46, size = 27, normalized size = 1.59

$$\frac{(fx+e)b + a\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*b + a*log(abs(tan(1/2*f*x + 1/2*e))))/f
```

**Mupad** [B]
time = 6.80, size = 85, normalized size = 5.00

$$\frac{a\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f} + \frac{2b\,\mathrm{atan}\left(\frac{b\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+a\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{a\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-b\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x),x)
```

```
[Out] (a*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f + (2*b*atan((b*cos(e/2 + (
f*x)/2) + a*sin(e/2 + (f*x)/2))/(a*cos(e/2 + (f*x)/2) - b*sin(e/2 + (f*x)/2
))))/f
```

# 3.154 $\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=26

$$-\frac{b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a\cot(e+fx)}{f}$$

```
[Out] -b*arctanh(cos(f*x+e))/f-a*cot(f*x+e)/f
```

**Rubi** [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2827, 3852, 8, 3855}

$$-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*ArcTanh[Cos[e + f*x]])/f) - (a*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^2(e+fx)\,dx + b\int \csc(e+fx)\,dx$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{f} - \frac{a\text{Subst}(\int 1\,dx, x, \cot(e+fx))}{f}$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{f} - \frac{a\cot(e+fx)}{f}$$

**Mathematica [A]**
time = 0.02, size = 52, normalized size = 2.00

$$-\frac{a\cot(e+fx)}{f} - \frac{b\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{b\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]
[Out] -((a*Cot[e + f*x])/f) - (b*Log[Cos[e/2 + (f*x)/2]])/f + (b*Log[Sin[e/2 + (f
*x)/2]])/f
```

**Maple [A]**
time = 0.17, size = 33, normalized size = 1.27

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\cot(fx+e)a+b\ln(\csc(fx+e)-\cot(fx+e))}{f}$ | 33 |
| default | $\frac{-\cot(fx+e)a+b\ln(\csc(fx+e)-\cot(fx+e))}{f}$ | 33 |
| risch | $-\frac{2ia}{f\left(e^{2i(fx+e)}-1\right)} - \frac{b\ln\left(e^{i(fx+e)}+1\right)}{f} + \frac{b\ln\left(e^{i(fx+e)}-1\right)}{f}$ | 57 |
| norman | $\frac{-\frac{a}{2f}+\frac{a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)} + \frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$ | 68 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(-cot(f*x+e)*a+b*ln(csc(f*x+e)-cot(f*x+e)))
```

**Maxima [A]**
time = 0.28, size = 43, normalized size = 1.65

$$-\frac{b(\log(\cos(fx+e)+1) - \log(\cos(fx+e)-1)) + \frac{2\,a}{\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) + 2*a/tan(f*x + e))
/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

time = 0.39, size = 68, normalized size = 2.62

$$-\frac{b\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-b\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2\,a\cos\left(fx+e\right)}{2\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - b*log(-1/2*cos(f*x + e)
+ 1/2)*sin(f*x + e) + 2*a*cos(f*x + e))/(f*sin(f*x + e))
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(28) = 56$.

time = 0.45, size = 62, normalized size = 2.38

$$\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*f*x + 1/2*e))) + a*tan(1/2*f*x + 1/2*e) - (2*b*tan
(1/2*f*x + 1/2*e) + a)/tan(1/2*f*x + 1/2*e))/f
```

**Mupad** [B]

time = 6.76, size = 28, normalized size = 1.08

$$\frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\cot(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^2,x)
```

```
[Out] (b*log(tan(e/2 + (f*x)/2)))/f - (a*cot(e + f*x))/f
```

# 3.155 $\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=48

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -1/2*a*arctanh(cos(f*x+e))/f-b*cot(f*x+e)/f-1/2*a*cot(f*x+e)*csc(f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2827, 3853, 3855, 3852, 8}

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Cos[e + f*x]])/f - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc
[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^3(e+fx)\,dx + b\int \csc^2(e+fx)\,dx$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}a\int \csc(e+fx)\,dx - \frac{b\text{Subst}(\int 1\,d$$
$$= -\frac{a\tanh^{-1}(\cos(e+fx))}{2f} - \frac{b\cot(e+fx)}{f} - \frac{a\cot(e+fx)\csc(e+f}{2f}$$

**Mathematica** [A]
time = 0.03, size = 91, normalized size = 1.90

$$-\frac{b\cot(e+fx)}{f} - \frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
[Out] -((b*Cot[e + f*x])/f) - (a*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(e + f*x)
/2]])/(2*f) + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)/(8*f
)
```

**Maple** [A]
time = 0.23, size = 50, normalized size = 1.04

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-b\cot(fx+e)}{f}$ | 50 |
| default | $\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-b\cot(fx+e)}{f}$ | 50 |
| risch | $-\frac{i\left(ia\,e^{3i(fx+e)}+ia\,e^{i(fx+e)}+2b\,e^{2i(fx+e)}-2b\right)}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{a\ln\left(e^{i(fx+e)}+1\right)}{2f}+\frac{a\ln\left(e^{i(fx+e)}-1\right)}{2f}$ | 99 |
| norman | $\frac{-\frac{a}{8f}+\frac{a\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{b\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$ | 118 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-b*cot(f*x
+e))
```

**Maxima** [A]
time = 0.29, size = 65, normalized size = 1.35

$$\frac{a\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{4b}{\tan(fx+e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(c
os(f*x + e) - 1)) - 4*b/tan(f*x + e))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.
time = 0.36, size = 104, normalized size = 2.17

$$\frac{4b\cos\left(fx+e\right)\sin\left(fx+e\right)+2a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(a\cos\left(fx+e\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*cos(f*x + e)*sin(f*x + e) + 2*a*cos(f*x + e) - (a*cos(f*x + e)^2 -
 a)*log(1/2*cos(f*x + e) + 1/2) + (a*cos(f*x + e)^2 - a)*log(-1/2*cos(f*x +
 e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**3, x)
```

**Giac** [A]
time = 0.46, size = 92, normalized size = 1.92

$$\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\frac{6a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*f*x + 1/2*e)^2 + 4*a*log(abs(tan(1/2*f*x + 1/2*e))) + 4*b*ta
n(1/2*f*x + 1/2*e) - (6*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
 + a)/tan(1/2*f*x + 1/2*e)^2)/f
```

**Mupad** [B]
time = 6.72, size = 81, normalized size = 1.69

$$\frac{b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}-\frac{{\mathrm{cot}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{a}{2}+2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}+\frac{a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2}{8\,f}+\frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^3,x)
```

```
[Out] (b*tan(e/2 + (f*x)/2))/(2*f) - (cot(e/2 + (f*x)/2)^2*(a/2 + 2*b*tan(e/2 + (
f*x)/2)))/(4*f) + (a*tan(e/2 + (f*x)/2)^2)/(8*f) + (a*log(tan(e/2 + (f*x)/2
)))/(2*f)
```

# 3.156 $\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=64

$$-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -1/2*b*arctanh(cos(f*x+e))/f-a*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f-1/2*b*cot(
f*x+e)*csc(f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2827, 3852, 3853, 3855}

$$-\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/2*(b*ArcTanh[Cos[e + f*x]])/f - (a*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/
(3*f) - (b*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^4(e+fx)\,dx + b\int \csc^3(e+fx)\,dx$$
$$= -\frac{b\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}b\int \csc(e+fx)\,dx - \frac{a\text{Subst}\left(\int (1\right.}{}$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)}{f} - \frac{a\cot^3(e+fx)}{3f} - \frac{b\cot(}{}$$

**Mathematica [A]**
time = 0.02, size = 115, normalized size = 1.80

$$-\frac{2a\cot(e+fx)}{3f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Cot[e + f*x]*
Csc[e + f*x]^2)/(3*f) - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f
*x)/2]])/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

**Maple [A]**
time = 0.25, size = 61, normalized size = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$ |
| default | $\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$ |
| risch | $\frac{3b\,e^{5i(fx+e)}+12ia\,e^{2i(fx+e)}-4ia-3b\,e^{i(fx+e)}}{3f\left(e^{2i(fx+e)}-1\right)^3}+\frac{b\ln\left(e^{i(fx+e)}-1\right)}{2f}-\frac{b\ln\left(e^{i(fx+e)}+1\right)}{2f}$ |
| norman | $\frac{-\frac{a}{24f}-\frac{5a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{5a\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{a\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{b\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2
*ln(csc(f*x+e)-cot(f*x+e)))
```

**Maxima** [A]
time = 0.28, size = 79, normalized size = 1.23

$$\frac{3\,b\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{4\left(3\,\tan\left(fx+e\right)^{2}+1\right)a}{\tan\left(fx+e\right)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + lo
g(cos(f*x + e) - 1)) - 4*(3*tan(f*x + e)^2 + 1)*a/tan(f*x + e)^3)/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(63) = 126$.
time = 0.34, size = 140, normalized size = 2.19

$$-\frac{8\,a\cos\left(fx+e\right)^{3}-6\,b\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(b\cos\left(fx+e\right)^{2}-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\left(b\cos\left(fx+e\right)^{2}-b\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-12\,a\cos\left(fx+e\right)}{12\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*a*cos(f*x + e)^3 - 6*b*cos(f*x + e)*sin(f*x + e) + 3*(b*cos(f*x +
e)^2 - b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(b*cos(f*x + e)^2 -
b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 12*a*cos(f*x + e))/((f*cos(f
*x + e)^2 - f)*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**4, x)
```

**Giac** [A]
time = 0.49, size = 122, normalized size = 1.91

$$\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+12\,b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+9\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{22\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+9\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + 3*b*tan(1/2*f*x + 1/2*e)^2 + 12*b*log(abs(
tan(1/2*f*x + 1/2*e))) + 9*a*tan(1/2*f*x + 1/2*e) - (22*b*tan(1/2*f*x + 1/2
*e)^3 + 9*a*tan(1/2*f*x + 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e) + a)/tan(1/2*
f*x + 1/2*e)^3)/f
```

**Mupad [B]**
time = 6.74, size = 111, normalized size = 1.73

$$\frac{3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8\,f}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24\,f}+\frac{b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a}{3}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^4,x)
```

```
[Out] (3*a*tan(e/2 + (f*x)/2))/(8*f) + (a*tan(e/2 + (f*x)/2)^3)/(24*f) + (b*tan(e
/2 + (f*x)/2)^2)/(8*f) + (b*log(tan(e/2 + (f*x)/2)))/(2*f) - (cot(e/2 + (f*
x)/2)^3*(a/3 + b*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2))/(8*f)
```

# 3.157 $\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=112

$$\frac{3abx}{4}-\frac{(a^2+b^2)\cos(e+fx)}{f}+\frac{(a^2+2b^2)\cos^3(e+fx)}{3f}-\frac{b^2\cos^5(e+fx)}{5f}-\frac{3ab\cos(e+fx)\sin(e+fx)}{4f}-\frac{ab\cos$$

```
[Out] 3/4*a*b*x-(a^2+b^2)*cos(f*x+e)/f+1/3*(a^2+2*b^2)*cos(f*x+e)^3/f-1/5*b^2*cos
(f*x+e)^5/f-3/4*a*b*cos(f*x+e)*sin(f*x+e)/f-1/2*a*b*cos(f*x+e)*sin(f*x+e)^3
/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 2715, 8, 3092, 380}

$$\frac{(a^2+2b^2)\cos^3(e+fx)}{3f}-\frac{(a^2+b^2)\cos(e+fx)}{f}-\frac{ab\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3ab\sin(e+fx)\cos(e+fx)}{4f}+\frac{3abx}{4}-\frac{b^2\cos^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - ((a^2 + b^2)*Cos[e + f*x])/f + ((a^2 + 2*b^2)*Cos[e + f*x]^3)
/(3*f) - (b^2*Cos[e + f*x]^5)/(5*f) - (3*a*b*Cos[e + f*x]*Sin[e + f*x])/(4*
f) - (a*b*Cos[e + f*x]*Sin[e + f*x]^3)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
```

```
, f, m}, x]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^4(e+fx)\,dx + \int \sin^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,d$$

$$= -\frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{2}(3ab)\int \sin^2(e+fx)\,dx - \frac{\text{Subs}}{}$$

$$= -\frac{3ab\cos(e+fx)\sin(e+fx)}{4f} - \frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{4}($$

$$= \frac{3abx}{4} - \frac{(a^2+b^2)\cos(e+fx)}{f} + \frac{(a^2+2b^2)\cos^3(e+fx)}{3f} - \frac{b^2\cos^5}{5}$$

**Mathematica** [A]
time = 0.23, size = 91, normalized size = 0.81

$$\frac{-30(6a^2+5b^2)\cos(e+fx)+5(4a^2+5b^2)\cos(3(e+fx))-3b(b\cos(5(e+fx))-5a(12(e+fx)-8\sin(2(e+fx))+\sin(4(e+fx))))}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-30*(6*a^2 + 5*b^2)*Cos[e + f*x] + 5*(4*a^2 + 5*b^2)*Cos[3*(e + f*x)] - 3*
b*(b*Cos[5*(e + f*x)] - 5*a*(12*(e + f*x) - 8*Sin[2*(e + f*x)] + Sin[4*(e +
 f*x)])))/(240*f)
```

**Maple** [A]
time = 0.30, size = 95, normalized size = 0.85

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2ab\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2\left(2+\sin^2(fx\right.}{}}{f}$ |

| | |
|---|---|
| default | $\frac{-\frac{b^{2}\left(\frac{8}{3}+\sin^{4}(fx+e)+\frac{4\left(\sin^{2}(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2ab\left(-\frac{\left(\sin^{3}(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^{2}\left(2+\sin^{2}(fx+e\right.}{3}}{f}$ |
| risch | $\frac{3abx}{4}-\frac{3a^{2}\cos(fx+e)}{4f}-\frac{5b^{2}\cos(fx+e)}{8f}-\frac{b^{2}\cos(5fx+5e)}{80f}+\frac{ab\sin(4fx+4e)}{16f}+\frac{a^{2}\cos(3fx+3e)}{12f}+\frac{5\cos(3fx+3}{48f}$ |
| norman | $\frac{-\frac{20a^{2}+16b^{2}}{15f}+\frac{3abx}{4}-\frac{4a^{2}\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2\left(14a^{2}+16b^{2}\right)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{\left(20a^{2}+16b^{2}\right)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{3ab\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}-7}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

**Maxima [A]**
time = 0.27, size = 102, normalized size = 0.91

$$\frac{80\left(\cos(fx+e)^{3}-3\cos(fx+e)\right)a^{2}+15\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)ab-16\left(3\cos(fx+e)^{5}-10\cos(fx+e)^{3}+15\cos(fx+e)\right)b^{2}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*b - 16*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*b^2)/f
```

**Fricas [A]**
time = 0.37, size = 96, normalized size = 0.86

$$-\frac{12b^{2}\cos(fx+e)^{5}-45abfx-20\left(a^{2}+2b^{2}\right)\cos(fx+e)^{3}+60\left(a^{2}+b^{2}\right)\cos(fx+e)-15\left(2ab\cos(fx+e)^{3}-5ab\cos(fx+e)\right)\sin(fx+e)}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^2*cos(f*x + e)^5 - 45*a*b*f*x - 20*(a^2 + 2*b^2)*cos(f*x + e)^3
 + 60*(a^2 + b^2)*cos(f*x + e) - 15*(2*a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x +
 e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(104) = 208$.
time = 0.30, size = 221, normalized size = 1.97

$$\begin{cases} -\frac{a^{2}\sin^{2}(e+fx)\cos(e+fx)}{f}-\frac{2a^{2}\cos^{3}(e+fx)}{3f}+\frac{3abx\sin^{4}(e+fx)}{4}+\frac{3abx\sin^{2}(e+fx)\cos^{2}(e+fx)}{2}+\frac{3abx\cos^{4}(e+fx)}{4}-\frac{5ab\sin^{3}(e+fx)\cos(e+fx)}{4f}-\frac{3ab\sin(e+fx)\cos^{3}(e+fx)}{4f}-\frac{b^{2}\sin^{4}(e+fx)\cos(e+fx)}{f}-\frac{4b^{2}\sin^{2}(e+fx)\cos^{3}(e+fx)}{3f}-\frac{8b^{2}\cos^{5}(e+fx)}{15f} & \text{for } f\neq 0 \\ x(a+b\sin(e))^{2}\sin^{3}(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*cos(e + f*x)**3/(3
*f) + 3*a*b*x*sin(e + f*x)**4/4 + 3*a*b*x*sin(e + f*x)**2*cos(e + f*x)**2/2
 + 3*a*b*x*cos(e + f*x)**4/4 - 5*a*b*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 3
*a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**2*sin(e + f*x)**4*cos(e + f*x)
/f - 4*b**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*cos(e + f*x)**5/
(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e)**3, True))
```

**Giac** [A]
time = 0.46, size = 128, normalized size = 1.14

$$\frac{3}{4}abx-\frac{b^2\cos(5fx+5e)}{80f}+\frac{ab\sin(4fx+4e)}{16f}-\frac{ab\sin(2fx+2e)}{2f}+\frac{(4a^2+5b^2)\cos(3fx+3e)}{48f}-\frac{(2a^2+3b^2)\cos(fx+e)}{8f}-\frac{(2a^2+b^2)\cos(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x - 1/80*b^2*cos(5*f*x + 5*e)/f + 1/16*a*b*sin(4*f*x + 4*e)/f - 1/2
*a*b*sin(2*f*x + 2*e)/f + 1/48*(4*a^2 + 5*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*
a^2 + 3*b^2)*cos(f*x + e)/f - 1/4*(2*a^2 + b^2)*cos(f*x + e)/f
```

**Mupad** [B]
time = 10.37, size = 157, normalized size = 1.40

$$\frac{3\,a\,b\,x}{4}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{20\,a^2}{3}+\frac{16\,b^2}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{28\,a^2}{3}+\frac{32\,b^2}{3}\right)+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\frac{4\,a^2}{3}+\frac{16\,b^2}{15}+7\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-7\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7-\frac{3\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2}+\frac{3\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x))^2,x)
```

```
[Out] (3*a*b*x)/4 - (tan(e/2 + (f*x)/2)^2*((20*a^2)/3 + (16*b^2)/3) + tan(e/2 + (
f*x)/2)^4*((28*a^2)/3 + (32*b^2)/3) + 4*a^2*tan(e/2 + (f*x)/2)^6 + (4*a^2)/
3 + (16*b^2)/15 + 7*a*b*tan(e/2 + (f*x)/2)^3 - 7*a*b*tan(e/2 + (f*x)/2)^7 -
 (3*a*b*tan(e/2 + (f*x)/2)^9)/2 + (3*a*b*tan(e/2 + (f*x)/2))/2)/(f*(tan(e/2
 + (f*x)/2)^2 + 1)^5)
```

# 3.158 $\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=101

$$\frac{1}{8}\left(4a^2+3b^2\right)x-\frac{2ab\cos(e+fx)}{f}+\frac{2ab\cos^3(e+fx)}{3f}-\frac{(4a^2+3b^2)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\sin}{4f}$$

```
[Out] 1/8*(4*a^2+3*b^2)*x-2*a*b*cos(f*x+e)/f+2/3*a*b*cos(f*x+e)^3/f-1/8*(4*a^2+3*
b^2)*cos(f*x+e)*sin(f*x+e)/f-1/4*b^2*cos(f*x+e)*sin(f*x+e)^3/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 2713, 3093, 2715, 8}

$$-\frac{(4a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x\left(4a^2+3b^2\right)+\frac{2ab\cos^3(e+fx)}{3f}-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*x)/8 - (2*a*b*Cos[e + f*x])/f + (2*a*b*Cos[e + f*x]^3)/(3*
f) - ((4*a^2 + 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b^2*Cos[e + f*x]*
Sin[e + f*x]^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
```

```
, f, m}, x]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^3(e+fx)\,dx + \int \sin^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,d$$

$$= -\frac{b^2\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sin^2(e+fx)\,dx -$$

$$= -\frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{(4a^2+3b^2)\cos(e+fx)\mathrm{si}}{8f}$$

$$= \frac{1}{8}\left(4a^2+3b^2\right)x - \frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{(4a^2+3b}{}$$

**Mathematica [A]**
time = 0.11, size = 117, normalized size = 1.16

$$\frac{a^2(e+fx)}{2f} + \frac{3b^2(e+fx)}{8f} - \frac{3ab\cos(e+fx)}{2f} + \frac{ab\cos(3(e+fx))}{6f} - \frac{a^2\sin(2(e+fx))}{4f} - \frac{b^2\sin(2(e+fx))}{4f} + \frac{b^2\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x))/(2*f) + (3*b^2*(e + f*x))/(8*f) - (3*a*b*Cos[e + f*x])/(2*f
) + (a*b*Cos[3*(e + f*x)])/(6*f) - (a^2*Sin[2*(e + f*x)])/(4*f) - (b^2*Sin[
2*(e + f*x)])/(4*f) + (b^2*Sin[4*(e + f*x)])/(32*f)
```

**Maple [A]**
time = 0.24, size = 89, normalized size = 0.88

| method | result |
|---|---|
| derivativedivides | $\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2ab\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |

| | |
|---|---|
| default | $\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2ab\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$ |
| risch | $\frac{a^2x}{2}+\frac{3b^2x}{8}-\frac{3ab\cos(fx+e)}{2f}+\frac{b^2\sin(4fx+4e)}{32f}+\frac{ab\cos(3fx+3e)}{6f}-\frac{a^2\sin(2fx+2e)}{4f}-\frac{\sin(2fx+2e)b^2}{4f}$ |
| norman | $\frac{\left(\frac{a^2}{2}+\frac{3b^2}{8}\right)x+\left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3a^2+\frac{9b^2}{4}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{a^2}{2}+\frac{3b^2}{8}\right)}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*
a*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2
*e))
```

**Maxima** [A]
time = 0.30, size = 91, normalized size = 0.90

$$\frac{24\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))a^2+64\,\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ab+3\,(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right))b^2}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2 + 64*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a*b + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b
^2)/f
```

**Fricas** [A]
time = 0.37, size = 89, normalized size = 0.88

$$\frac{16\,ab\cos\left(fx+e\right)^3+3\,(4\,a^2+3\,b^2)fx-48\,ab\cos\left(fx+e\right)+3\,\left(2\,b^2\cos\left(fx+e\right)^3-(4\,a^2+5\,b^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*b*cos(f*x + e)^3 + 3*(4*a^2 + 3*b^2)*f*x - 48*a*b*cos(f*x + e) +
 3*(2*b^2*cos(f*x + e)^3 - (4*a^2 + 5*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(92) = 184$.
time = 0.20, size = 211, normalized size = 2.09

$$\begin{cases} \frac{a^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2x\cos^2\left(e+fx\right)}{2}-\frac{a^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ab\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ab\cos^3\left(e+fx\right)}{3f}+\frac{3b^2x\sin^4\left(e+fx\right)}{8}+\frac{3b^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{3b^2x\cos^4\left(e+fx\right)}{8}-\frac{5b^2\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{3b^2\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f} & \text{for}\: f \neq 0 \\ x(a+b\sin\left(e\right))^2\sin^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 - a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 2*a*b*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c
os(e + f*x)**3/(3*f) + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)**3*c
os(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (
x*(a + b*sin(e))**2*sin(e)**2, True))
```

**Giac** [A]
time = 0.52, size = 86, normalized size = 0.85

$$\frac{1}{8}\left(4a^2+3b^2\right)x+\frac{ab\cos\left(3fx+3e\right)}{6f}-\frac{3ab\cos\left(fx+e\right)}{2f}+\frac{b^2\sin\left(4fx+4e\right)}{32f}-\frac{\left(a^2+b^2\right)\sin\left(2fx+2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/6*a*b*cos(3*f*x + 3*e)/f - 3/2*a*b*cos(f*x + e)/f
 + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(a^2 + b^2)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 6.93, size = 85, normalized size = 0.84

$$\frac{\frac{3b^2\sin\left(4e+4fx\right)}{4}-6b^2\sin\left(2e+2fx\right)-6a^2\sin\left(2e+2fx\right)-36ab\cos\left(e+fx\right)+4ab\cos\left(3e+3fx\right)+12a^2fx+9b^2fx}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((3*b^2*sin(4*e + 4*f*x))/4 - 6*b^2*sin(2*e + 2*f*x) - 6*a^2*sin(2*e + 2*f*
x) - 36*a*b*cos(e + f*x) + 4*a*b*cos(3*e + 3*f*x) + 12*a^2*f*x + 9*b^2*f*x)
/(24*f)
```

# 3.159 $\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=71

$$abx-\frac{2(a^2+b^2)\cos(e+fx)}{3f}-\frac{ab\cos(e+fx)\sin(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] a*b*x-2/3*(a^2+b^2)*cos(f*x+e)/f-1/3*a*b*cos(f*x+e)*sin(f*x+e)/f-1/3*cos(f*
x+e)*(a+b*sin(f*x+e))^2/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2832, 2813}

$$-\frac{2(a^2+b^2)\cos(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}-\frac{ab\sin(e+fx)\cos(e+fx)}{3f}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] a*b*x - (2*(a^2 + b^2)*Cos[e + f*x])/(3*f) - (a*b*Cos[e + f*x]*Sin[e + f*x]
)/(3*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx=-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int(2b+2a\sin(e+fx))(a+b$$

$$=abx-\frac{2(a^2+b^2)\cos(e+fx)}{3f}-\frac{ab\cos(e+fx)\sin(e+fx)}{3f}-\cos(e-$$

**Mathematica [A]**
time = 0.14, size = 59, normalized size = 0.83

$$\frac{-3(4a^2+3b^2)\cos(e+fx)+b(12a(e+fx)+b\cos(3(e+fx))-6a\sin(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-3*(4*a^2 + 3*b^2)*Cos[e + f*x] + b*(12*a*(e + f*x) + b*Cos[3*(e + f*x)] -
 6*a*Sin[2*(e + f*x)]))/(12*f)
```

**Maple [A]**
time = 0.19, size = 64, normalized size = 0.90

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos(fx+e)a^2}{f}$ |
| default | $\frac{-\frac{b^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos(fx+e)a^2}{f}$ |
| risch | $abx-\frac{a^2\cos(fx+e)}{f}-\frac{3b^2\cos(fx+e)}{4f}+\frac{\cos(3fx+3e)b^2}{12f}-\frac{ab\sin(2fx+2e)}{2f}$ |
| norman | $\frac{abx+abx\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{6a^2+4b^2}{3f}-\frac{2a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{\left(4a^2+4b^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2ab\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2ab\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*b^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-cos(f*x+e)*a^2)
```

**Maxima [A]**
time = 0.27, size = 67, normalized size = 0.94

$$\frac{3\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))ab+2\,\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^2-6\,a^2\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b + 2*(cos(f*x + e)^3 - 3*cos(f*x
 + e))*b^2 - 6*a^2*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.35, size = 59, normalized size = 0.83

$$\frac{b^2\cos(fx+e)^3+3\,abfx-3\,ab\cos(fx+e)\sin(fx+e)-3\,(a^2+b^2)\cos(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(f*x + e)^3 + 3*a*b*f*x - 3*a*b*cos(f*x + e)*sin(f*x + e) - 3*(
a^2 + b^2)*cos(f*x + e))/f
```

**Sympy** [A]
time = 0.12, size = 107, normalized size = 1.51

$$\begin{cases} -\frac{a^2\cos(e+fx)}{f}+abx\sin^2(e+fx)+abx\cos^2(e+fx)-\frac{ab\sin(e+fx)\cos(e+fx)}{f}-\frac{b^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2b^2\cos^3(e+fx)}{3f} & \text{for } f\neq 0\\ x(a+b\sin(e))^2\sin(e) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*cos(e + f*x)/f + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x
)**2 - a*b*sin(e + f*x)*cos(e + f*x)/f - b**2*sin(e + f*x)**2*cos(e + f*x)/
f - 2*b**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e), T
rue))
```

**Giac** [A]
time = 0.47, size = 76, normalized size = 1.07

$$abx+\frac{b^2\cos(3\,fx+3\,e)}{12\,f}-\frac{b^2\cos(fx+e)}{4\,f}-\frac{ab\sin(2\,fx+2\,e)}{2\,f}-\frac{(2\,a^2+b^2)\cos(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*b*x + 1/12*b^2*cos(3*f*x + 3*e)/f - 1/4*b^2*cos(f*x + e)/f - 1/2*a*b*sin(
2*f*x + 2*e)/f - 1/2*(2*a^2 + b^2)*cos(f*x + e)/f
```

**Mupad** [B]
time = 8.99, size = 103, normalized size = 1.45

$$a\,b\,x-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(4\,a^2+4\,b^2)+2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\,a^2+\frac{4\,b^2}{3}-2\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+2\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^2,x)
```

```
[Out] a*b*x - (tan(e/2 + (f*x)/2)^2*(4*a^2 + 4*b^2) + 2*a^2*tan(e/2 + (f*x)/2)^4
+ 2*a^2 + (4*b^2)/3 - 2*a*b*tan(e/2 + (f*x)/2)^5 + 2*a*b*tan(e/2 + (f*x)/2)
)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^3)
```

# 3.160 $\int (a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=50

$$\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*(2*a^2+b^2)*x-2*a*b*cos(f*x+e)/f-1/2*b^2*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2723}

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.07, size = 46, normalized size = 0.92

$$-\frac{-2(2a^2+b^2)\left(e+fx\right)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)
])/f
```

**Maple [A]**
time = 0.12, size = 51, normalized size = 1.02

| method | result |
|---|---|
| risch | $a^2x+\frac{b^2x}{2}-\frac{2ab\cos(fx+e)}{f}-\frac{\sin(2fx+2e)b^2}{4f}$ |
| derivativedivides | $\frac{b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\cos(fx+e)+a^2(fx+e)}{f}$ |
| default | $\frac{b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\cos(fx+e)+a^2(fx+e)}{f}$ |
| norman | $\frac{\left(a^2+\frac{b^2}{2}\right)x+\frac{b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\left(a^2+\frac{b^2}{2}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2a^2+b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{4ab\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{b^2\tan\left(\frac{f}{2}\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

**Maxima [A]**
time = 0.30, size = 49, normalized size = 0.98

$$a^2x+\frac{(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2}{4\,f}-\frac{2\,ab\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

**Fricas [A]**
time = 0.34, size = 48, normalized size = 0.96

$$-\frac{b^2\cos(fx+e)\sin(fx+e)-(2\,a^2+b^2)fx+4\,ab\cos(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

**Sympy** [A]
time = 0.08, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos(e+fx)}{f} + \frac{b^2x\sin^2(e+fx)}{2} + \frac{b^2x\cos^2(e+fx)}{2} - \frac{b^2\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\sin(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

**Giac** [A]
time = 0.49, size = 45, normalized size = 0.90

$$\frac{1}{2}\left(2\,a^2 + b^2\right)x - \frac{2\,ab\cos\left(fx + e\right)}{f} - \frac{b^2\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 6.79, size = 44, normalized size = 0.88

$$-\frac{\frac{b^2\,\sin(2\,e+2\,f\,x)}{2} + 4\,a\,b\,\cos\left(e + f\,x\right) - 2\,a^2\,f\,x - b^2\,f\,x}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2,x)
```

```
[Out] -((b^2*sin(2*e + 2*f*x))/2 + 4*a*b*cos(e + f*x) - 2*a^2*f*x - b^2*f*x)/(2*f
)
```

# 3.161 $\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=35

$$2abx-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}-\frac{b^2\cos(e+fx)}{f}$$

```
[Out] 2*a*b*x-a^2*arctanh(cos(f*x+e))/f-b^2*cos(f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2825, 2814, 3855}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+2abx-\frac{b^2\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (a^2*ArcTanh[Cos[e + f*x]])/f - (b^2*Cos[e + f*x])/f
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx = -\frac{b^2\cos(e+fx)}{f} + \int \csc(e+fx)\left(a^2+2ab\sin(e+fx)\right)\,dx$$
$$= 2abx - \frac{b^2\cos(e+fx)}{f} + a^2\int \csc(e+fx)\,dx$$
$$= 2abx - \frac{a^2\tanh^{-1}(\cos(e+fx))}{f} - \frac{b^2\cos(e+fx)}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.
time = 0.02, size = 76, normalized size = 2.17

$$2abx - \frac{b^2\cos(e)\cos(fx)}{f} - \frac{a^2\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{a^2\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{b^2\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (b^2*Cos[e]*Cos[f*x])/f - (a^2*Log[Cos[e/2 + (f*x)/2]])/f + (a^2*
Log[Sin[e/2 + (f*x)/2]])/f + (b^2*Sin[e]*Sin[f*x])/f
```

**Maple** [A]
time = 0.20, size = 46, normalized size = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab(fx+e)-\cos(fx+e)b^2}{f}$ |
| default | $\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab(fx+e)-\cos(fx+e)b^2}{f}$ |
| risch | $2abx - \frac{b^2 e^{i(fx+e)}}{2f} - \frac{b^2 e^{-i(fx+e)}}{2f} + \frac{a^2\ln(e^{i(fx+e)}-1)}{f} - \frac{a^2\ln(e^{i(fx+e)}+1)}{f}$ |
| norman | $\frac{\frac{2b^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+2abx+\frac{2b^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+4abx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2abx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} + \frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*ln(csc(f*x+e)-cot(f*x+e))+2*a*b*(f*x+e)-cos(f*x+e)*b^2)
```

**Maxima** [A]
time = 0.28, size = 48, normalized size = 1.37

$$\frac{2\,(fx+e)ab - b^2\cos\left(fx+e\right) - a^2\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*(f*x + e)*a*b - b^2*cos(f*x + e) - a^2*log(cot(f*x + e) + csc(f*x + e)))
/f
```

**Fricas** [A]
time = 0.39, size = 57, normalized size = 1.63

$$\frac{4\,abfx - 2\,b^2\cos\left(fx+e\right) - a^2\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right) + a^2\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*f*x - 2*b^2*cos(f*x + e) - a^2*log(1/2*cos(f*x + e) + 1/2) + a^2
*log(-1/2*cos(f*x + e) + 1/2))/f
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin\left(e+fx\right))^2\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x), x)
```

**Giac** [A]
time = 0.46, size = 52, normalized size = 1.49

$$\frac{2\left(fx+e\right)ab + a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right) - \frac{2\,b^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(f*x + e)*a*b + a^2*log(abs(tan(1/2*f*x + 1/2*e))) - 2*b^2/(tan(1/2*f*x
+ 1/2*e)^2 + 1))/f
```

**Mupad** [B]
time = 6.48, size = 125, normalized size = 3.57

$$\frac{a^2\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f} - \frac{2\,b^2}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{16\,a^2\,b^2}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}+\frac{8\,a^3\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x),x)
```

```
[Out] (a^2*log(tan(e/2 + (f*x)/2)))/f - (2*b^2)/(f*(tan(e/2 + (f*x)/2)^2 + 1)) +
(4*a*b*atan((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(e/2 + (f*x)/2)) + (8*a^3
*b*tan(e/2 + (f*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(e/2 + (f*x)/2))))/f
```

# 3.162 $\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=34

$$b^2x-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)}{f}$$

```
[Out] b^2*x-2*a*b*arctanh(cos(f*x+e))/f-a^2*cot(f*x+e)/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2868, 3855, 3091, 8}

$$-\frac{a^2\cot(e+fx)}{f}-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] b^2*x - (2*a*b*ArcTanh[Cos[e + f*x]])/f - (a^2*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc(e+fx)\,dx + \int \csc^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,d$$

$$= -\frac{2ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{a^2\cot(e+fx)}{f} + b^2\int 1\,dx$$

$$= b^2x - \frac{2ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{a^2\cot(e+fx)}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.
time = 0.17, size = 76, normalized size = 2.24

$$\frac{-a^2\cot\left(\frac{1}{2}(e+fx)\right)+2b\left(be+bfx-2a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+a^2\tan\left(\frac{1}{2}(e+fx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-(a^2*Cot[(e + f*x)/2]) + 2*b*(b*e + b*f*x - 2*a*Log[Cos[(e + f*x)/2]] + 2
*a*Log[Sin[(e + f*x)/2]]) + a^2*Tan[(e + f*x)/2])/(2*f)
```

**Maple** [A]
time = 0.23, size = 46, normalized size = 1.35

| method | result |
|---|---|
| derivativedivides | $\frac{-a^2\cot(fx+e)+2ab\ln(\csc(fx+e)-\cot(fx+e))+b^2(fx+e)}{f}$ |
| default | $\frac{-a^2\cot(fx+e)+2ab\ln(\csc(fx+e)-\cot(fx+e))+b^2(fx+e)}{f}$ |
| risch | $b^2x-\frac{2ia^2}{f\left(e^{2i(fx+e)}-1\right)}-\frac{2ab\ln\left(e^{i(fx+e)}+1\right)}{f}+\frac{2ab\ln\left(e^{i(fx+e)}-1\right)}{f}$ |
| norman | $\frac{b^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+b^2x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{a^2}{2f}-\frac{a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}+\frac{a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}+\frac{a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}+2b^2x\left(\tan^3\left(\frac{fx}{2}\right.\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*ln(csc(f*x+e)-cot(f*x+e))+b^2*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 56, normalized size = 1.65

$$\frac{(fx+e)b^2 - ab(\log(\cos(fx+e)+1) - \log(\cos(fx+e)-1)) - \frac{a^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b^2 - a*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) - a^2/
tan(f*x + e))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.
time = 0.41, size = 84, normalized size = 2.47

$$\frac{b^2fx\sin(fx+e) - ab\log\left(\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right)\sin(fx+e) + ab\log\left(-\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right)\sin(fx+e) - a^2\cos(fx+e)}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (b^2*f*x*sin(f*x + e) - a*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + a*b*
log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - a^2*cos(f*x + e))/(f*sin(f*x +
e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^2 \csc^2(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.
time = 0.46, size = 79, normalized size = 2.32

$$\frac{2(fx+e)b^2 + 4ab\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + a^2\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{4ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a^2}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)*b^2 + 4*a*b*log(abs(tan(1/2*f*x + 1/2*e))) + a^2*tan(1/2*f
*x + 1/2*e) - (4*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1/2*f*x + 1/2*e))/f
```

**Mupad** [B]
time = 6.83, size = 105, normalized size = 3.09

$$\frac{2\,b^2\,\mathrm{atan}\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f} - \frac{a^2\cot(e+f\,x)}{f} + \frac{2\,a\,b\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^2,x)
```

```
[Out] (2*b^2*atan((b*cos(e/2 + (f*x)/2) + 2*a*sin(e/2 + (f*x)/2))/(2*a*cos(e/2 +
(f*x)/2) - b*sin(e/2 + (f*x)/2))))/f - (a^2*cot(e + f*x))/f + (2*a*b*log(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f
```

# 3.163 $\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=59

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{2ab\cot(e+fx)}{f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -1/2*(a^2+2*b^2)*arctanh(cos(f*x+e))/f-2*a*b*cot(f*x+e)/f-1/2*a^2*cot(f*x+e
)*csc(f*x+e)/f
```

**Rubi** [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 3852, 8, 3091, 3855}

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}-\frac{2ab\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*((a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/f - (2*a*b*Cot[e + f*x])/f - (a^
2*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^2(e+fx)\,dx + \int \csc^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,d$$

$$= -\frac{a^2\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}\left(a^2+2b^2\right)\int \csc(e+fx)\,dx - \frac{(2}{}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{2ab\cot(e+fx)}{f} - \frac{a^2\cot(e+}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 133 vs. 2(59) = 118.
time = 0.30, size = 133, normalized size = 2.25

$$\frac{-8ab\cot\left(\frac{1}{2}(e+fx)\right)-a^2\csc^2\left(\frac{1}{2}(e+fx)\right)-4a^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(e+fx)\right)+8ab\tan\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-8*a*b*Cot[(e + f*x)/2] - a^2*Csc[(e + f*x)/2]^2 - 4*a^2*Log[Cos[(e + f*x)
/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] + 4*a^2*Log[Sin[(e + f*x)/2]] + 8*b^2*Lo
g[Sin[(e + f*x)/2]] + a^2*Sec[(e + f*x)/2]^2 + 8*a*b*Tan[(e + f*x)/2])/(8*f
)
```

**Maple** [A]
time = 0.34, size = 73, normalized size = 1.24

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-2ab\cot(fx+e)+b^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$ |
| default | $\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-2ab\cot(fx+e)+b^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$ |
| risch | $-\frac{ia\left(ia\,e^{3i(fx+e)}+ia\,e^{i(fx+e)}+4b\,e^{2i(fx+e)}-4b\right)}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{a^2\ln\left(e^{i(fx+e)}+1\right)}{2f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b^2}{f}+\frac{a^2\ln\left(e^{i(fx+e)}-1\right)}{2f}$ |

| | |
|---|---|
| norman | $\frac{\frac{ab\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{ab\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2}{8f}+\frac{a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{ab\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\ldots}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-2*a*b*c
ot(f*x+e)+b^2*ln(csc(f*x+e)-cot(f*x+e)))
```

**Maxima [A]**
time = 0.28, size = 96, normalized size = 1.63

$$\frac{a^2\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-2\,b^2(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right))-\frac{8\,ab}{\tan(fx+e)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log
(cos(f*x + e) - 1)) - 2*b^2*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1))
 - 8*a*b/tan(f*x + e))/f
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(59) = 118$.
time = 0.36, size = 137, normalized size = 2.32

$$\frac{8\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+2\,a^2\cos\left(fx+e\right)-\left((a^2+2\,b^2)\cos\left(fx+e\right)^2-a^2-2\,b^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a^2+2\,b^2)\cos\left(fx+e\right)^2-a^2-2\,b^2\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(f*x + e)*sin(f*x + e) + 2*a^2*cos(f*x + e) - ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)
^2 - f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^2\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**3, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(59) = 118.
time = 0.45, size = 125, normalized size = 2.12

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{6\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*f*x + 1/2*e)^2 + 8*a*b*tan(1/2*f*x + 1/2*e) + 4*(a^2 + 2*b
^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (6*a^2*tan(1/2*f*x + 1/2*e)^2 + 12*b^2
*tan(1/2*f*x + 1/2*e)^2 + 8*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1/2*f*x + 1
/2*e)^2)/f
```

**Mupad** [B]
time = 6.49, size = 92, normalized size = 1.56

$$\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{a^2}{2}+b^2\right)}{f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a^2}{8}+b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a\right)}{f}+\frac{a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^3,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^2)/(8*f) + (log(tan(e/2 + (f*x)/2))*(a^2/2 + b^2))/
f - (cot(e/2 + (f*x)/2)^2*(a^2/8 + a*b*tan(e/2 + (f*x)/2)))/f + (a*b*tan(e/
2 + (f*x)/2))/f
```

# 3.164 $\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=82

$$-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}$$

```
[Out] -a*b*arctanh(cos(f*x+e))/f-1/3*(2*a^2+3*b^2)*cot(f*x+e)/f-a*b*cot(f*x+e)*cs
c(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2868, 3853, 3855, 3091, 3852, 8}

$$-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((a*b*ArcTanh[Cos[e + f*x]])/f) - ((2*a^2 + 3*b^2)*Cot[e + f*x])/(3*f) - (
a*b*Cot[e + f*x]*Csc[e + f*x])/f - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^3(e+fx)\,dx + \int \csc^4(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,$$
$$= -\frac{ab\cot(e+fx)\csc(e+fx)}{f} - \frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f} + (ab$$
$$= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{ab\cot(e+fx)\csc(e+fx)}{f} - \frac{a^2\cot(e}{}$$
$$= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{(2a^2+3b^2)\cot(e+fx)}{3f} - \frac{ab\cot(e+}{}$$

**Mathematica** [A]
time = 0.03, size = 132, normalized size = 1.61

$$-\frac{2a^2\cot(e+fx)}{3f} - \frac{b^2\cot(e+fx)}{f} - \frac{ab\csc^2\left(\frac{1}{2}(e+fx)\right)}{4f} - \frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{ab\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{ab\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{ab\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a^2*Cot[e + f*x])/(3*f) - (b^2*Cot[e + f*x])/f - (a*b*Csc[(e + f*x)/2]^
2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*b*Log[Cos[(e + f*x)
/2]])/f + (a*b*Log[Sin[(e + f*x)/2]])/f + (a*b*Sec[(e + f*x)/2]^2)/(4*f)
```

**Maple** [A]
time = 0.36, size = 76, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+2ab\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-\cot(fx+e)b^2}{f}$ |

| | |
|---|---|
| default | $\frac{a^{2}\left(-\frac{2}{3}-\frac{\left(\csc^{2}(fx+e)\right)}{3}\right)\cot(fx+e)+2ab\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-\cot(fx+e)b^{2}}{f}$ |
| risch | $\frac{-2ib^{2}e^{4i(fx+e)}+2ab\,e^{5i(fx+e)}+4ia^{2}e^{2i(fx+e)}+4ib^{2}e^{2i(fx+e)}-\frac{4ia^{2}}{3}-2ib^{2}-2ab\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^{3}}+\frac{ab\ln\left(e^{i(fx+e)}-1\right)}{f}-\frac{ab\ln(}{}$ |
| norman | $\frac{\frac{ab\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{ab\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^{2}}{24f}+\frac{a^{2}\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{\left(5a^{2}+6b^{2}\right)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{\left(5a^{2}+6b^{2}\right)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/2*csc(f*x+e)*cot(f*x+
e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-cot(f*x+e)*b^2)
```

**Maxima [A]**
time = 0.28, size = 96, normalized size = 1.17

$$\frac{3\,ab\left(\frac{2\cos(fx+e)}{\cos(fx+e)^{2}-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{6\,b^{2}}{\tan(fx+e)}-\frac{2\left(3\tan(fx+e)^{2}+1\right)a^{2}}{\tan(fx+e)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + l
og(cos(f*x + e) - 1)) - 6*b^2/tan(f*x + e) - 2*(3*tan(f*x + e)^2 + 1)*a^2/t
an(f*x + e)^3)/f
```

**Fricas [A]**
time = 0.53, size = 161, normalized size = 1.96

$$-\frac{2\left(2\,a^{2}+3\,b^{2}\right)\cos\left(fx+e\right)^{3}-6\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(ab\cos\left(fx+e\right)^{2}-ab\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\left(ab\cos\left(fx+e\right)^{2}-ab\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-6\left(a^{2}+b^{2}\right)\cos\left(fx+e\right)}{6\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^2 + 3*b^2)*cos(f*x + e)^3 - 6*a*b*cos(f*x + e)*sin(f*x + e) +
3*(a*b*cos(f*x + e)^2 - a*b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(
a*b*cos(f*x + e)^2 - a*b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 6*(a^
2 + b^2)*cos(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^{2}\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**4, x)
```

**Giac [A]**
time = 0.43, size = 166, normalized size = 2.02

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,ab\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+9\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{44\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 + 24*a*b*lo
g(abs(tan(1/2*f*x + 1/2*e))) + 9*a^2*tan(1/2*f*x + 1/2*e) + 12*b^2*tan(1/2*
f*x + 1/2*e) - (44*a*b*tan(1/2*f*x + 1/2*e)^3 + 9*a^2*tan(1/2*f*x + 1/2*e)^
2 + 12*b^2*tan(1/2*f*x + 1/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1
/2*f*x + 1/2*e)^3)/f
```

**Mupad [B]**
time = 6.78, size = 136, normalized size = 1.66

$$\frac{a^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{24\,f}+\frac{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,a^2}{8}+\frac{b^2}{2}\right)}{f}-\frac{{\mathrm{cot}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(3\,a^2+4\,b^2\right)+\frac{a^2}{3}+2\,a\,b\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}+\frac{a\,b\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2}{4\,f}+\frac{a\,b\,\ln\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^4,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^3)/(24*f) + (tan(e/2 + (f*x)/2)*((3*a^2)/8 + b^2/2)
)/f - (cot(e/2 + (f*x)/2)^3*(tan(e/2 + (f*x)/2)^2*(3*a^2 + 4*b^2) + a^2/3 +
 2*a*b*tan(e/2 + (f*x)/2)))/(8*f) + (a*b*tan(e/2 + (f*x)/2)^2)/(4*f) + (a*b
*log(tan(e/2 + (f*x)/2)))/f
```

# 3.165 $\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=110

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{2ab\cot(e+fx)}{f}-\frac{2ab\cot^3(e+fx)}{3f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}$$

```
[Out] -1/8*(3*a^2+4*b^2)*arctanh(cos(f*x+e))/f-2*a*b*cot(f*x+e)/f-2/3*a*b*cot(f*x
+e)^3/f-1/8*(3*a^2+4*b^2)*cot(f*x+e)*csc(f*x+e)/f-1/4*a^2*cot(f*x+e)*csc(f*
x+e)^3/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 3852, 3091, 3853, 3855}

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{2ab\cot^3(e+fx)}{3f}-\frac{2ab\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/8*((3*a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/f - (2*a*b*Cot[e + f*x])/f - (
2*a*b*Cot[e + f*x]^3)/(3*f) - ((3*a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(
8*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f)
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx &= (2ab)\int \csc^4(e+fx)\,dx + \int \csc^5(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,d\\ &= -\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{1}{4}\left(3a^2+4b^2\right)\int \csc^3(e+fx)\,dx -\\ &= -\frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e+fx)}{3f} - \frac{\left(3a^2+4b^2\right)\cot(e+fx)\,c}{8f}\\ &= -\frac{\left(3a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e}{3f}\end{aligned}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(110) = 220.
time = 0.04, size = 255, normalized size = 2.32

$$-\frac{4ab\cot(e+fx)}{3f} - \frac{3a^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} - \frac{b^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{2ab\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{3a^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{b^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{3a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{b^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{3a^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{b^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-4*a*b*Cot[e + f*x])/(3*f) - (3*a^2*Csc[(e + f*x)/2]^2)/(32*f) - (b^2*Csc[
(e + f*x)/2]^2)/(8*f) - (a^2*Csc[(e + f*x)/2]^4)/(64*f) - (2*a*b*Cot[e + f*
x]*Csc[e + f*x]^2)/(3*f) - (3*a^2*Log[Cos[(e + f*x)/2]])/(8*f) - (b^2*Log[C
os[(e + f*x)/2]])/(2*f) + (3*a^2*Log[Sin[(e + f*x)/2]])/(8*f) + (b^2*Log[Si
n[(e + f*x)/2]])/(2*f) + (3*a^2*Sec[(e + f*x)/2]^2)/(32*f) + (b^2*Sec[(e +
f*x)/2]^2)/(8*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f)
```

**Maple** [**A**]
time = 0.37, size = 114, normalized size = 1.04

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{a^2\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b^2\left(-\right.}{f}$ |
| default | $\frac{a^2\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b^2\left(-\right.}{f}$ |
| risch | $\frac{9a^2e^{7i(fx+e)}+12b^2e^{7i(fx+e)}+96iab\,e^{4i(fx+e)}-33a^2e^{5i(fx+e)}-12b^2e^{5i(fx+e)}-128iab\,e^{2i(fx+e)}-33a^2e^{3i(fx+e)}-12b^2e^{3i}}{12f\left(e^{2i(fx+e)}-1\right)^4}$ |
| norman | $\frac{-\frac{a^2}{64f}+\frac{a^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}-\frac{\left(5a^2+4b^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}+\frac{\left(5a^2+4b^2\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{\left(17a^2+16b^2\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{\left(17a^2+\right.}{}}{\tan\left(\frac{f}{2}\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-c
ot(f*x+e)))+2*a*b*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b^2*(-1/2*csc(f*x+e)*c
ot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e))))
```

**Maxima** [A]
time = 0.28, size = 159, normalized size = 1.45

$$\frac{3\,a^2\left(\frac{2\left(3\cos(fx+e)^3-5\cos(fx+e)\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}-3\log(\cos(fx+e)+1)+3\log(\cos(fx+e)-1)\right)+12\,b^2\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log(\cos(fx+e)+1)+\log(\cos(fx+e)-1)\right)-\frac{32\left(3\tan(fx+e)^2+1\right)ab}{\tan(fx+e)^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 32*(3*tan(f*x + e)^2 + 1)*a*b/tan(f*x + e)^3)/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(109) = 218.
time = 0.36, size = 242, normalized size = 2.20

$$\frac{6\left(3a^2+4b^2\right)\cos(fx+e)^3-6\left(5a^2+4b^2\right)\cos(fx+e)-3\left(\left(3a^2+4b^2\right)\cos(fx+e)^4-2\left(3a^2+4b^2\right)\cos(fx+e)^2+3a^2+4b^2\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+3\left(\left(3a^2+4b^2\right)\cos(fx+e)^4-2\left(3a^2+4b^2\right)\cos(fx+e)^2+3a^2+4b^2\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+32\left(2ab\cos(fx+e)^3-3ab\cos(fx+e)\right)\sin(fx+e)}{48\left(f\cos(fx+e)^4-2f\cos(fx+e)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 4*b^2)*cos(f*x + e)^3 - 6*(5*a^2 + 4*b^2)*cos(f*x + e) - 3
*((3*a^2 + 4*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2
 + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((3*a^2 + 4*b^2)*cos(f*x + e)^4 -
```

```
 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2 + 4*b^2)*log(-1/2*cos(f*x + e) +
1/2) + 32*(2*a*b*cos(f*x + e)^3 - 3*a*b*cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^2 \csc^5(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**5, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(109) = 218.
time = 0.47, size = 230, normalized size = 2.09

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+16\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+144\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,(3\,a^2+4\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{150\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+200\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+144\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+16\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*f*x + 1/2*e)^4 + 16*a*b*tan(1/2*f*x + 1/2*e)^3 + 24*a^
2*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*tan(1/2*f*x + 1/2*e)^2 + 144*a*b*tan(1/2*
f*x + 1/2*e) + 24*(3*a^2 + 4*b^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (150*a^2
*tan(1/2*f*x + 1/2*e)^4 + 200*b^2*tan(1/2*f*x + 1/2*e)^4 + 144*a*b*tan(1/2*
f*x + 1/2*e)^3 + 24*a^2*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*tan(1/2*f*x + 1/2*e
)^2 + 16*a*b*tan(1/2*f*x + 1/2*e) + 3*a^2)/tan(1/2*f*x + 1/2*e)^4)/f
```

**Mupad** [**B**]
time = 6.88, size = 178, normalized size = 1.62

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{8}+\frac{b^2}{2}\right)}{f}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^2+2\,b^2\right)+\frac{a^2}{4}+12\,a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\frac{4\,a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{3}\right)}{16\,f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a^2}{8}+\frac{b^2}{8}\right)}{f}+\frac{a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12\,f}+\frac{3\,a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^5,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*((3*a^2)/8 + b^2/2))/f + (a^2*tan(e/2 + (f*x)/2)^4
)/(64*f) - (cot(e/2 + (f*x)/2)^4*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 2*b^2) + a^
2/4 + 12*a*b*tan(e/2 + (f*x)/2)^3 + (4*a*b*tan(e/2 + (f*x)/2))/3))/(16*f) +
 (tan(e/2 + (f*x)/2)^2*(a^2/8 + b^2/8))/f + (a*b*tan(e/2 + (f*x)/2)^3)/(12*
f) + (3*a*b*tan(e/2 + (f*x)/2))/(4*f)
```

# 3.166 $\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=171

$$\frac{1}{16}b\left(18a^2+5b^2\right)x-\frac{a(a^2+3b^2)\cos(e+fx)}{f}+\frac{a(a^2+6b^2)\cos^3(e+fx)}{3f}-\frac{3ab^2\cos^5(e+fx)}{5f}-\frac{b(18a^2+5b^2)\mathrm{co}}{}$$

```
[Out] 1/16*b*(18*a^2+5*b^2)*x-a*(a^2+3*b^2)*cos(f*x+e)/f+1/3*a*(a^2+6*b^2)*cos(f*
x+e)^3/f-3/5*a*b^2*cos(f*x+e)^5/f-1/16*b*(18*a^2+5*b^2)*cos(f*x+e)*sin(f*x+
e)/f-1/24*b*(18*a^2+5*b^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*b^3*cos(f*x+e)*sin
(f*x+e)^5/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{2872, 3102, 2827, 2713, 2715, 8\}$

$$\frac{a(5a^2+12b^2)\cos^3(e+fx)}{15f}-\frac{a(5a^2+12b^2)\cos(e+fx)}{5f}-\frac{b(18a^2+5b^2)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{b(18a^2+5b^2)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}bx(18a^2+5b^2)-\frac{13ab^2\sin^4(e+fx)\cos(e+fx)}{30f}-\frac{b^2\sin^4(e+fx)\cos(e+fx)(a+b\sin(e+fx))}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 - (a*(5*a^2 + 12*b^2)*Cos[e + f*x])/(5*f) + (a*(5
*a^2 + 12*b^2)*Cos[e + f*x]^3)/(15*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f
) - (13*a*b^2*Cos[e + f*x]*Sin[e + f*x]^4)/(30*f) - (b^2*Cos[e + f*x]*Sin[e
 + f*x]^4*(a + b*Sin[e + f*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx &= -\frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin(e+fx))}{6f}+\frac{1}{6}\int \sin^3(e+f\\
&= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f}-\frac{b^2\cos(e+fx)\sin^4(e+fx)(a\\
&= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f}-\frac{b^2\cos(e+fx)\sin^4(e+fx)(a\\
&= -\frac{b(18a^2+5b^2)\cos(e+fx)\sin^3(e+fx)}{24f}-\frac{13ab^2\cos(e+fx)\sin^4}{30f}\\
&= -\frac{a(5a^2+12b^2)\cos(e+fx)}{5f}+\frac{a(5a^2+12b^2)\cos^3(e+fx)}{15f}-\frac{b(18\\
&= \frac{1}{16}b\left(18a^2+5b^2\right)x-\frac{a(5a^2+12b^2)\cos(e+fx)}{5f}+\frac{a(5a^2+12b^2)\,c}{15f}\end{aligned}$$

**Mathematica** [A]

time = 0.49, size = 147, normalized size = 0.86

$$\frac{-360a(2a^2+5b^2)\cos(e+fx)+20(4a^3+15ab^2)\cos(3(e+fx))+b(-36ab\cos(5(e+fx))+5(216a^2e+60b^2e+216a^2fx+60b^2fx-9(16a^2+5b^2)\sin(2(e+fx))+9(2a^2+b^2)\sin(4(e+fx))-b^2\sin(6(e+fx))))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-360*a*(2*a^2 + 5*b^2)*Cos[e + f*x] + 20*(4*a^3 + 15*a*b^2)*Cos[3*(e + f*x
)] + b*(-36*a*b*Cos[5*(e + f*x)] + 5*(216*a^2*e + 60*b^2*e + 216*a^2*f*x +
60*b^2*f*x - 9*(16*a^2 + 5*b^2)*Sin[2*(e + f*x)] + 9*(2*a^2 + b^2)*Sin[4*(e
 + f*x)] - b^2*Sin[6*(e + f*x)])))/(960*f)
```

**Maple [A]**
time = 0.41, size = 145, normalized size = 0.85

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a\,b^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}}{f}$ |
| default | $\frac{b^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a\,b^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}}{f}$ |
| risch | $\frac{9a^2bx}{8}+\frac{5b^3x}{16}-\frac{3a^3\cos(fx+e)}{4f}-\frac{15a\,b^2\cos(fx+e)}{8f}-\frac{b^3\sin(6fx+6e)}{192f}-\frac{3a\,b^2\cos(5fx+5e)}{80f}+\frac{3b\sin(4fx+4e)a^2}{32f}$ |
| norman | $-\frac{20a^3+48a\,b^2}{15f}+\frac{b\left(18a^2+5b^2\right)x}{16}-\frac{4a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4\left(10a^3+24a\,b^2\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{\left(16a^3+48a\,b^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(20a}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*
a^2*b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima [A]**
time = 0.28, size = 157, normalized size = 0.92

$$\frac{320\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3+90\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^2b-192\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)ab^2+5\left(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e)\right)b^3}{960f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b - 192*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*a*b^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*b^3)/f
```

**Fricas [A]**
time = 0.34, size = 145, normalized size = 0.85

$$-\frac{144\,ab^2\cos\left(fx+e\right)^5-80\,(a^3+6\,ab^2)\cos\left(fx+e\right)^3-15\,(18\,a^2b+5\,b^3)fx+240\,(a^3+3\,ab^2)\cos\left(fx+e\right)+5\,\left(8\,b^3\cos\left(fx+e\right)^5-2\,(18\,a^2b+13\,b^3)\cos\left(fx+e\right)^3+3\,(30\,a^2b+11\,b^3)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(f*x + e)^5 - 80*(a^3 + 6*a*b^2)*cos(f*x + e)^3 - 15*(
18*a^2*b + 5*b^3)*f*x + 240*(a^3 + 3*a*b^2)*cos(f*x + e) + 5*(8*b^3*cos(f*x
 + e)^5 - 2*(18*a^2*b + 13*b^3)*cos(f*x + e)^3 + 3*(30*a^2*b + 11*b^3)*cos(
f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(155) = 310$.
time = 0.46, size = 393, normalized size = 2.30

$$\begin{cases} -\frac{a^3\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2a^3\cos^3(e+fx)}{3f}+\frac{9a^2bx\sin^4(e+fx)}{8}+\frac{9a^2bx\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{9a^2bx\cos^4(e+fx)}{8}-\frac{15a^2b\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{9a^2b\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)}{f}-\frac{4ab^2\sin^2(e+fx)\cos^3(e+fx)}{f}-\frac{8ab^2\cos^5(e+fx)}{5f}+\frac{5b^3x\sin^6(e+fx)}{16}+\frac{15b^3x\sin^4(e+fx)\cos^2(e+fx)}{16}+\frac{15b^3x\sin^2(e+fx)\cos^4(e+fx)}{16}+\frac{5b^3x\cos^6(e+fx)}{16}-\frac{11b^3\sin^5(e+fx)\cos(e+fx)}{16f}-\frac{5b^3\sin^3(e+fx)\cos^3(e+fx)}{6f}-\frac{5b^3\sin(e+fx)\cos^5(e+fx)}{16f} & \text{for}\ f\neq 0 \\ x(a+b\sin(e))^3\sin^3(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3*cos(e + f*x)**3/(3
*f) + 9*a**2*b*x*sin(e + f*x)**4/8 + 9*a**2*b*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 + 9*a**2*b*x*cos(e + f*x)**4/8 - 15*a**2*b*sin(e + f*x)**3*cos(e + f
*x)/(8*f) - 9*a**2*b*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*sin(e +
f*x)**4*cos(e + f*x)/f - 4*a*b**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b
**2*cos(e + f*x)**5/(5*f) + 5*b**3*x*sin(e + f*x)**6/16 + 15*b**3*x*sin(e +
 f*x)**4*cos(e + f*x)**2/16 + 15*b**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16
+ 5*b**3*x*cos(e + f*x)**6/16 - 11*b**3*sin(e + f*x)**5*cos(e + f*x)/(16*f)
 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*b**3*sin(e + f*x)*cos(e
 + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e)**3, True))
```

**Giac [A]**
time = 0.46, size = 180, normalized size = 1.05

$$-\frac{3\,ab^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{b^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\,(18\,a^2b+5\,b^3)x+\frac{(4\,a^3+15\,ab^2)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{(2\,a^3+9\,ab^2)\cos\left(fx+e\right)}{8\,f}-\frac{(2\,a^3+3\,ab^2)\cos\left(fx+e\right)}{4\,f}+\frac{3\,(2\,a^2b+b^3)\sin\left(4\,fx+4\,e\right)}{64\,f}-\frac{3\,(16\,a^2b+5\,b^3)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -3/80*a*b^2*cos(5*f*x + 5*e)/f - 1/192*b^3*sin(6*f*x + 6*e)/f + 1/16*(18*a^
2*b + 5*b^3)*x + 1/48*(4*a^3 + 15*a*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*a^3 +
9*a*b^2)*cos(f*x + e)/f - 1/4*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f + 3/64*(2*a^
2*b + b^3)*sin(4*f*x + 4*e)/f - 3/64*(16*a^2*b + 5*b^3)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.40, size = 417, normalized size = 2.44

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(18\,a^2+5\,b^2\right)}{8\,\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)}\right)\,\left(18\,a^2+5\,b^2\right)}{8\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)+4\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\frac{16\,a\,b^2}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(16\,a^3+48\,a\,b^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{40\,a^3}{3}+32\,a\,b^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(8\,a^3+\frac{96\,a\,b^2}{5}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}\,\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(\frac{21\,a^2\,b}{2}+\frac{33\,b^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(\frac{21\,a^2\,b}{2}+\frac{33\,b^3}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{51\,a^2\,b}{4}+\frac{85\,b^3}{24}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\,\left(\frac{51\,a^2\,b}{4}+\frac{85\,b^3}{24}\right)+\frac{4\,a^3}{3}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+20\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{b\,\left(18\,a^2+5\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (b*atan((b*tan(e/2 + (f*x)/2)*(18*a^2 + 5*b^2))/(8*((9*a^2*b)/4 + (5*b^3)/8
)))*(18*a^2 + 5*b^2))/(8*f) - (tan(e/2 + (f*x)/2)*((9*a^2*b)/4 + (5*b^3)/8)
 + 4*a^3*tan(e/2 + (f*x)/2)^8 + (16*a*b^2)/5 + tan(e/2 + (f*x)/2)^4*(48*a*b
^2 + 16*a^3) + tan(e/2 + (f*x)/2)^6*(32*a*b^2 + (40*a^3)/3) + tan(e/2 + (f*
x)/2)^2*((96*a*b^2)/5 + 8*a^3) - tan(e/2 + (f*x)/2)^11*((9*a^2*b)/4 + (5*b^
3)/8) + tan(e/2 + (f*x)/2)^5*((21*a^2*b)/2 + (33*b^3)/4) - tan(e/2 + (f*x)/
2)^7*((21*a^2*b)/2 + (33*b^3)/4) + tan(e/2 + (f*x)/2)^3*((51*a^2*b)/4 + (85
*b^3)/24) - tan(e/2 + (f*x)/2)^9*((51*a^2*b)/4 + (85*b^3)/24) + (4*a^3)/3)/
(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2
)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2
)^12 + 1)) - (b*(18*a^2 + 5*b^2)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(8*f
)
```

# 3.167 $\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=160

$$\frac{1}{8}a\left(4a^2+9b^2\right)x-\frac{b(15a^2+4b^2)\cos(e+fx)}{5f}+\frac{b(15a^2+4b^2)\cos^3(e+fx)}{15f}-\frac{a(4a^2+9b^2)\cos(e+fx)\sin(e+$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*x-1/5*b*(15*a^2+4*b^2)*cos(f*x+e)/f+1/15*b*(15*a^2+4*b^
2)*cos(f*x+e)^3/f-1/8*a*(4*a^2+9*b^2)*cos(f*x+e)*sin(f*x+e)/f-11/20*a*b^2*c
os(f*x+e)*sin(f*x+e)^3/f-1/5*b^2*cos(f*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2870, 2832, 2813}

$$\frac{(3a^2-16b^2)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf}+\frac{a(6a^2-71b^2)\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}ax\left(4a^2+9b^2\right)+\frac{(3a^4-52a^2b^2-16b^4)\cos(e+fx)}{30bf}-\frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf}+\frac{a\cos(e+fx)(a+b\sin(e+fx))^3}{20bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 + ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Cos[e + f*x])/(30*
b*f) + (a*(6*a^2 - 71*b^2)*Cos[e + f*x]*Sin[e + f*x])/(120*f) + ((3*a^2 - 1
6*b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(60*b*f) + (a*Cos[e + f*x]*(a +
 b*Sin[e + f*x])^3)/(20*b*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f
)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
```

```
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int (4b-a\sin(e+fx))(a+b\sin}{5b}$$

$$= \frac{a\cos(e+fx)(a+b\sin(e+fx))^3}{20bf} - \frac{\cos(e+fx)(a+b\sin(e+fx))}{5bf}$$

$$= \frac{(3a^2-16b^2)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf} + \frac{a\cos(e+fx)(a+b}{20bf}$$

$$= \frac{1}{8}a\left(4a^2+9b^2\right)x + \frac{(3a^4-52a^2b^2-16b^4)\cos(e+fx)}{30bf} + \frac{a(6a^2-71}{}$$

**Mathematica [A]**
time = 0.45, size = 117, normalized size = 0.73

$$\frac{-60b(18a^2+5b^2)\cos(e+fx)+10(12a^2b+5b^3)\cos(3(e+fx))-6b^3\cos(5(e+fx))+15a(4(4a^2+9b^2)(e+fx)-8(a^2+3b^2)\sin(2(e+fx))+3b^2\sin(4(e+fx)))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-60*b*(18*a^2 + 5*b^2)*Cos[e + f*x] + 10*(12*a^2*b + 5*b^3)*Cos[3*(e + f*x
)] - 6*b^3*Cos[5*(e + f*x)] + 15*a*(4*(4*a^2 + 9*b^2)*(e + f*x) - 8*(a^2 +
3*b^2)*Sin[2*(e + f*x)] + 3*b^2*Sin[4*(e + f*x)]))/(480*f)
```

**Maple [A]**
time = 0.33, size = 124, normalized size = 0.78

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+3a\,b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a^2b\left(2+\sin^2(f\right.}{f}$ |
| default | $\frac{-\frac{b^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+3a\,b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a^2b\left(2+\sin^2(f\right.}{f}$ |
| risch | $\frac{a^3x}{2}+\frac{9a\,b^2x}{8}-\frac{9b\cos(fx+e)a^2}{4f}-\frac{5b^3\cos(fx+e)}{8f}-\frac{b^3\cos(5fx+5e)}{80f}+\frac{3a\,b^2\sin(4fx+4e)}{32f}+\frac{b\cos(3fx+3e)a^2}{4f}+$ |

| | |
|---|---|
| norman | $\frac{-\frac{60a^2b+16b^3}{15f}+\frac{a(4a^2+9b^2)x}{8}-\frac{12a^2b(\tan^6(\frac{fx}{2}+\frac{e}{2}))}{f}-\frac{2(42a^2b+16b^3)(\tan^4(\frac{fx}{2}+\frac{e}{2}))}{3f}-\frac{(60a^2b+16b^3)(\tan^2(\frac{fx}{2}+\frac{e}{2}))}{3f}-\frac{a(4\ldots}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*b^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a^2*b*(2+sin(f*x+e)
^2)*cos(f*x+e)+a^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

**Maxima** [A]
time = 0.28, size = 131, normalized size = 0.82

$$\frac{120\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^3+480\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2b+45\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)ab^2-32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)b^3}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3 + 480*(cos(f*x + e)^3 - 3*c
os(f*x + e))*a^2*b + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2
*e))*a*b^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b^
3)/f
```

**Fricas** [A]
time = 0.45, size = 124, normalized size = 0.78

$$-\frac{24b^3\cos\left(fx+e\right)^5-40\left(3a^2b+2b^3\right)\cos\left(fx+e\right)^3-15\left(4a^3+9ab^2\right)fx+120\left(3a^2b+b^3\right)\cos\left(fx+e\right)-15\left(6ab^2\cos\left(fx+e\right)^3-\left(4a^3+15ab^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*cos(f*x + e)^5 - 40*(3*a^2*b + 2*b^3)*cos(f*x + e)^3 - 15*(4
*a^3 + 9*a*b^2)*f*x + 120*(3*a^2*b + b^3)*cos(f*x + e) - 15*(6*a*b^2*cos(f*
x + e)^3 - (4*a^3 + 15*a*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [A]
time = 0.37, size = 284, normalized size = 1.78

$$\begin{cases} \frac{a^3x\sin^2(e+fx)}{2}+\frac{a^3x\cos^2(e+fx)}{2}-\frac{a^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{3a^2b\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2a^2b\cos^3(e+fx)}{f}+\frac{9ab^2x\sin^4(e+fx)}{8}+\frac{9ab^2x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{9ab^2x\cos^4(e+fx)}{8}-\frac{15ab^2\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{9ab^2\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{b^3\sin^4(e+fx)\cos(e+fx)}{f}-\frac{4b^3\sin^2(e+fx)\cos^3(e+fx)}{3f}-\frac{8b^3\cos^5(e+fx)}{15f} & \text{for } f\neq 0 \\ x(a+b\sin(e))^3\sin^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(e + f*x)**2/2 + a**3*x*cos(e + f*x)**2/2 - a**3*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 3*a**2*b*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*
```

```
*2*b*cos(e + f*x)**3/f + 9*a*b**2*x*sin(e + f*x)**4/8 + 9*a*b**2*x*sin(e +
f*x)**2*cos(e + f*x)**2/4 + 9*a*b**2*x*cos(e + f*x)**4/8 - 15*a*b**2*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - 9*a*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f)
- b**3*sin(e + f*x)**4*cos(e + f*x)/f - 4*b**3*sin(e + f*x)**2*cos(e + f*x)
**3/(3*f) - 8*b**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*
sin(e)**2, True))
```

**Giac [A]**
time = 0.46, size = 129, normalized size = 0.81

$$-\frac{b^3\cos(5fx+5e)}{80f}+\frac{3ab^2\sin(4fx+4e)}{32f}+\frac{1}{8}\left(4a^3+9ab^2\right)x+\frac{(12a^2b+5b^3)\cos(3fx+3e)}{48f}-\frac{(18a^2b+5b^3)\cos(fx+e)}{8f}-\frac{(a^3+3ab^2)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^3*cos(5*f*x + 5*e)/f + 3/32*a*b^2*sin(4*f*x + 4*e)/f + 1/8*(4*a^3 +
 9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*cos(3*f*x + 3*e)/f - 1/8*(18*a^2*b +
5*b^3)*cos(f*x + e)/f - 1/4*(a^3 + 3*a*b^2)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.17, size = 328, normalized size = 2.05

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4a^2+9b^2)}{4\left(a^3+\frac{9ab^2}{4}\right)}\right)(4a^2+9b^2)}{4f}-\frac{4a^2b-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(a^3+\frac{9ab^2}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(2a^3+\frac{21ab^2}{2}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(2a^3+\frac{21ab^2}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(20a^2b+\frac{16b^3}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(28a^2b+\frac{32b^3}{3}\right)+\frac{16b^3}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a^3+\frac{9ab^2}{4}\right)+12a^2b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}-\frac{a(4a^2+9b^2)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(4*a^2 + 9*b^2))/(4*((9*a*b^2)/4 + a^3)))*(4*
a^2 + 9*b^2))/(4*f) - (4*a^2*b - tan(e/2 + (f*x)/2)^9*((9*a*b^2)/4 + a^3) +
 tan(e/2 + (f*x)/2)^3*((21*a*b^2)/2 + 2*a^3) - tan(e/2 + (f*x)/2)^7*((21*a*
b^2)/2 + 2*a^3) + tan(e/2 + (f*x)/2)^2*(20*a^2*b + (16*b^3)/3) + tan(e/2 +
(f*x)/2)^4*(28*a^2*b + (32*b^3)/3) + (16*b^3)/15 + tan(e/2 + (f*x)/2)*((9*a
*b^2)/4 + a^3) + 12*a^2*b*tan(e/2 + (f*x)/2)^6)/(f*(5*tan(e/2 + (f*x)/2)^2
+ 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a*(4*a^2 + 9*b^2)*(atan(tan(e/2 + (f*x)/
2)) - (f*x)/2))/(4*f)
```

# 3.168 $\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=121

$$\frac{3}{8}b\left(4a^2+b^2\right)x-\frac{a(a^2+4b^2)\cos(e+fx)}{2f}-\frac{b(2a^2+3b^2)\cos(e+fx)\sin(e+fx)}{8f}-\frac{a\cos(e+fx)(a+b\sin(e-}{4f}$$

```
[Out] 3/8*b*(4*a^2+b^2)*x-1/2*a*(a^2+4*b^2)*cos(f*x+e)/f-1/8*b*(2*a^2+3*b^2)*cos(
f*x+e)*sin(f*x+e)/f-1/4*a*cos(f*x+e)*(a+b*sin(f*x+e))^2/f-1/4*cos(f*x+e)*(a
+b*sin(f*x+e))^3/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2832, 2813}

$$-\frac{a(a^2+4b^2)\cos(e+fx)}{2f}-\frac{b(2a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}bx(4a^2+b^2)-\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f}-\frac{a\cos(e+fx)(a+b\sin(e+fx))^2}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 - (a*(a^2 + 4*b^2)*Cos[e + f*x])/(2*f) - (b*(2*a^2
+ 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*Cos[e + f*x]*(a + b*Sin[e +
f*x])^2)/(4*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{4}\int (3b+3a\sin(e+fx))(a+b$$

$$= -\frac{a\cos(e+fx)(a+b\sin(e+fx))^2}{4f} - \frac{\cos(e+fx)(a+b\sin(e+fx)}{4f}$$

$$= \frac{3}{8}b(4a^2+b^2)\,x - \frac{a(a^2+4b^2)\cos(e+fx)}{2f} - \frac{b(2a^2+3b^2)\cos(e+fx}{8f}$$

**Mathematica [A]**
time = 0.24, size = 100, normalized size = 0.83

$$\frac{-8a(4a^2+9b^2)\cos(e+fx)+b(48a^2e+12b^2e+48a^2fx+12b^2fx+8ab\cos(3(e+fx))-8(3a^2+b^2)\sin(2(e+fx))+b^2\sin(4(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-8*a*(4*a^2 + 9*b^2)*Cos[e + f*x] + b*(48*a^2*e + 12*b^2*e + 48*a^2*f*x +
12*b^2*f*x + 8*a*b*Cos[3*(e + f*x)] - 8*(3*a^2 + b^2)*Sin[2*(e + f*x)] + b^
2*Sin[4*(e + f*x)]))/(32*f)
```

**Maple [A]**
time = 0.27, size = 104, normalized size = 0.86

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a\,b^2(2+\sin^2(fx+e))\cos(fx+e)+3a^2b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}\right.}{f}$ |
| default | $\frac{b^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a\,b^2(2+\sin^2(fx+e))\cos(fx+e)+3a^2b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}\right.}{f}$ |
| risch | $\frac{3a^2bx}{2}+\frac{3b^3x}{8}-\frac{a^3\cos(fx+e)}{f}-\frac{9a\,b^2\cos(fx+e)}{4f}+\frac{b^3\sin(4fx+4e)}{32f}+\frac{\cos(3fx+3e)a\,b^2}{4f}-\frac{3b\sin(2fx+2e)a^2}{4f}-$ |
| norman | $\frac{\frac{(2a^3+4a\,b^2)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3b(4a^2+b^2)x}{8}+\frac{2a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{6(a^3+2a\,b^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2(3a^3+8a\,b^2)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a*b^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1
/2*e)-a^3*cos(f*x+e))
```

**Maxima [A]**
time = 0.28, size = 105, normalized size = 0.87

$$\frac{24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2b+32\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)ab^2+(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))b^3-32\,a^3\cos(fx+e)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b + 32*(cos(f*x + e)^3 - 3*co
s(f*x + e))*a*b^2 + (12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*b^3 - 32*a^3*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.35, size = 98, normalized size = 0.81

$$\frac{8\,ab^2\cos(fx+e)^3+3\,(4\,a^2b+b^3)fx-8\,(a^3+3\,ab^2)\cos(fx+e)+\left(2\,b^3\cos(fx+e)^3-(12\,a^2b+5\,b^3)\cos(fx+e)\right)\sin(fx+e)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a*b^2*cos(f*x + e)^3 + 3*(4*a^2*b + b^3)*f*x - 8*(a^3 + 3*a*b^2)*cos
(f*x + e) + (2*b^3*cos(f*x + e)^3 - (12*a^2*b + 5*b^3)*cos(f*x + e))*sin(f*
x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.
time = 0.21, size = 233, normalized size = 1.93

$$\begin{cases} -\frac{a^3\cos(e+fx)}{f}+\frac{3a^2bx\sin^2(e+fx)}{2}+\frac{3a^2bx\cos^2(e+fx)}{2}-\frac{3a^2b\sin(e+fx)\cos(e+fx)}{2f}-\frac{3ab^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2ab^2\cos^3(e+fx)}{f}+\frac{3b^3x\sin^4(e+fx)}{8}+\frac{3b^3x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{3b^3x\cos^4(e+fx)}{8}-\frac{5b^3\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{3b^3\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for}\ f\neq 0 \\ x(a+b\sin(e))^3\sin(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*cos(e + f*x)/f + 3*a**2*b*x*sin(e + f*x)**2/2 + 3*a**2*b*x
*cos(e + f*x)**2/2 - 3*a**2*b*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*b**2*si
n(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*cos(e + f*x)**3/f + 3*b**3*x*sin(e
+ f*x)**4/8 + 3*b**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**3*x*cos(e +
 f*x)**4/8 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**3*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e), True))
```

**Giac [A]**
time = 0.44, size = 116, normalized size = 0.96

$$\frac{ab^2\cos(3\,fx+3\,e)}{4\,f}-\frac{3\,ab^2\cos(fx+e)}{4\,f}+\frac{b^3\sin(4\,fx+4\,e)}{32\,f}+\frac{3}{8}\,(4\,a^2b+b^3)x-\frac{(2\,a^3+3\,ab^2)\cos(fx+e)}{2\,f}-\frac{(3\,a^2b+b^3)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*b^2*cos(3*f*x + 3*e)/f - 3/4*a*b^2*cos(f*x + e)/f + 1/32*b^3*sin(4*f*
x + 4*e)/f + 3/8*(4*a^2*b + b^3)*x - 1/2*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f -
 1/4*(3*a^2*b + b^3)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 8.08, size = 313, normalized size = 2.59

$$\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,a^2+b^2\right)}{4\,\left(3\,a^2\,b+\frac{3\,b^3}{4}\right)}\right)\,\left(4\,a^2+b^2\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,a^2\,b+\frac{3\,b^3}{4}\right)+2\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+4\,a\,b^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(6\,a^3+12\,a\,b^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(6\,a^3+16\,a\,b^2\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(3\,a^2\,b+\frac{3\,b^3}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,a^2\,b+\frac{11\,b^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(3\,a^2\,b+\frac{11\,b^3}{4}\right)+2\,a^3}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}-\frac{3\,b\,\left(4\,a^2+b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (3*b*atan((3*b*tan(e/2 + (f*x)/2)*(4*a^2 + b^2))/(4*(3*a^2*b + (3*b^3)/4)))
*(4*a^2 + b^2))/(4*f) - (tan(e/2 + (f*x)/2)*(3*a^2*b + (3*b^3)/4) + 2*a^3*t
an(e/2 + (f*x)/2)^6 + 4*a*b^2 + tan(e/2 + (f*x)/2)^4*(12*a*b^2 + 6*a^3) + t
an(e/2 + (f*x)/2)^2*(16*a*b^2 + 6*a^3) - tan(e/2 + (f*x)/2)^7*(3*a^2*b + (3
*b^3)/4) + tan(e/2 + (f*x)/2)^3*(3*a^2*b + (11*b^3)/4) - tan(e/2 + (f*x)/2)
^5*(3*a^2*b + (11*b^3)/4) + 2*a^3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 +
 (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (3*b*(4
*a^2 + b^2)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

# 3.169 $\int (a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=90

$$\frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))}{3f}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*x-2/3*b*(4*a^2+b^2)*cos(f*x+e)/f-5/6*a*b^2*cos(f*x+e)*s
in(f*x+e)/f-1/3*b*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2735, 2813}

$$-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))^3\,dx &= -\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\right.\\ &= \frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\end{aligned}$$

**Mathematica [A]**
time = 0.12, size = 71, normalized size = 0.79

$$\frac{6a(2a^2+3b^2)(e+fx)-9b(4a^2+b^2)\cos(e+fx)+b^3\cos(3(e+fx))-9ab^2\sin(2(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

**Maple [A]**
time = 0.20, size = 76, normalized size = 0.84

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a\,b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos(fx+e)+a^3(fx+e)}{f}$ |
| default | $\frac{-\frac{b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a\,b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos(fx+e)+a^3(fx+e)}{f}$ |
| risch | $a^3x+\frac{3a\,b^2x}{2}-\frac{3b\cos(fx+e)a^2}{f}-\frac{3b^3\cos(fx+e)}{4f}+\frac{b^3\cos(3fx+3e)}{12f}-\frac{3\sin(2fx+2e)a\,b^2}{4f}$ |
| norman | $\frac{(a^3+\frac{3}{2}a\,b^2)x+(a^3+\frac{3}{2}a\,b^2)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{18a^2b+4}{3f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+a^3*(f*x+e))
```

**Maxima [A]**
time = 0.28, size = 79, normalized size = 0.88

$$a^3x+\frac{3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ab^2}{4\,f}+\frac{\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)b^3}{3\,f}-\frac{3\,a^2b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.41, size = 75, normalized size = 0.83

$$\frac{2\,b^3\cos\left(fx+e\right)^3-9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,(2\,a^3+3\,ab^2)fx-6\,(3\,a^2b+b^3)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

**Sympy** [A]
time = 0.13, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+fx\right)}{3f} & \text{for}\ f\neq 0\\ x(a+b\sin\left(e\right))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

**Giac** [A]
time = 0.45, size = 75, normalized size = 0.83

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\,(2\,a^3+3\,ab^2)x-\frac{3\,(4\,a^2b+b^3)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

**Mupad** [B]
time = 6.74, size = 127, normalized size = 1.41

$$a^3\,x-\frac{4\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f}+\frac{8\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3\,f}+\frac{3\,a\,b^2\,x}{2}-\frac{6\,a^2\,b\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f}-\frac{6\,a\,b^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}+\frac{3\,a\,b^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3,x)
```

```
[Out] a^3*x - (4*b^3*cos(e/2 + (f*x)/2)^4)/f + (8*b^3*cos(e/2 + (f*x)/2)^6)/(3*f)
 + (3*a*b^2*x)/2 - (6*a^2*b*cos(e/2 + (f*x)/2)^2)/f - (6*a*b^2*cos(e/2 + (f
*x)/2)^3*sin(e/2 + (f*x)/2))/f + (3*a*b^2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x
)/2))/f
```

# 3.170 $\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=74

$$\frac{1}{2}b\left(6a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

```
[Out] 1/2*b*(6*a^2+b^2)*x-a^3*arctanh(cos(f*x+e))/f-5/2*a*b^2*cos(f*x+e)/f-1/2*b^
2*cos(f*x+e)*(a+b*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2872, 3102, 2814, 3855}

$$-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}+\frac{1}{2}bx\left(6a^2+b^2\right)-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/2 - (a^3*ArcTanh[Cos[e + f*x]])/f - (5*a*b^2*Cos[e + f*
x])/(2*f) - (b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx &= -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e+fx)\left(2a^3+b(6a\right.\\ &= -\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(\\ &= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+\\ &= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2}{}\end{aligned}$$

**Mathematica [A]**
time = 0.11, size = 81, normalized size = 1.09

$$-\frac{-2b(6a^2+b^2)(e+fx)+12ab^2\cos(e+fx)+4a^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+b^3\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
[Out] -1/4*(-2*b*(6*a^2 + b^2)*(e + f*x) + 12*a*b^2*Cos[e + f*x] + 4*a^3*Log[Cos[
(e + f*x)/2]] - 4*a^3*Log[Sin[(e + f*x)/2]] + b^3*Sin[2*(e + f*x)])/f
```

**Maple [A]**
time = 0.26, size = 75, normalized size = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{a^3\ln(\csc(fx+e)-\cot(fx+e))+3a^2b(fx+e)-3a\,b^2\cos(fx+e)+b^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$ |
| default | $\frac{a^3\ln(\csc(fx+e)-\cot(fx+e))+3a^2b(fx+e)-3a\,b^2\cos(fx+e)+b^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$ |

| | |
|---|---|
| risch | $3a^2bx+\frac{b^3x}{2}-\frac{3a\,b^2\mathrm{e}^{i(fx+e)}}{2f}-\frac{3a\,b^2\mathrm{e}^{-i(fx+e)}}{2f}-\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{f}+\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{f}-\frac{b^3\sin(2fx+2e)}{4f}$ |
| norman | $\frac{(3a^2b+\frac{1}{2}b^3)x+\frac{b^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+(3a^2b+\frac{1}{2}b^3)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(9a^2b+\frac{3}{2}b^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(9a^2b+\frac{3}{2}b^3)x\left(\tan^4\left(\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*ln(csc(f*x+e)-cot(f*x+e))+3*a^2*b*(f*x+e)-3*a*b^2*cos(f*x+e)+b^3*(
-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

**Maxima [A]**
time = 0.28, size = 77, normalized size = 1.04

$$\frac{12\,(fx+e)a^2b+(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))b^3-12\,ab^2\cos\left(fx+e\right)-4\,a^3\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*(f*x + e)*a^2*b + (2*f*x + 2*e - sin(2*f*x + 2*e))*b^3 - 12*a*b^2*c
os(f*x + e) - 4*a^3*log(cot(f*x + e) + csc(f*x + e)))/f
```

**Fricas [A]**
time = 0.39, size = 84, normalized size = 1.14

$$-\frac{b^3\cos\left(fx+e\right)\sin\left(fx+e\right)+6\,ab^2\cos\left(fx+e\right)+a^3\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-a^3\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-(6\,a^2b+b^3)fx}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*cos(f*x + e)*sin(f*x + e) + 6*a*b^2*cos(f*x + e) + a^3*log(1/2*co
s(f*x + e) + 1/2) - a^3*log(-1/2*cos(f*x + e) + 1/2) - (6*a^2*b + b^3)*f*x)
/f
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^3\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x), x)
```

**Giac [A]**
time = 0.46, size = 114, normalized size = 1.54

$$\frac{2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+(6\,a^{2}b+b^{3})(fx+e)+\frac{2\left(b^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-6\,ab^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-b^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*f*x + 1/2*e))) + (6*a^2*b + b^3)*(f*x + e) + 2*(
b^3*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*tan(1/2*f*x + 1/2*e)^2 - b^3*tan(1/2*f
*x + 1/2*e) - 6*a*b^2)/(tan(1/2*f*x + 1/2*e)^2 + 1)^2)/f
```

**Mupad [B]**
time = 6.79, size = 259, normalized size = 3.50

$$\frac{a^{3}\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}-\frac{b^{3}\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{3}+6\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{2}\,b+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^{3}}{-2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{3}+6\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{2}\,b+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^{3}}\right)}{f}-\frac{b^{3}\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{6\,a^{2}\,b\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{3}+6\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{2}\,b+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^{3}}{-2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{3}+6\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^{2}\,b+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^{3}}\right)}{f}-\frac{3\,a\,b^{2}\,\cos\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x),x)
```

```
[Out] (a^3*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f - (b^3*atan((b^3*cos(e/2
 + (f*x)/2) + 2*a^3*sin(e/2 + (f*x)/2) + 6*a^2*b*cos(e/2 + (f*x)/2))/(b^3*s
in(e/2 + (f*x)/2) - 2*a^3*cos(e/2 + (f*x)/2) + 6*a^2*b*sin(e/2 + (f*x)/2)))
)/f - (b^3*sin(2*e + 2*f*x))/(4*f) - (6*a^2*b*atan((b^3*cos(e/2 + (f*x)/2)
+ 2*a^3*sin(e/2 + (f*x)/2) + 6*a^2*b*cos(e/2 + (f*x)/2))/(b^3*sin(e/2 + (f*
x)/2) - 2*a^3*cos(e/2 + (f*x)/2) + 6*a^2*b*sin(e/2 + (f*x)/2))))/f - (3*a*b
^2*cos(e + f*x))/f
```

# 3.171 $\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=68

$$3ab^2x-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(a^2-b^2)\cos(e+fx)}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}$$

```
[Out] 3*a*b^2*x-3*a^2*b*arctanh(cos(f*x+e))/f+b*(a^2-b^2)*cos(f*x+e)/f-a^2*cot(f*
x+e)*(a+b*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2871, 3102, 2814, 3855}

$$\frac{b(a^2-b^2)\cos(e+fx)}{f}-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}+3ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] 3*a*b^2*x - (3*a^2*b*ArcTanh[Cos[e + f*x]])/f + (b*(a^2 - b^2)*Cos[e + f*x]
)/f - (a^2*Cot[e + f*x]*(a + b*Sin[e + f*x]))/f
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int \csc(e+fx)\left(3a^2b+3ab^2\right.$$

$$= \frac{b(a^2-b^2)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int c$$

$$= 3ab^2x + \frac{b(a^2-b^2)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx)}{f}$$

$$= 3ab^2x - \frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f} + \frac{b(a^2-b^2)\cos(e+fx)}{f} - \frac{a^2}{}$$

**Mathematica [A]**
time = 0.38, size = 87, normalized size = 1.28

$$\frac{-2b^3\cos(e+fx) - a^3\cot\left(\frac{1}{2}(e+fx)\right) + 6ab\left(b(e+fx) - a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right) + a^3\tan\left(\frac{1}{2}(e+fx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-2*b^3*Cos[e + f*x] - a^3*Cot[(e + f*x)/2] + 6*a*b*(b*(e + f*x) - a*Log[Co
s[(e + f*x)/2]] + a*Log[Sin[(e + f*x)/2]]) + a^3*Tan[(e + f*x)/2])/(2*f)
```

**Maple [A]**
time = 0.29, size = 61, normalized size = 0.90

| method | result |
|---|---|
| derivativedivides | $\frac{-a^3\cot(fx+e)+3a^2b\ln(\csc(fx+e)-\cot(fx+e))+3a\,b^2(fx+e)-b^3\cos(fx+e)}{f}$ |
| default | $\frac{-a^3\cot(fx+e)+3a^2b\ln(\csc(fx+e)-\cot(fx+e))+3a\,b^2(fx+e)-b^3\cos(fx+e)}{f}$ |
| risch | $3a\,b^2x - \frac{b^3e^{i(fx+e)}}{2f} - \frac{b^3e^{-i(fx+e)}}{2f} - \frac{2ia^3}{f\left(e^{2i(fx+e)}-1\right)} + \frac{3a^2b\ln\left(e^{i(fx+e)}-1\right)}{f} - \frac{3a^2b\ln\left(e^{i(fx+e)}+1\right)}{f}$ |

| | |
|---|---|
| norman | $\frac{\frac{a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2b^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2b^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4b^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^3}{2f}-\frac{a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{2f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\tan^2\left(\frac{fx}{2}+\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^3*cot(f*x+e)+3*a^2*b*ln(csc(f*x+e)-cot(f*x+e))+3*a*b^2*(f*x+e)-b^3*
cos(f*x+e))
```

**Maxima** [A]
time = 0.29, size = 73, normalized size = 1.07

$$\frac{6\,(fx+e)ab^2-3\,a^2b(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right))-2\,b^3\cos\left(fx+e\right)-\frac{2\,a^3}{\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(f*x + e)*a*b^2 - 3*a^2*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 2*b^3*cos(f*x + e) - 2*a^3/tan(f*x + e))/f
```

**Fricas** [A]
time = 0.36, size = 107, normalized size = 1.57

$$-\frac{3\,a^2b\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\,a^2b\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2\,a^3\cos\left(fx+e\right)-2\,(3\,ab^2fx-b^3\cos\left(fx+e\right))\sin\left(fx+e\right)}{2\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*a^2*b*log(-1/2*c
os(f*x + e) + 1/2)*sin(f*x + e) + 2*a^3*cos(f*x + e) - 2*(3*a*b^2*f*x - b^3
*cos(f*x + e))*sin(f*x + e))/(f*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^3\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x)**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(72) = 144.
time = 0.44, size = 146, normalized size = 2.15

$$\frac{6\,(fx+e)ab^2+6\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{2\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^3}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(f*x + e)*a*b^2 + 6*a^2*b*log(abs(tan(1/2*f*x + 1/2*e))) + a^3*tan(1
/2*f*x + 1/2*e) - (2*a^2*b*tan(1/2*f*x + 1/2*e)^3 + a^3*tan(1/2*f*x + 1/2*e
)^2 + 2*a^2*b*tan(1/2*f*x + 1/2*e) + 4*b^3*tan(1/2*f*x + 1/2*e) + a^3)/(tan
(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))/f
```

**Mupad** [B]
time = 6.72, size = 194, normalized size = 2.85

$$\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}-\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3+4\,b^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{36\,a^2\,b^4}{36\,a^3\,b^3-36\,a^2\,b^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}+\frac{36\,a^3\,b^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{36\,a^3\,b^3-36\,a^2\,b^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^2,x)
```

```
[Out] (a^3*tan(e/2 + (f*x)/2))/(2*f) - (a^3*tan(e/2 + (f*x)/2)^2 + a^3 + 4*b^3*ta
n(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2) + 2*tan(e/2 + (f*x)/2)^3)) + (6*
a*b^2*atan((36*a^2*b^4)/(36*a^3*b^3 - 36*a^2*b^4*tan(e/2 + (f*x)/2)) + (36*
a^3*b^3*tan(e/2 + (f*x)/2))/(36*a^3*b^3 - 36*a^2*b^4*tan(e/2 + (f*x)/2))))/
f + (3*a^2*b*log(tan(e/2 + (f*x)/2)))/f
```

# 3.172 $\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=79

$$b^3x-\frac{a(a^2+6b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}$$

```
[Out] b^3*x-1/2*a*(a^2+6*b^2)*arctanh(cos(f*x+e))/f-5/2*a^2*b*cot(f*x+e)/f-1/2*a^
2*cot(f*x+e)*csc(f*x+e)*(a+b*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2871, 3100, 2814, 3855}

$$-\frac{a(a^2+6b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] b^3*x - (a*(a^2 + 6*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (5*a^2*b*Cot[e + f*
x])/(2*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc^2(e+f$$

$$= -\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx)}{2f}$$

$$= b^3x-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+}{2f}$$

$$= b^3x-\frac{a(a^2+6b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2 c}{}$$

**Mathematica [A]**
time = 0.45, size = 152, normalized size = 1.92

$$\frac{8b^3e+8b^3fx-12a^2b\cot\left(\frac{1}{2}(e+fx)\right)-a^3\csc^2\left(\frac{1}{2}(e+fx)\right)-4a^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-24ab^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+24ab^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+a^3\sec^2\left(\frac{1}{2}(e+fx)\right)+12a^2b\tan\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (8*b^3*e + 8*b^3*f*x - 12*a^2*b*Cot[(e + f*x)/2] - a^3*Csc[(e + f*x)/2]^2 -
 4*a^3*Log[Cos[(e + f*x)/2]] - 24*a*b^2*Log[Cos[(e + f*x)/2]] + 4*a^3*Log[S
in[(e + f*x)/2]] + 24*a*b^2*Log[Sin[(e + f*x)/2]] + a^3*Sec[(e + f*x)/2]^2
+ 12*a^2*b*Tan[(e + f*x)/2])/(8*f)
```

**Maple [A]**
time = 0.38, size = 86, normalized size = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{a^3\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-3a^2b\cot(fx+e)+3a\,b^2\ln(\csc(fx+e)-\cot(fx+e))+b^3(fx+e)}{f}$ |

| | |
|---|---|
| default | $\frac{a^3\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-3a^2b\cot(fx+e)+3a\,b^2\ln(\csc(fx+e)-\cot(fx+e))+b^3(fx+e)}{f}$ |
| risch | $b^3x-\frac{ia^2\left(ia\,\mathrm{e}^{3i(fx+e)}+ia\,\mathrm{e}^{i(fx+e)}+6b\,\mathrm{e}^{2i(fx+e)}-6b\right)}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)^2}+\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{2f}+\frac{3a\ln\left(\mathrm{e}^{i(fx+e)}-1\right)b^2}{f}-\frac{a^3\ln\left(\mathrm{e}^{i(fx+\right.}}{2f}$ |
| norman | $\frac{b^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+b^3x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{a^3}{8f}+\frac{a^3\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}+3b^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3b^3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{3a^3(}{}}{\tan\left(\frac{fx}{2}\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-3*a^2*b
*cot(f*x+e)+3*a*b^2*ln(csc(f*x+e)-cot(f*x+e))+b^3*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 110, normalized size = 1.39

$$\frac{4\,(fx+e)b^3+a^3\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-6\,ab^2(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right))-\frac{12\,a^2b}{\tan(fx+e)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*b^3 + a^3*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f
*x + e) + 1) + log(cos(f*x + e) - 1)) - 6*a*b^2*(log(cos(f*x + e) + 1) - lo
g(cos(f*x + e) - 1)) - 12*a^2*b/tan(f*x + e))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(78) = 156$.
time = 0.52, size = 164, normalized size = 2.08

$$\frac{4\,b^3fx\cos\left(fx+e\right)^2-4\,b^3fx+12\,a^2b\cos\left(fx+e\right)\sin\left(fx+e\right)+2\,a^3\cos\left(fx+e\right)+\left(a^3+6\,ab^2-\left(a^3+6\,ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(a^3+6\,ab^2-\left(a^3+6\,ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*f*x*cos(f*x + e)^2 - 4*b^3*f*x + 12*a^2*b*cos(f*x + e)*sin(f*x +
 e) + 2*a^3*cos(f*x + e) + (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) - (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e
)^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^3\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x)**3, x)
```

**Giac [A]**
time = 0.46, size = 142, normalized size = 1.80

$$\frac{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,(fx+e)b^3+12\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,(a^3+6\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^3}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*f*x + 1/2*e)^2 + 8*(f*x + e)*b^3 + 12*a^2*b*tan(1/2*f*x +
1/2*e) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (6*a^3*tan(1/2*
f*x + 1/2*e)^2 + 36*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 12*a^2*b*tan(1/2*f*x + 1
/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^2)/f
```

**Mupad [B]**
time = 6.96, size = 234, normalized size = 2.96

$$\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^3+6\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a\,b^2+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^3}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^3+6\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a\,b^2-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^3}\right)}{f}-\frac{a^3\,\mathrm{cot}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{2\,f}-\frac{3\,a^2\,b\,\mathrm{cot}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}+\frac{3\,a\,b^2\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}+\frac{3\,a^2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^3,x)
```

```
[Out] (2*b^3*atan((2*b^3*cos(e/2 + (f*x)/2) + a^3*sin(e/2 + (f*x)/2) + 6*a*b^2*si
n(e/2 + (f*x)/2))/(a^3*cos(e/2 + (f*x)/2) - 2*b^3*sin(e/2 + (f*x)/2) + 6*a*
b^2*cos(e/2 + (f*x)/2))))/f - (a^3*cot(e/2 + (f*x)/2)^2)/(8*f) + (a^3*tan(e
/2 + (f*x)/2)^2)/(8*f) + (a^3*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(
2*f) - (3*a^2*b*cot(e/2 + (f*x)/2))/(2*f) + (3*a*b^2*log(sin(e/2 + (f*x)/2)
/cos(e/2 + (f*x)/2)))/f + (3*a^2*b*tan(e/2 + (f*x)/2))/(2*f)
```

# 3.173 $\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=109

$$-\frac{b(3a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a(2a^2+9b^2)\cot(e+fx)}{3f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx}{}$$

```
[Out] -1/2*b*(3*a^2+2*b^2)*arctanh(cos(f*x+e))/f-1/3*a*(2*a^2+9*b^2)*cot(f*x+e)/f
-7/6*a^2*b*cot(f*x+e)*csc(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2*(a+b*sin
(f*x+e))/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2871, 3100, 2827, 3852, 8, 3855}

$$-\frac{a(2a^2+9b^2)\cot(e+fx)}{3f}-\frac{b(3a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -1/2*(b*(3*a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/f - (a*(2*a^2 + 9*b^2)*Cot[e
 + f*x])/(3*f) - (7*a^2*b*Cot[e + f*x]*Csc[e + f*x])/(6*f) - (a^2*Cot[e + f
*x]*Csc[e + f*x]^2*(a + b*Sin[e + f*x]))/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
```

```
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx &= -\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx))}{3f}+\frac{1}{3}\int \csc^3(e+f\\ &= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+}{3f}\\ &= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+}{3f}\\ &= -\frac{b(3a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx}{6f}\\ &= -\frac{b(3a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a(2a^2+9b^2)\cot(e+fx)}{3f}-\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 525 vs. 2(109) = 218.
time = 6.14, size = 525, normalized size = 4.82

$$\frac{\left(-2a^3\cos\left(\frac{1}{2}(e+fx)\right)-9ab^2\cos\left(\frac{1}{2}(e+fx)\right)\right)\csc\left(\frac{1}{2}(e+fx)\right)(b+a\csc(e+fx))^3\sin^3(e+fx)}{6f(a+b\sin(e+fx))^3}-\frac{3a^2b\csc^2\left(\frac{1}{2}(e+fx)\right)(b+a\csc(e+fx))^3\sin^3(e+fx)}{8f(a+b\sin(e+fx))^3}-\frac{a^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)(b+a\csc(e+fx))^3\sin^3(e+fx)}{24f(a+b\sin(e+fx))^3}+\frac{(-3a^2b-2b^3)(b+a\csc(e+fx))^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\sin^3(e+fx)}{2f(a+b\sin(e+fx))^3}+\frac{(3a^2b+2b^3)(b+a\csc(e+fx))^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^3(e+fx)}{2f(a+b\sin(e+fx))^3}+\frac{3a^2b(b+a\csc(e+fx))^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin^3(e+fx)}{8f(a+b\sin(e+fx))^3}+\frac{(b+a\csc(e+fx))^3\sec\left(\frac{1}{2}(e+fx)\right)\left(2a^3\sin\left(\frac{1}{2}(e+fx)\right)+9ab^2\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^3(e+fx)}{6f(a+b\sin(e+fx))^3}+\frac{a^3(b+a\csc(e+fx))^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin^3(e+fx)\tan\left(\frac{1}{2}(e+fx)\right)}{24f(a+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^3*Cos[(e + f*x)/2] - 9*a*b^2*Cos[(e + f*x)/2])*Csc[(e + f*x)/2]*(b +
 a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) - (3*a^2*b*
Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e
 + f*x])^3) - (a^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])
^3*Sin[e + f*x]^3)/(24*f*(a + b*Sin[e + f*x])^3) + ((-3*a^2*b - 2*b^3)*(b +
 a*Csc[e + f*x])^3*Log[Cos[(e + f*x)/2]]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e
+ f*x])^3) + ((3*a^2*b + 2*b^3)*(b + a*Csc[e + f*x])^3*Log[Sin[(e + f*x)/2]
]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e + f*x])^3) + (3*a^2*b*(b + a*Csc[e + f*
x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e + f*x])^3) + ((b
 + a*Csc[e + f*x])^3*Sec[(e + f*x)/2]*(2*a^3*Sin[(e + f*x)/2] + 9*a*b^2*Sin
[(e + f*x)/2])*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) + (a^3*(b + a*C
sc[e + f*x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3*Tan[(e + f*x)/2])/(24*f*(a
 + b*Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.46, size = 99, normalized size = 0.91

| method | result |
|---|---|
| derivativedivides | $\frac{a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+3a^2b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-3\cot(fx+e)a\,b^2+b^3\ln(\csc(}{f}$ |
| default | $\frac{a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+3a^2b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-3\cot(fx+e)a\,b^2+b^3\ln(\csc(}{f}$ |
| risch | $\frac{a\left(-18ib^2e^{4i(fx+e)}+9ab\,e^{5i(fx+e)}+12ia^2e^{2i(fx+e)}+36ib^2e^{2i(fx+e)}-4ia^2-18ib^2-9ab\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}-\frac{3a^2b\ln\left(e^{i(fx+e)}+1\right)}{2f}$ |
| norman | $\frac{-\frac{a^3}{24f}+\frac{a^3\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{21a^2b\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{9a^2b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{33a^2b\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{a\left(a^2+3b^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+3*a^2*b*(-1/2*csc(f*x+e)*cot(f*
x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-3*cot(f*x+e)*a*b^2+b^3*ln(csc(f*x+e)-co
t(f*x+e)))
```

**Maxima [A]**
time = 0.28, size = 127, normalized size = 1.17

$$\frac{9\,a^2b\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-6\,b^3(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right))-\frac{36\,ab^2}{\tan(fx+e)}-\frac{4\left(3\tan(fx+e)^2+1\right)a^3}{\tan(fx+e)^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^2*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1)
+ log(cos(f*x + e) - 1)) - 6*b^3*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 36*a*b^2/tan(f*x + e) - 4*(3*tan(f*x + e)^2 + 1)*a^3/tan(f*x + e)^3
)/f
```

**Fricas [A]**
time = 0.40, size = 203, normalized size = 1.86

$$\frac{18\,a^2b\cos\left(fx+e\right)\sin\left(fx+e\right)-4\,\left(2\,a^3+9\,ab^2\right)\cos\left(fx+e\right)^3+3\,\left(3\,a^2b+2\,b^3-\left(3\,a^2b+2\,b^3\right)\cos\left(fx+e\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\,\left(3\,a^2b+2\,b^3-\left(3\,a^2b+2\,b^3\right)\cos\left(fx+e\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+12\,\left(a^3+3\,ab^2\right)\cos\left(fx+e\right)}{12\,\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^2*b*cos(f*x + e)*sin(f*x + e) - 4*(2*a^3 + 9*a*b^2)*cos(f*x + e)
^3 + 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x
 + e) + 1/2)*sin(f*x + e) - 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x
+ e)^2)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + 12*(a^3 + 3*a*b^2)*cos(
f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^3\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x)**4, x)
```

**Giac [A]**
time = 0.45, size = 201, normalized size = 1.84

$$\frac{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+36\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,\left(3\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{66\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+44\,b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^3}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*f*x + 1/2*e)^3 + 9*a^2*b*tan(1/2*f*x + 1/2*e)^2 + 9*a^3*t
an(1/2*f*x + 1/2*e) + 36*a*b^2*tan(1/2*f*x + 1/2*e) + 12*(3*a^2*b + 2*b^3)*
log(abs(tan(1/2*f*x + 1/2*e))) - (66*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 44*b^3*
tan(1/2*f*x + 1/2*e)^3 + 9*a^3*tan(1/2*f*x + 1/2*e)^2 + 36*a*b^2*tan(1/2*f*
x + 1/2*e)^2 + 9*a^2*b*tan(1/2*f*x + 1/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^3)/
f
```

**Mupad [B]**
time = 6.78, size = 150, normalized size = 1.38

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{3\,a^2\,b}{2}+b^3\right)}{f}+\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(3\,a^3+12\,a\,b^2\right)+\frac{a^3}{3}+3\,a^2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,a^3}{8}+\frac{3\,a\,b^2}{2}\right)}{f}+\frac{3\,a^2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^4,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*((3*a^2*b)/2 + b^3))/f + (a^3*tan(e/2 + (f*x)/2)^3
)/(24*f) - (cot(e/2 + (f*x)/2)^3*(tan(e/2 + (f*x)/2)^2*(12*a*b^2 + 3*a^3) +
 a^3/3 + 3*a^2*b*tan(e/2 + (f*x)/2)))/(8*f) + (tan(e/2 + (f*x)/2)*((3*a*b^2
)/2 + (3*a^3)/8))/f + (3*a^2*b*tan(e/2 + (f*x)/2)^2)/(8*f)
```

# 3.174 $\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=134

$$-\frac{3a(a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{b(2a^2+b^2)\cot(e+fx)}{f}-\frac{3a(a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b}{}$$

```
[Out] -3/8*a*(a^2+4*b^2)*arctanh(cos(f*x+e))/f-b*(2*a^2+b^2)*cot(f*x+e)/f-3/8*a*(
a^2+4*b^2)*cot(f*x+e)*csc(f*x+e)/f-3/4*a^2*b*cot(f*x+e)*csc(f*x+e)^2/f-1/4*
a^2*cot(f*x+e)*csc(f*x+e)^3*(a+b*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2871, 3100, 2827, 3853, 3855, 3852, 8}

$$-\frac{b(2a^2+b^2)\cot(e+fx)}{f}-\frac{3a(a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{3a(a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*(a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (b*(2*a^2 + b^2)*Cot[e +
 f*x])/f - (3*a*(a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (3*a^2*b*C
ot[e + f*x]*Csc[e + f*x]^2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3*(a + b
*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+fx))}{4f} + \frac{1}{4}\int \csc^4(e+f$$

$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f} - \frac{a^2\cot(e+fx)\csc^3(e+fx)(a+}{4f}$$

$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f} - \frac{a^2\cot(e+fx)\csc^3(e+fx)(a+}{4f}$$

$$= -\frac{3a(a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f} - \frac{3a^2b\cot(e+fx)\csc^2(e-}{4f}$$

$$= -\frac{3a(a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{b(2a^2+b^2)\cot(e+fx)}{f} - \frac{3}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 322 vs. 2(134) = 268.
time = 6.14, size = 322, normalized size = 2.40

$$\frac{\left(-2a^2b\cos\left(\frac{1}{2}(e+fx)\right)-b^3\cos\left(\frac{1}{2}(e+fx)\right)\right)\csc\left(\frac{1}{2}(e+fx)\right)}{2f} - \frac{3(a^3+4ab^2)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} - \frac{a^2b\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a^3\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{3(a^3+4ab^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{3(a^3+4ab^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{3(a^3+4ab^2)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a^3\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(2a^2b\sin\left(\frac{1}{2}(e+fx)\right)+b^3\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{a^2b\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^2*b*Cos[(e + f*x)/2] - b^3*Cos[(e + f*x)/2])*Csc[(e + f*x)/2])/(2*f)
 - (3*(a^3 + 4*a*b^2)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*b*Cot[(e + f*x)/2]*
Csc[(e + f*x)/2]^2)/(8*f) - (a^3*Csc[(e + f*x)/2]^4)/(64*f) - (3*(a^3 + 4*a
*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + (3*(a^3 + 4*a*b^2)*Log[Sin[(e + f*x)/2
]])/(8*f) + (3*(a^3 + 4*a*b^2)*Sec[(e + f*x)/2]^2)/(32*f) + (a^3*Sec[(e + f
*x)/2]^4)/(64*f) + (Sec[(e + f*x)/2]*(2*a^2*b*Sin[(e + f*x)/2] + b^3*Sin[(e
 + f*x)/2]))/(2*f) + (a^2*b*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(8*f)
```

**Maple** [A]
time = 0.51, size = 129, normalized size = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{a^3\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+3a^2b\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+3a}{f}$ |
| default | $\frac{a^3\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+3a^2b\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+3a}{f}$ |
| risch | $-\frac{i\left(3ia^3e^{7i(fx+e)}+12ia\,b^2e^{7i(fx+e)}-11ia^3e^{5i(fx+e)}-12ia\,b^2e^{5i(fx+e)}+8b^3e^{6i(fx+e)}-11ia^3e^{3i(fx+e)}-12ia\,b^2e^{3i(fx+}\right.}{4f\left(e^{2i(fx-}\right.}$ |

| | |
|---|---|
| norman | $-\frac{a^3}{64f}+\frac{a^3\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}-\frac{\left(8a^3+21a\,b^2\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{\left(27a^3+72a\,b^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{\left(49a^3+132a\,b^2\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-c
ot(f*x+e)))+3*a^2*b*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+3*a*b^2*(-1/2*csc(f*
x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))-b^3*cot(f*x+e))
```

**Maxima** [A]
time = 0.29, size = 175, normalized size = 1.31

$$\frac{a^3\left(\frac{2\left(3\cos(fx+e)^3-5\cos(fx+e)\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}-3\log\left(\cos\left(fx+e\right)+1\right)+3\log\left(\cos\left(fx+e\right)-1\right)\right)+12\,ab^2\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{16\,b^3}{\tan(fx+e)}-\frac{16\left(3\tan(fx+e)^2+1\right)a^2b}{\tan(fx+e)^3}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*a*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 16*b^3/tan(f*x + e) - 16*(3*tan(f*x + e)^2 + 1)*a^2*b/tan(f
*x + e)^3)/f
```

**Fricas** [A]
time = 0.35, size = 251, normalized size = 1.87

$$\frac{6\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^3-2\left(5\,a^3+12\,ab^2\right)\cos\left(fx+e\right)-3\left(\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^4+a^3+4\,ab^2-2\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+3\left(\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^4+a^3+4\,ab^2-2\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+16\left(\left(2\,a^2b+b^3\right)\cos\left(fx+e\right)^3-\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{16\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 4*a*b^2)*cos(f*x + e)^3 - 2*(5*a^3 + 12*a*b^2)*cos(f*x + e)
- 3*((a^3 + 4*a*b^2)*cos(f*x + e)^4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos
(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 + 4*a*b^2)*cos(f*x + e)^
4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
 + 1/2) + 16*((2*a^2*b + b^3)*cos(f*x + e)^3 - (3*a^2*b + b^3)*cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.48, size = 269, normalized size = 2.01

$$\frac{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+8\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+72\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+32\,b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,(a^3+4\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{50\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+200\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+72\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+32\,b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^3}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*f*x + 1/2*e)^4 + 8*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 8*a^3*t
an(1/2*f*x + 1/2*e)^2 + 24*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 72*a^2*b*tan(1/2*
f*x + 1/2*e) + 32*b^3*tan(1/2*f*x + 1/2*e) + 24*(a^3 + 4*a*b^2)*log(abs(tan
(1/2*f*x + 1/2*e))) - (50*a^3*tan(1/2*f*x + 1/2*e)^4 + 200*a*b^2*tan(1/2*f*
x + 1/2*e)^4 + 72*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 32*b^3*tan(1/2*f*x + 1/2*e
)^3 + 8*a^3*tan(1/2*f*x + 1/2*e)^2 + 24*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 8*a^
2*b*tan(1/2*f*x + 1/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^4)/f
```

**Mupad** [B]
time = 6.86, size = 203, normalized size = 1.51

$$\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(2\,a^3+6\,a\,b^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(18\,a^2\,b+8\,b^3)+\frac{a^3}{4}+2\,a^2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{16\,f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a^3}{8}+\frac{3\,a\,b^2}{8}\right)}{f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\left(\frac{3\,a^3}{8}+\frac{3\,a\,b^2}{2}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{9\,a^2\,b}{8}+\frac{b^3}{2}\right)}{f}+\frac{a^2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^5,x)
```

```
[Out] (a^3*tan(e/2 + (f*x)/2)^4)/(64*f) - (cot(e/2 + (f*x)/2)^4*(tan(e/2 + (f*x)/
2)^2*(6*a*b^2 + 2*a^3) + tan(e/2 + (f*x)/2)^3*(18*a^2*b + 8*b^3) + a^3/4 +
2*a^2*b*tan(e/2 + (f*x)/2)))/(16*f) + (tan(e/2 + (f*x)/2)^2*((3*a*b^2)/8 +
a^3/8))/f + (log(tan(e/2 + (f*x)/2))*((3*a*b^2)/2 + (3*a^3)/8))/f + (tan(e/
2 + (f*x)/2)*((9*a^2*b)/8 + b^3/2))/f + (a^2*b*tan(e/2 + (f*x)/2)^3)/(8*f)
```

# 3.175 $\int (a+b\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=137

$$\frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x-\frac{ab(19a^2+16b^2)\cos(e+fx)}{6f}-\frac{b^2(26a^2+9b^2)\cos(e+fx)\sin(e+fx)}{24f}-\frac{7ab\cos(e+$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*x-1/6*a*b*(19*a^2+16*b^2)*cos(f*x+e)/f-1/24*b^
2*(26*a^2+9*b^2)*cos(f*x+e)*sin(f*x+e)/f-7/12*a*b*cos(f*x+e)*(a+b*sin(f*x+e
))^2/f-1/4*b*cos(f*x+e)*(a+b*sin(f*x+e))^3/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2735, 2832, 2813}

$$-\frac{ab(19a^2+16b^2)\cos(e+fx)}{6f}-\frac{b^2(26a^2+9b^2)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(8a^4+24a^2b^2+3b^4\right)-\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f}-\frac{7ab\cos(e+fx)(a+b\sin(e+fx))^2}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 - (a*b*(19*a^2 + 16*b^2)*Cos[e + f*x])/(
6*f) - (b^2*(26*a^2 + 9*b^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - (7*a*b*Cos
[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f
*x])^3)/(4*f)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
```

```
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))^4\,dx = -\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{4}\int (a+b\sin(e+fx))^2\left(4a^2+3b^2+7a\right.$$
$$= -\frac{7ab\cos(e+fx)(a+b\sin(e+fx))^2}{12f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{12}$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x-\frac{ab(19a^2+16b^2)\cos(e+fx)}{6f}-\frac{b^2(26a^2+9b^2)\cos(e}{24}$$

**Mathematica [A]**
time = 0.24, size = 106, normalized size = 0.77

$$\frac{-96ab(4a^2+3b^2)\cos(e+fx)+32ab^3\cos(3(e+fx))+3(4(8a^4+24a^2b^2+3b^4)(e+fx)-8(6a^2b^2+b^4)\sin(2(e+fx))+b^4\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^4,x]
[Out] (-96*a*b*(4*a^2 + 3*b^2)*Cos[e + f*x] + 32*a*b^3*Cos[3*(e + f*x)] + 3*(4*(8
*a^4 + 24*a^2*b^2 + 3*b^4)*(e + f*x) - 8*(6*a^2*b^2 + b^4)*Sin[2*(e + f*x)]
 + b^4*Sin[4*(e + f*x)]))/(96*f)
```

**Maple [A]**
time = 0.27, size = 116, normalized size = 0.85

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{4a\,b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+6a^2b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{}\right.}{f}$ |
| default | $\frac{b^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{4a\,b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+6a^2b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{}\right.}{f}$ |
| risch | $a^4x+3x\,a^2b^2+\frac{3x\,b^4}{8}-\frac{4a^3b\cos(fx+e)}{f}-\frac{3a\,b^3\cos(fx+e)}{f}+\frac{b^4\sin(4fx+4e)}{32f}+\frac{a\,b^3\cos(3fx+3e)}{3f}-\frac{3\sin}{}$ |
| norman | $\frac{(a^4+3a^2b^2+\frac{3}{8}b^4)x+(a^4+3a^2b^2+\frac{3}{8}b^4)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a^4+12a^2b^2+\frac{3}{2}b^4)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a^4+12a^2b^2+\frac{3}{2}b^4)}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4/3*
a*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+6*a^2*b^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)-4*b*a^3*cos(f*x+e)+a^4*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 121, normalized size = 0.88

$$a^4x + \frac{3\,(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right))a^2b^2}{2\,f} + \frac{4\,\left(\cos\left(fx + e\right)^3 - 3\,\cos\left(fx + e\right)\right)ab^3}{3\,f} + \frac{(12\,fx + 12\,e + \sin\left(4\,fx + 4\,e\right) - 8\,\sin\left(2\,fx + 2\,e\right))b^4}{32\,f} - \frac{4\,a^3b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 3/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b^2/f + 4/3*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a*b^3/f + 1/32*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*b^4/f - 4*a^3*b*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.53, size = 111, normalized size = 0.81

$$\frac{32\,ab^3\cos\left(fx + e\right)^3 + 3\,(8\,a^4 + 24\,a^2b^2 + 3\,b^4)fx - 96\,(a^3b + ab^3)\cos\left(fx + e\right) + 3\,\left(2\,b^4\cos\left(fx + e\right)^3 - (24\,a^2b^2 + 5\,b^4)\cos\left(fx + e\right)\right)\sin\left(fx + e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a*b^3*cos(f*x + e)^3 + 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*f*x - 96*(a^
3*b + a*b^3)*cos(f*x + e) + 3*(2*b^4*cos(f*x + e)^3 - (24*a^2*b^2 + 5*b^4)*
cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [A]
time = 0.20, size = 240, normalized size = 1.75

$$\begin{cases} a^4x - \frac{4a^3b\cos\left(e+fx\right)}{f} + 3a^2b^2x\sin^2\left(e + fx\right) + 3a^2b^2x\cos^2\left(e + fx\right) - \frac{3a^2b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{4ab^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{8ab^3\cos^3\left(e+fx\right)}{3f} + \frac{3b^4x\sin^4\left(e+fx\right)}{8} + \frac{3b^4x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} + \frac{3b^4x\cos^4\left(e+fx\right)}{8} - \frac{5b^4\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f} - \frac{3b^4\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f} & \text{for}\: f \neq 0 \\ x(a + b\sin\left(e\right))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*cos(e + f*x)/f + 3*a**2*b**2*x*sin(e + f*x)**2
 + 3*a**2*b**2*x*cos(e + f*x)**2 - 3*a**2*b**2*sin(e + f*x)*cos(e + f*x)/f
- 4*a*b**3*sin(e + f*x)**2*cos(e + f*x)/f - 8*a*b**3*cos(e + f*x)**3/(3*f)
+ 3*b**4*x*sin(e + f*x)**4/8 + 3*b**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 +
 3*b**4*x*cos(e + f*x)**4/8 - 5*b**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**4*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**4,
True))
```

**Giac** [A]
time = 0.43, size = 112, normalized size = 0.82

$$\frac{ab^3\cos\left(3\,fx + 3\,e\right)}{3\,f} + \frac{b^4\sin\left(4\,fx + 4\,e\right)}{32\,f} + \frac{1}{8}\,(8\,a^4 + 24\,a^2b^2 + 3\,b^4)x - \frac{(4\,a^3b + 3\,ab^3)\cos\left(fx + e\right)}{f} - \frac{(6\,a^2b^2 + b^4)\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a*b^3*cos(3*f*x + 3*e)/f + 1/32*b^4*sin(4*f*x + 4*e)/f + 1/8*(8*a^4 + 2
4*a^2*b^2 + 3*b^4)*x - (4*a^3*b + 3*a*b^3)*cos(f*x + e)/f - 1/4*(6*a^2*b^2
+ b^4)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 6.95, size = 114, normalized size = 0.83

$$\frac{\frac{3\,b^4\,\sin\left(4\,e+4\,f\,x\right)}{4}-6\,b^4\,\sin\left(2\,e+2\,f\,x\right)+8\,a\,b^3\,\cos\left(3\,e+3\,f\,x\right)-36\,a^2\,b^2\,\sin\left(2\,e+2\,f\,x\right)-72\,a\,b^3\,\cos\left(e+f\,x\right)-96\,a^3\,b\,\cos\left(e+f\,x\right)+24\,a^4\,f\,x+9\,b^4\,f\,x+72\,a^2\,b^2\,f\,x}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^4,x)
```

```
[Out] ((3*b^4*sin(4*e + 4*f*x))/4 - 6*b^4*sin(2*e + 2*f*x) + 8*a*b^3*cos(3*e + 3*
f*x) - 36*a^2*b^2*sin(2*e + 2*f*x) - 72*a*b^3*cos(e + f*x) - 96*a^3*b*cos(e
 + f*x) + 24*a^4*f*x + 9*b^4*f*x + 72*a^2*b^2*f*x)/(24*f)
```

# 3.176 $\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=110

$$-\frac{a(2a^2+b^2)\,x}{2b^4}+\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{(3a^2+2b^2)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4-1/3*(3*a^2+2*b^2)*cos(x)/b^3+1/2*a*cos(x)*sin(x)/b
^2-1/3*cos(x)*sin(x)^2/b+2*a^4*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^4
/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2872, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{ax(2a^2+b^2)}{2b^4}-\frac{(3a^2+2b^2)\cos(x)}{3b^3}+\frac{2a^4\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\sin(x)\cos(x)}{2b^2}-\frac{\sin^2(x)\cos(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + b^2)*x)/b^4 + (2*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^4*Sqrt[a^2 - b^2]) - ((3*a^2 + 2*b^2)*Cos[x])/(3*b^3) + (a*Cos[x]*S
in[x])/(2*b^2) - (Cos[x]*Sin[x]^2)/(3*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx = -\frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{\sin(x)(2a+2b\sin(x)-3a\sin^2(x))}{a+b\sin(x)}dx}{3b}$$
$$= \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{-3a^2+ab\sin(x)+2(3a^2+2b^2)\sin^2(x)}{a+b\sin(x)}dx}{6b^2}$$
$$= -\frac{(3a^2+2b^2)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{-3a^2b-3a(2a^2+b^2)\sin(x)}{a+b\sin(x)}dx}{6b^3}$$
$$= -\frac{a(2a^2+b^2)x}{2b^4} - \frac{(3a^2+2b^2)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{a^4\int \frac{1}{a+b\sin(x)}$$
$$= -\frac{a(2a^2+b^2)x}{2b^4} - \frac{(3a^2+2b^2)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{(2a^4)\,\text{Subst}(}{}$$
$$= -\frac{a(2a^2+b^2)x}{2b^4} - \frac{(3a^2+2b^2)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} - \frac{(4a^4)\,\text{Subst}(}{}$$
$$= -\frac{a(2a^2+b^2)x}{2b^4} + \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}} - \frac{(3a^2+2b^2)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \text{cos}$$

**Mathematica** [A]
time = 0.18, size = 98, normalized size = 0.89

$$\frac{-6a(2a^2+b^2)x + \frac{24a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 3b(4a^2+3b^2)\cos(x) + b^3\cos(3x) + 3ab^2\sin(2x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*x + (24*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 3*b*(4*a^2 + 3*b^2)*Cos[x] + b^3*Cos[3*x] + 3*a*b^2*Sin[2*
x])/(12*b^4)
```

**Maple** [A]
time = 0.19, size = 143, normalized size = 1.30

| method | result |
|---|---|
| default | $\frac{2a^4\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}} - \frac{2\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}+a^2b\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(2a^2b+2b^3\right)\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{x}{2}\right)}{2}+a^2b+\frac{2b^3}{3}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{a\left(2a^2+b^2\right)\arctan(}{2}\right.}{b^4}$ |

| | |
|---|---|
| risch | $-\frac{a^3 x}{b^4}-\frac{a x}{2 b^2}-\frac{e^{i x} a^2}{2 b^3}-\frac{3 e^{i x}}{8 b}-\frac{e^{-i x} a^2}{2 b^3}-\frac{3 e^{-i x}}{8 b}-\frac{a^4 \ln\left(e^{i x}+\frac{i a \sqrt{-a^2+b^2}-a^2+b^2}{b \sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\, b^4}+\frac{a^4 \ln\left(e^{i x}+\frac{i a \sqrt{-a^2+}}{b \sqrt{-a^2}}\right.}{\sqrt{-a^2+b^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-
2/b^4*((1/2*a*b^2*tan(1/2*x)^5+a^2*b*tan(1/2*x)^4+(2*a^2*b+2*b^3)*tan(1/2*x
)^2-1/2*a*b^2*tan(1/2*x)+a^2*b+2/3*b^3)/(tan(1/2*x)^2+1)^3+1/2*a*(2*a^2+b^2
)*arctan(tan(1/2*x)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 333, normalized size = 3.03

$$\left[-\frac{3 \sqrt{-a^2+b^2}\, a^4 \log\left(\frac{(2 a^2-b^2) \cos(x)^2-2 a b \sin(x)-a^2-b^2+2 (a \cos(x) \sin(x)+b \cos(x)) \sqrt{-a^2+b^2}}{b^2 \cos(x)^2-2 a b \sin(x)-a^2-b^2}\right)-2 (a^2 b^3-b^5) \cos(x)^3-3 (a^3 b^2-a b^4) \cos(x) \sin(x)+3 (2 a^5-a^3 b^2-a b^4) x+6 (a^4 b-b^5) \cos(x)}{6 (a^2 b^4-b^6)}, -\frac{6 \sqrt{a^2-b^2}\, a^4 \arctan\left(-\frac{a \sin(x)+b}{\sqrt{a^2-b^2} \cos(x)}\right)-2 (a^2 b^3-b^5) \cos(x)^3-3 (a^3 b^2-a b^4) \cos(x) \sin(x)+3 (2 a^5-a^3 b^2-a b^4) x+6 (a^4 b-b^5) \cos(x)}{6 (a^2 b^4-b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
- 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*
b^4)*cos(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x)
)/(a^2*b^4 - b^6), -1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(x) + b)/(sqrt
(a^2 - b^2)*cos(x))) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*b^4)*cos
(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x))/(a^2*b
^4 - b^6)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.44, size = 149, normalized size = 1.35

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}\,b^4}-\frac{(2a^3+ab^2)x}{2b^4}-\frac{3ab\tan\left(\frac{1}{2}x\right)^5+6a^2\tan\left(\frac{1}{2}x\right)^4+12a^2\tan\left(\frac{1}{2}x\right)^2+12b^2\tan\left(\frac{1}{2}x\right)^2-3ab\tan\left(\frac{1}{2}x\right)+6a^2+4b^2}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^4/(sqrt(a^2 - b^2)*b^4) - 1/2*(2*a^3 + a*b^2)*x/b^4 - 1/3*(3*a*b*ta
n(1/2*x)^5 + 6*a^2*tan(1/2*x)^4 + 12*a^2*tan(1/2*x)^2 + 12*b^2*tan(1/2*x)^2
 - 3*a*b*tan(1/2*x) + 6*a^2 + 4*b^2)/((tan(1/2*x)^2 + 1)^3*b^3)
```

**Mupad [B]**
time = 7.22, size = 1075, normalized size = 9.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*sin(x)),x)
```

```
[Out] - ((2*(3*a^2 + 2*b^2))/(3*b^3) + (a*tan(x/2)^5)/b^2 + (2*a^2*tan(x/2)^4)/b^
3 + (4*tan(x/2)^2*(a^2 + b^2))/b^3 - (a*tan(x/2))/b^2)/(3*tan(x/2)^2 + 3*ta
n(x/2)^4 + tan(x/2)^6 + 1) - (a*atan((8*a^4*tan(x/2))/(8*a^4 + (40*a^6)/b^2
 + (48*a^8)/b^4) + (40*a^6*tan(x/2))/(40*a^6 + 8*a^4*b^2 + (48*a^8)/b^2) +
(48*a^8*tan(x/2))/(48*a^8 + 8*a^4*b^4 + 40*a^6*b^2))*(2*a^2 + b^2))/b^4 - (
a^4*atan(((a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(2*
a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 + (a^4*((8*(2*a^2*b^10 +
2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b + (a^4*(32*a^2*b^3 + (8*tan(x/2)*(12*
a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1/2
)))*1i)/(b^4*(b^2 - a^2)^(1/2)) + (a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3
))/b^8 + (8*tan(x/2)*(2*a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 -
 (a^4*((8*(2*a^2*b^10 + 2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b - (a^4*(32*a^
2*b^3 + (8*tan(x/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))
))/(b^4*(b^2 - a^2)^(1/2)))*1i)/(b^4*(b^2 - a^2)^(1/2)))/((16*(2*a^10 + a^8
*b^2))/b^8 + (16*tan(x/2)*(8*a^11 + 2*a^7*b^4 + 8*a^9*b^2))/b^9 + (a^4*((8*
(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(2*a^3*b^9 + 7*a^5*b^7
```

```
 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 + (a^4*((8*(2*a^2*b^10 + 2*a^4*b^8))/b^8 + (
64*a^5*tan(x/2))/b + (a^4*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b^13 - 8*a^3*b^11
))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^
2)^(1/2)) - (a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(
2*a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 - (a^4*((8*(2*a^2*b^10
+ 2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b - (a^4*(32*a^2*b^3 + (8*tan(x/2)*(1
2*a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1
/2))))/(b^4*(b^2 - a^2)^(1/2))))*2i)/(b^4*(b^2 - a^2)^(1/2))
```

# 3.177 $\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=82

$$\frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3+a*cos(x)/b^2-1/2*cos(x)*sin(x)/b-2*a^3*arctan((b+a*ta
n(1/2*x))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2872, 3102, 2814, 2739, 632, 210}

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(b^3*Sqrt[a^2 - b^2]) + (a*Cos[x])/b^2 - (Cos[x]*Sin[x])/(2*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)\sin(x)}{2b}+\frac{\int \frac{a+b\sin(x)-2a\sin^2(x)}{a+b\sin(x)}\,dx}{2b}\\
&= \frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}+\frac{\int \frac{ab+(2a^2+b^2)\sin(x)}{a+b\sin(x)}\,dx}{2b^2}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}-\frac{a^3\int \frac{1}{a+b\sin(x)}\,dx}{b^3}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}-\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}+\frac{(4a^3)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\right.}{b^3}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}
\end{aligned}
$$

**Mathematica [A]**
time = 0.08, size = 78, normalized size = 0.95

$$\frac{4a^2x+2b^2x-\frac{8a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+4ab\cos(x)-b^2\sin(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x - (8*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + 4*a*b*Cos[x] - b^2*Sin[2*x])/(4*b^3)
```

**Maple [A]**
time = 0.15, size = 112, normalized size = 1.37

| method | result |
|---|---|
| default | $-\frac{2a^3\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{\frac{2\left(\frac{b^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}+ab\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+(2a^2+b^2)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^3}$ |
| risch | $\frac{x\,a^2}{b^3}+\frac{x}{2b}+\frac{a\,e^{ix}}{2b^2}+\frac{a\,e^{-ix}}{2b^2}+\frac{ia^3\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{ia^3\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{\sin(2}{4b}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
+2/b^3*((1/2*b^2*tan(1/2*x)^3+a*b*tan(1/2*x)^2-1/2*b^2*tan(1/2*x)+a*b)/(tan
(1/2*x)^2+1)^2+1/2*(2*a^2+b^2)*arctan(tan(1/2*x)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.37, size = 291, normalized size = 3.55

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^3\log\left(-\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+(a^2b^2-b^4)\cos(x)\sin(x)-(2\,a^4-a^2b^2-b^4)x-2\,(a^3b-ab^3)\cos(x)}{2\,(a^2b^3-b^5)},\ \frac{2\,\sqrt{a^2-b^2}\,a^3\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)-(a^2b^2-b^4)\cos(x)\sin(x)+(2\,a^4-a^2b^2-b^4)x+2\,(a^3b-ab^3)\cos(x)}{2\,(a^2b^3-b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
 2*a*b*sin(x) - a^2 - b^2)) + (a^2*b^2 - b^4)*cos(x)*sin(x) - (2*a^4 - a^2*
b^2 - b^4)*x - 2*(a^3*b - a*b^3)*cos(x))/(a^2*b^3 - b^5), 1/2*(2*sqrt(a^2 -
 b^2)*a^3*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b^2 - b^4
)*cos(x)*sin(x) + (2*a^4 - a^2*b^2 - b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^
2*b^3 - b^5)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.45, size = 112, normalized size = 1.37

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}\,b^3}+\frac{(2\,a^2+b^2)x}{2\,b^3}+\frac{b\tan\left(\frac{1}{2}\,x\right)^3+2\,a\tan\left(\frac{1}{2}\,x\right)^2-b\tan\left(\frac{1}{2}\,x\right)+2\,a}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) + 1/2*(2*a^2 + b^2)*x/b^3 + (b*tan(1/2*x)^
3 + 2*a*tan(1/2*x)^2 - b*tan(1/2*x) + 2*a)/((tan(1/2*x)^2 + 1)^2*b^2)
```

**Mupad [B]**
time = 7.08, size = 1004, normalized size = 12.24

$$\frac{\frac{2\,a}{b^2}-\frac{\tan\left(\frac{x}{2}\right)}{b}+\frac{\tan\left(\frac{x}{2}\right)^3}{b}+\frac{2\,a\tan\left(\frac{x}{2}\right)^2}{b^2}}{\tan\left(\frac{x}{2}\right)^4+2\tan\left(\frac{x}{2}\right)^2+1}-\frac{\mathrm{atan}\left(\frac{40\,a^3\tan\left(\frac{x}{2}\right)}{8\,a\,b^2+40\,a^3+\frac{48\,a^5}{b^2}}+\frac{48\,a^5\tan\left(\frac{x}{2}\right)}{48\,a^5+40\,a^3\,b^2+8\,a\,b^4}+\frac{8\,a\,b\tan\left(\frac{x}{2}\right)}{8\,a\,b+\frac{40\,a^3}{b}+\frac{48\,a^5}{b^3}}\right)\left(a^2\,2\mathrm{i}+b^2\,1\mathrm{i}\right)\,1\mathrm{i}}{b^3}+\frac{a^3\,\mathrm{atan}\left(\frac{\frac{a^3\left(\frac{8\left(4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6\right)}{b^5}+\frac{8\tan\left(\frac{x}{2}\right)\left(-8\,a^7\,b^2+4\,a^5\,b^4+7\,a^3\,b^6+2\,a\,b^8\right)}{b^6}+\frac{a^3\left(64\,a^4\tan\left(\frac{x}{2}\right)+\frac{8\left(2\,a^3\,b^6+2\,a\,b^8\right)}{b^5}+\frac{a^3\left(32\,a^2\,b^3+\frac{8\tan\left(\frac{x}{2}\right)\left(12\,a\,b^{10}-8\,a^3\,b^8\right)}{b^6}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}\right)1\mathrm{i}}{b^3\sqrt{b^2-a^2}}+\frac{a^3\left(\frac{8\left(4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6\right)}{b^5}+\frac{8\tan\left(\frac{x}{2}\right)\left(-8\,a^7\,b^2+4\,a^5\,b^4+7\,a^3\,b^6+2\,a\,b^8\right)}{b^6}-\frac{a^3\left(64\,a^4\tan\left(\frac{x}{2}\right)+\frac{8\left(2\,a^3\,b^6+2\,a\,b^8\right)}{b^5}-\frac{a^3\left(32\,a^2\,b^3+\frac{8\tan\left(\frac{x}{2}\right)\left(12\,a\,b^{10}-8\,a^3\,b^8\right)}{b^6}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}\right)1\mathrm{i}}{b^3\sqrt{b^2-a^2}}}{\frac{16\left(2\,a^7+a^5\,b^2\right)}{b^5}+\frac{16\tan\left(\frac{x}{2}\right)\left(8\,a^8+8\,a^6\,b^2+2\,a^4\,b^4\right)}{b^6}+\frac{a^3\left(\frac{8\left(4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6\right)}{b^5}+\frac{8\tan\left(\frac{x}{2}\right)\left(-8\,a^7\,b^2+4\,a^5\,b^4+7\,a^3\,b^6+2\,a\,b^8\right)}{b^6}+\frac{a^3\left(64\,a^4\tan\left(\frac{x}{2}\right)+\frac{8\left(2\,a^3\,b^6+2\,a\,b^8\right)}{b^5}+\frac{a^3\left(32\,a^2\,b^3+\frac{8\tan\left(\frac{x}{2}\right)\left(12\,a\,b^{10}-8\,a^3\,b^8\right)}{b^6}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}-\frac{a^3\left(\frac{8\left(4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6\right)}{b^5}+\frac{8\tan\left(\frac{x}{2}\right)\left(-8\,a^7\,b^2+4\,a^5\,b^4+7\,a^3\,b^6+2\,a\,b^8\right)}{b^6}-\frac{a^3\left(64\,a^4\tan\left(\frac{x}{2}\right)+\frac{8\left(2\,a^3\,b^6+2\,a\,b^8\right)}{b^5}-\frac{a^3\left(32\,a^2\,b^3+\frac{8\tan\left(\frac{x}{2}\right)\left(12\,a\,b^{10}-8\,a^3\,b^8\right)}{b^6}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}\right)}{b^3\sqrt{b^2-a^2}}}\right)2\mathrm{i}}{b^3\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x)),x)
```

```
[Out] ((2*a)/b^2 - tan(x/2)/b + tan(x/2)^3/b + (2*a*tan(x/2)^2)/b^2)/(2*tan(x/2)^
2 + tan(x/2)^4 + 1) - (atan((40*a^3*tan(x/2))/(8*a*b^2 + 40*a^3 + (48*a^5)/
b^2) + (48*a^5*tan(x/2))/(8*a*b^4 + 48*a^5 + 40*a^3*b^2) + (8*a*b*tan(x/2))
/(8*a*b + (40*a^3)/b + (48*a^5)/b^3))*(a^2*2i + b^2*1i)*1i)/b^3 + (a^3*atan
(((a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2))/b^5 + (8*tan(x/2)*(2*a*b^8 +
7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 + (a^3*(64*a^4*tan(x/2) + (8*(2*a*b
^8 + 2*a^3*b^6))/b^5 + (a^3*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^
8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)))*1i)/(b^3*(b^2
- a^2)^(1/2)) + (a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2))/b^5 + (8*tan(x/
2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 - (a^3*(64*a^4*tan(x/
2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 - (a^3*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b
^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)))*
1i)/(b^3*(b^2 - a^2)^(1/2)))/((16*(2*a^7 + a^5*b^2))/b^5 + (16*tan(x/2)*(8*
a^8 + 2*a^4*b^4 + 8*a^6*b^2))/b^6 + (a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b
^2))/b^5 + (8*tan(x/2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 +
 (a^3*(64*a^4*tan(x/2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 + (a^3*(32*a^2*b^3 +
 (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*
(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)) - (a^3*((8*(a^2*b^6 + 4*a^4*b^
4 + 4*a^6*b^2))/b^5 + (8*tan(x/2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*
b^2))/b^6 - (a^3*(64*a^4*tan(x/2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 - (a^3*(3
2*a^2*b^3 + (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/
2))))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2))))*2i)/(b^3*(b^2 - a
^2)^(1/2))
```

# 3.178 $\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=61

$$-\frac{ax}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b}$$

```
[Out] -a*x/b^2-cos(x)/b+2*a^2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^2/(a^2-b
^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2825, 12, 2814, 2739, 632, 210}

$$\frac{2a^2\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a
^2 - b^2]) - Cos[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)}{b}-\frac{\int \frac{a\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{\cos(x)}{b}-\frac{a\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}+\frac{a^2\int \frac{1}{a+b\sin(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}-\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b}
\end{aligned}
$$

**Mathematica** [A]
time = 0.06, size = 56, normalized size = 0.92

$$
-\frac{ax-\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+b\cos(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x - (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 b*Cos[x])/b^2)
```

**Maple [A]**
time = 0.14, size = 71, normalized size = 1.16

| method | result | size |
|---|---|---|
| default | $\frac{2a^{2}\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{b^{2}\sqrt{a^{2}-b^{2}}}-\frac{2\left(\frac{b}{\tan^{2}\left(\frac{x}{2}\right)+1}+a\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^{2}}$ | 71 |
| risch | $-\frac{ax}{b^{2}}-\frac{e^{ix}}{2b}-\frac{e^{-ix}}{2b}+\frac{a^{2}\ln\left(e^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}-\frac{a^{2}\ln\left(e^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}$ | 158 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-
2/b^2*(b/(tan(1/2*x)^2+1)+a*arctan(tan(1/2*x)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.46, size = 231, normalized size = 3.79

$$\left[-\frac{\sqrt{-a^{2}+b^{2}}\,a^{2}\log\left(\frac{(2a^{2}-b^{2})\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}}\right)+2(a^{3}-ab^{2})x+2(a^{2}b-b^{3})\cos(x)}{2(a^{2}b^{2}-b^{4})},\ -\frac{\sqrt{a^{2}-b^{2}}\,a^{2}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^{2}-b^{2}}\cos(x)}\right)+(a^{3}-ab^{2})x+(a^{2}b-b^{3})\cos(x)}{a^{2}b^{2}-b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
```

```
2*a*b*sin(x) - a^2 - b^2)) + 2*(a^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a
^2*b^2 - b^4), -(sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2
)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b^3)*cos(x))/(a^2*b^2 - b^4)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(49) = 98$.

time = 146.75, size = 1192, normalized size = 19.54

$$\begin{cases} \tilde{\infty} \cos(x) & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{bx\tan^2(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} - \frac{bx}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} - \frac{2b\tan(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} + \frac{x\sqrt{b^2}\tan^3(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} + \frac{x\sqrt{b^2}\tan(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} + \frac{2\sqrt{b^2}\tan^2(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} + \frac{4\sqrt{b^2}}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})-b\sqrt{b^2}\tan^2(\frac{x}{2})-b\sqrt{b^2}} & \text{for}\: a = -\sqrt{b^2} \\ -\frac{bx\tan^2(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{bx}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{2b\tan(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{x\sqrt{b^2}\tan^3(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{x\sqrt{b^2}\tan(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{2\sqrt{b^2}\tan^2(\frac{x}{2})}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} - \frac{4\sqrt{b^2}}{b^2\tan^3(\frac{x}{2})+b^2\tan(\frac{x}{2})+b\sqrt{b^2}\tan^2(\frac{x}{2})+b\sqrt{b^2}} & \text{for}\: a = \sqrt{b^2} \\ \frac{\frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}-\frac{\sin(x)\cos(x)}{2}}{a} & \text{for}\: b = 0 \\ -\frac{\cos(x)}{b} & \text{for}\: a = 0 \\ \frac{a^2\log\left(\tan(\frac{x}{2})+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)\tan^2(\frac{x}{2})}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} + \frac{a^2\log\left(\tan(\frac{x}{2})+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} - \frac{a^2\log\left(\tan(\frac{x}{2})+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)\tan^2(\frac{x}{2})}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} - \frac{a^2\log\left(\tan(\frac{x}{2})+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} - \frac{ax\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} - \frac{ax\sqrt{-a^2+b^2}}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} - \frac{2b\sqrt{-a^2+b^2}}{b^2\sqrt{-a^2+b^2}\tan^2(\frac{x}{2})+b^2\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*cos(x), Eq(a, 0) & Eq(b, 0)), (-b*x*tan(x/2)**2/(b**2*tan(x/
2)**3 + b**2*tan(x/2) - b*sqrt(b**2)*tan(x/2)**2 - b*sqrt(b**2)) - b*x/(b**
2*tan(x/2)**3 + b**2*tan(x/2) - b*sqrt(b**2)*tan(x/2)**2 - b*sqrt(b**2)) -
2*b*tan(x/2)/(b**2*tan(x/2)**3 + b**2*tan(x/2) - b*sqrt(b**2)*tan(x/2)**2 -
 b*sqrt(b**2)) + x*sqrt(b**2)*tan(x/2)**3/(b**2*tan(x/2)**3 + b**2*tan(x/2)
 - b*sqrt(b**2)*tan(x/2)**2 - b*sqrt(b**2)) + x*sqrt(b**2)*tan(x/2)/(b**2*t
an(x/2)**3 + b**2*tan(x/2) - b*sqrt(b**2)*tan(x/2)**2 - b*sqrt(b**2)) + 2*s
qrt(b**2)*tan(x/2)**2/(b**2*tan(x/2)**3 + b**2*tan(x/2) - b*sqrt(b**2)*tan(
x/2)**2 - b*sqrt(b**2)) + 4*sqrt(b**2)/(b**2*tan(x/2)**3 + b**2*tan(x/2) -
b*sqrt(b**2)*tan(x/2)**2 - b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (-b*x*tan(x/
2)**2/(b**2*tan(x/2)**3 + b**2*tan(x/2) + b*sqrt(b**2)*tan(x/2)**2 + b*sqrt
(b**2)) - b*x/(b**2*tan(x/2)**3 + b**2*tan(x/2) + b*sqrt(b**2)*tan(x/2)**2
+ b*sqrt(b**2)) - 2*b*tan(x/2)/(b**2*tan(x/2)**3 + b**2*tan(x/2) + b*sqrt(b
**2)*tan(x/2)**2 + b*sqrt(b**2)) - x*sqrt(b**2)*tan(x/2)**3/(b**2*tan(x/2)*
*3 + b**2*tan(x/2) + b*sqrt(b**2)*tan(x/2)**2 + b*sqrt(b**2)) - x*sqrt(b**2
)*tan(x/2)/(b**2*tan(x/2)**3 + b**2*tan(x/2) + b*sqrt(b**2)*tan(x/2)**2 + b
*sqrt(b**2)) - 2*sqrt(b**2)*tan(x/2)**2/(b**2*tan(x/2)**3 + b**2*tan(x/2) +
 b*sqrt(b**2)*tan(x/2)**2 + b*sqrt(b**2)) - 4*sqrt(b**2)/(b**2*tan(x/2)**3
+ b**2*tan(x/2) + b*sqrt(b**2)*tan(x/2)**2 + b*sqrt(b**2)), Eq(a, sqrt(b**2
))), ((x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2)/a, Eq(b, 0)), (-cos
(x)/b, Eq(a, 0)), (a**2*log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(x/2)
**2/(b**2*sqrt(-a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + a**2*
log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b**2*sqrt(-a**2 + b**2)*tan(x/2
)**2 + b**2*sqrt(-a**2 + b**2)) - a**2*log(tan(x/2) + b/a + sqrt(-a**2 + b*
*2)/a)*tan(x/2)**2/(b**2*sqrt(-a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(-a**2 +
 b**2)) - a**2*log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b**2*sqrt(-a**2
+ b**2)*tan(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - a*x*sqrt(-a**2 + b**2)*tan
(x/2)**2/(b**2*sqrt(-a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(-a**2 + b**2)) -
a*x*sqrt(-a**2 + b**2)/(b**2*sqrt(-a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(-a*
```

```
*2 + b**2)) - 2*b*sqrt(-a**2 + b**2)/(b**2*sqrt(-a**2 + b**2)*tan(x/2)**2 +
 b**2*sqrt(-a**2 + b**2)), True))
```

**Giac [A]**
time = 0.45, size = 77, normalized size = 1.26

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}\,b^2}-\frac{ax}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - a*x/b^2 - 2/((tan(1/2*x)^2 + 1)*b)
```

**Mupad [B]**
time = 6.91, size = 623, normalized size = 10.21

$$-\frac{2}{b\,\left({\tan\left(\frac{x}{2}\right)}^2+1\right)}-\frac{a\,x}{b^2}-\frac{a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,\left(\frac{32\,a^4}{b}-\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}+\frac{a^2\,\left(32\,a^2\,b^2+64\,a^3\,b\,\tan\left(\frac{x}{2}\right)+\frac{a^2\,\left(32\,a^2\,b^3+\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}\right)\,1{}\mathrm{i}}{b^2\,\sqrt{b^2-a^2}}-\frac{a^2\,\left(\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}-\frac{32\,a^4}{b}+\frac{a^2\,\left(32\,a^2\,b^2+64\,a^3\,b\,\tan\left(\frac{x}{2}\right)-\frac{a^2\,\left(32\,a^2\,b^3+\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}\right)\,1{}\mathrm{i}}{b^2\,\sqrt{b^2-a^2}}}{\frac{128\,a^5\,\tan\left(\frac{x}{2}\right)}{b^3}+\frac{a^2\,\left(\frac{32\,a^4}{b}-\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}+\frac{a^2\,\left(32\,a^2\,b^2+64\,a^3\,b\,\tan\left(\frac{x}{2}\right)+\frac{a^2\,\left(32\,a^2\,b^3+\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}+\frac{a^2\,\left(\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}-\frac{32\,a^4}{b}+\frac{a^2\,\left(32\,a^2\,b^2+64\,a^3\,b\,\tan\left(\frac{x}{2}\right)-\frac{a^2\,\left(32\,a^2\,b^3+\frac{32\,\tan\left(\frac{x}{2}\right)\,\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}\right)}{b^2\,\sqrt{b^2-a^2}}}\right)\,2{}\mathrm{i}}{b^2\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x)),x)
```

```
[Out] - 2/(b*(tan(x/2)^2 + 1)) - (a*x)/b^2 - (a^2*atan(((a^2*((32*a^4)/b - (32*ta
n(x/2)*(2*a^5*b - 2*a^3*b^3))/b^3 + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) +
(a^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^
2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^(1/2)) - (a^2*((3
2*tan(x/2)*(2*a^5*b - 2*a^3*b^3))/b^3 - (32*a^4)/b + (a^2*(32*a^2*b^2 + 64*
a^3*b*tan(x/2) - (a^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3
))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^
(1/2)))/((128*a^5*tan(x/2))/b^3 + (a^2*((32*a^4)/b - (32*tan(x/2)*(2*a^5*b
- 2*a^3*b^3))/b^3 + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) + (a^2*(32*a^2*b^3
 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2
*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) + (a^2*((32*tan(x/2)*(2*a^5*b
 - 2*a^3*b^3))/b^3 - (32*a^4)/b + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) - (a
^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)
^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))*2i)/(b^2*(b^2
 - a^2)^(1/2))
```

# 3.179 $\int \frac{\sin(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=50

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

```
[Out] x/b-2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2814, 2739, 632, 210}

$$\frac{x}{b}-\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{a+b\sin(x)}\,dx &= \frac{x}{b} - \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b} \\
&= \frac{x}{b} - \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{x}{b} + \frac{(4a)\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{x}{b} - \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}
\end{aligned}
$$

**Mathematica** [A]
time = 0.03, size = 47, normalized size = 0.94

$$
\frac{x - \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x]),x]
[Out] (x - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2])/b
```

**Maple** [A]
time = 0.11, size = 54, normalized size = 1.08

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b} - \frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$ | 54 |
| risch | $\frac{x}{b} - \frac{ia\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b} + \frac{ia\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}$ | 135 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*arctan(tan(1/2*x))-2*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b
)/(a^2-b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.37, size = 192, normalized size = 3.84

$$\left[-\frac{\sqrt{-a^2+b^2}\,a\log\left(-\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2-2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2(a^2-b^2)x}{2(a^2b-b^3)},\ \frac{\sqrt{a^2-b^2}\,a\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+(a^2-b^2)x}{a^2b-b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) - 2*(a^2 - b^2)*x)/(a^2*b - b^3), (sqrt(a^2 - b^2
)*a*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^2*
b - b^3)]
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(37) = 74$.
time = 32.28, size = 236, normalized size = 4.72

$$\begin{cases} \tilde{\infty}x & \text{for}\ a=0 \wedge b=0 \\ \frac{x}{b} & \text{for}\ a=0 \\ -\frac{\cos(x)}{a} & \text{for}\ b=0 \\ \frac{bx\tan\left(\frac{x}{2}\right)}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}}+\frac{2b}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}}-\frac{x\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}} & \text{for}\ a=-\sqrt{b^2} \\ \frac{bx\tan\left(\frac{x}{2}\right)}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}}+\frac{2b}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}}+\frac{x\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}} & \text{for}\ a=\sqrt{b^2} \\ -\frac{a\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{b\sqrt{-a^2+b^2}}+\frac{a\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{b\sqrt{-a^2+b^2}}+\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-cos(x)/a, Eq(b,
0)), (b*x*tan(x/2)/(b**2*tan(x/2) - b*sqrt(b**2)) + 2*b/(b**2*tan(x/2) - b*
sqrt(b**2)) - x*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2
))), (b*x*tan(x/2)/(b**2*tan(x/2) + b*sqrt(b**2)) + 2*b/(b**2*tan(x/2) + b*
sqrt(b**2)) + x*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2)
)), (-a*log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b*sqrt(-a**2 + b**2)) +
 a*log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b*sqrt(-a**2 + b**2)) + x/b,
 True))
```

**Giac** [A]
time = 0.44, size = 58, normalized size = 1.16

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a/(sqrt(a^2 - b^2)*b) + x/b
```

**Mupad** [B]
time = 6.70, size = 101, normalized size = 2.02

$$\frac{x}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\,a^4-\cos\left(\frac{x}{2}\right)\,a^3\,b-3\,\sin\left(\frac{x}{2}\right)\,a^2\,b^2+\cos\left(\frac{x}{2}\right)\,a\,b^3+2\,\sin\left(\frac{x}{2}\right)\,b^4}{\left(b^2-a^2\right)^{3/2}\,\left(2\,b\,\sin\left(\frac{x}{2}\right)+a\,\cos\left(\frac{x}{2}\right)\right)}\right)}{b\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x)),x)
```

```
[Out] x/b - (2*a*atanh((a^4*sin(x/2) + 2*b^4*sin(x/2) - 3*a^2*b^2*sin(x/2) + a*b^
3*cos(x/2) - a^3*b*cos(x/2))/((b^2 - a^2)^(3/2)*(2*b*sin(x/2) + a*cos(x/2))
)))/(b*(b^2 - a^2)^(1/2))
```

# 3.180 $\int \frac{1}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] 2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin(x)}\,dx = 2\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\left(4\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

**Mathematica** [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

**Maple** [A]
time = 0.09, size = 39, normalized size = 0.98

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$ | 39 |
| risch | $-\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$ | 119 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.35, size = 148, normalized size = 3.70

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)}{2(a^2-b^2)}, -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.
time = 3.82, size = 141, normalized size = 3.52

$$\begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ \frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a,
 0)), (2*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (-
2*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(x
/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/2) + b/a +
 sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

**Giac** [A]
time = 0.41, size = 48, normalized size = 1.20

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

**Mupad** [B]
time = 6.88, size = 45, normalized size = 1.12

$$\frac{2\operatorname{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^
(1/2)
```

# 3.181 $\int \frac{\csc(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=53

$$-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] -arctanh(cos(x))/a-2*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a/(a^2-b^2)
^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2826, 3855, 2739, 632, 210}

$$-\frac{2b\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]) - ArcTa
nh[Cos[x]]/a
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+b\sin(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a}\\ &=-\frac{\tanh^{-1}(\cos(x))}{a}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a}\\ &=-\frac{\tanh^{-1}(\cos(x))}{a}+\frac{(4b)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a}\\ &=-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 62, normalized size = 1.17

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - Log[Cos[
x/2]] + Log[Sin[x/2]])/a
```

**Maple** [A]
time = 0.18, size = 53, normalized size = 1.00

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| default | $-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 53 |
| risch | $\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,a}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,a}+\frac{\ln(\mathrm{e}^{ix}-1)}{a}-\frac{\ln(\mathrm{e}^{ix}+1)}{a}$ | 155 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/a
*ln(tan(1/2*x))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.62, size = 239, normalized size = 4.51

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2-2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+(a^2-b^2)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(a^2-b^2)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a^3-ab^2)},\ \frac{2\sqrt{a^2-b^2}\,b\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)-(a^2-b^2)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+(a^2-b^2)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a^3-ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) + (a^2 - b^2)*log(1/2*cos(x) + 1/2) - (a^2 - b^2)
*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a
*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2 - b^2)*log(1/2*cos(x) + 1/2)
+ (a^2 - b^2)*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x)), x)
```

**Giac** [A]
time = 0.45, size = 63, normalized size = 1.19

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(sqrt(a^2 - b^2)*a) + log(abs(tan(1/2*x)))/a
```

**Mupad** [B]
time = 6.86, size = 122, normalized size = 2.30

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{b^2-a^2}\,\left(-1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{1\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a^3+3\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2\,b-2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b^2-4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^3}\right)}{a\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*sin(x))),x)
```

```
[Out] log(sin(x/2)/cos(x/2))/a + (2*b*atanh(((b^2 - a^2)^(1/2)*(b^2*sin(x/2)*4i -
 a^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*cos(x/2)*1i - b^3*sin(x/2)*4i - a
*b^2*cos(x/2)*2i + a^2*b*sin(x/2)*3i)))/(a*(b^2 - a^2)^(1/2))
```

# 3.182 $\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=62

$$\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

```
[Out] b*arctanh(cos(x))/a^2-cot(x)/a+2*b^2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2
))/a^2/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 12, 2826, 3855, 2739, 632, 210}

$$\frac{2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2]) + (b
*ArcTanh[Cos[x]])/a^2 - Cot[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)}{a} - \frac{\int \frac{b\csc(x)}{a+b\sin(x)}\,dx}{a}$$
$$= -\frac{\cot(x)}{a} - \frac{b\int \frac{\csc(x)}{a+b\sin(x)}\,dx}{a}$$
$$= -\frac{\cot(x)}{a} - \frac{b\int \csc(x)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sin(x)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}} + \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a}$$

**Mathematica [A]**
time = 0.18, size = 91, normalized size = 1.47

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(-a\cos(x) + \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\sin(x)}{\sqrt{a^2-b^2}} + b\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*(-(a*Cos[x]) + (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
 b^2]]*Sin[x])/Sqrt[a^2 - b^2] + b*(Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x]))
/(2*a^2)
```

**Maple [A]**
time = 0.18, size = 77, normalized size = 1.24

| method | result |
|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)}{2a} + \frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}} - \frac{1}{2a\tan\left(\frac{x}{2}\right)} - \frac{b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$ |
| risch | $-\frac{2i}{a\left(e^{2ix}-1\right)} + \frac{b^2\ln\left(e^{ix} + \frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2} - \frac{b^2\ln\left(e^{ix} + \frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2} - \frac{b\ln\left(e^{ix}-1\right)}{a^2} + \frac{b\ln\left(e^{ix}+1\right)}{a^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*tan(1/2*x)+2/a^2*b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/
(a^2-b^2)^(1/2))-1/2/a/tan(1/2*x)-1/a^2*b*ln(tan(1/2*x))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.
time = 0.40, size = 302, normalized size = 4.87

$$\left[-\frac{\sqrt{-a^2+b^2}\,b^2\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)\sin(x)-(a^2b-b^3)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+(a^2b-b^3)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2(a^3-ab^2)\cos(x)}{2(a^4-a^2b^2)\sin(x)},\ -\frac{2\sqrt{a^2-b^2}\,b^2\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)\sin(x)-(a^2b-b^3)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+(a^2b-b^3)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2(a^3-ab^2)\cos(x)}{2(a^4-a^2b^2)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*sin
(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x))
/((a^4 - a^2*b^2)*sin(x)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(x) +
b)/(sqrt(a^2 - b^2)*cos(x)))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*s
in(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x
))/((a^4 - a^2*b^2)*sin(x))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x)), x)
```

**Giac [A]**
time = 0.45, size = 98, normalized size = 1.58

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}\,a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}x\right)}{2a}+\frac{2b\tan\left(\frac{1}{2}x\right)-a}{2a^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - b*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*
x)/a + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

**Mupad [B]**
time = 7.01, size = 179, normalized size = 2.89

$$\frac{b^3\ln\left(\tan\left(\frac{x}{2}\right)\right)-a^2b\ln\left(\tan\left(\frac{x}{2}\right)\right)+b^2\operatorname{atan}\left(\frac{-a^2\tan\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^2\tan\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a\,b\sqrt{b^2-a^2}\,2\mathrm{i}}{-a^3-3\tan\left(\frac{x}{2}\right)a^2b+2a\,b^2+4\tan\left(\frac{x}{2}\right)b^3}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{a^4-a^2b^2}+\frac{a\,b^2-a^3}{a^4\tan\left(x\right)-a^2b^2\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))),x)
```

```
[Out] (b^3*log(tan(x/2)) - a^2*b*log(tan(x/2)) + b^2*atan((b^2*tan(x/2)*(b^2 - a^
2)^(1/2)*4i - a^2*tan(x/2)*(b^2 - a^2)^(1/2)*1i + a*b*(b^2 - a^2)^(1/2)*2i)
/(4*b^3*tan(x/2) + 2*a*b^2 - a^3 - 3*a^2*b*tan(x/2)))*(b^2 - a^2)^(1/2)*2i)
/(a^4 - a^2*b^2) + (a*b^2 - a^3)/(a^4*tan(x) - a^2*b^2*tan(x))
```

# 3.183 $\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=84

$$-\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}$$

```
[Out] -1/2*(a^2+2*b^2)*arctanh(cos(x))/a^3+b*cot(x)/a^2-1/2*cot(x)*csc(x)/a-2*b^3
*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(x)}{a^2}-\frac{2b^3\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3}-\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (-2*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*Sqrt[a^2 - b^2]) - (
(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^3) + (b*Cot[x])/a^2 - (Cot[x]*Csc[x])/(
2*a)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)\csc(x)}{2a} + \frac{\int \frac{\csc^2(x)(-2b+a\sin(x)+b\sin^2(x))}{a+b\sin(x)}\,dx}{2a}$$
$$= \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} + \frac{\int \frac{\csc(x)(a^2+2b^2+ab\sin(x))}{a+b\sin(x)}\,dx}{2a^2}$$
$$= \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} - \frac{b^3\int \frac{1}{a+b\sin(x)}\,dx}{a^3} + \frac{(a^2+2b^2)\int \csc(x)\,dx}{2a^3}$$
$$= -\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,\right.}{a^3}$$
$$= -\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} + \frac{(4b^3)\operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\right.}{a^3}$$
$$= -\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}} - \frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a}$$

## Mathematica [A]

time = 0.35, size = 144, normalized size = 1.71

$$\frac{-\frac{16b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 4ab\cot\left(\frac{x}{2}\right) - a^2\csc^2\left(\frac{x}{2}\right) - 4a^2\log\left(\cos\left(\frac{x}{2}\right)\right) - 8b^2\log\left(\cos\left(\frac{x}{2}\right)\right) + 4a^2\log\left(\sin\left(\frac{x}{2}\right)\right) + 8b^2\log\left(\sin\left(\frac{x}{2}\right)\right) + a^2\sec^2\left(\frac{x}{2}\right) - 4ab\tan\left(\frac{x}{2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((-16*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b
*Cot[x/2] - a^2*Csc[x/2]^2 - 4*a^2*Log[Cos[x/2]] - 8*b^2*Log[Cos[x/2]] + 4*
a^2*Log[Sin[x/2]] + 8*b^2*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - 4*a*b*Tan[x/2])/
(8*a^3)
```

## Maple [A]

time = 0.26, size = 112, normalized size = 1.33

| method | result |
|---|---|
| default | $\frac{\frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-2b\tan\left(\frac{x}{2}\right)}{4a^2} - \frac{2b^3\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}} - \frac{1}{8a\tan\left(\frac{x}{2}\right)^2} + \frac{(2a^2+4b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a^3} + \frac{b}{2a^2\tan\left(\frac{x}{2}\right)}$ |
| risch | $\frac{i(-ia\,e^{3ix}-ia\,e^{ix}+2b\,e^{2ix}-2b)}{(e^{2ix}-1)^2a^2} - \frac{\ln(e^{ix}+1)}{2a} - \frac{\ln(e^{ix}+1)b^2}{a^3} - \frac{ib^3\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,a^3} + \frac{ib^3\ln\left(e^{ix}+\frac{i(\sqrt{}\right.}{\sqrt{a^{}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/2*a*tan(1/2*x)^2-2*b*tan(1/2*x))-2/a^3*b^3/(a^2-b^2)^(1/2)*arcta
n(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/8/a/tan(1/2*x)^2+1/4/a^3*(2*a
^2+4*b^2)*ln(tan(1/2*x))+1/2/a^2*b/tan(1/2*x)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(74) = 148$.
time = 0.47, size = 490, normalized size = 5.83

$$\left[\frac{4\left(a^{3}b-ab^{3}\right)\cos\left(x\right)\sin\left(x\right)+2\left(b^{3}\cos\left(x\right)^{2}-b^{3}\right)\sqrt{-a^{2}+b^{2}}\log\left(-\frac{\left(2a^{2}-b^{2}\right)\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}}\right)-2\left(a^{4}-a^{2}b^{2}\right)\cos\left(x\right)-\left(a^{4}+a^{2}b^{2}-2b^{4}-\left(a^{4}+a^{2}b^{2}-2b^{4}\right)\cos\left(x\right)^{2}\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\left(a^{4}+a^{2}b^{2}-2b^{4}-\left(a^{4}+a^{2}b^{2}-2b^{4}\right)\cos\left(x\right)^{2}\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{4\left(a^{5}-a^{3}b^{2}-\left(a^{5}-a^{3}b^{2}\right)\cos\left(x\right)^{2}\right)}, \frac{4\left(a^{3}b-ab^{3}\right)\cos\left(x\right)\sin\left(x\right)-4\left(b^{3}\cos\left(x\right)^{2}-b^{3}\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^{2}-b^{2}}\cos(x)}\right)-2\left(a^{4}-a^{2}b^{2}\right)\cos\left(x\right)-\left(a^{4}+a^{2}b^{2}-2b^{4}-\left(a^{4}+a^{2}b^{2}-2b^{4}\right)\cos\left(x\right)^{2}\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\left(a^{4}+a^{2}b^{2}-2b^{4}-\left(a^{4}+a^{2}b^{2}-2b^{4}\right)\cos\left(x\right)^{2}\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{4\left(a^{5}-a^{3}b^{2}-\left(a^{5}-a^{3}b^{2}\right)\cos\left(x\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) + 2*(b^3*cos(x)^2 - b^3)*sqrt(-a^2 +
b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*
sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2)) - 2*(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
 2*b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a
^2*b^2 - 2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a
^3*b^2)*cos(x)^2), 1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 4*(b^3*cos(x)^2 -
 b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*
(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 - 2*b^4)*c
os(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a^3*b^2)*c
os(x)^2)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(x\right)}{a+b\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x)), x)
```

**Giac [A]**
time = 0.43, size = 141, normalized size = 1.68

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}\,a^3}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4\,b\tan\left(\frac{1}{2}x\right)}{8\,a^2}+\frac{(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,a^3}-\frac{6\,a^2\tan\left(\frac{1}{2}x\right)^2+12\,b^2\tan\left(\frac{1}{2}x\right)^2-4\,ab\tan\left(\frac{1}{2}x\right)+a^2}{8\,a^3\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b^3/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*tan(1/2*x)^2 - 4*b*tan(1/2*x))/a^
2 + 1/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^3 - 1/8*(6*a^2*tan(1/2*x)^2 +
12*b^2*tan(1/2*x)^2 - 4*a*b*tan(1/2*x) + a^2)/(a^3*tan(1/2*x)^2)
```

**Mupad [B]**
time = 7.39, size = 531, normalized size = 6.32

$$\frac{a^4\left(\frac{\cos(x)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}+\frac{\cos(2\,x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}\right)-a^2\left(\frac{b^2\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}+\frac{b^2\cos(x)}{2}-\frac{b^2\cos(2\,x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}\right)+\frac{b^4\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}-b^3\,\mathrm{atan}\left(\frac{-a^4\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^4\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}+a\,b^3\cos\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a^3\,b\cos\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}}{\cos\left(\frac{x}{2}\right)a^5+2\sin\left(\frac{x}{2}\right)a^4\,b+\cos\left(\frac{x}{2}\right)a^3\,b^2+4\sin\left(\frac{x}{2}\right)a^2\,b^3-4\cos\left(\frac{x}{2}\right)a\,b^4-8\sin\left(\frac{x}{2}\right)b^5}\right)\sqrt{b^2-a^2}\,1\mathrm{i}-\frac{b^4\cos(2\,x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}+\frac{a\,b^3\sin(2\,x)}{2}-\frac{a^3\,b\sin(2\,x)}{2}+b^3\cos(2\,x)\,\mathrm{atan}\left(\frac{-a^4\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^4\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}+a\,b^3\cos\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a^3\,b\cos\left(\frac{x}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}}{\cos\left(\frac{x}{2}\right)a^5+2\sin\left(\frac{x}{2}\right)a^4\,b+\cos\left(\frac{x}{2}\right)a^3\,b^2+4\sin\left(\frac{x}{2}\right)a^2\,b^3-4\cos\left(\frac{x}{2}\right)a\,b^4-8\sin\left(\frac{x}{2}\right)b^5}\right)\sqrt{b^2-a^2}\,1\mathrm{i}}{\frac{a^5\cos(2\,x)}{2}-\frac{a^5}{2}+\frac{a^3\,b^2}{2}-\frac{a^3\,b^2\cos(2\,x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))),x)
```

```
[Out] (a^4*(cos(x)/2 - log(sin(x/2)/cos(x/2))/4 + (cos(2*x)*log(sin(x/2)/cos(x/2)
))/4) - a^2*((b^2*log(sin(x/2)/cos(x/2)))/4 + (b^2*cos(x))/2 - (b^2*cos(2*x
)*log(sin(x/2)/cos(x/2)))/4) + (b^4*log(sin(x/2)/cos(x/2)))/2 - b^3*atan((b
^4*sin(x/2)*(b^2 - a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^
3*cos(x/2)*(b^2 - a^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5
*cos(x/2) - 8*b^5*sin(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^
4*cos(x/2) + 2*a^4*b*sin(x/2)))*(b^2 - a^2)^(1/2)*1i - (b^4*cos(2*x)*log(si
n(x/2)/cos(x/2)))/2 + (a*b^3*sin(2*x))/2 - (a^3*b*sin(2*x))/2 + b^3*cos(2*x
)*atan((b^4*sin(x/2)*(b^2 - a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*
1i + a*b^3*cos(x/2)*(b^2 - a^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)
*1i)/(a^5*cos(x/2) - 8*b^5*sin(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2)
 - 4*a*b^4*cos(x/2) + 2*a^4*b*sin(x/2)))*(b^2 - a^2)^(1/2)*1i)/((a^5*cos(2*
x))/2 - a^5/2 + (a^3*b^2)/2 - (a^3*b^2*cos(2*x))/2)
```

# 3.184 $\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=112

$$\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}+\frac{b(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^4}-\frac{(2a^2+3b^2)\cot(x)}{3a^3}+\frac{b\cot(x)\csc(x)}{2a^2}-\frac{\cot(x)\csc^2(x)}{3a}$$

```
[Out] 1/2*b*(a^2+2*b^2)*arctanh(cos(x))/a^4-1/3*(2*a^2+3*b^2)*cot(x)/a^3+1/2*b*co
t(x)*csc(x)/a^2-1/3*cot(x)*csc(x)^2/a+2*b^4*arctan((b+a*tan(1/2*x))/(a^2-b^
2)^(1/2))/a^4/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(x)\csc(x)}{2a^2}+\frac{2b^4\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}+\frac{b(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^4}-\frac{(2a^2+3b^2)\cot(x)}{3a^3}-\frac{\cot(x)\csc^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*Sqrt[a^2 - b^2]) + (b
*(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^4) - ((2*a^2 + 3*b^2)*Cot[x])/(3*a^3)
+ (b*Cot[x]*Csc[x])/(2*a^2) - (Cot[x]*Csc[x]^2)/(3*a)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^3(x)(-3b+2a\sin(x)+2b\sin^2(x))}{a+b\sin(x)}\,dx}{3a}$$
$$= \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^2(x)(2(2a^2+3b^2)+ab\sin(x)-3b^2\sin^2(x))}{a+b\sin(x)}\,dx}{6a^2}$$
$$= -\frac{(2a^2+3b^2)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc(x)(-3b(a^2+2b^2)-3ab^2\sin(x))}{a+b\sin(x)}\,d}{6a^3}$$
$$= -\frac{(2a^2+3b^2)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sin(x)}\,dx}{a^4} - \frac{(b(a^2+2b}{}$$
$$= \frac{b(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^4} - \frac{(2a^2+3b^2)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{(}{}$$
$$= \frac{b(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^4} - \frac{(2a^2+3b^2)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} - \frac{(}{}$$
$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}} + \frac{b(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^4} - \frac{(2a^2+3b^2)\cot(x)}{3a^3} + \frac{b\cot(x)\,c}{2a^2}$$

**Mathematica** [A]
time = 1.14, size = 125, normalized size = 1.12

$$\frac{\frac{24b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a(2a^2+3b^2)\cos(3x)\csc^3(x) - 3a\cot(x)\csc(x)\left(-2ab+(2a^2+b^2)\csc(x)\right) + 6b(a^2+2b^2)\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*(2*a
^2 + 3*b^2)*Cos[3*x]*Csc[x]^3 - 3*a*Cot[x]*Csc[x]*(-2*a*b + (2*a^2 + b^2)*C
sc[x]) + 6*b*(a^2 + 2*b^2)*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(12*a^4)
```

**Maple** [A]
time = 0.29, size = 156, normalized size = 1.39

| method | result |
|---|---|
| default | $\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)a^2}{3} - ab\left(\tan^2\left(\frac{x}{2}\right)\right) + 3a^2\tan\left(\frac{x}{2}\right) + 4b^2\tan\left(\frac{x}{2}\right)}{8a^3} - \frac{1}{24a\tan\left(\frac{x}{2}\right)^3} - \frac{3a^2+4b^2}{8a^3\tan\left(\frac{x}{2}\right)} + \frac{b}{8a^2\tan\left(\frac{x}{2}\right)^2} - \frac{b(a^2+2b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^4}$ |

| | |
|---|---|
| risch | $-\frac{6ib^2 e^{4ix}+3ab\,e^{5ix}-12ia^2 e^{2ix}-12ib^2 e^{2ix}+4ia^2+6ib^2-3ab\,e^{ix}}{3a^3(e^{2ix}-1)^3}-\frac{b^4\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^4}+\frac{b^4\ln\left(e^{ix}+\frac{ia\sqrt{-}}{b\sqrt{}}\right.}{\sqrt{-a^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^3*(1/3*tan(1/2*x)^3*a^2-a*b*tan(1/2*x)^2+3*a^2*tan(1/2*x)+4*b^2*tan(1
/2*x))-1/24/a/tan(1/2*x)^3-1/8*(3*a^2+4*b^2)/a^3/tan(1/2*x)+1/8/a^2*b/tan(1
/2*x)^2-1/2/a^4*b*(a^2+2*b^2)*ln(tan(1/2*x))+2/a^4*b^4/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(98) = 196$.
time = 0.51, size = 577, normalized size = 5.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 + 6*(b^4*cos(x)^2 - b^4)*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*
cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b^3
 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(x)
 - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-1/
2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6 -
a^4*b^2)*cos(x)^2)*sin(x)), 1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 +
12*(b^4*cos(x)^2 - b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 -
b^2)*cos(x)))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b
^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(
```

```
x) - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-
1/2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6
- a^4*b^2)*cos(x)^2)*sin(x))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*sin(x)), x)
```

**Giac** [A]
time = 0.43, size = 194, normalized size = 1.73

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-3\,ab\tan\left(\frac{1}{2}x\right)^2+9\,a^2\tan\left(\frac{1}{2}x\right)+12\,b^2\tan\left(\frac{1}{2}x\right)}{24\,a^3}-\frac{(a^2b+2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,a^4}+\frac{22\,a^2b\tan\left(\frac{1}{2}x\right)^3+44\,b^3\tan\left(\frac{1}{2}x\right)^3-9\,a^3\tan\left(\frac{1}{2}x\right)^2-12\,ab^2\tan\left(\frac{1}{2}x\right)^2+3\,a^2b\tan\left(\frac{1}{2}x\right)-a^3}{24\,a^4\tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^4/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2*tan(1/2*x)^3 - 3*a*b*tan(1/2*x)
^2 + 9*a^2*tan(1/2*x) + 12*b^2*tan(1/2*x))/a^3 - 1/2*(a^2*b + 2*b^3)*log(ab
s(tan(1/2*x)))/a^4 + 1/24*(22*a^2*b*tan(1/2*x)^3 + 44*b^3*tan(1/2*x)^3 - 9*
a^3*tan(1/2*x)^2 - 12*a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - a^3)/(a^4*t
an(1/2*x)^3)
```

**Mupad** [B]
time = 7.52, size = 586, normalized size = 5.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b*sin(x))),x)
```

```
[Out] (a^5*(cos(3*x)/12 - cos(x)/4) - a*((b^4*cos(3*x))/8 - (b^4*cos(x))/8) + a^4
*((b*sin(2*x))/8 - (3*b*log(sin(x/2)/cos(x/2))*sin(x))/16 + (b*sin(3*x)*log
(sin(x/2)/cos(x/2)))/16) - a^2*((b^3*sin(2*x))/8 - (b^3*sin(3*x)*log(sin(x/
2)/cos(x/2)))/16 + (3*b^3*log(sin(x/2)/cos(x/2))*sin(x))/16) + a^3*((b^2*co
s(3*x))/24 + (b^2*cos(x))/8) - (b^5*sin(3*x)*log(sin(x/2)/cos(x/2)))/8 + (3
*b^5*log(sin(x/2)/cos(x/2))*sin(x))/8 + (b^4*atan((b^4*sin(x/2)*(b^2 - a^2)
^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^3*cos(x/2)*(b^2 - a^2)^
(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5*cos(x/2) - 8*b^5*sin(x
```

```
/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^4*cos(x/2) + 2*a^4*b*si
n(x/2)))*sin(3*x)*(b^2 - a^2)^(1/2)*1i)/4 - (b^4*atan((b^4*sin(x/2)*(b^2 -
a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^3*cos(x/2)*(b^2 - a
^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5*cos(x/2) - 8*b^5*s
in(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^4*cos(x/2) + 2*a^4*
b*sin(x/2)))*sin(x)*(b^2 - a^2)^(1/2)*3i)/4)/((3*a^6*sin(x))/8 - (a^6*sin(3
*x))/8 + (a^4*b^2*sin(3*x))/8 - (3*a^4*b^2*sin(x))/8)
```

# 3.185 $\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=169

$$\frac{(6a^2+b^2)\,x}{2b^4}-\frac{2a^3(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\,(a^2-b^2)^{3/2}}+\frac{a(3a^2-2b^2)\cos(x)}{b^3\,(a^2-b^2)}-\frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2\,(a^2-b^2)}+\frac{a^2\cos}{b\,(a^2-b^2}$$

```
[Out] 1/2*(6*a^2+b^2)*x/b^4-2*a^3*(3*a^2-4*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)
^(1/2))/b^4/(a^2-b^2)^(3/2)+a*(3*a^2-2*b^2)*cos(x)/b^3/(a^2-b^2)-1/2*(3*a^2
-b^2)*cos(x)*sin(x)/b^2/(a^2-b^2)+a^2*cos(x)*sin(x)^2/b/(a^2-b^2)/(a+b*sin(
x))
```

**Rubi** [A]
time = 0.28, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,
Rules used = {2871, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{a^2\sin^2(x)\cos(x)}{b\,(a^2-b^2)\,(a+b\sin(x))}-\frac{(3a^2-b^2)\sin(x)\cos(x)}{2b^2\,(a^2-b^2)}+\frac{x(6a^2+b^2)}{2b^4}+\frac{a(3a^2-2b^2)\cos(x)}{b^3\,(a^2-b^2)}-\frac{2a^3(3a^2-4b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\,(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)) + (a*(3*a^2 - 2*b^2)*Cos[x])/(b^3
*(a^2 - b^2)) - ((3*a^2 - b^2)*Cos[x]*Sin[x])/(2*b^2*(a^2 - b^2)) + (a^2*Co
s[x]*Sin[x]^2)/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx = \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin(x))} - \frac{\int \frac{\sin(x)(2a^2-ab\sin(x)-(3a^2-b^2)\sin^2(x))}{a+b\sin(x)}\,dx}{b(a^2-b^2)}$$

$$= -\frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin(x))} - \frac{\int \frac{-a(3a^2-b^2)+b(a^2+b^2)\sin(x)+2a(3}{a+b\sin(x)}}{2b^2(a^2-b^2)}$$

$$= \frac{a(3a^2-2b^2)\cos(x)}{b^3(a^2-b^2)} - \frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin(x))} - \int \frac{-ab(3a}{}$$

$$= \frac{(6a^2+b^2)x}{2b^4} + \frac{a(3a^2-2b^2)\cos(x)}{b^3(a^2-b^2)} - \frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin}$$

$$= \frac{(6a^2+b^2)x}{2b^4} + \frac{a(3a^2-2b^2)\cos(x)}{b^3(a^2-b^2)} - \frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin}$$

$$= \frac{(6a^2+b^2)x}{2b^4} + \frac{a(3a^2-2b^2)\cos(x)}{b^3(a^2-b^2)} - \frac{(3a^2-b^2)\cos(x)\sin(x)}{2b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{b(a^2-b^2)(a+b\sin}$$

$$= \frac{(6a^2+b^2)x}{2b^4} - \frac{2a^3(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4(a^2-b^2)^{3/2}} + \frac{a(3a^2-2b^2)\cos(x)}{b^3(a^2-b^2)} - \frac{(3a^2-b}{2b^2}$$

**Mathematica** [A]
time = 0.39, size = 115, normalized size = 0.68

$$\frac{12a^2x+2b^2x-\frac{8a^3(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+4ab\cos(x)\left(2+\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}\right)-b^2\sin(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] (12*a^2*x + 2*b^2*x - (8*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(3/2) + 4*a*b*Cos[x]*(2 + a^3/((a - b)*(a + b)*(a +
 b*Sin[x]))) - b^2*Sin[2*x])/(4*b^4)
```

**Maple** [A]
time = 0.30, size = 183, normalized size = 1.08

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\frac{2\left(\frac{b^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}+2ab\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+(6a^2+b^2)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^4}-\frac{2a^3\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{ab}{a^2-b^2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{(3a^2-4b^2)\arctan}{(a^2}\right.}{b^4}$ |
| risch | $\frac{3x\,a^2}{b^4}+\frac{x}{2b^2}+\frac{i\mathrm{e}^{2ix}}{8b^2}+\frac{a\,\mathrm{e}^{ix}}{b^3}+\frac{a\,\mathrm{e}^{-ix}}{b^3}-\frac{i\mathrm{e}^{-2ix}}{8b^2}-\frac{2ia^4(ib+a\,\mathrm{e}^{ix})}{b^4(a^2-b^2)(-ib\,\mathrm{e}^{2ix}+ib+2a\,\mathrm{e}^{ix})}-\frac{3a^5\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-b)b^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*((1/2*b^2*tan(1/2*x)^3+2*a*b*tan(1/2*x)^2-1/2*b^2*tan(1/2*x)+2*a*b)/(
tan(1/2*x)^2+1)^2+1/2*(6*a^2+b^2)*arctan(tan(1/2*x)))-2*a^3/b^4*((-b^2/(a^2
-b^2)*tan(1/2*x)-a*b/(a^2-b^2))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+(3*a^2-4*
b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.46, size = 580, normalized size = 3.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 - (3*a^6 - 4*a^4*b^2 + (3*a^5*b
- 4*a^3*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a
*b^6)*x + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*
a^4*b^3 + 4*a^2*b^5 + b^7)*x + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(
x))/(a^5*b^4 - 2*a^3*b^6 + a*b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x)), 1/2
```

```
*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 + 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b -
4*a^3*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*
cos(x))) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*x + (6*a^6*b - 11*a^4*b
^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*x
+ 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(x))/(a^5*b^4 - 2*a^3*b^6 + a*
b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x))]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.45, size = 184, normalized size = 1.09

$$-\frac{2\,(3\,a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\,\left(a^3b\tan\left(\frac{1}{2}\,x\right)+a^4\right)}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{(6\,a^2+b^2)x}{2\,b^4}+\frac{b\tan\left(\frac{1}{2}\,x\right)^3+4\,a\tan\left(\frac{1}{2}\,x\right)^2-b\tan\left(\frac{1}{2}\,x\right)+4\,a}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*b*tan
(1/2*x) + a^4)/((a^2*b^3 - b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
2*(6*a^2 + b^2)*x/b^4 + (b*tan(1/2*x)^3 + 4*a*tan(1/2*x)^2 - b*tan(1/2*x) +
 4*a)/((tan(1/2*x)^2 + 1)^2*b^3)
```

**Mupad [B]**
time = 11.88, size = 2500, normalized size = 14.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*sin(x))^2,x)
```

```
[Out] (atan((((a^2*6i + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*
b^5 + 36*a^10*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 +
 19*a^3*b^11 + 16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13
 - 2*a^2*b^11 + a^4*b^9) - ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 -
14*a^5*b^10 + 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(4*a^2*b^15
- 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12
```

*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4
*b^9))*(a^2*6i + b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10
+ 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4))*1i)/(2*b^4) + ((a^2
*6i + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^1
0*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11
 + 16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^1
1 + a^4*b^9) + ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10
+ 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8*(4*a^2*b^15 - 8*a^4*b^13
 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32
*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*
6i + b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8
))/(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4))*1i)/(2*b^4))/((16*(54*a^11 + 4*
a^5*b^6 + 9*a^7*b^4 - 81*a^9*b^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) - ((a^2*6i
 + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b
^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 +
16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 +
 a^4*b^9) - ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 + 6
*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(4*a^2*b^15 - 8*a^4*b^13 +
4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^
3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*6i
+ b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/
(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4)))/(2*b^4) + ((a^2*6i + b^2*1i)*((8*
(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b^3))/(b^12 - 2*
a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 + 16*a^5*b^9 - 19
7*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((a
^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^1
2 - 2*a^2*b^10 + a^4*b^8) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b
^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5
*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*6i + b^2*1i))/(2*b
^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b
^11 + a^4*b^9)))/(2*b^4)))/(2*b^4) + (16*tan(x/2)*(216*a^12 + 8*a^4*b^8 + 8
2*a^6*b^6 + 126*a^8*b^4 - 432*a^10*b^2))/(b^13 - 2*a^2*b^11 + a^4*b^9)))*(a
^2*6i + b^2*1i)*1i)/b^4 - ((tan(x/2)*(7*a*b^2 - 9*a^3))/(b^2*(a^2 - b^2)) -
 (2*(3*a^4 - 2*a^2*b^2))/(b^3*(a^2 - b^2)) + (4*tan(x/2)^3*(2*a*b^2 - 3*a^3
))/(b^2*(a^2 - b^2)) + (tan(x/2)^5*(a*b^2 - 3*a^3))/(b^2*(a^2 - b^2)) + (2*
tan(x/2)^4*(b^4 - 3*a^4 + a^2*b^2))/(b^3*(a^2 - b^2)) - (2*tan(x/2)^2*(6*a^
4 + b^4 - 5*a^2*b^2))/(b^3*(a^2 - b^2)))/(a + 2*b*tan(x/2) + 3*a*tan(x/2)^2
 + 3*a*tan(x/2)^4 + a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2)^5) + (a^3*
atan(((a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(a^2*b^11 + 10*
a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b^3))/(b^12 - 2*a^2*b^10 + a^4*
b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 + 16*a^5*b^9 - 197*a^7*b^7 + 228
*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a^3*(3*a^2 - 4*b^
2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 +
6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(32*a^4*b^12 - 56*a
^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a^3*((8*(4*a^2*b^15

```
 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(1
2*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^
4*b^9))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3
*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 -
 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b
)^3)^(1/2)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b
^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 +
16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 +
 a^4*b^9) - (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14
 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (
8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a
^4*b^9) - (a^3*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*
b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a
^4*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^...
```

# 3.186 $\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=124

$$-\frac{2ax}{b^3}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}}-\frac{(2a^2-b^2)\cos(x)}{b^2\left(a^2-b^2\right)}+\frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] -2*a*x/b^3+2*a^2*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^3
/(a^2-b^2)^(3/2)-(2*a^2-b^2)*cos(x)/b^2/(a^2-b^2)+a^2*cos(x)*sin(x)/b/(a^2-
b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2871, 3102, 2814, 2739, 632, 210}

$$\frac{2a^2(2a^2-3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}}-\frac{(2a^2-b^2)\cos(x)}{b^2\left(a^2-b^2\right)}+\frac{a^2\sin(x)\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)) - ((2*a^2 - b^2)*Cos[x])/(b^2*(a^2 - b^2)) + (
a^2*Cos[x]*Sin[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx = \frac{a^2\cos(x)\sin(x)}{b(a^2-b^2)(a+b\sin(x))} - \frac{\int \frac{a^2-ab\sin(x)-(2a^2-b^2)\sin^2(x)}{a+b\sin(x)}\,dx}{b(a^2-b^2)}$$

$$= -\frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin(x)}{b(a^2-b^2)(a+b\sin(x))} - \frac{\int \frac{a^2b+2a(a^2-b^2)\sin(x)}{a+b\sin(x)}\,dx}{b^2(a^2-b^2)}$$

$$= -\frac{2ax}{b^3} - \frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin(x)}{b(a^2-b^2)(a+b\sin(x))} + \frac{(a^2(2a^2-3b^2))\int \frac{1}{a+b\sin(x)}\,\text{d}}{b^3(a^2-b^2)}$$

$$= -\frac{2ax}{b^3} - \frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin(x)}{b(a^2-b^2)(a+b\sin(x))} + \frac{(2a^2(2a^2-3b^2))\,\text{Subst}\left(\int \frac{}{a}\right.}{b^3(a^2-}$$

$$= -\frac{2ax}{b^3} - \frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)} + \frac{a^2\cos(x)\sin(x)}{b(a^2-b^2)(a+b\sin(x))} - \frac{(4a^2(2a^2-3b^2))\,\text{Subst}\left(\int\right.}{b^3}$$

$$= -\frac{2ax}{b^3} + \frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}} - \frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)} + \frac{a^2\cos(x)\,\text{si}}{b(a^2-b^2)(a+}$$

**Mathematica** [A]
time = 0.29, size = 94, normalized size = 0.76

$$\frac{-2ax + \frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + b\cos(x)\left(-1-\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) + b*Cos[x]*(-1 - a^3/((a - b)*(a + b)*(a + b*Sin[x]))))/b
^3
```

**Maple** [A]
time = 0.28, size = 142, normalized size = 1.15

| method | result |
|---|---|
| default | $\frac{4a^2\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{2(a^2-b^2)}-\frac{ab}{2(a^2-b^2)}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{(2a^2-3b^2)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{\frac{3}{2}}}\right)}{b^3}-\frac{4\left(\frac{b}{2\left(\tan^2\left(\frac{x}{2}\right)\right)+2}+a\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^3}$ |

| | |
|---|---|
| risch | $-\frac{2ax}{b^3}-\frac{e^{ix}}{2b^2}-\frac{e^{-ix}}{2b^2}+\frac{2ia^3(-ia\,e^{ix}+b)}{b^3(-a^2+b^2)(b\,e^{2ix}-b+2ia\,e^{ix})}+\frac{2ia^4\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)b^3}-\frac{3ia^2\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2\right.}{\sqrt{}}\right)}{\sqrt{a^2-b^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*a^2/b^3*((-1/2*b^2/(a^2-b^2)*tan(1/2*x)-1/2*a*b/(a^2-b^2))/(a*tan(1/2*x)^
2+2*b*tan(1/2*x)+a)+1/2*(2*a^2-3*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1
/2*x)+2*b)/(a^2-b^2)^(1/2)))-4/b^3*(1/2*b/(tan(1/2*x)^2+1)+a*arctan(tan(1/2
*x)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.40, size = 483, normalized size = 3.90

$$\left[-\frac{(2a^5-3a^3b^2+(2a^4b-3a^2b^3)\sin(x))\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+4(a^6-2a^4b^2+a^2b^4)x+2(2a^5b-3a^3b^3+ab^5)\cos(x)+2(2(a^5b-2a^3b^3+ab^5)x+(a^4b^2-2a^2b^4+b^6)\cos(x))\sin(x)}{2(a^5b^3-2a^3b^5+ab^7+(a^4b^4-2a^2b^6+b^8)\sin(x))},-\frac{(2a^5-3a^3b^2+(2a^4b-3a^2b^3)\sin(x))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+2(a^6-2a^4b^2+a^2b^4)x+(2a^5b-3a^3b^3+ab^5)\cos(x)+(2(a^5b-2a^3b^3+ab^5)x+(a^4b^2-2a^2b^4+b^6)\cos(x))\sin(x)}{a^5b^3-2a^3b^5+ab^7+(a^4b^4-2a^2b^6+b^8)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(-a^2 + b^2)*
log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x)
 + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) +
 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*x + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) +
 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*x + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*s
in(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x)),
-((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*
x + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*x
 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*sin(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.45, size = 204, normalized size = 1.65

$$\frac{2\,(2\,a^4-3\,a^2b^2)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\,\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}}-\frac{2\left(a^2b\tan\left(\frac{1}{2}\,x\right)^3+2\,a^3\tan\left(\frac{1}{2}\,x\right)^2-ab^2\tan\left(\frac{1}{2}\,x\right)^2+3\,a^2b\tan\left(\frac{1}{2}\,x\right)-2\,b^3\tan\left(\frac{1}{2}\,x\right)+2\,a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^4+2\,b\tan\left(\frac{1}{2}\,x\right)^3+2\,a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)(a^2b^2-b^4)}-\frac{2\,ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(a^2*b*tan(
1/2*x)^3 + 2*a^3*tan(1/2*x)^2 - a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2
*b^3*tan(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)*(a^2*b^2 - b^4)) - 2*a*x/b^3
```

**Mupad** [**B**]
time = 9.98, size = 2578, normalized size = 20.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x))^2,x)
```

```
[Out] ((2*(a*b^2 - 2*a^3))/(b^2*(a^2 - b^2)) - (2*a^2*tan(x/2)^3)/(b*(a^2 - b^2))
 + (2*tan(x/2)^2*(a*b^2 - 2*a^3))/(b^2*(a^2 - b^2)) - (2*tan(x/2)*(3*a^2 -
2*b^2))/(b*(a^2 - b^2)))/(a + 2*b*tan(x/2) + 2*a*tan(x/2)^2 + a*tan(x/2)^4
+ 2*b*tan(x/2)^3) - (4*a*atan((512*a^4*b^5*tan(x/2))/((512*a^4*b^14)/(b^9 -
 2*a^2*b^7 + a^4*b^5) - (1408*a^6*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (1280
*a^8*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) - (384*a^10*b^8)/(b^9 - 2*a^2*b^7 +
a^4*b^5)) - (384*a^6*b^3*tan(x/2))/((512*a^4*b^14)/(b^9 - 2*a^2*b^7 + a^4*b
^5) - (1408*a^6*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (1280*a^8*b^10)/(b^9 -
2*a^2*b^7 + a^4*b^5) - (384*a^10*b^8)/(b^9 - 2*a^2*b^7 + a^4*b^5))))/b^3 -
(a^2*atan(((a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^
6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(8*a
^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6)
```

+ (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(4*a^8 - 6*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(x/2)*(16*a^9 + 24*a^5*b^4 - 40*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)

# 3.187 $\int \frac{\sin^2(x)}{(a+b\sin(x))^2} dx$

**Optimal**. Leaf size=87

$$\frac{x}{b^2} - \frac{2a(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}} + \frac{a^2\cos(x)}{b\left(a^2-b^2\right)\left(a+b\sin(x)\right)}$$

```
[Out] x/b^2-2*a*(a^2-2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2
)^(3/2)+a^2*cos(x)/b/(a^2-b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2869, 2814, 2739, 632, 210}

$$-\frac{2a(a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}} + \frac{a^2\cos(x)}{b\left(a^2-b^2\right)\left(a+b\sin(x)\right)} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*(
a^2 - b^2)^(3/2)) + (a^2*Cos[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{(a+b\sin(x))^2}\,dx &= \frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{ab+\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(a(a^2-2b^2)\right)\int \frac{1}{a+b\sin(x)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2a(a^2-2b^2)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4a(a^2-2b^2)\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\dots\right.}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}-\frac{2a(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}
$$

**Mathematica** [A]
time = 0.16, size = 83, normalized size = 0.95

$$
\frac{x-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a^2b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (x - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + (a^2*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])))/b^2
```

**Maple [A]**
time = 0.23, size = 121, normalized size = 1.39

| method | result |
|---|---|
| default | $\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2a\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{ab}{a^2-b^2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^2}$ |
| risch | $\frac{x}{b^2}-\frac{2ia^2\left(ib+a\,e^{ix}\right)}{b^2\left(a^2-b^2\right)\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)}+\frac{a^3\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)b^2}-\frac{2a\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*arctan(tan(1/2*x))-2/b^2*a*((-b^2/(a^2-b^2)*tan(1/2*x)-a*b/(a^2-b^2))
/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+(a^2-2*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(81) = 162$.
time = 0.44, size = 403, normalized size = 4.63

$$\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)x\sin\left(x\right)-\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\sin\left(x\right)\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(x\right)^2-2ab\sin\left(x\right)-a^2-b^2-2\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(x\right)^2-2ab\sin\left(x\right)-a^2-b^2}\right)+2\left(a^5-2a^3b^2+ab^4\right)x+2\left(a^4b-a^2b^3\right)\cos\left(x\right)}{2\left(a^5b^2-2a^3b^4+ab^6+\left(a^4b^3-2a^2b^5+b^7\right)\sin\left(x\right)\right)}, \frac{\left(a^4b-2a^2b^3+b^5\right)x\sin\left(x\right)+\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\sin\left(x\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(x\right)+b}{\sqrt{a^2-b^2}\cos\left(x\right)}\right)+\left(a^5-2a^3b^2+ab^4\right)x+\left(a^4b-a^2b^3\right)\cos\left(x\right)}{a^5b^2-2a^3b^4+ab^6+\left(a^4b^3-2a^2b^5+b^7\right)\sin\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) - (a^4 - 2*a^2*b^2 + (a^3*b - 2*
a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x)
```

```
 - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)
^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x + 2*(a^4*b
 - a^2*b^3)*cos(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^
7)*sin(x)), ((a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) + (a^4 - 2*a^2*b^2 + (a^3*b
 - 2*a*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x))) + (a^5 - 2*a^3*b^2 + a*b^4)*x + (a^4*b - a^2*b^3)*cos(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sin(x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.46, size = 124, normalized size = 1.43

$$-\frac{2\left(a^{3}-2\,ab^{2}\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{2}-b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(ab\tan\left(\frac{1}{2}\,x\right)+a^{2}\right)}{\left(a^{2}b-b^{3}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 - 2*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(a*b*tan(1/2*x
) + a^2)/((a^2*b - b^3)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + x/b^2
```

**Mupad** [**B**]
time = 9.95, size = 2562, normalized size = 29.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x))^2,x)
```

```
[Out] ((2*a^2)/(b*(a^2 - b^2)) + (2*a*tan(x/2))/(a^2 - b^2))/(a + 2*b*tan(x/2) +
a*tan(x/2)^2) - (2*atan((64*a^3*b^3*tan(x/2))/((64*a^3*b^9)/(b^6 - 2*a^2*b^
4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (192*a^7*b^5)/(b
^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (64*
a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2)) + (64*a*b^5*tan(x/2))/((64*a^3*b^9)/(b
^6 - 2*a^2*b^4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (19
```

2*a^7*b^5)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2*a^2*b^4 + a^
4*b^2) - (64*a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2)) - (64*a^5*b*tan(x/2))/((6
4*a^3*b^9)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a
^4*b^2) - (192*a^7*b^5)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2
*a^2*b^4 + a^4*b^2) - (64*a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2))))/b^2 + (a*a
tan(((a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 -
2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b -
9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^
2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a
^4*b^3) + (a*(a^2 - 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a
^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*
b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a
^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i
)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b
^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^
2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^
8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4
*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*((32*(a^2
*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*
a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(
-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8
- 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6
*b^2))/((64*(a^5 - 2*a^3*b^2))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*tan(x/2)*(
2*a^6 + 4*a^2*b^4 - 6*a^4*b^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 -
2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5
*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*
(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2
- 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2)
+ (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2
*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4
- a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 +
3*a^4*b^4 - a^6*b^2) - (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(
a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2
*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (a
*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 -
2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b
^7 - 2*a^2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^
6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 +
7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)
^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*
b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2 - 2*b^2)*(-

```
(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)
```

# 3.188 $\int \frac{\sin(x)}{(a+b\sin(x))^2} dx$

**Optimal**. Leaf size=66

$$-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}$$

```
[Out] -2*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-a*cos(x)/(a^2
-b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2833, 12, 2739, 632, 210}

$$-\frac{2b\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx &= -\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}+\frac{\int \frac{b}{a+b\sin(x)}\,dx}{-a^2+b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}-\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}+\frac{(4b)\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}
\end{aligned}
$$

**Mathematica** [A]
time = 0.08, size = 67, normalized size = 1.02

$$
-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a-b)(a+b)(a+b\sin(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a - b)*(a + b)*(a + b*Sin[x]))
```

**Maple** [A]
time = 0.18, size = 99, normalized size = 1.50

| method | result | size |
|---|---|---|
| default | $\frac{-8b\tan\left(\frac{x}{2}\right)-8a}{\left(4a^2-4b^2\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}-\frac{8b\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(4a^2-4b^2\right)\sqrt{a^2-b^2}}$ | 99 |
| risch | $\frac{2ia\left(-ia\,\mathrm{e}^{ix}+b\right)}{b\left(-a^2+b^2\right)\left(b\,\mathrm{e}^{2ix}-b+2ia\,\mathrm{e}^{ix}\right)}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$ | 199 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(-2*b*tan(1/2*x)-2*a)/(4*a^2-4*b^2)/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)-8*b
/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1
/2))
```

**<u>Maxima</u> [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**<u>Fricas</u> [A]**
time = 0.38, size = 266, normalized size = 4.03

$$\left[\frac{\left(b^2\sin\left(x\right)+ab\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(x\right)^2-2ab\sin\left(x\right)-a^2-b^2+2\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(x\right)^2-2ab\sin\left(x\right)-a^2-b^2}\right)-2\left(a^3-ab^2\right)\cos\left(x\right)}{2\left(a^5-2a^3b^2+ab^4+\left(a^4b-2a^2b^3+b^5\right)\sin\left(x\right)\right)},\ \frac{\left(b^2\sin\left(x\right)+ab\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(x\right)+b}{\sqrt{a^2-b^2}\cos\left(x\right)}\right)-\left(a^3-ab^2\right)\cos\left(x\right)}{a^5-2a^3b^2+ab^4+\left(a^4b-2a^2b^3+b^5\right)\sin\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*sin(x) + a*b)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b
^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(x))/(a^5 - 2
*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), ((b^2*sin(x) + a*b)*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^3 - a*
b^2)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**2,x)
```

```
[Out] Integral(sin(x)/(a + b*sin(x))**2, x)
```

**Giac [A]**
time = 0.43, size = 90, normalized size = 1.36

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{(a^2-b^2)^{\frac{3}{2}}}-\frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2+2\,b\tan\left(\frac{1}{2}x\right)+a\right)(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(a^2 - b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 + 2*b*ta
n(1/2*x) + a)*(a^2 - b^2))
```

**Mupad [B]**
time = 6.49, size = 123, normalized size = 1.86

$$-\frac{\frac{2\,a}{a^2-b^2}+\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2-b^2}}{a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{(a^2-b^2)\left(\frac{2\,b^2}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,b\tan\left(\frac{x}{2}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)}{2\,b}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x))^2,x)
```

```
[Out] - ((2*a)/(a^2 - b^2) + (2*b*tan(x/2))/(a^2 - b^2))/(a + 2*b*tan(x/2) + a*ta
n(x/2)^2) - (2*b*atan(((a^2 - b^2)*((2*b^2)/((a + b)^(3/2)*(a - b)^(3/2)) +
 (2*a*b*tan(x/2))/((a + b)^(3/2)*(a - b)^(3/2))))/(2*b)))/((a + b)^(3/2)*(a
 - b)^(3/2))
```

# 3.189 $\int \frac{1}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=65

$$\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)\left(a+b\sin(x)\right)}$$

```
[Out] 2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+b*cos(x)/(a^2-
b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2743, 12, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)\left(a+b\sin(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(x))^2}\,dx &= \frac{b\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}-\frac{\int \frac{a}{a+b\sin(x)}\,dx}{-a^2+b^2}\\
&= \frac{b\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}+\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2}\\
&= \frac{b\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{b\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b\cos(x)}{(a^2-b^2)\,(a+b\sin(x))}
\end{aligned}$$

**Mathematica** [A]
time = 0.07, size = 66, normalized size = 1.02

$$\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a - b)*(a + b)*(a + b*Sin[x]))
```

**Maple** [A]
time = 0.14, size = 98, normalized size = 1.51

| method | result | size |
|---|---|---|
| default | $\frac{\frac{2b^{2}\tan\left(\frac{x}{2}\right)}{a\left(a^{2}-b^{2}\right)}+\frac{2b}{a^{2}-b^{2}}}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}$ | 98 |
| risch | $\frac{2ib+2a\,\mathrm{e}^{ix}}{\left(a^{2}-b^{2}\right)\left(b\,\mathrm{e}^{2ix}-b+2ia\,\mathrm{e}^{ix}\right)}-\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$ | 192 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tan(1/2*x)+b/(a^2-b^2))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a
)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.35, size = 268, normalized size = 4.12

$$\left[\frac{\left(ab\sin\left(x\right)+a^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(-\frac{\left(2a^{2}-b^{2}\right)\cos\left(x\right)^{2}-2ab\sin\left(x\right)-a^{2}-b^{2}-2\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(x\right)^{2}-2ab\sin\left(x\right)-a^{2}-b^{2}}\right)+2\left(a^{2}b-b^{3}\right)\cos\left(x\right)}{2\left(a^{5}-2a^{3}b^{2}+ab^{4}+\left(a^{4}b-2a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)},-\frac{\left(ab\sin\left(x\right)+a^{2}\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin\left(x\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(x\right)}\right)-\left(a^{2}b-b^{3}\right)\cos\left(x\right)}{a^{5}-2a^{3}b^{2}+ab^{4}+\left(a^{4}b-2a^{2}b^{3}+b^{5}\right)\sin\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(
b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(x))/(a^5 -
2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), -((a*b*sin(x) + a^2)
*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b
- b^3)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))
]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**2,x)
```

```
[Out] Integral((a + b*sin(x))**(-2), x)
```

**Giac** [A]
time = 0.44, size = 95, normalized size = 1.46

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)a}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{2\left(b^{2}\tan\left(\frac{1}{2}x\right)+ab\right)}{\left(a^{3}-ab^{2}\right)\left(a\tan\left(\frac{1}{2}x\right)^{2}+2\,b\tan\left(\frac{1}{2}x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(a^2 - b^2)^(3/2) + 2*(b^2*tan(1/2*x) + a*b)/((a^3 - a*b^2)*(a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a))
```

**Mupad** [B]
time = 6.82, size = 148, normalized size = 2.28

$$\frac{\frac{2\,b}{a^2-b^2}+\frac{2\,b^2\,\tan\left(\frac{x}{2}\right)}{a\,\left(a^2-b^2\right)}}{a\,{\tan\left(\frac{x}{2}\right)}^2+2\,b\,\tan\left(\frac{x}{2}\right)+a}+\frac{2\,a\,\mathrm{atan}\left(\frac{\left(a^2-b^2\right)\,\left(\frac{2\,a^2\,\tan\left(\frac{x}{2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a\,\left(a^2\,b-b^3\right)}{{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}\right)}{2\,a}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x))^2,x)
```

```
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(x/2))/(a*(a^2 - b^2)))/(a + 2*b*tan(x/2) +
a*tan(x/2)^2) + (2*a*atan(((a^2 - b^2)*((2*a^2*tan(x/2))/((a + b)^(3/2)*(a
- b)^(3/2)) + (2*a*(a^2*b - b^3))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))
))/(2*a)))/((a + b)^(3/2)*(a - b)^(3/2))
```

# 3.190 $\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=93

$$-\frac{2b(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] -2*b*(2*a^2-b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/
2)-arctanh(cos(x))/a^2-b^2*cos(x)/a/(a^2-b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2881, 3080, 3855, 2739, 632, 210}

$$-\frac{2b(2a^2-b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\tanh^{-1}(\cos(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b
^2)^(3/2)) - ArcTanh[Cos[x]]/a^2 - (b^2*Cos[x])/(a*(a^2 - b^2)*(a + b*Sin[x
]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx &= -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{\csc(x)\left(a^2-b^2-ab\sin(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)}\\
&= -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{a^2\left(a^2-b^2\right)}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2b\left(2a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,\right.}{a^2\left(a^2-b^2\right)}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4b\left(2a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\right.}{a^2\left(a^2-b^2\right)}\\
&= -\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}
$$

**Mathematica [A]**

time = 0.18, size = 99, normalized size = 1.06

$$\frac{\frac{2b(-2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{ab^2\cos(x)}{(a-b)(a+b)(a+b\sin(x))}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*b*(-2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(3/2) - Log[Cos[x/2]] + Log[Sin[x/2]] - (a*b^2*Cos[x])/((a - b)*(a + b)*(a
+ b*Sin[x])))/a^2
```

**Maple [A]**
time = 0.31, size = 123, normalized size = 1.32

| method | result |
|---|---|
| default | $\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{\frac{b^2\tan\left(\frac{x}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{a^2}$ |
| risch | $\frac{2ib\left(-ia\,e^{ix}+b\right)}{a(-a^2+b^2)(b\,e^{2ix}-b+2ia\,e^{ix})}+\frac{2ib\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{ib^3\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)a^2}-\frac{2ib\ln}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*ln(tan(1/2*x))-4/a^2*b*((1/2*b^2/(a^2-b^2)*tan(1/2*x)+1/2*a*b/(a^2-b^
2))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+1/2*(2*a^2-b^2)/(a^2-b^2)^(3/2)*arcta
n(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(87) = 174$.
time = 0.69, size = 511, normalized size = 5.49

$$\left[-\frac{\left(2\,a^{3}b-ab^{3}+\left(2\,a^{2}b^{2}-b^{4}\right)\sin\left(x\right)\right)\sqrt{-a^{2}+b^{2}}\log\left(-\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}-2\,\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}}\right)+2\,\left(a^{3}b^{2}-ab^{4}\right)\cos\left(x\right)+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-\left(a^{5}-2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\,\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}+\left(a^{6}b-2\,a^{4}b^{3}+a^{2}b^{5}\right)\sin\left(x\right)\right)},\ \frac{2\,\left(2\,a^{3}b-ab^{3}+\left(2\,a^{2}b^{2}-b^{4}\right)\sin\left(x\right)\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin\left(x\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(x\right)}\right)-2\,\left(a^{3}b^{2}-ab^{4}\right)\cos\left(x\right)-\left(a^{5}-2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\,\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}+\left(a^{6}b-2\,a^{4}b^{3}+a^{2}b^{5}\right)\sin\left(x\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*sin(x))*sqrt(-a^2 + b^2)*log(-(
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3*b^2 - a*b^4)*cos(x) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^
5)*sin(x))*log(1/2*cos(x) + 1/2) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^
2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 + a^3*b^4 + (
a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x)), 1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2
- b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(
x))) - 2*(a^3*b^2 - a*b^4)*cos(x) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a
^2*b^3 + b^5)*sin(x))*log(1/2*cos(x) + 1/2) + (a^5 - 2*a^3*b^2 + a*b^4 + (a
^4*b - 2*a^2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 +
a^3*b^4 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(x\right)}{\left(a+b\sin\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x))**2, x)
```

**Giac [A]**
time = 0.45, size = 134, normalized size = 1.44

$$-\frac{2\,\left(2\,a^{2}b-b^{3}\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\,\left(b^{3}\tan\left(\frac{1}{2}\,x\right)+ab^{2}\right)}{\left(a^{4}-a^{2}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(2*a^2*b - b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) - 2*(b^3*tan(1/2*x
```

```
) + a*b^2)/((a^4 - a^2*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + log(ab
s(tan(1/2*x)))/a^2
```

**Mupad [B]**
time = 7.82, size = 1356, normalized size = 14.58

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\frac{2b^2}{a\left(a^2-b^2\right)}+\frac{2b^3\tan\left(\frac{x}{2}\right)}{a^2\left(a^2-b^2\right)}}{a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a}-\frac{b\,\mathrm{atan}\left(\frac{\frac{b\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\tan\left(\frac{x}{2}\right)\left(a^6-8a^4b^2+11a^2b^4-4b^6\right)}{a^5-2a^3b^2+ab^4}-\frac{2\left(3a^4b-2a^2b^3\right)}{a^4-a^2b^2}+\frac{b\left(\frac{2\left(a^6b-a^4b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^8-10a^6b^2+11a^4b^4-4a^2b^6\right)}{a^5-2a^3b^2+ab^4}\right)\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}}{a^8-3a^6b^2+3a^4b^4-a^2b^6}\right)1i}{a^8-3a^6b^2+3a^4b^4-a^2b^6}-\frac{b\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(3a^4b-2a^2b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(a^6-8a^4b^2+11a^2b^4-4b^6\right)}{a^5-2a^3b^2+ab^4}+\frac{b\left(\frac{2\left(a^6b-a^4b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^8-10a^6b^2+11a^4b^4-4a^2b^6\right)}{a^5-2a^3b^2+ab^4}\right)\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}}{a^8-3a^6b^2+3a^4b^4-a^2b^6}\right)1i}{a^8-3a^6b^2+3a^4b^4-a^2b^6}}{\frac{4\left(2a^2b-b^3\right)}{a^4-a^2b^2}+\frac{4\tan\left(\frac{x}{2}\right)\left(2b^4-4a^2b^2\right)}{a^5-2a^3b^2+ab^4}+\frac{b\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\tan\left(\frac{x}{2}\right)\left(a^6-8a^4b^2+11a^2b^4-4b^6\right)}{a^5-2a^3b^2+ab^4}-\frac{2\left(3a^4b-2a^2b^3\right)}{a^4-a^2b^2}+\frac{b\left(\frac{2\left(a^6b-a^4b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^8-10a^6b^2+11a^4b^4-4a^2b^6\right)}{a^5-2a^3b^2+ab^4}\right)\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}}{a^8-3a^6b^2+3a^4b^4-a^2b^6}\right)}{a^8-3a^6b^2+3a^4b^4-a^2b^6}+\frac{b\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(3a^4b-2a^2b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(a^6-8a^4b^2+11a^2b^4-4b^6\right)}{a^5-2a^3b^2+ab^4}+\frac{b\left(\frac{2\left(a^6b-a^4b^3\right)}{a^4-a^2b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^8-10a^6b^2+11a^4b^4-4a^2b^6\right)}{a^5-2a^3b^2+ab^4}\right)\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}}{a^8-3a^6b^2+3a^4b^4-a^2b^6}\right)}{a^8-3a^6b^2+3a^4b^4-a^2b^6}}\right)\left(2a^2-b^2\right)\sqrt{-(a+b)^3(a-b)^3}\,2i}{a^8-3a^6b^2+3a^4b^4-a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*sin(x))^2),x)
```

```
[Out] log(tan(x/2))/a^2 - ((2*b^2)/(a*(a^2 - b^2)) + (2*b^3*tan(x/2))/(a^2*(a^2 -
 b^2)))/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (b*atan(((b*(2*a^2 - b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2)*((2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 - 8*a^4*b^2)
)/(a*b^4 + a^5 - 2*a^3*b^2) - (2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) + (
b*((2*(a^6*b - a^4*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 +
 11*a^4*b^4 - 10*a^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a +
b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*1i)/(a^8 -
a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (b*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(a^6 - 4*b^6
 + 11*a^2*b^4 - 8*a^4*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*((2*(a^6*b - a^4
*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4 - 10*a
^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1
/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*1i)/(a^8 - a^2*b^6 + 3*a^4*b^
4 - 3*a^6*b^2))/((4*(2*a^2*b - b^3))/(a^4 - a^2*b^2) + (4*tan(x/2)*(2*b^4 -
 4*a^2*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*(2*a^2 - b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*((2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 - 8*a^4*b^2))/(a*b^4 + a
^5 - 2*a^3*b^2) - (2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) + (b*((2*(a^6*b
 - a^4*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4
- 10*a^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)
^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4
*b^4 - 3*a^6*b^2) + (b*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(3*a^
4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 -
 8*a^4*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*((2*(a^6*b - a^4*b^3))/(a^4 - a
^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4 - 10*a^6*b^2))/(a*b^4
 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2
*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(
2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^8 - a^2*b^6 + 3*a^4*b^4 -
3*a^6*b^2)
```

# 3.191 $\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=123

$$\frac{2b^2(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\,(a^2-b^2)^{3/2}}+\frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{(a^2-2b^2)\cot(x)}{a^2\,(a^2-b^2)}-\frac{b^2\cot(x)}{a\,(a^2-b^2)\,(a+b\sin(x))}$$

```
[Out] 2*b^2*(3*a^2-2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^
(3/2)+2*b*arctanh(cos(x))/a^3-(a^2-2*b^2)*cot(x)/a^2/(a^2-b^2)-b^2*cot(x)/a
/(a^2-b^2)/(a+b*sin(x))
```

**Rubi** [A]
time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{(a^2-2b^2)\cot(x)}{a^2\,(a^2-b^2)}-\frac{b^2\cot(x)}{a\,(a^2-b^2)\,(a+b\sin(x))}+\frac{2b^2(3a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\,(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2
 - b^2)^(3/2)) + (2*b*ArcTanh[Cos[x]])/a^3 - ((a^2 - 2*b^2)*Cot[x])/(a^2*(a^
2 - b^2)) - (b^2*Cot[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)}{a(a^2-b^2)(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)(a^2-2b^2-ab\sin(x)+b^2\sin^2(x))}{a+b\sin(x)}\,dx}{a(a^2-b^2)}$$

$$= -\frac{(a^2-2b^2)\cot(x)}{a^2(a^2-b^2)} - \frac{b^2\cot(x)}{a(a^2-b^2)(a+b\sin(x))} + \frac{\int \frac{\csc(x)(-2b(a^2-b^2)+ab^2\sin(x))}{a+b\sin(x)}\,dx}{a^2(a^2-b^2)}$$

$$= -\frac{(a^2-2b^2)\cot(x)}{a^2(a^2-b^2)} - \frac{b^2\cot(x)}{a(a^2-b^2)(a+b\sin(x))} - \frac{(2b)\int \csc(x)\,dx}{a^3} + \frac{(b^2(3a^2-2b^2))\,\int}{a^3(a^2-}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{(a^2-2b^2)\cot(x)}{a^2(a^2-b^2)} - \frac{b^2\cot(x)}{a(a^2-b^2)(a+b\sin(x))} + \frac{(2b^2(3a^2-2b^2)}{}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{(a^2-2b^2)\cot(x)}{a^2(a^2-b^2)} - \frac{b^2\cot(x)}{a(a^2-b^2)(a+b\sin(x))} - \frac{(4b^2(3a^2-2b^2)}{}$$

$$= \frac{2b^2(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan(\frac{x}{2})}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{3/2}} + \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{(a^2-2b^2)\cot(x)}{a^2(a^2-b^2)} - \frac{}{a(a^2}$$

**Mathematica [A]**
time = 0.49, size = 127, normalized size = 1.03

$$\frac{\frac{4b^2(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan(\frac{x}{2})}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - a\cot\left(\frac{x}{2}\right) + 4b\log\left(\cos\left(\frac{x}{2}\right)\right) - 4b\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{2ab^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))} + a\tan\left(\frac{x}{2}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] ((4*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) - a*Cot[x/2] + 4*b*Log[Cos[x/2]] - 4*b*Log[Sin[x/2]] + (2*a*b^3*Co
s[x])/((a - b)*(a + b)*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^3)
```

**Maple [A]**
time = 0.33, size = 144, normalized size = 1.17

| method | result |
|---|---|
| default | $\frac{\tan(\frac{x}{2})}{2a^2} - \frac{1}{2a^2\tan(\frac{x}{2})} - \frac{2b\ln(\tan(\frac{x}{2}))}{a^3} + \frac{2b^2\left(\frac{\frac{b^2\tan(\frac{x}{2})}{a^2-b^2}+\frac{ab}{a^2-b^2}}{a(\tan^2(\frac{x}{2}))+2b\tan(\frac{x}{2})+a} + \frac{(3a^2-2b^2)\arctan\left(\frac{2a\tan(\frac{x}{2})+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}\right)}{a^3}$ |

| | |
|---|---|
| risch | $-\frac{2i(2a^3 e^{ix} - 3b^2 a e^{ix} - ia^2 b e^{2ix} + 2ib^3 e^{2ix} + ia^2 b - 2ib^3 + a b^2 e^{3ix})}{(e^{2ix} - 1)(a^2 - b^2)(-ib e^{2ix} + ib + 2a e^{ix}) a^2} + \frac{3b^2 \ln\left(e^{ix} + \frac{ia\sqrt{-a^2+b^2} + a^2 - b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)a} - \frac{2b^4 \ln\left(e^{ix} + \frac{ia\sqrt{}}{}\right.}{\sqrt{-a^2+}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*tan(1/2*x)-1/2/a^2/tan(1/2*x)-2/a^3*b*ln(tan(1/2*x))+2/a^3*b^2*((b^
2/(a^2-b^2)*tan(1/2*x)+a*b/(a^2-b^2))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+(3*
a^2-2*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(117) = 234$.
time = 0.68, size = 784, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x)*sin(x) + (3*a^2*b^3 - 2*b^5 -
 (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 - 2*a*b^4)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6 -
(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*
log(1/2*cos(x) + 1/2) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/
2))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
 (a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x)), -((a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x
)*sin(x) + (3*a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 -
```

```
 2*a*b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*c
os(x))) + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - (a^4*b^2 - 2*a^2*b^4 + b^6 -
 (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))
*log(1/2*cos(x) + 1/2) + (a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
+ b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/2
))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
(a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**2, x)
```

**Giac [A]**
time = 0.43, size = 234, normalized size = 1.90

$$\frac{2\,(3\,a^2b^2-2\,b^4)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{a^2-b^2}}+\frac{4\,a^3b\tan\left(\frac{1}{2}x\right)^3-4\,ab^3\tan\left(\frac{1}{2}x\right)^3-3\,a^4\tan\left(\frac{1}{2}x\right)^2+11\,a^2b^2\tan\left(\frac{1}{2}x\right)^2+4\,b^4\tan\left(\frac{1}{2}x\right)^2-2\,a^3b\tan\left(\frac{1}{2}x\right)+14\,ab^3\tan\left(\frac{1}{2}x\right)-3\,a^4+3\,a^2b^2}{6\,(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}x\right)^3+2\,b\tan\left(\frac{1}{2}x\right)^2+a\tan\left(\frac{1}{2}x\right)\right)}-\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 1/6*(4*a^3*b*
tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 3*a^4*tan(1/2*x)^2 + 11*a^2*b^2*tan(1
/2*x)^2 + 4*b^4*tan(1/2*x)^2 - 2*a^3*b*tan(1/2*x) + 14*a*b^3*tan(1/2*x) - 3
*a^4 + 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*x)^3 + 2*b*tan(1/2*x)^2 + a*t
an(1/2*x))) - 2*b*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^2
```

**Mupad [B]**
time = 7.63, size = 1471, normalized size = 11.96

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^2}-\frac{a-\frac{\tan\left(\frac{x}{2}\right)^2\left(-a^4+a^2b^2+4b^4\right)}{a\left(a^2-b^2\right)}+\frac{2b\tan\left(\frac{x}{2}\right)\left(a^2-3b^2\right)}{a^2-b^2}}{2a^3\tan\left(\frac{x}{2}\right)^3+2a^3\tan\left(\frac{x}{2}\right)+4ba^2\tan\left(\frac{x}{2}\right)^2}-\frac{2b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{b^2\,\mathrm{atan}\left(\frac{\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\tan\left(\frac{x}{2}\right)\left(-2a^7b+14a^5b^3-20a^3b^5+8ab^7\right)}{a^7-2a^5b^2+a^3b^4}-\frac{2\left(4a^3b^4-5a^5b^2\right)}{a^6-a^4b^2}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(a^8b-a^6b^3\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^{10}-10a^8b^2+11a^6b^4-4a^4b^6\right)}{a^7-2a^5b^2+a^3b^4}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}\right)1i}{a^9-3a^7b^2+3a^5b^4-a^3b^6}-\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(4a^3b^4-5a^5b^2\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(-2a^7b+14a^5b^3-20a^3b^5+8ab^7\right)}{a^7-2a^5b^2+a^3b^4}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(a^8b-a^6b^3\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^{10}-10a^8b^2+11a^6b^4-4a^4b^6\right)}{a^7-2a^5b^2+a^3b^4}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}\right)1i}{a^9-3a^7b^2+3a^5b^4-a^3b^6}}{\frac{4\tan\left(\frac{x}{2}\right)\left(4b^6-6a^2b^4\right)}{a^7-2a^5b^2+a^3b^4}-\frac{4\left(4b^5-6a^2b^3\right)}{a^6-a^4b^2}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\tan\left(\frac{x}{2}\right)\left(-2a^7b+14a^5b^3-20a^3b^5+8ab^7\right)}{a^7-2a^5b^2+a^3b^4}-\frac{2\left(4a^3b^4-5a^5b^2\right)}{a^6-a^4b^2}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(a^8b-a^6b^3\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^{10}-10a^8b^2+11a^6b^4-4a^4b^6\right)}{a^7-2a^5b^2+a^3b^4}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(4a^3b^4-5a^5b^2\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(-2a^7b+14a^5b^3-20a^3b^5+8ab^7\right)}{a^7-2a^5b^2+a^3b^4}+\frac{b^2\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\left(\frac{2\left(a^8b-a^6b^3\right)}{a^6-a^4b^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(3a^{10}-10a^8b^2+11a^6b^4-4a^4b^6\right)}{a^7-2a^5b^2+a^3b^4}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}\right)}{a^9-3a^7b^2+3a^5b^4-a^3b^6}}\right)\left(3a^2-2b^2\right)\sqrt{-(a+b)^3(a-b)^3}\,2i}{a^9-3a^7b^2+3a^5b^4-a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - (a - (tan(x/2)^2*(4*b^4 - a^4 + a^2*b^2))/(a*(a^2 - b^2)
) + (2*b*tan(x/2)*(a^2 - 3*b^2))/(a^2 - b^2))/(2*a^3*tan(x/2) + 2*a^3*tan(x
/2)^3 + 4*a^2*b*tan(x/2)^2) - (2*b*log(tan(x/2)))/a^3 - (b^2*atan(((b^2*(3*
```

```
a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*tan(x/2)*(8*a*b^7 - 2*a^7*b -
 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (2*(4*a^3*b^4 - 5*
a^5*b^2))/(a^6 - a^4*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/
2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6
 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 +
 3*a^5*b^4 - 3*a^7*b^2))*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) - (b^2
*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(4*a^3*b^4 - 5*a^5*b^2))/
(a^6 - a^4*b^2) - (2*tan(x/2)*(8*a*b^7 - 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3)
)/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4
*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b
^6 + 3*a^5*b^4 - 3*a^7*b^2))*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/(
(4*tan(x/2)*(4*b^6 - 6*a^2*b^4))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (4*(4*b^5 -
6*a^2*b^3))/(a^6 - a^4*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((2*tan(x/2)*(8*a*b^7 - 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3
*b^4 - 2*a^5*b^2) - (2*(4*a^3*b^4 - 5*a^5*b^2))/(a^6 - a^4*b^2) + (b^2*(3*a
^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*
b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a
^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/(a^9 - a^3*
b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((2*(4*a^3*b^4 - 5*a^5*b^2))/(a^6 - a^4*b^2) - (2*tan(x/2)*(8*a*b^7 -
 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b^2*(3*
a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4
*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 +
a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/(a^9 - a^3
*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2
)*2i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)
```

# 3.192 $\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx$

**Optimal**. Leaf size=168

$$-\frac{2b^3(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}-\frac{(a^2+6b^2)\tanh^{-1}(\cos(x))}{2a^4}+\frac{b(2a^2-3b^2)\cot(x)}{a^3\left(a^2-b^2\right)}-\frac{(a^2-3b^2)\cot(x)\,\mathrm{csc}}{2a^2\left(a^2-b^2\right)}$$

```
[Out] -2*b^3*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^4/(a^2-b^2)
^(3/2)-1/2*(a^2+6*b^2)*arctanh(cos(x))/a^4+b*(2*a^2-3*b^2)*cot(x)/a^3/(a^2-
b^2)-1/2*(a^2-3*b^2)*cot(x)*csc(x)/a^2/(a^2-b^2)-b^2*cot(x)*csc(x)/a/(a^2-b
^2)/(a+b*sin(x))
```

**Rubi** [A]

time = 0.41, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$-\frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{2b^3(4a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}-\frac{(a^2+6b^2)\tanh^{-1}(\cos(x))}{2a^4}+\frac{b(2a^2-3b^2)\cot(x)}{a^3\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2
 - b^2)^(3/2)) - ((a^2 + 6*b^2)*ArcTanh[Cos[x]])/(2*a^4) + (b*(2*a^2 - 3*b^
2)*Cot[x])/(a^3*(a^2 - b^2)) - ((a^2 - 3*b^2)*Cot[x]*Csc[x])/(2*a^2*(a^2 -
b^2)) - (b^2*Cot[x]*Csc[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)\csc(x)}{a\,(a^2-b^2)\,(a+b\sin(x))} + \frac{\int \frac{\csc^3(x)(a^2-3b^2-ab\sin(x)+2b^2\sin^2(x))}{a+b\sin(x)}\,dx}{a\,(a^2-b^2)}$$

$$= -\frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\,(a^2-b^2)} - \frac{b^2\cot(x)\csc(x)}{a\,(a^2-b^2)\,(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)(-2b(2a^2-3b^2)+a(a^2+b^2)\,\mathrm{s}}{a+b\sin(x}}{2a^2\,(a^2-}$$

$$= \frac{b(2a^2-3b^2)\cot(x)}{a^3\,(a^2-b^2)} - \frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\,(a^2-b^2)} - \frac{b^2\cot(x)\csc(x)}{a\,(a^2-b^2)\,(a+b\sin(x))} + \frac{\int \frac{\csc(x)(}{}}{}$$

$$= \frac{b(2a^2-3b^2)\cot(x)}{a^3\,(a^2-b^2)} - \frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\,(a^2-b^2)} - \frac{b^2\cot(x)\csc(x)}{a\,(a^2-b^2)\,(a+b\sin(x))} - \frac{(b^3(4a^2}{}$$

$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b(2a^2-3b^2)\cot(x)}{a^3\,(a^2-b^2)} - \frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\,(a^2-b^2)} - \frac{}{a\,(a^2}$$

$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b(2a^2-3b^2)\cot(x)}{a^3\,(a^2-b^2)} - \frac{(a^2-3b^2)\cot(x)\csc(x)}{2a^2\,(a^2-b^2)} - \frac{}{a\,(a^2}$$

$$= -\frac{2b^3(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\,(a^2-b^2)^{3/2}} - \frac{(a^2+6b^2)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b(2a^2-3b^2)\cot}{a^3\,(a^2-b^2)}$$

**Mathematica** [A]
time = 0.63, size = 171, normalized size = 1.02

$$\frac{\frac{16b^3(-4a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 8ab\cot\left(\frac{x}{2}\right) - a^2\csc^2\left(\frac{x}{2}\right) - 4(a^2+6b^2)\log\left(\cos\left(\frac{x}{2}\right)\right) + 4(a^2+6b^2)\log\left(\sin\left(\frac{x}{2}\right)\right) + a^2\sec^2\left(\frac{x}{2}\right) - \frac{8ab^4\cos(x)}{(a-b)(a+b)(a+b\sin(x))} - 8ab\tan\left(\frac{x}{2}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] ((16*b^3*(-4*a^2 + 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(3/2) + 8*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 6*b^2)*Log[Cos[x/2]
] + 4*(a^2 + 6*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (8*a*b^4*Cos[x])/((a -
 b)*(a + b)*(a + b*Sin[x])) - 8*a*b*Tan[x/2])/(8*a^4)
```

**Maple** [A]
time = 0.36, size = 181, normalized size = 1.08

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-4b\tan\left(\frac{x}{2}\right)}{4a^3}-\frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2}+\frac{\left(2a^2+12b^2\right)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a^4}+\frac{b}{a^3\tan\left(\frac{x}{2}\right)}-\frac{4b^3\left(\frac{\frac{b^2\tan\left(\frac{x}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{\left(4a^2-3b^2\right.}{}\right.}{a^4}$ |
| risch | $\frac{2a^4e^{4ix}+2a^2b^2e^{4ix}-ia^3b\,e^{5ix}+3ia\,b^3e^{5ix}+8ia^3b\,e^{3ix}-12ia\,b^3e^{3ix}+2a^4e^{2ix}-10a^2b^2e^{2ix}-7ia^3b\,e^{ix}+9ia\,b^3e^{ix}-6b^4e^{4ix}+12b^4e^{2ix}+}{\left(e^{2ix}-1\right)^2\left(a^2-b^2\right)\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)a^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(1/2*a*tan(1/2*x)^2-4*b*tan(1/2*x))-1/8/a^2/tan(1/2*x)^2+1/4/a^4*(2
*a^2+12*b^2)*ln(tan(1/2*x))+1/a^3*b/tan(1/2*x)-4/a^4*b^3*((1/2*b^2/(a^2-b^2
)*tan(1/2*x)+1/2*a*b/(a^2-b^2))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+1/2*(4*a^
2-3*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(158) = 316$.
time = 0.77, size = 1174, normalized size = 6.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3
+ a^2*b^5)*cos(x)*sin(x) + 2*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*c
os(x)^2 + (4*a^2*b^4 - 3*b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*c
```

```
os(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2)) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6)*cos(x) + (a^7 + 4*a
^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*co
s(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*
a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^7 + 4*a^5*b^2
 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(x)^2
 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*a^2*b^
5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^9 - 2*a^7*b^2 + a^5
*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + a^4*b^5
- (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x)), -1/4*(4*(2*a^5*b^2 - 5*a
^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(x)*sin(x)
- 4*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*cos(x)^2 + (4*a^2*b^4 - 3*
b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(
x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b
^6)*cos(x) + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 1
1*a^3*b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (
a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) +
1/2) - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*
b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b
+ 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))
/(a^9 - 2*a^7*b^2 + a^5*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b
 - 2*a^6*b^3 + a^4*b^5 - (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a + b\sin(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**2, x)
```

**Giac** [A]
time = 0.44, size = 215, normalized size = 1.28

$$-\frac{2\,(4\,a^2b^3 - 3\,b^5)\left(\pi\left\lfloor\frac{x}{2\,\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right) + b}{\sqrt{a^2 - b^2}}\right)\right)}{(a^6 - a^4b^2)\sqrt{a^2 - b^2}} - \frac{2\,(b^5\tan\left(\frac{1}{2}\,x\right) + ab^4)}{(a^6 - a^4b^2)\left(a\tan\left(\frac{1}{2}\,x\right)^2 + 2\,b\tan\left(\frac{1}{2}\,x\right) + a\right)} + \frac{(a^2 + 6\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^4} + \frac{a^2\tan\left(\frac{1}{2}\,x\right)^2 - 8\,ab\tan\left(\frac{1}{2}\,x\right)}{8\,a^4} - \frac{6\,a^2\tan\left(\frac{1}{2}\,x\right)^2 + 36\,b^2\tan\left(\frac{1}{2}\,x\right)^2 - 8\,ab\tan\left(\frac{1}{2}\,x\right) + a^2}{8\,a^4\tan\left(\frac{1}{2}\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

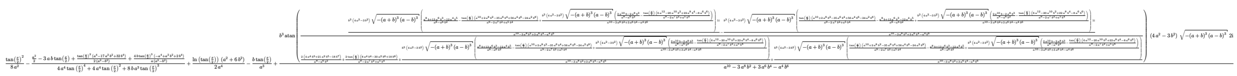
```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) - 2*(b^5*tan(1
/2*x) + a*b^4)/((a^6 - a^4*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
```

```
2*(a^2 + 6*b^2)*log(abs(tan(1/2*x)))/a^4 + 1/8*(a^2*tan(1/2*x)^2 - 8*a*b*ta
n(1/2*x))/a^4 - 1/8*(6*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2 - 8*a*b*tan(1
/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

**Mupad** [B]

time = 7.67, size = 1576, normalized size = 9.38



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))^2),x)
```

```
[Out] tan(x/2)^2/(8*a^2) - (a^2/2 - 3*a*b*tan(x/2) + (tan(x/2)^2*(a^4 + 32*b^4 -
17*a^2*b^2))/(2*(a^2 - b^2)) + (4*b*tan(x/2)^3*(2*b^4 - a^4 + a^2*b^2))/(a*
(a^2 - b^2)))/(4*a^4*tan(x/2)^2 + 4*a^4*tan(x/2)^4 + 8*a^3*b*tan(x/2)^3) +
(log(tan(x/2))*(a^2 + 6*b^2))/(2*a^4) - (b*tan(x/2))/a^3 + (b^3*atan(((b^3*
(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((a^8*b - 12*a^4*b^5 + 13*a^6*
b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b
^4 + 2*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(
6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2)
))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2) - (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(x/
2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2*a^8*b^2))/(a^9 + a^5*b^
4 - 2*a^7*b^2) - (a^8*b - 12*a^4*b^5 + 13*a^6*b^3)/(a^8 - a^6*b^2) + (b^3*(
4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a^8 -
a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9
+ a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^1
0 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((2*(21*a^2*b^5 - 18*b^7 + 4*a^4*b^3)
)/(a^8 - a^6*b^2) + (2*tan(x/2)*(18*b^8 - 30*a^2*b^6 + 8*a^4*b^4))/(a^9 + a
^5*b^4 - 2*a^7*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((a
^8*b - 12*a^4*b^5 + 13*a^6*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(a^10 - 24*a^2*
b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (
b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a
^8 - a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/
(a^9 + a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a
^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((tan(x/2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2
*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (a^8*b - 12*a^4*b^5 + 13*a^6*b^3)/
(a^8 - a^6*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^1
0*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b
^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)
*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
```

# 3.193 $\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=243

$$\frac{(12a^2+b^2)\,x}{2b^5}-\frac{a^3(12a^4-29a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^5\,(a^2-b^2)^{5/2}}+\frac{3a(4a^4-7a^2b^2+2b^4)\cos(x)}{2b^4\,(a^2-b^2)^2}-\frac{(6a^4-10a^2b^2}{2b^3\,(}$$

```
[Out] 1/2*(12*a^2+b^2)*x/b^5-a^3*(12*a^4-29*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*x
))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(5/2)+3/2*a*(4*a^4-7*a^2*b^2+2*b^4)*cos(x
)/b^4/(a^2-b^2)^2-1/2*(6*a^4-10*a^2*b^2+b^4)*cos(x)*sin(x)/b^3/(a^2-b^2)^2+
1/2*a^2*cos(x)*sin(x)^3/b/(a^2-b^2)/(a+b*sin(x))^2+1/2*a^2*(4*a^2-7*b^2)*co
s(x)*sin(x)^2/b^2/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.46, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2871, 3126, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{a^2\sin^3(x)\cos(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2}+\frac{a^2(4a^2-7b^2)\sin^2(x)\cos(x)}{2b^2\,(a^2-b^2)^2\,(a+b\sin(x))}+\frac{x(12a^2+b^2)}{2b^5}+\frac{3a(4a^4-7a^2b^2+2b^4)\cos(x)}{2b^4\,(a^2-b^2)^2}-\frac{(6a^4-10a^2b^2+b^4)\sin(x)\cos(x)}{2b^3\,(a^2-b^2)^2}-\frac{a^3(12a^4-29a^2b^2+20b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5\,(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b
+ a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(5/2)) + (3*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[x])/(2*b^4*(a^2 - b^2)^2) - ((6*a^4 - 10*a^2*b^2 + b^4)
*Cos[x]*Sin[x])/(2*b^3*(a^2 - b^2)^2) + (a^2*Cos[x]*Sin[x]^3)/(2*b*(a^2 - b
^2)*(a + b*Sin[x])^2) + (a^2*(4*a^2 - 7*b^2)*Cos[x]*Sin[x]^2)/(2*b^2*(a^2 -
 b^2)^2*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{\sin^2(x)\left(3a^2-2ab\sin(x)-2\left(2a^2-b^2\right)\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos(x)\sin^2(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\sin(x)\left(-2a^2\left(4a^2-7b^2\right)+ab(a\right.}{}}{2}$$

$$= -\frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos}{2b^2\left(a^2-b^2\right)^2(a}$$

$$= \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin}{2b\left(a^2-b^2\right)(a+}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} +$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} +$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} +$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}} + \frac{3a\left(4a^4-7a^2b^2+2\right.}{2b^4\left(a^2-b^2\right.}$$

**Mathematica** [A]

time = 0.65, size = 164, normalized size = 0.67

$$\frac{2(12a^2+b^2)x - \frac{4a^3(12a^4-29a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + 12ab\cos(x) - \frac{2a^5b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{2a^4b(7a^2-10b^2)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - b^2\sin(2x)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] (2*(12*a^2 + b^2)*x - (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b + a*T
an[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cos[x] - (2*a^5*b*Cos
[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (2*a^4*b*(7*a^2 - 10*b^2)*Cos[x])
/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - b^2*Sin[2*x])/(4*b^5)
```

**Maple [A]**
time = 0.52, size = 330, normalized size = 1.36

| method | result |
|---|---|
| default | $\frac{\frac{4\left(\frac{b^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3ab\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{b^2\tan\left(\frac{x}{2}\right)}{4}+\frac{3ab}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\left(12a^2+b^2\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^5} - \frac{4a^3\left(\frac{-\frac{ab^2\left(5a^2-8b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{4\left(a^4-2a^2b^2+b^4\right)}-\frac{3b\left(2a^4+a^2b^2\cdots\right.}{4\left(a^4-\cdots\right.}}{\left(a\left(\tan\cdots\right.\right.}\right.}{\cdots}$ |
| risch | $\frac{6xa^2}{b^5}+\frac{x}{2b^3}+\frac{ie^{2ix}}{8b^3}+\frac{3a\,e^{ix}}{2b^4}+\frac{3a\,e^{-ix}}{2b^4}-\frac{ie^{-2ix}}{8b^3}-\frac{ia^4\left(-8ia^3b\,e^{3ix}+11ia\,b^3e^{3ix}+20ia^3b\,e^{ix}-29ia\,b^3e^{ix}+14a^4e^{2ix}-13a^2\cdots\right.}{\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)^2\left(a^2-b^2\right)^2b^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/b^5*((1/4*b^2*tan(1/2*x)^3+3/2*a*b*tan(1/2*x)^2-1/4*b^2*tan(1/2*x)+3/2*a*
b)/(tan(1/2*x)^2+1)^2+1/4*(12*a^2+b^2)*arctan(tan(1/2*x)))-4/b^5*a^3*((-1/4
*a*b^2*(5*a^2-8*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-3/4*b*(2*a^4+a^2*b^2-
6*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/4*a*b^2*(19*a^2-28*b^2)/(a^4-2*a^
2*b^2+b^4)*tan(1/2*x)-3/4*a^2*b*(2*a^2-3*b^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1
/2*x)^2+2*b*tan(1/2*x)+a)^2+1/4*(12*a^4-29*a^2*b^2+20*b^4)/(a^4-2*a^2*b^2+b
^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

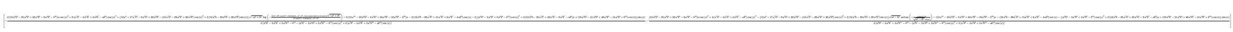
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(227) = 454$.
time = 0.44, size = 1090, normalized size = 4.49



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)
^2 + 8*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(x)^3 + (12*a^9 - 17*a^
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos
(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*sin(x))*sqrt(-a^2 + b^2)*log
(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) +
 b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2
*(12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x - 2*
(12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - 2*((a^6
*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 -
 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^1
3 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*sin(x)), -1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^
4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)^2 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 -
 a*b^9)*cos(x)^3 - (12*a^9 - 17*a^7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*
b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4
*b^5)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) - (12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x
 - (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - ((a^
6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6
- 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^
13 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^
5*b^8 + 3*a^3*b^10 - a*b^12)*sin(x))]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(227) = 454.
time = 0.44, size = 516, normalized size = 2.12

$$-\frac{\left(12\,a^{7}-29\,a^{5}b^{2}+20\,a^{3}b^{4}\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{5}-2\,a^{2}b^{7}+b^{9}\right)\sqrt{a^{2}-b^{2}}}+\frac{6\,a^{6}b\tan\left(\frac{1}{2}x\right)^{7}-10\,a^{4}b^{3}\tan\left(\frac{1}{2}x\right)^{7}+a^{2}b^{5}\tan\left(\frac{1}{2}x\right)^{7}+12\,a^{7}\tan\left(\frac{1}{2}x\right)^{6}-5\,a^{5}b^{2}\tan\left(\frac{1}{2}x\right)^{6}-20\,a^{3}b^{4}\tan\left(\frac{1}{2}x\right)^{6}+4\,ab^{6}\tan\left(\frac{1}{2}x\right)^{6}+54\,a^{6}b\tan\left(\frac{1}{2}x\right)^{5}-90\,a^{4}b^{3}\tan\left(\frac{1}{2}x\right)^{5}+17\,a^{2}b^{5}\tan\left(\frac{1}{2}x\right)^{5}+4\,b^{7}\tan\left(\frac{1}{2}x\right)^{5}+36\,a^{7}\tan\left(\frac{1}{2}x\right)^{4}-15\,a^{5}b^{2}\tan\left(\frac{1}{2}x\right)^{4}-66\,a^{3}b^{4}\tan\left(\frac{1}{2}x\right)^{4}+24\,ab^{6}\tan\left(\frac{1}{2}x\right)^{4}+90\,a^{6}b\tan\left(\frac{1}{2}x\right)^{3}-162\,a^{4}b^{3}\tan\left(\frac{1}{2}x\right)^{3}+55\,a^{2}b^{5}\tan\left(\frac{1}{2}x\right)^{3}-4\,b^{7}\tan\left(\frac{1}{2}x\right)^{3}+36\,a^{7}\tan\left(\frac{1}{2}x\right)^{2}-31\,a^{5}b^{2}\tan\left(\frac{1}{2}x\right)^{2}-40\,a^{3}b^{4}\tan\left(\frac{1}{2}x\right)^{2}+20\,ab^{6}\tan\left(\frac{1}{2}x\right)^{2}+42\,a^{6}b\tan\left(\frac{1}{2}x\right)-74\,a^{4}b^{3}\tan\left(\frac{1}{2}x\right)+23\,a^{2}b^{5}\tan\left(\frac{1}{2}x\right)+12\,a^{7}-21\,a^{5}b^{2}+6\,a^{3}b^{4}}{\left(a^{4}b^{4}-2\,a^{2}b^{6}+b^{8}\right)\left(a\tan\left(\frac{1}{2}x\right)^{4}+2\,b\tan\left(\frac{1}{2}x\right)^{3}+2\,a\tan\left(\frac{1}{2}x\right)^{2}+2\,b\tan\left(\frac{1}{2}x\right)+a\right)^{2}}+\frac{\left(12\,a^{2}+b^{2}\right)x}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(12*a^7 - 29*a^5*b^2 + 20*a^3*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a
^2 - b^2)) + (6*a^6*b*tan(1/2*x)^7 - 10*a^4*b^3*tan(1/2*x)^7 + a^2*b^5*tan(
1/2*x)^7 + 12*a^7*tan(1/2*x)^6 - 5*a^5*b^2*tan(1/2*x)^6 - 20*a^3*b^4*tan(1/
2*x)^6 + 4*a*b^6*tan(1/2*x)^6 + 54*a^6*b*tan(1/2*x)^5 - 90*a^4*b^3*tan(1/2*
x)^5 + 17*a^2*b^5*tan(1/2*x)^5 + 4*b^7*tan(1/2*x)^5 + 36*a^7*tan(1/2*x)^4 -
 15*a^5*b^2*tan(1/2*x)^4 - 66*a^3*b^4*tan(1/2*x)^4 + 24*a*b^6*tan(1/2*x)^4
+ 90*a^6*b*tan(1/2*x)^3 - 162*a^4*b^3*tan(1/2*x)^3 + 55*a^2*b^5*tan(1/2*x)^
3 - 4*b^7*tan(1/2*x)^3 + 36*a^7*tan(1/2*x)^2 - 31*a^5*b^2*tan(1/2*x)^2 - 40
*a^3*b^4*tan(1/2*x)^2 + 20*a*b^6*tan(1/2*x)^2 + 42*a^6*b*tan(1/2*x) - 74*a^
4*b^3*tan(1/2*x) + 23*a^2*b^5*tan(1/2*x) + 12*a^7 - 21*a^5*b^2 + 6*a^3*b^4)
/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + 1/2*(12*a^2 + b^2)*x/b^5
```

**Mupad** [B]
time = 15.81, size = 2500, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + b*sin(x))^3,x)
```

```
[Out] ((3*(4*a^7 + 2*a^3*b^4 - 7*a^5*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x
/2)^7*(6*a^6 + a^2*b^4 - 10*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(
x/2)^5*(54*a^6 + 4*b^6 + 17*a^2*b^4 - 90*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*
b^2)) + (tan(x/2)^3*(90*a^6 - 4*b^6 + 55*a^2*b^4 - 162*a^4*b^2))/(b^3*(a^4
+ b^4 - 2*a^2*b^2)) + (tan(x/2)^6*(4*a*b^6 + 12*a^7 - 20*a^3*b^4 - 5*a^5*b^
2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)^2*(20*a*b^6 + 36*a^7 - 40*a^3
*b^4 - 31*a^5*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)*(42*a^6 + 23*
a^2*b^4 - 74*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*b^2)) + (3*tan(x/2)^4*(3*a^2
 + 4*b^2)*(2*a*b^4 + 4*a^5 - 7*a^3*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)))/(ta
n(x/2)^2*(4*a^2 + 4*b^2) + tan(x/2)^6*(4*a^2 + 4*b^2) + tan(x/2)^4*(6*a^2 +
 8*b^2) + a^2 + a^2*tan(x/2)^8 + 4*a*b*tan(x/2) + 12*a*b*tan(x/2)^3 + 12*a*
b*tan(x/2)^5 + 4*a*b*tan(x/2)^7) + (atan((((a^2*12i + b^2*1i)*((4*(2*a^2*b^
16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*
```

b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^1
1) - ((a^2*12i + b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a
^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4
*a^6*b^13 + a^8*b^11) - (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a
^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*
b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 +
44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 +
a^8*b^12))*(a^2*12i + b^2*1i))/(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^
6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 +
6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39
*a^3*b^16 + 88*a^5*b^14 - 1326*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1
536*a^13*b^6 - 288*a^15*b^4))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14
+ a^8*b^12))*1i)/(2*b^5) + ((a^2*12i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^1
4 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*
b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + ((a^2*12i
+ b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a
^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8
*b^11) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^1
0*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(
x/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 -
8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^
2*12i + b^2*1i))/(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^
8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4
*a^6*b^14 + a^8*b^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*
a^5*b^14 - 1326*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 -
288*a^15*b^4))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*1i
)/(2*b^5))/((8*(864*a^15 + 20*a^5*b^10 + 11*a^7*b^8 - 2326*a^9*b^6 + 4770*a
^11*b^4 - 3456*a^13*b^2))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^
8*b^11) - ((a^2*12i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12
- 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a
^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - ((a^2*12i + b^2*1i)*((4*(4*
a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11
*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (((4*(8*
a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 -
4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 -
56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^
20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*12i + b^2*1i))/
(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10
*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b
^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 1326*a
^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4))/(b
^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + ((a^2*12
i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1
538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^
15 - 4*a^6*b^13 + a^8*b^11) + ((a^2*12i + b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^

```
18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*
a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^4*
b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4
*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104
*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 +
6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*12i + b^2*1i))/(2*b^5) + (8*tan(x
/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^
10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + (
8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^1...
```

# 3.194 $\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=179

$$-\frac{3ax}{b^4}+\frac{3a^2(2a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}-\frac{(3a^2-2b^2)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)\left(a+b\sin(x)\right)^2}-\frac{3a}{2b^3\,(a}$$

```
[Out] -3*a*x/b^4+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^
(1/2))/b^4/(a^2-b^2)^(5/2)-1/2*(3*a^2-2*b^2)*cos(x)/b^3/(a^2-b^2)+1/2*a^2*c
os(x)*sin(x)^2/b/(a^2-b^2)/(a+b*sin(x))^2-3/2*a^3*(a^2-2*b^2)*cos(x)/b^3/(a
^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]

time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2871, 3110, 3102, 2814, 2739, 632, 210}

$$\frac{a^2\sin^2(x)\cos(x)}{2b\left(a^2-b^2\right)\left(a+b\sin(x)\right)^2}-\frac{(3a^2-2b^2)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{3a^2(2a^4-5a^2b^2+4b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}-\frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\left(a^2-b^2\right)^2\left(a+b\sin(x)\right)}-\frac{3ax}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/S
qrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)) - ((3*a^2 - 2*b^2)*Cos[x])/(2*b^3*
(a^2 - b^2)) + (a^2*Cos[x]*Sin[x]^2)/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (
3*a^3*(a^2 - 2*b^2)*Cos[x])/(2*b^3*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{\int \frac{\sin(x)(2a^2-2ab\sin(x)-(3a^2-2b^2)\sin^2(x))}{(a+b\sin(x))^2}\,dx}{2b\,(a^2-b^2)}$$

$$= \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)^2\,(a+b\sin(x))} - \frac{\int \frac{3a^2b(a^2-2b^2)+a(3a^2-4b^2)(a \cdots}{\cdots}}{2b \cdots}$$

$$= -\frac{(3a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)^2\,(a+b\sin(x))} - \cdots$$

$$= -\frac{3ax}{b^4} - \frac{(3a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)^2\,(a+b\sin(\cdots}$$

$$= -\frac{3ax}{b^4} - \frac{(3a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)^2\,(a+b\sin(\cdots}$$

$$= -\frac{3ax}{b^4} - \frac{(3a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)} + \frac{a^2\cos(x)\sin^2(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{3a^3(a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)^2\,(a+b\sin(\cdots}$$

$$= -\frac{3ax}{b^4} + \frac{3a^2(2a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\,(a^2-b^2)^{5/2}} - \frac{(3a^2-2b^2)\cos(x)}{2b^3\,(a^2-b^2)} + \frac{a^2 \cdots}{2b\,(a^2-\cdots}$$

**Mathematica [A]**
time = 0.55, size = 144, normalized size = 0.80

$$\frac{-6ax + \frac{6a^2(2a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - 2b\cos(x) + \frac{a^4b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{a^3b(-5a^2+8b^2)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-6*a*x + (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*b*Cos[x] + (a^4*b*Cos[x])/((a - b)*(a + b
)*(a + b*Sin[x])^2) + (a^3*b*(-5*a^2 + 8*b^2)*Cos[x])/((a - b)^2*(a + b)^2*
(a + b*Sin[x])))/(2*b^4)
```

**Maple [A]**
time = 0.46, size = 286, normalized size = 1.60

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{2a^2\left(\frac{-\frac{3ab^2(a^2-2b^2)(\tan^3(\frac{x}{2}))}{2(a^4-2a^2b^2+b^4)}-\frac{b(4a^4+a^2b^2-14b^4)(\tan^2(\frac{x}{2}))}{2(a^4-2a^2b^2+b^4)}-\frac{ab^2(13a^2-22b^2)\tan(\frac{x}{2})}{2(a^4-2a^2b^2+b^4)}-\frac{a^2b(4a^2-7b^2)}{2(a^4-2a^2b^2+b^4)}}{(a(\tan^2(\frac{x}{2}))+2b\tan(\frac{x}{2})+a)^2}+\frac{3(2a^4-5a^2b^2+4b^4)\,\mathrm{arct}\ldots}{2(a^4-2a^2b^2+b^4)\ldots}\ldots\right.}{b^4}$ |
| risch | $-\frac{3ax}{b^4}-\frac{e^{ix}}{2b^3}-\frac{e^{-ix}}{2b^3}+\frac{ia^3(-6ia^3b\,e^{3ix}+9iab^3e^{3ix}+14ia^3b\,e^{ix}-23iab^3e^{ix}+10a^4e^{2ix}-11a^2b^2e^{2ix}-8b^4e^{2ix}-5a^2b^2+8b^4)}{(-ib\,e^{2ix}+ib+2a\,e^{ix})^2(a^2-b^2)^2b^4}-\ldots$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/b^4*((-3/2*a*b^2*(a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-1/2*b*(
4*a^4+a^2*b^2-14*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/2*a*b^2*(13*a^2-22
*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-1/2*a^2*b*(4*a^2-7*b^2)/(a^4-2*a^2*b^2
+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+3/2*(2*a^4-5*a^2*b^2+4*b^4)/(a^4
-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(
1/2)))-2/b^4*(b/(tan(1/2*x)^2+1)+3*a*arctan(tan(1/2*x)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(167) = 334$.
time = 0.45, size = 945, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 4*(a^6*b^3
- 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 +
4*a^2*b^6 - (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a
^5*b^3 + 4*a^3*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 -
```

```
2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))
/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^9 - 2*a^7*b^2 + 2*a^3*b
^6 - a*b^8)*x - 2*(6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*co
s(x) - 2*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (9*a^7*b^2 - 25*
a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2*a^6*b^6 + 2*a^
2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*cos(x)^2 + 2*(a^7
*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x)), 1/2*(6*(a^7*b^2 - 3*a^5*b^4
 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6 - (2*a^6*b^2 - 5*a^
4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*sin(x))*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 6*(a^9 -
2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*x - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^
2*b^7 - 2*b^9)*cos(x) - (12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (
9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2
*a^6*b^6 + 2*a^2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*co
s(x)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.44, size = 256, normalized size = 1.43

$$\frac{3\,(2\,a^6-5\,a^4b^2+4\,a^2b^4)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^4-2\,a^2b^6+b^8)\sqrt{a^2-b^2}}-\frac{3\,a^5b\tan\left(\frac{1}{2}\,x\right)^3-6\,a^3b^3\tan\left(\frac{1}{2}\,x\right)^3+4\,a^6\tan\left(\frac{1}{2}\,x\right)^2+a^4b^2\tan\left(\frac{1}{2}\,x\right)^2-14\,a^2b^4\tan\left(\frac{1}{2}\,x\right)^2+13\,a^5b\tan\left(\frac{1}{2}\,x\right)-22\,a^3b^3\tan\left(\frac{1}{2}\,x\right)+4\,a^6-7\,a^4b^2}{(a^4b^3-2\,a^2b^5+b^7)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)^2}-\frac{3\,ax}{b^4}-\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2
 - b^2)) - (3*a^5*b*tan(1/2*x)^3 - 6*a^3*b^3*tan(1/2*x)^3 + 4*a^6*tan(1/2*x
)^2 + a^4*b^2*tan(1/2*x)^2 - 14*a^2*b^4*tan(1/2*x)^2 + 13*a^5*b*tan(1/2*x)
- 22*a^3*b^3*tan(1/2*x) + 4*a^6 - 7*a^4*b^2)/((a^4*b^3 - 2*a^2*b^5 + b^7)*(
a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*a*x/b^4 - 2/((tan(1/2*x)^2 + 1)
*b^3)
```

**Mupad** [**B**]
time = 14.87, size = 2500, normalized size = 13.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4/(a + b*sin(x))^3,x)

[Out] - ((a^2*(6*a^4 + 2*b^4 - 11*a^2*b^2))/(b^3*(a^2 - b^2)^2) + (3*tan(x/2)^5*(a^5 - 2*a^3*b^2))/(b^2*(a^2 - b^2)^2) - (3*tan(x/2)^4*(4*a^2*b^4 - 2*a^6 + a^4*b^2))/(b^3*(a^2 - b^2)^2) + (2*tan(x/2)^2*(6*a^6 + 4*b^6 - 13*a^2*b^4 - 3*a^4*b^2))/(b^3*(a^2 - b^2)^2) + (4*a*tan(x/2)^3*(6*a^4 + 2*b^4 - 11*a^2*b^2))/(b^2*(a^2 - b^2)^2) + (a*tan(x/2)*(21*a^4 + 8*b^4 - 38*a^2*b^2))/(b^2*(a^2 - b^2)^2))/(tan(x/2)^2*(3*a^2 + 4*b^2) + tan(x/2)^4*(3*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^6 + 4*a*b*tan(x/2) + 8*a*b*tan(x/2)^3 + 4*a*b*tan(x/2)^5) - (6*a*atan(((3*a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (a*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4))/b^4 + (3*a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (a*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4))/b^4)/((16*(54*a^12 - 216*a^6*b^6 + 378*a^8*b^4 - 243*a^10*b^2))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (16*tan(x/2)*(216*a^13 + 432*a^5*b^8 - 1404*a^7*b^6 + 1728*a^9*b^4 - 972*a^11*b^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (a*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b

^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)
*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^
11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4
+ (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 2
4*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b
^4)*3i)/b^4 + (a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^
5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) +
 (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*
a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8
*b^9) + (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a
^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (a*((8*
(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16
 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21
 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(
b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*tan(x/
2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8)
)/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4)*3i)/b^4
)))/b^4 - (a^2*atan(((a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a
^2*b^2)*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^
12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x
/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5
- 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (
3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*
b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b
^10 + a^8*b^8) + (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 1
08*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^15 ...

# 3.195 $\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=144

$$\frac{x}{b^3}-\frac{a(2a^4-5a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{a^2(2a^2-5b^2)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}$$

```
[Out] x/b^3-a*(2*a^4-5*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^
3/(a^2-b^2)^(5/2)+1/2*a^2*cos(x)*sin(x)/b/(a^2-b^2)/(a+b*sin(x))^2+1/2*a^2*
(2*a^2-5*b^2)*cos(x)/b^2/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2871, 3100, 2814, 2739, 632, 210}

$$\frac{a^2(2a^2-5b^2)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{a^2\sin(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a(2a^4-5a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(b^3*(a^2 - b^2)^(5/2)) + (a^2*Cos[x]*Sin[x])/(2*b*(a^2 - b^2)*(a + b
*Sin[x])^2) + (a^2*(2*a^2 - 5*b^2)*Cos[x])/(2*b^2*(a^2 - b^2)^2*(a + b*Sin[
x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{\int \frac{a^2-2ab\sin(x)-2(a^2-b^2)\sin^2(x)}{(a+b\sin(x))^2}\,dx}{2b(a^2-b^2)}$$

$$= \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{a^2(2a^2-5b^2)\cos(x)}{2b^2(a^2-b^2)^2(a+b\sin(x))} + \frac{\int \frac{ab(a^2-4b^2)+2(a^2-b^2)^2\sin(}{a+b\sin(x)}}{2b^2(a^2-b^2)^2}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{a^2(2a^2-5b^2)\cos(x)}{2b^2(a^2-b^2)^2(a+b\sin(x))} - \frac{(a(2a^4-5a^2b^2+}{2b^3(a^2}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{a^2(2a^2-5b^2)\cos(x)}{2b^2(a^2-b^2)^2(a+b\sin(x))} - \frac{(a(2a^4-5a^2b^2+}{}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{a^2(2a^2-5b^2)\cos(x)}{2b^2(a^2-b^2)^2(a+b\sin(x))} + \frac{(2a(2a^4-5a^2b^2+}{}$$

$$= \frac{x}{b^3} - \frac{a(2a^4-5a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{5/2}} + \frac{a^2\cos(x)\sin(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{}{2b^2}$$

**Mathematica [A]**
time = 0.38, size = 136, normalized size = 0.94

$$\frac{2x - \frac{2a(2a^4-5a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^3b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{3a^2b(a^2-2b^2)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] (2*x - (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) - (a^3*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2
) + (3*a^2*b*(a^2 - 2*b^2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])))/(2
*b^3)
```

**Maple [A]**
time = 0.38, size = 269, normalized size = 1.87

| method | result |
|---|---|
| default | $\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^3} - \frac{2a\left(\frac{-\frac{a b^2\left(a^2-4b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)} - \frac{b\left(2a^4-a^2b^2-10b^4\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)} - \frac{a b^2\left(7a^2-16b^2\right)\tan\left(\frac{x}{2}\right)}{2\left(a^4-2a^2b^2+b^4\right)} - \frac{a^2b\left(2a^2-5b^2\right)}{2\left(a^4-2a^2b^2+b^4\right)}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^2} + \frac{(2a}{}\right.}{b^3}$ |

| | |
|---|---|
| risch | $\frac{x}{b^3} - \frac{ia^2(-4ia^3b\,e^{3ix}+7ia\,b^3e^{3ix}+8ia^3b\,e^{ix}-17ia\,b^3e^{ix}+6a^4e^{2ix}-9a^2b^2e^{2ix}-6b^4e^{2ix}-3a^2b^2+6b^4)}{(-ib\,e^{2ix}+ib+2a\,e^{ix})^2(a^2-b^2)^2b^3} + \frac{ia^5\ln\left(e^{ix}+\frac{i(\sqrt{a^2-b^2}}{\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*arctan(tan(1/2*x))-2/b^3*a*((-1/2*a*b^2*(a^2-4*b^2)/(a^4-2*a^2*b^2+b^
4)*tan(1/2*x)^3-1/2*b*(2*a^4-a^2*b^2-10*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)
^2-1/2*a*b^2*(7*a^2-16*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-1/2*a^2*b*(2*a^2
-5*b^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+1/2*(2*a^4
-5*a^2*b^2+6*b^4)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(134) = 268.
time = 0.50, size = 819, normalized size = 5.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos(x)^2 + (2*a^7 - 3*a^
5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(x)^2 + 2*
(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b
^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sq
rt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 4*(a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8)*x - 2*(2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - 2*
(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a
^2*b^6)*cos(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5
```

```
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b
^8 - a*b^10)*sin(x)), -1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos
(x)^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6
*a*b^6)*cos(x)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*(a^8 - 2*a^6*b^2 +
 2*a^2*b^6 - b^8)*x - (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - (4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*co
s(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5 - 3*a^4*b^
7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^1
0)*sin(x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.46, size = 234, normalized size = 1.62

$$-\frac{(2a^5 - 5a^3b^2 + 6ab^4)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^3 - 2a^2b^5 + b^7)\sqrt{a^2-b^2}} + \frac{a^4b\tan\left(\frac{1}{2}x\right)^3 - 4a^2b^3\tan\left(\frac{1}{2}x\right)^3 + 2a^5\tan\left(\frac{1}{2}x\right)^2 - a^3b^2\tan\left(\frac{1}{2}x\right)^2 - 10ab^4\tan\left(\frac{1}{2}x\right)^2 + 7a^4b\tan\left(\frac{1}{2}x\right) - 16a^2b^3\tan\left(\frac{1}{2}x\right) + 2a^5 - 5a^3b^2}{(a^4b^2 - 2a^2b^4 + b^6)\left(a\tan\left(\frac{1}{2}x\right)^2 + 2b\tan\left(\frac{1}{2}x\right) + a\right)^2} + \frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
b^2)) + (a^4*b*tan(1/2*x)^3 - 4*a^2*b^3*tan(1/2*x)^3 + 2*a^5*tan(1/2*x)^2 -
 a^3*b^2*tan(1/2*x)^2 - 10*a*b^4*tan(1/2*x)^2 + 7*a^4*b*tan(1/2*x) - 16*a^2
*b^3*tan(1/2*x) + 2*a^5 - 5*a^3*b^2)/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/
2*x)^2 + 2*b*tan(1/2*x) + a)^2) + x/b^3
```

**Mupad** [**B**]
time = 14.90, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x))^3,x)
```

```
[Out] (2*atan((((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^
2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a*b^14
```

- 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))/b^3 + ((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (((((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))/b^3)/((((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (((((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 - (((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (16*(2*a^9 - 24*a^3*b^6 + 26*a^5*b^4 - 13*a^7*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (16*tan(x/2)*(8*a^10 + 24*a^2*b^8

```
- 68*a^4*b^6 + 72*a^6*b^4 - 36*a^8*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 -
4*a^6*b^8 + a^8*b^6))))/b^3 + ((2*a^5 - 5*a^3*b^2)/(b^2*(a^4 + b^4 - 2*a^2*
b^2)) + (tan(x/2)*(7*a^4 - 16*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (tan(
x/2)^3*(a^4 - 4*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) - (tan(x/2)^2*(5*a*b^
2 - 2*a^3)*(a^2 + 2*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2
 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + (a*
atan(((a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6
*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7
 + a^8*b^5) + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b
^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8
+ a^8*b^6) - (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14 - 8*a^3*b^12 + 6
*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*
b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*
a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^
6) - (a*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*
b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(
12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11
*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(-(a + b)^5*
(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^
4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2...
```

# 3.196 $\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=118

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2\cos(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2}-\frac{a(a^2-4b^2)\cos(x)}{2b\,(a^2-b^2)^2\,(a+b\sin(x))}$$

```
[Out] (a^2+2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/2*a^
2*cos(x)/b/(a^2-b^2)/(a+b*sin(x))^2-1/2*a*(a^2-4*b^2)*cos(x)/b/(a^2-b^2)^2/
(a+b*sin(x))
```

**Rubi** [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2869, 2833, 12, 2739, 632, 210}

$$\frac{(a^2+2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2\cos(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2}-\frac{a(a^2-4b^2)\cos(x)}{2b\,(a^2-b^2)^2\,(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cos[x])/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (a*(a^2 - 4*b^2)*Cos[x]
)/(2*b*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{\int \frac{2ab+(a^2-2b^2)\sin(x)}{(a+b\sin(x))^2}\,dx}{2b(a^2-b^2)}$$

$$= \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))} + \frac{\int \frac{b(a^2+2b^2)}{a+b\sin(x)}\,dx}{2b(a^2-b^2)^2}$$

$$= \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))} + \frac{(a^2+2b^2)\int \frac{1}{a+b\sin(x)}\,dx}{2(a^2-b^2)^2}$$

$$= \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))} + \frac{(a^2+2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+}\right.}{(a^2-b^2)}$$

$$= \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))} - \frac{(2(a^2+2b^2))\,\text{Subst}\left(\int \frac{}{-4}\right.}{(}$$

$$= \frac{(a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2} - \frac{a(a^2-4b^2)\cos(x}{2b(a^2-b^2)^2(a+b\sin}$$

**Mathematica** [A]
time = 0.25, size = 94, normalized size = 0.80

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a\cos(x)\left(3ab-(a^2-4b^2)\sin(x)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a*Cos[x]*(3*a*b - (a^2 - 4*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*S
in[x])^2)
```

**Maple** [A]
time = 0.32, size = 213, normalized size = 1.81

| method | result |
|---|---|
| default | $\frac{\frac{8(a^2+2b^2)a\left(\tan^3\left(\frac{x}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{3b(a^2+2b^2)\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^4-2a^2b^2+b^4}-\frac{a(a^2-10b^2)\tan\left(\frac{x}{2}\right)}{a^4-2a^2b^2+b^4}+\frac{3a^2b}{a^4-2a^2b^2+b^4}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^2}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}$ |

| | |
|---|---|
| risch | $\frac{ia\left(-2ia^3b\,\mathrm{e}^{3ix}+5ia\,b^3\mathrm{e}^{3ix}+2ia^3b\,\mathrm{e}^{ix}-11ia\,b^3\mathrm{e}^{ix}+2a^4\mathrm{e}^{2ix}-7a^2b^2\mathrm{e}^{2ix}-4b^4\mathrm{e}^{2ix}-a^2b^2+4b^4\right)}{\left(-ib\,\mathrm{e}^{2ix}+ib+2a\,\mathrm{e}^{ix}\right)^2\left(a^2-b^2\right)^2b^2}+\frac{a^2\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-}{b\sqrt{-a^2+b^2}}\right.}{2\sqrt{-a^2+b^2}\,(a+b)^2(a-b)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8*(1/8*(a^2+2*b^2)*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2+2*b^2)/(a^
4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*a*(a^2-10*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/
2*x)+3/8*a^2*b/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+(a^
2+2*b^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b
)/(a^2-b^2)^(1/2))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.
time = 0.44, size = 516, normalized size = 4.37

$$\left[-\frac{2\,(a^5-5\,a^3b^2+4\,ab^4)\cos(x)\sin(x)+\left(a^4+3\,a^2b^2+2\,b^4-(a^2b^2+2\,b^4)\cos(x)^2+2\,(a^3b+2\,ab^3)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)-6\,(a^4b-a^2b^3)\cos(x)}{4\,\left(a^8-2\,a^6b^2+2\,a^2b^6-b^8-(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)\cos(x)^2+2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)\sin(x)\right)},\ -\frac{(a^5-5\,a^3b^2+4\,ab^4)\cos(x)\sin(x)+\left(a^4+3\,a^2b^2+2\,b^4-(a^2b^2+2\,b^4)\cos(x)^2+2\,(a^3b+2\,ab^3)\sin(x)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)-3\,(a^4b-a^2b^3)\cos(x)}{2\,\left(a^8-2\,a^6b^2+2\,a^2b^6-b^8-(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)\cos(x)^2+2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^4 + 3*a^2*b^2 + 2*b
^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) - 6*(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*sin(x)), -1/2*((a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^
4 + 3*a^2*b^2 + 2*b^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*si
n(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 3*
(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3
```

```
*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a
*b^7)*sin(x))]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.44, size = 182, normalized size = 1.54

$$\frac{\left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}x\right) + b}{\sqrt{a^2 - b^2}}\right)\right)(a^2 + 2b^2)}{(a^4 - 2a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{a^3 \tan\left(\frac{1}{2}x\right)^3 + 2ab^2 \tan\left(\frac{1}{2}x\right)^3 + 3a^2b \tan\left(\frac{1}{2}x\right)^2 + 6b^3 \tan\left(\frac{1}{2}x\right)^2 - a^3 \tan\left(\frac{1}{2}x\right) + 10ab^2 \tan\left(\frac{1}{2}x\right) + 3a^2b}{(a^4 - 2a^2b^2 + b^4)\left(a \tan\left(\frac{1}{2}x\right)^2 + 2b \tan\left(\frac{1}{2}x\right) + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(a^2 + 2*b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*
x)^3 + 2*a*b^2*tan(1/2*x)^3 + 3*a^2*b*tan(1/2*x)^2 + 6*b^3*tan(1/2*x)^2 - a
^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4 - 2*a^2*b^2 + b^4)*(a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

**Mupad [B]**
time = 7.23, size = 318, normalized size = 2.69

$$\frac{\frac{3a^2b}{a^4-2a^2b^2+b^4} - \frac{a\tan\left(\frac{x}{2}\right)(a^2-10b^2)}{a^4-2a^2b^2+b^4} + \frac{a\tan\left(\frac{x}{2}\right)^3(a^2+2b^2)}{a^4-2a^2b^2+b^4} + \frac{3b\tan\left(\frac{x}{2}\right)^2(a^2+2b^2)}{a^4-2a^2b^2+b^4}}{\tan\left(\frac{x}{2}\right)^2(2a^2+4b^2) + a^2 + a^2\tan\left(\frac{x}{2}\right)^4 + 4ab\tan\left(\frac{x}{2}\right) + 4ab\tan\left(\frac{x}{2}\right)^3} + \frac{\operatorname{atan}\left(\frac{\left(\frac{(a^2+2b^2)(2a^4b-4a^2b^3+2b^5)}{2(a+b)^{5/2}(a-b)^{5/2}(a^4-2a^2b^2+b^4)} + \frac{a\tan\left(\frac{x}{2}\right)(a^2+2b^2)}{(a+b)^{5/2}(a-b)^{5/2}}\right)(a^4-2a^2b^2+b^4)}{a^2+2b^2}\right)(a^2+2b^2)}{(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x))^3,x)
```

```
[Out] ((3*a^2*b)/(a^4 + b^4 - 2*a^2*b^2) - (a*tan(x/2)*(a^2 - 10*b^2))/(a^4 + b^4
 - 2*a^2*b^2) + (a*tan(x/2)^3*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (3*b
*tan(x/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2))/(tan(x/2)^2*(2*a^2 + 4*
b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + (atan(((
((a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2
)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(x/2)*(a^2 + 2*b^2))/((a + b)^(5/2)*(a -
 b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2))/(a^2 + 2*b^2))*(a^2 + 2*b^2))/((a + b)
^(5/2)*(a - b)^(5/2))
```

# 3.197 $\int \frac{\sin(x)}{(a+b\sin(x))^3} dx$

**Optimal**. Leaf size=103

$$-\frac{3ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}$$

```
[Out] -3*a*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-1/2*a*cos(x
)/(a^2-b^2)/(a+b*sin(x))^2-1/2*(a^2+2*b^2)*cos(x)/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2833, 12, 2739, 632, 210}

$$-\frac{3ab\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}-\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a*Co
s[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) - ((a^2 + 2*b^2)*Cos[x])/(2*(a^2 - b
^2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{(a+b\sin(x))^3}\,dx &= -\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{\int \frac{2b-a\sin(x)}{(a+b\sin(x))^2}\,dx}{2(a^2-b^2)}\\
&= -\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{\int -\frac{3ab}{a+b\sin(x)}\,dx}{2(a^2-b^2)^2}\\
&= -\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}-\frac{(3ab)\int \frac{1}{a+b\sin(x)}\,dx}{2(a^2-b^2)^2}\\
&= -\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}-\frac{(3ab)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,d\right.}{(a^2-b^2)^2}\\
&= -\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{(6ab)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\right.}{(a^2-}\\
&= -\frac{3ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{a\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}-\frac{(a^2+2b^2)\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}
\end{aligned}$$

**Mathematica [A]**
time = 0.20, size = 94, normalized size = 0.91

$$-\frac{3ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{\cos(x)\left(a(2a^2+b^2)+b(a^2+2b^2)\sin(x)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + b*Sin[x])^3,x]

[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (Cos[x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*Sin[x])^2)

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(93) = 186$.
time = 0.28, size = 221, normalized size = 2.15

| method | result |
|---|---|
| default | $\frac{-\frac{3a^2 b\left(\tan^3\left(\frac{x}{2}\right)\right)}{a^4-2a^2b^2+b^4}-\frac{\left(2a^4+5a^2b^2+2b^4\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{a\left(a^4-2a^2b^2+b^4\right)}-\frac{\left(5a^2+4b^2\right)b\tan\left(\frac{x}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{\left(2a^2+b^2\right)a}{a^4-2a^2b^2+b^4}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^2}-\frac{3ab\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}$ |
| risch | $\frac{i\left(-3iab^3e^{3ix}+4ia^3b\,e^{ix}+5iab^3e^{ix}+2a^4e^{2ix}+5a^2b^2e^{2ix}+2b^4e^{2ix}-a^2b^2-2b^4\right)}{\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)^2\left(a^2-b^2\right)^2b}-\frac{3iab\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}+ -$ 3a |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)

[Out] 4*(-3/4*a^2*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-1/4*(2*a^4+5*a^2*b^2+2*b^4)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/4*(5*a^2+4*b^2)*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-1/4*(2*a^2+b^2)*a/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(93) = 186$.
time = 0.39, size = 490, normalized size = 4.76

$$\left[-\frac{2\left(a^4b+a^2b^3-2b^5\right)\cos(x)\sin(x)-3\left(ab^3\cos(x)^2-2a^2b^2\sin(x)-a^3b-ab^3\right)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2-2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2\left(2a^5-a^3b^2-ab^4\right)\cos(x)}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos(x)^2+2\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)\sin(x)\right)},-\frac{\left(a^4b+a^2b^3-2b^5\right)\cos(x)\sin(x)+3\left(ab^3\cos(x)^2-2a^2b^2\sin(x)-a^3b-ab^3\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+\left(2a^5-a^3b^2-ab^4\right)\cos(x)}{2\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos(x)^2+2\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) - 3*(a*b^3*cos(x)^2 - 2*a^
2*b^2*sin(x) - a^3*b - a*b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^
2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(2*a^5 - a^3*b^2 - a*b^4
)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x)), -
1/2*((a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) + 3*(a*b^3*cos(x)^2 - 2*a^2*b^
2*sin(x) - a^3*b - a*b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2
- b^2)*cos(x))) + (2*a^5 - a^3*b^2 - a*b^4)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^
2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(93) = 186.
time = 0.43, size = 189, normalized size = 1.83

$$-\frac{3\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}x\right)^3+2a^4\tan\left(\frac{1}{2}x\right)^2+5a^2b^2\tan\left(\frac{1}{2}x\right)^2+2b^4\tan\left(\frac{1}{2}x\right)^2+5a^3b\tan\left(\frac{1}{2}x\right)+4ab^3\tan\left(\frac{1}{2}x\right)+2a^4+a^2b^2}{(a^5-2a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) - (3*a^3*b*tan(1/2*x)^
3 + 2*a^4*tan(1/2*x)^2 + 5*a^2*b^2*tan(1/2*x)^2 + 2*b^4*tan(1/2*x)^2 + 5*a^
3*b*tan(1/2*x) + 4*a*b^3*tan(1/2*x) + 2*a^4 + a^2*b^2)/((a^5 - 2*a^3*b^2 +
a*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

**Mupad** [**B**]
time = 7.51, size = 310, normalized size = 3.01

$$-\frac{\frac{2a^3+ab^2}{a^4-2a^2b^2+b^4}+\frac{3a^2b\tan\left(\frac{x}{2}\right)^3}{a^4-2a^2b^2+b^4}+\frac{b\tan\left(\frac{x}{2}\right)(5a^2+4b^2)}{a^4-2a^2b^2+b^4}+\frac{\tan\left(\frac{x}{2}\right)^2(2a^2+b^2)(a^2+2b^2)}{a(a^4-2a^2b^2+b^4)}}{\tan\left(\frac{x}{2}\right)^2(2a^2+4b^2)+a^2+a^2\tan\left(\frac{x}{2}\right)^4+4ab\tan\left(\frac{x}{2}\right)+4ab\tan\left(\frac{x}{2}\right)^3}-\frac{3ab\,\mathrm{atan}\left(\frac{\left(\frac{3a^2b\tan\left(\frac{x}{2}\right)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{3ab^2(2a^4-4a^2b^2+2b^4)}{2(a+b)^{5/2}(a-b)^{5/2}(a^4-2a^2b^2+b^4)}\right)(a^4-2a^2b^2+b^4)}{3ab}\right)}{(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x))^3,x)
```

```
[Out] - ((a*b^2 + 2*a^3)/(a^4 + b^4 - 2*a^2*b^2) + (3*a^2*b*tan(x/2)^3)/(a^4 + b^
4 - 2*a^2*b^2) + (b*tan(x/2)*(5*a^2 + 4*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (ta
n(x/2)^2*(2*a^2 + b^2)*(a^2 + 2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2
)^2*(2*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2
)^3) - (3*a*b*atan((((3*a^2*b*tan(x/2))/((a + b)^(5/2)*(a - b)^(5/2)) + (3*
a*b^2*(2*a^4 + 2*b^4 - 4*a^2*b^2))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^
4 - 2*a^2*b^2)))*(a^4 + b^4 - 2*a^2*b^2))/(3*a*b)))/((a + b)^(5/2)*(a - b)^
(5/2))
```

# 3.198 $\int \frac{1}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=102

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}$$

```
[Out] (2*a^2+b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/2*b*
cos(x)/(a^2-b^2)/(a+b*sin(x))^2+3/2*a*b*cos(x)/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$\frac{(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) + (3*a*b*Cos[x])/(2*(a^2 - b^
2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(x))^3}\,dx &= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\int \frac{-2a+b\sin(x)}{(a+b\sin(x))^2}\,dx}{2\left(a^2-b^2\right)}\\
&= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\int \frac{2a^2+b^2}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}\\
&= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\left(2a^2+b^2\right)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}\\
&= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\left(2a^2+b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+\cdots}\right.}{\left(a^2-b^2\right)\cdots}\\
&= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{\left(2\left(2a^2+b^2\right)\right)\text{Subst}\left(\int \frac{\cdots}{-4(\cdots}\right.}{(a\cdots}\\
&= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin\cdots}
\end{aligned}$$

**Mathematica [A]**

time = 0.14, size = 93, normalized size = 0.91

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b\cos(x)\left(4a^2-b^2+3ab\sin(x)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x]*(4*a^2 - b^2 + 3*a*b*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*Sin
[x])^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(92) = 184.
time = 0.25, size = 248, normalized size = 2.43

| method | result |
|---|---|
| default | $\frac{\frac{b^2(5a^2-2b^2)\left(\tan^3\left(\frac{x}{2}\right)\right)}{(a^4-2a^2b^2+b^4)a}+\frac{b(4a^4+7a^2b^2-2b^4)\left(\tan^2\left(\frac{x}{2}\right)\right)}{(a^4-2a^2b^2+b^4)a^2}+\frac{b^2(11a^2-2b^2)\tan\left(\frac{x}{2}\right)}{a(a^4-2a^2b^2+b^4)}+\frac{2b(4a^2-b^2)}{2a^4-4a^2b^2+2b^4}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^2}+\frac{(2a^2+b^2)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}$ |
| risch | $-\frac{i(-2ib\,a^2e^{3ix}-ie^{3ix}b^3+10ia^2b\,e^{ix}-ib^3e^{ix}+6a^3e^{2ix}+3b^2a\,e^{2ix}-3a\,b^2)}{(-ib\,e^{2ix}+ib+2a\,e^{ix})^2(a^2-b^2)^2}-\frac{a^2\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}-\frac{\ln\left(e^{ix}+\frac{ia}{}\right.}{2\sqrt{-a}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2*b^2*(5*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*x)^3+1/2*b*(4*a^4+7*
a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*x)^2+1/2*b^2*(11*a^2-2*b^2)/
a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+1/2*b*(4*a^2-b^2)/(a^4-2*a^2*b^2+b^4))/(a*
tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(92) = 184.
time = 0.50, size = 516, normalized size = 5.06

$$\left[\frac{6\,(a^{3}b^{2}-ab^{4})\cos(x)\sin(x)-\left(2\,a^{4}+3\,a^{2}b^{2}+b^{4}-(2\,a^{2}b^{2}+b^{4})\cos(x)^{2}+2\,(2\,a^{3}b+ab^{3})\sin(x)\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{(2\,a^{2}-b^{2})\cos(x)^{2}-2\,ab\sin(x)-a^{2}-b^{2}+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(x)^{2}-2\,ab\sin(x)-a^{2}-b^{2}}\right)+2\,(4\,a^{4}b-5\,a^{2}b^{3}+b^{5})\cos(x)}{4\,\left(a^{8}-2\,a^{6}b^{2}+2\,a^{2}b^{6}-b^{8}-(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8})\cos(x)^{2}+2\,(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7})\sin(x)\right)},\ \frac{3\,(a^{3}b^{2}-ab^{4})\cos(x)\sin(x)-\left(2\,a^{4}+3\,a^{2}b^{2}+b^{4}-(2\,a^{2}b^{2}+b^{4})\cos(x)^{2}+2\,(2\,a^{3}b+ab^{3})\sin(x)\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^{2}-b^{2}}\cos(x)}\right)+(4\,a^{4}b-5\,a^{2}b^{3}+b^{5})\cos(x)}{2\,\left(a^{8}-2\,a^{6}b^{2}+2\,a^{2}b^{6}-b^{8}-(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8})\cos(x)^{2}+2\,(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7})\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2
*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(((2
*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*co
s(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(4*a
^4*b - 5*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*sin(x)), 1/2*(3*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^
2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt
(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a
^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(x))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin(x))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**3,x)
```

```
[Out] Integral((a + b*sin(x))**(-3), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(92) = 184.
time = 0.43, size = 215, normalized size = 2.11

$$\frac{\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)(2\,a^{2}+b^{2})}{(a^{4}-2\,a^{2}b^{2}+b^{4})\sqrt{a^{2}-b^{2}}}+\frac{5\,a^{3}b^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,ab^{4}\tan\left(\frac{1}{2}\,x\right)^{3}+4\,a^{4}b\tan\left(\frac{1}{2}\,x\right)^{2}+7\,a^{2}b^{3}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b^{5}\tan\left(\frac{1}{2}\,x\right)^{2}+11\,a^{3}b^{2}\tan\left(\frac{1}{2}\,x\right)-2\,ab^{4}\tan\left(\frac{1}{2}\,x\right)+4\,a^{4}b-a^{2}b^{3}}{(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4})\left(a\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*ta
n(1/2*x)^3 - 2*a*b^4*tan(1/2*x)^3 + 4*a^4*b*tan(1/2*x)^2 + 7*a^2*b^3*tan(1/
2*x)^2 - 2*b^5*tan(1/2*x)^2 + 11*a^3*b^2*tan(1/2*x) - 2*a*b^4*tan(1/2*x) +
4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a)^2)
```

**Mupad [B]**
time = 7.32, size = 349, normalized size = 3.42

$$\frac{\frac{4\,a^2\,b-b^3}{a^4-2\,a^2\,b^2+b^4}+\frac{{\mathrm{tan}\left(\frac{x}{2}\right)}^2\,\left(4\,a^2\,b-b^3\right)\,\left(a^2+2\,b^2\right)}{a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3\,\left(5\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\mathrm{tan}\left(\frac{x}{2}\right)\,\left(11\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{{\mathrm{tan}\left(\frac{x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2+a^2\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4+4\,a\,b\,\mathrm{tan}\left(\frac{x}{2}\right)+4\,a\,b\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3}+\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,a^2+b^2\right)\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\mathrm{tan}\left(\frac{x}{2}\right)\,\left(2\,a^2+b^2\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,a^2+b^2}\right)\,\left(2\,a^2+b^2\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x))^3,x)
```

```
[Out] ((4*a^2*b - b^3)/(a^4 + b^4 - 2*a^2*b^2) + (tan(x/2)^2*(4*a^2*b - b^3)*(a^2
 + 2*b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)^3*(5*a^2*b - 2*b^3))
/(a*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)*(11*a^2*b - 2*b^3))/(a*(a^4 + b^
4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b
*tan(x/2) + 4*a*b*tan(x/2)^3) + (atan(((((2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4
*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan
(x/2)*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2)
)/(2*a^2 + b^2))*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2))
```

# 3.199 $\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=145

$$-\frac{b(6a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}-\frac{\tanh^{-1}(\cos(x))}{a^3}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{b^2(5a^2-2b}{2a^2\left(a^2-b^2\right)^2(a}$$

```
[Out] -b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^
2-b^2)^(5/2)-arctanh(cos(x))/a^3-1/2*b^2*cos(x)/a/(a^2-b^2)/(a+b*sin(x))^2-
1/2*b^2*(5*a^2-2*b^2)*cos(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.25, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$-\frac{\tanh^{-1}(\cos(x))}{a^3}-\frac{b^2(5a^2-2b^2)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{b(6a^4-5a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2))) - ArcTanh[Cos[x]]/a^3 - (b^2*Cos[x])/(2*a*(a^2 - b
^2)*(a + b*Sin[x])^2) - (b^2*(5*a^2 - 2*b^2)*Cos[x])/(2*a^2*(a^2 - b^2)^2*(
a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\int \frac{\csc(x)\left(2(a^2-b^2)-2ab\sin(x)+b^2\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2a\,(a^2-b^2)}$$

$$= -\frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{b^2(5a^2-2b^2)\cos(x)}{2a^2\,(a^2-b^2)^2\,(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(2(a^2-b^2)^2-ab(4\cdots\right.}{a+b\sin(x)}}{2a^2\,(a^2-\cdots}$$

$$= -\frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{b^2(5a^2-2b^2)\cos(x)}{2a^2\,(a^2-b^2)^2\,(a+b\sin(x))} + \frac{\int \csc(x)\,dx}{a^3} - \frac{(b(6a^4\cdots}{\cdots}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{b^2(5a^2-2b^2)\cos(x)}{2a^2\,(a^2-b^2)^2\,(a+b\sin(x))} - \frac{(b\cdots}{\cdots}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\sin(x))^2} - \frac{b^2(5a^2-2b^2)\cos(x)}{2a^2\,(a^2-b^2)^2\,(a+b\sin(x))} + \frac{(2\cdots}{\cdots}$$

$$= -\frac{b(6a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\,(a^2-b^2)^{5/2}} - \frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\,(a^2-b^2)\,(a+b\cdots}$$

**Mathematica** [A]
time = 0.57, size = 140, normalized size = 0.97

$$-\frac{\frac{2b(6a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + 2\log\left(\cos\left(\frac{x}{2}\right)\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{ab^2\cos(x)(6a^3-3ab^2+b(5a^2-2b^2)\sin(x))}{(a-b)^2(a+b)^2(a+b\sin(x))^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -1/2*((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(a^2 - b^2)^(5/2) + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (a*b^2*Cos[x]
*(6*a^3 - 3*a*b^2 + b*(5*a^2 - 2*b^2)*Sin[x]))/((a - b)^2*(a + b)^2*(a + b*
Sin[x])^2))/a^3
```

**Maple** [A]
time = 0.45, size = 270, normalized size = 1.86

| method | result |
|---|---|
| default | $\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3} - \frac{2b\left(\frac{\frac{a\,b^2(7a^2-4b^2)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a^4-4a^2b^2+2b^4} + \frac{3b(2a^4+3a^2b^2-2b^4)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2(a^4-2a^2b^2+b^4)} + \frac{a\,b^2(17a^2-8b^2)\tan\left(\frac{x}{2}\right)}{2a^4-4a^2b^2+2b^4} + \frac{3a^2b(2a^2-b^2)}{2(a^4-2a^2b^2+b^4)}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^2} + \frac{(6a^4-5a^2\cdots}{2(a\cdots}\right.}{a^3}$ |

| | |
|---|---|
| risch | $\frac{ib\left(-4ia^3b\,e^{3ix}+ia\,b^3e^{3ix}+16ia^3b\,e^{ix}-7ia\,b^3e^{ix}+10a^4e^{2ix}+a^2b^2e^{2ix}-2b^4e^{2ix}-5a^2b^2+2b^4\right)}{\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)^2a^2\left(a^2-b^2\right)^2}+\frac{3iab\ln\left(e^{ix}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*ln(tan(1/2*x))-2/a^3*b*((1/2*a*b^2*(7*a^2-4*b^2)/(a^4-2*a^2*b^2+b^4)*
tan(1/2*x)^3+3/2*b*(2*a^4+3*a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2
+1/2*a*b^2*(17*a^2-8*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+3/2*a^2*b*(2*a^2-b
^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+1/2*(6*a^4-5*a
^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x
)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(135) = 270$.
time = 0.86, size = 1027, normalized size = 7.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) + (6*a^6*b + a^4*b
^3 - 3*a^2*b^5 + 2*b^7 - (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^
5*b^2 - 5*a^3*b^4 + 2*a*b^6)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a
^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 6*(2*a^6*b^2 - 3*a^
4*b^4 + a^2*b^6)*cos(x) + 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
```

```
8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^11 - 2*a^9*b^2 + 2
*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*cos(x)^2 +
 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x)), -1/2*((5*a^5*b^3 - 7
*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7
- (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a
*b^6)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(x) + (a^8 - 2*a^6*b^2 + 2*a^2
*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - (a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 +
 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/
(a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6
- a^3*b^8)*cos(x)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**3,x)
[Out] Integral(csc(x)/(a + b*sin(x))**3, x)
```

**Giac [A]**
time = 0.44, size = 246, normalized size = 1.70

$$-\frac{(6a^4b-5a^2b^3+2b^5)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^7-2a^5b^2+a^3b^4)\sqrt{a^2-b^2}}-\frac{7a^3b^3\tan\left(\frac{1}{2}x\right)^3-4ab^5\tan\left(\frac{1}{2}x\right)^3+6a^4b^2\tan\left(\frac{1}{2}x\right)^2+9a^2b^4\tan\left(\frac{1}{2}x\right)^2-6b^6\tan\left(\frac{1}{2}x\right)^2+17a^3b^3\tan\left(\frac{1}{2}x\right)-8ab^5\tan\left(\frac{1}{2}x\right)+6a^4b^2-3a^2b^4}{(a^7-2a^5b^2+a^3b^4)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="giac")
[Out] -(6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) - (7*a^3*b^3*tan(1/2*x)^3 - 4*a*b^5*tan(1/2*x)^3 + 6*a^4*b^2*tan(1/2*
x)^2 + 9*a^2*b^4*tan(1/2*x)^2 - 6*b^6*tan(1/2*x)^2 + 17*a^3*b^3*tan(1/2*x)
- 8*a*b^5*tan(1/2*x) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*
(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + log(abs(tan(1/2*x)))/a^3
```

**Mupad [B]**
time = 11.39, size = 2191, normalized size = 15.11

$$\frac{\frac{3(b^4-2a^2b^2)}{a(a^4-2a^2b^2+b^4)}-\frac{3\tan\left(\frac{x}{2}\right)^2(2a^4b^2+3a^2b^4-2b^6)}{a^3(a^4-2a^2b^2+b^4)}+\frac{\tan\left(\frac{x}{2}\right)(8b^5-17a^2b^3)}{a^2(a^4-2a^2b^2+b^4)}+\frac{b\tan\left(\frac{x}{2}\right)^3(4b^4-7a^2b^2)}{a^2(a^4-2a^2b^2+b^4)}}{\tan\left(\frac{x}{2}\right)^2(2a^2+4b^2)+a^2+a^2\tan\left(\frac{x}{2}\right)^4+4ab\tan\left(\frac{x}{2}\right)+4ab\tan\left(\frac{x}{2}\right)^3}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{b\,\mathrm{atan}\left([illegible]\right)\sqrt{-(a+b)^5(a-b)^5}\,(6a^4-5a^2b^2+2b^4)\,1i}{a^{13}-5a^{11}b^2+10a^9b^4-10a^7b^6+5a^5b^8-a^3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)*(a + b*sin(x))^3),x)

[Out] ((3*(b^4 - 2*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) - (3*tan(x/2)^2*(3*a^2*b
^4 - 2*b^6 + 2*a^4*b^2))/(a^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)*(8*b^5 -
 17*a^2*b^3))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)^3*(4*b^4 - 7*a^2*
b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2 + 4*b^2) + a^2 + a^
2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + log(tan(x/2))/a^3 + (b*
atan(((b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a
^8 + a^4*b^4 - 2*a^6*b^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*
a^5*b^6 - 62*a^7*b^4 + 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4
 - 4*a^9*b^2) - (b*((2*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a
^6*b^2) - (tan(x/2)*(6*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^1
0*b^4 - 32*a^12*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))
*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b
^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 -
5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 -
5*a^11*b^2)) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^
5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*
b^8 + 68*a^5*b^6 - 62*a^7*b^4 + 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 +
6*a^7*b^4 - 4*a^9*b^2) + (b*((2*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*
b^4 - 2*a^6*b^2) - (tan(x/2)*(6*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6
 + 68*a^10*b^4 - 32*a^12*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*
a^9*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^1
3 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 +
 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a
^9*b^4 - 5*a^11*b^2)))/((2*(6*a^4*b + 2*b^5 - 5*a^2*b^3))/(a^8 + a^4*b^4 -
2*a^6*b^2) + (2*tan(x/2)*(2*b^8 - 13*a^2*b^6 + 26*a^4*b^4 - 24*a^6*b^2))/(a
^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) - (b*(-(a + b)^5*(a - b)
^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) +
(tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*a^5*b^6 - 62*a^7*b^4 + 24*a^
9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) - (b*((2*a^10*
b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) - (tan(x/2)*(6*a^14
- 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^10*b^4 - 32*a^12*b^2))/(a^11
+ a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))*(-(a + b)^5*(a - b)^5)^(1/2
)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6
 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b
^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + (b*(-(a + b)^5*(
a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b
^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*a^5*b^6 - 62*a^7*b^4 +
 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (b*((2
*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) - (tan(x/2)*(6
*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^10*b^4 - 32*a^12*b^2))/
(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))*(-(a + b)^5*(a - b)^5

```
)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a
^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 -
 a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))))*(-(a + b)^
5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(a^13 - a^3*b^10 + 5*a^5
*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)
```

# 3.200 $\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=187

$$\frac{3b^2(4a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}+\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\,\mathrm{co}\ldots}{2a\left(a^2-b^2\right)(\ldots}$$

```
[Out] 3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^4/
(a^2-b^2)^(5/2)+3*b*arctanh(cos(x))/a^4-1/2*(2*a^4-11*a^2*b^2+6*b^4)*cot(x)
/a^3/(a^2-b^2)^2-1/2*b^2*cot(x)/a/(a^2-b^2)/(a+b*sin(x))^2-3/2*b^2*(2*a^2-b
^2)*cot(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.42, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{3b^2(2a^2-b^2)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3b^2(4a^4-5a^2b^2+2b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}-\frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(a^4*(a^2 - b^2)^(5/2)) + (3*b*ArcTanh[Cos[x]])/a^4 - ((2*a^4 - 11*a^2*b^
2 + 6*b^4)*Cot[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x])/(2*a*(a^2 - b^2)*(a
 + b*Sin[x])^2) - (3*b^2*(2*a^2 - b^2)*Cot[x])/(2*a^2*(a^2 - b^2)^2*(a + b*
Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} + \frac{\int \frac{\csc^2(x)(2a^2-3b^2-2ab\sin(x)+2b^2\sin^2(x))}{(a+b\sin(x))^2}\,dx}{2a(a^2-b^2)}$$

$$= -\frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} - \frac{3b^2(2a^2-b^2)\cot(x)}{2a^2(a^2-b^2)^2(a+b\sin(x))} + \frac{\int \csc^2(x)(2a^4-11a^2b^2+6b^4-$$

$$= -\frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3(a^2-b^2)^2} - \frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} - \frac{3b^2(2a^2-b^2)\cot(x}{2a^2(a^2-b^2)^2(a+b\sin$$

$$= -\frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3(a^2-b^2)^2} - \frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} - \frac{3b^2(2a^2-b^2)\cot(x}{2a^2(a^2-b^2)^2(a+b\sin$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3(a^2-b^2)^2} - \frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} - \frac{}{2a}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{(2a^4-11a^2b^2+6b^4)\cot(x)}{2a^3(a^2-b^2)^2} - \frac{b^2\cot(x)}{2a(a^2-b^2)(a+b\sin(x))^2} - \frac{}{2a}$$

$$= \frac{3b^2(4a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}} + \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{(2a^4-11a^2b^2+}{2a^3(a^2-}$$

**Mathematica** [A]
time = 0.88, size = 174, normalized size = 0.93

$$\frac{\frac{6b^2(4a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - a\cot\left(\frac{x}{2}\right) + 6b\log\left(\cos\left(\frac{x}{2}\right)\right) - 6b\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{a^2b^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{ab^3(7a^2-4b^2)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} + a\tan\left(\frac{x}{2}\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]
])/(a^2 - b^2)^(5/2) - a*Cot[x/2] + 6*b*Log[Cos[x/2]] - 6*b*Log[Sin[x/2]] +
 (a^2*b^3*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (a*b^3*(7*a^2 - 4*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^4)
```

**Maple** [A]
time = 0.50, size = 294, normalized size = 1.57

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)}{2a^{3}}+\frac{4b^{2}\left(\frac{\frac{3a\,b^{2}\left(3a^{2}-2b^{2}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}+\frac{b\left(8a^{4}+11a^{2}b^{2}-10b^{4}\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{4a^{4}-8a^{2}b^{2}+4b^{4}}+\frac{a\,b^{2}\left(23a^{2}-14b^{2}\right)\tan\left(\frac{x}{2}\right)}{4a^{4}-8a^{2}b^{2}+4b^{4}}+\frac{a^{2}b\left(8a^{2}-5b^{2}\right)}{4a^{4}-8a^{2}b^{2}+4b^{4}}}{\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^{2}}+\frac{3\left(4a^{4}-5a^{2}\right.}{4\left(a\right.}\right.}{a^{4}}$ |
| risch | $-\frac{i\left(3ia\,b^{5}e^{5ix}+44ia^{3}b^{3}e^{3ix}-24ia\,b^{5}e^{3ix}+21ia\,b^{5}e^{ix}-8ia^{5}b\,e^{3ix}+12a^{4}b^{2}e^{4ix}+3a^{2}b^{4}e^{4ix}-6b^{6}e^{4ix}-6ia^{3}b^{3}e^{5ix}+8ia^{5}b\,e^{ix}-38ia^{3}e^{ix}\right.}{\left(e^{2ix}-1\right)\left(-ib\,e^{2ix}+ib+2a\,e^{ix}\right)^{2}\left(a^{2}-b^{2}\right)^{2}a^{3}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*tan(1/2*x)+4/a^4*b^2*((3/4*a*b^2*(3*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)*
tan(1/2*x)^3+1/4*b*(8*a^4+11*a^2*b^2-10*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)
^2+1/4*a*b^2*(23*a^2-14*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+1/4*a^2*b*(8*a^
2-5*b^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^2+3/4*(4*a^
4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*x)+2*b)/(a^2-b^2)^(1/2)))-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(175) = 350$.
time = 0.87, size = 1436, normalized size = 7.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - 2*(4*a^8
*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 10
*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6*
```

```
b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)^
2)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) - 2*(2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6
- 6*a*b^8)*cos(x) + 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
1/2*cos(x) + 1/2) - 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
-1/2*cos(x) + 1/2))/(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11
*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6
*b^6 - a^4*b^8 - (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin
(x)), 1/2*((2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - (4*a^
8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 1
0*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6
*b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)
^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))
) - (2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6 - 6*a*b^8)*cos(x) + 3*(2*a^
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*(2*a^
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(2*a^11
*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5
- a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6*b^6 - a^4*b^8 - (a^10*b^2
- 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin(x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**3, x)
```

**Giac [A]**
time = 0.44, size = 280, normalized size = 1.50

$$\frac{3\left(4a^4b^2-5a^2b^4+2b^6\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{a^2-b^2}}+\frac{9a^3b^4\tan\left(\frac{1}{2}x\right)^3-6ab^6\tan\left(\frac{1}{2}x\right)^3+8a^4b^3\tan\left(\frac{1}{2}x\right)^2+11a^2b^5\tan\left(\frac{1}{2}x\right)^2-10b^7\tan\left(\frac{1}{2}x\right)^2+23a^3b^4\tan\left(\frac{1}{2}x\right)-14ab^6\tan\left(\frac{1}{2}x\right)+8a^4b^3-5a^2b^5}{\left(a^8-2a^6b^2+a^4b^4\right)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)^2}-\frac{3b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^4}+\frac{\tan\left(\frac{1}{2}x\right)}{2a^3}+\frac{6b\tan\left(\frac{1}{2}x\right)-a}{2a^4\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2
 - b^2)) + (9*a^3*b^4*tan(1/2*x)^3 - 6*a*b^6*tan(1/2*x)^3 + 8*a^4*b^3*tan(1
/2*x)^2 + 11*a^2*b^5*tan(1/2*x)^2 - 10*b^7*tan(1/2*x)^2 + 23*a^3*b^4*tan(1/
2*x) - 14*a*b^6*tan(1/2*x) + 8*a^4*b^3 - 5*a^2*b^5)/((a^8 - 2*a^6*b^2 + a^4
*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*b*log(abs(tan(1/2*x)))/a
^4 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan(1/2*x))
```

**Mupad [B]**
time = 9.02, size = 2295, normalized size = 12.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))^3),x)
```

```
[Out] tan(x/2)/(2*a^3) - (a^2 + (2*tan(x/2)*(7*a*b^5 + 2*a^5*b - 12*a^3*b^3))/(a^
4 + b^4 - 2*a^2*b^2) + (tan(x/2)^4*(a^6 + 12*b^6 - 17*a^2*b^4 - 2*a^4*b^2))
/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^2*(a^6 + 16*b^6 - 26*a^2*b^4))/(a^4
+ b^4 - 2*a^2*b^2) + (2*tan(x/2)^3*(2*a^6*b + 10*b^7 - 9*a^2*b^5 - 12*a^4*b
^3))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^3*(4*a^5 + 8*a^3*b^2) + 2*a^5*t
an(x/2) + 2*a^5*tan(x/2)^5 + 8*a^4*b*tan(x/2)^2 + 8*a^4*b*tan(x/2)^4) - (3*
b*log(tan(x/2)))/a^4 - (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4
+ 2*b^4 - 5*a^2*b^2)*((12*a^4*b^6 - 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^
4 - 2*a^8*b^2) - (tan(x/2)*(6*a^12*b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*
b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 -
 4*a^11*b^2) + (3*b^2*((2*a^12*b + 2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4
- 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 +
68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^1
1*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14
- a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*3i)/(2*(a
^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) - (b^2
*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*((tan(x/2)*(6*a^1
2*b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))
/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) - (12*a^4*b^6 - 27*a
^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4 - 2*a^8*b^2) + (3*b^2*((2*a^12*b + 2*a
^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a
^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 + 68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^
5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(
4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 +
10*a^10*b^4 - 5*a^12*b^2)))*3i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^
6 + 10*a^10*b^4 - 5*a^12*b^2)))/((2*(18*b^7 - 45*a^2*b^5 + 36*a^4*b^3))/(a^
10 + a^6*b^4 - 2*a^8*b^2) + (2*tan(x/2)*(18*b^10 - 81*a^2*b^8 + 126*a^4*b^6
 - 72*a^6*b^4))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (3*
```

```
b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*((12*a^4*b^6 -
 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^12*
b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(
a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (3*b^2*((2*a^12*b +
2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 -
8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 + 68*a^12*b^4 - 32*a^14*b^2))/(a^13 +
 a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2
)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6
 + 10*a^10*b^4 - 5*a^12*b^2))))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^
6 + 10*a^10*b^4 - 5*a^12*b^2)) + (3*b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4
 + 2*b^4 - 5*a^2*b^2)*((tan(x/2)*(6*a^12*b - 24*a^2*b^11 + 108*a^4*b^9 - 19
2*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9
*b^4 - 4*a^11*b^2) - (12*a^4*b^6 - 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4
 - 2*a^8*b^2) + (3*b^2*((2*a^12*b + 2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4
 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 +
 68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^
11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14
 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))/(2*(a^1
4 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))*(-(a +
 b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(a^14 - a^4*b^10 + 5
*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)
```

# 3.201 $\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx$

**Optimal**. Leaf size=241

$$-\frac{b^3(20a^4-29a^2b^2+12b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}-\frac{(a^2+12b^2)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b(2a^4-7a^2b^2+4b^4)\cot(x)}{2a^4\left(a^2-b^2\right)^2}$$

```
[Out] -b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^
5/(a^2-b^2)^(5/2)-1/2*(a^2+12*b^2)*arctanh(cos(x))/a^5+3/2*b*(2*a^4-7*a^2*b
^2+4*b^4)*cot(x)/a^4/(a^2-b^2)^2-1/2*(a^4-10*a^2*b^2+6*b^4)*cot(x)*csc(x)/a
^3/(a^2-b^2)^2-1/2*b^2*cot(x)*csc(x)/a/(a^2-b^2)/(a+b*sin(x))^2-1/2*b^2*(7*
a^2-4*b^2)*cot(x)*csc(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

**Rubi** [A]
time = 0.59, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$-\frac{b^2(7a^2-4b^2)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{(a^2+12b^2)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b(2a^4-7a^2b^2+4b^4)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{b^3(20a^4-29a^2b^2+12b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}-\frac{(a^4-10a^2b^2+6b^4)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(a^5*(a^2 - b^2)^(5/2))) - ((a^2 + 12*b^2)*ArcTanh[Cos[x]])/(2*a^5) +
(3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[x])/(2*a^4*(a^2 - b^2)^2) - ((a^4 - 10
*a^2*b^2 + 6*b^4)*Cot[x]*Csc[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x]*Csc[x]
)/(2*a*(a^2 - b^2)*(a + b*Sin[x])^2) - (b^2*(7*a^2 - 4*b^2)*Cot[x]*Csc[x])/
(2*a^2*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\int \frac{\csc^3(x)\left(2\left(a^2-2b^2\right)-2ab\sin(x)+3b^2\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{b^2\left(7a^2-4b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\int \frac{\csc^3(x)\left(2\left(a^4-10a^2b^2+6b\right.\right.}{}}{}$$

$$= -\frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{b^2\left(7a^2-4b^2\right)\mathrm{c}}{2a^2\left(a^2-b^2\right)^2(}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^4-10a^2b^2+6b^4\right)}{2a^3\left(a^2-\right.}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^4-10a^2b^2+6b^4\right)}{2a^3\left(a^2-\right.}$$

$$= -\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}-\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b}{}$$

**Mathematica [A]**
time = 1.26, size = 220, normalized size = 0.91

$$\frac{-\frac{8b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+12ab\cot\left(\frac{x}{2}\right)-a^2\csc^2\left(\frac{x}{2}\right)-4\left(a^2+12b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right)+4\left(a^2+12b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)+a^2\sec^2\left(\frac{x}{2}\right)-\frac{4a^2b^4\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2}+\frac{12ab^4\left(-3a^2+2b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}-12ab\tan\left(\frac{x}{2}\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] ((-8*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 12*b
^2)*Log[Cos[x/2]] + 4*(a^2 + 12*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (4*a^
2*b^4*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (12*a*b^4*(-3*a^2 + 2*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - 12*a*b*Tan[x/2])/(8*a^5)
```

**Maple [A]**
time = 0.54, size = 329, normalized size = 1.37

| method | result |
|---|---|
| default | $\frac{\frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-6b\tan\left(\frac{x}{2}\right)}{4a^4}-\frac{1}{8a^3\tan\left(\frac{x}{2}\right)^2}+\frac{(2a^2+24b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a^5}+\frac{3b}{2a^4\tan\left(\frac{x}{2}\right)}-\frac{4b^3\left(\frac{a\,b^2\left(11a^2-8b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a^4-8a^2b^2+4b^4}+\frac{b\left(10a^4+13\right.}{4}\right.}{}$ |
| risch | $5ia^2b^5\mathrm{e}^{6ix}+24ia^6b\,\mathrm{e}^{4ix}+40ia^4b^3\mathrm{e}^{2ix}+12ib^7+4a^7\mathrm{e}^{5ix}+4a^7\mathrm{e}^{3ix}-4ia^6b\,\mathrm{e}^{6ix}+20ia^4b^3\mathrm{e}^{6ix}-20ia^6b\,\mathrm{e}^{2ix}-66ia^4b^3\mathrm{e}^{4ix}-15ia^2b^5\mathrm{e}^{4ix}+31i$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^4*(1/2*a*tan(1/2*x)^2-6*b*tan(1/2*x))-1/8/a^3/tan(1/2*x)^2+1/4/a^5*(2
*a^2+24*b^2)*ln(tan(1/2*x))+3/2/a^4*b/tan(1/2*x)-4/a^5*b^3*((1/4*a*b^2*(11*
a^2-8*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+1/4*b*(10*a^4+13*a^2*b^2-14*b^4
)/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+1/4*a*b^2*(29*a^2-20*b^2)/(a^4-2*a^2*b^2
+b^4)*tan(1/2*x)+1/4*a^2*b*(10*a^2-7*b^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*x
)^2+2*b*tan(1/2*x)+a)^2+1/4*(20*a^4-29*a^2*b^2+12*b^4)/(a^4-2*a^2*b^2+b^4)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(225) = 450$.
time = 1.46, size = 2005, normalized size = 8.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 + (20
*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 12*
b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2 +
2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8
)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*s
in(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 -
51*a^4*b^6 + 18*a^2*b^8)*cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b
^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8
- 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2
*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 -
 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)
^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4
*b^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^
8 - 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a
^2*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7
 - 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(
x)^2)*sin(x))*log(-1/2*cos(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3
*b^7 - 4*a*b^9)*cos(x)^3 - (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 -
 12*a*b^9)*cos(x))*sin(x))/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11
*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9
*b^4 + 5*a^7*b^6 - 2*a^5*b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5
 - a^6*b^7 - (a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x)),
 -1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 - 2*(
20*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 1
2*b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2
+ 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b
^8)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^
2)*cos(x))) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 - 51*a^4*b^6 + 18*a^2*b^8)*
cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^10
 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (a^
10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^2
+ 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*a^
7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x
) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^
10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (
a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^
2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*
a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(-1/2*co
s(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(x)^3
- (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 - 12*a*b^9)*cos(x))*sin(x)
)/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*
b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9*b^4 + 5*a^7*b^6 - 2*a^5*
b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7 - (a^12*b - 3*
a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**3, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(225) = 450.
time = 0.49, size = 514, normalized size = 2.13

$$-\frac{(20\,a^4b^3 - 29\,a^2b^5 + 12\,b^7)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^9 - 2\,a^7b^2 + a^5b^4)\sqrt{a^2-b^2}} - \frac{2\,a^8\tan\left(\frac{1}{2}x\right)^6 + 20\,a^6b^2\tan\left(\frac{1}{2}x\right)^6 - 46\,a^4b^4\tan\left(\frac{1}{2}x\right)^6 + 24\,a^2b^6\tan\left(\frac{1}{2}x\right)^6 - 4\,a^7b\tan\left(\frac{1}{2}x\right)^5 + 104\,a^5b^3\tan\left(\frac{1}{2}x\right)^5 - 108\,a^3b^5\tan\left(\frac{1}{2}x\right)^5 + 32\,ab^7\tan\left(\frac{1}{2}x\right)^5 + 5\,a^8\tan\left(\frac{1}{2}x\right)^4 - 2\,a^6b^2\tan\left(\frac{1}{2}x\right)^4 + 165\,a^4b^4\tan\left(\frac{1}{2}x\right)^4 - 80\,a^2b^6\tan\left(\frac{1}{2}x\right)^4 - 16\,b^8\tan\left(\frac{1}{2}x\right)^4 - 12\,a^7b\tan\left(\frac{1}{2}x\right)^3 + 72\,a^5b^3\tan\left(\frac{1}{2}x\right)^3 + 124\,a^3b^5\tan\left(\frac{1}{2}x\right)^3 - 112\,ab^7\tan\left(\frac{1}{2}x\right)^3 + 4\,a^8\tan\left(\frac{1}{2}x\right)^2 - 28\,a^6b^2\tan\left(\frac{1}{2}x\right)^2 + 124\,a^4b^4\tan\left(\frac{1}{2}x\right)^2 - 76\,a^2b^6\tan\left(\frac{1}{2}x\right)^2 - 8\,a^7b\tan\left(\frac{1}{2}x\right) + 16\,a^5b^3\tan\left(\frac{1}{2}x\right) - 8\,a^3b^5\tan\left(\frac{1}{2}x\right) + a^8 - 2\,a^6b^2 + a^4b^4}{8\,(a^9 - 2\,a^7b^2 + a^5b^4)\left(a\tan\left(\frac{1}{2}x\right)^3 + 2\,b\tan\left(\frac{1}{2}x\right)^2 + a\tan\left(\frac{1}{2}x\right)\right)^2} + \frac{(a^2+12\,b^2)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,a^5} + \frac{a^3\tan\left(\frac{1}{2}x\right)^2 - 12\,a^2b\tan\left(\frac{1}{2}x\right)}{8\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a
^2 - b^2)) - 1/8*(2*a^8*tan(1/2*x)^6 + 20*a^6*b^2*tan(1/2*x)^6 - 46*a^4*b^4
*tan(1/2*x)^6 + 24*a^2*b^6*tan(1/2*x)^6 - 4*a^7*b*tan(1/2*x)^5 + 104*a^5*b^
3*tan(1/2*x)^5 - 108*a^3*b^5*tan(1/2*x)^5 + 32*a*b^7*tan(1/2*x)^5 + 5*a^8*t
an(1/2*x)^4 - 2*a^6*b^2*tan(1/2*x)^4 + 165*a^4*b^4*tan(1/2*x)^4 - 80*a^2*b^
6*tan(1/2*x)^4 - 16*b^8*tan(1/2*x)^4 - 12*a^7*b*tan(1/2*x)^3 + 72*a^5*b^3*t
an(1/2*x)^3 + 124*a^3*b^5*tan(1/2*x)^3 - 112*a*b^7*tan(1/2*x)^3 + 4*a^8*tan
(1/2*x)^2 - 28*a^6*b^2*tan(1/2*x)^2 + 124*a^4*b^4*tan(1/2*x)^2 - 76*a^2*b^6
*tan(1/2*x)^2 - 8*a^7*b*tan(1/2*x) + 16*a^5*b^3*tan(1/2*x) - 8*a^3*b^5*tan(
1/2*x) + a^8 - 2*a^6*b^2 + a^4*b^4)/((a^9 - 2*a^7*b^2 + a^5*b^4)*(a*tan(1/2
*x)^3 + 2*b*tan(1/2*x)^2 + a*tan(1/2*x))^2) + 1/2*(a^2 + 12*b^2)*log(abs(ta
n(1/2*x)))/a^5 + 1/8*(a^3*tan(1/2*x)^2 - 12*a^2*b*tan(1/2*x))/a^6
```

**Mupad** [B]
time = 9.28, size = 2405, normalized size = 9.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))^3),x)
```

```
[Out] (4*a^2*b*tan(x/2) - a^3/2 + (tan(x/2)^2*(50*a*b^6 - a^7 - 85*a^3*b^4 + 24*a
^5*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^5*(3*a^6*b + 16*b^7 - 19*a^2
*b^5 - 6*a^4*b^3))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^3*(5*a^6*b + 52*b^
```

```
7 - 77*a^2*b^5 + 2*a^4*b^3))/(a^4 + b^4 - 2*a^2*b^2) - (tan(x/2)^4*(a^8 - 1
12*b^8 + 56*a^2*b^6 + 177*a^4*b^4 - 50*a^6*b^2))/(2*a*(a^4 + b^4 - 2*a^2*b^
2)))/(tan(x/2)^4*(8*a^6 + 16*a^4*b^2) + 4*a^6*tan(x/2)^2 + 4*a^6*tan(x/2)^6
 + 16*a^5*b*tan(x/2)^3 + 16*a^5*b*tan(x/2)^5) + tan(x/2)^2/(8*a^3) + (log(t
an(x/2))*(a^2 + 12*b^2))/(2*a^5) - (3*b*tan(x/2))/(2*a^4) + (b^3*atan(((b^3
*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((a^11*b + 24*
a^5*b^7 - 52*a^7*b^5 + 30*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2
)*(a^15 - 48*a^3*b^12 + 212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*
b^4 + 6*a^13*b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) +
 (b^3*((2*a^14*b + 2*a^10*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) -
 (tan(x/2)*(6*a^18 - 8*a^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 -
 32*a^16*b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a
 + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^1
0 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*1i)/(2*(a^15 - a^5
*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) - (b^3*(-(a + b
)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((tan(x/2)*(a^15 - 48*a
^3*b^12 + 212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b
^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) - (a^11*b + 24*
a^5*b^7 - 52*a^7*b^5 + 30*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (b^3*((2
*a^14*b + 2*a^10*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2
)*(6*a^18 - 8*a^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*
b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(
a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7
*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*1i)/(2*(a^15 - a^5*b^10 + 5
*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))/((144*b^9 - 336*a^2*b^7
 + 211*a^4*b^5 + 20*a^6*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (2*tan(x/2)*(7
2*b^12 - 294*a^2*b^10 + 422*a^4*b^8 - 229*a^6*b^6 + 20*a^8*b^4))/(a^15 + a^
7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) + (b^3*(-(a + b)^5*(a - b)^5)^
(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((a^11*b + 24*a^5*b^7 - 52*a^7*b^5 + 3
0*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(a^15 - 48*a^3*b^12 + 
212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b^2))/(a^15
 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) + (b^3*((2*a^14*b + 2*a^1
0*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(6*a^18 - 8*a
^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*b^2))/(a^15 + a
^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)
*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b
^6 + 10*a^11*b^4 - 5*a^13*b^2))))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*
b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^
4 + 12*b^4 - 29*a^2*b^2)*((tan(x/2)*(a^15 - 48*a^3*b^12 + 212*a^5*b^10 - 36
3*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b^2))/(a^15 + a^7*b^8 - 4*a^
9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) - (a^11*b + 24*a^5*b^7 - 52*a^7*b^5 + 30*a
^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (b^3*((2*a^14*b + 2*a^10*b^5 - 4*a^
12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(6*a^18 - 8*a^8*b^10 + 38
*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*b^2))/(a^15 + a^7*b^8 - 4*a
^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 1
```

```
2*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11
*b^4 - 5*a^13*b^2))))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^1
1*b^4 - 5*a^13*b^2))))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a
^2*b^2)*1i)/(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^1
3*b^2)
```

# 3.202 $\int \frac{1}{(a+b\sin(c+dx))^4}\,dx$

**Optimal**. Leaf size=182

$$\frac{a(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d}+\frac{b\cos(c+dx)}{3\,(a^2-b^2)\,d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sin(c+dx))^2}+$$

```
[Out] a*(2*a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(7/2)/d+1/3*b*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^3+5/6*a*b*cos(d*x+c)/
(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2+1/6*b*(11*a^2+4*b^2)*cos(d*x+c)/(a^2-b^2)^
3/d/(a+b*sin(d*x+c))
```

**Rubi** [A]
time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$\frac{a(2a^2+3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{7/2}}+\frac{b(11a^2+4b^2)\cos(c+dx)}{6d\,(a^2-b^2)^3\,(a+b\sin(c+dx))}+\frac{5ab\cos(c+dx)}{6d\,(a^2-b^2)^2\,(a+b\sin(c+dx))^2}+\frac{b\cos(c+dx)}{3d\,(a^2-b^2)\,(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(7/2)*d) + (b*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^3)
 + (5*a*b*Cos[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (b*(11
*a^2 + 4*b^2)*Cos[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(c+dx))^4}\,dx = \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+2b\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{3(a^2-b^2)}$$

$$= \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{\int \frac{2(3a^2\text{-}}{(}}{}$$

$$= \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{b(11}{6(a^2-}$$

$$= \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{b(11}{6(a^2-}$$

$$= \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{b(11}{6(a^2-}$$

$$= \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{b(11}{6(a^2-}$$

$$= \frac{a(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^3} + \frac{}{6(}$$

**Mathematica** [A]
time = 0.65, size = 157, normalized size = 0.86

$$\frac{\frac{6a(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{b\cos(c+dx)\left(18a^4-5a^2b^2+2b^4+3ab(9a^2+b^2)\sin(c+dx)+b^2(11a^2+4b^2)\sin^2(c+dx)\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2 - b^2)^(7/2) + (b*Cos[c + d*x]*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2
 + b^2)*Sin[c + d*x] + b^2*(11*a^2 + 4*b^2)*Sin[c + d*x]^2))/((a - b)^3*(a
+ b)^3*(a + b*Sin[c + d*x])^3))/(6*d)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(171) = 342$.
time = 0.53, size = 510, normalized size = 2.80

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{b^2\left(9a^4-6a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{b\left(6a^6+27a^4b^2-12a^2b^4+4b^6\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2b^2\left(54a^6+21a^4b^2-4a^2b^4+4b^6\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{3a^3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\right.}$ |
| default | $\frac{\frac{b^2\left(9a^4-6a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{b\left(6a^6+27a^4b^2-12a^2b^4+4b^6\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2b^2\left(54a^6+21a^4b^2-4a^2b^4+4b^6\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{3a^3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\right.}$ |
| risch | $-\frac{i\left(-30ib\,a^4e^{4i(dx+c)}-45ia^2b^3e^{4i(dx+c)}-6a^3b^2e^{5i(dx+c)}-9a\,b^4e^{5i(dx+c)}+102ia^4b\,e^{2i(dx+c)}+36ia^2b^3e^{2i(dx+c)}+12ib^5e^{2}\right.}{3\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(d}\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/2*b^2*(9*a^4-6*a^2*b^2+2*b^4)/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan
(1/2*d*x+1/2*c)^5+1/2*b*(6*a^6+27*a^4*b^2-12*a^2*b^4+4*b^6)/a^2/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6+21*a^4*b^2-4*a^
2*b^4+4*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3+1/a^2*b*(6*
a^6+20*a^4*b^2-3*a^2*b^4+2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(27*a^4-4*a^2*b^2+2*b^4)/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ta
n(1/2*d*x+1/2*c)+1/6*b*(18*a^4-5*a^2*b^2+2*b^4)/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^3+a*(2*a^2+3*b^2)/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(171) = 342$.
time = 0.43, size = 965, normalized size = 5.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 - 6*(9*a^5*b^2 - 8
*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) - 3*(2*a^6 + 9*a^4*b^2 + 9*a^2*
b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b + 11*a^3*b^3 + 3*
a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 12*(3*a^6*b - 2*a^4*b^3 - b^7
)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d
*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3
*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*
x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11
)*d)*sin(d*x + c)), 1/6*((11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 -
3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b +
 11*a^3*b^3 + 3*a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 6*(3*a^6*b - 2*a^4*b^3 - b^7)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*
b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6
*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d)*sin(d*x + c))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(171) = 342.
time = 0.49, size = 510, normalized size = 2.80

$$\frac{\frac{3\,(2\,a^3+3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{27\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^7b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+81\,a^5b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^3b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,ab^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+108\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+42\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,b^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,a^7b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^5b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-18\,a^3b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+81\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,a^7b-5\,a^5b^3+2\,a^3b^5}{(a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
```

```
4 - b^6)*sqrt(a^2 - b^2)) + (27*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^4*b^4
*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 18*a^7*b*tan(1
/2*d*x + 1/2*c)^4 + 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*b^5*tan(1/2*
d*x + 1/2*c)^4 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 + 108*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^2*b^6*tan(1/2*d*x + 1/
2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b*tan(1/2*d*x + 1/2*c)^2 + 1
20*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 12*
a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*x + 1/2*c) - 12*a^4*b^4
*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 18*a^7*b - 5*a^5*b
^3 + 2*a^3*b^5)/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^3))/d
```

**Mupad [B]**
time = 10.38, size = 708, normalized size = 3.89

$$\frac{\frac{18\,a^4\,b-5\,a^2\,b^3+2\,b^5}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^6\,b+20\,a^4\,b^3-3\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(6\,a^6\,b+27\,a^4\,b^3-12\,a^2\,b^5+4\,b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(27\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(9\,a^4\,b-6\,a^2\,b^3+2\,b^5\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^2+2\,b^2\right)\,\left(18\,a^4\,b-5\,a^2\,b^3+2\,b^5\right)}{3\,a^3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(3\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\,a^2\,b+8\,b^3\right)+a^3+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{a\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{2\,a^3+3\,a\,b^2}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x))^4,x)
```

```
[Out] ((18*a^4*b + 2*b^5 - 5*a^2*b^3)/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (
2*tan(c/2 + (d*x)/2)^2*(6*a^6*b + 2*b^7 - 3*a^2*b^5 + 20*a^4*b^3))/(a^2*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^4*(6*a^6*b + 4*b^7
- 12*a^2*b^5 + 27*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b*
tan(c/2 + (d*x)/2)*(27*a^4*b + 2*b^5 - 4*a^2*b^3))/(a*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(9*a^4*b + 2*b^5 - 6*a^2*b^3))/(a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)^3*(3*a^2 +
2*b^2)*(18*a^4*b + 2*b^5 - 5*a^2*b^3))/(3*a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)))/(d*(a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 3*
a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 3*a^3) + tan(c/2 + (d*x)/2)^3*(12*a
^2*b + 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/
2)^5)) + (a*atan((((a^2*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2))/((a + b)^(7/2)*
(a - b)^(7/2)) + (a*(2*a^2 + 3*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(3*a*b^2 + 2*a^3))*(2*a^2 + 3*b^2))/(d*(
a + b)^(7/2)*(a - b)^(7/2))
```

# 3.203 $\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=172

$$-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2(a^2-b^2)\,F\left(\frac{1}{2}(e-\right.}{3bf\,\surd}$$

```
[Out] -2/3*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f-2/3*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/b/f/((a+b*sin(f*x+e))/(a+b))^(1/
2)+2/3*(a^2-b^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
f*x+e))/(a+b))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2-b^2)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3bf\sqrt{a+b\sin(e+fx)}}-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2a\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*f) + (2*a*EllipticE[(e - Pi/2
 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(3*b*f*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a
 + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,d$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{a\int \sqrt{a+b\sin(e+fx)}\,dx}{3b}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{\left(a\sqrt{a+b\sin(e+fx)}\right)\int}{3b\sqrt{\frac{a+bs}{}}}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)}{3bf\sqrt{\frac{a+b\sin(}{a+}}}$$

**Mathematica [A]**
time = 2.67, size = 143, normalized size = 0.83

$$-\frac{2\left(b\cos(e+fx)(a+b\sin(e+fx))+a(a+b)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}-(a^2-b^2)\,F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)}{3bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*Cos[e + f*x]*(a + b*Sin[e + f*x]) + a*(a + b)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)] - (a^2 - b^2)*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)]))/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(222) = 444.
time = 3.42, size = 460, normalized size = 2.67

| method | result |
|---|---|
| default | $\frac{2\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^2b}{3} - \frac{2\sqrt{\frac{a+b\sin(f}{a-b}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1+si
n(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2*b-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^
(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^3-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1
)*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(f*x+e)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(s
in(f*x+e)-1)*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+sin(f*x+e)^3*b^3+sin(f
*x+e)^2*a*b^2-b^3*sin(f*x+e)-a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 420, normalized size = 2.44

$$\frac{-3i\sqrt{2}\,a\sqrt{i\,b}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)-3i\,b\sin(fx+e)-2i\,a}{3\,b}\right)\right)+3i\sqrt{2}\,a\sqrt{-i\,b}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)+3i\,b\sin(fx+e)+2i\,a}{3\,b}\right)\right)-6\sqrt{b\sin(fx+e)+a}\,b^2\cos(fx+e)-\sqrt{2}\,(2\,a^2-3\,b^2)\sqrt{i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)-3i\,b\sin(fx+e)-2i\,a}{3\,b}\right)-\sqrt{2}\,(2\,a^2-3\,b^2)\sqrt{-i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)+3i\,b\sin(fx+e)+2i\,a}{3\,b}\right)}{9\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(-3*I*sqrt(2)*a*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*b*sin(f*x +
 e) - 2*I*a)/b)) + 3*I*sqrt(2)*a*sqrt(-I*b)*b*weierstrassZeta(-4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) +
 3*I*b*sin(f*x + e) + 2*I*a)/b)) - 6*sqrt(b*sin(f*x + e) + a)*b^2*cos(f*x +
 e) - sqrt(2)*(2*a^2 - 3*b^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*b*s
in(f*x + e) - 2*I*a)/b) - sqrt(2)*(2*a^2 - 3*b^2)*sqrt(-I*b)*weierstrassPIn
verse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*
cos(f*x + e) + 3*I*b*sin(f*x + e) + 2*I*a)/b))/(b^2*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(e+fx)}\,\sin(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sin(e + f*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2), x)
```

# 3.204 $\int \sqrt{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=62

$$\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/f
/((a+b*sin(f*x+e))/(a+b))^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2734, 2732}

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,dx = \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

**Mathematica** [**A**]
time = 0.05, size = 61, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]
])/(f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(91) = 182.
time = 3.32, size = 239, normalized size = 3.85

| method | result |
|---|---|
| default | $-\frac{2(a-b)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\left(\text{EllipticE}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a+\text{Ellipt}\right.}{b\cos(fx+e)\sqrt{a+}}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(
-(1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*b-a*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-E
llipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/cos(f*x+e)/
(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 371, normalized size = 5.98

$$\frac{\sqrt{2}\,a\sqrt{i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)-3i\,b\sin(fx+e)-2i\,a}{3\,b}\right)+\sqrt{2}\,a\sqrt{-i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)+3i\,b\sin(fx+e)+2i\,a}{3\,b}\right)-3i\,\sqrt{2}\,\sqrt{i\,b}\,b\text{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)-3i\,b\sin(fx+e)-2i\,a}{3\,b}\right)\right)+3i\,\sqrt{2}\,\sqrt{-i\,b}\,b\text{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)+3i\,b\sin(fx+e)+2i\,a}{3\,b}\right)\right)}{3\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*a*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*b*sin(f*x + e) - 2
*I*a)/b) + sqrt(2)*a*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) + 3*I*b*sin(f*x
+ e) + 2*I*a)/b) - 3*I*sqrt(2)*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*
b*sin(f*x + e) - 2*I*a)/b)) + 3*I*sqrt(2)*sqrt(-I*b)*b*weierstrassZeta(-4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f
*x + e) + 3*I*b*sin(f*x + e) + 2*I*a)/b)))/(b*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin{\left(e + fx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [B]
time = 6.87, size = 55, normalized size = 0.89

$$\frac{2\,\mathrm{E}\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}{f\,\sqrt{\frac{a+b\,\sin\left(e+f\,x\right)}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] (2*ellipticE(e/2 - pi/4 + (f*x)/2, (2*b)/(a + b))*(a + b*sin(e + f*x))^(1/2
))/(f*((a + b*sin(e + f*x))/(a + b))^(1/2))
```

# 3.205 $\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=128

$$\frac{2bF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b
))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)-2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/
(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2882, 2742, 2740, 2886, 2884}

$$\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*b*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])
/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (2*a*EllipticPi[2, (e - Pi/2 + f*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[
e + f*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = a\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx + b\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}} + \frac{\left(b\sqrt{\right.}}{}$$

$$= \frac{2bF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}} + \frac{2a\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}\right.\right.}{f}$$

**Mathematica [A]**
time = 13.75, size = 89, normalized size = 0.70

$$-\frac{2\left(bF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2b}{a+b}\right)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
[Out] (-2*(b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] + a*EllipticPi[2, (-
2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f
*Sqrt[a + b*Sin[e + f*x]])
```

**Maple [A]**
time = 3.22, size = 169, normalized size = 1.32

| method | result |
|---|---|
| default | $\frac{2(a-b)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)-\mathrm{EllipticPi}\left(\right.\right.}{\cos(fx+e)\sqrt{a+b\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1
+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))-EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(e+fx)}\csc(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x), x)
```

# 3.206 $\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=213

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}+\frac{aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f+(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*sin(f*x+e))^(1/2)/f/((a+b*sin(f*x+e))/(a+b))^(1/2)-a*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(
a+b*sin(f*x+e))^(1/2)-b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2))*
((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2875, 3139, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}+\frac{a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}+\frac{b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/f) - (EllipticE[(e - Pi/2 + f*x)/
2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[(a + b*Sin[e + f*x])/(a
 + b)]) + (a*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e
 + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (b*EllipticPi[2, (e - Pi/
2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a +
b*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2875

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
 + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \int \frac{\csc(e+fx)\left(\frac{b}{2} - \frac{1}{2}b\sin^2(e\right.}{\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{1}{2}\int \sqrt{a+b\sin(e+fx)}\,dx$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+}}{f\sqrt{\frac{a+b\sin(e+}{a+b}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+}}{f\sqrt{\frac{a+b\sin(e+}{a+b}}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 16.07, size = 312, normalized size = 1.46

$$\frac{\frac{2i\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(e+fx)\sqrt{-\frac{b(-1+\sin(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sin(e+fx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}} - 4\cot(e+fx)\sqrt{a+b\sin(e+fx)} - \frac{2b\Pi\left(2;\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{\sqrt{a+b\sin(e+fx)}}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] - (2*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*f)
```

**Maple [A]**
time = 3.12, size = 456, normalized size = 2.14

| method | result |
|---|---|
| default | $-\frac{a b^2 \sin(fx+e)(\cos^2(fx+e))+\sqrt{\frac{b\sin(fx+e)}{a-b}+\frac{a}{a-b}}\,\sqrt{-\frac{b\sin(fx+e)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(fx+e)}{a+b}+\frac{b}{a+b}}\,\left(\text{EllipticF}\left(\sqrt{\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*b^2*sin(f*x+e)*cos(f*x+e)^2+(b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2)*(-b/(a
-b)*sin(f*x+e)-b/(a-b))^(1/2)*(-b/(a+b)*sin(f*x+e)+b/(a+b))^(1/2)*(Elliptic
F((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b-EllipticF
((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+EllipticPi
((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^2-El
lipticPi((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
b^3-EllipticE((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3
+EllipticE((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2)
*sin(f*x+e)+a^2*b*cos(f*x+e)^2)/a/b/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e))^
(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(e+fx)}\,\csc^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin(e+f\,x)}}{\sin(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

# 3.207 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=132

$$\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{bf\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/b
/f/((a+b*sin(f*x+e))/(a+b))^(1/2)+2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
b*f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - (2*a*EllipticF[(e - Pi/2 + f*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(b*f*Sqrt[a + b*Sin[e
+ f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right.}{b\sqrt{a+}}$$

$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{2aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a}{}}}{bf\sqrt{a+b\sin(e+}}$$

**Mathematica [A]**
time = 2.27, size = 94, normalized size = 0.71

$$-\frac{2\left((a+b)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*((a + b)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] - a*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/
(b*f*Sqrt[a + b*Sin[e + f*x]])
```

**Maple [A]**
time = 2.99, size = 202, normalized size = 1.53

| method | result |
|---|---|
| default | $-\frac{2(a-b)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\left(\mathrm{EllipticE}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a+\mathrm{Ellipt}\right.}{b^2\cos(fx+e)\sqrt{a+b\sin(fx+e)}\ f}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*b-EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/b^
2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 373, normalized size = 2.83

$$-\frac{2\sqrt{2}\,a\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(fx+e)-3ib\sin(fx+e)-2ia}{3b}\right)+2\sqrt{2}\,a\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(fx+e)+3ib\sin(fx+e)+2ia}{3b}\right)+3i\sqrt{2}\sqrt{ib}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(fx+e)-3ib\sin(fx+e)-2ia}{3b}\right)\right)-3i\sqrt{2}\sqrt{-ib}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(fx+e)+3ib\sin(fx+e)+2ia}{3b}\right)\right)}{3b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*a*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*b*sin(f*x + e)
```

```
- 2*I*a)/b) + 2*sqrt(2)*a*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) + 3*I*b*sin
(f*x + e) + 2*I*a)/b) + 3*I*sqrt(2)*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) -
 3*I*b*sin(f*x + e) - 2*I*a)/b)) - 3*I*sqrt(2)*sqrt(-I*b)*b*weierstrassZeta
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPIn
verse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*
cos(f*x + e) + 3*I*b*sin(f*x + e) + 2*I*a)/b)))/(b^2*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [B]
time = 7.21, size = 118, normalized size = 0.89

$$\frac{\left(2\,a\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|\frac{2\,b}{a+b}\right)-2\,(a+b)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\cos(e+f\,x)^2}\,\sqrt{\frac{a+b\,\sin(e+f\,x)}{a+b}}}{b\,f\,\cos(e+f\,x)\,\sqrt{a+b\,\sin(e+f\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x))^(1/2),x)
[Out] ((2*a*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*b)/(a + b))
- 2*(a + b)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*b)/(a
+ b)))*(cos(e + f*x)^2)^(1/2)*((a + b*sin(e + f*x))/(a + b))^(1/2))/(b*f*co
s(e + f*x)*(a + b*sin(e + f*x))^(1/2))
```

# 3.208 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=62

$$\frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))
^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2742, 2740}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}$$
$$= \frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\mid\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

**Mathematica** [**A**]
time = 0.04, size = 61, normalized size = 0.98

$$-\frac{2F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

**Maple** [**A**]
time = 1.98, size = 126, normalized size = 2.03

| method | result | size |
|---|---|---|
| default | $\frac{2(a-b)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)}{b\cos(fx+e)\sqrt{a+b\sin\left(fx+e\right)}\,f}$ | 126 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1
+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))/b/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 152, normalized size = 2.45

$$\frac{\sqrt{2}\sqrt{i b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)-3i\,b\sin(fx+e)-2i\,a}{3\,b}\right)+\sqrt{2}\sqrt{-i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(fx+e)+3i\,b\sin(fx+e)+2i\,a}{3\,b}\right)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I
*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) - 3*I*b*sin(f*x + e) - 2*I*a)/
b) + sqrt(2)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(f*x + e) + 3*I*b*sin(f*x + e) + 2
*I*a)/b))/(b*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sin{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [B]
time = 6.91, size = 55, normalized size = 0.89

$$-\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\sin\left(e+f\,x\right)}{a+b}}}{f\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] -(2*ellipticF(pi/4 - e/2 - (f*x)/2, (2*b)/(a + b))*((a + b*sin(e + f*x))/(a
 + b))^(1/2))/(f*(a + b*sin(e + f*x))^(1/2))
```

# 3.209 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=63

$$\frac{2\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+
b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2886, 2884}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}$$
$$= \frac{2\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

**Mathematica [A]**
time = 0.06, size = 62, normalized size = 0.98

$$-\frac{2\Pi\left(2;\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e +
 f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

**Maple [A]**
time = 2.22, size = 135, normalized size = 2.14

| method | result | si |
|---|---|---|
| default | $-\frac{2(a-b)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\frac{a-b}{a},\sqrt{\frac{a-b}{a+b}}\right)}{a\cos(fx+e)\sqrt{a+b\sin\left(fx+e\right)}\,f}$ | 13 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-b
)/a,((a-b)/(a+b))^(1/2))/a/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maple [F]** 

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+2)),failed)
cannot
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (e + fx)}{\sqrt{a + b \sin (e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin (e + f\,x)\,\sqrt{a + b\,\sin (e + f\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)), x)
```

# 3.210 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=222

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}+\frac{F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{}}{f\sqrt{a+b\sin(e}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e))^(1/2)/a/f+(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/
sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/a/f/((a+b*sin(f*x+e))/(a+b))^(1/2)-(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f
/(a+b*sin(f*x+e))^(1/2)+b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(f*x+e))/(a+b))^(1/2)/a/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.35, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2881, 3139, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{af\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(a*f)) - (EllipticE[(e - Pi/2 + f
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(a*f*Sqrt[(a + b*Sin[e + f*
x])/(a + b)]) + (EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) - (b*EllipticPi[2, (e -
 Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(a*f*Sqr
t[a + b*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\int \frac{\csc(e+fx)\left(-\frac{b}{2}-\frac{1}{2}b\sin^2(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}-\frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{2a}-\frac{\int \frac{\csc(e+fx)\left(\frac{b^2}{2}\cdots\right.}{\sqrt{a+b\sin\cdots}}}{\cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx-\frac{b\int \frac{\cdots}{\sqrt{a+\cdots}}}{\cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+f\cdots}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+f\cdots}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 16.10, size = 315, normalized size = 1.42

$$\frac{\frac{2i\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(e+fx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(e+fx)\sqrt{-\frac{b(-1+\sin(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sin(e+fx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}}-4\cot(e+fx)\sqrt{a+b\sin(e+fx)}+\frac{6b\Pi\left(2;\frac{1}{4}(-2e+\pi-2fx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{\sqrt{a+b\sin(e+fx)}}}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] + (6*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*a*f)
```

**Maple** [A]
time = 6.77, size = 412, normalized size = 1.86

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-b\sin\left(fx+e\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-b\sin\left(fx+e\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}{a\sin(fx+e)}-\frac{b\left(\frac{a}{b}-1\right)\cdots}{\cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*(-1/a*(-(-b*sin(f*x+e)-a)*cos(f*x+e
)^2)^(1/2)/sin(f*x+e)-1/a*b*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-si
n(f*x+e))/(a+b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*
cos(f*x+e)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2)))+1/a^2*b^2*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-sin(f*x+e))/(a+
b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)
^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b)
)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)^2\,\sqrt{a+b\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x))^(1/2)), x)
```

# 3.211 $\int \sqrt{\sin(c+dx)}\,\sqrt{a+b\sin(c+dx)}\,dx$

**Optimal**. Leaf size=371

$$-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{(a-b)\sqrt{a+b}\,\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\,E\Big(\sin^{-1}\Big(}{ad}$$

```
[Out] -cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)+(a-b)*EllipticE((a+b*
sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a-b))^(1/2)*tan(d*x
+c)/a/d-EllipticF((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c
))/(a-b))^(1/2)*tan(d*x+c)/d+a*EllipticPi((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2
)/sin(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+
c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a-b))^(1/2)*tan(d*x+c)/b/d
```

**Rubi** [A]

time = 0.39, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2900, 3133, 2888, 12, 2880, 2895, 3073}

$$-\frac{\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{a\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\text{ArcSin}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])) + ((a - b
)*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x
]))/(a - b)]*EllipticE[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Si
n[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d) - (Sqrt[a + b]*Sqrt[
(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*Ellipt
icF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a
 + b)/(a - b))]*Tan[c + d*x])/d + (a*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a + b)/(a
- b))]*Tan[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
```

```
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3133

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\sin(c+dx)}\,\sqrt{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\sin^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)\sqrt{a+b\sin(c+}}}{b}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+}{a+b}}}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+}{a+b}}}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(a-b)\sqrt{a+b}\sqrt{\frac{a(1-\csc}{a}}}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 27.11, size = 10847, normalized size = 29.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] Result too large to show
```

**Maple** [C] Result contains complex when optimal does not.
time = 14.15, size = 9513, normalized size = 25.64

| method | result | size |
|---|---|---|

| default | Expression too large to display | 9513 |
|---|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin(c + dx)}\,\sqrt{\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/2)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x)), x)
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARc near
 0Simplification assuming sageVARc near 0ext_reduce Error: Bad Argument Typ
eSimpl
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sin\left(c+d\,x\right)}\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2), x)
```

# 3.212 $\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx$

**Optimal**. Leaf size=109

$$-\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\tan(c+}{ad}$$

```
[Out] -2*EllipticF((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a
-b))^(1/2)*tan(d*x+c)/a/d
```

**Rubi** [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2895}

$$-\frac{2\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d
*x]))/(a - b)]*EllipticF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[
Sin[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\,F\left(\sin^{-1}\right.}{ad}$$

**Mathematica [A]**
time = 4.21, size = 172, normalized size = 1.58

$$\frac{8a\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2c-\pi+2dx)\right)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(c+dx)}{a(-1+\sin(c+dx))}}\right)|\frac{2a}{a-b}\right)\sec(c+dx)\sqrt{-\frac{(a+b)\sin(c+dx)(a+b\sin(c+dx))}{a^2(-1+\sin(c+dx))^2}}\,\sin^4\left(\frac{1}{4}(2c-\pi+2dx)\right)}{(a+b)d\sqrt{\sin(c+dx)}\,\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (8*a*Sqrt[-(((a + b)*Cot[(2*c - Pi + 2*d*x)/4]^2)/(a - b))]*EllipticF[ArcSi
n[Sqrt[-((a + b*Sin[c + d*x])/(a*(-1 + Sin[c + d*x])))]], (2*a)/(a - b)]*Se
c[c + d*x]*Sqrt[-(((a + b)*Sin[c + d*x]*(a + b*Sin[c + d*x]))/(a^2*(-1 + Si
n[c + d*x])^2))]*Sin[(2*c - Pi + 2*d*x)/4]^4)/((a + b)*d*Sqrt[Sin[c + d*x]]
*Sqrt[a + b*Sin[c + d*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(101) = 202$.
time = 13.29, size = 310, normalized size = 2.84

| method | result |
|---|---|
| default | $-\frac{\sqrt{\frac{\sqrt{-a^2+b^2}\,\sin(dx+c)-a\cos(dx+c)+b\sin(dx+c)+a}{\left(b+\sqrt{-a^2+b^2}\right)\sin(dx+c)}}\;\sqrt{\frac{\sqrt{-a^2+b^2}\,\sin(dx+c)+a\cos(dx+c)-b\sin(dx+c)-a}{\sqrt{-a^2+b^2}\,\sin(dx+c)}}\;\sqrt{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/(a+b*sin(d*x+c))^(1/2)*(((-a^2+b^2)^(1/2)*sin(d*x+c)-a*cos(d*x+c)+b*si
n(d*x+c)+a)/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((-a^2+b^2)^(1/2)*sin(d
*x+c)+a*cos(d*x+c)-b*sin(d*x+c)-a)/(-a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-
1+cos(d*x+c))/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticF((((-a^2+b^2)
^(1/2)*sin(d*x+c)-a*cos(d*x+c)+b*sin(d*x+c)+a)/(b+(-a^2+b^2)^(1/2))/sin(d*x
+c))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*sin(d
*x+c)^(3/2)*2^(1/2)/(-1+cos(d*x+c))*(b+(-a^2+b^2)^(1/2))/a
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))/(b*cos(d*x + c)^2 - a
*sin(d*x + c) - b), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}\,\sqrt{\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)**(1/2)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x))), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin\left(c+d\,x\right)}\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2)), x)
```

# 3.213 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=270

$$-\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)}+\frac{a(3b^2(1+m)+a^2(2+m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin\right.}{df(1+m)(2+m)\sqrt{\cos^2(e+f}}$$

```
[Out] -a*b^2*(7+2*m)*cos(f*x+e)*(d*sin(f*x+e))^(1+m)/d/f/(2+m)/(3+m)-b^2*cos(f*x+
e)*(d*sin(f*x+e))^(1+m)*(a+b*sin(f*x+e))/d/f/(3+m)+a*(3*b^2*(1+m)+a^2*(2+m)
)*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*
x+e))^(1+m)/d/f/(1+m)/(2+m)/(cos(f*x+e)^2)^(1/2)+b*(b^2*(2+m)+3*a^2*(3+m))*
cos(f*x+e)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^
(2+m)/d^2/f/(2+m)/(3+m)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.27, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2872, 3102, 2827, 2722}

$$\frac{b(3a^2(m+3)+b^2(m+2))\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)(m+3)\sqrt{\cos^2(e+fx)}}+\frac{a(a^2(m+2)+3b^2(m+1))\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}-\frac{ab^2(2m+7)\cos(e+fx)(d\sin(e+fx))^{m+1}}{df(m+2)(m+3)}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))(d\sin(e+fx))^{m+1}}{df(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*m)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m)*(3 +
 m))) + (a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 +
 m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[
e + f*x])^(2 + m))/(d^2*f*(2 + m)*(3 + m)*Sqrt[Cos[e + f*x]^2]) - (b^2*Cos[
e + f*x]*(d*Sin[e + f*x])^(1 + m)*(a + b*Sin[e + f*x]))/(d*f*(3 + m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}(a+b\sin(e+fx))}{df(3+m)} + \frac{\int (d\sin}{}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(}{}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(}{}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} + \frac{a\left(a^2+\frac{3b^2(1+m)}{2+m}\right.}{}$$

**Mathematica [A]**
time = 0.52, size = 199, normalized size = 0.74

$$\frac{\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^m\left(-\frac{ab^2(7+2m)}{2+m}+\frac{a(3+m)(3b^2(1+m)+a^2(2+m))\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)}{(1+m)(2+m)\sqrt{\cos^2(e+fx)}}+\frac{b(b^2(2+m)+3a^2(3+m))\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(e+fx)\right)\sin(e+fx)}{(2+m)\sqrt{\cos^2(e+fx)}}-b^2(a+b\sin(e+fx))\right)}{f(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(-((a*b^2*(7 + 2*m))/(2 + m))
 + (a*(3 + m)*(3*b^2*(1 + m) + a^2*(2 + m))*Hypergeometric2F1[1/2, (1 + m)/
```

```
2, (3 + m)/2, Sin[e + f*x]^2])/((1 + m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*
(b^2*(2 + m) + 3*a^2*(3 + m))*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Sin[e + f*x]^2]*Sin[e + f*x])/((2 + m)*Sqrt[Cos[e + f*x]^2]) - b^2*(a + b*S
in[e + f*x])))/(f*(3 + m))
```

**Maple** [**F**]
time = 1.01, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)\right)^{m} \left(a+b \sin \left(f x+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*sin(f*x + e))^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,\sin\left(e+f\,x\right))^m\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^3, x)
```

# 3.214 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=194

$$-\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)}+\frac{(b^2(1+m)+a^2(2+m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)(d}{df(1+m)(2+m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -b^2*cos(f*x+e)*(d*sin(f*x+e))^(1+m)/d/f/(2+m)+(b^2*(1+m)+a^2*(2+m))*cos(f*
x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(1
+m)/d/f/(1+m)/(2+m)/(cos(f*x+e)^2)^(1/2)+2*a*b*cos(f*x+e)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+m)/d^2/f/(2+m)/(cos(f*x+e
)^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2868, 2722, 3093}

$$\frac{(a^2(m+2)+b^2(m+1))\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}+\frac{2ab\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}-\frac{b^2\cos(e+fx)(d\sin(e+fx))^{m+1}}{df(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m))) + ((b^2*(1 + m
) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*(2 + m)*Sqrt[Cos[e +
 f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x
]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx = \frac{(2ab)\int (d\sin(e+fx))^{1+m}\,dx}{d} + \int (d\sin(e+fx))^m\left(a^2+b^2\sin^2\right.$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{2ab\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+}{2}\right.}{d^2f(2}$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{\left(a^2+\frac{b^2(1+m)}{2+m}\right)\cos(e+f}{}$$

**Mathematica [A]**
time = 0.22, size = 144, normalized size = 0.74

$$-\frac{\cos(e+fx)(d\sin(e+fx))^m\sin^2(e+fx)^{\frac{1}{2}(-1-m)}\left(b^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{3}{2};\cos^2(e+fx)\right)\sin(e+fx)+a\left(a\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\cos^2(e+fx)\right)\sin(e+fx)+2b\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};\frac{3}{2};\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(d*Sin[e + f*x])^m*(Sin[e + f*x]^2)^((-1 - m)/2)*(b^2*Hyper
geometric2F1[1/2, (-1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + a*(a*Hype
rgeometric2F1[1/2, (1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + 2*b*Hyper
geometric2F1[1/2, -1/2*m, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/f)
```

**Maple [F]**
time = 1.07, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*sin(f*x
+ e))^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin\left(e+f\,x\right))^m\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^2, x)
```

# 3.215 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=139

$$\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+m}}{df(1+m)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(e+fx\right.}{d^2f(2+m)\sqrt{\cos^2(e+f}}$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*
x+e))^(1+m)/d/f/(1+m)/(cos(f*x+e)^2)^(1/2)+b*cos(f*x+e)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+m)/d^2/f/(2+m)/(cos(f*x+e)^
2)^(1/2)
```

**Rubi** [A]

time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2827, 2722}

$$\frac{a\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2
]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*Sqrt[Cos[e + f*x]^2]) + (b*Cos[e +
 f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[e
 + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx &= a\int (d\sin(e+fx))^m\,dx+\frac{b\int (d\sin(e+fx))^{1+m}\,dx}{d}\\ &= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+m}}{df(1+m)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.11, size = 111, normalized size = 0.80

$$\frac{\sqrt{\cos^2(e+fx)}\,(d\sin(e+fx))^m\left(a(2+m)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)+b(1+m)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(e+fx)\right)\sin(e+fx)\right)\tan(e+fx)}{f(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
[Out] (Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a*(2 + m)*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[e + f*x]^2] + b*(1 + m)*Hypergeometric2F1[1/2, (2
 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e + f*x])/(f*(1 + m)*
(2 + m))
```

**Maple [F]**
time = 0.14, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m\,(a+b\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^m\,(a+b\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*sin(e + f*x))**m*(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin\left(e+f\,x\right))^m\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)), x)
```

# 3.216 $\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=195

$$-\frac{adF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1-m}{2}}}{(a^2-b^2)\,f}+\frac{bF_1\left(\frac{1}{2};\right.}{}$$

```
[Out] -a*d*AppellF1(1/2,1/2-1/2*m,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b^2)/f+b*
AppellF1(1/2,-1/2*m,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x
+e)*(d*sin(f*x+e))^m/(a^2-b^2)/f/((sin(f*x+e)^2)^(1/2*m))
```

**Rubi** [A]
time = 0.17, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2902, 3268, 440}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^m F_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{ad\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*d*AppellF1[1/2, (1 - m)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]
^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^(
(1 - m)/2))/((a^2 - b^2)*f)) + (b*AppellF1[1/2, -1/2*m, 1, 3/2, Cos[e + f*x
]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/
((a^2 - b^2)*f*(Sin[e + f*x]^2)^(m/2))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
```

```
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx = a\int \frac{(d\sin(e+fx))^m}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{b\int \frac{(d\sin(e+fx))^{1+m}}{a^2-b^2\sin^2(e+fx)}\,dx}{d}$$

$$= -\frac{\left(ad(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^{\frac{1}{2}(-1+m)}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e}{f}$$

$$= -\frac{adF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\text{s}}{(a^2-b^2)\,f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1590 vs. $2(195) = 390$.
time = 15.61, size = 1590, normalized size = 8.15

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[
Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan
[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*Appe
llF1[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Ta
n[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])
*(((Sec[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2
 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 + m)/2
, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hyper
geometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*(1 + m)*(2 + m)) + (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan
[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1,
(3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((
a^2 - b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (
-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2,
```

```
 (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(1 + m)*(2 + m)) + (m*(
Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1
+ m)*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2
, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
 - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])
*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])
)/(a^2*b*(1 + m)*(2 + m)) + ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f
*x]/Sqrt[Sec[e + f*x]^2])^m*((1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 +
 m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*
Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Sec[e
+ f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m/2, 1, 1
+ (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*
x]^2*Tan[e + f*x])/(3 + m)) + (2*(-a^2 + b^2)*(1 + m)*AppellF1[1 + (1 + m)/
2, m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + m))) + (1 + m)*Tan[e + f*x]*((a^2
 - b^2)*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 1, 1 +
 (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(4 + m)) + (2*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 + (2 + m)/2,
(-1 + m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)) - a^2*(2 + m)*Csc[e + f*x]*Sec[e +
 f*x]*(-Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]
 + (1 + Tan[e + f*x]^2)^((-1 - m)/2)))))/(a^2*b*(1 + m)*(2 + m))))
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^m}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\sin\left(e+f\,x\right))^m}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x)), x)
```

# 3.217 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=306

$$-\frac{b^2 F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1+m}\sin^2(e+fx)^{\frac{1}{2}(-1-m)}}{\left(a^2-b^2\right)^2 df}$$

```
[Out] -b^2*AppellF1(1/2,-1/2-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2)
)*cos(f*x+e)*(d*sin(f*x+e))^(1+m)*(sin(f*x+e)^2)^(-1/2-1/2*m)/(a^2-b^2)^2/d
/f-a^2*d*AppellF1(1/2,1/2-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b
^2))*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b^2)^
2/f+2*a*b*AppellF1(1/2,-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^2/f/((sin(f*x+e)^2)^(1/2*m))
```

**Rubi** [A]
time = 0.29, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2903, 3268, 440, 16}

$$-\frac{b^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^2}-\frac{a^2d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^mF_1\left(\frac{1}{2};-\frac{m}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2)^(
(-1 - m)/2))/((a^2 - b^2)^2*d*f)) - (a^2*d*AppellF1[1/2, (1 - m)/2, 2, 3/2,
 Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e
 + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^2*f) + (2*a*b*
AppellF1[1/2, -1/2*m, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^2*f*(Sin[e + f*x]^2)^
(m/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx = \int \left(\frac{a^2(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} - \frac{2ab\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} + \frac{b^2\sin^2(e+f}{(-a^2+b^2}\right.$$

$$= a^2\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx - (2ab)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx + b^2$$

$$= \frac{b^2\int \frac{(d\sin(e+fx))^{2+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{(2ab)\int \frac{(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx}{d} - \frac{\left(a^2d(d\sin(e+fx))^2\right.}{}$$

$$= -\frac{a^2dF_1\left(\frac{1}{2};\frac{1-m}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+}}{\left(a^2-b^2\right)^2f}$$

$$= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+f}{\left(a^2-b^2\right)^2df}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1790 vs. $2(306) = 612$.
time = 16.48, size = 1790, normalized size = 5.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (
3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 +
m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] + (a^2 - b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^
3*(a^2 - b^2)*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])^2*(-(((Sec[e + f*x]^2)
^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*A
ppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(1 + m)*AppellF1[(
2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m))) - (m*(Sec[e + f
*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2
 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2,
 (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(
1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 +
 b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m))
 - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2
])^(-1 + m)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-
(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2,
m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 -
 b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(
2 + m)) - ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1
+ m/2, 1, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (2*(-1 + b^2/a^2)*(1 + m)*AppellF1[1
+ (1 + m)/2, m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m))) + 2*b*((a^2 - b^2)*(1 + m)*
AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^
2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 +
(1 + m)/2, 1 + m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (4*(-1 + b^2/a^2)*(1 + m
)*AppellF1[1 + (1 + m)/2, m/2, 3, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2
/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (a^2 - b^2)*(
1 + m)*Tan[e + f*x]*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 +
m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(4 + m)) + (4*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 +
 (2 + m)/2, (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)))))/(a^3*(a^2 - b^2)*(1
 + m)*(2 + m)))))

**Maple** [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (fx+e))^{m}}{(a+b \sin (fx+e))^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(d*sin(f*x + e))^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d \sin(e+f x))^m}{(a+b \sin(e+f x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^2, x)
```

# 3.218 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=406

$$-\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1+m}\sin^2(e+fx)^{\frac{1}{2}(-1-m)}}{(a^2-b^2)^3\,df}$$

```
[Out] -3*a*b^2*AppellF1(1/2,-1/2-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*sin(f*x+e))^(1+m)*(sin(f*x+e)^2)^(-1/2-1/2*m)/(a^2-b^2)
^3/d/f-a^3*d*AppellF1(1/2,1/2-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b
^2)^3/f+b^3*AppellF1(1/2,-1-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2
-b^2))*cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*m))+3
*a^2*b*AppellF1(1/2,-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*m))
```

**Rubi** [A]
time = 0.39, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2903, 3268, 440, 16}

$$-\frac{3ab^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^3}+\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^mF_1\left(\frac{1}{2};-\frac{m}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}+\frac{b^3\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^mF_1\left(\frac{1}{2};\frac{1}{2}(-m-2),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}-\frac{a^3d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2
)^((-1 - m)/2))/((a^2 - b^2)^3*d*f) - (a^3*d*AppellF1[1/2, (1 - m)/2, 3, 3/
2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin
[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^3*f) + (b^3*
AppellF1[1/2, (-2 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a
^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3*f*(Sin[e + f*x]
^2)^(m/2)) + (3*a^2*b*AppellF1[1/2, -1/2*m, 3, 3/2, Cos[e + f*x]^2, -((b^2*
Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)
^3*f*(Sin[e + f*x]^2)^(m/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx = \int \left(\frac{a^3(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} - \frac{3a^2b\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} + \frac{3ab^2\sin^2(e\right.$$

$$= a^3\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2b\right)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx +$$

$$= \frac{b^3\int \frac{(d\sin(e+fx))^{3+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2\right)\int \frac{(d\sin(e+fx))^{2+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d^2} - \frac{\left(3a^2b\right)\int \frac{(d\sin(e+fx}{(a^2-b^2\sin^2(e}}{d}$$

$$= -\frac{a^3dF_1\left(\frac{1}{2};\frac{1-m}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-}}{\left(a^2-b^2\right)^3f}$$

$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e}{\left(a^2-b^2\right)^3df}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2298 vs. 2(406) = 812.
time = 15.69, size = 2298, normalized size = 5.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m
)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*
a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2,
 (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
- 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*f*(1 + m)*(2 +
 m)*(a + b*Sin[e + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m
)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*
a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2,
 (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
- 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m
))) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 2, (3
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(2 + m)
*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2, (-1 + m)/2
, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*App
ellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*
Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - (m*(Se
c[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 +
m)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-(a*(a^2 +
 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 2, (3 + m)/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(2 + m)*AppellF1[(1 + m)/2,
(-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] +
 (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(2 + m)/2, (-1
+ m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[
e + f*x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - ((Sec[e + f*x]^2)^(m/2)*Tan
[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*
(-(((-2 + m)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 2, 1 + (3 + m)
/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e +
f*x])/(3 + m)) + (4*(-1 + b^2/a^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m)
/2, 3, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e
 + f*x]^2*Tan[e + f*x])/(3 + m))) + b*((1 + m)*((3*a^2 + b^2)*AppellF1[(2 +
 m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Sec[e + f*x]^2 + 4*a*b*(2 + m)*(-(((-2 +
m)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 3, 1 + (3 + m)/2, -Tan[e
```

```
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 +
 m)) + (6*(-1 + b^2/a^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m)/2, 4, 1 +
 (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(3 + m)) + (1 + m)*Tan[e + f*x]*((3*a^2 + b^2)*(-(((-1 + m)*(
2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 2, 1 + (4 + m)/2, -Tan[e + f
*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m))
 + (4*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 + (2 + m)/2, (-1 + m)/2, 3, 1 + (4
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(4 + m)) - 4*b^2*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 +
 (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)) + (6*(-1 + b^2/a^2)*(2 + m)*Appel
lF1[1 + (2 + m)/2, (-1 + m)/2, 4, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2
/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m))))))/(a^4*(a^2 -
 b^2)*(1 + m)*(2 + m)))))
```

**Maple [F]**
time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (fx+e))^m}{(a+b \sin (fx+e))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(d*sin(f*x + e))^m/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\sin(e+f\,x))^m}{(a+b\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^3, x)
```

# 3.219 $\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx$

**Optimal**. Leaf size=142

$$-\frac{(a^2+b^2)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)}{d}+\frac{2a(a^2+b^2)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2(a^2+b^2)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)\ldots}{bd\sqrt{\cos^2(c+dx)}}$$

```
[Out] -(a^2+b^2)*cos(d*x+c)/d/(sin(d*x+c)^(a^2/(a^2+b^2)))+2*a*(a^2+b^2)*cos(d*x+
c)*hypergeom([1/2, 1/2*b^2/(a^2+b^2)],[3/2-1/2*a^2/(a^2+b^2)],sin(d*x+c)^2)
*sin(d*x+c)^(b^2/(a^2+b^2))/b/d/(cos(d*x+c)^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2868, 2722, 3090}

$$\frac{2a(a^2+b^2)\cos(c+dx)\sin^{\frac{b^2}{a^2+b^2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2(a^2+b^2)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)}{bd\sqrt{\cos^2(c+dx)}}-\frac{(a^2+b^2)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cos[c + d*x])/(d*Sin[c + d*x]^(a^2/(a^2 + b^2)))) + (2*a*(a^
2 + b^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, b^2/(2*(a^2 + b^2)), (3 - a^2/
(a^2 + b^2))/2, Sin[c + d*x]^2]*Sin[c + d*x]^(b^2/(a^2 + b^2)))/(b*d*Sqrt[C
os[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\,dx + \int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)\left(a^2+\right.$$

$$= -\frac{(a^2+b^2)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)}{d} + \frac{2a(a^2+b^2)\cos(c}{}$$

**Mathematica** [A]
time = 0.21, size = 188, normalized size = 1.32

$$-\frac{\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\sin^2(c+dx)^{-\frac{b^2}{2(a^2+b^2)}}\left(2ab\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{a^2}{a^2+b^2}\right);\frac{3}{2};\cos^2(c+dx)\right)\sin(c+dx)+\left(b^2\,{}_2F_1\left(\frac{1}{2},\frac{a^2}{2(a^2+b^2)};\frac{3}{2};\cos^2(c+dx)\right)+a^2\,{}_2F_1\left(\frac{1}{2},1+\frac{a^2}{2(a^2+b^2)};\frac{3}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(2*a*b*Hypergeometric2F1[1/2, (1 + a^2/(a^2 + b^2))/2, 3/2,
 Cos[c + d*x]^2]*Sin[c + d*x] + (b^2*Hypergeometric2F1[1/2, a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2] + a^2*Hypergeometric2F1[1/2, 1 + a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]))/(d*Sin[c + d*x]^(a^2/(a^
2 + b^2))*(Sin[c + d*x]^2)^(b^2/(2*(a^2 + b^2)))))
```

**Maple** [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \left(\sin^{-1-\frac{a^2}{a^2+b^2}}(dx+c)\right)(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sin(d*x + c
)^(-(2*a^2 + b^2)/(a^2 + b^2)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2 \sin^{-\frac{a^2}{a^2+b^2}-1}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-1-a**2/(a**2+b**2))*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)**(-a**2/(a**2 + b**2) - 1), x
)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^2}{\sin(c+d\,x)^{\frac{a^2}{a^2+b^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/sin(c + d*x)^(a^2/(a^2 + b^2) + 1),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/sin(c + d*x)^(a^2/(a^2 + b^2) + 1), x)
```

# 3.220 $\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx$

**Optimal**. Leaf size=73

$$-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}+\frac{5\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)\sin^{\frac{4}{5}}(c+dx)}{d\sqrt{\cos^2(c+dx)}}$$

```
[Out] -5*cos(d*x+c)/d/sin(d*x+c)^(1/5)+5*cos(d*x+c)*hypergeom([2/5, 1/2],[7/5],si
n(d*x+c)^2)*sin(d*x+c)^(4/5)/d/(cos(d*x+c)^2)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2868, 2722, 3090}

$$\frac{5\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)}{d\sqrt{\cos^2(c+dx)}}-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) + (5*Cos[c + d*x]*Hypergeometric2F
1[2/5, 1/2, 7/5, Sin[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*Sqrt[Cos[c + d*x]^2
])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
```

]

Rubi steps

$$\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx = 4\int \frac{1}{\sqrt[5]{\sin(c+dx)}}\,dx + \int \frac{1+4\sin^2(c+dx)}{\sin^{\frac{6}{5}}(c+dx)}\,dx$$
$$= -\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}} + \frac{5\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)\sin^{\frac{4}{5}}(c+dx)}{d\sqrt{\cos^2(c+dx)}}$$

**Mathematica** [A]
time = 0.07, size = 73, normalized size = 1.00

$$-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}} - \frac{4\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{5};\frac{3}{2};\cos^2(c+dx)\right)\sin^{\frac{4}{5}}(c+dx)}{d\sin^2(c+dx)^{2/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) - (4*Cos[c + d*x]*Hypergeometric2F
1[1/2, 3/5, 3/2, Cos[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*(Sin[c + d*x]^2)^(2
/5))
```

**Maple** [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(1+2\sin(dx+c))^2}{\sin(dx+c)^{\frac{6}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

```
[Out] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="maxima")
```

[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="fricas")

[Out] integral((4*cos(d*x + c)^2 - 4*sin(d*x + c) - 5)*sin(d*x + c)^(4/5)/(cos(d*x + c)^2 - 1), x)

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*sin(d*x+c))**2/sin(d*x+c)**(6/5),x)

[Out] Timed out

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="giac")

[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)

**Mupad** [B]
time = 7.47, size = 127, normalized size = 1.74

$$-\frac{4\,\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{4/5}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{5};\ \frac{3}{2};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\left({\sin\left(c+d\,x\right)}^2\right)}^{2/5}}-\frac{\cos\left(c+d\,x\right)\,{\left({\sin\left(c+d\,x\right)}^2\right)}^{1/10}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{10};\ \frac{3}{2};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\sin\left(c+d\,x\right)}^{1/5}}-\frac{4\,\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{9/5}\,{}_2\mathrm{F}_1\left(\frac{1}{10},\frac{1}{2};\ \frac{3}{2};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\left({\sin\left(c+d\,x\right)}^2\right)}^{9/10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*sin(c + d*x) + 1)^2/sin(c + d*x)^(6/5),x)

[Out] - (4*cos(c + d*x)*sin(c + d*x)^(4/5)*hypergeom([1/2, 3/5], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(2/5)) - (cos(c + d*x)*(sin(c + d*x)^2)^(1/10)*hypergeom([1/2, 11/10], 3/2, cos(c + d*x)^2))/(d*sin(c + d*x)^(1/5)) - (4*cos(c + d*x)*sin(c + d*x)^(9/5)*hypergeom([1/10, 1/2], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(9/10))

# 3.221 $\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=24

$$\text{Int}(\sin^m(c+dx)(a+b\sin(c+dx))^n,x)$$

```
[Out] Unintegrable(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx=\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

**Mathematica** [A]
time = 1.53, size = 0, normalized size = 0.00

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

**Maple** [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sin^m\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(c + dx)})^n \sin^m{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n*sin(c + d*x)**m, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin\left(c+d\,x\right)^m\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^m*(a + b*sin(c + d*x))^n, x)
```

# 3.222 $\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=351

$$\frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)}-\frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(3+n)}-\frac{\sqrt{2}\,(a+b)\,(2a^2+b^2(2$$

```
[Out] 2*a*cos(d*x+c)*(a+b*sin(d*x+c))^(1+n)/b^2/d/(2+n)/(3+n)-cos(d*x+c)*sin(d*x+
c)*(a+b*sin(d*x+c))^(1+n)/b/d/(3+n)-(a+b)*(2*a^2+b^2*(2+n)^2)*AppellF1(1/2,
-1-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin
(d*x+c))^n*2^(1/2)/b^3/d/(2+n)/(3+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*
x+c))^(1/2)+a*(2*a^2+b^2*(n^2+5*n+4))*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+
c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b^3/d/(
2+n)/(3+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2872, 3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\,a(2a^2+b^2(n^2+5n+4))\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}\,(a+b)\,(2a^2+b^2(n+2)^2)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}+\frac{2a\cos(c+dx)(a+b\sin(c+dx))^{n+1}}{b^2d(n+2)(n+3)}-\frac{\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{n+1}}{bd(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (2*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b^2*d*(2 + n)*(3 + n)) - (
Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(3 + n)) - (Sq
rt[2]*(a + b)*(2*a^2 + b^2*(2 + n)^2)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 -
Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*((a + b*Sin[c + d*
x])/(a + b))^n) + (Sqrt[2]*a*(2*a^2 + b^2*(4 + 5*n + n^2))*AppellF1[1/2, 1/
2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d
*x]*(a + b*Sin[c + d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*(
(a + b*Sin[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(3+n)} + \frac{\int(a+b\sin(c+d\ldots}{}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+\ldots}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+\ldots}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+\ldots}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+\ldots}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+\ldots}{bd(3+n)}$$

**Mathematica** [**F**]
time = 2.66, size = 0, normalized size = 0.00

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n, x]
```

**Maple** [**F**]
time = 0.49, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + b*sin(c + d*x))^n, x)
```

# 3.223 $\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=274

$$-\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)}+\frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c}{b^2d(2+n)\sqrt{1+\sin(c+dx)}}$$

```
[Out] -cos(d*x+c)*(a+b*sin(d*x+c))^(1+n)/b/d/(2+n)+a*(a+b)*AppellF1(1/2,-1-n,1/2,
3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^
n*2^(1/2)/b^2/d/(2+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c))^(1/2)-(a^
2+b^2*(1+n))*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x
+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b^2/d/(2+n)/(((a+b*sin(d*x+c))/(
a+b))^n)/(1+sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2870, 2835, 2744, 144, 143}

$$-\frac{\sqrt{2}\left(a^2+b^2(n+1)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}+\frac{\sqrt{2}\,a(a+b)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}-\frac{\cos(c+dx)(a+b\sin(c+dx))^{n+1}}{bd(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(2 + n))) + (Sqrt[2]*a*(
a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[
1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n) - (Sqrt[2]*(a^2 + b^2*(
1 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d
*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[1 +
 Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2835

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2870

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\int (b(1+n)-a\sin(c+dx))(a}{b(2+n)}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{a\int (a+b\sin(c+dx))^{1+n}\,dx}{b^2(2+n)}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{(a\cos(c+dx))\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{b^2d(2+n)\sqrt{1-\sin(c}}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\left(a(-a-b)\cos(c+dx)(a+\right.}{}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\right.}{}$$

**Mathematica [F]**
time = 3.33, size = 0, normalized size = 0.00

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n, x]
```

**Maple [F]**
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\sin^2(dx+c)\right)(a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^2*(a + b*sin(c + d*x))^n, x)
```

# 3.224 $\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=220

$$-\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+d}{a+b}\right.}{bd\sqrt{1+\sin(c+dx)}}$$

```
[Out] -(a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))
*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b/d/(((a+b*sin(d*x+c))/(a+b))^n)/(1+
sin(d*x+c))^(1/2)+a*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*
sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b/d/(((a+b*sin(d*x+c))/(a
+b))^n)/(1+sin(d*x+c))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2835, 2744, 144, 143}

$$\frac{\sqrt{2}\,a\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}\,(a+b)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b
*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqr
t[1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)) + (Sqrt[2]*a*AppellF
1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*
Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqrt[1 + Sin[c + d*x]]*((a + b*Si
n[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx = \frac{\int (a+b\sin(c+dx))^{1+n}\,dx}{b} - \frac{a\int (a+b\sin(c+dx))^n\,dx}{b}$$

$$= \frac{\cos(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{bd\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}} - \frac{(a\cos(\ldots}{\ldots}$$

$$= -\frac{\left(a\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\text{Subst}\left(\int \ldots\right.}{bd\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+\ldots}}$$

$$= -\frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\text{c}\ldots}{bd\sqrt{1+\sin(c+dx)}\ldots}$$

**Mathematica [A]**
time = 0.33, size = 193, normalized size = 0.88

$$\frac{\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a+b\sin(c+dx))^{1+n}\left(-a(2+n)F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)+(1+n)F_1\left(2+n;\frac{1}{2},\frac{1}{2};3+n;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))\right)}{b^2d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c +
 d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n)*(-(a*(2 + n)*AppellF1[1 + n,
 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c + d*x])/(a + b
)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (a + b*Sin[c + d*x])/(a - b)
, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])))/(b^2*d*(1 + n)*(2 +
n))
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \sin \left(d x+c\right) \left(a +b \sin \left(d x+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin{\left(c + d x \right)}\right)^{n} \sin{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n*sin(c + d*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin \left(c+d\,x\right)\,{\left(a+b\,\sin \left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)*(a + b*sin(c + d*x))^n, x)
```

# 3.225 $\int (a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=104

$$-\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}}{d\sqrt{1+\sin(c+dx)}}$$

```
[Out] -AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+b*sin(d*x+c))^n*2^(1/2)/d/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c)
)^(1/2)
```

**Rubi** [A]

time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2744, 144, 143}

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(d*Sqrt[1 + Sin[c +
d*x]]*((a + b*Sin[c + d*x])/(a + b))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sin(c+dx))^n\,dx = \frac{\cos(c+dx)\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}}$$

$$= \frac{\left(\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\sqrt{1+x}} \right.}{d\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+}{d\sqrt{1+\sin(c+dx)}}$$

**Mathematica [A]**
time = 0.16, size = 120, normalized size = 1.15

$$\frac{F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a+b\sin(c+dx))^{1+n}}{bd(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(1 +
 n))
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^n,x)
```

```
[Out] int((a+b*sin(d*x+c))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**n,x)
[Out] Integral((a + b*sin(c + d*x))**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \sin (c + d x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^n,x)
[Out] int((a + b*sin(c + d*x))^n, x)
```

# 3.226 $\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$

**Optimal**. Leaf size=22

$$\text{Int}(\csc(c+dx)(a+b\sin(c+dx))^n, x)$$

```
[Out] Unintegrable(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx = \int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

**Mathematica** [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

**Maple** [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \csc(dx+c)\,(a+b\sin(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin{(c + dx)})^n \csc{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))**n,x)
[Out] Integral((a + b*sin(c + d*x))**n*csc(c + d*x), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b\,\sin{(c + d\,x)})^n}{\sin{(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b*sin(c + d*x))^n/sin(c + d*x), x)
```

# 3.227 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=116

$$\frac{7}{8}ac^4x+\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{7a\cos^3(e+$$

```
[Out] 7/8*a*c^4*x+7/12*a*c^4*cos(f*x+e)^3/f+7/8*a*c^4*cos(f*x+e)*sin(f*x+e)/f+1/5
*a*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/20*a*cos(f*x+e)^3*(c^4-c^4*sin(f
*x+e))/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{20f}+\frac{7ac^4\sin(e+fx)\cos(e+fx)}{8f}+\frac{7}{8}ac^4x+\frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*x)/8 + (7*a*c^4*Cos[e + f*x]^3)/(12*f) + (7*a*c^4*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^2 - c^2*Sin[e + f*x])^2)/(5*f) + (7
*a*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))(c-c\sin(e+fx))^4\,dx &= (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^3\,dx \\
&= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{1}{5}\left(7ac^2\right)\int \cos^2(e+ \\
&= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{7a\cos^3(e+fx)\left(c^4 -\right.}{20f} \\
&= \frac{7ac^4\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+ \\
&= \frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e}{} \\
&= \frac{7}{8}ac^4x+\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f}
\end{aligned}$$

**Mathematica [A]**
time = 0.37, size = 64, normalized size = 0.55

$$\frac{ac^4(420fx+420\cos(e+fx)+130\cos(3(e+fx))-6\cos(5(e+fx))+120\sin(2(e+fx))-45\sin(4(e+fx)))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
[Out] (a*c^4*(420*f*x + 420*Cos[e + f*x] + 130*Cos[3*(e + f*x)] - 6*Cos[5*(e + f*
x)] + 120*Sin[2*(e + f*x)] - 45*Sin[4*(e + f*x)]))/(480*f)
```

**Maple [A]**
time = 0.32, size = 149, normalized size = 1.28

| method | result |
|---|---|
| risch | $\frac{7a\,c^4x}{8}+\frac{7a\,c^4\cos(fx+e)}{8f}-\frac{a\,c^4\cos(5fx+5e)}{80f}-\frac{3a\,c^4\sin(4fx+4e)}{32f}+\frac{13a\,c^4\cos(3fx+3e)}{48f}+\frac{a\,c^4\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-\frac{a\,c^4\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3a\,c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^4\left(2+\sin^2\right.}{}}{f}$ |
| default | $\frac{-\frac{a\,c^4\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3a\,c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^4\left(2+\sin^2\right.}{}}{f}$ |
| norman | $\frac{6a\,c^4\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{34a\,c^4}{15f}+\frac{7a\,c^4x}{8}+\frac{20a\,c^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{16a\,c^4\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{16a\,c^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{a\,c^4\tan\left(\frac{fx}{2}+\right.}{4f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a*c^4*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*a*c^4*(2+sin(
f*x+e)^2)*cos(f*x+e)+2*a*c^4*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3*a
*c^4*cos(f*x+e)+a*c^4*(f*x+e))
```

**Maxima** [A]
time = 0.30, size = 158, normalized size = 1.36

$$-\frac{32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)ac^4-320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)ac^4+45\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)ac^4-240\left(2fx+2e-\sin\left(2fx+2e\right)\right)ac^4-480\left(fx+e\right)ac^4-1440ac^4\cos\left(fx+e\right)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*c^4 -
 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^4 + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^4 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*a*c^4 - 480*(f*x + e)*a*c^4 - 1440*a*c^4*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.33, size = 82, normalized size = 0.71

$$-\frac{24\,ac^4\cos\left(fx+e\right)^5-160\,ac^4\cos\left(fx+e\right)^3-105\,ac^4fx+15\left(6\,ac^4\cos\left(fx+e\right)^3-7\,ac^4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*c^4*cos(f*x + e)^5 - 160*a*c^4*cos(f*x + e)^3 - 105*a*c^4*f*x
+ 15*(6*a*c^4*cos(f*x + e)^3 - 7*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(109) = 218$.

time = 0.34, size = 314, normalized size = 2.71

$$\begin{cases} -\frac{9ac^4x\sin^4(e+fx)}{8} - \frac{9ac^4x\sin^2(e+fx)\cos^2(e+fx)}{4} + ac^4x\sin^2(e+fx) - \frac{9ac^4x\cos^4(e+fx)}{8} + ac^4x\cos^2(e+fx) + ac^4x - \frac{ac^4\sin^4(e+fx)\cos(e+fx)}{f} + \frac{15ac^4\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{4ac^4\sin^2(e+fx)\cos^3(e+fx)}{3f} - \frac{2ac^4\sin^2(e+fx)\cos(e+fx)}{f} + \frac{9ac^4\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{ac^4\sin(e+fx)\cos(e+fx)}{f} - \frac{8ac^4\cos^5(e+fx)}{15f} - \frac{4ac^4\cos^3(e+fx)}{3f} + \frac{3ac^4\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sin(e)+a)(-c\sin(e)+c)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*a*c**4*x*sin(e + f*x)**4/8 - 9*a*c**4*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + a*c**4*x*sin(e + f*x)**2 - 9*a*c**4*x*cos(e + f*x)**4/8 + a*
c**4*x*cos(e + f*x)**2 + a*c**4*x - a*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 15*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*c**4*sin(e + f*x)**2*co
s(e + f*x)**3/(3*f) - 2*a*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 9*a*c**4*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a*c**4*sin(e + f*x)*cos(e + f*x)/f - 8*a
*c**4*cos(e + f*x)**5/(15*f) - 4*a*c**4*cos(e + f*x)**3/(3*f) + 3*a*c**4*co
s(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**4, True))
```

**Giac** [A]

time = 0.50, size = 100, normalized size = 0.86

$$\frac{7}{8}\,ac^4x - \frac{ac^4\cos(5\,fx+5\,e)}{80\,f} + \frac{13\,ac^4\cos(3\,fx+3\,e)}{48\,f} + \frac{7\,ac^4\cos(fx+e)}{8\,f} - \frac{3\,ac^4\sin(4\,fx+4\,e)}{32\,f} + \frac{ac^4\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/8*a*c^4*x - 1/80*a*c^4*cos(5*f*x + 5*e)/f + 13/48*a*c^4*cos(3*f*x + 3*e)/
f + 7/8*a*c^4*cos(f*x + e)/f - 3/32*a*c^4*sin(4*f*x + 4*e)/f + 1/4*a*c^4*si
n(2*f*x + 2*e)/f
```

**Mupad** [B]

time = 8.86, size = 292, normalized size = 2.52

$$\frac{7\,a\,c^4\,x}{8} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a\,c^4\,(105\,e+105\,f\,x)}{24} - \frac{a\,c^4\,(525\,e+525\,f\,x+640)}{120}\right) + \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\left(\frac{a\,c^4\,(105\,e+105\,f\,x)}{24} - \frac{a\,c^4\,(525\,e+525\,f\,x+720)}{120}\right) + \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a\,c^4\,(105\,e+105\,f\,x)}{12} - \frac{a\,c^4\,(1050\,e+1050\,f\,x+800)}{120}\right) + \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(\frac{a\,c^4\,(105\,e+105\,f\,x)}{12} - \frac{a\,c^4\,(1050\,e+1050\,f\,x+1920)}{120}\right) - \frac{a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4} - \frac{13\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2} + \frac{13\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2} + \frac{a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4} + \frac{a\,c^4\,(105\,e+105\,f\,x)}{120} - \frac{a\,c^4\,(105\,e+105\,f\,x+272)}{120}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (7*a*c^4*x)/8 - (tan(e/2 + (f*x)/2)^2*((a*c^4*(105*e + 105*f*x))/24 - (a*c^
4*(525*e + 525*f*x + 640))/120) + tan(e/2 + (f*x)/2)^8*((a*c^4*(105*e + 105
*f*x))/24 - (a*c^4*(525*e + 525*f*x + 720))/120) + tan(e/2 + (f*x)/2)^4*((a
*c^4*(105*e + 105*f*x))/12 - (a*c^4*(1050*e + 1050*f*x + 800))/120) + tan(e
/2 + (f*x)/2)^6*((a*c^4*(105*e + 105*f*x))/12 - (a*c^4*(1050*e + 1050*f*x +
 1920))/120) - (a*c^4*tan(e/2 + (f*x)/2))/4 - (13*a*c^4*tan(e/2 + (f*x)/2)^
3)/2 + (13*a*c^4*tan(e/2 + (f*x)/2)^7)/2 + (a*c^4*tan(e/2 + (f*x)/2)^9)/4 +
 (a*c^4*(105*e + 105*f*x))/120 - (a*c^4*(105*e + 105*f*x + 272))/120)/(f*(t
an(e/2 + (f*x)/2)^2 + 1)^5)
```

# 3.228 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=83

$$\frac{5}{8}ac^3x+\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\,(c^3-c^3\sin(e+fx))}{4f}$$

```
[Out] 5/8*a*c^3*x+5/12*a*c^3*cos(f*x+e)^3/f+5/8*a*c^3*cos(f*x+e)*sin(f*x+e)/f+1/4
*a*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\,(c^3-c^3\sin(e+fx))}{4f}+\frac{5ac^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*x)/8 + (5*a*c^3*Cos[e + f*x]^3)/(12*f) + (5*a*c^3*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^3 - c^3*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^2\,dx\\
&= \frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5ac^2\right)\int \cos^2(e+f\\
&= \frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\\
&= \frac{5ac^3\cos^3(e+fx)}{12f}+\frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e\\
&= \frac{5}{8}ac^3x+\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f}\end{aligned}$$

**Mathematica** [A]
time = 0.26, size = 54, normalized size = 0.65

$$\frac{ac^3(60fx+48\cos(e+fx)+16\cos(3(e+fx))+24\sin(2(e+fx))-3\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c^3*(60*f*x + 48*Cos[e + f*x] + 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

**Maple** [A]
time = 0.24, size = 89, normalized size = 1.07

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{5a\,c^3x}{8}+\frac{a\,c^3\cos(fx+e)}{2f}-\frac{a\,c^3\sin(4fx+4e)}{32f}+\frac{a\,c^3\cos(3fx+3e)}{6f}+\frac{a\,c^3\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-a\,c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2a\,c^3\cos(fx+e)+a\,c^3(fx+e)}{f}$ |
| default | $\frac{-a\,c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2a\,c^3\cos(fx+e)+a\,c^3(fx+e)}{f}$ |
| norman | $\frac{\frac{4a\,c^3}{3f}+\frac{5a\,c^3x}{8}+\frac{4a\,c^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4a\,c^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{4a\,c^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{11a\,c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2
/3*a*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*c^3*cos(f*x+e)+a*c^3*(f*x+e))
```

**Maxima [A]**
time = 0.31, size = 93, normalized size = 1.12

$$\frac{64\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)ac^3-3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)ac^3+96\left(fx+e\right)ac^3+192\,ac^3\cos\left(fx+e\right)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^3 - 3*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^3 + 96*(f*x + e)*a*c^3 + 192*a*c^3*co
s(f*x + e))/f
```

**Fricas [A]**
time = 0.33, size = 67, normalized size = 0.81

$$\frac{16\,ac^3\cos\left(fx+e\right)^3+15\,ac^3fx-3\left(2\,ac^3\cos\left(fx+e\right)^3-5\,ac^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*c^3*cos(f*x + e)^3 + 15*a*c^3*f*x - 3*(2*a*c^3*cos(f*x + e)^3 -
5*a*c^3*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.
time = 0.21, size = 196, normalized size = 2.36

$$\begin{cases}-\frac{3ac^3x\sin^4(e+fx)}{8}-\frac{3ac^3x\sin^2(e+fx)\cos^2(e+fx)}{4}-\frac{3ac^3x\cos^4(e+fx)}{8}+ac^3x+\frac{5ac^3\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{2ac^3\sin^2(e+fx)\cos(e+fx)}{f}+\frac{3ac^3\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{4ac^3\cos^3(e+fx)}{3f}+\frac{2ac^3\cos(e+fx)}{f} & \text{for}\ f\neq 0\\ x(a\sin(e)+a)(-c\sin(e)+c)^3 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*a*c**3*x*sin(e + f*x)**4/8 - 3*a*c**3*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a*c**3*x*cos(e + f*x)**4/8 + a*c**3*x + 5*a*c**3*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 2*a*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 3*a*c
**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*a*c**3*cos(e + f*x)**3/(3*f) + 2
*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**3, Tr
ue))
```

**Giac** [A]
time = 0.44, size = 81, normalized size = 0.98

$$\frac{5}{8}\,ac^3x+\frac{ac^3\cos\left(3\,fx+3\,e\right)}{6\,f}+\frac{ac^3\cos\left(fx+e\right)}{2\,f}-\frac{ac^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac^3\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/8*a*c^3*x + 1/6*a*c^3*cos(3*f*x + 3*e)/f + 1/2*a*c^3*cos(f*x + e)/f - 1/3
2*a*c^3*sin(4*f*x + 4*e)/f + 1/4*a*c^3*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 9.00, size = 250, normalized size = 3.01

$$\frac{5\,a\,c^3\,x}{8}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a\,c^3\,\left(60\,e+60\,f\,x+32\right)}{24}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a\,c^3\,\left(60\,e+60\,f\,x+96\right)}{24}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{4}-\frac{a\,c^3\,\left(90\,e+90\,f\,x+96\right)}{24}\right)-\frac{3\,a\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}-\frac{11\,a\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{4}+\frac{11\,a\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{4}+\frac{3\,a\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{4}+\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{24}-\frac{a\,c^3\,\left(15\,e+15\,f\,x+32\right)}{24}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*a*c^3*x)/8 - (tan(e/2 + (f*x)/2)^2*((a*c^3*(15*e + 15*f*x))/6 - (a*c^3*(
60*e + 60*f*x + 32))/24) + tan(e/2 + (f*x)/2)^6*((a*c^3*(15*e + 15*f*x))/6
- (a*c^3*(60*e + 60*f*x + 96))/24) + tan(e/2 + (f*x)/2)^4*((a*c^3*(15*e + 1
5*f*x))/4 - (a*c^3*(90*e + 90*f*x + 96))/24) - (3*a*c^3*tan(e/2 + (f*x)/2))
/4 - (11*a*c^3*tan(e/2 + (f*x)/2)^3)/4 + (11*a*c^3*tan(e/2 + (f*x)/2)^5)/4
+ (3*a*c^3*tan(e/2 + (f*x)/2)^7)/4 + (a*c^3*(15*e + 15*f*x))/24 - (a*c^3*(1
5*e + 15*f*x + 32))/24)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)
```

# 3.229 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=52

$$\frac{1}{2}ac^2x+\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*c^2*x+1/3*a*c^2*cos(f*x+e)^3/f+1/2*a*c^2*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2815, 2748, 2715, 8}

$$\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c^2*x)/2 + (a*c^2*Cos[e + f*x]^3)/(3*f) + (a*c^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx &= (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx\\ &= \frac{ac^2\cos^3(e+fx)}{3f}+\left(ac^2\right)\int \cos^2(e+fx)\,dx\\ &= \frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(ac^2\right)\int\\ &= \frac{1}{2}ac^2x+\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

**Mathematica [A]**
time = 0.20, size = 42, normalized size = 0.81

$$\frac{ac^2(6fx+3\cos(e+fx)+\cos(3(e+fx))+3\sin(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
[Out] (a*c^2*(6*f*x + 3*Cos[e + f*x] + Cos[3*(e + f*x)] + 3*Sin[2*(e + f*x)]))/(1
2*f)
```

**Maple [A]**
time = 0.16, size = 77, normalized size = 1.48

| method | result |
|---|---|
| risch | $\frac{a\,c^2x}{2}+\frac{a\,c^2\cos(fx+e)}{4f}+\frac{a\,c^2\cos(3fx+3e)}{12f}+\frac{a\,c^2\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-\frac{a\,c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a\,c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a\,c^2\cos(fx+e)+a\,c^2(fx+e)}{f}$ |
| default | $\frac{-\frac{a\,c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a\,c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a\,c^2\cos(fx+e)+a\,c^2(fx+e)}{f}$ |
| norman | $\frac{\frac{2a\,c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\,c^2}{3f}+\frac{a\,c^2x}{2}-\frac{a\,c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a\,c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a\,c^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-a*c^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)+a*c^2*cos(f*x+e)+a*c^2*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 83, normalized size = 1.60

$$\frac{4\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)ac^{2}-3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac^{2}+12\left(fx+e\right)ac^{2}+12\,ac^{2}\cos\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2 - 3*(2*f*x + 2*e - sin(2*f*
x + 2*e))*a*c^2 + 12*(f*x + e)*a*c^2 + 12*a*c^2*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.34, size = 49, normalized size = 0.94

$$\frac{2\,ac^{2}\cos\left(fx+e\right)^{3}+3\,ac^{2}fx+3\,ac^{2}\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*cos(f*x + e)^3 + 3*a*c^2*f*x + 3*a*c^2*cos(f*x + e)*sin(f*x +
e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(46) = 92$.
time = 0.13, size = 133, normalized size = 2.56

$$\begin{cases} -\frac{ac^{2}x\sin^{2}\left(e+fx\right)}{2}-\frac{ac^{2}x\cos^{2}\left(e+fx\right)}{2}+ac^{2}x-\frac{ac^{2}\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{ac^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ac^{2}\cos^{3}\left(e+fx\right)}{3f}+\frac{ac^{2}\cos\left(e+fx\right)}{f} & \text{for}\: f\neq 0 \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a*c**2*x*sin(e + f*x)**2/2 - a*c**2*x*cos(e + f*x)**2/2 + a*c**
2*x - a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + a*c**2*sin(e + f*x)*cos(e + f
*x)/(2*f) - 2*a*c**2*cos(e + f*x)**3/(3*f) + a*c**2*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

**Giac** [A]
time = 0.44, size = 62, normalized size = 1.19

$$\frac{1}{2}\,ac^{2}x+\frac{ac^{2}\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{ac^{2}\cos\left(fx+e\right)}{4\,f}+\frac{ac^{2}\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*c^2*x + 1/12*a*c^2*cos(3*f*x + 3*e)/f + 1/4*a*c^2*cos(f*x + e)/f + 1/
4*a*c^2*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.96, size = 125, normalized size = 2.40

$$\frac{a\,c^2\,x}{2}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{3\,a\,c^2\,\left(e+f\,x\right)}{2}-\frac{a\,c^2\,\left(9\,e+9\,f\,x+12\right)}{6}\right)-a\,c^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a\,c^2\,\left(e+f\,x\right)}{2}+a\,c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5-\frac{a\,c^2\,\left(3\,e+3\,f\,x+4\right)}{6}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a*c^2*x)/2 - (tan(e/2 + (f*x)/2)^4*((3*a*c^2*(e + f*x))/2 - (a*c^2*(9*e +
9*f*x + 12))/6) - a*c^2*tan(e/2 + (f*x)/2) + (a*c^2*(e + f*x))/2 + a*c^2*ta
n(e/2 + (f*x)/2)^5 - (a*c^2*(3*e + 3*f*x + 4))/6)/(f*(tan(e/2 + (f*x)/2)^2
+ 1)^3)
```

# 3.230 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx$

**Optimal**. Leaf size=29

$$\frac{acx}{2}+\frac{ac\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*c*x+1/2*a*c*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{2813\}$

$$\frac{ac\sin(e+fx)\cos(e+fx)}{2f}+\frac{acx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*x)/2 + (a*c*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx=\frac{acx}{2}+\frac{ac\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.02, size = 25, normalized size = 0.86

$$\frac{ac(2(e+fx)+\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*(2*(e + f*x) + Sin[2*(e + f*x)]))/(4*f)
```

**Maple** [A]
time = 0.10, size = 40, normalized size = 1.38

| method | result | size |
|---|---|---|
| risch | $\frac{acx}{2}+\frac{ca\sin(2fx+2e)}{4f}$ | 23 |
| derivativedivides | $\frac{-ca\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+ca(fx+e)}{f}$ | 40 |
| default | $\frac{-ca\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+ca(fx+e)}{f}$ | 40 |
| norman | $\frac{acx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ca\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{acx}{2}+\frac{acx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{ca\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ | 87 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(-c*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+c*a*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 40, normalized size = 1.38

$$-\frac{(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))ac-4\,(fx+e)ac}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")
[Out] -1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*a*c - 4*(f*x + e)*a*c)/f
```

**Fricas** [A]
time = 0.32, size = 28, normalized size = 0.97

$$\frac{acfx+ac\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")
[Out] 1/2*(a*c*f*x + a*c*cos(f*x + e)*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26)=52$.
time = 0.08, size = 70, normalized size = 2.41

$$\begin{cases}-\frac{acx\sin^2\left(e+fx\right)}{2}-\frac{acx\cos^2\left(e+fx\right)}{2}+acx+\frac{ac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f\neq 0\\ x(a\sin\left(e\right)+a)\left(-c\sin\left(e\right)+c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*c*x*sin(e + f*x)**2/2 - a*c*x*cos(e + f*x)**2/2 + a*c*x + a*c
*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) +
 c), True))
```

**Giac [A]**
time = 0.41, size = 23, normalized size = 0.79

$$\frac{1}{2}\,acx + \frac{ac\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*c*x + 1/4*a*c*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 7.15, size = 54, normalized size = 1.86

$$\frac{a\,c\,x}{2} - \frac{a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 - a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

```
[Out] (a*c*x)/2 - (a*c*tan(e/2 + (f*x)/2)^3 - a*c*tan(e/2 + (f*x)/2))/(f*(tan(e/2
 + (f*x)/2)^2 + 1)^2)
```

# 3.231 $\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=33

$$-\frac{ax}{c}+\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}$$

```
[Out] -a*x/c+2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

**Rubi** [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2814, 2727}

$$\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]
[Out] -((a*x)/c) + (2*a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx &= -\frac{ax}{c}+(2a)\int \frac{1}{c-c\sin(e+fx)}\,dx\\ &= -\frac{ax}{c}+\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(33) = 66.
time = 0.13, size = 83, normalized size = 2.52

$$\frac{a\left(-fx\cos\left(\frac{fx}{2}\right)+4\sin\left(\frac{fx}{2}\right)+fx\sin\left(e+\frac{fx}{2}\right)\right)}{cf\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*(-(f*x*Cos[(f*x)/2]) + 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(c*f*(Cos
[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 0.24, size = 38, normalized size = 1.15

| method | result | size |
|---|---|---|
| risch | $-\frac{ax}{c}+\frac{4a}{fc\left(e^{i(fx+e)}-i\right)}$ | 32 |
| derivativedivides | $\frac{2a\left(-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$ | 38 |
| default | $\frac{2a\left(-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$ | 38 |
| norman | $\frac{\frac{ax}{c}+\frac{ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4a}{cf}-\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{ax\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$ | 117 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a/c*(-2/(tan(1/2*f*x+1/2*e)-1)-arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(36) = 72$.
time = 0.49, size = 88, normalized size = 2.67

$$-\frac{2\left(a\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{a}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(c
os(f*x + e) + 1))) - a/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas [A]**
time = 0.33, size = 70, normalized size = 2.12

$$-\frac{afx+(afx-2a)\cos(fx+e)-(afx+2a)\sin(fx+e)-2a}{cf\cos(fx+e)-cf\sin(fx+e)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(a*f*x + (a*f*x - 2*a)*cos(f*x + e) - (a*f*x + 2*a)*sin(f*x + e) - 2*a)/(c
*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(26) = 52$.
time = 0.66, size = 88, normalized size = 2.67

$$\begin{cases} -\frac{afx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} + \frac{afx}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} - \frac{4a}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} & \text{for } f \neq 0 \\ \frac{x(a\sin(e)+a)}{-c\sin(e)+c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2) - c*f) + a*f*x/(c*
f*tan(e/2 + f*x/2) - c*f) - 4*a/(c*f*tan(e/2 + f*x/2) - c*f), Ne(f, 0)), (x
*(a*sin(e) + a)/(-c*sin(e) + c), True))
```

**Giac** [A]
time = 0.42, size = 37, normalized size = 1.12

$$-\frac{\frac{(fx+e)a}{c} + \frac{4a}{c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*a/c + 4*a/(c*(tan(1/2*f*x + 1/2*e) - 1)))/f
```

**Mupad** [B]
time = 6.81, size = 46, normalized size = 1.39

$$-\frac{a\,x}{c} - \frac{a\,(e+f\,x) - a\,(e+f\,x-4)}{c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x)),x)
```

```
[Out] - (a*x)/c - (a*(e + f*x) - a*(e + f*x - 4))/(c*f*(tan(e/2 + (f*x)/2) - 1))
```

# 3.232 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=30

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] 1/3*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2815, 2750}

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(30) = 60.
time = 0.20, size = 74, normalized size = 2.47

$$-\frac{a\left(-3\cos\left(e+\frac{fx}{2}\right)+\cos\left(e+\frac{3fx}{2}\right)\right)}{3c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(a*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(c^2*f*(Cos[e/2] - Sin[
e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

**Maple** [A]
time = 0.28, size = 56, normalized size = 1.87

| method | result | size |
|---|---|---|
| risch | $-\frac{2\left(3a\,e^{2i(fx+e)}-a\right)}{3\left(e^{i(fx+e)}-i\right)^3 f\,c^2}$ | 39 |
| derivativedivides | $\frac{2a\left(-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$ | 56 |
| default | $\frac{2a\left(-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$ | 56 |
| norman | $\frac{-\frac{2a}{3cf}-\frac{8a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{2a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{c\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a/c^2*(-4/3/(tan(1/2*f*x+1/2*e)-1)^3-2/(tan(1/2*f*x+1/2*e)-1)^2-1/(tan(
1/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(31) = 62.
time = 0.29, size = 235, normalized size = 7.83

$$-\frac{2\left(\frac{a\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{a\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - a*(
3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.
time = 0.32, size = 112, normalized size = 3.73

$$\frac{a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2\,a\right)\sin\left(fx+e\right)-2\,a}{3\,\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2\,c^{2}f+\left(c^{2}f\cos\left(fx+e\right)+2\,c^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x + e
) - 2*a)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(
f*x + e) + 2*c^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(26) = 52$.
time = 1.31, size = 158, normalized size = 5.27

$$\begin{cases} -\frac{6a\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^{2}f}-\frac{2a}{3c^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^{2}f} & \text{for}\: f\neq 0\\ \frac{x\left(a\sin\left(e\right)+a\right)}{\left(-c\sin\left(e\right)+c\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*
f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*a/(3*c**2
*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

**Giac** [A]
time = 0.42, size = 39, normalized size = 1.30

$$-\frac{2\,\left(3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a\right)}{3\,c^{2}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*f*x + 1/2*e)^2 + a)/(c^2*f*(tan(1/2*f*x + 1/2*e) - 1)^3)
```

**Mupad** [B]
time = 6.73, size = 56, normalized size = 1.87

$$-\frac{2\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\right)}{3\,c^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] -(2*a*cos(e/2 + (f*x)/2)*(2*cos(e/2 + (f*x)/2)^2 - 3))/(3*c^2*f*(cos(e/2 +
(f*x)/2) - sin(e/2 + (f*x)/2))^3)
```

# 3.233 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=60

$$\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}$$

```
[Out] 1/5*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^4+1/15*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^3
```

**Rubi** [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2815, 2751, 2750}

$$\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}+\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])^4) + (a*Cos[e + f*x]^3)/(15*
f*(c - c*Sin[e + f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\ &= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}+\frac{1}{5}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}\end{aligned}$$

**Mathematica [A]**
time = 0.23, size = 96, normalized size = 1.60

$$\frac{a\left(15\cos\left(e+\frac{fx}{2}\right)-5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)+\sin\left(2e+\frac{5fx}{2}\right)\right)}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(15*Cos[e + (f*x)/2] - 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e +
 (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5)
```

**Maple [A]**
time = 0.32, size = 86, normalized size = 1.43

| method | result |
|---|---|
| risch | $\frac{2ia\left(5i\,e^{2i(fx+e)}+15\,e^{3i(fx+e)}+i-5\,e^{i(fx+e)}\right)}{15fc^3\left(e^{i(fx+e)}-i\right)^5}$ |
| derivativedivides | $\frac{2a\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}\right)}{fc^3}$ |
| default | $\frac{2a\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}\right)}{fc^3}$ |
| norman | $\frac{\frac{2a\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8a}{15cf}-\frac{2a\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}+\frac{8a\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{16a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{58a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{15cf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

[Out] 2/f*a/c^3*(-1/(tan(1/2*f*x+1/2*e)-1)-8/5/(tan(1/2*f*x+1/2*e)-1)^5-3/(tan(1/2*f*x+1/2*e)-1)^2-4/(tan(1/2*f*x+1/2*e)-1)^4-14/3/(tan(1/2*f*x+1/2*e)-1)^3)

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(62) = 124$.
time = 0.31, size = 423, normalized size = 7.05

$$-\frac{2\left(\frac{a\left(\frac{20\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5\,c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10\,c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5\,c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3\,a\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^3-\frac{5\,c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10\,c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5\,c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 3*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(62) = 124$.
time = 0.33, size = 166, normalized size = 2.77

$$-\frac{a\cos\left(fx+e\right)^3-2\,a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)+6\,a\right)\sin\left(fx+e\right)+6\,a}{15\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a*cos(f*x + e)^3 - 2*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) + (a*cos(f*
x + e)^2 + 3*a*cos(f*x + e) + 6*a)*sin(f*x + e) + 6*a)/(c^3*f*cos(f*x + e)^
3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(51) = 102$.
time = 2.90, size = 571, normalized size = 9.52

$$\begin{cases} -\frac{30a\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f}+\frac{30a\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f}-\frac{50a\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f}+\frac{10a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f}-\frac{8a}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f} & \text{for}\: f \neq 0 \\ \frac{x\left(a\sin\left(e\right)+a\right)}{\left(-c\sin\left(e\right)+c\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c*
*3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(
e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*a*tan(e/2 +
f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 +
150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) - 50*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/
2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)
**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3
*f) + 10*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(
e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x
/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*a/(15*c**3*f*tan(e/2 +
 f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3
 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f)
, Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**3, True))
```

**Giac** [A]

time = 0.43, size = 84, normalized size = 1.40

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,a\right)}{15\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*f*x + 1/2*e)^4 - 15*a*tan(1/2*f*x + 1/2*e)^3 + 25*a*tan
(1/2*f*x + 1/2*e)^2 - 5*a*tan(1/2*f*x + 1/2*e) + 4*a)/(c^3*f*(tan(1/2*f*x +
 1/2*e) - 1)^5)
```

**Mupad** [B]

time = 7.10, size = 136, normalized size = 2.27

$$\frac{2\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+25\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-15\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+15\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right)}{15\,c^3\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*a*cos(e/2 + (f*x)/2)*(4*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4 -
 15*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 - 5*cos(e/2 + (f*x)/2)^3*sin(e/
2 + (f*x)/2) + 25*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*c^3*f*(co
s(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2))^5)
```

# 3.234 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=92

$$\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}$$

```
[Out] 1/7*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^5+2/35*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^4+2/105*a*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2815, 2751, 2750}

$$\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(3
5*f*(c - c*Sin[e + f*x])^4) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e +
f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{1}{7}(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{35c} \\
&= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}
\end{aligned}$$

**Mathematica [A]**
time = 0.32, size = 109, normalized size = 1.18

$$\frac{a\left(70\cos\left(e+\frac{fx}{2}\right)-21\cos\left(e+\frac{3fx}{2}\right)+\cos\left(3e+\frac{7fx}{2}\right)+35\sin\left(\frac{fx}{2}\right)+7\sin\left(2e+\frac{5fx}{2}\right)\right)}{210c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*(70*Cos[e + (f*x)/2] - 21*Cos[e + (3*f*x)/2] + Cos[3*e + (7*f*x)/2] + 35
*Sin[(f*x)/2] + 7*Sin[2*e + (5*f*x)/2]))/(210*c^4*f*(Cos[e/2] - Sin[e/2])*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

**Maple [A]**
time = 0.34, size = 116, normalized size = 1.26

| method | result |
|---|---|
| risch | $\frac{\frac{4a}{105}+\frac{4ia\,e^{3i(fx+e)}}{3}+\frac{8a\,e^{4i(fx+e)}}{3}+\frac{4ia\,e^{i(fx+e)}}{15}-\frac{4a\,e^{2i(fx+e)}}{5}}{\left(e^{i(fx+e)}-i\right)^7 f\,c^4}$ |
| derivativedivides | $\frac{2a\left(-\frac{28}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{68}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{16}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots}\right.}{f\,c^4}$ |
| default | $\frac{2a\left(-\frac{28}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{68}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{16}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots}\right.}{f\,c^4}$ |

| | |
|---|---|
| norman | $\frac{-\frac{46a}{105cf}-\frac{2a\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{4a\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{32a\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{32a\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{16a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}-\frac{208a\left(\tan^{4}\left(\frac{fx}{2}\right.\right.}{15cf}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a/c^4*(-28/3/(tan(1/2*f*x+1/2*e)-1)^3-4/(tan(1/2*f*x+1/2*e)-1)^2-68/5/(
tan(1/2*f*x+1/2*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1)-8/(tan(1/2*f*x+1/2*e)-1)^6
-16/7/(tan(1/2*f*x+1/2*e)-1)^7-14/(tan(1/2*f*x+1/2*e)-1)^4)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(95) = 190$.
time = 0.30, size = 611, normalized size = 6.64

$$\frac{2\left(\frac{a\left(\frac{91\sin(fx+e)}{\cos(fx+e)+1}-\frac{168\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{280\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{175\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{105\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-13\right)}{c^{4}-\frac{7c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{35c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{21c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{7c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}-\frac{3a\left(\frac{49\sin(fx+e)}{\cos(fx+e)+1}-\frac{147\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{210\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{210\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{105\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{35\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-12\right)}{c^{4}-\frac{7c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{35c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{21c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{7c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 -
 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^
4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
1)^7) - 3*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^
4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(95) = 190$.
time = 0.32, size = 222, normalized size = 2.41

$$\frac{2a\cos(fx+e)^{4}+8a\cos(fx+e)^{3}-9a\cos(fx+e)^{2}+15a\cos(fx+e)-\left(2a\cos(fx+e)^{3}-6a\cos(fx+e)^{2}-15a\cos(fx+e)-30a\right)\sin(fx+e)+30a}{105\left(c^{4}f\cos(fx+e)^{4}-3c^{4}f\cos(fx+e)^{3}-8c^{4}f\cos(fx+e)^{2}+4c^{4}f\cos(fx+e)+8c^{4}f+\left(c^{4}f\cos(fx+e)^{3}+4c^{4}f\cos(fx+e)^{2}-4c^{4}f\cos(fx+e)-8c^{4}f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a*cos(f*x + e)^4 + 8*a*cos(f*x + e)^3 - 9*a*cos(f*x + e)^2 + 15*a*
cos(f*x + e) - (2*a*cos(f*x + e)^3 - 6*a*cos(f*x + e)^2 - 15*a*cos(f*x + e)
 - 30*a)*sin(f*x + e) + 30*a)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^
3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*
x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x
 + e))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(82) = 164$.
time = 6.51, size = 1061, normalized size = 11.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735
*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f
*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/
2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*a*tan(e/2 +
 f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6
 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675
*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*
tan(e/2 + f*x/2) - 105*c**4*f) - 910*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(
e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*
x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3
 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4
*f) + 700*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*
f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 546*a*tan(e/2 + f*x/2
)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 220
5*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*
f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/
2 + f*x/2) - 105*c**4*f) + 112*a*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x
/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 -
 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c
**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 46*
a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c
**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*t
an(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 +
 f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**4, Tru
e))
```

**Giac** [**A**]

time = 0.51, size = 114, normalized size = 1.24

$$-\frac{2\left(105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-350\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+273\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-56\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+23\,a\right)}{105\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*a*tan(1/2*f*x + 1/2*e)^6 - 210*a*tan(1/2*f*x + 1/2*e)^5 + 455*a
*tan(1/2*f*x + 1/2*e)^4 - 350*a*tan(1/2*f*x + 1/2*e)^3 + 273*a*tan(1/2*f*x
+ 1/2*e)^2 - 56*a*tan(1/2*f*x + 1/2*e) + 23*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

**Mupad [B]**
time = 7.35, size = 97, normalized size = 1.05

$$\frac{\sqrt{2}\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{25\,\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{595\,\sin\left(e+f\,x\right)}{8}-\frac{43\,\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{353\,\cos\left(e+f\,x\right)}{8}+\frac{77\,\sin\left(2\,e+2\,f\,x\right)}{4}+\frac{21\,\sin\left(3\,e+3\,f\,x\right)}{8}+\frac{171}{2}\right)}{840\,c^4\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2^(1/2)*a*cos(e/2 + (f*x)/2)*((25*cos(3*e + 3*f*x))/8 - (595*sin(e + f*x))
/8 - (43*cos(2*e + 2*f*x))/2 - (353*cos(e + f*x))/8 + (77*sin(2*e + 2*f*x))
/4 + (21*sin(3*e + 3*f*x))/8 + 171/2))/(840*c^4*f*cos(e/2 + pi/4 + (f*x)/2)
^7)
```

# 3.235 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=126

$$\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}$$

```
[Out] 1/9*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^6+1/21*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^5+2/105*a*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^4+2/315*a*c*cos(f*x+e)^3/
f/(c^2-c^2*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2815, 2751, 2750}

$$\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^6) + (a*Cos[e + f*x]^3)/(21*
f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e + f*
x])^4) + (2*a*c*Cos[e + f*x]^3)/(315*f*(c^2 - c^2*Sin[e + f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{1}{3}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{21c}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}$$

**Mathematica [A]**
time = 0.40, size = 124, normalized size = 0.98

$$\frac{a\left(315\cos\left(e+\frac{fx}{2}\right)-84\cos\left(e+\frac{3fx}{2}\right)+9\cos\left(3e+\frac{7fx}{2}\right)+189\sin\left(\frac{fx}{2}\right)+36\sin\left(2e+\frac{5fx}{2}\right)-\sin\left(4e+\frac{9fx}{2}\right)\right)}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
[Out] (a*(315*Cos[e + (f*x)/2] - 84*Cos[e + (3*f*x)/2] + 9*Cos[3*e + (7*f*x)/2] +
 189*Sin[(f*x)/2] + 36*Sin[2*e + (5*f*x)/2] - Sin[4*e + (9*f*x)/2]))/(1260*
c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

**Maple [A]**
time = 0.39, size = 146, normalized size = 1.16

| method | result |
|---|---|
| risch | $-\frac{4ia\left(189i\,e^{4i(fx+e)}+315\,e^{5i(fx+e)}+36i\,e^{2i(fx+e)}-84\,e^{3i(fx+e)}-i+9\,e^{i(fx+e)}\right)}{315f\,c^5\left(e^{i(fx+e)}-i\right)^9}$ |
| derivativedivides | $\frac{2a\left(-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{236}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{248}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{148}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}\right.}{f\,c^5}$ |

| | |
|---|---|
| default | $\frac{2a\left(-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{236}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{248}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{148}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,c^{5}}$ |
| norman | $\frac{\frac{6a\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{116a}{315cf}-\frac{2a\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{28a\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{616a\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{56a\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{46a\tan\left(\right.}{35c}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c^{4}\left(\tan\left(\frac{fx}{2}-\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)

[Out] 2/f*a/c^5*(-16/(tan(1/2*f*x+1/2*e)-1)^8-236/5/(tan(1/2*f*x+1/2*e)-1)^5-5/(tan(1/2*f*x+1/2*e)-1)^2-248/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(tan(1/2*f*x+1/2*e)-1)-148/3/(tan(1/2*f*x+1/2*e)-1)^6-32/9/(tan(1/2*f*x+1/2*e)-1)^9-32/(tan(1/2*f*x+1/2*e)-1)^4-46/3/(tan(1/2*f*x+1/2*e)-1)^3)

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(130) = 260$.

time = 0.32, size = 799, normalized size = 6.34

$$-\frac{2\left(\frac{a\left(\frac{432\sin(fx+e)}{\cos(fx+e)+1}-\frac{1728\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{3612\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{5418\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{5040\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{3360\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{1260\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{315\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-83\right)}{c^{5}-\frac{9c^{5}\sin(fx+e)}{\cos(fx+e)+1}+\frac{36c^{5}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{84c^{5}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{126c^{5}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{126c^{5}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{84c^{5}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{36c^{5}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{9c^{5}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{c^{5}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}}-\frac{5a\left(\frac{45\sin(fx+e)}{\cos(fx+e)+1}-\frac{117\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{273\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{315\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{315\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{147\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{63\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-5\right)}{c^{5}-\frac{9c^{5}\sin(fx+e)}{\cos(fx+e)+1}+\frac{36c^{5}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{84c^{5}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{126c^{5}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{126c^{5}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{84c^{5}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{36c^{5}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{9c^{5}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{c^{5}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}}\right)}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out] -2/315*(a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(130) = 260$.
time = 0.33, size = 276, normalized size = 2.19

$$-\frac{2\,a\cos\left(fx+e\right)^{5}-8\,a\cos\left(fx+e\right)^{4}-25\,a\cos\left(fx+e\right)^{3}+20\,a\cos\left(fx+e\right)^{2}-35\,a\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^{4}+10\,a\cos\left(fx+e\right)^{3}-15\,a\cos\left(fx+e\right)^{2}-35\,a\cos\left(fx+e\right)-70\,a\right)\sin\left(fx+e\right)-70\,a}{315\,\left(c^{5}f\cos\left(fx+e\right)^{5}+5\,c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{3}-20\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f\cos\left(fx+e\right)+16\,c^{5}f-\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}-12\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f\cos\left(fx+e\right)+16\,c^{5}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a*cos(f*x + e)^5 - 8*a*cos(f*x + e)^4 - 25*a*cos(f*x + e)^3 + 20*
a*cos(f*x + e)^2 - 35*a*cos(f*x + e) + (2*a*cos(f*x + e)^4 + 10*a*cos(f*x +
 e)^3 - 15*a*cos(f*x + e)^2 - 35*a*cos(f*x + e) - 70*a)*sin(f*x + e) - 70*a
)/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 -
 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*
x + e) + 16*c^5*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1700 vs. $2(112) = 224$.
time = 13.11, size = 1700, normalized size = 13.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*a*tan(e/2
 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*a*tan(e/2 + f*x/2)**6/(315*c**5*f*
tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/
2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)
**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315
*c**5*f) + 6930*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 7686*a*tan(e/2
```

```
 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 4494*a*tan(e/2 + f*x/2)**3/(315*c**5*f*
tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/
2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)
**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315
*c**5*f) - 2286*a*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 414*a*tan(e/2
+ f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8
+ 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 396
90*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c*
*5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*t
an(e/2 + f*x/2) - 315*c**5*f) - 116*a/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f), Ne(f, 0)), (x*(
a*sin(e) + a)/(-c*sin(e) + c)**5, True))
```

**Giac [A]**
time = 0.43, size = 144, normalized size = 1.14

$$-\frac{2\left(315\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3465\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3843\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2247\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+1143\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-207\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+58\,a\right)}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/315*(315*a*tan(1/2*f*x + 1/2*e)^8 - 945*a*tan(1/2*f*x + 1/2*e)^7 + 2625*
a*tan(1/2*f*x + 1/2*e)^6 - 3465*a*tan(1/2*f*x + 1/2*e)^5 + 3843*a*tan(1/2*f
*x + 1/2*e)^4 - 2247*a*tan(1/2*f*x + 1/2*e)^3 + 1143*a*tan(1/2*f*x + 1/2*e)
^2 - 207*a*tan(1/2*f*x + 1/2*e) + 58*a)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9
)
```

**Mupad [B]**
time = 8.77, size = 119, normalized size = 0.94

$$\frac{\sqrt{2}\,a\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{121\,\cos\left(3\,e+3\,f\,x\right)}{4}-\frac{1575\,\sin\left(e+f\,x\right)}{4}-\frac{625\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{635\,\cos\left(e+f\,x\right)}{4}+\frac{7\,\cos\left(4\,e+4\,f\,x\right)}{2}+\frac{399\,\sin\left(2\,e+2\,f\,x\right)}{4}+\frac{141\,\sin\left(3\,e+3\,f\,x\right)}{4}-\frac{15\,\sin\left(4\,e+4\,f\,x\right)}{4}+\frac{1357}{4}\right)}{5040\,c^5\,f\,{\cos\left(\frac{e}{2}+\frac{\pi }{4}+\frac{f\,x}{2}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a*cos(e/2 + (f*x)/2)*((121*cos(3*e + 3*f*x))/4 - (1575*sin(e + f*x
))/4 - (625*cos(2*e + 2*f*x))/4 - (635*cos(e + f*x))/4 + (7*cos(4*e + 4*f*x
))/2 + (399*sin(2*e + 2*f*x))/4 + (141*sin(3*e + 3*f*x))/4 - (15*sin(4*e +
4*f*x))/4 + 1357/4))/(5040*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

# 3.236 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx$

**Optimal**. Leaf size=152

$$\frac{9}{16}a^2c^5x+\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f}+\frac{3a^2c^5\cos^3(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^3\cos^5(e-$$

```
[Out] 9/16*a^2*c^5*x+3/10*a^2*c^5*cos(f*x+e)^5/f+9/16*a^2*c^5*cos(f*x+e)*sin(f*x+
e)/f+3/8*a^2*c^5*cos(f*x+e)^3*sin(f*x+e)/f+1/7*a^2*c^3*cos(f*x+e)^5*(c-c*si
n(f*x+e))^2/f+3/14*a^2*cos(f*x+e)^5*(c^5-c^5*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{3a^2\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{14f}+\frac{3a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{9a^2c^5\sin(e+fx)\cos(e+fx)}{16f}+\frac{9}{16}a^2c^5x+\frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*x)/16 + (3*a^2*c^5*Cos[e + f*x]^5)/(10*f) + (9*a^2*c^5*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (3*a^2*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(8*f) +
(a^2*c^3*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (3*a^2*Cos[e + f*x]
^5*(c^5 - c^5*Sin[e + f*x]))/(14*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{1}{7}\left(9a^2c^3\right)\int \cos^4(e+$$
$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{3a^2\cos^5(e+fx)\left(c^5\right.}{14f}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{3a^2c^5\cos^3(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^3}{}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f} + \frac{3a^2c^5}{}$$
$$= \frac{9}{16}a^2c^5x + \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+f}{16f}$$

## **Mathematica** [A]

time = 0.72, size = 89, normalized size = 0.59

$$\frac{a^2c^5(1260e+1260fx+945\cos(e+fx)+455\cos(3(e+fx))+77\cos(5(e+fx))-5\cos(7(e+fx))+665\sin(2(e+fx))-35\sin(4(e+fx))-35\sin(6(e+fx)))}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(1260*e + 1260*f*x + 945*Cos[e + f*x] + 455*Cos[3*(e + f*x)] + 77*
Cos[5*(e + f*x)] - 5*Cos[7*(e + f*x)] + 665*Sin[2*(e + f*x)] - 35*Sin[4*(e
+ f*x)] - 35*Sin[6*(e + f*x)]))/(2240*f)
```

**Maple [A]**

time = 0.46, size = 255, normalized size = 1.68

| method | result |
|---|---|
| risch | $\frac{9a^2c^5x}{16}+\frac{27c^5a^2\cos(fx+e)}{64f}-\frac{c^5a^2\cos(7fx+7e)}{448f}-\frac{c^5a^2\sin(6fx+6e)}{64f}+\frac{11c^5a^2\cos(5fx+5e)}{320f}-\frac{c^5a^2\sin(4fx+}{64f}$ |
| derivativedivides | $\frac{\frac{c^5a^2\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3c^5a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{}$ |
| default | $\frac{\frac{c^5a^2\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3c^5a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{}$ |
| norman | $\frac{6c^5a^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{16c^5a^2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{32c^5a^2\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{46c^5a^2}{35f}+\frac{9a^2c^5x}{16}+\frac{14c^5a^2\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{16c^5}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/7*c^5*a^2*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(
f*x+e)+3*c^5*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(
f*x+e)+5/16*f*x+5/16*e)+1/5*c^5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos
(f*x+e)-5*c^5*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e)-5/3*c^5*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+c^5*a^2*(-1/2*cos(f*x+e)*sin(f
*x+e)+1/2*f*x+1/2*e)+3*c^5*a^2*cos(f*x+e)+c^5*a^2*(f*x+e))
```

**Maxima [A]**

time = 0.29, size = 276, normalized size = 1.82

$$-\frac{192\,(5\cos(fx+e)^7-21\cos(fx+e)^5+35\cos(fx+e)^3-35\cos(fx+e))a^2c^5-448\,(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e))a^2c^5-11200\,(\cos(fx+e)^3-3\cos(fx+e))a^2c^5-105\,(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e))a^2c^5+1050\,(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e))a^2c^5-1680\,(2fx+2e-\sin(2fx+2e))a^2c^5-6720\,(fx+e)a^2c^5-20160\,a^2c^5\cos(fx+e)}{6720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35
*cos(f*x + e))*a^2*c^5 - 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^2*c^5 - 11200*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^5 - 105*
(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x +
 2*e))*a^2*c^5 + 1050*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*a^2*c^5 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^5 - 6720*(f*x + e)
*a^2*c^5 - 20160*a^2*c^5*cos(f*x + e))/f
```

**Fricas [A]**

time = 0.33, size = 109, normalized size = 0.72

$$-\frac{80\,a^2c^5\cos(fx+e)^7-448\,a^2c^5\cos(fx+e)^5-315\,a^2c^5fx+35\,(8\,a^2c^5\cos(fx+e)^5-6\,a^2c^5\cos(fx+e)^3-9\,a^2c^5\cos(fx+e))\sin(fx+e)}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] -1/560*(80*a^2*c^5*cos(f*x + e)^7 - 448*a^2*c^5*cos(f*x + e)^5 - 315*a^2*c^5*f*x + 35*(8*a^2*c^5*cos(f*x + e)^5 - 6*a^2*c^5*cos(f*x + e)^3 - 9*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(144) = 288$.
time = 0.77, size = 629, normalized size = 4.14

$$\begin{cases} \frac{15a^2c^5x\sin^6(e+fx)}{16} + \frac{45a^2c^5x\sin^4(e+fx)\cos^2(e+fx)}{16} - \frac{15a^2c^5x\sin^4(e+fx)}{8} + \frac{45a^2c^5x\sin^2(e+fx)\cos^4(e+fx)}{16} - \frac{15a^2c^5x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{a^2c^5x\sin^2(e+fx)}{2} + \frac{15a^2c^5x\cos^6(e+fx)}{16} - \frac{15a^2c^5x\cos^4(e+fx)}{8} + \frac{a^2c^5x\cos^2(e+fx)}{2} + a^2c^5x + \frac{a^2c^5\sin^6(e+fx)\cos(e+fx)}{f} - \frac{33a^2c^5\sin^5(e+fx)\cos(e+fx)}{16f} + \frac{2a^2c^5\sin^4(e+fx)\cos^3(e+fx)}{f} + \frac{a^2c^5\sin^4(e+fx)\cos(e+fx)}{f} - \frac{5a^2c^5\sin^3(e+fx)\cos^3(e+fx)}{2f} + \frac{25a^2c^5\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{8a^2c^5\sin^2(e+fx)\cos^5(e+fx)}{5f} + \frac{4a^2c^5\sin^2(e+fx)\cos^3(e+fx)}{3f} - \frac{5a^2c^5\sin^2(e+fx)\cos(e+fx)}{f} - \frac{15a^2c^5\sin(e+fx)\cos^5(e+fx)}{16f} + \frac{15a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{a^2c^5\sin(e+fx)\cos(e+fx)}{2f} + \frac{16a^2c^5\cos^7(e+fx)}{35f} + \frac{8a^2c^5\cos^5(e+fx)}{15f} - \frac{10a^2c^5\cos^3(e+fx)}{3f} + \frac{3a^2c^5\cos(e+fx)}{f} & \text{for}\ f \neq 0 \\ x(a\sin(e)+a)^2(-c\sin(e)+c)^5 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**5,x)

[Out] Piecewise((15*a**2*c**5*x*sin(e + f*x)**6/16 + 45*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 - 15*a**2*c**5*x*sin(e + f*x)**4/8 + 45*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a**2*c**5*x*sin(e + f*x)**2/2 + 15*a**2*c**5*x*cos(e + f*x)**6/16 - 15*a**2*c**5*x*cos(e + f*x)**4/8 + a**2*c**5*x*cos(e + f*x)**2/2 + a**2*c**5*x + a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f - 33*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*a**2*c**5*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 15*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2*c**5*cos(e + f*x)**7/(35*f) + 8*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*a**2*c**5*cos(e + f*x)**3/(3*f) + 3*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))

**Giac** [A]
time = 0.50, size = 154, normalized size = 1.01

$$\frac{9}{16}a^2c^5x - \frac{a^2c^5\cos(7fx+7e)}{448f} + \frac{11a^2c^5\cos(5fx+5e)}{320f} + \frac{13a^2c^5\cos(3fx+3e)}{64f} + \frac{27a^2c^5\cos(fx+e)}{64f} - \frac{a^2c^5\sin(6fx+6e)}{64f} - \frac{a^2c^5\sin(4fx+4e)}{64f} + \frac{19a^2c^5\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] 9/16*a^2*c^5*x - 1/448*a^2*c^5*cos(7*f*x + 7*e)/f + 11/320*a^2*c^5*cos(5*f*x + 5*e)/f + 13/64*a^2*c^5*cos(3*f*x + 3*e)/f + 27/64*a^2*c^5*cos(f*x + e)/f - 1/64*a^2*c^5*sin(6*f*x + 6*e)/f - 1/64*a^2*c^5*sin(4*f*x + 4*e)/f + 19/64*a^2*c^5*sin(2*f*x + 2*e)/f

**Mupad [B]**
time = 9.23, size = 452, normalized size = 2.97

$$\frac{9\,a^2\,c^5\,x}{16}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,\left(2205\,e+2205\,f\,x+1792\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,\left(2205\,e+2205\,f\,x+3360\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{3\,a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,\left(6615\,e+6615\,f\,x+6496\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\,\left(\frac{3\,a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,\left(6615\,e+6615\,f\,x+8960\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{16}-\frac{a^2\,c^5\,\left(11025\,e+11025\,f\,x+7840\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{16}-\frac{a^2\,c^5\,\left(11025\,e+11025\,f\,x+17920\right)}{560}\right)-\frac{17\,a^2\,c^5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{2}+\frac{13\,a^2\,c^5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{8}-\frac{13\,a^2\,c^5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{8}+\frac{17\,a^2\,c^5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}}{2}+\frac{7\,a^2\,c^5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{13}}{8}+\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{560}-\frac{a^2\,c^5\,\left(315\,e+315\,f\,x+736\right)}{560}-\frac{7\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (9*a^2*c^5*x)/16 - (tan(e/2 + (f*x)/2)^2*((a^2*c^5*(315*e + 315*f*x))/80 -
(a^2*c^5*(2205*e + 2205*f*x + 1792))/560) + tan(e/2 + (f*x)/2)^12*((a^2*c^5
*(315*e + 315*f*x))/80 - (a^2*c^5*(2205*e + 2205*f*x + 3360))/560) + tan(e/
2 + (f*x)/2)^4*((3*a^2*c^5*(315*e + 315*f*x))/80 - (a^2*c^5*(6615*e + 6615*
f*x + 6496))/560) + tan(e/2 + (f*x)/2)^10*((3*a^2*c^5*(315*e + 315*f*x))/80
 - (a^2*c^5*(6615*e + 6615*f*x + 8960))/560) + tan(e/2 + (f*x)/2)^8*((a^2*c
^5*(315*e + 315*f*x))/16 - (a^2*c^5*(11025*e + 11025*f*x + 7840))/560) + ta
n(e/2 + (f*x)/2)^6*((a^2*c^5*(315*e + 315*f*x))/16 - (a^2*c^5*(11025*e + 11
025*f*x + 17920))/560) - (17*a^2*c^5*tan(e/2 + (f*x)/2)^3)/2 + (13*a^2*c^5*
tan(e/2 + (f*x)/2)^5)/8 - (13*a^2*c^5*tan(e/2 + (f*x)/2)^9)/8 + (17*a^2*c^5
*tan(e/2 + (f*x)/2)^11)/2 + (7*a^2*c^5*tan(e/2 + (f*x)/2)^13)/8 + (a^2*c^5*
(315*e + 315*f*x))/560 - (a^2*c^5*(315*e + 315*f*x + 736))/560 - (7*a^2*c^5
*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

# 3.237 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=118

$$\frac{7}{16}a^2c^4x+\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{7a^2c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^2\cos^5(e+f\ldots}{}$$

```
[Out] 7/16*a^2*c^4*x+7/30*a^2*c^4*cos(f*x+e)^5/f+7/16*a^2*c^4*cos(f*x+e)*sin(f*x+
e)/f+7/24*a^2*c^4*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^2*cos(f*x+e)^5*(c^4-c^4*s
in(f*x+e))/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{7a^2c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{7a^2c^4\sin(e+fx)\cos(e+fx)}{16f}+\frac{7}{16}a^2c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*x)/16 + (7*a^2*c^4*Cos[e + f*x]^5)/(30*f) + (7*a^2*c^4*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (7*a^2*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) +
 (a^2*Cos[e + f*x]^5*(c^4 - c^4*Sin[e + f*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^2\,dx\\ &= \frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{1}{6}\left(7a^2c^3\right)\int \cos^4(e\\ &= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}\\ &= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^2}{}\\ &= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{7a^2}{}\\ &= \frac{7}{16}a^2c^4x+\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos(e+fx)\sin(e+}{16f}\end{aligned}$$

**Mathematica** [A]
time = 0.49, size = 79, normalized size = 0.67

$$\frac{a^2c^4(420e+420fx+240\cos(e+fx)+120\cos(3(e+fx))+24\cos(5(e+fx))+255\sin(2(e+fx))+15\sin(4(e+fx))-5\sin(6(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(420*e + 420*f*x + 240*Cos[e + f*x] + 120*Cos[3*(e + f*x)] + 24*Co
s[5*(e + f*x)] + 255*Sin[2*(e + f*x)] + 15*Sin[4*(e + f*x)] - 5*Sin[6*(e +
f*x)]))/(960*f)
```

**Maple** [A]
time = 0.40, size = 211, normalized size = 1.79

| method | result |
|---|---|
| risch | $\frac{7a^2c^4x}{16}+\frac{c^4a^2\cos(fx+e)}{4f}-\frac{c^4a^2\sin(6fx+6e)}{192f}+\frac{c^4a^2\cos(5fx+5e)}{40f}+\frac{c^4a^2\sin(4fx+4e)}{64f}+\frac{c^4a^2\cos(3fx+3e)}{8f}+$ |
| derivativedivides | $\frac{c^4a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{2c^4a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+}{5}}{}$ |
| default | $\frac{c^4a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{2c^4a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+}{5}}{}$ |
| norman | $\frac{\frac{4c^4a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{8c^4a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4c^4a^2}{5f}+\frac{7a^2c^4x}{16}+\frac{4c^4a^2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{8c^4a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4c^4a^2\left(\tan^2\left(\right.\right.}{5f}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^4*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+
e)+5/16*f*x+5/16*e)+2/5*c^4*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e)-c^4*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4
/3*c^4*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)-c^4*a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+
1/2*f*x+1/2*e)+2*c^4*a^2*cos(f*x+e)+c^4*a^2*(f*x+e))
```

## **Maxima** [A]

time = 0.30, size = 225, normalized size = 1.91

$$\frac{128\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^2c^4+1280\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2c^4+5\left(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e)\right)a^2c^4-30\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^2c^4-240\left(2fx+2e-\sin(2fx+2e)\right)a^2c^4+960(fx+e)a^2c^4+1920a^2c^4\cos(fx+e)}{960f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^4 + 5*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*c^4 - 30*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^4 - 240*(2*f*x
 + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4 + 1920*a^2*c^4*c
os(f*x + e))/f
```

## **Fricas** [A]

time = 0.35, size = 92, normalized size = 0.78

$$\frac{96a^2c^4\cos(fx+e)^5+105a^2c^4fx-5\left(8a^2c^4\cos(fx+e)^5-14a^2c^4\cos(fx+e)^3-21a^2c^4\cos(fx+e)\right)\sin(fx+e)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(96*a^2*c^4*cos(f*x + e)^5 + 105*a^2*c^4*f*x - 5*(8*a^2*c^4*cos(f*x +
 e)^5 - 14*a^2*c^4*cos(f*x + e)^3 - 21*a^2*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(112) = 224$.

time = 0.49, size = 530, normalized size = 4.49

$$\begin{cases} \frac{5a^2c^4x\sin^6(e+fx)}{16} + \frac{15a^2c^4x\sin^4(e+fx)\cos^2(e+fx)}{16} - \frac{3a^2c^4x\sin^4(e+fx)}{8} + \frac{15a^2c^4x\sin^2(e+fx)\cos^4(e+fx)}{16} - \frac{3a^2c^4x\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{a^2c^4x\sin^2(e+fx)}{2} + \frac{5a^2c^4x\cos^6(e+fx)}{16} - \frac{3a^2c^4x\cos^4(e+fx)}{8} - \frac{a^2c^4x\cos^2(e+fx)}{2} + a^2c^4x - \frac{11a^2c^4\sin^5(e+fx)\cos(e+fx)}{16f} + \frac{2a^2c^4\sin^4(e+fx)\cos(e+fx)}{f} - \frac{5a^2c^4\sin^3(e+fx)\cos^3(e+fx)}{6f} + \frac{5a^2c^4\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{8a^2c^4\sin^2(e+fx)\cos^3(e+fx)}{3f} - \frac{4a^2c^4\sin^2(e+fx)\cos(e+fx)}{f} - \frac{5a^2c^4\sin(e+fx)\cos^5(e+fx)}{16f} + \frac{3a^2c^4\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{a^2c^4\sin(e+fx)\cos(e+fx)}{2f} + \frac{16a^2c^4\cos^5(e+fx)}{15f} - \frac{8a^2c^4\cos^3(e+fx)}{3f} + \frac{2a^2c^4\cos(e+fx)}{f} & \text{for}\ f \neq 0 \\ x(a\sin(e)+a)^2(-c\sin(e)+c)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*a**2*c**4*x*sin(e + f*x)**6/16 + 15*a**2*c**4*x*sin(e + f*x)**
4*cos(e + f*x)**2/16 - 3*a**2*c**4*x*sin(e + f*x)**4/8 + 15*a**2*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 - 3*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 - a**2*c**4*x*sin(e + f*x)**2/2 + 5*a**2*c**4*x*cos(e + f*x)**6/16 -
 3*a**2*c**4*x*cos(e + f*x)**4/8 - a**2*c**4*x*cos(e + f*x)**2/2 + a**2*c**
4*x - 11*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**4*sin(e
+ f*x)**4*cos(e + f*x)/f - 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f
) + 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f -
5*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*a**2*c**4*sin(e + f*x)*
cos(e + f*x)**3/(8*f) + a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2
*c**4*cos(e + f*x)**5/(15*f) - 8*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*a**2*c
**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c)**4, Tru
e))
```

**Giac** [A]

time = 0.45, size = 133, normalized size = 1.13

$$\frac{7}{16}\,a^2c^4x+\frac{a^2c^4\cos\left(5\,fx+5\,e\right)}{40\,f}+\frac{a^2c^4\cos\left(3\,fx+3\,e\right)}{8\,f}+\frac{a^2c^4\cos\left(fx+e\right)}{4\,f}-\frac{a^2c^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{a^2c^4\sin\left(4\,fx+4\,e\right)}{64\,f}+\frac{17\,a^2c^4\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/16*a^2*c^4*x + 1/40*a^2*c^4*cos(5*f*x + 5*e)/f + 1/8*a^2*c^4*cos(3*f*x +
3*e)/f + 1/4*a^2*c^4*cos(f*x + e)/f - 1/192*a^2*c^4*sin(6*f*x + 6*e)/f + 1/
64*a^2*c^4*sin(4*f*x + 4*e)/f + 17/64*a^2*c^4*sin(2*f*x + 2*e)/f
```

**Mupad** [B]

time = 8.90, size = 284, normalized size = 2.41

$$\frac{a^2\,c^4\,\left(105\,e+270\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+192\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+890\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+1920\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-660\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+1920\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+660\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+960\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-890\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+960\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-270\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+105\,f\,x+630\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(e+f\,x\right)+1575\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(e+f\,x\right)+2100\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(e+f\,x\right)+1575\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(e+f\,x\right)+630\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(e+f\,x\right)+105\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}\,\left(e+f\,x\right)+192\right)}{240\,f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (a^2*c^4*(105*e + 270*tan(e/2 + (f*x)/2) + 192*tan(e/2 + (f*x)/2)^2 + 890*t
an(e/2 + (f*x)/2)^3 + 1920*tan(e/2 + (f*x)/2)^4 - 660*tan(e/2 + (f*x)/2)^5
+ 1920*tan(e/2 + (f*x)/2)^6 + 660*tan(e/2 + (f*x)/2)^7 + 960*tan(e/2 + (f*x
)/2)^8 - 890*tan(e/2 + (f*x)/2)^9 + 960*tan(e/2 + (f*x)/2)^10 - 270*tan(e/2
 + (f*x)/2)^11 + 105*f*x + 630*tan(e/2 + (f*x)/2)^2*(e + f*x) + 1575*tan(e/
2 + (f*x)/2)^4*(e + f*x) + 2100*tan(e/2 + (f*x)/2)^6*(e + f*x) + 1575*tan(e
/2 + (f*x)/2)^8*(e + f*x) + 630*tan(e/2 + (f*x)/2)^10*(e + f*x) + 105*tan(e
/2 + (f*x)/2)^12*(e + f*x) + 192))/(240*f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

# 3.238 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=85

$$\frac{3}{8}a^2c^3x+\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^3\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 3/8*a^2*c^3*x+1/5*a^2*c^3*cos(f*x+e)^5/f+3/8*a^2*c^3*cos(f*x+e)*sin(f*x+e)/
f+1/4*a^2*c^3*cos(f*x+e)^3*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2815, 2748, 2715, 8}

$$\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*x)/8 + (a^2*c^3*Cos[e + f*x]^5)/(5*f) + (3*a^2*c^3*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^2*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\left(a^2c^3\right)\int \cos^4(e+fx)\,dx\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}\left(3a^2c\right.\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^3\,\mathrm{co}}{}\\ &= \frac{3}{8}a^2c^3x+\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f}\end{aligned}$$

**Mathematica [A]**
time = 1.02, size = 69, normalized size = 0.81

$$\frac{a^2c^3(60e+60fx+20\cos(e+fx)+10\cos(3(e+fx))+2\cos(5(e+fx))+40\sin(2(e+fx))+5\sin(4(e+fx)))}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(60*e + 60*f*x + 20*Cos[e + f*x] + 10*Cos[3*(e + f*x)] + 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(77) = 154$.
time = 0.31, size = 159, normalized size = 1.87

| method | result |
|---|---|
| risch | $\frac{3a^2c^3x}{8}+\frac{c^3a^2\cos(fx+e)}{8f}+\frac{c^3a^2\cos(5fx+5e)}{80f}+\frac{c^3a^2\sin(4fx+4e)}{32f}+\frac{c^3a^2\cos(3fx+3e)}{16f}+\frac{c^3a^2\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{\frac{c^3a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+c^3a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2c^3a^2\left(2+\sin^2(\right.}{}}{f}$ |
| default | $\frac{\frac{c^3a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+c^3a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2c^3a^2\left(2+\sin^2(\right.}{}}{f}$ |

| | |
|---|---|
| norman | $\frac{\frac{2c^3a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2c^3a^2}{5f}+\frac{3a^2c^3x}{8}+\frac{4c^3a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{15a^2c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{15a^2c^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+15a^2c^3}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*c^3*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^3*a^2*(-1
/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*c^3*a^2*(2+s
in(f*x+e)^2)*cos(f*x+e)-2*c^3*a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e
)+c^3*a^2*cos(f*x+e)+c^3*a^2*(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.
time = 0.28, size = 170, normalized size = 2.00

$$\frac{32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)a^2c^3+320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c^3+15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^2c^3-240\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c^3+480\left(fx+e\right)a^2c^3+480a^2c^3\cos\left(fx+e\right)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^3
+ 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3 + 480*a^2*c^3*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.33, size = 75, normalized size = 0.88

$$\frac{8\,a^2c^3\cos\left(fx+e\right)^5+15\,a^2c^3fx+5\left(2\,a^2c^3\cos\left(fx+e\right)^3+3\,a^2c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/40*(8*a^2*c^3*cos(f*x + e)^5 + 15*a^2*c^3*f*x + 5*(2*a^2*c^3*cos(f*x + e)
^3 + 3*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(80) = 160$.
time = 0.33, size = 340, normalized size = 4.00

$$\begin{cases}\frac{3a^2c^3x\sin^4\left(e+fx\right)}{8}+\frac{3a^2c^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^2c^3x\sin^2\left(e+fx\right)+\frac{3a^2c^3x\cos^4\left(e+fx\right)}{8}-a^2c^3x\cos^2\left(e+fx\right)+a^2c^3x+\frac{a^2c^3\sin^4\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{5a^2c^3\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{4a^2c^3\sin^2\left(e+fx\right)\cos^3\left(e+fx\right)}{3f}-\frac{2a^2c^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^2c^3\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f}+\frac{a^2c^3\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{8a^2c^3\cos^5\left(e+fx\right)}{15f}-\frac{4a^2c^3\cos^3\left(e+fx\right)}{3f}+\frac{a^2c^3\cos\left(e+fx\right)}{f} & \text{for}\: f\neq 0\\x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^3 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**2*c**3*x*sin(e + f*x)**4/8 + 3*a**2*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**3*x*sin(e + f*x)**2 + 3*a**2*c**3*x*cos(e + f*x
)**4/8 - a**2*c**3*x*cos(e + f*x)**2 + a**2*c**3*x + a**2*c**3*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 4*a**
2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*a**2*c**3*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**2*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**2*c**3
*sin(e + f*x)*cos(e + f*x)/f + 8*a**2*c**3*cos(e + f*x)**5/(15*f) - 4*a**2*
c**3*cos(e + f*x)**3/(3*f) + a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**2*(-c*sin(e) + c)**3, True))
```

**Giac** [A]
time = 0.47, size = 112, normalized size = 1.32

$$\frac{3}{8}\,a^2c^3x+\frac{a^2c^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{a^2c^3\cos\left(3\,fx+3\,e\right)}{16\,f}+\frac{a^2c^3\cos\left(fx+e\right)}{8\,f}+\frac{a^2c^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a^2c^3\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 3/8*a^2*c^3*x + 1/80*a^2*c^3*cos(5*f*x + 5*e)/f + 1/16*a^2*c^3*cos(3*f*x +
3*e)/f + 1/8*a^2*c^3*cos(f*x + e)/f + 1/32*a^2*c^3*sin(4*f*x + 4*e)/f + 1/4
*a^2*c^3*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 10.00, size = 220, normalized size = 2.59

$$\frac{3\,a^2\,c^3\,x}{8}+\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{a^2\,c^3\,\left(75\,e+75\,f\,x+80\right)}{40}-\frac{15\,a^2\,c^3\,\left(e+f\,x\right)}{8}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{a^2\,c^3\,\left(150\,e+150\,f\,x+160\right)}{40}-\frac{15\,a^2\,c^3\,\left(e+f\,x\right)}{4}\right)+\frac{a^2\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{2}-\frac{a^2\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{2}-\frac{5\,a^2\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{4}+\frac{a^2\,c^3\,\left(15\,e+15\,f\,x+16\right)}{40}+\frac{5\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}-\frac{3\,a^2\,c^3\,\left(e+f\,x\right)}{8}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (3*a^2*c^3*x)/8 + (tan(e/2 + (f*x)/2)^8*((a^2*c^3*(75*e + 75*f*x + 80))/40
- (15*a^2*c^3*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^3*(150*e + 150*f
*x + 160))/40 - (15*a^2*c^3*(e + f*x))/4) + (a^2*c^3*tan(e/2 + (f*x)/2)^3)/
2 - (a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (5*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4 +
 (a^2*c^3*(15*e + 15*f*x + 16))/40 + (5*a^2*c^3*tan(e/2 + (f*x)/2))/4 - (3*
a^2*c^3*(e + f*x))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^5)
```

# 3.239 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=64

$$\frac{3}{8}a^2c^2x+\frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 3/8*a^2*c^2*x+3/8*a^2*c^2*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2*c^2*cos(f*x+e)^3*
sin(f*x+e)/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2715, 8}

$$\frac{a^2c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*x)/8 + (3*a^2*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a^2*c^2*Co
s[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)\,dx\\ &= \frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}\left(3a^2c^2\right)\int \cos^2(e+fx)\,dx\\ &= \frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^2\cos^3(e+fx)\sin(e+f}{4f}\\ &= \frac{3}{8}a^2c^2x+\frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^2\cos^3(e+fx)}{4f}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 39, normalized size = 0.61

$$\frac{a^2c^2(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(32*f)
```

**Maple** [A]
time = 0.21, size = 88, normalized size = 1.38

| method | result |
|---|---|
| risch | $\frac{3a^2c^2x}{8}+\frac{c^2a^2\sin(4fx+4e)}{32f}+\frac{c^2a^2\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{c^2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2c^2a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+c^2a^2(fx+e)}{f}$ |
| default | $\frac{c^2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2c^2a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+c^2a^2(fx+e)}{f}$ |
| norman | $\frac{\frac{3a^2c^2x}{8}+\frac{3a^2c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{9a^2c^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{3a^2c^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a^2c^2x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{5c^2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{4f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^2*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
2*c^2*a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+c^2*a^2*(f*x+e))
```

**Maxima** [A]
time = 0.28, size = 87, normalized size = 1.36

$$\frac{(12\,fx + 12\,e + \sin(4\,fx + 4\,e) - 8\,\sin(2\,fx + 2\,e))a^2c^2 - 16\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))a^2c^2 + 32\,(fx + e)a^2c^2}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/32*((12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2 - 16*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 32*(f*x + e)*a^2*c^2)/f
```

**Fricas** [A]
time = 0.33, size = 57, normalized size = 0.89

$$\frac{3\,a^2c^2fx + \left(2\,a^2c^2\cos\left(fx + e\right)^3 + 3\,a^2c^2\cos\left(fx + e\right)\right)\sin\left(fx + e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*c^2*f*x + (2*a^2*c^2*cos(f*x + e)^3 + 3*a^2*c^2*cos(f*x + e))*si
n(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(61) = 122$.
time = 0.20, size = 206, normalized size = 3.22

$$\begin{cases} \frac{3a^2c^2x\sin^4(e+fx)}{8} + \frac{3a^2c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - a^2c^2x\sin^2(e+fx) + \frac{3a^2c^2x\cos^4(e+fx)}{8} - a^2c^2x\cos^2(e+fx) + a^2c^2x - \frac{5a^2c^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3a^2c^2\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{a^2c^2\sin(e+fx)\cos(e+fx)}{f} & \text{for}\ f \neq 0 \\ x(a\sin(e)+a)^2(-c\sin(e)+c)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*c**2*x*sin(e + f*x)**4/8 + 3*a**2*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**2*x*sin(e + f*x)**2 + 3*a**2*c**2*x*cos(e + f*x
)**4/8 - a**2*c**2*x*cos(e + f*x)**2 + a**2*c**2*x - 5*a**2*c**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 3*a**2*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) +
 a**2*c**2*sin(e + f*x)*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c
*sin(e) + c)**2, True))
```

**Giac** [A]
time = 0.43, size = 52, normalized size = 0.81

$$\frac{3}{8}\,a^2c^2x + \frac{a^2c^2\sin\left(4\,fx + 4\,e\right)}{32\,f} + \frac{a^2c^2\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*c^2*x + 1/32*a^2*c^2*sin(4*f*x + 4*e)/f + 1/4*a^2*c^2*sin(2*f*x + 2
*e)/f
```

**Mupad [B]**
time = 6.74, size = 36, normalized size = 0.56

$$\frac{a^2\,c^2\,\left(8\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+12\,f\,x\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^2*c^2*(8*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) + 12*f*x))/(32*f)
```

# 3.240 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

**Optimal**. Leaf size=52

$$\frac{1}{2}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f+1/2*a^2*c*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2815, 2748, 2715, 8}

$$-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/2 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx &= (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx\\ &= -\frac{a^2c\cos^3(e+fx)}{3f}+\left(a^2c\right)\int \cos^2(e+fx)\,dx\\ &= -\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int\\ &= \frac{1}{2}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

**Mathematica [A]**
time = 0.23, size = 43, normalized size = 0.83

$$-\frac{a^2c(3\cos(e+fx)+\cos(3(e+fx))-3(2fx+\sin(2(e+fx))))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/12*(a^2*c*(3*Cos[e + f*x] + Cos[3*(e + f*x)] - 3*(2*f*x + Sin[2*(e + f*x
)])))/f
```

**Maple [A]**
time = 0.18, size = 78, normalized size = 1.50

| method | result |
|---|---|
| risch | $\frac{a^2cx}{2}-\frac{a^2c\cos(fx+e)}{4f}-\frac{a^2c\cos(3fx+3e)}{12f}+\frac{a^2c\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{\frac{a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2c\cos(fx+e)+a^2c(fx+e)}{f}$ |
| default | $\frac{\frac{a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2c\cos(fx+e)+a^2c(fx+e)}{f}$ |
| norman | $\frac{\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a^2c}{3f}+\frac{a^2cx}{2}-\frac{2a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a^2cx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/3*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^2*c*cos(f*x+e)+a^2*c*(f*x+e))

**Maxima** [A]
time = 0.29, size = 83, normalized size = 1.60

$$-\frac{4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)a^{2}c+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^{2}c-12\left(fx+e\right)a^{2}c+12\,a^{2}c\cos\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c - 12*(f*x + e)*a^2*c + 12*a^2*c*cos(f*x + e))/f

**Fricas** [A]
time = 0.34, size = 49, normalized size = 0.94

$$-\frac{2\,a^{2}c\cos\left(fx+e\right)^{3}-3\,a^{2}cfx-3\,a^{2}c\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] -1/6*(2*a^2*c*cos(f*x + e)^3 - 3*a^2*c*f*x - 3*a^2*c*cos(f*x + e)*sin(f*x + e))/f

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(46) = 92.
time = 0.13, size = 133, normalized size = 2.56

$$\begin{cases}-\frac{a^{2}cx\sin^{2}\left(e+fx\right)}{2}-\frac{a^{2}cx\cos^{2}\left(e+fx\right)}{2}+a^{2}cx+\frac{a^{2}c\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{a^{2}c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2a^{2}c\cos^{3}\left(e+fx\right)}{3f}-\frac{a^{2}c\cos\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\left(a\sin\left(e\right)+a\right)^{2}\left(-c\sin\left(e\right)+c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)

[Out] Piecewise((-a**2*c*x*sin(e + f*x)**2/2 - a**2*c*x*cos(e + f*x)**2/2 + a**2*c*x + a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*cos(e + f*x)**3/(3*f) - a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c), True))

**Giac** [A]
time = 0.41, size = 62, normalized size = 1.19

$$\frac{1}{2}\,a^{2}cx-\frac{a^{2}c\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^{2}c\cos\left(fx+e\right)}{4\,f}+\frac{a^{2}c\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c*x - 1/12*a^2*c*cos(3*f*x + 3*e)/f - 1/4*a^2*c*cos(f*x + e)/f + 1/
4*a^2*c*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 9.02, size = 125, normalized size = 2.40

$$\frac{a^2\,c\,x}{2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{3\,a^2\,c\,\left(e+f\,x\right)}{2}-\frac{a^2\,c\,\left(9\,e+9\,f\,x-12\right)}{6}\right)-a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a^2\,c\,\left(e+f\,x\right)}{2}+a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{a^2\,c\,\left(3\,e+3\,f\,x-4\right)}{6}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((3*a^2*c*(e + f*x))/2 - (a^2*c*(9*e +
9*f*x - 12))/6) - a^2*c*tan(e/2 + (f*x)/2) + (a^2*c*(e + f*x))/2 + a^2*c*ta
n(e/2 + (f*x)/2)^5 - (a^2*c*(3*e + 3*f*x - 4))/6)/(f*(tan(e/2 + (f*x)/2)^2
+ 1)^3)
```

# 3.241 $\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=57

$$-\frac{3a^2x}{c}+\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}$$

```
[Out] -3*a^2*x/c+3*a^2*cos(f*x+e)/c/f+2*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2815, 2759, 2761, 8}

$$\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{3a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*x)/c + (3*a^2*Cos[e + f*x])/(c*f) + (2*a^2*c*Cos[e + f*x]^3)/(f*(c
 - c*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\left(3a^2\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx\\ &= \frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{\left(3a^2\right)\int 1\,dx}{c}\\ &= -\frac{3a^2x}{c}+\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 130 vs. 2(57) = 114.
time = 0.27, size = 130, normalized size = 2.28

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(3(e+fx)-\cos(e+fx))+(-8-3e-3fx+\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x) -
 Cos[e + f*x]) + (-8 - 3*e - 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(1 + S
in[e + f*x])^2)/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e +
f*x]))
```

**Maple [A]**
time = 0.29, size = 55, normalized size = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{2a^2\left(\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$ |
| default | $\frac{2a^2\left(\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$ |
| risch | $-\frac{3a^2x}{c}+\frac{a^2e^{i(fx+e)}}{2cf}+\frac{a^2e^{-i(fx+e)}}{2cf}+\frac{8a^2}{fc\left(e^{i(fx+e)}-i\right)}$ |

| | |
|---|---|
| norman | $\frac{\frac{3a^2x}{c}-\frac{8a^2}{cf}-\frac{3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{6a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{6a^2x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{3a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{3a^2x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c*(1/(1+tan(1/2*f*x+1/2*e)^2)-3*arctan(tan(1/2*f*x+1/2*e))-4/(tan(1
/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(61) = 122$.
time = 0.53, size = 236, normalized size = 4.14

$$-\frac{2\left(a^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+2a^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{a^2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + 2*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
 - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - a^2/(c - c*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

**Fricas** [A]
time = 0.32, size = 111, normalized size = 1.95

$$-\frac{3a^2fx-a^2\cos(fx+e)^2-4a^2+(3a^2fx-5a^2)\cos(fx+e)-(3a^2fx-a^2\cos(fx+e)+4a^2)\sin(fx+e)}{cf\cos(fx+e)-cf\sin(fx+e)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*a^2*f*x - a^2*cos(f*x + e)^2 - 4*a^2 + (3*a^2*f*x - 5*a^2)*cos(f*x + e)
 - (3*a^2*f*x - a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) -
 c*f*sin(f*x + e) + c*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(51) = 102$.
time = 1.17, size = 454, normalized size = 7.96

$$\begin{cases} -\frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{3a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{3a^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{3a^2fx}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{8a^2\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{2a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{10a^2}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} & \text{for}\: f \neq 0 \\ \frac{x(a\sin(e)+a)^2}{-c\sin(e)+c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**2*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x*tan(e/2 + f*x
/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f
*x/2) - c*f) - 3*a**2*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x/(c*f*tan(e/2
+ f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 8*a**
2*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
c*f*tan(e/2 + f*x/2) - c*f) + 2*a**2*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)
**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 10*a**2/(c*f*
tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f)
, Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c), True))
```

**Giac [A]**
time = 0.45, size = 103, normalized size = 1.81

$$-\frac{\frac{3\,(fx+e)a^2}{c}+\frac{2\left(4\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*a^2/c + 2*(4*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2*tan(1/2*f*x + 1
/2*e) + 5*a^2)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) - 1)*c))/f
```

**Mupad [B]**
time = 6.91, size = 118, normalized size = 2.07

$$\frac{3\,\sqrt{2}\,a^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(e+f\,x\right)-\frac{\sqrt{2}\,a^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,e+6\,f\,x-16\right)}{2}}{c\,f\,\left(\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sqrt{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{3\,a^2\,x}{c}+\frac{2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x)),x)
```

```
[Out] (3*2^(1/2)*a^2*sin(e/2 + (f*x)/2)*(e + f*x) - (2^(1/2)*a^2*sin(e/2 + (f*x)/
2)*(6*e + 6*f*x - 16))/2)/(c*f*(2^(1/2)*cos(e/2 + (f*x)/2) - 2^(1/2)*sin(e/
2 + (f*x)/2))) - (3*a^2*x)/c + (2*a^2*cos(e/2 + (f*x)/2)^2)/(c*f)
```

# 3.242 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=72

$$\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}$$

```
[Out] a^2*x/c^2+2/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3-2*a^2*cos(f*x+e)/f/(c
^2-c^2*sin(f*x+e))
```

**Rubi** [A]

time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {2815, 2759, 8}

$$-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}+\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) - (2*a^
2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)} + \frac{a^2\int 1\,dx}{c^2} \\
&= \frac{a^2x}{c^2} + \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}
$$

**Mathematica** [A]
time = 0.43, size = 121, normalized size = 1.68

$$
-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(8+3e+3fx)\cos\left(\frac{1}{2}(e+fx)\right)+(16+3e+3fx)\cos\left(\frac{3}{2}(e+fx)\right)+6(2(2+e+fx)+(e+fx)\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^2f(-1+\sin(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/6*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-3*(8 + 3*e + 3*f*x)*Cos[(
e + f*x)/2] + (16 + 3*e + 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(2 + e + f*x)
+ (e + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(c^2*f*(-1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.33, size = 53, normalized size = 0.74

| method | result |
|---|---|
| derivativedivides | $\frac{2a^2\left(-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^2}$ |
| default | $\frac{2a^2\left(-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^2}$ |
| risch | $\frac{a^2x}{c^2}-\frac{8\left(-3ia^2e^{i(fx+e)}+3a^2e^{2i(fx+e)}-2a^2\right)}{3\left(e^{i(fx+e)}-i\right)^3f\,c^2}$ |
| norman | $\frac{\frac{a^2x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{a^2x}{c}+\frac{8a^2}{3cf}+\frac{3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{5a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{7a^2x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{7a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+5a^2 \cdots}{\left(1+\tan^2\left(\cdots\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^2*(-8/3/(tan(1/2*f*x+1/2*e)-1)^3-4/(tan(1/2*f*x+1/2*e)-1)^2+arcta
n(tan(1/2*f*x+1/2*e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(76) = 152$.

time = 0.55, size = 394, normalized size = 5.47

$$\frac{2\left(a^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{a^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{2a^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.

time = 0.32, size = 166, normalized size = 2.31

$$-\frac{6\,a^2fx-(3\,a^2fx+8\,a^2)\cos(fx+e)^2+4\,a^2+(3\,a^2fx-4\,a^2)\cos(fx+e)-(6\,a^2fx-4\,a^2+(3\,a^2fx-8\,a^2)\cos(fx+e))\sin(fx+e)}{3\,(c^2f\cos(fx+e)^2-c^2f\cos(fx+e)-2\,c^2f+(c^2f\cos(fx+e)+2\,c^2f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^2*f*x - (3*a^2*f*x + 8*a^2)*cos(f*x + e)^2 + 4*a^2 + (3*a^2*f*x -
 4*a^2)*cos(f*x + e) - (6*a^2*f*x - 4*a^2 + (3*a^2*f*x - 8*a^2)*cos(f*x + e
))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^
2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(65) = 130$.

time = 2.56, size = 473, normalized size = 6.57

$$\begin{cases}\frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{9a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}+\frac{9a^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{3a^2fx}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{24a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}+\frac{8a^2}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} & \text{for}\: f \neq 0 \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9
*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*a**
```

```
2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*a**2*f*x*tan(e/2 +
f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**3
 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2
4*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*a**2/(3*c**2*f*tan(e/
2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

**Giac** [A]
time = 0.42, size = 60, normalized size = 0.83

$$\frac{\frac{3\,(fx+e)a^2}{c^2}-\frac{8\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 - 8*(3*a^2*tan(1/2*f*x + 1/2*e) - a^2)/(c^2*(tan(1
/2*f*x + 1/2*e) - 1)^3))/f
```

**Mupad** [B]
time = 6.87, size = 90, normalized size = 1.25

$$\frac{a^2\,x}{c^2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,a^2\,\left(e+f\,x\right)-\frac{a^2\,\left(9\,e+9\,f\,x-24\right)}{3}\right)-a^2\,\left(e+f\,x\right)+\frac{a^2\,\left(3\,e+3\,f\,x-8\right)}{3}}{c^2\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^2*x)/c^2 - (tan(e/2 + (f*x)/2)*(3*a^2*(e + f*x) - (a^2*(9*e + 9*f*x - 24
))/3) - a^2*(e + f*x) + (a^2*(3*e + 3*f*x - 8))/3)/(c^2*f*(tan(e/2 + (f*x)/
2) - 1)^3)
```

# 3.243 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=34

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[Out] 1/5*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

**Rubi** [A]
time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2815, 2750}

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(34) = 68.
time = 0.28, size = 81, normalized size = 2.38

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-10\sin\left(\frac{1}{2}(e+fx)\right)-5\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)\right)}{10c^3f(-1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-10*Sin[(e + f*x)/2] - 5*Sin[(3
*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(10*c^3*f*(-1 + Sin[e + f*x])^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(32) = 64.
time = 0.38, size = 88, normalized size = 2.59

| method | result | size |
|---|---|---|
| risch | $\frac{2a^2e^{4i(fx+e)}-4a^2e^{2i(fx+e)}+\frac{2a^2}{5}}{\left(e^{i(fx+e)}-i\right)^5 f\,c^3}$ | 55 |
| derivativedivides | $\frac{2a^2\left(-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}\right)}{f\,c^3}$ | 88 |
| default | $\frac{2a^2\left(-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}\right)}{f\,c^3}$ | 88 |
| norman | $\frac{-\frac{2a^2}{5cf}-\frac{24a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cf}-\frac{52a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cf}-\frac{8a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$ | 133 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^3*(-8/(tan(1/2*f*x+1/2*e)-1)^3-16/5/(tan(1/2*f*x+1/2*e)-1)^5-1/(t
an(1/2*f*x+1/2*e)-1)-4/(tan(1/2*f*x+1/2*e)-1)^2-8/(tan(1/2*f*x+1/2*e)-1)^4)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(35) = 70.
time = 0.40, size = 605, normalized size = 17.79

$$-\frac{2\left(\frac{a^2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{6a^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2a^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 6*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2
*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(35) = 70$.
time = 0.33, size = 180, normalized size = 5.29

$$\frac{a^2\cos(fx+e)^3 + 3a^2\cos(fx+e)^2 - 2a^2\cos(fx+e) - 4a^2 + (a^2\cos(fx+e)^2 - 2a^2\cos(fx+e) - 4a^2)\sin(fx+e)}{5\left(c^3f\cos(fx+e)^3 + 3c^3f\cos(fx+e)^2 - 2c^3f\cos(fx+e) - 4c^3f - (c^3f\cos(fx+e)^2 - 2c^3f\cos(fx+e) - 4c^3f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2
 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))/(c^3*f*c
os(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(29) = 58$.
time = 5.58, size = 354, normalized size = 10.41

$$\begin{cases} -\frac{10a^2\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-50c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5c^3f} - \frac{20a^2\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-50c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5c^3f} - \frac{2a^2}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-50c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5c^3f} & \text{for } f \neq 0 \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*a**2*tan(e/2 + f*x/2)**4/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*
c**3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(
e/2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f) - 20*a**2*tan(e/2
+ f*x/2)**2/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c**3*f*tan(e/2 + f*x/2)**4 +
 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e/2 + f*x/2)**2 + 25*c**3*f*
tan(e/2 + f*x/2) - 5*c**3*f) - 2*a**2/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c*
```

```
*3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e/
2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f), Ne(f, 0)), (x*(a*si
n(e) + a)**2/(-c*sin(e) + c)**3, True))
```

**Giac** [A]
time = 0.48, size = 60, normalized size = 1.76

$$-\frac{2\left(5\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{5\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2*tan(1/2*f*x + 1/2*e)^4 + 10*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(
c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

**Mupad** [B]
time = 6.99, size = 92, normalized size = 2.71

$$\frac{2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left({\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\right)}{5\,c^3\,f\,{\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*a^2*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^4 + 5*sin(e/2 + (f*x)/2)^4 +
10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(5*c^3*f*(cos(e/2 + (f*x)/2)
 - sin(e/2 + (f*x)/2))^5)
```

# 3.244 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=67

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*c*cos(f*x+e)^5/f/(c-
c*sin(f*x+e))^5
```

**Rubi** [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*c*Cos[e + f*x]
^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}\end{aligned}$$

**Mathematica [A]**
time = 0.43, size = 117, normalized size = 1.75

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-35\cos\left(\frac{1}{2}(e+fx)\right)+14\cos\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)-70\sin\left(\frac{1}{2}(e+fx)\right)-35\sin\left(\frac{3}{2}(e+fx)\right)+7\sin\left(\frac{5}{2}(e+fx)\right)\right)}{140c^4f(-1+\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
[Out] -1/140*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-35*Cos[(e + f*x)/2] + 1
4*Cos[(3*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 70*Sin[(e + f*x)/2] - 35*Si
n[(3*(e + f*x))/2] + 7*Sin[(5*(e + f*x))/2]))/(c^4*f*(-1 + Sin[e + f*x])^4)
```

**Maple [A]**
time = 0.39, size = 118, normalized size = 1.76

| method | result |
|---|---|
| risch | $-\frac{2ia^2\left(35i e^{4i(fx+e)}+35\,e^{5i(fx+e)}-14i e^{2i(fx+e)}-70\,e^{3i(fx+e)}-i+7\,e^{i(fx+e)}\right)}{35f\,c^4\left(e^{i(fx+e)}-i\right)^7}$ |
| derivativedivides | $\frac{2a^2\left(-\frac{24}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{14}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,c^4}$ |
| default | $\frac{2a^2\left(-\frac{24}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{14}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,c^4}$ |
| norman | $\frac{\frac{2a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{12a^2}{35cf}-\frac{2a^2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5cf}+\frac{8a^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{12a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{24a^2\left(\tan^3\left(\right.\right.}{5cf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2c^3\left(\tan\left(\frac{fx}{2}\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^4*(-24/(tan(1/2*f*x+1/2*e)-1)^4-32/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(
tan(1/2*f*x+1/2*e)-1)-16/(tan(1/2*f*x+1/2*e)-1)^6-14/(tan(1/2*f*x+1/2*e)-1)
^3-128/5/(tan(1/2*f*x+1/2*e)-1)^5-5/(tan(1/2*f*x+1/2*e)-1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(69) = 138$.
time = 0.32, size = 888, normalized size = 13.25

$$\frac{2\left(\frac{2\,a^{2}\left(\frac{91\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{168\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{280\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{175\,\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{105\,\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-13\right)}{c^{4}-\frac{7\,c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\,c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35\,c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{35\,c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{21\,c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{7\,c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}-\frac{3\,a^{2}\left(\frac{49\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{147\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{210\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{210\,\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{105\,\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{35\,\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-12\right)}{c^{4}-\frac{7\,c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\,c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35\,c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{35\,c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{21\,c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{7\,c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}-\frac{4\,a^{2}\left(\frac{14\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{42\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{35\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{35\,\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-2\right)}{c^{4}-\frac{7\,c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\,c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35\,c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{35\,c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{21\,c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{7\,c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(2*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7) - 3*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3
5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^2*(14*sin(f*x
 + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(69) = 138$.
time = 0.33, size = 238, normalized size = 3.55

$$-\frac{a^{2}\cos(fx+e)^{4}+4\,a^{2}\cos(fx+e)^{3}+13\,a^{2}\cos(fx+e)^{2}-10\,a^{2}\cos(fx+e)-20\,a^{2}-(a^{2}\cos(fx+e)^{3}-3\,a^{2}\cos(fx+e)^{2}+10\,a^{2}\cos(fx+e)+20\,a^{2})\sin(fx+e)}{35\,(c^{4}f\cos(fx+e)^{4}-3\,c^{4}f\cos(fx+e)^{3}-8\,c^{4}f\cos(fx+e)^{2}+4\,c^{4}f\cos(fx+e)+8\,c^{4}f+(c^{4}f\cos(fx+e)^{3}+4\,c^{4}f\cos(fx+e)^{2}-4\,c^{4}f\cos(fx+e)-8\,c^{4}f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(a^2*cos(f*x + e)^4 + 4*a^2*cos(f*x + e)^3 + 13*a^2*cos(f*x + e)^2 -
10*a^2*cos(f*x + e) - 20*a^2 - (a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 +
 10*a^2*cos(f*x + e) + 20*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*
f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f
+ (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8
*c^4*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(58) = 116$.
time = 10.86, size = 1074, normalized size = 16.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 - 24
5*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f
*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2
 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*a**2*tan(e/2 +
 f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6
+ 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c
**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan
(e/2 + f*x/2) - 35*c**4*f) - 280*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/
2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2
)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 -
735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) +
 140*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*t
an(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 +
 f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)*
*2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*a**2*tan(e/2 + f*x/2)**
2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**
4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan
(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*
x/2) - 35*c**4*f) + 14*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7
 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c
**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*ta
n(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 12*a**2/(35*
c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*ta
n(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 +
 f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) -
 35*c**4*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))
```

**Giac** [A]
time = 0.45, size = 128, normalized size = 1.91

$$-\frac{2\left(35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+140\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-70\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+91\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-7\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,a^{2}\right)}{35\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*
a^2*tan(1/2*f*x + 1/2*e)^4 - 70*a^2*tan(1/2*f*x + 1/2*e)^3 + 91*a^2*tan(1/2
*f*x + 1/2*e)^2 - 7*a^2*tan(1/2*f*x + 1/2*e) + 6*a^2)/(c^4*f*(tan(1/2*f*x +
 1/2*e) - 1)^7)
```

**Mupad** [B]
time = 7.30, size = 99, normalized size = 1.48

$$\frac{\sqrt{2}\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{105\,\sin\left(e+f\,x\right)}{8}-\frac{27\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{121\,\cos\left(e+f\,x\right)}{8}+\frac{7\,\sin\left(2\,e+2\,f\,x\right)}{2}+\frac{7\,\sin\left(3\,e+3\,f\,x\right)}{8}+\frac{109}{4}\right)}{280\,c^4\,f\,{\cos\left(\frac{e}{2}+\frac{\pi }{4}+\frac{f\,x}{2}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2^(1/2)*a^2*cos(e/2 + (f*x)/2)*((5*cos(3*e + 3*f*x))/8 - (105*sin(e + f*x)
)/8 - (27*cos(2*e + 2*f*x))/4 - (121*cos(e + f*x))/8 + (7*sin(2*e + 2*f*x))
/2 + (7*sin(3*e + 3*f*x))/8 + 109/4))/(280*c^4*f*cos(e/2 + pi/4 + (f*x)/2)^
7)
```

## 3.245 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=98

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

```
[Out] 1/9*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/63*a^2*c*cos(f*x+e)^5/f/(c-
c*sin(f*x+e))^6+2/315*a^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

**Rubi** [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (2*a^2*c*Cos[e + f*
x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(315*f*(c - c*
Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{1}{9}\left(2a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{1}{63}\left(2a^2\right)\int \frac{\cos^4(e+f}{(c-c\sin(e+}\\ &= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}\end{aligned}$$

**Mathematica [A]**
time = 0.40, size = 121, normalized size = 1.23

$$\frac{a^2\left(315\cos\left(\frac{1}{2}(e+fx)\right)-126\cos\left(\frac{3}{2}(e+fx)\right)-9\cos\left(\frac{7}{2}(e+fx)\right)+441\sin\left(\frac{1}{2}(e+fx)\right)+210\sin\left(\frac{3}{2}(e+fx)\right)-36\sin\left(\frac{5}{2}(e+fx)\right)+\sin\left(\frac{9}{2}(e+fx)\right)\right)}{1260c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
[Out] (a^2*(315*Cos[(e + f*x)/2] - 126*Cos[(3*(e + f*x))/2] - 9*Cos[(7*(e + f*x))
/2] + 441*Sin[(e + f*x)/2] + 210*Sin[(3*(e + f*x))/2] - 36*Sin[(5*(e + f*x)
)/2] + Sin[(9*(e + f*x))/2]))/(1260*c^5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^9)
```

**Maple [A]**
time = 0.43, size = 148, normalized size = 1.51

| method | result |
|---|---|
| risch | $-\frac{4\left(-126ia^2e^{3i(fx+e)}-441a^2e^{4i(fx+e)}-9ia^2e^{i(fx+e)}+36a^2e^{2i(fx+e)}-a^2+315ia^2e^{5i(fx+e)}+210a^2e^{6i(fx+e)}\right)}{315\left(e^{i(fx+e)}-i\right)^9fc^5}$ |
| derivativedivides | $\frac{2a^2\left(-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{404}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{272}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{fc^5}$ |
| default | $\frac{2a^2\left(-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{404}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{272}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{fc^5}$ |

| | |
|---|---|
| norman | $\frac{\frac{24a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{94a^2}{315cf}-\frac{2a^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{4a^2\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{24a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{35cf}-\frac{56a^2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{232a^2\left(\right.}{}}{\left(1+\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^5*(-6/(tan(1/2*f*x+1/2*e)-1)^2-32/(tan(1/2*f*x+1/2*e)-1)^8-1/(tan
(1/2*f*x+1/2*e)-1)-404/5/(tan(1/2*f*x+1/2*e)-1)^5-272/3/(tan(1/2*f*x+1/2*e)
-1)^6-50/(tan(1/2*f*x+1/2*e)-1)^4-64/3/(tan(1/2*f*x+1/2*e)-1)^3-64/9/(tan(1
/2*f*x+1/2*e)-1)^9-480/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(101) = 202$.
time = 0.40, size = 1169, normalized size = 11.93

$$-\frac{2\left(\frac{a^2\left(\frac{432\sin(fx+e)}{\cos(fx+e)+1}-\frac{1728\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3612\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{5418\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5040\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{3360\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1260\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-83\right)}{c^5-\frac{9c^5\sin(fx+e)}{\cos(fx+e)+1}+\frac{36c^5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{84c^5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126c^5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{126c^5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84c^5\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{36c^5\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{9c^5\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{c^5\sin(fx+e)^9}{(\cos(fx+e)+1)^9}}-\frac{10a^2\left(\frac{45\sin(fx+e)}{\cos(fx+e)+1}-\frac{117\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{273\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{315\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{147\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-5\right)}{c^5-\frac{9c^5\sin(fx+e)}{\cos(fx+e)+1}+\frac{36c^5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{84c^5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126c^5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{126c^5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84c^5\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{36c^5\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{9c^5\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{c^5\sin(fx+e)^9}{(\cos(fx+e)+1)^9}}+\frac{14a^2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{36\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{54\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{81\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{45\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{30\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-1\right)}{c^5-\frac{9c^5\sin(fx+e)}{\cos(fx+e)+1}+\frac{36c^5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{84c^5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126c^5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{126c^5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84c^5\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{36c^5\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{9c^5\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{c^5\sin(fx+e)^9}{(\cos(fx+e)+1)^9}}\right)}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 10*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*a^2*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 -
9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e)
```

```
 + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*
c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(101) = 202$.
time = 0.34, size = 298, normalized size = 3.04

$$\frac{2\,a^2\cos(fx+e)^5 - 8\,a^2\cos(fx+e)^4 - 25\,a^2\cos(fx+e)^3 - 85\,a^2\cos(fx+e)^2 + 70\,a^2\cos(fx+e) + 140\,a^2 + \left(2\,a^2\cos(fx+e)^4 + 10\,a^2\cos(fx+e)^3 - 15\,a^2\cos(fx+e)^2 + 70\,a^2\cos(fx+e) + 140\,a^2\right)\sin(fx+e)}{315\left(c^5 f\cos(fx+e)^5 + 5\,c^5 f\cos(fx+e)^4 - 8\,c^5 f\cos(fx+e)^3 - 20\,c^5 f\cos(fx+e)^2 + 8\,c^5 f\cos(fx+e) + 16\,c^5 f - \left(c^5 f\cos(fx+e)^4 - 4\,c^5 f\cos(fx+e)^3 - 12\,c^5 f\cos(fx+e)^2 + 8\,c^5 f\cos(fx+e) + 16\,c^5 f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(2*a^2*cos(f*x + e)^5 - 8*a^2*cos(f*x + e)^4 - 25*a^2*cos(f*x + e)^3
- 85*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e) + 140*a^2 + (2*a^2*cos(f*x +
e)^4 + 10*a^2*cos(f*x + e)^3 - 15*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e)
+ 140*a^2)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8
*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16
*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(88) = 176$.
time = 20.92, size = 1717, normalized size = 17.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 -
2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*
c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*
f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan
(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260*a**2*t
an(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2
)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
+ 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 283
5*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*a**2*tan(e/2 + f*x/2)**6/(31
5*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5
*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*ta
n(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2
 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x
```

```
/2) - 315*c**5*f) + 5040*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x
/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7
 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39
690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c
**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 68
04*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*t
an(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/
2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f
*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)
**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276*a**2*tan(e/2 + f*x/
2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 1
1340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*
c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*
f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(
e/2 + f*x/2) - 315*c**5*f) - 2124*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(
e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 +
f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2
)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3
- 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**
5*f) + 216*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**
5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*t
an(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/
2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f
*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 94*a**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e
/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 +
f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 31
5*c**5*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**5, True))
```

**Giac** [A]

time = 0.46, size = 162, normalized size = 1.65

$$-\frac{2\left(315\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-630\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2310\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-2520\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3402\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1638\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+1062\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-108\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+47\,a^{2}\right)}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*a^2*tan(1/2*f*x + 1/2*e)^7 + 2
310*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*a^2
*tan(1/2*f*x + 1/2*e)^4 - 1638*a^2*tan(1/2*f*x + 1/2*e)^3 + 1062*a^2*tan(1/
2*f*x + 1/2*e)^2 - 108*a^2*tan(1/2*f*x + 1/2*e) + 47*a^2)/(c^5*f*(tan(1/2*f
*x + 1/2*e) - 1)^9)
```

**Mupad** [B]

time = 8.81, size = 121, normalized size = 1.23

$$\frac{\sqrt{2}\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{89\cos(3\,e+3\,f\,x)}{4}-\frac{2205\sin(e+f\,x)}{8}-\frac{265\cos(2\,e+2\,f\,x)}{2}-\frac{625\cos(e+f\,x)}{4}+\frac{49\cos(4\,e+4\,f\,x)}{16}+\frac{567\sin(2\,e+2\,f\,x)}{8}+\frac{243\sin(3\,e+3\,f\,x)}{8}-\frac{45\sin(4\,e+4\,f\,x)}{16}+\frac{4967}{16}\right)}{5040\,c^5\,f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a^2*cos(e/2 + (f*x)/2)*((89*cos(3*e + 3*f*x))/4 - (2205*sin(e + f*
x))/8 - (265*cos(2*e + 2*f*x))/2 - (625*cos(e + f*x))/4 + (49*cos(4*e + 4*f
*x))/16 + (567*sin(2*e + 2*f*x))/8 + (243*sin(3*e + 3*f*x))/8 - (45*sin(4*e
 + 4*f*x))/16 + 4967/16))/(5040*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

# 3.246 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx$

**Optimal**. Leaf size=132

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}$$

```
[Out] 1/11*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/33*a^2*c*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^7+2/231*a^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+2/1155*a^2*cos
(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

**Rubi** [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*c*Cos[e + f*x
]^5)/(33*f*(c - c*Sin[e + f*x])^7) + (2*a^2*Cos[e + f*x]^5)/(231*f*(c - c*S
in[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(1155*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(3a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{1}{33}\left(2a^2\right)\int \frac{\cos^4(e+}{(c-c\sin(e-}$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}$$

**Mathematica [A]**
time = 0.49, size = 133, normalized size = 1.01

$$\frac{a^2\left(2079\cos\left(\frac{1}{2}(e+fx)\right)-825\cos\left(\frac{3}{2}(e+fx)\right)-55\cos\left(\frac{7}{2}(e+fx)\right)+\cos\left(\frac{11}{2}(e+fx)\right)+2541\sin\left(\frac{1}{2}(e+fx)\right)+1155\sin\left(\frac{3}{2}(e+fx)\right)-165\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)\right)}{9240c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
[Out] (a^2*(2079*Cos[(e + f*x)/2] - 825*Cos[(3*(e + f*x))/2] - 55*Cos[(7*(e + f*x
))/2] + Cos[(11*(e + f*x))/2] + 2541*Sin[(e + f*x)/2] + 1155*Sin[(3*(e + f*
x))/2] - 165*Sin[(5*(e + f*x))/2] + 11*Sin[(9*(e + f*x))/2]))/(9240*c^6*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

**Maple [A]**
time = 0.50, size = 178, normalized size = 1.35

| method | result |
|---|---|
| risch | $\frac{4ia^2\left(2079i\,e^{6i(fx+e)}+1155\,e^{7i(fx+e)}-825i\,e^{4i(fx+e)}-2541\,e^{5i(fx+e)}-55i\,e^{2i(fx+e)}+165\,e^{3i(fx+e)}+i-11\,e^{i(fx+e)}\right)}{1155f\,c^6\left(e^{i(fx+e)}-i\right)^{11}}$ |
| derivativedivides | $\frac{2a^2\left(-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{88}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{30}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{512}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{292}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,c^6}$ |

| | |
|---|---|
| default | $\frac{2a^2\left(-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{88}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{30}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{512}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{292}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}\right.}{f\,c^6}$ |
| norman | $\frac{52a^2\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{304a^2}{1155cf}-\frac{2a^2\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{6a^2\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{28a^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{94a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{105cf}-\frac{574a}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^6*(-7/(tan(1/2*f*x+1/2*e)-1)^2-88/(tan(1/2*f*x+1/2*e)-1)^4-1/(tan
(1/2*f*x+1/2*e)-1)-30/(tan(1/2*f*x+1/2*e)-1)^3-512/3/(tan(1/2*f*x+1/2*e)-1)
^9-292/(tan(1/2*f*x+1/2*e)-1)^6-128/11/(tan(1/2*f*x+1/2*e)-1)^11-2376/7/(ta
n(1/2*f*x+1/2*e)-1)^7-288/(tan(1/2*f*x+1/2*e)-1)^8-932/5/(tan(1/2*f*x+1/2*e
)-1)^5-64/(tan(1/2*f*x+1/2*e)-1)^10)
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 1452 vs. $2(136) = 272$.
time = 0.35, size = 1452, normalized size = 11.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/3465*(5*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 14
6)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*a^2*(671*sin(f
*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 660
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
```

```
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11))/f
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(136) = 272$.
time = 0.32, size = 356, normalized size = 2.70

$$-\frac{2\,a^{2}\cos\left(fx+e\right)^{6}+12\,a^{2}\cos\left(fx+e\right)^{5}-25\,a^{2}\cos\left(fx+e\right)^{4}-70\,a^{2}\cos\left(fx+e\right)^{3}-245\,a^{2}\cos\left(fx+e\right)^{2}+210\,a^{2}\cos\left(fx+e\right)+420\,a^{2}-\left(2\,a^{2}\cos\left(fx+e\right)^{5}-10\,a^{2}\cos\left(fx+e\right)^{4}-35\,a^{2}\cos\left(fx+e\right)^{3}+35\,a^{2}\cos\left(fx+e\right)^{2}-210\,a^{2}\cos\left(fx+e\right)-420\,a^{2}\right)\sin\left(fx+e\right)}{1155\,\left(c^{6}f\cos\left(fx+e\right)^{6}-5\,c^{6}f\cos\left(fx+e\right)^{5}-18\,c^{6}f\cos\left(fx+e\right)^{4}+20\,c^{6}f\cos\left(fx+e\right)^{3}+48\,c^{6}f\cos\left(fx+e\right)^{2}-16\,c^{6}f\cos\left(fx+e\right)-32\,c^{6}f+\left(c^{6}f\cos\left(fx+e\right)^{5}+6\,c^{6}f\cos\left(fx+e\right)^{4}-12\,c^{6}f\cos\left(fx+e\right)^{3}-32\,c^{6}f\cos\left(fx+e\right)^{2}+16\,c^{6}f\cos\left(fx+e\right)+32\,c^{6}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^2*cos(f*x + e)^6 + 12*a^2*cos(f*x + e)^5 - 25*a^2*cos(f*x + e)
^4 - 70*a^2*cos(f*x + e)^3 - 245*a^2*cos(f*x + e)^2 + 210*a^2*cos(f*x + e)
+ 420*a^2 - (2*a^2*cos(f*x + e)^5 - 10*a^2*cos(f*x + e)^4 - 35*a^2*cos(f*x
+ e)^3 + 35*a^2*cos(f*x + e)^2 - 210*a^2*cos(f*x + e) - 420*a^2)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
*sin(f*x + e))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2509 vs. $2(117) = 234$.
time = 39.37, size = 2509, normalized size = 19.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-2310*a**2*tan(e/2 + f*x/2)**10/(1155*c**6*f*tan(e/2 + f*x/2)**1
1 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 -
190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533
610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150
*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**
6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 69
30*a**2*tan(e/2 + f*x/2)**9/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*
f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*t
an(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(
e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2
 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f
*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) - 27720*a**2*tan(e/
2 + f*x/2)**8/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*
x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2
)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**
6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 +
 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 127
05*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 46200*a**2*tan(e/2 + f*x/2)**7/
(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 635
25*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*
c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**
6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f
*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(
e/2 + f*x/2) - 1155*c**6*f) - 74844*a**2*tan(e/2 + f*x/2)**6/(1155*c**6*f*t
an(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(
e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2
 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 +
f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x
/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) -
 1155*c**6*f) + 65604*a**2*tan(e/2 + f*x/2)**5/(1155*c**6*f*tan(e/2 + f*x/2
)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**
9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 -
 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 38
1150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525
*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f)
- 54120*a**2*tan(e/2 + f*x/2)**4/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*
c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**
6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f
*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*ta
n(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/
2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 22440*a**2*t
an(e/2 + f*x/2)**3/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2
 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 +
f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x
/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)
```

```
**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2
+ 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) - 9790*a**2*tan(e/2 + f*x/2)
**2/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 +
 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381
150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610
*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c*
*6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*
tan(e/2 + f*x/2) - 1155*c**6*f) + 1034*a**2*tan(e/2 + f*x/2)/(1155*c**6*f*t
an(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(
e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2
 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 +
f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x
/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) -
 1155*c**6*f) - 304*a**2/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*t
an(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(
e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2
 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 +
f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/
2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f), Ne(f, 0)), (x*(a*sin(
e) + a)**2/(-c*sin(e) + c)**6, True))
```

**Giac [A]**
time = 0.49, size = 196, normalized size = 1.48

$$-\frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-3465\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+13860\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-23100\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+37422\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-32802\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+27060\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-11220\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+4895\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-517\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+152\,a^2\right)}{1155\,c^6 f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^2*tan(1/2*f*x + 1/2*e)^10 - 3465*a^2*tan(1/2*f*x + 1/2*e)^9
 + 13860*a^2*tan(1/2*f*x + 1/2*e)^8 - 23100*a^2*tan(1/2*f*x + 1/2*e)^7 + 37
422*a^2*tan(1/2*f*x + 1/2*e)^6 - 32802*a^2*tan(1/2*f*x + 1/2*e)^5 + 27060*a
^2*tan(1/2*f*x + 1/2*e)^4 - 11220*a^2*tan(1/2*f*x + 1/2*e)^3 + 4895*a^2*tan
(1/2*f*x + 1/2*e)^2 - 517*a^2*tan(1/2*f*x + 1/2*e) + 152*a^2)/(c^6*f*(tan(1
/2*f*x + 1/2*e) - 1)^11)
```

**Mupad [B]**
time = 9.36, size = 143, normalized size = 1.08

$$-\frac{\sqrt{2}\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(697\,\cos\left(e+f\,x\right)+\frac{7623\,\sin(e+f\,x)}{4}+\frac{3977\,\cos(2\,e+2\,f\,x)}{4}-\frac{3203\,\cos(3\,e+3\,f\,x)}{16}-\frac{461\,\cos(4\,e+4\,f\,x)}{8}+\frac{75\,\cos(5\,e+5\,f\,x)}{16}-462\,\sin\left(2\,e+2\,f\,x\right)-\frac{4983\,\sin(3\,e+3\,f\,x)}{16}+\frac{187\,\sin(4\,e+4\,f\,x)}{4}+\frac{77\,\sin(5\,e+5\,f\,x)}{16}-\frac{12721}{8}\right)}{36960\,c^6\,f\,{\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2^(1/2)*a^2*cos(e/2 + (f*x)/2)*(697*cos(e + f*x) + (7623*sin(e + f*x))/4
+ (3977*cos(2*e + 2*f*x))/4 - (3203*cos(3*e + 3*f*x))/16 - (461*cos(4*e + 4
```

```
*f*x))/8 + (75*cos(5*e + 5*f*x))/16 - 462*sin(2*e + 2*f*x) - (4983*sin(3*e
+ 3*f*x))/16 + (187*sin(4*e + 4*f*x))/4 + (77*sin(5*e + 5*f*x))/16 - 12721/
8))/(36960*c^6*f*cos(e/2 + pi/4 + (f*x)/2)^11)
```

# 3.247 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx$

**Optimal**. Leaf size=180

$$\frac{55}{128}a^3c^6x+\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f}+\frac{55a^3c^6\cos^3(e+fx)\sin(e+fx)}{192f}+\frac{11a^3c^6}{}$$

```
[Out] 55/128*a^3*c^6*x+11/56*a^3*c^6*cos(f*x+e)^7/f+55/128*a^3*c^6*cos(f*x+e)*sin
(f*x+e)/f+55/192*a^3*c^6*cos(f*x+e)^3*sin(f*x+e)/f+11/48*a^3*c^6*cos(f*x+e)
^5*sin(f*x+e)/f+1/9*a^3*cos(f*x+e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/72*a^3*cos
(f*x+e)^7*(c^6-c^6*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{72f}+\frac{11a^3c^6\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{55a^3c^6\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{55a^3c^6\sin(e+fx)\cos(e+fx)}{128f}+\frac{55}{128}a^3c^6x+\frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*x)/128 + (11*a^3*c^6*Cos[e + f*x]^7)/(56*f) + (55*a^3*c^6*Cos[e
 + f*x]*Sin[e + f*x])/(128*f) + (55*a^3*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(1
92*f) + (11*a^3*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) + (a^3*Cos[e + f*x]
^7*(c^3 - c^3*Sin[e + f*x])^2)/(9*f) + (11*a^3*Cos[e + f*x]^7*(c^6 - c^6*Si
n[e + f*x]))/(72*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx &= \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^3\,dx \\
&= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} + \frac{1}{9}\left(11a^3c^4\right)\int \cos^6(e \\
&= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} + \frac{11a^3\cos^7(e+fx)\,(c}{72} \\
&= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} \\
&= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{11a^3c^6\cos^5(e+fx)\sin(e+fx)}{48f} + \frac{a^3}{} \\
&= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{11}{} \\
&= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f} + \frac{55a}{} \\
&= \frac{55}{128}a^3c^6x + \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos(e+fx)\sin(e}{128f}
\end{aligned}$$

## **Mathematica** [A]

time = 1.37, size = 109, normalized size = 0.61

$$\frac{a^3c^6(27720e+27720fx+16632\cos(e+fx)+9744\cos(3(e+fx))+3024\cos(5(e+fx))+324\cos(7(e+fx))-28\cos(9(e+fx))+18144\sin(2(e+fx))+1512\sin(4(e+fx))-672\sin(6(e+fx))-189\sin(8(e+fx)))}{64512f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(27720*e + 27720*f*x + 16632*Cos[e + f*x] + 9744*Cos[3*(e + f*x)]
+ 3024*Cos[5*(e + f*x)] + 324*Cos[7*(e + f*x)] - 28*Cos[9*(e + f*x)] + 1814
4*Sin[2*(e + f*x)] + 1512*Sin[4*(e + f*x)] - 672*Sin[6*(e + f*x)] - 189*Sin
[8*(e + f*x)]))/(64512*f)
```

**Maple** [A]
time = 0.60, size = 297, normalized size = 1.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/9*c^6*a^3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+
64/35*sin(f*x+e)^2)*cos(f*x+e)-3*c^6*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)
^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+8*c
^6*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+6/5*c^6*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*
c^6*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-8/3*c
^6*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*c^6*a^3*cos(f*x+e)+c^6*a^3*(f*x+e))
```

**Maxima** [A]
time = 0.30, size = 325, normalized size = 1.81

$$-\frac{1024\,\left(35\,\cos\left(fx+e\right)^{9}-180\,\cos\left(fx+e\right)^{7}+378\,\cos\left(fx+e\right)^{5}-420\,\cos\left(fx+e\right)^{3}+315\,\cos\left(fx+e\right)\right)a^{3}c^{6}-129024\,\left(3\,\cos\left(fx+e\right)^{5}-10\,\cos\left(fx+e\right)^{3}+15\,\cos\left(fx+e\right)\right)a^{3}c^{6}-860160\,\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)a^{3}c^{6}+315\,\left(128\,\sin\left(2\,fx+2\,e\right)^{3}+840\,fx+840\,e+3\,\sin\left(8\,fx+8\,e\right)+168\,\sin\left(4\,fx+4\,e\right)-768\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{6}-13440\,\left(4\,\sin\left(2\,fx+2\,e\right)^{3}+60\,fx+60\,e+9\,\sin\left(4\,fx+4\,e\right)-48\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{6}+60480\,\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{6}-322560\,\left(fx+e\right)a^{3}c^{6}-967680\,a^{3}c^{6}\cos\left(fx+e\right)}{322560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*a^3*c^6 - 129024*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^6 - 860160*(cos(f*x + e)^3
- 3*cos(f*x + e))*a^3*c^6 + 315*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e +
 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^6
- 13440*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^6 + 60480*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*a^3*c^6 - 322560*(f*x + e)*a^3*c^6 - 967680*a^3*c^6*cos(f*x + e
))/f
```

**Fricas** [A]
time = 0.35, size = 126, normalized size = 0.70

$$-\frac{896\,a^{3}c^{6}\cos\left(fx+e\right)^{9}-4608\,a^{3}c^{6}\cos\left(fx+e\right)^{7}-3465\,a^{3}c^{6}fx+21\,\left(144\,a^{3}c^{6}\cos\left(fx+e\right)^{7}-88\,a^{3}c^{6}\cos\left(fx+e\right)^{5}-110\,a^{3}c^{6}\cos\left(fx+e\right)^{3}-165\,a^{3}c^{6}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/8064*(896*a^3*c^6*cos(f*x + e)^9 - 4608*a^3*c^6*cos(f*x + e)^7 - 3465*a^
3*c^6*f*x + 21*(144*a^3*c^6*cos(f*x + e)^7 - 88*a^3*c^6*cos(f*x + e)^5 - 11
0*a^3*c^6*cos(f*x + e)^3 - 165*a^3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(172) = 344$.
time = 1.58, size = 838, normalized size = 4.66

$$\begin{cases} -\frac{105a^3c^6x\sin^8(e+fx)}{128} - \frac{105a^3c^6x\sin^6(e+fx)\cos^2(e+fx)}{32} + \frac{5a^3c^6x\sin^6(e+fx)}{2} - \frac{315a^3c^6x\sin^4(e+fx)\cos^4(e+fx)}{64} + \frac{15a^3c^6x\sin^4(e+fx)\cos^2(e+fx)}{2} - \frac{9a^3c^6x\sin^4(e+fx)}{4} - \frac{105a^3c^6x\sin^2(e+fx)\cos^6(e+fx)}{32} + \frac{15a^3c^6x\sin^2(e+fx)\cos^4(e+fx)}{2} - \frac{9a^3c^6x\sin^2(e+fx)\cos^2(e+fx)}{2} - \frac{105a^3c^6x\cos^8(e+fx)}{128} + \frac{5a^3c^6x\cos^6(e+fx)}{2} - \frac{9a^3c^6x\cos^4(e+fx)}{4} + a^3c^6x - \frac{a^3c^6\sin^8(e+fx)\cos(e+fx)}{f} + \frac{279a^3c^6\sin^7(e+fx)\cos(e+fx)}{128f} - \frac{8a^3c^6\sin^6(e+fx)\cos^3(e+fx)}{3f} + \frac{511a^3c^6\sin^5(e+fx)\cos^3(e+fx)}{128f} - \frac{11a^3c^6\sin^5(e+fx)\cos(e+fx)}{2f} - \frac{16a^3c^6\sin^4(e+fx)\cos^5(e+fx)}{5f} + \frac{6a^3c^6\sin^4(e+fx)\cos(e+fx)}{f} + \frac{385a^3c^6\sin^3(e+fx)\cos^5(e+fx)}{128f} - \frac{20a^3c^6\sin^3(e+fx)\cos^3(e+fx)}{3f} + \frac{15a^3c^6\sin^3(e+fx)\cos(e+fx)}{4f} - \frac{64a^3c^6\sin^2(e+fx)\cos^7(e+fx)}{35f} + \frac{8a^3c^6\sin^2(e+fx)\cos^3(e+fx)}{f} - \frac{8a^3c^6\sin^2(e+fx)\cos(e+fx)}{f} + \frac{105a^3c^6\sin(e+fx)\cos^7(e+fx)}{128f} - \frac{5a^3c^6\sin(e+fx)\cos^5(e+fx)}{2f} + \frac{9a^3c^6\sin(e+fx)\cos^3(e+fx)}{4f} - \frac{128a^3c^6\cos^9(e+fx)}{315f} + \frac{16a^3c^6\cos^5(e+fx)}{5f} - \frac{16a^3c^6\cos^3(e+fx)}{3f} + \frac{3a^3c^6\cos(e+fx)}{f} & \text{for}\: f \neq 0 \\ x(a\sin(e)+a)^3(-c\sin(e)+c)^6 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*a**3*c**6*x*sin(e + f*x)**8/128 - 105*a**3*c**6*x*sin(e + f
*x)**6*cos(e + f*x)**2/32 + 5*a**3*c**6*x*sin(e + f*x)**6/2 - 315*a**3*c**6
*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**6*x*sin(e + f*x)**4*cos(
e + f*x)**2/2 - 9*a**3*c**6*x*sin(e + f*x)**4/4 - 105*a**3*c**6*x*sin(e + f
*x)**2*cos(e + f*x)**6/32 + 15*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/
2 - 9*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105*a**3*c**6*x*cos(e
 + f*x)**8/128 + 5*a**3*c**6*x*cos(e + f*x)**6/2 - 9*a**3*c**6*x*cos(e + f*
x)**4/4 + a**3*c**6*x - a**3*c**6*sin(e + f*x)**8*cos(e + f*x)/f + 279*a**3
*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 8*a**3*c**6*sin(e + f*x)**6*co
s(e + f*x)**3/(3*f) + 511*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f)
 - 11*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(2*f) - 16*a**3*c**6*sin(e + f
*x)**4*cos(e + f*x)**5/(5*f) + 6*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f +
 385*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 20*a**3*c**6*sin(e
 + f*x)**3*cos(e + f*x)**3/(3*f) + 15*a**3*c**6*sin(e + f*x)**3*cos(e + f*x
)/(4*f) - 64*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*a**3*c**6
*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a**3*c**6*sin(e + f*x)**2*cos(e + f*
x)/f + 105*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**6*sin
(e + f*x)*cos(e + f*x)**5/(2*f) + 9*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
(4*f) - 128*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*a**3*c**6*cos(e + f*x)**
5/(5*f) - 16*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*a**3*c**6*cos(e + f*x)/f,
Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

**Giac** [**A**]
time = 0.47, size = 196, normalized size = 1.09

$$\frac{55}{128}a^3c^6x - \frac{a^3c^6\cos(9fx+9e)}{2304f} + \frac{9a^3c^6\cos(7fx+7e)}{1792f} + \frac{3a^3c^6\cos(5fx+5e)}{64f} + \frac{29a^3c^6\cos(3fx+3e)}{192f} + \frac{33a^3c^6\cos(fx+e)}{128f} - \frac{3a^3c^6\sin(8fx+8e)}{1024f} - \frac{a^3c^6\sin(6fx+6e)}{96f} + \frac{3a^3c^6\sin(4fx+4e)}{128f} + \frac{9a^3c^6\sin(2fx+2e)}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 55/128*a^3*c^6*x - 1/2304*a^3*c^6*cos(9*f*x + 9*e)/f + 9/1792*a^3*c^6*cos(7
*f*x + 7*e)/f + 3/64*a^3*c^6*cos(5*f*x + 5*e)/f + 29/192*a^3*c^6*cos(3*f*x
```

```
+ 3*e)/f + 33/128*a^3*c^6*cos(f*x + e)/f - 3/1024*a^3*c^6*sin(8*f*x + 8*e)/
f - 1/96*a^3*c^6*sin(6*f*x + 6*e)/f + 3/128*a^3*c^6*sin(4*f*x + 4*e)/f + 9/
32*a^3*c^6*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 9.31, size = 403, normalized size = 2.24

$$\frac{a^3\,c^6\,\left(3465\,e+9198\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+18432\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+79716\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+138240\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4284\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+387072\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+176148\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+290304\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+645120\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-176148\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+236544\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+4284\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{13}+129024\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}-79716\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{15}+48384\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{16}-9198\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{17}+3465\,f\,x+31185\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(e+f\,x\right)+124740\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(e+f\,x\right)+291060\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(e+f\,x\right)+436590\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(e+f\,x\right)+436590\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(e+f\,x\right)+291060\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}\,\left(e+f\,x\right)+124740\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}\,\left(e+f\,x\right)+31185\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{16}\,\left(e+f\,x\right)+3465\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{18}\,\left(e+f\,x\right)+7424\right)}{8064\,f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

```
[Out] (a^3*c^6*(3465*e + 9198*tan(e/2 + (f*x)/2) + 18432*tan(e/2 + (f*x)/2)^2 + 7
9716*tan(e/2 + (f*x)/2)^3 + 138240*tan(e/2 + (f*x)/2)^4 - 4284*tan(e/2 + (f
*x)/2)^5 + 387072*tan(e/2 + (f*x)/2)^6 + 176148*tan(e/2 + (f*x)/2)^7 + 2903
04*tan(e/2 + (f*x)/2)^8 + 645120*tan(e/2 + (f*x)/2)^10 - 176148*tan(e/2 + (
f*x)/2)^11 + 236544*tan(e/2 + (f*x)/2)^12 + 4284*tan(e/2 + (f*x)/2)^13 + 12
9024*tan(e/2 + (f*x)/2)^14 - 79716*tan(e/2 + (f*x)/2)^15 + 48384*tan(e/2 +
(f*x)/2)^16 - 9198*tan(e/2 + (f*x)/2)^17 + 3465*f*x + 31185*tan(e/2 + (f*x)
/2)^2*(e + f*x) + 124740*tan(e/2 + (f*x)/2)^4*(e + f*x) + 291060*tan(e/2 +
(f*x)/2)^6*(e + f*x) + 436590*tan(e/2 + (f*x)/2)^8*(e + f*x) + 436590*tan(e
/2 + (f*x)/2)^10*(e + f*x) + 291060*tan(e/2 + (f*x)/2)^12*(e + f*x) + 12474
0*tan(e/2 + (f*x)/2)^14*(e + f*x) + 31185*tan(e/2 + (f*x)/2)^16*(e + f*x) +
 3465*tan(e/2 + (f*x)/2)^18*(e + f*x) + 7424))/(8064*f*(tan(e/2 + (f*x)/2)^
2 + 1)^9)
```

# 3.248 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx$

**Optimal**. Leaf size=145

$$\frac{45}{128}a^3c^5x+\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{45a^3c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{15a^3c^5\cos^3(e+fx)\sin(e+fx)}{64f}+\frac{3a^3c^5\cos^5}{}$$

```
[Out] 45/128*a^3*c^5*x+9/56*a^3*c^5*cos(f*x+e)^7/f+45/128*a^3*c^5*cos(f*x+e)*sin(
f*x+e)/f+15/64*a^3*c^5*cos(f*x+e)^3*sin(f*x+e)/f+3/16*a^3*c^5*cos(f*x+e)^5*
sin(f*x+e)/f+1/8*a^3*cos(f*x+e)^7*(c^5-c^5*sin(f*x+e))/f
```

**Rubi** [A]

time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2757, 2748, 2715, 8}

$$\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{3a^3c^5\sin(e+fx)\cos^5(e+fx)}{16f}+\frac{15a^3c^5\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{45a^3c^5\sin(e+fx)\cos(e+fx)}{128f}+\frac{45}{128}a^3c^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (45*a^3*c^5*x)/128 + (9*a^3*c^5*Cos[e + f*x]^7)/(56*f) + (45*a^3*c^5*Cos[e
+ f*x]*Sin[e + f*x])/(128*f) + (15*a^3*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64
*f) + (3*a^3*c^5*Cos[e + f*x]^5*Sin[e + f*x])/(16*f) + (a^3*Cos[e + f*x]^7*
(c^5 - c^5*Sin[e + f*x]))/(8*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{1}{8}\left(9a^3c^4\right)\int \cos^6(e$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{3a^3c^5\cos^5(e+fx)\sin(e+fx)}{16f}+\frac{a^3}{}$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{15a^3c^5\cos^3(e+fx)\sin(e+fx)}{64f}+\frac{3a}{}$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{45a^3c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{15a}{}$$
$$= \frac{45}{128}a^3c^5x+\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{45a^3c^5\cos(e+fx)\sin(e}{128f}$$

**Mathematica** [A]
time = 0.79, size = 89, normalized size = 0.61

$$\frac{a^3c^5(2520e+2520fx+1120\cos(e+fx)+672\cos(3(e+fx))+224\cos(5(e+fx))+32\cos(7(e+fx))+1792\sin(2(e+fx))+280\sin(4(e+fx))-7\sin(8(e+fx)))}{7168f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*(2520*e + 2520*f*x + 1120*Cos[e + f*x] + 672*Cos[3*(e + f*x)] + 22
4*Cos[5*(e + f*x)] + 32*Cos[7*(e + f*x)] + 1792*Sin[2*(e + f*x)] + 280*Sin[
4*(e + f*x)] - 7*Sin[8*(e + f*x)]))/(7168*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(133) = 266$.
time = 0.57, size = 276, normalized size = 1.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c^5*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16
*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*c^5*a^3*(16/5+sin(f*x+e)^6
+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*c^5*a^3*(-1/6*(sin(f*x+e)^
5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*c^5*a^3
*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*c^5*a^3*(2+sin(f*x+e)^2)*
cos(f*x+e)-2*c^5*a^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2*c^5*a^3*c
os(f*x+e)+c^5*a^3*(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(143) = 286$.
time = 0.28, size = 304, normalized size = 2.10

$$\frac{6144\,\left(5\,\cos\left(fx+e\right)^{7}-21\,\cos\left(fx+e\right)^{5}+35\,\cos\left(fx+e\right)^{3}-35\,\cos\left(fx+e\right)\right)a^{3}c^{5}+43008\,\left(3\,\cos\left(fx+e\right)^{5}-10\,\cos\left(fx+e\right)^{3}+15\,\cos\left(fx+e\right)\right)a^{3}c^{5}+215040\,\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)a^{3}c^{5}-35\,\left(128\,\sin\left(2\,fx+2\,e\right)^{3}+840\,fx+840\,e+3\,\sin\left(8\,fx+8\,e\right)+168\,\sin\left(4\,fx+4\,e\right)-768\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{5}+1120\,\left(4\,\sin\left(2\,fx+2\,e\right)^{3}+60\,fx+60\,e+9\,\sin\left(4\,fx+4\,e\right)-48\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{5}-53760\,\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{5}+107520\,\left(fx+e\right)a^{3}c^{5}+215040\,a^{3}c^{5}\cos\left(fx+e\right)}{107520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/107520*(6144*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*a^3*c^5 + 43008*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^3*c^5 + 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^5 -
 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*si
n(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^5 + 1120*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*c^5 - 5376
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^5 + 107520*(f*x + e)*a^3*c^5 + 215
040*a^3*c^5*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.34, size = 109, normalized size = 0.75

$$\frac{256\,a^{3}c^{5}\cos\left(fx+e\right)^{7}+315\,a^{3}c^{5}fx-7\,\left(16\,a^{3}c^{5}\cos\left(fx+e\right)^{7}-24\,a^{3}c^{5}\cos\left(fx+e\right)^{5}-30\,a^{3}c^{5}\cos\left(fx+e\right)^{3}-45\,a^{3}c^{5}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{896\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/896*(256*a^3*c^5*cos(f*x + e)^7 + 315*a^3*c^5*f*x - 7*(16*a^3*c^5*cos(f*x
 + e)^7 - 24*a^3*c^5*cos(f*x + e)^5 - 30*a^3*c^5*cos(f*x + e)^3 - 45*a^3*c^
5*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(139) = 278$.

time = 1.06, size = 740, normalized size = 5.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*a**3*c**5*x*sin(e + f*x)**8/128 - 35*a**3*c**5*x*sin(e + f*x
)**6*cos(e + f*x)**2/32 + 5*a**3*c**5*x*sin(e + f*x)**6/8 - 105*a**3*c**5*x
*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**5*x*sin(e + f*x)**4*cos(e
+ f*x)**2/8 - 35*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*a**3*c
**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - a**3*c**5*x*sin(e + f*x)**2 - 35*
a**3*c**5*x*cos(e + f*x)**8/128 + 5*a**3*c**5*x*cos(e + f*x)**6/8 - a**3*c*
*5*x*cos(e + f*x)**2 + a**3*c**5*x + 93*a**3*c**5*sin(e + f*x)**7*cos(e + f
*x)/(128*f) - 2*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)/f + 511*a**3*c**5*si
n(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*a**3*c**5*sin(e + f*x)**5*cos(e
+ f*x)/(8*f) - 4*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + 6*a**3*c**5*
sin(e + f*x)**4*cos(e + f*x)/f + 385*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)
**5/(384*f) - 5*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*a**3*c
**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 8*a**3*c**5*sin(e + f*x)**2*cos
(e + f*x)**3/f - 6*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*a**3*c**5*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**5*sin(e + f*x)*cos(e + f*x
)**5/(8*f) + a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*a**3*c**5*cos(e + f
*x)**7/(35*f) + 16*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*a**3*c**5*cos(e + f*
x)**3/f + 2*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*s
in(e) + c)**5, True))
```

**Giac** [A]

time = 0.46, size = 154, normalized size = 1.06

$$\frac{45}{128}a^3c^5x + \frac{a^3c^5\cos(7fx+7e)}{224f} + \frac{a^3c^5\cos(5fx+5e)}{32f} + \frac{3a^3c^5\cos(3fx+3e)}{32f} + \frac{5a^3c^5\cos(fx+e)}{32f} - \frac{a^3c^5\sin(8fx+8e)}{1024f} + \frac{5a^3c^5\sin(4fx+4e)}{128f} + \frac{a^3c^5\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 45/128*a^3*c^5*x + 1/224*a^3*c^5*cos(7*f*x + 7*e)/f + 1/32*a^3*c^5*cos(5*f*
x + 5*e)/f + 3/32*a^3*c^5*cos(3*f*x + 3*e)/f + 5/32*a^3*c^5*cos(f*x + e)/f
- 1/1024*a^3*c^5*sin(8*f*x + 8*e)/f + 5/128*a^3*c^5*sin(4*f*x + 4*e)/f + 1/
4*a^3*c^5*sin(2*f*x + 2*e)/f
```

**Mupad** [B]

time = 9.12, size = 372, normalized size = 2.57

$$\frac{a^3c^5\left(\frac{315e}{2}+581\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+256\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+2065\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+5376\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+5376\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+5705\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+8960\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-5705\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+8960\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+1792\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-2065\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}+1792\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}-581\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{15}+\frac{315fx}{2}+1260\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(e+fx)+4410\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(e+fx)+8820\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(e+fx)+11025\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(e+fx)+8820\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(e+fx)+4410\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(e+fx)+1260\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}(e+fx)+\frac{315\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}(e+fx)}{2}+256\right)}{448f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (a^3*c^5*((315*e)/2 + 581*tan(e/2 + (f*x)/2) + 256*tan(e/2 + (f*x)/2)^2 + 2
065*tan(e/2 + (f*x)/2)^3 + 5376*tan(e/2 + (f*x)/2)^4 + 21*tan(e/2 + (f*x)/2
)^5 + 5376*tan(e/2 + (f*x)/2)^6 + 5705*tan(e/2 + (f*x)/2)^7 + 8960*tan(e/2
+ (f*x)/2)^8 - 5705*tan(e/2 + (f*x)/2)^9 + 8960*tan(e/2 + (f*x)/2)^10 - 21*
tan(e/2 + (f*x)/2)^11 + 1792*tan(e/2 + (f*x)/2)^12 - 2065*tan(e/2 + (f*x)/2
)^13 + 1792*tan(e/2 + (f*x)/2)^14 - 581*tan(e/2 + (f*x)/2)^15 + (315*f*x)/2
 + 1260*tan(e/2 + (f*x)/2)^2*(e + f*x) + 4410*tan(e/2 + (f*x)/2)^4*(e + f*x
) + 8820*tan(e/2 + (f*x)/2)^6*(e + f*x) + 11025*tan(e/2 + (f*x)/2)^8*(e + f
*x) + 8820*tan(e/2 + (f*x)/2)^10*(e + f*x) + 4410*tan(e/2 + (f*x)/2)^12*(e
+ f*x) + 1260*tan(e/2 + (f*x)/2)^14*(e + f*x) + (315*tan(e/2 + (f*x)/2)^16*
(e + f*x))/2 + 256))/(448*f*(tan(e/2 + (f*x)/2)^2 + 1)^8)
```

# 3.249 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=112

$$\frac{5}{16}a^3c^4x+\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3c^4\cos^5(e+$$

```
[Out] 5/16*a^3*c^4*x+1/7*a^3*c^4*cos(f*x+e)^7/f+5/16*a^3*c^4*cos(f*x+e)*sin(f*x+e
)/f+5/24*a^3*c^4*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*c^4*cos(f*x+e)^5*sin(f*x
+e)/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2748, 2715, 8}

$$\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^4\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*x)/16 + (a^3*c^4*Cos[e + f*x]^7)/(7*f) + (5*a^3*c^4*Cos[e + f*x]
*Sin[e + f*x])/(16*f) + (5*a^3*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a
^3*c^4*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx &= \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))\,dx\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\left(a^3c^4\right)\int \cos^6(e+fx)\,dx\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}\left(5a^3c\right.\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3c^4\,\text{co}}{}\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3c^4\,\text{c}}{}\\ &= \frac{5}{16}a^3c^4x+\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos(e+fx)\sin(e+fx}{16f}\end{aligned}$$

**Mathematica [A]**
time = 0.68, size = 89, normalized size = 0.79

$$\frac{a^3c^4(420e+420fx+105\cos(e+fx)+63\cos(3(e+fx))+21\cos(5(e+fx))+3\cos(7(e+fx))+315\sin(2(e+fx))+63\sin(4(e+fx))+7\sin(6(e+fx)))}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(420*e + 420*f*x + 105*Cos[e + f*x] + 63*Cos[3*(e + f*x)] + 21*Cos
[5*(e + f*x)] + 3*Cos[7*(e + f*x)] + 315*Sin[2*(e + f*x)] + 63*Sin[4*(e + f
*x)] + 7*Sin[6*(e + f*x)]))/(1344*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(102) = 204$.
time = 0.47, size = 255, normalized size = 2.28

| method | result |
|---|---|
| risch | $\frac{5a^3c^4x}{16}+\frac{5c^4a^3\cos(fx+e)}{64f}+\frac{c^4a^3\cos(7fx+7e)}{448f}+\frac{c^4a^3\sin(6fx+6e)}{192f}+\frac{c^4a^3\cos(5fx+5e)}{64f}+\frac{3c^4a^3\sin(4fx+4e)}{64f}$ |

| | |
|---|---|
| derivativedivides | $\frac{-\frac{c^4a^3\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}-c^4a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{}$ |
| default | $\frac{-\frac{c^4a^3\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}-c^4a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{}$ |
| norman | $\frac{\frac{2c^4a^3\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2c^4a^3}{7f}+\frac{5a^3c^4x}{16}+\frac{6c^4a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{10c^4a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{35a^3c^4x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16}+105a^3c}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/7*c^4*a^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos
(f*x+e)-c^4*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f
*x+e)+5/16*f*x+5/16*e)+3/5*c^4*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(
f*x+e)+3*c^4*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
*e)-c^4*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*c^4*a^3*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)+c^4*a^3*cos(f*x+e)+c^4*a^3*(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(109) = 218$.
time = 0.29, size = 276, normalized size = 2.46

$$\frac{192\left(5\cos(fx+e)^7-21\cos(fx+e)^5+35\cos(fx+e)^3-35\cos(fx+e)\right)a^3c^4+1344\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^3c^4+6720\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3c^4-35\left(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e)\right)a^3c^4+630\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^3c^4-5040\left(2fx+2e-\sin(2fx+2e)\right)a^3c^4+6720(fx+e)a^3c^4+6720a^3c^4\cos(fx+e)}{6720f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*
cos(f*x + e))*a^3*c^4 + 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^3*c^4 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^4 - 35*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*a^3*c^4 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
a^3*c^4 - 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^4 + 6720*(f*x + e)*a^
3*c^4 + 6720*a^3*c^4*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.33, size = 92, normalized size = 0.82

$$\frac{48\,a^3c^4\cos(fx+e)^7+105\,a^3c^4fx+7\left(8\,a^3c^4\cos(fx+e)^5+10\,a^3c^4\cos(fx+e)^3+15\,a^3c^4\cos(fx+e)\right)\sin(fx+e)}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/336*(48*a^3*c^4*cos(f*x + e)^7 + 105*a^3*c^4*f*x + 7*(8*a^3*c^4*cos(f*x +
 e)^5 + 10*a^3*c^4*cos(f*x + e)^3 + 15*a^3*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(107) = 214$.
time = 0.73, size = 631, normalized size = 5.63

$$\begin{cases} - \frac{5a^3c^4x\sin^6(e+fx)}{16} - \frac{15a^3c^4x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9a^3c^4x\sin^4(e+fx)}{8} - \frac{15a^3c^4x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9a^3c^4x\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3a^3c^4x\sin^2(e+fx)}{2} - \frac{5a^3c^4x\cos^6(e+fx)}{16} + \frac{9a^3c^4x\cos^4(e+fx)}{8} - \frac{3a^3c^4x\cos^2(e+fx)}{2} + a^3c^4x - \frac{a^3c^4\sin^6(e+fx)\cos(e+fx)}{f} + \frac{11a^3c^4\sin^5(e+fx)\cos(e+fx)}{16f} - \frac{2a^3c^4\sin^4(e+fx)\cos^3(e+fx)}{f} + \frac{3a^3c^4\sin^4(e+fx)\cos(e+fx)}{f} + \frac{5a^3c^4\sin^3(e+fx)\cos^3(e+fx)}{6f} - \frac{15a^3c^4\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{8a^3c^4\sin^2(e+fx)\cos^5(e+fx)}{5f} + \frac{4a^3c^4\sin^2(e+fx)\cos^3(e+fx)}{f} - \frac{3a^3c^4\sin^2(e+fx)\cos(e+fx)}{f} + \frac{5a^3c^4\sin(e+fx)\cos^5(e+fx)}{16f} - \frac{9a^3c^4\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{3a^3c^4\sin(e+fx)\cos(e+fx)}{2f} - \frac{16a^3c^4\cos^7(e+fx)}{35f} + \frac{8a^3c^4\cos^5(e+fx)}{5f} - \frac{2a^3c^4\cos^3(e+fx)}{f} + \frac{a^3c^4\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sin(e)+a)^3(-c\sin(e)+c)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*a**3*c**4*x*sin(e + f*x)**6/16 - 15*a**3*c**4*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**4*x*sin(e + f*x)**4/8 - 15*a**3*c**4*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 - 3*a**3*c**4*x*sin(e + f*x)**2/2 - 5*a**3*c**4*x*cos(e + f*x)**6/1
6 + 9*a**3*c**4*x*cos(e + f*x)**4/8 - 3*a**3*c**4*x*cos(e + f*x)**2/2 + a**
3*c**4*x - a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f + 11*a**3*c**4*sin(e +
f*x)**5*cos(e + f*x)/(16*f) - 2*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
 + 3*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f + 5*a**3*c**4*sin(e + f*x)**3
*cos(e + f*x)**3/(6*f) - 15*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
8*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**3*c**4*sin(e + f*x
)**2*cos(e + f*x)**3/f - 3*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*a**
3*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**4*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + 3*a**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 16*a**3*c*
*4*cos(e + f*x)**7/(35*f) + 8*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*a**3*c**4
*cos(e + f*x)**3/f + a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)
**3*(-c*sin(e) + c)**4, True))
```

**Giac** [A]
time = 0.43, size = 154, normalized size = 1.38

$$\frac{5}{16}a^3c^4x + \frac{a^3c^4\cos(7fx+7e)}{448f} + \frac{a^3c^4\cos(5fx+5e)}{64f} + \frac{3a^3c^4\cos(3fx+3e)}{64f} + \frac{5a^3c^4\cos(fx+e)}{64f} + \frac{a^3c^4\sin(6fx+6e)}{192f} + \frac{3a^3c^4\sin(4fx+4e)}{64f} + \frac{15a^3c^4\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^4*x + 1/448*a^3*c^4*cos(7*f*x + 7*e)/f + 1/64*a^3*c^4*cos(5*f*x
+ 5*e)/f + 3/64*a^3*c^4*cos(3*f*x + 3*e)/f + 5/64*a^3*c^4*cos(f*x + e)/f +
1/192*a^3*c^4*sin(6*f*x + 6*e)/f + 3/64*a^3*c^4*sin(4*f*x + 4*e)/f + 15/64*
a^3*c^4*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 10.45, size = 301, normalized size = 2.69

$$\frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{12}\left(\frac{a^3c^4(735e+735fx+672)}{336}-\frac{35a^3c^4(e+fx)}{16}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4\left(\frac{a^3c^4(2205e+2205fx+2016)}{336}-\frac{105a^3c^4(e+fx)}{16}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8\left(\frac{a^3c^4(3675e+3675fx+3360)}{336}-\frac{175a^3c^4(e+fx)}{16}\right)+\frac{7a^3c^4{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3}{6}+\frac{85a^3c^4{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5}{24}-\frac{85a^3c^4{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9}{24}-\frac{7a^3c^4{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{11}}{6}-\frac{11a^3c^4{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{13}}{8}+\frac{a^3c^4(105e+105fx+96)}{336}+\frac{11a^3c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{8}-\frac{5a^3c^4(e+fx)}{16}}{f{\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+1\right)}^7}+\frac{5a^3c^4x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)

[Out] (tan(e/2 + (f*x)/2)^12*((a^3*c^4*(735*e + 735*f*x + 672))/336 - (35*a^3*c^4
*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^4*(2205*e + 2205*f*x + 2016)
)/336 - (105*a^3*c^4*(e + f*x))/16) + tan(e/2 + (f*x)/2)^8*((a^3*c^4*(3675*
e + 3675*f*x + 3360))/336 - (175*a^3*c^4*(e + f*x))/16) + (7*a^3*c^4*tan(e/
2 + (f*x)/2)^3)/6 + (85*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 - (85*a^3*c^4*tan(
e/2 + (f*x)/2)^9)/24 - (7*a^3*c^4*tan(e/2 + (f*x)/2)^11)/6 - (11*a^3*c^4*ta
n(e/2 + (f*x)/2)^13)/8 + (a^3*c^4*(105*e + 105*f*x + 96))/336 + (11*a^3*c^4
*tan(e/2 + (f*x)/2))/8 - (5*a^3*c^4*(e + f*x))/16)/(f*(tan(e/2 + (f*x)/2)^2
 + 1)^7) + (5*a^3*c^4*x)/16

# 3.250 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=91

$$\frac{5}{16}a^3c^3x+\frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f}$$

```
[Out] 5/16*a^3*c^3*x+5/16*a^3*c^3*cos(f*x+e)*sin(f*x+e)/f+5/24*a^3*c^3*cos(f*x+e)
^3*sin(f*x+e)/f+1/6*a^3*c^3*cos(f*x+e)^5*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2715, 8}

$$\frac{a^3c^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*x)/16 + (5*a^3*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (5*a^3*c^
3*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a^3*c^3*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx = (a^3c^3)\int \cos^6(e+fx)\,dx$$
$$= \frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{6}(5a^3c^3)\int \cos^4(e+fx)\,d$$
$$= \frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3c^3\cos^5(e+fx)\sin(e+$$
$$= \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+fx)\sin(e+}{24f}$$
$$= \frac{5}{16}a^3c^3x + \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+}{2}$$

**Mathematica [A]**
time = 0.04, size = 49, normalized size = 0.54

$$\frac{a^3c^3(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx)))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
[Out] (a^3*c^3*(60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(e + f*x)] + Sin[6*
(e + f*x)]))/(192*f)
```

**Maple [A]**
time = 0.26, size = 140, normalized size = 1.54

| method | result |
|---|---|
| risch | $\frac{5a^3c^3x}{16} + \frac{c^3a^3\sin(6fx+6e)}{192f} + \frac{3c^3a^3\sin(4fx+4e)}{64f} + \frac{15c^3a^3\sin(2fx+2e)}{64f}$ |
| derivativedivides | $\frac{-c^3a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+3c^3a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx}{4}\right.}{f}$ |
| default | $\frac{-c^3a^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+3c^3a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx}{4}\right.}{f}$ |
| norman | $\frac{\frac{5a^3c^3x}{16}+\frac{15a^3c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{75a^3c^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16}+\frac{25a^3c^3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{75a^3c^3x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16}+\frac{15a^3c^3x(t}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c^3*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x
+e)+5/16*f*x+5/16*e)+3*c^3*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-3*c^3*a^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+c^3*
a^3*(f*x+e))
```

**Maxima [A]**
time = 0.28, size = 142, normalized size = 1.56

$$-\frac{\left(4\,\sin\left(2\,fx+2\,e\right)^3+60\,fx+60\,e+9\,\sin\left(4\,fx+4\,e\right)-48\,\sin\left(2\,fx+2\,e\right)\right)a^3c^3-18\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)a^3c^3+144\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^3c^3-192\left(fx+e\right)a^3c^3}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/192*((4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^3 - 18*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
 + 2*e))*a^3*c^3 + 144*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^3 - 192*(f*x
+ e)*a^3*c^3)/f
```

**Fricas [A]**
time = 0.32, size = 74, normalized size = 0.81

$$\frac{15\,a^3c^3fx+\left(8\,a^3c^3\cos\left(fx+e\right)^5+10\,a^3c^3\cos\left(fx+e\right)^3+15\,a^3c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*a^3*c^3*f*x + (8*a^3*c^3*cos(f*x + e)^5 + 10*a^3*c^3*cos(f*x + e)^
3 + 15*a^3*c^3*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(88) = 176$.
time = 0.57, size = 398, normalized size = 4.37

$$\begin{cases} -\frac{5a^3c^3x\sin^6(e+fx)}{16} - \frac{15a^3c^3x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9a^3c^3x\sin^4(e+fx)}{8} - \frac{15a^3c^3x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9a^3c^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3a^3c^3x\sin^2(e+fx)}{2} - \frac{5a^3c^3x\cos^6(e+fx)}{16} + \frac{9a^3c^3x\cos^4(e+fx)}{8} - \frac{3a^3c^3x\cos^2(e+fx)}{2} + a^3c^3x + \frac{11a^3c^3\sin^5(e+fx)\cos(e+fx)}{16f} + \frac{5a^3c^3\sin^3(e+fx)\cos^3(e+fx)}{6f} - \frac{15a^3c^3\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{5a^3c^3\sin(e+fx)\cos^5(e+fx)}{16f} - \frac{9a^3c^3\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{3a^3c^3\sin(e+fx)\cos(e+fx)}{2f} & \text{for}\: f \neq 0 \\ x(a\sin(e)+a)^3(-c\sin(e)+c)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*a**3*c**3*x*sin(e + f*x)**6/16 - 15*a**3*c**3*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**3*x*sin(e + f*x)**4/8 - 15*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 - 3*a**3*c**3*x*sin(e + f*x)**2/2 - 5*a**3*c**3*x*cos(e + f*x)**6/1
6 + 9*a**3*c**3*x*cos(e + f*x)**4/8 - 3*a**3*c**3*x*cos(e + f*x)**2/2 + a**
3*c**3*x + 11*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a**3*c**3*s
```

```
in(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*c**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + 5*a**3*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a**3*c**3*sin(e + f*x)*cos(e + f*x)
/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**3, True))
```

**Giac** [A]
time = 0.45, size = 73, normalized size = 0.80

$$\frac{5}{16}\,a^3c^3x+\frac{a^3c^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{3\,a^3c^3\sin\left(4\,fx+4\,e\right)}{64\,f}+\frac{15\,a^3c^3\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^3*x + 1/192*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*a^3*c^3*sin(4*f*x
+ 4*e)/f + 15/64*a^3*c^3*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 10.14, size = 143, normalized size = 1.57

$$\frac{5\,a^3\,c^3\,x}{16}-\frac{\frac{11\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{15\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4}-\frac{15\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4}+\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}-\frac{11\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*c^3*x)/16 - ((5*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 - (15*a^3*c^3*tan(e
/2 + (f*x)/2)^5)/4 + (15*a^3*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^3*c^3*tan(e
/2 + (f*x)/2)^9)/24 + (11*a^3*c^3*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^3*ta
n(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

# 3.251 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=85

$$\frac{3}{8}a^3c^2x-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^3c^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 3/8*a^3*c^2*x-1/5*a^3*c^2*cos(f*x+e)^5/f+3/8*a^3*c^2*cos(f*x+e)*sin(f*x+e)/
f+1/4*a^3*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2748, 2715, 8}

$$-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^3c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*x)/8 - (a^3*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^3*c^2*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^3*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))\,dx\\ &= -\frac{a^3c^2\cos^5(e+fx)}{5f}+\left(a^3c^2\right)\int \cos^4(e+fx)\,dx\\ &= -\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}(3\\ &= -\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^3c^2}{}\\ &= \frac{3}{8}a^3c^2x-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{3a^3c^2\cos(e+fx)\sin(e+fx}{8f}\end{aligned}$$

**Mathematica** [A]
time = 1.02, size = 69, normalized size = 0.81

$$\frac{a^3c^2(60e+60fx-20\cos(e+fx)-10\cos(3(e+fx))-2\cos(5(e+fx))+40\sin(2(e+fx))+5\sin(4(e+fx)))}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(60*e + 60*f*x - 20*Cos[e + f*x] - 10*Cos[3*(e + f*x)] - 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(77) = 154.
time = 0.29, size = 160, normalized size = 1.88

| method | result |
|---|---|
| risch | $\frac{3a^3c^2x}{8}-\frac{c^2a^3\cos(fx+e)}{8f}-\frac{c^2a^3\cos(5fx+5e)}{80f}+\frac{c^2a^3\sin(4fx+4e)}{32f}-\frac{c^2a^3\cos(3fx+3e)}{16f}+\frac{c^2a^3\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-\frac{c^2a^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+c^2a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2c^2a^3(2+}{}}{f}$ |
| default | $\frac{-\frac{c^2a^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+c^2a^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2c^2a^3(2+}{}}{f}$ |

| | |
|---|---|
| norman | $-\frac{2c^2a^3}{5f}+\frac{3a^3c^2x}{8}-\frac{4c^2a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2c^2a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{15a^3c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{15a^3c^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{15a^3c^2}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*c^2*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^2*a^3*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*c^2*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e)-2*c^2*a^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*
e)-c^2*a^3*cos(f*x+e)+c^2*a^3*(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.
time = 0.32, size = 170, normalized size = 2.00

$$-\frac{32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)a^3c^2+320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3c^2-15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^3c^2+240\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^3c^2-480\left(fx+e\right)a^3c^2+480a^3c^2\cos\left(fx+e\right)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^2
 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 - 15*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c^2 + 240*(2*f*x + 2*e - sin(2*f*x +
 2*e))*a^3*c^2 - 480*(f*x + e)*a^3*c^2 + 480*a^3*c^2*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.34, size = 75, normalized size = 0.88

$$-\frac{8\,a^3c^2\cos\left(fx+e\right)^5-15\,a^3c^2fx-5\left(2\,a^3c^2\cos\left(fx+e\right)^3+3\,a^3c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^3*c^2*cos(f*x + e)^5 - 15*a^3*c^2*f*x - 5*(2*a^3*c^2*cos(f*x + e
)^3 + 3*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(80) = 160$.
time = 0.36, size = 340, normalized size = 4.00

$$\begin{cases}\frac{3a^3c^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^3c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^3c^2x\sin^2\left(e+fx\right)+\frac{3a^3c^2x\cos^4\left(e+fx\right)}{8}-a^3c^2x\cos^2\left(e+fx\right)+a^3c^2x-\frac{a^3c^2\sin^4\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{5a^3c^2\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{4a^3c^2\sin^2\left(e+fx\right)\cos^3\left(e+fx\right)}{3f}+\frac{2a^3c^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^3c^2\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f}+\frac{a^3c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{8a^3c^2\cos^5\left(e+fx\right)}{15f}+\frac{4a^3c^2\cos^3\left(e+fx\right)}{3f}-\frac{a^3c^2\cos\left(e+fx\right)}{f} & \text{for}\: f\neq 0\\x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**4/8 + 3*a**3*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**3*c**2*x*sin(e + f*x)**2 + 3*a**3*c**2*x*cos(e + f*x
)**4/8 - a**3*c**2*x*cos(e + f*x)**2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**
3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*a**3*c**2*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**3*c**2
*sin(e + f*x)*cos(e + f*x)/f - 8*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*a**3*
c**2*cos(e + f*x)**3/(3*f) - a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**3*(-c*sin(e) + c)**2, True))
```

**Giac** [A]
time = 0.43, size = 112, normalized size = 1.32

$$\frac{3}{8}a^3c^2x - \frac{a^3c^2\cos(5fx+5e)}{80f} - \frac{a^3c^2\cos(3fx+3e)}{16f} - \frac{a^3c^2\cos(fx+e)}{8f} + \frac{a^3c^2\sin(4fx+4e)}{32f} + \frac{a^3c^2\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^3*c^2*x - 1/80*a^3*c^2*cos(5*f*x + 5*e)/f - 1/16*a^3*c^2*cos(3*f*x +
3*e)/f - 1/8*a^3*c^2*cos(f*x + e)/f + 1/32*a^3*c^2*sin(4*f*x + 4*e)/f + 1/4
*a^3*c^2*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 10.32, size = 220, normalized size = 2.59

$$\frac{3a^3c^2x}{8} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(\frac{a^3c^2(75e+75fx-80)}{40} - \frac{15a^3c^2(e+fx)}{8}\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{a^3c^2(150e+150fx-160)}{40} - \frac{15a^3c^2(e+fx)}{4}\right) + \frac{a^3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{2} - \frac{a^3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{2} - \frac{5a^3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{4} + \frac{a^3c^2(15e+15fx-16)}{40} + \frac{5a^3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4} - \frac{3a^3c^2(e+fx)}{8}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (3*a^3*c^2*x)/8 + (tan(e/2 + (f*x)/2)^8*((a^3*c^2*(75*e + 75*f*x - 80))/40
- (15*a^3*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^3*c^2*(150*e + 150*f
*x - 160))/40 - (15*a^3*c^2*(e + f*x))/4) + (a^3*c^2*tan(e/2 + (f*x)/2)^3)/
2 - (a^3*c^2*tan(e/2 + (f*x)/2)^7)/2 - (5*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 +
 (a^3*c^2*(15*e + 15*f*x - 16))/40 + (5*a^3*c^2*tan(e/2 + (f*x)/2))/4 - (3*
a^3*c^2*(e + f*x))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^5)
```

# 3.252 $\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))\,dx$

**Optimal**. Leaf size=82

$$\frac{5}{8}a^3cx - \frac{5a^3c\cos^3(e+fx)}{12f} + \frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f} - \frac{c\cos^3(e+fx)\,(a^3+a^3\sin(e+fx))}{4f}$$

```
[Out] 5/8*a^3*c*x-5/12*a^3*c*cos(f*x+e)^3/f+5/8*a^3*c*cos(f*x+e)*sin(f*x+e)/f-1/4
*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2815, 2757, 2748, 2715, 8}

$$-\frac{5a^3c\cos^3(e+fx)}{12f} - \frac{c\cos^3(e+fx)\,(a^3\sin(e+fx)+a^3)}{4f} + \frac{5a^3c\sin(e+fx)\cos(e+fx)}{8f} + \frac{5}{8}a^3cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*c*x)/8 - (5*a^3*c*Cos[e + f*x]^3)/(12*f) + (5*a^3*c*Cos[e + f*x]*Sin
[e + f*x])/(8*f) - (c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2\,dx$$
$$= -\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5a^2c\right)\int \cos^2(e+$$
$$= -\frac{5a^3c\cos^3(e+fx)}{12f}-\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}$$
$$= -\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}-\frac{c\cos^3}{}$$
$$= \frac{5}{8}a^3cx-\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}$$

**Mathematica [A]**
time = 0.27, size = 54, normalized size = 0.66

$$\frac{a^3c(60fx-48\cos(e+fx)-16\cos(3(e+fx))+24\sin(2(e+fx))-3\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*c*(60*f*x - 48*Cos[e + f*x] - 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

**Maple [A]**
time = 0.22, size = 89, normalized size = 1.09

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{5a^3cx}{8}-\frac{a^3c\cos(fx+e)}{2f}-\frac{a^3c\sin(4fx+4e)}{32f}-\frac{a^3c\cos(3fx+3e)}{6f}+\frac{a^3c\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2a^3c\cos(fx+e)+a^3c(fx+e)}{f}$ |
| default | $\frac{-a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2a^3c\cos(fx+e)+a^3c(fx+e)}{f}$ |
| norman | $\frac{-\frac{4a^3c}{3f}+\frac{5a^3cx}{8}-\frac{4a^3c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4a^3c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4a^3c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{11a^3c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(1\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2
/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^3*c*cos(f*x+e)+a^3*c*(f*x+e))
```

**Maxima [A]**
time = 0.31, size = 93, normalized size = 1.13

$$-\frac{64\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3c+3\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^3c-96\left(fx+e\right)a^3c+192a^3c\cos\left(fx+e\right)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 3*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c - 96*(f*x + e)*a^3*c + 192*a^3*c*c
os(f*x + e))/f
```

**Fricas [A]**
time = 0.33, size = 67, normalized size = 0.82

$$-\frac{16\,a^3c\cos\left(fx+e\right)^3-15\,a^3cfx+3\left(2\,a^3c\cos\left(fx+e\right)^3-5\,a^3c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^3*c*cos(f*x + e)^3 - 15*a^3*c*f*x + 3*(2*a^3*c*cos(f*x + e)^3 -
 5*a^3*c*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.
time = 0.21, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3a^3cx\sin^4(e+fx)}{8}-\frac{3a^3cx\sin^2(e+fx)\cos^2(e+fx)}{4}-\frac{3a^3cx\cos^4(e+fx)}{8}+a^3cx+\frac{5a^3c\sin^3(e+fx)\cos(e+fx)}{8f}+\frac{2a^3c\sin^2(e+fx)\cos(e+fx)}{f}+\frac{3a^3c\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{4a^3c\cos^3(e+fx)}{3f}-\frac{2a^3c\cos(e+fx)}{f} & \text{for } f\neq 0 \\ x(a\sin(e)+a)^3(-c\sin(e)+c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**3*c*x*sin(e + f*x)**4/8 - 3*a**3*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a**3*c*x*cos(e + f*x)**4/8 + a**3*c*x + 5*a**3*c*sin(e + f
*x)**3*cos(e + f*x)/(8*f) + 2*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*a**
3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*a**3*c*cos(e + f*x)**3/(3*f) - 2
*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c), Tr
ue))
```

**Giac** [A]
time = 0.45, size = 81, normalized size = 0.99

$$\frac{5}{8}\,a^3cx - \frac{a^3c\cos\left(3\,fx+3\,e\right)}{6\,f} - \frac{a^3c\cos\left(fx+e\right)}{2\,f} - \frac{a^3c\sin\left(4\,fx+4\,e\right)}{32\,f} + \frac{a^3c\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 5/8*a^3*c*x - 1/6*a^3*c*cos(3*f*x + 3*e)/f - 1/2*a^3*c*cos(f*x + e)/f - 1/3
2*a^3*c*sin(4*f*x + 4*e)/f + 1/4*a^3*c*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 8.81, size = 250, normalized size = 3.05

$$\frac{5\,a^3\,c\,x}{8} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{a^3\,c\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a^3\,c\,\left(60\,e+60\,f\,x-32\right)}{24}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{a^3\,c\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a^3\,c\,\left(60\,e+60\,f\,x-96\right)}{24}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{a^3\,c\,\left(15\,e+15\,f\,x\right)}{4}-\frac{a^3\,c\,\left(90\,e+90\,f\,x-96\right)}{24}\right)-\frac{3\,a^3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}-\frac{11\,a^3\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{4}+\frac{11\,a^3\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{4}+\frac{3\,a^3\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{4}+\frac{a^3\,c\,\left(15\,e+15\,f\,x\right)}{24}-\frac{a^3\,c\,\left(15\,e+15\,f\,x-32\right)}{24}}{f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

```
[Out] (5*a^3*c*x)/8 - (tan(e/2 + (f*x)/2)^2*((a^3*c*(15*e + 15*f*x))/6 - (a^3*c*(
60*e + 60*f*x - 32))/24) + tan(e/2 + (f*x)/2)^6*((a^3*c*(15*e + 15*f*x))/6
- (a^3*c*(60*e + 60*f*x - 96))/24) + tan(e/2 + (f*x)/2)^4*((a^3*c*(15*e + 1
5*f*x))/4 - (a^3*c*(90*e + 90*f*x - 96))/24) - (3*a^3*c*tan(e/2 + (f*x)/2))
/4 - (11*a^3*c*tan(e/2 + (f*x)/2)^3)/4 + (11*a^3*c*tan(e/2 + (f*x)/2)^5)/4
+ (3*a^3*c*tan(e/2 + (f*x)/2)^7)/4 + (a^3*c*(15*e + 15*f*x))/24 - (a^3*c*(1
5*e + 15*f*x - 32))/24)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)
```

# 3.253 $\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)} dx$

**Optimal**. Leaf size=94

$$-\frac{15a^3x}{2c}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}$$

```
[Out] -15/2*a^3*x/c+15/2*a^3*cos(f*x+e)/c/f+2*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x
+e))^3+5/2*a^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))
```

**Rubi** [A]
time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2758, 2761, 8}

$$\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{15a^3x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-15*a^3*x)/(2*c) + (15*a^3*Cos[e + f*x])/(2*c*f) + (2*a^3*c^2*Cos[e + f*x]
^5)/(f*(c - c*Sin[e + f*x])^3) + (5*a^3*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e +
 f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\ &= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx\\ &= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{1}{2}\left(15a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\\ &= \frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{(15a^3)\int}{2c}\\ &= -\frac{15a^3x}{2c}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}\end{aligned}$$

**Mathematica [A]**
time = 0.36, size = 153, normalized size = 1.63

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(e+fx)-16\cos(e+fx)-\sin(2(e+fx)))+\sin\left(\frac{1}{2}(e+fx)\right)(-64-30e-30fx+16\cos(e+fx)+\sin(2(e+fx)))\right)}{4cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(e + f*x) - 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Sin[(e + f*x)/
2]*(-64 - 30*e - 30*f*x + 16*Cos[e + f*x] + Sin[2*(e + f*x)])))/(4*c*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.28, size = 96, normalized size = 1.02

| method | result |
|---|---|
| derivativedivides | $\frac{2a^3\left(-\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-4}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{15\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$ |
| default | $\frac{2a^3\left(-\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-4}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{15\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$ |
| risch | $-\frac{15a^3x}{2c}+\frac{2a^3e^{i(fx+e)}}{cf}+\frac{2a^3e^{-i(fx+e)}}{cf}+\frac{16a^3}{fc\left(e^{i(fx+e)}-i\right)}+\frac{a^3\sin(2fx+2e)}{4cf}$ |
| norman | $\frac{15a^3x}{2c}-\frac{7a^3}{cf}-\frac{15a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c}+\frac{45a^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c}-\frac{45a^3x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c}+\frac{45a^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c}-\frac{45a^3x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{2c}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c*(-(1/2*tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1/2*e)^2-1/2*tan(1/2*f*
x+1/2*e)-4)/(1+tan(1/2*f*x+1/2*e)^2)^2-15/2*arctan(tan(1/2*f*x+1/2*e))-8/(t
an(1/2*f*x+1/2*e)-1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(95) = 190.
time = 0.53, size = 471, normalized size = 5.01

$$-\frac{6a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-4}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+6a^3\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{2a^3}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) +
 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 6*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) - 2*a^3/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [A]
time = 0.32, size = 137, normalized size = 1.46

$$\frac{a^3\cos(fx+e)^3-15\,a^3fx+8\,a^3\cos(fx+e)^2+16\,a^3-(15\,a^3fx-23\,a^3)\cos(fx+e)+(15\,a^3fx+a^3\cos(fx+e)^2-7\,a^3\cos(fx+e)+16\,a^3)\sin(fx+e)}{2\,(cf\cos(fx+e)-cf\sin(fx+e)+cf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(f*x + e)^3 - 15*a^3*f*x + 8*a^3*cos(f*x + e)^2 + 16*a^3 - (15*
a^3*f*x - 23*a^3)*cos(f*x + e) + (15*a^3*f*x + a^3*cos(f*x + e)^2 - 7*a^3*c
os(f*x + e) + 16*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) +
c*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(83) = 166$.
time = 2.47, size = 1168, normalized size = 12.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*a**3*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 15*a**3*f*x*tan(e/2 + f*x/2)**4/(2
*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/
2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 30*a*
*3*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x
/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(
e/2 + f*x/2) - 2*c*f) + 30*a**3*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*
x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan
(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 15*a**3*f*x*tan(e/2 +
f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e
/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f
) + 15*a**3*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) - 34*a**3*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f
*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**
2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*a**3*tan(e/2 + f*x/2)**3/(2*c*f*ta
n(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 -
 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 78*a**3*tan(
e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) + 14*a**3*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*ta
n(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
```

```
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 48*a**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c
*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)
**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*
sin(e) + c), True))
```

**Giac** [A]
time = 0.48, size = 117, normalized size = 1.24

$$-\frac{\frac{15\,(fx+e)a^3}{c}+\frac{32\,a^3}{c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)}+\frac{2\left(a^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3-8\,a^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-a^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-8\,a^3\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+1\right)^2c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*a^3/c + 32*a^3/(c*(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^3*t
an(1/2*f*x + 1/2*e)^3 - 8*a^3*tan(1/2*f*x + 1/2*e)^2 - a^3*tan(1/2*f*x + 1/
2*e) - 8*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

**Mupad** [B]
time = 9.12, size = 219, normalized size = 2.33

$$-\frac{15\,a^3\,x}{2\,c}-\frac{\frac{15\,a^3\,(e+f\,x)}{2}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{15\,a^3\,(e+f\,x)}{2}-\frac{a^3\,(15\,e+15\,f\,x-14)}{2}\right)-\frac{a^3\,(15\,e+15\,f\,x-48)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{15\,a^3\,(e+f\,x)}{2}-\frac{a^3\,(15\,e+15\,f\,x-34)}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(15\,a^3\,(e+f\,x)-\frac{a^3\,(30\,e+30\,f\,x-18)}{2}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(15\,a^3\,(e+f\,x)-\frac{a^3\,(30\,e+30\,f\,x-78)}{2}\right)}{c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x)),x)
```

```
[Out] - (15*a^3*x)/(2*c) - ((15*a^3*(e + f*x))/2 - tan(e/2 + (f*x)/2)*((15*a^3*(e
 + f*x))/2 - (a^3*(15*e + 15*f*x - 14))/2) - (a^3*(15*e + 15*f*x - 48))/2 +
 tan(e/2 + (f*x)/2)^4*((15*a^3*(e + f*x))/2 - (a^3*(15*e + 15*f*x - 34))/2)
 - tan(e/2 + (f*x)/2)^3*(15*a^3*(e + f*x) - (a^3*(30*e + 30*f*x - 18))/2) +
 tan(e/2 + (f*x)/2)^2*(15*a^3*(e + f*x) - (a^3*(30*e + 30*f*x - 78))/2))/(c
*f*(tan(e/2 + (f*x)/2) - 1)*(tan(e/2 + (f*x)/2)^2 + 1)^2)
```

# 3.254 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=92

$$\frac{5a^3x}{c^2}-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] 5*a^3*x/c^2-5*a^3*cos(f*x+e)/c^2/f+2/3*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+
e))^4-10/3*a^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2759, 2761, 8}

$$-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{5a^3x}{c^2}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*x)/c^2 - (5*a^3*Cos[e + f*x])/(c^2*f) + (2*a^3*c^2*Cos[e + f*x]^5)/(
3*f*(c - c*Sin[e + f*x])^4) - (10*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{1}{3}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx \\
&= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{c} \\
&= -\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(5a^3\right)\int}{c^2} \\
&= \frac{5a^3x}{c^2}-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}
\end{aligned}$$

**Mathematica [A]**
time = 0.65, size = 149, normalized size = 1.62

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(23+15e+15fx)\cos\left(\frac{1}{2}(e+fx)\right)-(121+30e+30fx)\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{5}{2}(e+fx)\right)-6(31+20e+20fx+2(-2+5e+5fx)\cos(e+fx)-\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{12c^2f(-1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*(23 + 15*e + 15*f*x)*Cos[(e +
 f*x)/2] - (121 + 30*e + 30*f*x)*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))
/2] - 6*(31 + 20*e + 20*f*x + 2*(-2 + 5*e + 5*f*x)*Cos[e + f*x] - Cos[2*(e
+ f*x)])*Sin[(e + f*x)/2]))/(12*c^2*f*(-1 + Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.32, size = 87, normalized size = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{2a^3\left(-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+5\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{16}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$ |

| | |
|---|---|
| default | $\frac{2a^3\left(-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+5\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{16}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$ |
| risch | $\frac{5a^3x}{c^2}-\frac{a^3\mathrm{e}^{i(fx+e)}}{2c^2f}-\frac{a^3\mathrm{e}^{-i(fx+e)}}{2c^2f}-\frac{8\left(-12ia^3\mathrm{e}^{i(fx+e)}+9a^3\mathrm{e}^{2i(fx+e)}-7a^3\right)}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3f\,c^2}$ |
| norman | $\frac{\frac{8a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{5a^3x}{c}+\frac{46a^3}{3cf}+\frac{15a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{30a^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{50a^3x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{60a^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{c}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^2*(-1/(1+tan(1/2*f*x+1/2*e)^2)+5*arctan(tan(1/2*f*x+1/2*e))-16/3/
(tan(1/2*f*x+1/2*e)-1)^3-8/(tan(1/2*f*x+1/2*e)-1)^2+4/(tan(1/2*f*x+1/2*e)-1
))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(95) = 190$.
time = 0.54, size = 644, normalized size = 7.00

$$\frac{2\left(2\,a^3\left(\frac{\frac{12\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{11\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-5}{c^2-\frac{3\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4\,c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4\,c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,c^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)+3\,a^3\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{a^3\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\,a^3\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 3*a^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(95) = 190$.
time = 0.34, size = 194, normalized size = 2.11

$$-\frac{3\,a^3\cos\left(fx+e\right)^3+30\,a^3fx+8\,a^3-\left(15\,a^3fx+31\,a^3\right)\cos\left(fx+e\right)^2+\left(15\,a^3fx-26\,a^3\right)\cos\left(fx+e\right)-\left(30\,a^3fx-3\,a^3\cos\left(fx+e\right)^2-8\,a^3+\left(15\,a^3fx-34\,a^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+2\,c^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(3*a^3*cos(f*x + e)^3 + 30*a^3*f*x + 8*a^3 - (15*a^3*f*x + 31*a^3)*cos(f*x + e)^2 + (15*a^3*f*x - 26*a^3)*cos(f*x + e) - (30*a^3*f*x - 3*a^3*cos(f*x + e)^2 - 8*a^3 + (15*a^3*f*x - 34*a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. 2(87) = 174.
time = 4.77, size = 1282, normalized size = 13.93

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)

[Out] Piecewise((15*a**3*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 45*a**3*f*x*tan(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 60*a**3*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 60*a**3*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 45*a**3*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 15*a**3*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*a**3*tan(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 102*a**3*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 82*a**3*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 114*a**3*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 46*a**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2

```
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(
e) + c)**2, True))
```

**Giac** [A]
time = 0.45, size = 101, normalized size = 1.10

$$\frac{\frac{15\,(fx+e)a^3}{c^2} - \frac{6\,a^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2} + \frac{8\left(3\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,a^3\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*a^3/c^2 - 6*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^2) + 8*(3
*a^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*tan(1/2*f*x + 1/2*e) + 5*a^3)/(c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3))/f
```

**Mupad** [B]
time = 9.75, size = 218, normalized size = 2.37

$$\frac{5\,a^3\,x}{c^2}+\frac{5\,a^3\,(e+f\,x)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,a^3\,(e+f\,x)-\frac{a^3\,(45\,e+45\,f\,x-114)}{3}\right)-\frac{a^3\,(15\,e+15\,f\,x-46)}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(15\,a^3\,(e+f\,x)-\frac{a^3\,(45\,e+45\,f\,x-24)}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(20\,a^3\,(e+f\,x)-\frac{a^3\,(60\,e+60\,f\,x-82)}{3}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(20\,a^3\,(e+f\,x)-\frac{a^3\,(60\,e+60\,f\,x-102)}{3}\right)}{c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (5*a^3*x)/c^2 + (5*a^3*(e + f*x) - tan(e/2 + (f*x)/2)*(15*a^3*(e + f*x) - (
a^3*(45*e + 45*f*x - 114))/3) - (a^3*(15*e + 15*f*x - 46))/3 + tan(e/2 + (f
*x)/2)^4*(15*a^3*(e + f*x) - (a^3*(45*e + 45*f*x - 24))/3) + tan(e/2 + (f*x
)/2)^2*(20*a^3*(e + f*x) - (a^3*(60*e + 60*f*x - 82))/3) - tan(e/2 + (f*x)/
2)^3*(20*a^3*(e + f*x) - (a^3*(60*e + 60*f*x - 102))/3))/(c^2*f*(tan(e/2 +
(f*x)/2) - 1)^3*(tan(e/2 + (f*x)/2)^2 + 1))
```

# 3.255 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=106

$$-\frac{a^3x}{c^3}+\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^3\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}$$

```
[Out] -a^3*x/c^3+2/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^3*cos(f*x+e)
^3/f/(c-c*sin(f*x+e))^3+2*a^3*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

**Rubi** [A]
time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2759, 8}

$$\frac{2a^3\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}-\frac{a^3x}{c^3}+\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*x)/c^3) + (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) -
(2*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^3*Cos[e + f*x])/
(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx &= (a^3c^3)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx \\
&= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - (a^3c)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{a^3\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{c} \\
&= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))} - \cdots \\
&= -\frac{a^3x}{c^3} + \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx}{f\,(c^3-c^3\sin(e+\cdots}
\end{aligned}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(106) = 212$.

time = 0.32, size = 249, normalized size = 2.35

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-44\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-15(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+48\sin\left(\frac{1}{2}(e+fx)\right)-88\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+92\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(a+a\sin(e+fx))^3}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]) - 44*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 15*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5 + 48*Sin[(e + f*x)/2] - 88*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 92*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4*Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^3)
```

**Maple** [**A**]

time = 0.36, size = 100, normalized size = 0.94

| method | result |
|---|---|
| risch | $-\frac{a^3x}{c^3}+\frac{-24ia^3e^{3i(fx+e)}+12a^3e^{4i(fx+e)}+\frac{56ia^3e^{i(fx+e)}}{3}-\frac{112a^3e^{2i(fx+e)}}{3}+\frac{92a^3}{15}}{\left(e^{i(fx+e)}-i\right)^5 f\,c^3}$ |
| derivativedivides | $\frac{2a^3\left(-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{40}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{\cdots}{\tan(\cdots}\right.}{f\,c^3}$ |
| default | $\frac{2a^3\left(-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{40}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{\cdots}{\tan(\cdots}\right.}{f\,c^3}$ |

| | |
|---|---|
| norman | $\frac{a^3x}{c}+\frac{8a^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{48a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{52a^3}{15cf}-\frac{5a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{13a^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{25a^3x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^3*(-arctan(tan(1/2*f*x+1/2*e))-32/5/(tan(1/2*f*x+1/2*e)-1)^5-16/(
tan(1/2*f*x+1/2*e)-1)^4-40/3/(tan(1/2*f*x+1/2*e)-1)^3-4/(tan(1/2*f*x+1/2*e)
-1)^2-2/(tan(1/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(111) = 222$.
time = 0.55, size = 853, normalized size = 8.05

$$-\frac{2\left(a^3\left(\frac{\frac{95\sin(fx+e)}{\cos(fx+e)+1}-\frac{145\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{75\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-22}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}\right)+\frac{a^3\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{9a^3\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{6a^3\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3)
+ a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) - 9*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
- 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*a^3*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(111) = 222$.
time = 0.34, size = 247, normalized size = 2.33

$$\frac{60\,a^3fx-(15\,a^3fx-46\,a^3)\cos(fx+e)^3-24\,a^3-(45\,a^3fx+2\,a^3)\cos(fx+e)^2+6\,(5\,a^3fx-12\,a^3)\cos(fx+e)-(60\,a^3fx+24\,a^3-(15\,a^3fx+46\,a^3)\cos(fx+e)^2+6\,(5\,a^3fx-8\,a^3)\cos(fx+e))\sin(fx+e)}{15\,(c^3f\cos(fx+e)^3+3\,c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)-4\,c^3f-(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)-4\,c^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*a^3*f*x - (15*a^3*f*x - 46*a^3)*cos(f*x + e)^3 - 24*a^3 - (45*a^3*
f*x + 2*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 12*a^3)*cos(f*x + e) - (60*a^3
*f*x + 24*a^3 - (15*a^3*f*x + 46*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 8*a^3
)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(95) = 190$.
time = 9.53, size = 1282, normalized size = 12.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5
- 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2
+ f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)*
*2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*a**3*f*x*tan(e/2 + f*x/2
)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c
**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(
e/2 + f*x/2) - 15*c**3*f) + 150*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan
(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x
/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c
**3*f) - 75*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c
**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*a**3*f*x/(1
5*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*t
an(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f
*x/2) - 15*c**3*f) - 60*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2
)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150
*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120
*a**3*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/
2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2
)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 400*a**3*tan(e/2 + f*x/2)*
*2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**
3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/
2 + f*x/2) - 15*c**3*f) + 200*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*
x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 -
```

```
150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
52*a**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 15
0*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) +
c)**3, True))
```

**Giac** [A]
time = 0.45, size = 111, normalized size = 1.05

$$-\frac{\frac{15\,(fx+e)a^3}{c^3}+\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-50\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,a^3\right)}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*a^3/c^3 + 4*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 30*a^3*tan
(1/2*f*x + 1/2*e)^3 + 100*a^3*tan(1/2*f*x + 1/2*e)^2 - 50*a^3*tan(1/2*f*x +
 1/2*e) + 13*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

**Mupad** [B]
time = 8.48, size = 203, normalized size = 1.92

$$-\frac{a^3\,x}{c^3}-\frac{a^3\,\left(e+f\,x\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(75\,e+75\,f\,x-200\right)}{15}\right)-\frac{a^3\,\left(15\,e+15\,f\,x-52\right)}{15}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(5\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(75\,e+75\,f\,x-60\right)}{15}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(10\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(150\,e+150\,f\,x-120\right)}{15}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(10\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(150\,e+150\,f\,x-400\right)}{15}\right)}{c^3\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^3,x)
```

```
[Out] - (a^3*x)/c^3 - (a^3*(e + f*x) - tan(e/2 + (f*x)/2)*(5*a^3*(e + f*x) - (a^3
*(75*e + 75*f*x - 200))/15) - (a^3*(15*e + 15*f*x - 52))/15 + tan(e/2 + (f*
x)/2)^4*(5*a^3*(e + f*x) - (a^3*(75*e + 75*f*x - 60))/15) - tan(e/2 + (f*x)
/2)^3*(10*a^3*(e + f*x) - (a^3*(150*e + 150*f*x - 120))/15) + tan(e/2 + (f*
x)/2)^2*(10*a^3*(e + f*x) - (a^3*(150*e + 150*f*x - 400))/15))/(c^3*f*(tan(
e/2 + (f*x)/2) - 1)^5)
```

# 3.256 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=34

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

[Out] 1/7*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7

**Rubi** [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2815, 2750}

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]

[Out] (a^3*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7)

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.
time = 0.52, size = 93, normalized size = 2.74

$$\frac{a^3\left(35\cos\left(\frac{1}{2}(e+fx)\right)-21\cos\left(\frac{3}{2}(e+fx)\right)-7\cos\left(\frac{5}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{28c^4f(-1+\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*(35*Cos[(e + f*x)/2] - 21*Cos[(3*(e + f*x))/2] - 7*Cos[(5*(e + f*x))/2
] + Cos[(7*(e + f*x))/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(28*c^4*f*
(-1 + Sin[e + f*x])^4)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(32) = 64$.
time = 0.42, size = 118, normalized size = 3.47

| method | result |
|---|---|
| risch | $-\frac{2\left(7a^3e^{6i(fx+e)}-35a^3e^{4i(fx+e)}+21a^3e^{2i(fx+e)}-a^3\right)}{7\left(e^{i(fx+e)}-i\right)^7 f\, c^4}$ |
| derivativedivides | $\frac{2a^3\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{40}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}\right.}{f\,c^4}$ |
| default | $\frac{2a^3\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{40}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}\right.}{f\,c^4}$ |
| norman | $\frac{-\frac{2a^3}{7cf}-\frac{48a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7cf}-\frac{202a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7cf}-\frac{352a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7cf}-\frac{42a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{16a^3\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-2}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^4*(-1/(tan(1/2*f*x+1/2*e)-1)-32/(tan(1/2*f*x+1/2*e)-1)^6-40/(tan(
1/2*f*x+1/2*e)-1)^4-20/(tan(1/2*f*x+1/2*e)-1)^3-64/7/(tan(1/2*f*x+1/2*e)-1)
^7-6/(tan(1/2*f*x+1/2*e)-1)^2-48/(tan(1/2*f*x+1/2*e)-1)^5)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(35) = 70$.
time = 0.35, size = 1137, normalized size = 33.44

$$\frac{2\left(\frac{a^3\left(\frac{91\sin(fx+e)}{\cos(fx+e)+1}-\frac{168\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{280\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{175\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{105\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-13\right)}{c^4-\frac{7c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{35c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{21c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{7c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}-\frac{a^3\left(\frac{49\sin(fx+e)}{\cos(fx+e)+1}-\frac{147\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{210\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{210\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{105\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-12\right)}{c^4-\frac{7c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{35c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{21c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{7c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}-\frac{4a^3\left(\frac{14\sin(fx+e)}{\cos(fx+e)+1}-\frac{42\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-2\right)}{c^4-\frac{7c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{35c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{21c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{7c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{2a^3\left(\frac{7\sin(fx+e)}{\cos(fx+e)+1}-\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^4-\frac{7c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{35c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{21c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{7c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}\right)}{35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/35*(a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
- 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c
^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7) - a^3*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^3*(14*sin(f*x + e)
/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7) + 2*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c
^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(35) = 70$.

time = 0.32, size = 238, normalized size = 7.00

$$\frac{a^3 \cos(fx+e)^4 - 3a^3\cos(fx+e)^3 - 8a^3\cos(fx+e)^2 + 4a^3\cos(fx+e) + 8a^3 - (a^3\cos(fx+e)^3 + 4a^3\cos(fx+e)^2 - 4a^3\cos(fx+e) - 8a^3)\sin(fx+e)}{7\left(c^4 f\cos(fx+e)^4 - 3c^4 f\cos(fx+e)^3 - 8c^4 f\cos(fx+e)^2 + 4c^4 f\cos(fx+e) + 8c^4 f + (c^4 f\cos(fx+e)^3 + 4c^4 f\cos(fx+e)^2 - 4c^4 f\cos(fx+e) - 8c^4 f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f
*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*
f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)
*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(29) = 58$.

time = 19.96, size = 619, normalized size = 18.21

$$\begin{cases} -\frac{14a^3\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{7c^4f\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)-49c^4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+147c^4f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-245c^4f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+245c^4f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-147c^4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+49c^4f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-7c^4f}-\frac{70a^3\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{7c^4f\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)-49c^4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+147c^4f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-245c^4f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+245c^4f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-147c^4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+49c^4f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-7c^4f}-\frac{42a^3\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{7c^4f\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)-49c^4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+147c^4f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-245c^4f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+245c^4f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-147c^4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+49c^4f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-7c^4f}-\frac{2a^3}{7c^4f\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)-49c^4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+147c^4f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-245c^4f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+245c^4f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-147c^4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+49c^4f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-7c^4f} & \text{for } f \neq 0 \\ \frac{x(a\sin(e)+a)^3}{(-c\sin(e)+c)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-14*a**3*tan(e/2 + f*x/2)**6/(7*c**4*f*tan(e/2 + f*x/2)**7 - 49*
c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f*x/2)**5 - 245*c**4*f*ta
n(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3 - 147*c**4*f*tan(e/2 + f
*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f) - 70*a**3*tan(e/2 + f*x/2
)**4/(7*c**4*f*tan(e/2 + f*x/2)**7 - 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c*
*4*f*tan(e/2 + f*x/2)**5 - 245*c**4*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(
e/2 + f*x/2)**3 - 147*c**4*f*tan(e/2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/
2) - 7*c**4*f) - 42*a**3*tan(e/2 + f*x/2)**2/(7*c**4*f*tan(e/2 + f*x/2)**7
- 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f*x/2)**5 - 245*c**4
*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3 - 147*c**4*f*tan(e/
2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f) - 2*a**3/(7*c**4*f*t
an(e/2 + f*x/2)**7 - 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f
*x/2)**5 - 245*c**4*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3
- 147*c**4*f*tan(e/2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f),
Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**4, True))
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.
time = 0.48, size = 77, normalized size = 2.26

$$-\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+21\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3\right)}{7\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/7*(7*a^3*tan(1/2*f*x + 1/2*e)^6 + 35*a^3*tan(1/2*f*x + 1/2*e)^4 + 21*a^3
*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

**Mupad [B]**
time = 7.01, size = 116, normalized size = 3.41

$$\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left({\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+21\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+35\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+7\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\right)}{7\,c^4\,f\,{\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2*a^3*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^6 + 7*sin(e/2 + (f*x)/2)^6 +
35*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 + 21*cos(e/2 + (f*x)/2)^4*sin(
e/2 + (f*x)/2)^2))/(7*c^4*f*(cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2))^7)
```

# 3.257 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=69

$$\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[Out] 1/9*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*c^2*cos(f*x+e)^7/f/(
c-c*sin(f*x+e))^7
```

**Rubi** [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*c^2*Cos[e + f*
x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{1}{9}\left(a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}
\end{aligned}
$$

**Mathematica [A]**
time = 0.50, size = 135, normalized size = 1.96

$$
\frac{a^3\left(315\cos\left(\frac{1}{2}(e+fx)\right)-189\cos\left(\frac{3}{2}(e+fx)\right)-63\cos\left(\frac{5}{2}(e+fx)\right)+9\cos\left(\frac{7}{2}(e+fx)\right)+189\sin\left(\frac{1}{2}(e+fx)\right)+105\sin\left(\frac{3}{2}(e+fx)\right)-27\sin\left(\frac{5}{2}(e+fx)\right)-\sin\left(\frac{9}{2}(e+fx)\right)\right)}{504c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(315*Cos[(e + f*x)/2] - 189*Cos[(3*(e + f*x))/2] - 63*Cos[(5*(e + f*x)
)/2] + 9*Cos[(7*(e + f*x))/2] + 189*Sin[(e + f*x)/2] + 105*Sin[(3*(e + f*x)
)/2] - 27*Sin[(5*(e + f*x))/2] - Sin[(9*(e + f*x))/2]))/(504*c^5*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(65) = 130$.
time = 0.52, size = 148, normalized size = 2.14

| method | result |
|---|---|
| risch | $\frac{2ia^3\left(105i\,e^{6i(fx+e)}+63\,e^{7i(fx+e)}-189i\,e^{4i(fx+e)}-315\,e^{5i(fx+e)}+27i\,e^{2i(fx+e)}+189\,e^{3i(fx+e)}+i-9\,e^{i(fx+e)}\right)}{63f\,c^5\left(e^{i(fx+e)}-i\right)^9}$ |
| derivativedivides | $\frac{2a^3\left(-\frac{928}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{86}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{496}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}\right.}{f\,c^5}$ |
| default | $\frac{2a^3\left(-\frac{928}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{86}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{496}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}\right.}{f\,c^5}$ |
| norman | $\frac{\frac{2a^3\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{16a^3\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{42a^3\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{16a^3}{63cf}-\frac{2a^3\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7cf}+\frac{48a^3\left(\tan\right.}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^5*(-928/7/(tan(1/2*f*x+1/2*e)-1)^7-7/(tan(1/2*f*x+1/2*e)-1)^2-1/(
tan(1/2*f*x+1/2*e)-1)-86/3/(tan(1/2*f*x+1/2*e)-1)^3-496/3/(tan(1/2*f*x+1/2*
e)-1)^6-64/(tan(1/2*f*x+1/2*e)-1)^8-76/(tan(1/2*f*x+1/2*e)-1)^4-128/9/(tan(
1/2*f*x+1/2*e)-1)^9-136/(tan(1/2*f*x+1/2*e)-1)^5)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1513 vs. $2(71) = 142$.
time = 0.37, size = 1513, normalized size = 21.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 15*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*a^3*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 3
6*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 42*a^3*(9*sin(f*x
 + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
```

```
45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(71) = 142$.
time = 0.32, size = 296, normalized size = 4.29

$$-\frac{a^3\cos(fx+e)^5 - 4a^3\cos(fx+e)^4 + 19a^3\cos(fx+e)^3 + 52a^3\cos(fx+e)^2 - 28a^3\cos(fx+e) - 56a^3 + (a^3\cos(fx+e)^4 + 5a^3\cos(fx+e)^3 + 24a^3\cos(fx+e)^2 - 28a^3\cos(fx+e) - 56a^3)\sin(fx+e)}{63\left(c^5f\cos(fx+e)^5 + 5c^5f\cos(fx+e)^4 - 8c^5f\cos(fx+e)^3 - 20c^5f\cos(fx+e)^2 + 8c^5f\cos(fx+e) + 16c^5f - (c^5f\cos(fx+e)^4 - 4c^5f\cos(fx+e)^3 - 12c^5f\cos(fx+e)^2 + 8c^5f\cos(fx+e) + 16c^5f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(a^3*cos(f*x + e)^5 - 4*a^3*cos(f*x + e)^4 + 19*a^3*cos(f*x + e)^3 +
52*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a^3 + (a^3*cos(f*x + e)^4
+ 5*a^3*cos(f*x + e)^3 + 24*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a
^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*
cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
- (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2
+ 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(60) = 120$.
time = 35.32, size = 1717, normalized size = 24.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9 - 5
67*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5
*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(
e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f
*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*a**3*tan(e/2 + f*
x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**
5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan
(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f
*x/2) - 63*c**5*f) - 966*a**3*tan(e/2 + f*x/2)**6/(63*c**5*f*tan(e/2 + f*x/
2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 -
5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c*
```

```
*5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*ta
n(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 630*a**3*tan
(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/
2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 +
 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c
**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*ta
n(e/2 + f*x/2) - 63*c**5*f) - 1386*a**3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(
e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*
x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5
 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268
*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 37
8*a**3*tan(e/2 + f*x/2)**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(
e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f
*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**
4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567
*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 450*a**3*tan(e/2 + f*x/2)**2/(63*c*
*5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan
(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 +
f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)*
*3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**
5*f) + 18*a**3*tan(e/2 + f*x/2)/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f
*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/
2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x
/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2
+ 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 16*a**3/(63*c**5*f*tan(e/2 + f
*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7
 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f), Ne(f, 0)),
 (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**5, True))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(71) =$ 142.
time = 0.50, size = 162, normalized size = 2.35

$$-\frac{2\left(63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+483\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+693\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-189\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+225\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,a^{3}\right)}{63\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/63*(63*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*a^3*tan(1/2*f*x + 1/2*e)^5 + 693*a^3*tan(1
/2*f*x + 1/2*e)^4 - 189*a^3*tan(1/2*f*x + 1/2*e)^3 + 225*a^3*tan(1/2*f*x +
1/2*e)^2 - 9*a^3*tan(1/2*f*x + 1/2*e) + 8*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e)
 - 1)^9)
```

**Mupad [B]**
time = 8.58, size = 121, normalized size = 1.75

$$\frac{\sqrt{2}\,a^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{37\cos(3\,e+3\,f\,x)}{8}-\frac{63\sin(e+f\,x)}{2}-\frac{113\cos(2\,e+2\,f\,x)}{4}-\frac{257\cos(e+f\,x)}{8}+\frac{7\cos(4\,e+4\,f\,x)}{16}+\frac{63\sin(2\,e+2\,f\,x)}{8}+\frac{9\sin(3\,e+3\,f\,x)}{2}-\frac{9\sin(4\,e+4\,f\,x)}{16}+\frac{1013}{16}\right)}{1008\,c^5\,f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((37*cos(3*e + 3*f*x))/8 - (63*sin(e + f*x)
)/2 - (113*cos(2*e + 2*f*x))/4 - (257*cos(e + f*x))/8 + (7*cos(4*e + 4*f*x)
)/16 + (63*sin(2*e + 2*f*x))/8 + (9*sin(3*e + 3*f*x))/2 - (9*sin(4*e + 4*f*
x))/16 + 1013/16))/(1008*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

# 3.258 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6} dx$

**Optimal**. Leaf size=101

$$\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/99*a^3*c^2*cos(f*x+e)^7/f/
(c-c*sin(f*x+e))^8+2/693*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

**Rubi** [A]
time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c^2*Cos[e +
 f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (2*a^3*c*Cos[e + f*x]^7)/(693*f*(c
 - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{1}{11}\left(2a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{1}{99}\left(2a^3c\right)\int \frac{\cos^6(e+ \cdots}{(c-c\sin(e+ \cdots} \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}
\end{aligned}$$

**Mathematica [A]**
time = 0.59, size = 145, normalized size = 1.44

$$-\frac{a^3\left(-2541\cos\left(\frac{1}{2}(e+fx)\right)+1485\cos\left(\frac{3}{2}(e+fx)\right)+462\cos\left(\frac{5}{2}(e+fx)\right)-55\cos\left(\frac{7}{2}(e+fx)\right)+\cos\left(\frac{11}{2}(e+fx)\right)-2079\sin\left(\frac{1}{2}(e+fx)\right)-1155\sin\left(\frac{3}{2}(e+fx)\right)+297\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)\right)}{5544c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] -1/5544*(a^3*(-2541*Cos[(e + f*x)/2] + 1485*Cos[(3*(e + f*x))/2] + 462*Cos[
(5*(e + f*x))/2] - 55*Cos[(7*(e + f*x))/2] + Cos[(11*(e + f*x))/2] - 2079*S
in[(e + f*x)/2] - 1155*Sin[(3*(e + f*x))/2] + 297*Sin[(5*(e + f*x))/2] + 11
*Sin[(9*(e + f*x))/2]))/(c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

**Maple [A]**
time = 0.54, size = 178, normalized size = 1.76

| method | result |
|---|---|
| risch | $\frac{\frac{60a^3e^{4i(fx+e)}}{7}-12ia^3e^{5i(fx+e)}-\frac{20a^3e^{2i(fx+e)}}{63}+\frac{12ia^3e^{3i(fx+e)}}{7}+\frac{4ia^3e^{i(fx+e)}}{63}-\frac{44a^3e^{6i(fx+e)}}{3}+\frac{4a^3}{693}+\frac{20ia^3e^{7i(fx+e)}}{3}+ \cdots}{\left(e^{i(fx+e)}-i\right)^{11}fc^6}$ |
| derivativedivides | $\frac{2a^3\left(-\frac{544}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{3008}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4272}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{126}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right) \cdots\right.} \cdots\right.}{fc^6}$ |
| default | $\frac{2a^3\left(-\frac{544}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{3008}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4272}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{126}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right) \cdots\right.} \cdots\right.}{fc^6}$ |

| | |
|---|---|
| norman | $-\frac{158a^3}{693cf}-\frac{2a^3\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{4a^3\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{88a^3\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{32a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{63cf}+\frac{128a^3\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^6*(-544/(tan(1/2*f*x+1/2*e)-1)^8-3008/9/(tan(1/2*f*x+1/2*e)-1)^9-
1/(tan(1/2*f*x+1/2*e)-1)-4272/7/(tan(1/2*f*x+1/2*e)-1)^7-126/(tan(1/2*f*x+1
/2*e)-1)^4-256/11/(tan(1/2*f*x+1/2*e)-1)^11-8/(tan(1/2*f*x+1/2*e)-1)^2-1480
/3/(tan(1/2*f*x+1/2*e)-1)^6-292/(tan(1/2*f*x+1/2*e)-1)^5-128/(tan(1/2*f*x+1
/2*e)-1)^10-116/3/(tan(1/2*f*x+1/2*e)-1)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. $2(104) = 208$.
time = 0.39, size = 1890, normalized size = 18.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/3465*(5*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 14
6)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*a^3*(671*sin(f
*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 660
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
```

```
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*a^3*(341*sin
(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*a^3*(253*sin(f*x + e)/(
cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5
313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55
*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(104) = 208$.
time = 0.33, size = 356, normalized size = 3.52

$$\frac{2\,a^{3}\cos\left(fx+e\right)^{6}+12\,a^{3}\cos\left(fx+e\right)^{5}-25\,a^{3}\cos\left(fx+e\right)^{4}+161\,a^{3}\cos\left(fx+e\right)^{3}+448\,a^{3}\cos\left(fx+e\right)^{2}-252\,a^{3}\cos\left(fx+e\right)-504\,a^{3}-\left(2\,a^{3}\cos\left(fx+e\right)^{5}-10\,a^{3}\cos\left(fx+e\right)^{4}-35\,a^{3}\cos\left(fx+e\right)^{3}-196\,a^{3}\cos\left(fx+e\right)^{2}+252\,a^{3}\cos\left(fx+e\right)+504\,a^{3}\right)\sin\left(fx+e\right)}{693\left(c^{6}f\cos\left(fx+e\right)^{6}-5\,c^{6}f\cos\left(fx+e\right)^{5}-18\,c^{6}f\cos\left(fx+e\right)^{4}+20\,c^{6}f\cos\left(fx+e\right)^{3}+48\,c^{6}f\cos\left(fx+e\right)^{2}-16\,c^{6}f\cos\left(fx+e\right)-32\,c^{6}f+\left(c^{6}f\cos\left(fx+e\right)^{5}+6\,c^{6}f\cos\left(fx+e\right)^{4}-12\,c^{6}f\cos\left(fx+e\right)^{3}-32\,c^{6}f\cos\left(fx+e\right)^{2}+16\,c^{6}f\cos\left(fx+e\right)+32\,c^{6}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(2*a^3*cos(f*x + e)^6 + 12*a^3*cos(f*x + e)^5 - 25*a^3*cos(f*x + e)^4
 + 161*a^3*cos(f*x + e)^3 + 448*a^3*cos(f*x + e)^2 - 252*a^3*cos(f*x + e) -
 504*a^3 - (2*a^3*cos(f*x + e)^5 - 10*a^3*cos(f*x + e)^4 - 35*a^3*cos(f*x +
 e)^3 - 196*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) + 504*a^3)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2509 vs. $2(92) = 184$.

time = 54.89, size = 2509, normalized size = 24.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-1386*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)**11
 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 11
4345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32016
6*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c
**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*
f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 2772*a
**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(
e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 +
f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x
/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)*
*2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*a**3*tan(e/2 + f*x/
2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228
690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166
*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c*
*6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*t
an(e/2 + f*x/2) - 693*c**6*f) + 21252*a**3*tan(e/2 + f*x/2)**7/(693*c**6*f*
tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(
e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2
 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 +
f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x
/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) -
693*c**6*f) - 42504*a**3*tan(e/2 + f*x/2)**6/(693*c**6*f*tan(e/2 + f*x/2)**
11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320
166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690
*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**
6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 3049
2*a**3*tan(e/2 + f*x/2)**5/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*t
an(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(
e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2
 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/
2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 30888*a**3*tan(e/2 + f
*x/2)**4/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**1
0 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 +
228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320
```

```
166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345
*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*
f*tan(e/2 + f*x/2) - 693*c**6*f) + 9900*a**3*tan(e/2 + f*x/2)**3/(693*c**6*
f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*ta
n(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e
/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2
+ f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f
*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2)
- 693*c**6*f) - 5918*a**3*tan(e/2 + f*x/2)**2/(693*c**6*f*tan(e/2 + f*x/2)*
*11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32
0166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 22869
0*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c*
*6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 352
*a**3*tan(e/2 + f*x/2)/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e
/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
+ f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f
*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/
2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**
2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 158*a**3/(693*c**6*f*tan(e
/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 +
 f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*
x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2
)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**
3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c
**6*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**6, True))
```

**Giac** [A]

time = 0.48, size = 196, normalized size = 1.94

$$-\frac{2\left(693\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+8085\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-10626\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+21252\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-15246\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+15444\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-4950\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2959\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-176\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+79\,a^{3}\right)}{693\,c^{6}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/693*(693*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*a^3*tan(1/2*f*x + 1/2*e)^9 +
 8085*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252
*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*a^3*
tan(1/2*f*x + 1/2*e)^4 - 4950*a^3*tan(1/2*f*x + 1/2*e)^3 + 2959*a^3*tan(1/2
*f*x + 1/2*e)^2 - 176*a^3*tan(1/2*f*x + 1/2*e) + 79*a^3)/(c^6*f*(tan(1/2*f*
x + 1/2*e) - 1)^11)
```

**Mupad** [B]

time = 9.34, size = 143, normalized size = 1.42

$$-\frac{\sqrt{2}\,a^{3}\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{6635\,\cos(e+f\,x)}{16}+\frac{13629\,\sin(e+f\,x)}{16}+565\,\cos\left(2\,e+2\,f\,x\right)-\frac{3527\,\cos(3\,e+3\,f\,x)}{32}-29\,\cos\left(4\,e+4\,f\,x\right)+\frac{81\,\cos(5\,e+5\,f\,x)}{32}-\frac{1617\,\sin(2\,e+2\,f\,x)}{8}-\frac{5049\,\sin(3\,e+3\,f\,x)}{32}+\frac{407\,\sin(4\,e+4\,f\,x)}{16}+\frac{77\,\sin(5\,e+5\,f\,x)}{32}-922\right)}{22176\,c^{6}\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((6635*cos(e + f*x))/16 + (13629*sin(e + f
*x))/16 + 565*cos(2*e + 2*f*x) - (3527*cos(3*e + 3*f*x))/32 - 29*cos(4*e +
4*f*x) + (81*cos(5*e + 5*f*x))/32 - (1617*sin(2*e + 2*f*x))/8 - (5049*sin(3
*e + 3*f*x))/32 + (407*sin(4*e + 4*f*x))/16 + (77*sin(5*e + 5*f*x))/32 - 92
2))/(22176*c^6*f*cos(e/2 + pi/4 + (f*x)/2)^11)
```

# 3.259 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx$

**Optimal**. Leaf size=132

$$\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}$$

```
[Out] 1/13*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+3/143*a^3*c^2*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^9+2/429*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+2/3003*a
^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

**Rubi** [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (3*a^3*c^2*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c*Cos[e + f*x]^7)/(429*f*
(c - c*Sin[e + f*x])^8) + (2*a^3*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x
])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{1}{13}\left(3a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{1}{143}\left(6a^3c\right)\int \frac{\cos^6(}{(c-c\sin}\\ &= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx}\\ &= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx}\end{aligned}$$

**Mathematica [A]**
time = 1.20, size = 157, normalized size = 1.19

$$\frac{a^3\left(18018\cos\left(\frac{1}{2}(e+fx)\right)-10296\cos\left(\frac{3}{2}(e+fx)\right)-3003\cos\left(\frac{5}{2}(e+fx)\right)+286\cos\left(\frac{7}{2}(e+fx)\right)-13\cos\left(\frac{11}{2}(e+fx)\right)+16302\sin\left(\frac{1}{2}(e+fx)\right)+9009\sin\left(\frac{3}{2}(e+fx)\right)-2288\sin\left(\frac{5}{2}(e+fx)\right)-78\sin\left(\frac{9}{2}(e+fx)\right)+\sin\left(\frac{13}{2}(e+fx)\right)\right)}{48048c^7f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(18018*Cos[(e + f*x)/2] - 10296*Cos[(3*(e + f*x))/2] - 3003*Cos[(5*(e
+ f*x))/2] + 286*Cos[(7*(e + f*x))/2] - 13*Cos[(11*(e + f*x))/2] + 16302*Si
n[(e + f*x)/2] + 9009*Sin[(3*(e + f*x))/2] - 2288*Sin[(5*(e + f*x))/2] - 78
*Sin[(9*(e + f*x))/2] + Sin[(13*(e + f*x))/2]))/(48048*c^7*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^13)
```

**Maple [A]**
time = 0.38, size = 208, normalized size = 1.58

| method | result |
|---|---|
| risch | $-\frac{4ia^3\left(9009i\mathrm{e}^{8i(fx+e)}+3003\,\mathrm{e}^{9i(fx+e)}-16302i\mathrm{e}^{6i(fx+e)}-18018\,\mathrm{e}^{7i(fx+e)}+2288i\mathrm{e}^{4i(fx+e)}+10296\,\mathrm{e}^{5i(fx+e)}+78i\mathrm{e}^{2i(f}}{3003f\,c^7\left(\mathrm{e}^{i(fx+e)}-i\right)^{13}}$ |
| derivativedivides | $\frac{2a^3\left(-\frac{192}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{540}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1148}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{512}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{1}{7(\tan(}\right.}{}$ |

| | |
|---|---|
| default | $\frac{2a^3\left(-\frac{192}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{540}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1148}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{512}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{131}{7\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^7*(-192/(tan(1/2*f*x+1/2*e)-1)^4-50/(tan(1/2*f*x+1/2*e)-1)^3-540/
(tan(1/2*f*x+1/2*e)-1)^5-1148/(tan(1/2*f*x+1/2*e)-1)^6-512/13/(tan(1/2*f*x+
1/2*e)-1)^13-13112/7/(tan(1/2*f*x+1/2*e)-1)^7-256/(tan(1/2*f*x+1/2*e)-1)^12
-6752/3/(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)-2352/(tan(1/2*f*x
+1/2*e)-1)^8-1600/(tan(1/2*f*x+1/2*e)-1)^10-8832/11/(tan(1/2*f*x+1/2*e)-1)^
11-9/(tan(1/2*f*x+1/2*e)-1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 2266 vs. $2(136) = 272$.
time = 0.44, size = 2266, normalized size = 17.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -2/15015*(2*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 187330
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75075*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^
7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11
 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(
f*x + e) + 1)^13) + 5*a^3*(3796*sin(f*x + e)/(cos(f*x + e) + 1) - 22776*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 444444*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 3003*sin(
f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*
```

```
x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*a^3*(611*sin(f*x + e)/(co
s(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33
462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e
)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*
x + e) + 1)^13) - 154*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*
c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13))/f
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(136) = 272$.
time = 0.33, size = 414, normalized size = 3.14

$$-\frac{2\,a^{3}\cos\left(fx+e\right)^{7}-12\,a^{3}\cos\left(fx+e\right)^{6}-49\,a^{3}\cos\left(fx+e\right)^{5}+70\,a^{3}\cos\left(fx+e\right)^{4}-567\,a^{3}\cos\left(fx+e\right)^{3}-1596\,a^{3}\cos\left(fx+e\right)^{2}+924\,a^{3}\cos\left(fx+e\right)+1848\,a^{3}+\left(2\,a^{3}\cos\left(fx+e\right)^{6}+14\,a^{3}\cos\left(fx+e\right)^{5}-35\,a^{3}\cos\left(fx+e\right)^{4}-105\,a^{3}\cos\left(fx+e\right)^{3}-672\,a^{3}\cos\left(fx+e\right)^{2}+924\,a^{3}\cos\left(fx+e\right)+1848\,a^{3}\right)\sin\left(fx+e\right)}{3003\,\left(c^{7}f\cos\left(fx+e\right)^{7}+7\,c^{7}f\cos\left(fx+e\right)^{6}-18\,c^{7}f\cos\left(fx+e\right)^{5}-56\,c^{7}f\cos\left(fx+e\right)^{4}+48\,c^{7}f\cos\left(fx+e\right)^{3}+112\,c^{7}f\cos\left(fx+e\right)^{2}-32\,c^{7}f\cos\left(fx+e\right)-64\,c^{7}f-\left(c^{7}f\cos\left(fx+e\right)^{6}-6\,c^{7}f\cos\left(fx+e\right)^{5}-24\,c^{7}f\cos\left(fx+e\right)^{4}+32\,c^{7}f\cos\left(fx+e\right)^{3}+80\,c^{7}f\cos\left(fx+e\right)^{2}-32\,c^{7}f\cos\left(fx+e\right)-64\,c^{7}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(2*a^3*cos(f*x + e)^7 - 12*a^3*cos(f*x + e)^6 - 49*a^3*cos(f*x + e)
^5 + 70*a^3*cos(f*x + e)^4 - 567*a^3*cos(f*x + e)^3 - 1596*a^3*cos(f*x + e)
^2 + 924*a^3*cos(f*x + e) + 1848*a^3 + (2*a^3*cos(f*x + e)^6 + 14*a^3*cos(f
*x + e)^5 - 35*a^3*cos(f*x + e)^4 - 105*a^3*cos(f*x + e)^3 - 672*a^3*cos(f*
x + e)^2 + 924*a^3*cos(f*x + e) + 1848*a^3)*sin(f*x + e))/(c^7*f*cos(f*x +
e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5 - 56*c^7*f*cos(f*x
+ e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(
f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f*x + e)^5 - 24*c
^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f*cos(f*x + e)^2 - 3
2*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3451 vs. $2(121) = 242$.
time = 93.29, size = 3451, normalized size = 26.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Piecewise((-6006*a**3*tan(e/2 + f*x/2)**12/(3003*c**7*f*tan(e/2 + f*x/2)**1
3 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11
- 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 -
 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 -
5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2
147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234
234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*
f) + 18018*a**3*tan(e/2 + f*x/2)**11/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39
039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 8588
58*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 38648
61*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 515314
8*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145
*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c*
*7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 1
02102*a**3*tan(e/2 + f*x/2)**10/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c
**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c*
*7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c*
*7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**
7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7
*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*
tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 198198
*a**3*tan(e/2 + f*x/2)**9/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*
tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*t
an(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*t
an(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*ta
```

```
n(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan
(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/
2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 432432*a**3*
tan(e/2 + f*x/2)**8/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/
2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2
 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2
 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2
+ f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 +
 f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*
x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 492492*a**3*tan(e/
2 + f*x/2)**7/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*
x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x
/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x
/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/
2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2
)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**
2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 571428*a**3*tan(e/2 + f*
x/2)**6/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**
12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**1
0 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8
 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6
+ 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 +
 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39
039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 365508*a**3*tan(e/2 + f*x/2)**
5/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 2
34234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 21
47145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 515
3148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864
861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 85885
8*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c*
*7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 245102*a**3*tan(e/2 + f*x/2)**4/(300
3*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*
c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*
c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c
**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c*
*7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7
*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*t
an(e/2 + f*x/2) - 3003*c**7*f) + 75218*a**3*tan(e/2 + f*x/2)**3/(3003*c**7*
f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*
tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*
tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*t
an(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f...
```

**Giac** [A]

time = 0.49, size = 230, normalized size = 1.74

$$-\frac{2\left(3003\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-9009\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+51051\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-99099\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+216216\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-246246\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+285714\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-182754\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+122551\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-37609\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15171\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1027\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+310\,a^{3}\right)}{3003\,c^{7}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -2/3003*(3003*a^3*tan(1/2*f*x + 1/2*e)^12 - 9009*a^3*tan(1/2*f*x + 1/2*e)^1
1 + 51051*a^3*tan(1/2*f*x + 1/2*e)^10 - 99099*a^3*tan(1/2*f*x + 1/2*e)^9 +
216216*a^3*tan(1/2*f*x + 1/2*e)^8 - 246246*a^3*tan(1/2*f*x + 1/2*e)^7 + 285
714*a^3*tan(1/2*f*x + 1/2*e)^6 - 182754*a^3*tan(1/2*f*x + 1/2*e)^5 + 122551
*a^3*tan(1/2*f*x + 1/2*e)^4 - 37609*a^3*tan(1/2*f*x + 1/2*e)^3 + 15171*a^3*
tan(1/2*f*x + 1/2*e)^2 - 1027*a^3*tan(1/2*f*x + 1/2*e) + 310*a^3)/(c^7*f*(t
an(1/2*f*x + 1/2*e) - 1)^13)
```

**Mupad** [B]
time = 10.17, size = 165, normalized size = 1.25

$$-\frac{\sqrt{2}\,a^{3}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{8993\cos(e+fx)}{4}+\frac{57915\sin(e+fx)}{8}+\frac{73423\cos(2e+2fx)}{16}-\frac{15365\cos(3e+3fx)}{16}-\frac{6943\cos(4e+4fx)}{16}+\frac{937\cos(5e+5fx)}{16}+\frac{77\cos(6e+6fx)}{16}-\frac{6435\sin(2e+2fx)}{4}-\frac{27027\sin(3e+3fx)}{16}+\frac{5005\sin(4e+4fx)}{16}+\frac{1079\sin(5e+5fx)}{16}-\frac{39\sin(6e+6fx)}{8}-\frac{93061}{16}\right)}{192192\,c^{7}f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^7,x)
```

```
[Out] -(2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((8993*cos(e + f*x))/4 + (57915*sin(e + f*
x))/8 + (73423*cos(2*e + 2*f*x))/16 - (15365*cos(3*e + 3*f*x))/16 - (6943*c
os(4*e + 4*f*x))/16 + (937*cos(5*e + 5*f*x))/16 + (77*cos(6*e + 6*f*x))/16
- (6435*sin(2*e + 2*f*x))/4 - (27027*sin(3*e + 3*f*x))/16 + (5005*sin(4*e +
 4*f*x))/16 + (1079*sin(5*e + 5*f*x))/16 - (39*sin(6*e + 6*f*x))/8 - 93061/
16))/(192192*c^7*f*cos(e/2 + pi/4 + (f*x)/2)^13)
```

# 3.260 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx$

**Optimal**. Leaf size=166

$$\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}$$

```
[Out] 1/15*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+4/195*a^3*c^2*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^10+4/715*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+8/6435*
a^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+8/45045*a^3*cos(f*x+e)^7/c/f/(c-c*sin
(f*x+e))^7
```

**Rubi** [A]
time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 2750}

$$\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{8a^3\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (4*a^3*c^2*Cos[e
+ f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (4*a^3*c*Cos[e + f*x]^7)/(715*f
*(c - c*Sin[e + f*x])^9) + (8*a^3*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*
x])^8) + (8*a^3*Cos[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11}}\,dx\\
&= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{1}{15}\left(4a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx\\
&= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{1}{65}\left(4a^3c\right)\int \frac{\cos^6(e}{(c-c\sin(}\\
&= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx)}\\
&= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx)}\\
&= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx)}
\end{aligned}$$

**Mathematica [A]**
time = 1.20, size = 209, normalized size = 1.26

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a+a\sin(e+fx))^3\left(115830\cos\left(\frac{1}{2}(e+fx)\right)-65065\cos\left(\frac{3}{2}(e+fx)\right)-18018\cos\left(\frac{5}{2}(e+fx)\right)+1365\cos\left(\frac{7}{2}(e+fx)\right)-105\cos\left(\frac{11}{2}(e+fx)\right)+\cos\left(\frac{15}{2}(e+fx)\right)+109395\sin\left(\frac{1}{2}(e+fx)\right)+60060\sin\left(\frac{3}{2}(e+fx)\right)-15015\sin\left(\frac{5}{2}(e+fx)\right)-455\sin\left(\frac{9}{2}(e+fx)\right)+15\sin\left(\frac{13}{2}(e+fx)\right)\right)}{360360f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(115830*Cos[(
e + f*x)/2] - 65065*Cos[(3*(e + f*x))/2] - 18018*Cos[(5*(e + f*x))/2] + 136
5*Cos[(7*(e + f*x))/2] - 105*Cos[(11*(e + f*x))/2] + Cos[(15*(e + f*x))/2]
+ 109395*Sin[(e + f*x)/2] + 60060*Sin[(3*(e + f*x))/2] - 15015*Sin[(5*(e +
f*x))/2] - 455*Sin[(9*(e + f*x))/2] + 15*Sin[(13*(e + f*x))/2]))/(360360*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^8)
```

**Maple [A]**
time = 0.39, size = 238, normalized size = 1.43

| method | result |
|---|---|
| risch | $-\frac{16\left(105a^3e^{2i(fx+e)}-1365a^3e^{4i(fx+e)}+65065a^3e^{6i(fx+e)}-115830a^3e^{8i(fx+e)}+455ia^3e^{3i(fx+e)}-15ia^3e^{i(fx+e)}-10939\right.}{45045\left(e^{i(fx+e)}-i\right)^{15}fc^8}$ |

| | |
|---|---|
| derivativedivides | $\frac{2a^3\left(-\frac{512}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{14}}-\frac{13184}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32288}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{47072}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{8}{11\left(\tan\left(\right.\right.}$ |
| default | $\frac{2a^3\left(-\frac{512}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{14}}-\frac{13184}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32288}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{47072}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{8}{11\left(\tan\left(\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^8*(-512/(tan(1/2*f*x+1/2*e)-1)^14-13184/3/(tan(1/2*f*x+1/2*e)-1)^
12-1/(tan(1/2*f*x+1/2*e)-1)-32288/7/(tan(1/2*f*x+1/2*e)-1)^7-47072/5/(tan(1
/2*f*x+1/2*e)-1)^10-81344/11/(tan(1/2*f*x+1/2*e)-1)^11-276/(tan(1/2*f*x+1/2
*e)-1)^4-1024/15/(tan(1/2*f*x+1/2*e)-1)^15-4536/5/(tan(1/2*f*x+1/2*e)-1)^5-
2304/(tan(1/2*f*x+1/2*e)-1)^6-188/3/(tan(1/2*f*x+1/2*e)-1)^3-24320/13/(tan(
1/2*f*x+1/2*e)-1)^13-84112/9/(tan(1/2*f*x+1/2*e)-1)^9-7352/(tan(1/2*f*x+1/2
*e)-1)^8-10/(tan(1/2*f*x+1/2*e)-1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 2642 vs. $2(171) = 342$.
time = 0.47, size = 2642, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 2/45045*(3*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 89134
5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f
*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*sin
(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*
c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f
*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x +
e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14
- c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 7*a^3*(7845*sin(f*x + e)/(co
```

s(f*x + e) + 1) - 54915*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 668850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 1444443*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 3063060*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2357355*sin(f*x + e)^9/(cos(f*x + e) +
1)^9 - 1414413*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 630630*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 210210*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4
5045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 952)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x
+ e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) +
1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*a^3*(1740*sin(f*x +
 e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116
)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455
*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f
*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*
x + e)^15/(cos(f*x + e) + 1)^15) + 6*a^3*(675*sin(f*x + e)/(cos(f*x + e) +
1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20475*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 102102*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 122265*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 33033*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x + e)^11/(cos(f*x + e) + 1)^11
- 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365
*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(

```
f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 ...
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(171) = 342$.
time = 0.33, size = 472, normalized size = 2.84

$$\frac{8\,a^3\cos(fx+e)^8+64\,a^3\cos(fx+e)^7-196\,a^3\cos(fx+e)^6-672\,a^3\cos(fx+e)^5+735\,a^3\cos(fx+e)^4-7161\,a^3\cos(fx+e)^3-20328\,a^3\cos(fx+e)^2+12012\,a^3\cos(fx+e)+24024\,a^3-\left(8\,a^3\cos(fx+e)^7-56\,a^3\cos(fx+e)^6-252\,a^3\cos(fx+e)^5+420\,a^3\cos(fx+e)^4+1155\,a^3\cos(fx+e)^3+8316\,a^3\cos(fx+e)^2-12012\,a^3\cos(fx+e)-24024\,a^3\right)\sin(fx+e)}{45045\left(c^8f\cos(fx+e)^8-7\,c^8f\cos(fx+e)^7-32\,c^8f\cos(fx+e)^6+56\,c^8f\cos(fx+e)^5+160\,c^8f\cos(fx+e)^4-112\,c^8f\cos(fx+e)^3-256\,c^8f\cos(fx+e)^2+64\,c^8f\cos(fx+e)+128\,c^8f+\left(c^8f\cos(fx+e)^7+8\,c^8f\cos(fx+e)^6-24\,c^8f\cos(fx+e)^5-80\,c^8f\cos(fx+e)^4+80\,c^8f\cos(fx+e)^3+192\,c^8f\cos(fx+e)^2-64\,c^8f\cos(fx+e)-128\,c^8f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/45045*(8*a^3*cos(f*x + e)^8 + 64*a^3*cos(f*x + e)^7 - 196*a^3*cos(f*x + e
)^6 - 672*a^3*cos(f*x + e)^5 + 735*a^3*cos(f*x + e)^4 - 7161*a^3*cos(f*x +
e)^3 - 20328*a^3*cos(f*x + e)^2 + 12012*a^3*cos(f*x + e) + 24024*a^3 - (8*a
^3*cos(f*x + e)^7 - 56*a^3*cos(f*x + e)^6 - 252*a^3*cos(f*x + e)^5 + 420*a^
3*cos(f*x + e)^4 + 1155*a^3*cos(f*x + e)^3 + 8316*a^3*cos(f*x + e)^2 - 1201
2*a^3*cos(f*x + e) - 24024*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8
*f*cos(f*x + e)^7 - 32*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160
*c^8*f*cos(f*x + e)^4 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2
 + 64*c^8*f*cos(f*x + e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(
f*x + e)^6 - 24*c^8*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*c
os(f*x + e)^3 + 192*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*
f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 4542 vs. $2(151) = 302$.
time = 148.30, size = 4542, normalized size = 27.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Piecewise((-90090*a**3*tan(e/2 + f*x/2)**14/(45045*c**8*f*tan(e/2 + f*x/2)*
*15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)*
*13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/
2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 +
 f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/
2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan
(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*ta
n(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(
e/2 + f*x/2) - 45045*c**8*f) + 360360*a**3*tan(e/2 + f*x/2)**13/(45045*c**8
*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8
```

*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c
**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450
225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289
864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 +
135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4
+ 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2
+ 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 2132130*a**3*tan(e/2 + f
*x/2)**12/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/
2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*
x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2
+ f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(
e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*t
an(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f
*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f
*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 540
5400*a**3*tan(e/2 + f*x/2)**11/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*
c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 2049547
5*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 1352
70135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 -
289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7
 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)
**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2
)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2)
 - 45045*c**8*f) - 13351338*a**3*tan(e/2 + f*x/2)**10/(45045*c**8*f*tan(e/2
+ f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2
+ f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e
/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*
tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8
*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c
**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*
c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c*
*8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 20420400*a**3*tan(e/2 + f*x/2)**9/(
45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 47
29725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 +
61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**1
0 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2
)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*
x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 +
f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 +
f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 28249650*a**3*
tan(e/2 + f*x/2)**8/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(
e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*ta
n(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*
f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c
**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 22545022

```
5*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486
425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729
725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**
8*f) + 26357760*a**3*tan(e/2 + f*x/2)**7/(45045*c**8*f*tan(e/2 + f*x/2)**15
 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13
 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)*
*11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*
x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 +
 f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/
2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e
/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2
 + f*x/2) - 45045*c**8*f) - 22052030*a**3*tan(e...
```

**Giac** [A]
time = 0.50, size = 264, normalized size = 1.59

$$-\frac{2\left(45045\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-180180\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+1066065\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-2702700\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+6675669\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-10210200\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+14124825\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-13178880\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+11026015\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-6066060\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3088995\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-864500\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+265335\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-18600\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4243\,a^{3}\right)}{45045\,c^{8}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -2/45045*(45045*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*a^3*tan(1/2*f*x + 1/2*
e)^13 + 1066065*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*a^3*tan(1/2*f*x + 1/2
*e)^11 + 6675669*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*a^3*tan(1/2*f*x + 1
/2*e)^9 + 14124825*a^3*tan(1/2*f*x + 1/2*e)^8 - 13178880*a^3*tan(1/2*f*x +
1/2*e)^7 + 11026015*a^3*tan(1/2*f*x + 1/2*e)^6 - 6066060*a^3*tan(1/2*f*x +
1/2*e)^5 + 3088995*a^3*tan(1/2*f*x + 1/2*e)^4 - 864500*a^3*tan(1/2*f*x + 1/
2*e)^3 + 265335*a^3*tan(1/2*f*x + 1/2*e)^2 - 18600*a^3*tan(1/2*f*x + 1/2*e)
 + 4243*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

**Mupad** [B]
time = 11.20, size = 187, normalized size = 1.13

$$\frac{\sqrt{2}\,a^{3}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3497111\cos(3e+3fx)}{128}-\frac{25501905\sin(e+fx)}{128}-\frac{257861\cos(2e+2fx)}{2}-\frac{5734111\cos(e+fx)}{128}+\frac{72047\cos(4e+4fx)}{4}-\frac{378579\cos(5e+5fx)}{128}-\frac{1059\cos(6e+6fx)}{2}+\frac{4251\cos(7e+7fx)}{128}+\frac{2633345\sin(2e+2fx)}{64}+\frac{7210775\sin(3e+3fx)}{128}-\frac{89375\sin(4e+4fx)}{8}-\frac{504205\sin(5e+5fx)}{128}+\frac{29765\sin(6e+6fx)}{64}+\frac{4235\sin(7e+7fx)}{128}+\frac{544369}{4}\right)}{5765760\,c^{8}f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^8,x)
```

```
[Out] (2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((3497111*cos(3*e + 3*f*x))/128 - (25501905
*sin(e + f*x))/128 - (257861*cos(2*e + 2*f*x))/2 - (5734111*cos(e + f*x))/1
28 + (72047*cos(4*e + 4*f*x))/4 - (378579*cos(5*e + 5*f*x))/128 - (1059*cos
(6*e + 6*f*x))/2 + (4251*cos(7*e + 7*f*x))/128 + (2633345*sin(2*e + 2*f*x))
/64 + (7210775*sin(3*e + 3*f*x))/128 - (89375*sin(4*e + 4*f*x))/8 - (504205
*sin(5*e + 5*f*x))/128 + (29765*sin(6*e + 6*f*x))/64 + (4235*sin(7*e + 7*f*
x))/128 + 544369/4))/(5765760*c^8*f*cos(e/2 + pi/4 + (f*x)/2)^15)
```

# 3.261 $\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=118

$$-\frac{35c^4x}{2a}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{35c^4\cos(e+fx)\sin(e+fx)}{2af}-\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}$$

```
[Out] -35/2*c^4*x/a-35/3*c^4*cos(f*x+e)^3/a/f-35/2*c^4*cos(f*x+e)*sin(f*x+e)/a/f-
2*a^3*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4-14*a*c^4*cos(f*x+e)^5/f/(a+a*si
n(f*x+e))^2
```

**Rubi** [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2761, 2715, 8}

$$-\frac{2a^3c^4\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{14ac^4\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{35c^4\sin(e+fx)\cos(e+fx)}{2af}-\frac{35c^4x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-35*c^4*x)/(2*a) - (35*c^4*Cos[e + f*x]^3)/(3*a*f) - (35*c^4*Cos[e + f*x]*
Sin[e + f*x])/(2*a*f) - (2*a^3*c^4*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^
4) - (14*a*c^4*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}-\left(35c^4\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx}\\
&= -\frac{35c^4\cos^3(e+fx)}{3af}-\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}-\frac{(35c^4}{}\\
&= -\frac{35c^4\cos^3(e+fx)}{3af}-\frac{35c^4\cos(e+fx)\sin(e+fx)}{2af}-\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\\
&= -\frac{35c^4x}{2a}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{35c^4\cos(e+fx)\sin(e+fx)}{2af}-\frac{2a^3c^4\cos^7(e}{f(a+a\sin(e}\end{aligned}$$

**Mathematica [A]**
time = 0.91, size = 175, normalized size = 1.48

$$-\frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)(-384+210e+210fx+141\cos(e+fx)-\cos(3(e+fx))-15\sin(2(e+fx)))+\cos\left(\frac{1}{2}(e+fx)\right)(210e+210fx+141\cos(e+fx)-\cos(3(e+fx))-15\sin(2(e+fx)))\right)}{12af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/12*(c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*(Sin
[(e + f*x)/2]*(-384 + 210*e + 210*f*x + 141*Cos[e + f*x] - Cos[3*(e + f*x)]
 - 15*Sin[2*(e + f*x)]) + Cos[(e + f*x)/2]*(210*e + 210*f*x + 141*Cos[e + f
```

```
*x] - Cos[3*(e + f*x)] - 15*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.32, size = 109, normalized size = 0.92

| method | result |
|---|---|
| derivativedivides | $\frac{2c^4\left(-\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\frac{5\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+11\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+24\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+\frac{35}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{35\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right.}{fa}$ |
| default | $\frac{2c^4\left(-\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\frac{5\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+11\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+24\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+\frac{35}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{35\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right.}{fa}$ |
| risch | $-\frac{35c^4x}{2a}-\frac{47c^4\mathrm{e}^{i(fx+e)}}{8af}-\frac{47c^4\mathrm{e}^{-i(fx+e)}}{8af}-\frac{32c^4}{fa\left(\mathrm{e}^{i(fx+e)}+i\right)}+\frac{c^4\cos(3fx+3e)}{12af}+\frac{5c^4\sin(2fx+2e)}{4af}$ |
| norman | $\frac{-\frac{257c^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{155c^4\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{37c^4\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{27c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{35c^4x}{2a}-\frac{166c^4}{3af}-\frac{35c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{2a}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^4/a*(-16/(tan(1/2*f*x+1/2*e)+1)-(5/2*tan(1/2*f*x+1/2*e)^5+11*tan(1/2*
f*x+1/2*e)^4+24*tan(1/2*f*x+1/2*e)^2-5/2*tan(1/2*f*x+1/2*e)+35/3)/(1+tan(1/
2*f*x+1/2*e)^2)^3-35/2*arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(119) = 238$.
time = 0.57, size = 786, normalized size = 6.66

$$-\frac{c^4\left(\frac{\frac{7\sin(fx+e)}{\cos(fx+e)+1}+\frac{39\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{24\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{24\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{9\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+16}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{3a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{9\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+12c^4\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+36c^4\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+24c^4\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{6c^4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 12*c^4*(
(sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)
```

```
^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^4*((sin(f*x + e)/(cos(f*x + e) + 1
) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 24*c^4*(arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [**A**]
time = 0.33, size = 166, normalized size = 1.41

$$\frac{2\,c^4\cos\left(fx+e\right)^4-13\,c^4\cos\left(fx+e\right)^3-105\,c^4fx-72\,c^4\cos\left(fx+e\right)^2-96\,c^4-3\left(35\,c^4fx+51\,c^4\right)\cos\left(fx+e\right)+\left(2\,c^4\cos\left(fx+e\right)^3-105\,c^4fx+15\,c^4\cos\left(fx+e\right)^2-57\,c^4\cos\left(fx+e\right)+96\,c^4\right)\sin\left(fx+e\right)}{6\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c^4*cos(f*x + e)^4 - 13*c^4*cos(f*x + e)^3 - 105*c^4*f*x - 72*c^4*co
s(f*x + e)^2 - 96*c^4 - 3*(35*c^4*f*x + 51*c^4)*cos(f*x + e) + (2*c^4*cos(f
*x + e)^3 - 105*c^4*f*x + 15*c^4*cos(f*x + e)^2 - 57*c^4*cos(f*x + e) + 96*
c^4)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2108 vs. $2(112) = 224$.
time = 4.46, size = 2108, normalized size = 17.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-105*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2
+ f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a
*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**5/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e
/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x
/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315
*c**4*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
```

```
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f
*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 222*c**4*tan(e/2 + f*x/2)**6/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 162*c**4*tan(e/2 + f*x/2)**5/(
6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*
x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*
tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 708*c**4*tan(e/2 +
f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*t
an(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**
3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 288*c**4
*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 834*c**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
+ f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*
a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/
2) + 6*a*f) - 110*c**4*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) - 332*c**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**4/(a*sin(e) + a), True))
```

**Giac [A]**

time = 0.43, size = 135, normalized size = 1.14

$$-\frac{\frac{105\,(fx+e)c^4}{a}+\frac{192\,c^4}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+66\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+144\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+70\,c^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(105*(f*x + e)*c^4/a + 192*c^4/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(15*
c^4*tan(1/2*f*x + 1/2*e)^5 + 66*c^4*tan(1/2*f*x + 1/2*e)^4 + 144*c^4*tan(1/
```

```
2*f*x + 1/2*e)^2 - 15*c^4*tan(1/2*f*x + 1/2*e) + 70*c^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

**Mupad** [B]
time = 10.53, size = 290, normalized size = 2.46

$$\frac{\frac{35\,c^4\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{35\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(105\,e+105\,f\,x+110\right)}{6}\right)-\frac{c^4\,\left(105\,e+105\,f\,x+332\right)}{6}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{35\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(105\,e+105\,f\,x+222\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(\frac{105\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(315\,e+315\,f\,x+162\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{105\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(315\,e+315\,f\,x+288\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{105\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(315\,e+315\,f\,x+708\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{105\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(315\,e+315\,f\,x+834\right)}{6}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^3}-\frac{35\,c^4\,x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x)),x)
```

```
[Out] ((35*c^4*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((35*c^4*(e + f*x))/2 - (c^4*(10
5*e + 105*f*x + 110))/6) - (c^4*(105*e + 105*f*x + 332))/6 + tan(e/2 + (f*x
)/2)^6*((35*c^4*(e + f*x))/2 - (c^4*(105*e + 105*f*x + 222))/6) + tan(e/2 +
 (f*x)/2)^5*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 162))/6) + tan
(e/2 + (f*x)/2)^3*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 288))/6)
 + tan(e/2 + (f*x)/2)^4*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 70
8))/6) + tan(e/2 + (f*x)/2)^2*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*
x + 834))/6))/(a*f*(tan(e/2 + (f*x)/2) + 1)*(tan(e/2 + (f*x)/2)^2 + 1)^3) -
 (35*c^4*x)/(2*a)
```

# 3.262 $\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=92

$$-\frac{15c^3x}{2a}-\frac{15c^3\cos(e+fx)}{2af}-\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$

```
[Out] -15/2*c^3*x/a-15/2*c^3*cos(f*x+e)/a/f-2*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x
+e))^3-5/2*c^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))
```

**Rubi** [A]
time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2815, 2759, 2758, 2761, 8}

$$-\frac{2a^2c^3\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{15c^3\cos(e+fx)}{2af}-\frac{5c^3\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{15c^3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-15*c^3*x)/(2*a) - (15*c^3*Cos[e + f*x])/(2*a*f) - (2*a^2*c^3*Cos[e + f*x]
^5)/(f*(a + a*Sin[e + f*x])^3) - (5*c^3*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e +
 f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))} - \frac{1}{2}\left(15c^3\right)\int \frac{\cos^2(e+f}{a+a\sin(e+f}$$
$$= -\frac{15c^3\cos(e+fx)}{2af} - \frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))} - \frac{(15c^3)}{2}$$
$$= -\frac{15c^3x}{2a} - \frac{15c^3\cos(e+fx)}{2af} - \frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$

**Mathematica** [A]
time = 0.34, size = 155, normalized size = 1.68

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)(-64+30e+30fx+16\cos(e+fx)-\sin(2(e+fx)))+\cos\left(\frac{1}{2}(e+fx)\right)(30(e+fx)+16\cos(e+fx)-\sin(2(e+fx)))\right)}{4af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Sin[(e +
f*x)/2]*(-64 + 30*e + 30*f*x + 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Cos[(e
 + f*x)/2]*(30*(e + f*x) + 16*Cos[e + f*x] - Sin[2*(e + f*x)])))/(4*a*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.31, size = 96, normalized size = 1.04

| method | result |
|---|---|
| derivativedivides | $\frac{2c^{3}\left(-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+4\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+4}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}}-\frac{15\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{fa}$ |
| default | $\frac{2c^{3}\left(-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+4\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+4}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}}-\frac{15\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{fa}$ |
| risch | $-\frac{15c^{3}x}{2a}-\frac{2c^{3}e^{i(fx+e)}}{af}-\frac{2c^{3}e^{-i(fx+e)}}{af}-\frac{16c^{3}}{fa\left(e^{i(fx+e)}+i\right)}+\frac{c^{3}\sin(2fx+2e)}{4af}$ |
| norman | $-\frac{7c^{3}}{af}+\frac{10c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{17c^{3}\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{5c^{3}\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{35c^{3}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{15c^{3}x}{2a}-\frac{15c^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2a}-45c^{3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^3/a*(-8/(tan(1/2*f*x+1/2*e)+1)-(1/2*tan(1/2*f*x+1/2*e)^3+4*tan(1/2*f*
x+1/2*e)^2-1/2*tan(1/2*f*x+1/2*e)+4)/(1+tan(1/2*f*x+1/2*e)^2)^2-15/2*arctan
(tan(1/2*f*x+1/2*e)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(91) = 182$.
time = 0.54, size = 462, normalized size = 5.02

$$-\frac{c^{3}\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{3\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{2a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+6c^{3}\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+6c^{3}\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{2c^{3}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [A]
time = 0.32, size = 136, normalized size = 1.48

$$-\frac{c^3\cos(fx+e)^3+15\,c^3fx+8\,c^3\cos(fx+e)^2+16\,c^3+(15\,c^3fx+23\,c^3)\cos(fx+e)+(15\,c^3fx-c^3\cos(fx+e)^2+7\,c^3\cos(fx+e)-16\,c^3)\sin(fx+e)}{2\,(af\cos(fx+e)+af\sin(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(c^3*cos(f*x + e)^3 + 15*c^3*f*x + 8*c^3*cos(f*x + e)^2 + 16*c^3 + (15
*c^3*f*x + 23*c^3)*cos(f*x + e) + (15*c^3*f*x - c^3*cos(f*x + e)^2 + 7*c^3*
cos(f*x + e) - 16*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) +
 a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. 2(85) = 170.
time = 2.30, size = 1170, normalized size = 12.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*c**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 30*c*
*3*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) - 30*c**3*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 +
f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e
/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f
) - 15*c**3*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) - 34*c**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 18*c**3*tan(e/2 + f*x/2)**3/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 78*c**3*tan(
e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) - 14*c**3*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*ta
n(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
```

```
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 48*c**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a
*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*
sin(e) + a), True))
```

**Giac [A]**
time = 0.47, size = 117, normalized size = 1.27

$$-\frac{\frac{15\,(fx+e)c^3}{a}+\frac{32\,c^3}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,c^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*c^3/a + 32*c^3/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(c^3*t
an(1/2*f*x + 1/2*e)^3 + 8*c^3*tan(1/2*f*x + 1/2*e)^2 - c^3*tan(1/2*f*x + 1/
2*e) + 8*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

**Mupad [B]**
time = 8.71, size = 216, normalized size = 2.35

$$\frac{\frac{15\,c^3\,(e+f\,x)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{15\,c^3\,(e+f\,x)}{2}-\frac{c^3\,(15\,e+15\,f\,x+14)}{2}\right)-\frac{c^3\,(15\,e+15\,f\,x+48)}{2}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{15\,c^3\,(e+f\,x)}{2}-\frac{c^3\,(15\,e+15\,f\,x+34)}{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(15\,c^3\,(e+f\,x)-\frac{c^3\,(30\,e+30\,f\,x+18)}{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(15\,c^3\,(e+f\,x)-\frac{c^3\,(30\,e+30\,f\,x+78)}{2}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^2}-\frac{15\,c^3\,x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x)),x)
```

```
[Out] ((15*c^3*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((15*c^3*(e + f*x))/2 - (c^3*(15
*e + 15*f*x + 14))/2) - (c^3*(15*e + 15*f*x + 48))/2 + tan(e/2 + (f*x)/2)^4
*((15*c^3*(e + f*x))/2 - (c^3*(15*e + 15*f*x + 34))/2) + tan(e/2 + (f*x)/2)
^3*(15*c^3*(e + f*x) - (c^3*(30*e + 30*f*x + 18))/2) + tan(e/2 + (f*x)/2)^2
*(15*c^3*(e + f*x) - (c^3*(30*e + 30*f*x + 78))/2))/(a*f*(tan(e/2 + (f*x)/2
) + 1)*(tan(e/2 + (f*x)/2)^2 + 1)^2) - (15*c^3*x)/(2*a)
```

# 3.263 $\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=56

$$-\frac{3c^2x}{a}-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}$$

```
[Out] -3*c^2*x/a-3*c^2*cos(f*x+e)/a/f-2*a*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2759, 2761, 8}

$$-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{3c^2x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*c^2*x)/a - (3*c^2*Cos[e + f*x])/(a*f) - (2*a*c^2*Cos[e + f*x]^3)/(f*(a
+ a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}-\left(3c^2\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx\\ &= -\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}-\frac{\left(3c^2\right)\int 1\,dx}{a}\\ &= -\frac{3c^2x}{a}-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(56) = 112.
time = 0.25, size = 129, normalized size = 2.30

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(3(e+fx)+\cos(e+fx))+(-8+3e+3fx+\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2}{af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x)
 + Cos[e + f*x]) + (-8 + 3*e + 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(-1
+ Sin[e + f*x])^2)/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e
+ f*x])))
```

**Maple [A]**
time = 0.27, size = 57, normalized size = 1.02

| method | result |
|---|---|
| derivativedivides | $\frac{2c^2\left(-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$ |
| default | $\frac{2c^2\left(-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$ |
| risch | $-\frac{3c^2x}{a}-\frac{c^2e^{i(fx+e)}}{2af}-\frac{c^2e^{-i(fx+e)}}{2af}-\frac{8c^2}{fa\left(e^{i(fx+e)}+i\right)}$ |

| | |
|---|---|
| norman | $\frac{-\frac{10c^2}{af}-\frac{8c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{3c^2x}{a}-\frac{3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{6c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{6c^2x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-3c^2 \cdots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^2/a*(-1/(1+tan(1/2*f*x+1/2*e)^2)-3*arctan(tan(1/2*f*x+1/2*e))-4/(tan(
1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(59) = 118$.
time = 0.55, size = 228, normalized size = 4.07

$$-\frac{2\left(c^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+2\,c^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c^2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) + 2*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

**Fricas** [A]
time = 0.32, size = 107, normalized size = 1.91

$$-\frac{3\,c^2fx+c^2\cos\left(fx+e\right)^2+4\,c^2+\left(3\,c^2fx+5\,c^2\right)\cos\left(fx+e\right)+\left(3\,c^2fx+c^2\cos\left(fx+e\right)-4\,c^2\right)\sin\left(fx+e\right)}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*c^2*f*x + c^2*cos(f*x + e)^2 + 4*c^2 + (3*c^2*f*x + 5*c^2)*cos(f*x + e)
 + (3*c^2*f*x + c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) +
 a*f*sin(f*x + e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(53) = 106$.
time = 1.17, size = 456, normalized size = 8.14

$$\begin{cases} -\frac{3c^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{8c^2\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{10c^2}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\: f \neq 0 \\ \frac{x\left(-c\sin\left(e\right)+c\right)^2}{a\sin\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*c**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x
/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x/(a*f*tan(e/2
+ f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 8*c**
2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
a*f*tan(e/2 + f*x/2) + a*f) - 2*c**2*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 10*c**2/(a*f*
tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f)
, Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a), True))
```

**Giac [A]**
time = 0.43, size = 100, normalized size = 1.79

$$-\frac{\frac{3\,(fx+e)c^2}{a}+\frac{2\left(4\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,c^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*c^2/a + 2*(4*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2*tan(1/2*f*x + 1
/2*e) + 5*c^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) + 1)*a))/f
```

**Mupad [B]**
time = 6.99, size = 118, normalized size = 2.11

$$-\frac{3\,c^2\,x}{a}-\frac{3\,\sqrt{2}\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(e+f\,x\right)-\frac{\sqrt{2}\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,e+6\,f\,x+16\right)}{2}}{a\,f\,\left(\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sqrt{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{2\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x)),x)
```

```
[Out] - (3*c^2*x)/a - (3*2^(1/2)*c^2*sin(e/2 + (f*x)/2)*(e + f*x) - (2^(1/2)*c^2*
sin(e/2 + (f*x)/2)*(6*e + 6*f*x + 16))/2)/(a*f*(2^(1/2)*cos(e/2 + (f*x)/2)
+ 2^(1/2)*sin(e/2 + (f*x)/2))) - (2*c^2*cos(e/2 + (f*x)/2)^2)/(a*f)
```

# 3.264 $\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=32

$$-\frac{cx}{a}-\frac{2c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] -c*x/a-2*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

**Rubi** [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2814, 2727}

$$-\frac{2c\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
[Out] -((c*x)/a) - (2*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= -\frac{cx}{a}+(2c)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= -\frac{cx}{a}-\frac{2c\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.13, size = 79, normalized size = 2.47

$$-\frac{c\left(fx\cos\left(\frac{fx}{2}\right)-4\sin\left(\frac{fx}{2}\right)+fx\sin\left(e+\frac{fx}{2}\right)\right)}{af\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]

[Out] -((c*(f*x*Cos[(f*x)/2] - 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(a*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))

**Maple** [A]
time = 0.21, size = 38, normalized size = 1.19

| method | result | size |
|---|---|---|
| risch | $-\frac{cx}{a}-\frac{4c}{fa\left(e^{i(fx+e)}+i\right)}$ | 32 |
| derivativedivides | $\frac{2c\left(-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$ | 38 |
| default | $\frac{2c\left(-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$ | 38 |
| norman | $\frac{\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{4c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{cx}{a}-\frac{cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{cx\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{cx\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ | 128 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2/f*c/a*(-2/(tan(1/2*f*x+1/2*e)+1)-arctan(tan(1/2*f*x+1/2*e)))

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.53, size = 83, normalized size = 2.59

$$-\frac{2\left(c\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] -2*(c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f

**Fricas** [A]
time = 0.32, size = 68, normalized size = 2.12

$$-\frac{cfx+(cfx+2c)\cos(fx+e)+(cfx-2c)\sin(fx+e)+2c}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(c*f*x + (c*f*x + 2*c)*cos(f*x + e) + (c*f*x - 2*c)*sin(f*x + e) + 2*c)/(a
*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.
time = 0.64, size = 90, normalized size = 2.81

$$\begin{cases} -\frac{cfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} - \frac{cfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} - \frac{4c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f \neq 0 \\ \frac{x(-c\sin(e)+c)}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2) + a*f) - c*f*x/(a*
f*tan(e/2 + f*x/2) + a*f) - 4*c/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x
*(-c*sin(e) + c)/(a*sin(e) + a), True))
```

**Giac** [A]
time = 0.42, size = 37, normalized size = 1.16

$$-\frac{\frac{(fx+e)c}{a} + \frac{4c}{a(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*c/a + 4*c/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

**Mupad** [B]
time = 6.64, size = 45, normalized size = 1.41

$$\frac{c(e+fx) - c(e+fx+4)}{af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)} - \frac{cx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

```
[Out] (c*(e + f*x) - c*(e + f*x + 4))/(a*f*(tan(e/2 + (f*x)/2) + 1)) - (c*x)/a
```

# 3.265 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

**Optimal**. Leaf size=16

$$\frac{\tan(e+fx)}{acf}$$

[Out] tan(f*x+e)/a/c/f

**Rubi** [A]
time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 3852, 8}

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]

[Out] Tan[e + f*x]/(a*c*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{\int \sec^2(e+fx)\,dx}{ac}$$
$$= -\frac{\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{acf}$$
$$= \frac{\tan(e+fx)}{acf}$$

**Mathematica [A]**
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(a*c*f)
```

**Maple [A]**
time = 0.27, size = 17, normalized size = 1.06

| method | result | size |
|---|---|---|
| default | $\frac{\tan(fx+e)}{acf}$ | 17 |
| risch | $\frac{2i}{(e^{i(fx+e)}-i)(e^{i(fx+e)}+i)acf}$ | 41 |
| norman | $-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$ | 47 |
| derivativedivides | `error in RationalFunction: argument is not a rational function\` | N/A |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(f*x+e)/a/c/f
```

**Maxima [A]**
time = 0.32, size = 17, normalized size = 1.06

$$\frac{\tan(fx+e)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(a*c*f)
```

**Fricas [A]**
time = 0.31, size = 26, normalized size = 1.62

$$\frac{\sin(fx+e)}{acf\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] sin(f*x + e)/(a*c*f*cos(f*x + e))
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(10) = 20$.
time = 0.60, size = 49, normalized size = 3.06

$$\begin{cases} -\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f\neq 0 \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0
)), (x/((a*sin(e) + a)*(-c*sin(e) + c)), True))
```

**Giac [A]**
time = 0.43, size = 17, normalized size = 1.06

$$\frac{\tan(fx+e)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] tan(f*x + e)/(a*c*f)
```

**Mupad [B]**
time = 6.85, size = 35, normalized size = 2.19

$$-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

# 3.266 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=53

$$\frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{2\tan(e+fx)}{3ac^2f}$$

```
[Out] 1/3*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+2/3*tan(f*x+e)/a/c^2/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2815, 2751, 3852, 8}

$$\frac{2\tan(e+fx)}{3ac^2f}+\frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] Sec[e + f*x]/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*c^2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{2\int \sec^2(e+fx)\,dx}{3ac^2}\\ &= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}-\frac{2\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3ac^2f}\\ &= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{2\tan(e+fx)}{3ac^2f}\end{aligned}$$

**Mathematica [A]**
time = 0.30, size = 87, normalized size = 1.64

$$\frac{-2+4\cos(e+fx)-2\cos(2(e+fx))+4\cos(3(e+fx))+\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))}{24ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (-2 + 4*Cos[e + f*x] - 2*Cos[2*(e + f*x)] + 4*Cos[3*(e + f*x)] + Sin[e + f*
x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(24*a*c^2*f*(-1 + Sin[e + f*x])
^2*(1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.25, size = 73, normalized size = 1.38

| method | result | size |
|---|---|---|
| risch | $\frac{\frac{8\,\mathrm{e}^{i(fx+e)}}{3}-\frac{4i}{3}}{\left(\mathrm{e}^{i(fx+e)}-i\right)^3\left(\mathrm{e}^{i(fx+e)}+i\right)a\,c^2 f}$ | 54 |
| derivativedivides | $\frac{-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a\,c^2 f}$ | 73 |
| default | $\frac{-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a\,c^2 f}$ | 73 |

| | | |
|---|---|---|
| norman | $\frac{\frac{2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2}{3acf}-\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$ | 107 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a/c^2*(-1/4/(tan(1/2*f*x+1/2*e)+1)-1/3/(tan(1/2*f*x+1/2*e)-1)^3-1/2/(ta
n(1/2*f*x+1/2*e)-1)^2-3/4/(tan(1/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(53) = 106$.
time = 0.32, size = 154, normalized size = 2.91

$$\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{3\left(ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/((a*c^2 - 2*a*c^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

**Fricas** [A]
time = 0.32, size = 61, normalized size = 1.15

$$-\frac{2\cos\left(fx+e\right)^2+2\sin\left(fx+e\right)-1}{3\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 + 2*sin(f*x + e) - 1)/(a*c^2*f*cos(f*x + e)*sin(f*x
+ e) - a*c^2*f*cos(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(41) = 82$.
time = 1.37, size = 328, normalized size = 6.19

$$\begin{cases} -\frac{6\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{6\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{2}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f} & \text{for}\ f\neq 0 \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**
2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*tan
(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x
/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*tan(e/2 + f*x/2)/(3*
a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*
tan(e/2 + f*x/2) - 3*a*c**2*f) - 2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c*
*2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f), Ne(f,
 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c)**2), True))
```

**Giac** [A]
time = 0.44, size = 77, normalized size = 1.45

$$-\frac{\frac{3}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*tan(1/2*f*x + 1/2*e)^2 - 12
*tan(1/2*f*x + 1/2*e) + 7)/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

**Mupad** [B]
time = 7.01, size = 74, normalized size = 1.40

$$-\frac{2\,\left(3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}{3\,a\,c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(tan(e/2 + (f*x)/2) - 3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^3 +
 1))/(3*a*c^2*f*(tan(e/2 + (f*x)/2) - 1)^3*(tan(e/2 + (f*x)/2) + 1))
```

# 3.267 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=85

$$\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{\sec(e+fx)}{5af\,(c^3-c^3\sin(e+fx))}+\frac{2\tan(e+fx)}{5ac^3f}$$

```
[Out] 1/5*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/5*sec(f*x+e)/a/f/(c^3-c^3*sin(f*x
+e))+2/5*tan(f*x+e)/a/c^3/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2815, 2751, 3852, 8}

$$\frac{2\tan(e+fx)}{5ac^3f}+\frac{\sec(e+fx)}{5af\,(c^3-c^3\sin(e+fx))}+\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]/(5*a*c*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]/(5*a*f*(c^3 -
c^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{ac}\\ &= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{3\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}\\ &= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{2\int s\ldots\\ &= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}-\frac{2\mathrm{Sul}\ldots\\ &= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{2\tan\ldots\end{aligned}$$

**Mathematica [A]**
time = 0.47, size = 111, normalized size = 1.31

$$-\frac{-15+32\cos(e+fx)-12\cos(2(e+fx))+32\cos(3(e+fx))+3\cos(4(e+fx))+12\sin(e+fx)+32\sin(2(e+fx))+12\sin(3(e+fx))-8\sin(4(e+fx))}{160ac^3f(-1+\sin(e+fx))^3(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
[Out] -1/160*(-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] +
 3*Cos[4*(e + f*x)] + 12*Sin[e + f*x] + 32*Sin[2*(e + f*x)] + 12*Sin[3*(e +
 f*x)] - 8*Sin[4*(e + f*x)])/(a*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*
x]))
```

**Maple [A]**
time = 0.28, size = 103, normalized size = 1.21

| method | result |
|---|---|
| risch | $-\frac{4i\left(-4i e^{i(fx+e)}+5\,e^{2i(fx+e)}-1\right)}{5\left(e^{i(fx+e)}-i\right)^5\left(e^{i(fx+e)}+i\right)af\,c^3}$ |
| derivativedivides | $\frac{-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\ldots}}{af\,c^3}$ |
| default | $\frac{-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\ldots}}{af\,c^3}$ |

| | |
|---|---|
| norman | $\frac{\frac{4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{4}{5acf}-\frac{2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5acf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a/c^3*(-2/5/(tan(1/2*f*x+1/2*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1)^4-3/2/(ta
n(1/2*f*x+1/2*e)-1)^3-5/4/(tan(1/2*f*x+1/2*e)-1)^2-7/8/(tan(1/2*f*x+1/2*e)-
1)-1/8/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(86) = 172$.
time = 0.42, size = 229, normalized size = 2.69

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-2\right)}{5\left(ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 2)/((a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

**Fricas** [A]
time = 0.31, size = 90, normalized size = 1.06

$$-\frac{4\cos\left(fx+e\right)^2-\left(2\cos\left(fx+e\right)^2-3\right)\sin\left(fx+e\right)-2}{5\left(ac^3f\cos\left(fx+e\right)^3+2ac^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2ac^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a*c^3*f*
cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x +
e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(66) = 132$.
time = 2.81, size = 614, normalized size = 7.22

$$\begin{cases} -\frac{10\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f}+\frac{20\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f}-\frac{20\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f}+\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f}-\frac{4}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f} & \text{for}\ f\neq 0 \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*tan(e/2 + f*x/2)**5/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c
**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*t
an(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) + 20*tan(e/
2 + f*x/2)**4/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2
)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 +
20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) - 20*tan(e/2 + f*x/2)**3/(5*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*ta
n(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 +
 f*x/2) - 5*a*c**3*f) + 6*tan(e/2 + f*x/2)/(5*a*c**3*f*tan(e/2 + f*x/2)**6
- 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*
c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) - 4
/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c
**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*t
an(e/2 + f*x/2) - 5*a*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c
)**3), True))
```

**Giac** [A]
time = 0.44, size = 105, normalized size = 1.24

$$-\frac{\frac{5}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+120\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-70\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+21}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (35*tan(1/2*f*x + 1/2*e)^4 -
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 - 70*tan(1/2*f*x + 1
/2*e) + 21)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

**Mupad** [B]
time = 7.16, size = 89, normalized size = 1.05

$$-\frac{2\left(5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\right)}{5\,a\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^5\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(10*tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) - 10*tan(e/2 + (f*x)/2)
^4 + 5*tan(e/2 + (f*x)/2)^5 + 2))/(5*a*c^3*f*(tan(e/2 + (f*x)/2) - 1)^5*(ta
n(e/2 + (f*x)/2) + 1))
```

# 3.268 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=118

$$\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{8\tan(e+fx)}{35ac^4f}$$

```
[Out] 1/7*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+4/35*sec(f*x+e)/a/f/(c^2-c^2*sin(f*
x+e))^2+4/35*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+8/35*tan(f*x+e)/a/c^4/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2751, 3852, 8}

$$\frac{8\tan(e+fx)}{35ac^4f}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]/(7*a*c*f*(c - c*Sin[e + f*x])^3) + (4*Sec[e + f*x])/(35*a*f*(c
^2 - c^2*Sin[e + f*x])^2) + (4*Sec[e + f*x])/(35*a*f*(c^4 - c^4*Sin[e + f*x
])) + (8*Tan[e + f*x])/(35*a*c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{ac} \\
&= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2} \\
&= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{12}{\ } \\
&= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{}{35} \\
&= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{}{35} \\
&= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{}{35}
\end{aligned}$$

**Mathematica [A]**
time = 0.51, size = 131, normalized size = 1.11

$$\frac{-406+896\cos(e+fx)-232\cos(2(e+fx))+832\cos(3(e+fx))+174\cos(4(e+fx))-64\cos(5(e+fx))+406\sin(e+fx)+512\sin(2(e+fx))+377\sin(3(e+fx))-384\sin(4(e+fx))-29\sin(5(e+fx))}{4480ac^4f(-1+\sin(e+fx))^4(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
[Out] (-406 + 896*Cos[e + f*x] - 232*Cos[2*(e + f*x)] + 832*Cos[3*(e + f*x)] + 17
4*Cos[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 406*Sin[e + f*x] + 512*Sin[2*(e
+ f*x)] + 377*Sin[3*(e + f*x)] - 384*Sin[4*(e + f*x)] - 29*Sin[5*(e + f*x)]
)/(4480*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.31, size = 133, normalized size = 1.13

| method | result |
|---|---|
| risch | $-\frac{16\left(-6\,\mathrm{e}^{i(fx+e)}+i+14\,\mathrm{e}^{3i(fx+e)}-14i\mathrm{e}^{2i(fx+e)}\right)}{35\left(\mathrm{e}^{i(fx+e)}-i\right)^7\left(\mathrm{e}^{i(fx+e)}+i\right)af\,c^4}$ |

| | |
|---|---|
| derivativedivides | $\frac{-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{38}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{9}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{15}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)- \ldots\right.}}{a f c^{4}}$ |
| default | $\frac{-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{38}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{9}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{15}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)- \ldots\right.}}{a f c^{4}}$ |
| norman | $\frac{\frac{6\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{26}{35acf}-\frac{2\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{6\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{10\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}-\frac{22\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf} \ldots}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a/c^4*(-1/16/(tan(1/2*f*x+1/2*e)+1)-4/7/(tan(1/2*f*x+1/2*e)-1)^7-2/(tan
(1/2*f*x+1/2*e)-1)^6-19/5/(tan(1/2*f*x+1/2*e)-1)^5-9/2/(tan(1/2*f*x+1/2*e)-
1)^4-15/4/(tan(1/2*f*x+1/2*e)-1)^3-17/8/(tan(1/2*f*x+1/2*e)-1)^2-15/16/(tan
(1/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(120) = 240$.
time = 0.34, size = 347, normalized size = 2.94

$$-\frac{2\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{7\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{105\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{175\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{105\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{35\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-13\right)}{35\left(ac^{4}-\frac{6ac^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{14ac^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{14ac^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{14ac^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{6ac^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{ac^{4}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/((a*
c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*f)
```

**Fricas** [A]
time = 0.32, size = 120, normalized size = 1.02

$$\frac{8\cos(fx+e)^{4}-36\cos(fx+e)^{2}+4\left(6\cos(fx+e)^{2}-5\right)\sin(fx+e)+15}{35\left(3ac^{4}f\cos(fx+e)^{3}-4ac^{4}f\cos(fx+e)-\left(ac^{4}f\cos(fx+e)^{3}-4ac^{4}f\cos(fx+e)\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(8*cos(f*x + e)^4 - 36*cos(f*x + e)^2 + 4*(6*cos(f*x + e)^2 - 5)*sin(f
*x + e) + 15)/(3*a*c^4*f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f
*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1307 vs. $2(97) = 194$.
time = 6.24, size = 1307, normalized size = 11.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a
*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4
*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*tan(
e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f
*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)
**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 +
 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*tan(e/2 + f*x/2)**5/(35
*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c*
*4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*
tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e
/2 + f*x/2) - 35*a*c**4*f) + 210*tan(e/2 + f*x/2)**4/(35*a*c**4*f*tan(e/2 +
 f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/
2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3
 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*
c**4*f) + 14*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c
**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f
*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(
e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 154*tan(e/
2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x
/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**
5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*tan(e/2 + f*x/2)/(35*a*c**
4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*t
an(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/
2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f
*x/2) - 35*a*c**4*f) - 26/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*t
an(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f
*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f), Ne(f, 0)), (x/((a*
sin(e) + a)*(-c*sin(e) + c)**4), True))
```

**Giac** [**A**]

time = 0.45, size = 133, normalized size = 1.13

$$-\frac{\frac{35}{ac^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-1960\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+4025\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-4480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3143\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1176\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+243}{ac^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^7}}{280f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*tan(1/2*f*x + 1/2*e)^6
 - 1960*tan(1/2*f*x + 1/2*e)^5 + 4025*tan(1/2*f*x + 1/2*e)^4 - 4480*tan(1/2
*f*x + 1/2*e)^3 + 3143*tan(1/2*f*x + 1/2*e)^2 - 1176*tan(1/2*f*x + 1/2*e) +
 243)/(a*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

**Mupad [B]**
time = 7.27, size = 96, normalized size = 0.81

$$\frac{\frac{2\sin(e+fx)}{5}+\frac{2\cos(2e+2fx)}{5}-\frac{\cos(4e+4fx)}{35}-\frac{6\sin(3e+3fx)}{35}}{a\,c^4\,f\,\left(\frac{7\cos(e+fx)}{4}-\frac{3\cos(3e+3fx)}{4}-\frac{7\sin(2e+2fx)}{4}+\frac{\sin(4e+4fx)}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

```
[Out] ((2*sin(e + f*x))/5 + (2*cos(2*e + 2*f*x))/5 - cos(4*e + 4*f*x)/35 - (6*sin
(3*e + 3*f*x))/35)/(a*c^4*f*((7*cos(e + f*x))/4 - (3*cos(3*e + 3*f*x))/4 -
(7*sin(2*e + 2*f*x))/4 + sin(4*e + 4*f*x)/8))
```

# 3.269 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=148

$$\frac{105c^5x}{2a^2}+\frac{35c^5\cos^3(e+fx)}{a^2f}+\frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f}-\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}+\frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}+$$

```
[Out] 105/2*c^5*x/a^2+35*c^5*cos(f*x+e)^3/a^2/f+105/2*c^5*cos(f*x+e)*sin(f*x+e)/a
^2/f-2/3*a^4*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6+6*a^2*c^5*cos(f*x+e)^7/f
/(a+a*sin(f*x+e))^4+42*c^5*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]

time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2761, 2715, 8}

$$-\frac{2a^4c^5\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{35c^5\cos^3(e+fx)}{a^2f}+\frac{6a^2c^5\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{105c^5\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{105c^5x}{2a^2}+\frac{42c^5\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (105*c^5*x)/(2*a^2) + (35*c^5*Cos[e + f*x]^3)/(a^2*f) + (105*c^5*Cos[e + f*
x]*Sin[e + f*x])/(2*a^2*f) - (2*a^4*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e +
 f*x])^6) + (6*a^2*c^5*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (42*c^5
*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^7}\,dx \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(3a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \left(21ac^5\right)\int \frac{\cos^6(e+f}{(a+a\sin(e+} \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} + \\
&= \frac{35c^5\cos^3(e+fx)}{a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5}{f(a+} \\
&= \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} \\
&= \frac{105c^5x}{2a^2} + \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e}{3f(a+a\sin(e}
\end{aligned}
$$

**Mathematica [A]**
time = 0.49, size = 276, normalized size = 1.86

$$
\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(256\sin\left(\frac{1}{2}(e+fx)\right)-128\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-1664\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+630(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+285\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-21\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}(a+a\sin(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1664*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 630*(e + f*x)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 285*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3 - Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 21*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^10*(a + a*Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.41, size = 138, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $\frac{2c^5\left(-\frac{64}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{48}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{\frac{7\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+23\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+48\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{f\,a^2}$ |
| default | $\frac{2c^5\left(-\frac{64}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{48}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{\frac{7\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+23\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+48\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{f\,a^2}$ |
| risch | $\frac{105c^5x}{2a^2}+\frac{7ic^5e^{2i(fx+e)}}{8f\,a^2}+\frac{95c^5e^{i(fx+e)}}{8a^2f}+\frac{95c^5e^{-i(fx+e)}}{8a^2f}-\frac{7ic^5e^{-2i(fx+e)}}{8f\,a^2}+\frac{256ic^5e^{i(fx+e)}+160c^5e^{2i(fx+e)}-}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ |
| norman | $\frac{\frac{3675c^5x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2a}+\frac{2625c^5x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2a}+\frac{840c^5x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{420c^5x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{315c^5x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2a}+}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^5/a^2*(-64/3/(tan(1/2*f*x+1/2*e)+1)^3+32/(tan(1/2*f*x+1/2*e)+1)^2+48/
(tan(1/2*f*x+1/2*e)+1)+(7/2*tan(1/2*f*x+1/2*e)^5+23*tan(1/2*f*x+1/2*e)^4+48
*tan(1/2*f*x+1/2*e)^2-7/2*tan(1/2*f*x+1/2*e)+71/3)/(1+tan(1/2*f*x+1/2*e)^2)
^3+105/2*arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(151) = 302$.
time = 0.57, size = 1418, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
```

```
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) + 2*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3
*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 40*c^5*((12*si
n(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^5*((9*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3) + 10*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [A]
time = 0.33, size = 252, normalized size = 1.70

$$-\frac{2\,c^5\cos\left(fx+e\right)^5+19\,c^5\cos\left(fx+e\right)^4-106\,c^5\cos\left(fx+e\right)^3+630\,c^5fx-64\,c^5-7\left(45\,c^5fx-77\,c^5\right)\cos\left(fx+e\right)^2+\left(315\,c^5fx+598\,c^5\right)\cos\left(fx+e\right)-\left(2\,c^5\cos\left(fx+e\right)^4-17\,c^5\cos\left(fx+e\right)^3-630\,c^5fx-123\,c^5\cos\left(fx+e\right)^2-64\,c^5-\left(315\,c^5fx+662\,c^5\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*c^5*cos(f*x + e)^5 + 19*c^5*cos(f*x + e)^4 - 106*c^5*cos(f*x + e)^3
 + 630*c^5*f*x - 64*c^5 - 7*(45*c^5*f*x - 77*c^5)*cos(f*x + e)^2 + (315*c^5
*f*x + 598*c^5)*cos(f*x + e) - (2*c^5*cos(f*x + e)^4 - 17*c^5*cos(f*x + e)^
3 - 630*c^5*f*x - 123*c^5*cos(f*x + e)^2 - 64*c^5 - (315*c^5*f*x + 662*c^5)
*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2
*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3641 vs. $2(144) = 288$.

time = 15.58, size = 3641, normalized size = 24.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((315*c**5*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**9 +
 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*
tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f
*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 +
18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 945*c**5*f*x*tan(e/2 + f*x/2)**8/(
6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*ta
n(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x
/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36
*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1890
*c**5*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan
(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/
2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*
a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) + 3150*c**5*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(
e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2
)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a
**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3780*c**5*f*x*tan
(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)
**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a*
*2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/
2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) + 3780*c**5*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)*
*9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**
2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2
 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3150*c**5*f*x*tan(e/2 + f*x/2)
**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
 1890*c**5*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
 f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 945*c**5*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan
```

```
(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/
2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*
a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**5*f*x/(6*
a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(
e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2
)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 618*c*
*5*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 +
f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 +
 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*
tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) + 1938*c**5*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2
)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a
**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e
/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3386*c**5*tan(e/2 + f*x/2)**
6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f
*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 +
 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 6
054*c**5*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(
e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2
)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a
**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e
/2 + f*x/2) + 6*a**2*f) + 5802*c**5*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 +
 f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7
+ 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f
*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 6494*c**5*tan(e/2 + f*
x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2...
```

**Giac** [A]
time = 0.46, size = 203, normalized size = 1.37

$$\frac{\frac{315\,(fx+e)c^5}{a^2}+\frac{2\left(309\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+969\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+1693\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+3027\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+2901\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+3247\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+1995\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1173\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+494\,c^5\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(315*(f*x + e)*c^5/a^2 + 2*(309*c^5*tan(1/2*f*x + 1/2*e)^8 + 969*c^5*ta
n(1/2*f*x + 1/2*e)^7 + 1693*c^5*tan(1/2*f*x + 1/2*e)^6 + 3027*c^5*tan(1/2*f
*x + 1/2*e)^5 + 2901*c^5*tan(1/2*f*x + 1/2*e)^4 + 3247*c^5*tan(1/2*f*x + 1/
2*e)^3 + 1995*c^5*tan(1/2*f*x + 1/2*e)^2 + 1173*c^5*tan(1/2*f*x + 1/2*e) +
```

```
494*c^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x +
1/2*e) + 1)^3*a^2))/f
```

**Mupad** [B]
time = 10.86, size = 372, normalized size = 2.51

$$\frac{105\,c^5\,x}{2\,a^2}-\frac{\frac{105\,c^5\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{315\,c^5\,\left(e+f\,x\right)}{2}-\frac{c^5\,\left(945\,e+945\,f\,x+2346\right)}{6}\right)-\frac{c^5\,\left(315\,e+315\,f\,x+988\right)}{6}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{315\,c^5\,\left(e+f\,x\right)}{2}-\frac{c^5\,\left(945\,e+945\,f\,x+618\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(315\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(1890\,e+1890\,f\,x+1938\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(315\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(1890\,e+1890\,f\,x+3990\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(525\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(3150\,e+3150\,f\,x+3386\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(525\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(3150\,e+3150\,f\,x+6494\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(630\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(3780\,e+3780\,f\,x+5802\right)}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(630\,c^5\,\left(e+f\,x\right)-\frac{c^5\,\left(3780\,e+3780\,f\,x+6054\right)}{6}\right)}{a^2\,f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^5/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (105*c^5*x)/(2*a^2) - ((105*c^5*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((315*c^5
*(e + f*x))/2 - (c^5*(945*e + 945*f*x + 2346))/6) - (c^5*(315*e + 315*f*x +
 988))/6 + tan(e/2 + (f*x)/2)^8*((315*c^5*(e + f*x))/2 - (c^5*(945*e + 945*
f*x + 618))/6) + tan(e/2 + (f*x)/2)^7*(315*c^5*(e + f*x) - (c^5*(1890*e + 1
890*f*x + 1938))/6) + tan(e/2 + (f*x)/2)^2*(315*c^5*(e + f*x) - (c^5*(1890*
e + 1890*f*x + 3990))/6) + tan(e/2 + (f*x)/2)^6*(525*c^5*(e + f*x) - (c^5*(
3150*e + 3150*f*x + 3386))/6) + tan(e/2 + (f*x)/2)^3*(525*c^5*(e + f*x) - (
c^5*(3150*e + 3150*f*x + 6494))/6) + tan(e/2 + (f*x)/2)^4*(630*c^5*(e + f*x
) - (c^5*(3780*e + 3780*f*x + 5802))/6) + tan(e/2 + (f*x)/2)^5*(630*c^5*(e
+ f*x) - (c^5*(3780*e + 3780*f*x + 6054))/6))/(a^2*f*(tan(e/2 + (f*x)/2) +
tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^3 + 1)^3)
```

# 3.270 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=135

$$\frac{35c^4x}{2a^2}+\frac{35c^4\cos(e+fx)}{2a^2f}-\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)^3}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}$$

```
[Out] 35/2*c^4*x/a^2+35/2*c^4*cos(f*x+e)/a^2/f-2/3*a^3*c^4*cos(f*x+e)^7/f/(a+a*si
n(f*x+e))^5+14/3*a^4*c^4*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^3+35/6*c^4*cos
(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

**Rubi** [A]
time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2758, 2761, 8}

$$-\frac{2a^3c^4\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{35c^4\cos(e+fx)}{2a^2f}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{35c^4x}{2a^2}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (35*c^4*x)/(2*a^2) + (35*c^4*Cos[e + f*x])/(2*a^2*f) - (2*a^3*c^4*Cos[e + f
*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (14*a^4*c^4*Cos[e + f*x]^5)/(3*f*(a^2
 + a^2*Sin[e + f*x])^3) + (35*c^4*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{1}{3}\left(35c^4\right)\int \frac{\cos^4(e+f}{(a+a\sin(e+} \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)} \\
&= \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4}{6f\left(a^2\right.} \\
&= \frac{35c^4x}{2a^2} + \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}
\end{aligned}$$

**Mathematica [A]**
time = 0.34, size = 243, normalized size = 1.80

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(128\sin\left(\frac{1}{2}(e+fx)\right)-64\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-640\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+210(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+72\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(a+a\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*Sin[(e +
 f*x)/2] - 64*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 640*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3 + 72*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(a + a*Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.40, size = 125, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $\frac{2c^4\left(-\frac{32}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+6\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+6}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+35\cdots\right.}{f\,a^2}$ |
| default | $\frac{2c^4\left(-\frac{32}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+6\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+6}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+35\cdots\right.}{f\,a^2}$ |
| risch | $\frac{35c^4x}{2a^2}+\frac{ic^4e^{2i(fx+e)}}{8a^2f}+\frac{3c^4e^{i(fx+e)}}{a^2f}+\frac{3c^4e^{-i(fx+e)}}{a^2f}-\frac{ic^4e^{-2i(fx+e)}}{8a^2f}+\frac{96ic^4e^{i(fx+e)}+64c^4e^{2i(fx+e)}-\frac{160c^4}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ |
| norman | $\frac{\frac{111c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{131c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{500c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{35c^4x}{2a}+\frac{164c^4}{3af}+\frac{105c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2a}+\frac{245c^4x\left(\tan^2\left(\frac{fx}{2}+\cdots\right.\right.}{2a}\cdots}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^4/a^2*(-32/3/(tan(1/2*f*x+1/2*e)+1)^3+16/(tan(1/2*f*x+1/2*e)+1)^2+16/
(tan(1/2*f*x+1/2*e)+1)+(1/2*tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1/2*e)^2-1/2
*tan(1/2*f*x+1/2*e)+6)/(1+tan(1/2*f*x+1/2*e)^2)^2+35/2*arctan(tan(1/2*f*x+1
/2*e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(132) = 264$.
time = 0.55, size = 981, normalized size = 7.27

$$\frac{c^4\left(\frac{\frac{75\sin(fx+e)}{\cos(fx+e)+1}+\frac{97\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{126\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{98\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{63\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{21\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+32}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{3a^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{21\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+16c^4\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+12c^4\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{2c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{8c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
```

```
e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) + 16*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^2) + 12*c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 8*c^4*(3*sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3))/f
```

**Fricas** [**A**]
time = 0.32, size = 222, normalized size = 1.64

$$-\frac{3\,c^{4}\cos\left(fx+e\right)^{4}-30\,c^{4}\cos\left(fx+e\right)^{3}+210\,c^{4}fx-32\,c^{4}-\left(105\,c^{4}fx-193\,c^{4}\right)\cos\left(fx+e\right)^{2}+\left(105\,c^{4}fx+194\,c^{4}\right)\cos\left(fx+e\right)+\left(3\,c^{4}\cos\left(fx+e\right)^{3}+210\,c^{4}fx+33\,c^{4}\cos\left(fx+e\right)^{2}+32\,c^{4}+\left(105\,c^{4}fx+226\,c^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^4*cos(f*x + e)^4 - 30*c^4*cos(f*x + e)^3 + 210*c^4*f*x - 32*c^4 -
 (105*c^4*f*x - 193*c^4)*cos(f*x + e)^2 + (105*c^4*f*x + 194*c^4)*cos(f*x +
 e) + (3*c^4*cos(f*x + e)^3 + 210*c^4*f*x + 33*c^4*cos(f*x + e)^2 + 32*c^4
+ (105*c^4*f*x + 226*c^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2
 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x +
e))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2312 vs. $2(128) = 256$.
time = 8.82, size = 2312, normalized size = 17.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((105*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**4*f*x*tan(e/2 +
f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) + 525*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/
2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*
a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 735*c**4*f*x*tan
(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a*
*2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) + 735*c**4*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 525*c**4*
f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
 f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
+ 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**4*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 105*c
**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*
a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2
) + 6*a**2*f) + 198*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 666*c**4*tan(e/2 + f*x
/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) + 868*c**4*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1428*c**4*tan(e/2 + f*x
/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) + 974*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
```

```
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 786*c**4*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 328*c**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**4/(a*sin(e) + a)*
*2, True))
```

**Giac** [A]
time = 0.46, size = 152, normalized size = 1.13

$$\frac{\frac{105\,(fx+e)c^4}{a^2}+\frac{6\left(c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,c^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}+\frac{64\left(3\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,c^4\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(105*(f*x + e)*c^4/a^2 + 6*(c^4*tan(1/2*f*x + 1/2*e)^3 + 12*c^4*tan(1/2
*f*x + 1/2*e)^2 - c^4*tan(1/2*f*x + 1/2*e) + 12*c^4)/((tan(1/2*f*x + 1/2*e)
^2 + 1)^2*a^2) + 64*(3*c^4*tan(1/2*f*x + 1/2*e)^2 + 9*c^4*tan(1/2*f*x + 1/2
*e) + 4*c^4)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad** [B]
time = 10.09, size = 291, normalized size = 2.16

$$\frac{35\,c^4\,x}{2\,a^2}-\frac{\frac{35\,c^4\,(e+f\,x)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{105\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(315\,e+315\,f\,x+786)}{6}\right)-\frac{c^4\,(105\,e+105\,f\,x+328)}{6}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(\frac{105\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(315\,e+315\,f\,x+198)}{6}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(\frac{175\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(525\,e+525\,f\,x+666)}{6}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{175\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(525\,e+525\,f\,x+974)}{6}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{245\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(735\,e+735\,f\,x+868)}{6}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{245\,c^4\,(e+f\,x)}{2}-\frac{c^4\,(735\,e+735\,f\,x+1428)}{6}\right)}{a^2\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (35*c^4*x)/(2*a^2) - ((35*c^4*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((105*c^4*(
e + f*x))/2 - (c^4*(315*e + 315*f*x + 786))/6) - (c^4*(105*e + 105*f*x + 32
8))/6 + tan(e/2 + (f*x)/2)^6*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x
 + 198))/6) + tan(e/2 + (f*x)/2)^5*((175*c^4*(e + f*x))/2 - (c^4*(525*e + 5
25*f*x + 666))/6) + tan(e/2 + (f*x)/2)^2*((175*c^4*(e + f*x))/2 - (c^4*(525
*e + 525*f*x + 974))/6) + tan(e/2 + (f*x)/2)^4*((245*c^4*(e + f*x))/2 - (c^
4*(735*e + 735*f*x + 868))/6) + tan(e/2 + (f*x)/2)^3*((245*c^4*(e + f*x))/2
 - (c^4*(735*e + 735*f*x + 1428))/6))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3*(ta
n(e/2 + (f*x)/2)^2 + 1)^2)
```

# 3.271 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=90

$$\frac{5c^3x}{a^2}+\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] 5*c^3*x/a^2+5*c^3*cos(f*x+e)/a^2/f-2/3*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+
e))^4+10/3*c^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2759, 2761, 8}

$$\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{5c^3x}{a^2}+\frac{10c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*c^3*x)/a^2 + (5*c^3*Cos[e + f*x])/(a^2*f) - (2*a^2*c^3*Cos[e + f*x]^5)/(
3*f*(a + a*Sin[e + f*x])^4) + (10*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx \\
&= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} - \frac{1}{3}\left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{\left(5c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{a} \\
&= \frac{5c^3\cos(e+fx)}{a^2f} - \frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{\left(5c^3\right)\int 1}{a^2} \\
&= \frac{5c^3x}{a^2} + \frac{5c^3\cos(e+fx)}{a^2f} - \frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}
\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(90) = 180.
time = 0.25, size = 210, normalized size = 2.33

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(16\sin\left(\frac{1}{2}(e+fx)\right)-8\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-56\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+15(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+3\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)(c-c\sin(e+fx))^3}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(a+a\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*Sin[(e + f*x)/2] - 8*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) - 56*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^2 + 15*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*Co
s[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^3)
/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.34, size = 85, normalized size = 0.94

| method | result |
|---|---|
| derivativedivides | $\frac{2c^3\left(\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+5\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{16}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^2}$ |

| | |
|---|---|
| default | $\frac{2c^3\left(\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+5\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{16}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^2}$ |
| risch | $\frac{5c^3x}{a^2}+\frac{c^3 e^{i(fx+e)}}{2a^2f}+\frac{c^3 e^{-i(fx+e)}}{2a^2f}+\frac{32ic^3 e^{i(fx+e)}+24c^3 e^{2i(fx+e)}-\frac{56c^3}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ |
| norman | $\frac{\frac{8c^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{38c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{5c^3x}{a}+\frac{46c^3}{3af}+\frac{15c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{30c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{50c^3x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^3/a^2*(1/(1+tan(1/2*f*x+1/2*e)^2)+5*arctan(tan(1/2*f*x+1/2*e))-16/3/(
tan(1/2*f*x+1/2*e)+1)^3+8/(tan(1/2*f*x+1/2*e)+1)^2+4/(tan(1/2*f*x+1/2*e)+1)
)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(91) = 182$.

time = 0.55, size = 640, normalized size = 7.11

$$\frac{2\left(2c^3\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+3c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 3*c^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

time = 0.32, size = 195, normalized size = 2.17

$$\frac{3c^3\cos(fx+e)^3-30c^3fx+8c^3+(15c^3fx-31c^3)\cos(fx+e)^2-(15c^3fx+26c^3)\cos(fx+e)-\left(30c^3fx+3c^3\cos(fx+e)^2+8c^3+(15c^3fx+34c^3)\cos(fx+e)\right)\sin(fx+e)}{3\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2a^2f-(a^2f\cos(fx+e)+2a^2f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*(3*c^3*cos(f*x + e)^3 - 30*c^3*f*x + 8*c^3 + (15*c^3*f*x - 31*c^3)*cos(
f*x + e)^2 - (15*c^3*f*x + 26*c^3)*cos(f*x + e) - (30*c^3*f*x + 3*c^3*cos(f
*x + e)^2 + 8*c^3 + (15*c^3*f*x + 34*c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*
f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a
^2*f)*sin(f*x + e))

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(87) = 174$.
time = 4.94, size = 1282, normalized size = 14.24

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)

[Out] Piecewise((15*c**3*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan
(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)*
*4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 15*c**3*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*c**3*tan(e/2 +
f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12
*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) + 102*c**3*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 82
*c**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 114*c**3*tan(e/2 + f*x/2)/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) + 46*c**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2

```
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*sin(
e) + a)**2, True))
```

**Giac** [A]

time = 0.45, size = 101, normalized size = 1.12

$$\frac{\frac{15\,(fx+e)c^3}{a^2}+\frac{6\,c^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}+\frac{8\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,c^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*c^3/a^2 + 6*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^2) + 8*(3
*c^3*tan(1/2*f*x + 1/2*e)^2 + 12*c^3*tan(1/2*f*x + 1/2*e) + 5*c^3)/(a^2*(ta
n(1/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad** [B]

time = 9.62, size = 217, normalized size = 2.41

$$\frac{5\,c^3\,x}{a^2}-\frac{5\,c^3\,(e+f\,x)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,c^3\,(e+f\,x)-\frac{c^3\,(45\,e+45\,f\,x+114)}{3}\right)-\frac{c^3\,(15\,e+15\,f\,x+46)}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(15\,c^3\,(e+f\,x)-\frac{c^3\,(45\,e+45\,f\,x+24)}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(20\,c^3\,(e+f\,x)-\frac{c^3\,(60\,e+60\,f\,x+82)}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(20\,c^3\,(e+f\,x)-\frac{c^3\,(60\,e+60\,f\,x+102)}{3}\right)}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (5*c^3*x)/a^2 - (5*c^3*(e + f*x) + tan(e/2 + (f*x)/2)*(15*c^3*(e + f*x) - (
c^3*(45*e + 45*f*x + 114))/3) - (c^3*(15*e + 15*f*x + 46))/3 + tan(e/2 + (f
*x)/2)^4*(15*c^3*(e + f*x) - (c^3*(45*e + 45*f*x + 24))/3) + tan(e/2 + (f*x
)/2)^2*(20*c^3*(e + f*x) - (c^3*(60*e + 60*f*x + 82))/3) + tan(e/2 + (f*x)/
2)^3*(20*c^3*(e + f*x) - (c^3*(60*e + 60*f*x + 102))/3))/(a^2*f*(tan(e/2 +
(f*x)/2) + 1)^3*(tan(e/2 + (f*x)/2)^2 + 1))
```

# 3.272 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=70

$$\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2c^2\cos(e+fx)}{f\,(a^2+a^2\sin(e+fx))}$$

```
[Out] c^2*x/a^2-2/3*a*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*c^2*cos(f*x+e)/f/(a
^2+a^2*sin(f*x+e))
```

**Rubi** [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {2815, 2759, 8}

$$\frac{2c^2\cos(e+fx)}{f\,(a^2\sin(e+fx)+a^2)}+\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (2*a*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*c^
2*Cos[e + f*x])/(f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - c^2\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2c^2\cos(e+fx)}{f\,(a^2+a^2\sin(e+fx))} + \frac{c^2\int 1\,dx}{a^2}$$
$$= \frac{c^2x}{a^2} - \frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2c^2\cos(e+fx)}{f\,(a^2+a^2\sin(e+fx))}$$

## Mathematica [A]

time = 0.40, size = 119, normalized size = 1.70

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(-8+3e+3fx)\cos\left(\frac{1}{2}(e+fx)\right)+(16-3e-3fx)\cos\left(\frac{3}{2}(e+fx)\right)+6(2(-2+e+fx)+(e+fx)\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(-8 + 3*e + 3*f*x)*Cos[(e + f
*x)/2] + (16 - 3*e - 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(-2 + e + f*x) + (e
 + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(6*a^2*f*(1 + Sin[e + f*x])^2)
```

## Maple [A]

time = 0.29, size = 53, normalized size = 0.76

| method | result |
|---|---|
| derivativedivides | $\frac{2c^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}\right)}{f\,a^2}$ |
| default | $\frac{2c^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}\right)}{f\,a^2}$ |
| risch | $\frac{c^2x}{a^2}+\frac{8ic^2e^{i(fx+e)}+8c^2e^{2i(fx+e)}-\frac{16c^2}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ |
| norman | $\frac{\frac{c^2x}{a}+\frac{c^2x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{8c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{8c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{16c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{8c^2}{3af}+\frac{3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{5c^2x\left(\tan\right.}{}}{\left(1+\tan^2\left(\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^2/a^2*(arctan(tan(1/2*f*x+1/2*e))-8/3/(tan(1/2*f*x+1/2*e)+1)^3+4/(tan
(1/2*f*x+1/2*e)+1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(72) = 144$.
time = 0.52, size = 391, normalized size = 5.59

$$\frac{2\left(c^{2}\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+4}{a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{2}}\right)-\frac{c^{2}\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2\right)}{a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{2\,c^{2}\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(72) = 144$.
time = 0.32, size = 166, normalized size = 2.37

$$-\frac{6\,c^{2}fx-\left(3\,c^{2}fx-8\,c^{2}\right)\cos\left(fx+e\right)^{2}-4\,c^{2}+\left(3\,c^{2}fx+4\,c^{2}\right)\cos\left(fx+e\right)+\left(6\,c^{2}fx+4\,c^{2}+\left(3\,c^{2}fx+8\,c^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{3\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*c^2*f*x - (3*c^2*f*x - 8*c^2)*cos(f*x + e)^2 - 4*c^2 + (3*c^2*f*x +
 4*c^2)*cos(f*x + e) + (6*c^2*f*x + 4*c^2 + (3*c^2*f*x + 8*c^2)*cos(f*x + e
))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(65) = 130$.
time = 2.40, size = 473, normalized size = 6.76

$$\begin{cases} \frac{3c^{2}fx\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}+\frac{9c^{2}fx\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}+\frac{9c^{2}fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}+\frac{3c^{2}fx}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}+\frac{24c^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}+\frac{8c^{2}}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f} & \text{for}\: f \neq 0 \\ \frac{x\left(-c\sin\left(e\right)+c\right)^{2}}{\left(a\sin\left(e\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**
```

```
2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**2*f*x*tan(e/2 +
f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 2
4*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*c**2/(3*a**2*f*tan(e/
2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

**Giac [A]**
time = 0.42, size = 58, normalized size = 0.83

$$\frac{\frac{3\,(fx+e)c^2}{a^2}+\frac{8\,\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 8*(3*c^2*tan(1/2*f*x + 1/2*e) + c^2)/(a^2*(tan(1
/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad [B]**
time = 7.07, size = 89, normalized size = 1.27

$$\frac{c^2\,x}{a^2}-\frac{c^2\,(e+f\,x)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,c^2\,(e+f\,x)-\frac{c^2\,(9\,e+9\,f\,x+24)}{3}\right)-\frac{c^2\,(3\,e+3\,f\,x+8)}{3}}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (c^2*x)/a^2 - (c^2*(e + f*x) + tan(e/2 + (f*x)/2)*(3*c^2*(e + f*x) - (c^2*(
9*e + 9*f*x + 24))/3) - (c^2*(3*e + 3*f*x + 8))/3)/(a^2*f*(tan(e/2 + (f*x)/
2) + 1)^3)
```

# 3.273 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=29

$$-\frac{ac\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}$$

```
[Out] -1/3*a*c*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2815, 2750}

$$-\frac{ac\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(a*c*Cos[e + f*x]^3)/(f*(a + a*Sin[e + f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.
time = 0.19, size = 70, normalized size = 2.41

$$\frac{c\left(-3\cos\left(e+\frac{fx}{2}\right)+\cos\left(e+\frac{3fx}{2}\right)\right)}{3a^2f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(3*a^2*f*(Cos[e/2] + Sin[e/2
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.
time = 0.23, size = 56, normalized size = 1.93

| method | result | size |
|---|---|---|
| risch | $\frac{2c\,e^{2i(fx+e)}-\frac{2c}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ | 39 |
| derivativedivides | $\frac{2c\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}\right)}{f\,a^2}$ | 56 |
| default | $\frac{2c\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}\right)}{f\,a^2}$ | 56 |
| norman | $\frac{-\frac{2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c}{3af}-\frac{8c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c/a^2*(-1/(tan(1/2*f*x+1/2*e)+1)-4/3/(tan(1/2*f*x+1/2*e)+1)^3+2/(tan(1/
2*f*x+1/2*e)+1)^2)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(29) = 58.
time = 0.32, size = 233, normalized size = 8.03

$$-\frac{2\left(\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(29) = 58$.
time = 0.31, size = 112, normalized size = 3.86

$$-\frac{c\cos\left(fx+e\right)^{2}-c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)+2\,c\right)\sin\left(fx+e\right)-2\,c}{3\,\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(c*cos(f*x + e)^2 - c*cos(f*x + e) + (c*cos(f*x + e) + 2*c)*sin(f*x +
e) - 2*c)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.
time = 1.29, size = 158, normalized size = 5.45

$$\begin{cases} -\frac{6c\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{2c}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f} & \text{for}\: f \neq 0 \\ \frac{x\left(-c\sin\left(e\right)+c\right)}{\left(a\sin\left(e\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

**Giac** [A]
time = 0.45, size = 39, normalized size = 1.34

$$-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right)}{3\,a^{2}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + c)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

**Mupad** [B]
time = 7.03, size = 54, normalized size = 1.86

$$\frac{2\,c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\right)}{3\,a^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*c*cos(e/2 + (f*x)/2)*(2*cos(e/2 + (f*x)/2)^2 - 3))/(3*a^2*f*(cos(e/2 + (
f*x)/2) + sin(e/2 + (f*x)/2))^3)
```

# 3.274 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

**Optimal**. Leaf size=52

$$-\frac{\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{2\tan(e+fx)}{3a^2cf}$$

```
[Out] -1/3*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+2/3*tan(f*x+e)/a^2/c/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2751, 3852, 8}

$$\frac{2\tan(e+fx)}{3a^2cf}-\frac{\sec(e+fx)}{3cf\,(a^2\sin(e+fx)+a^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/3*Sec[e + f*x]/(c*f*(a^2 + a^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^2*
c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{ac}\\ &= -\frac{\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{2\int \sec^2(e+fx)\,dx}{3a^2c}\\ &= -\frac{\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}-\frac{2\text{Subst}(\int 1\,dx, x, -\tan(e+ \ldots}{3a^2cf}\\ &= -\frac{\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{2\tan(e+fx)}{3a^2cf}\end{aligned}$$

**Mathematica [A]**
time = 0.33, size = 87, normalized size = 1.67

$$-\frac{2-4\cos(e+fx)+2\cos(2(e+fx))-4\cos(3(e+fx))+\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))}{24a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/24*(2 - 4*Cos[e + f*x] + 2*Cos[2*(e + f*x)] - 4*Cos[3*(e + f*x)] + Sin[e
 + f*x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(a^2*c*f*(-1 + Sin[e + f*x
])*(1 + Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.25, size = 73, normalized size = 1.40

| method | result | size |
|---|---|---|
| risch | $-\frac{4\left(2\,e^{i(fx+e)}+i\right)}{3\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)a^2cf}$ | 54 |
| derivativedivides | $\frac{-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^2cf}$ | 73 |
| default | $\frac{-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^2cf}$ | 73 |

| | | |
|---|---|---|
| norman | $\frac{-\frac{2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2}{3acf}-\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$ | 107 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2/c*(-1/4/(tan(1/2*f*x+1/2*e)-1)-1/3/(tan(1/2*f*x+1/2*e)+1)^3+1/2/(ta
n(1/2*f*x+1/2*e)+1)^2-3/4/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(51) = 102$.
time = 0.32, size = 154, normalized size = 2.96

$$\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{3\left(a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/((a^2*c + 2*a^2*c*sin(f*x +
e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

**Fricas** [A]
time = 0.31, size = 60, normalized size = 1.15

$$-\frac{2\cos\left(fx+e\right)^2-2\sin\left(fx+e\right)-1}{3\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 - 2*sin(f*x + e) - 1)/(a^2*c*f*cos(f*x + e)*sin(f*x
+ e) + a^2*c*f*cos(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(41) = 82$.
time = 1.35, size = 328, normalized size = 6.31

$$\begin{cases} -\frac{6\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf}-\frac{6\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf}+\frac{2}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} & \text{for}\ f\neq 0 \\ \frac{x}{(a\sin(e)+a)^2(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*
c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*tan
(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x
/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*tan(e/2 + f*x/2)/(3*
a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*
tan(e/2 + f*x/2) - 3*a**2*c*f) + 2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2
*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f), Ne(f,
 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)), True))
```

**Giac** [**A**]
time = 0.43, size = 77, normalized size = 1.48

$$-\frac{\frac{3}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*tan(1/2*f*x + 1/2*e)^2 + 12
*tan(1/2*f*x + 1/2*e) + 7)/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad** [**B**]
time = 6.99, size = 74, normalized size = 1.42

$$-\frac{2\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}{3\,a^2\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(tan(e/2 + (f*x)/2) + 3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^3 -
 1))/(3*a^2*c*f*(tan(e/2 + (f*x)/2) - 1)*(tan(e/2 + (f*x)/2) + 1)^3)
```

# 3.275 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=38

$$\frac{\tan(e+fx)}{a^2c^2f}+\frac{\tan^3(e+fx)}{3a^2c^2f}$$

[Out] tan(f*x+e)/a^2/c^2/f+1/3*tan(f*x+e)^3/a^2/c^2/f

**Rubi** [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2815, 3852}

$$\frac{\tan^3(e+fx)}{3a^2c^2f}+\frac{\tan(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]

[Out] Tan[e + f*x]/(a^2*c^2*f) + Tan[e + f*x]^3/(3*a^2*c^2*f)

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(e+fx)\right)}{a^2c^2f}\\ &= \frac{\tan(e+fx)}{a^2c^2f}+\frac{\tan^3(e+fx)}{3a^2c^2f}\end{aligned}$$

**Mathematica [A]**
time = 0.04, size = 29, normalized size = 0.76

$$\frac{\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (Tan[e + f*x] + Tan[e + f*x]^3/3)/(a^2*c^2*f)
```

**Maple [A]**
time = 0.32, size = 30, normalized size = 0.79

| method | result | size |
|---|---|---|
| default | $-\frac{\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{a^2c^2f}$ | 30 |
| risch | $\frac{4i\left(3\,\mathrm{e}^{2i(fx+e)}+1\right)}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3\left(\mathrm{e}^{i(fx+e)}+i\right)^3 f\,a^2c^2}$ | 54 |
| norman | $\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3 c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$ | 99 |
| derivativedivides | `error in RationalFunction: argument is not a rational function\` | N/A |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/c^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

**Maxima [A]**
time = 0.30, size = 30, normalized size = 0.79

$$\frac{\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)}{3\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

**Fricas [A]**
time = 0.32, size = 40, normalized size = 1.05

$$\frac{\left(2\,\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(f*x + e)^2 + 1)*sin(f*x + e)/(a^2*c^2*f*cos(f*x + e)^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(32) = 64$.
time = 1.50, size = 286, normalized size = 7.53

$$\begin{cases} -\frac{6\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}c^{2}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^{2}c^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}c^{2}f}+\frac{4\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}c^{2}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^{2}c^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}c^{2}f}-\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}c^{2}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^{2}c^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}c^{2}f} & \text{for } f\neq 0 \\ \frac{x}{(a\sin(e)+a)^{2}(-c\sin(e)+c)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a*
*2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*
c**2*f) + 4*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2
*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c*
*2*f) - 6*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2
*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f)
, Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

**Giac** [A]
time = 0.44, size = 30, normalized size = 0.79

$$\frac{\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)}{3\,a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

**Mupad** [B]
time = 6.84, size = 63, normalized size = 1.66

$$-\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+3\right)}{3\,a^{2}\,c^{2}\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2)*(3*tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 3
))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

# 3.276 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=76

$$\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{4\tan^3(e+fx)}{15a^2c^3f}$$

```
[Out] 1/5*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+4/5*tan(f*x+e)/a^2/c^3/f+4/15*t
an(f*x+e)^3/a^2/c^3/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {2815, 2751, 3852}

$$\frac{4\tan^3(e+fx)}{15a^2c^3f}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]^3/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^2
*c^3*f) + (4*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx &= \frac{\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{a^2c^2} \\
&= \frac{\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))} + \frac{4\int \sec^4(e+fx)\,dx}{5a^2c^3} \\
&= \frac{\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))} - \frac{4\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan\right.}{5a^2c^3f} \\
&= \frac{\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))} + \frac{4\tan(e+fx)}{5a^2c^3f} + \frac{4\tan^3(e+fx}{15a^2c^3f}
\end{aligned}
$$

## Mathematica [A]

time = 0.58, size = 131, normalized size = 1.72

$$
-\frac{-54+128\cos(e+fx)-72\cos(2(e+fx))+192\cos(3(e+fx))-18\cos(4(e+fx))+64\cos(5(e+fx))+18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))}{1920a^2c^3f(-1+\sin(e+fx))^3(1+\sin(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] -1/1920*(-54 + 128*Cos[e + f*x] - 72*Cos[2*(e + f*x)] + 192*Cos[3*(e + f*x)
] - 18*Cos[4*(e + f*x)] + 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2
*(e + f*x)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x
)])/(a^2*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)
```

## Maple [A]

time = 0.31, size = 133, normalized size = 1.75

| method | result |
|---|---|
| risch | $\frac{\frac{32\,e^{3i(fx+e)}}{5}-\frac{32ie^{2i(fx+e)}}{15}+\frac{32\,e^{i(fx+e)}}{15}-\frac{16i}{15}}{\left(e^{i(fx+e)}-i\right)^5\left(e^{i(fx+e)}+i\right)^3 f\,c^3a^2}$ |
| derivativedivides | $\frac{-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots\right.}}{a^2f\,c^3}$ |
| default | $\frac{-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots\right.}}{a^2f\,c^3}$ |
| norman | $\frac{-\frac{10\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2}{5acf}+\frac{2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5acf}+\frac{14\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}+\frac{2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{26\cdots}{}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/c^3/a^2*(-1/5/(tan(1/2*f*x+1/2*e)-1)^5-1/2/(tan(1/2*f*x+1/2*e)-1)^4-5/6
/(tan(1/2*f*x+1/2*e)-1)^3-3/4/(tan(1/2*f*x+1/2*e)-1)^2-11/16/(tan(1/2*f*x+1
/2*e)-1)-1/12/(tan(1/2*f*x+1/2*e)+1)^3+1/8/(tan(1/2*f*x+1/2*e)+1)^2-5/16/(t
an(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(75) = 150$.
time = 0.34, size = 363, normalized size = 4.78

$$\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7} + 3\right)}{15\left(a^2c^3 - \frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{a^2c^3\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/((a^2*c^3
- 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

**Fricas** [A]
time = 0.31, size = 93, normalized size = 1.22

$$-\frac{8\cos(fx+e)^4 - 4\cos(fx+e)^2 + 4\left(2\cos(fx+e)^2 + 1\right)\sin(fx+e) - 1}{15\left(a^2c^3f\cos(fx+e)^3\sin(fx+e) - a^2c^3f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 + 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(66) = 132$.
time = 6.17, size = 1418, normalized size = 18.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30
*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*
a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a
**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*
c**3*f) + 30*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a
**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a*
*2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**
2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c*
*3*f) + 10*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**
2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2
*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*
c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3
*f) - 50*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*
c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c
**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c*
*3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f
) - 26*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c*
*3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**
3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3
*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f)
+ 42*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3
*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*
f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f
*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) -
18*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*ta
n(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan
(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(
e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6/(15
*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a
**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a*
*2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)
**2*(-c*sin(e) + c)**3), True))
```

**Giac [A]**
time = 0.43, size = 133, normalized size = 1.75

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13\right)}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{165\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+650\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-400\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+113}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 + 24*tan(1/2*f*x + 1/2*e) + 13)/(a^2*c
^3*(tan(1/2*f*x + 1/2*e) + 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 - 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 - 400*tan(1/2*f*x + 1/2*e) +
113)/(a^2*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

**Mupad [B]**
time = 7.83, size = 128, normalized size = 1.68

$$-\frac{2\left(15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7-15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+25\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+13\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3-21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3\right)}{15\,a^2\,c^3\,f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^5\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(9*tan(e/2 + (f*x)/2) - 21*tan(e/2 + (f*x)/2)^2 + 13*tan(e/2 + (f*x)/2)
^3 + 25*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^5 - 15*tan(e/2 + (f*x)/
2)^6 + 15*tan(e/2 + (f*x)/2)^7 + 3))/(15*a^2*c^3*f*(tan(e/2 + (f*x)/2) - 1)
^5*(tan(e/2 + (f*x)/2) + 1)^3)
```

# 3.277 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=111

$$\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{4\tan^3(e+fx)}{21a^2c^4f}$$

```
[Out] 1/7*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/7*sec(f*x+e)^3/a^2/f/(c^4-c
^4*sin(f*x+e))+4/7*tan(f*x+e)/a^2/c^4/f+4/21*tan(f*x+e)^3/a^2/c^4/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{4\tan^3(e+fx)}{21a^2c^4f}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^3/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + Sec[e + f*x]^3/(7*a^2
*f*(c^4 - c^4*Sin[e + f*x])) + (4*Tan[e + f*x])/(7*a^2*c^4*f) + (4*Tan[e +
f*x]^3)/(21*a^2*c^4*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{5\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} +$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} -$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} +$$

**Mathematica [A]**
time = 0.64, size = 151, normalized size = 1.36

$$\frac{-210+512\cos(e+fx)-255\cos(2(e+fx))+768\cos(3(e+fx))-30\cos(4(e+fx))+256\cos(5(e+fx))+15\cos(6(e+fx))+120\sin(e+fx)+1088\sin(2(e+fx))+180\sin(3(e+fx))+128\sin(4(e+fx))+60\sin(5(e+fx))-64\sin(6(e+fx))}{5376a^2c^4f(-1+\sin(e+fx))^4(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] (-210 + 512*Cos[e + f*x] - 255*Cos[2*(e + f*x)] + 768*Cos[3*(e + f*x)] - 30
*Cos[4*(e + f*x)] + 256*Cos[5*(e + f*x)] + 15*Cos[6*(e + f*x)] + 120*Sin[e
+ f*x] + 1088*Sin[2*(e + f*x)] + 180*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)
] + 60*Sin[5*(e + f*x)] - 64*Sin[6*(e + f*x)])/(5376*a^2*c^4*f*(-1 + Sin[e
+ f*x])^4*(1 + Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.38, size = 163, normalized size = 1.47

| method | result |
|---|---|
| risch | $-\frac{16i\left(-8i\,e^{3i(fx+e)}+14\,e^{4i(fx+e)}-4i\,e^{i(fx+e)}+3\,e^{2i(fx+e)}-1\right)}{21\left(e^{i(fx+e)}-i\right)^7\left(e^{i(fx+e)}+i\right)^3 f\,c^4a^2}$ |
| derivativedivides | $\frac{-\frac{4}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{55}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{23}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{a^2f\,c^4}$ |
| default | $\frac{-\frac{4}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{55}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{23}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{a^2f\,c^4}$ |

| | |
|---|---|
| norman | $\dfrac{\frac{4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{1}{14acf}-\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{2acf}+\frac{5\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{20\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7acf}-\frac{68\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{21acf}+5(\tan}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2/c^4*(-2/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(tan(1/2*f*x+1/2*e)-1)^6-2/(ta
n(1/2*f*x+1/2*e)-1)^5-5/2/(tan(1/2*f*x+1/2*e)-1)^4-55/24/(tan(1/2*f*x+1/2*e
)-1)^3-23/16/(tan(1/2*f*x+1/2*e)-1)^2-13/16/(tan(1/2*f*x+1/2*e)-1)-1/24/(ta
n(1/2*f*x+1/2*e)+1)^3+1/16/(tan(1/2*f*x+1/2*e)+1)^2-3/16/(tan(1/2*f*x+1/2*e
)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(111) = 222$.
time = 0.33, size = 463, normalized size = 4.17

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{24\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{76\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{28\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{42\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{56\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{28\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{42\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{21\sin(fx+e)^9}{(\cos(fx+e)+1)^9}-6\right)}{21\left(a^2c^4-\frac{4a^2c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{8a^2c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{14a^2c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{14a^2c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{8a^2c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{3a^2c^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{4a^2c^4\sin(fx+e)^9}{(\cos(fx+e)+1)^9}-\frac{a^2c^4\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 24*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 28*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 56*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 42*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 6)/(
(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a
^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

**Fricas** [A]
time = 0.31, size = 122, normalized size = 1.10

$$-\frac{16\cos(fx+e)^4-8\cos(fx+e)^2-\left(8\cos(fx+e)^4-12\cos(fx+e)^2-5\right)\sin(fx+e)-2}{21\left(a^2c^4f\cos(fx+e)^5+2a^2c^4f\cos(fx+e)^3\sin(fx+e)-2a^2c^4f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/21*(16*cos(f*x + e)^4 - 8*cos(f*x + e)^2 - (8*cos(f*x + e)^4 - 12*cos(f*
x + e)^2 - 5)*sin(f*x + e) - 2)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos
(f*x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2213 vs. $2(97) = 194$.

time = 12.96, size = 2213, normalized size = 19.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-42*tan(e/2 + f*x/2)**9/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 8
4*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 16
8*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 2
94*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3 -
63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21*a
**2*c**4*f) + 84*tan(e/2 + f*x/2)**8/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 -
 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 +
168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 +
 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3
- 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21
*a**2*c**4*f) - 56*tan(e/2 + f*x/2)**7/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10
 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8
+ 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6
 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**
3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) -
21*a**2*c**4*f) - 112*tan(e/2 + f*x/2)**6/(21*a**2*c**4*f*tan(e/2 + f*x/2)*
*10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)*
*8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)
**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2
)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2)
 - 21*a**2*c**4*f) + 84*tan(e/2 + f*x/2)**5/(21*a**2*c**4*f*tan(e/2 + f*x/2
)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2
)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/
2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x
/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/
2) - 21*a**2*c**4*f) + 56*tan(e/2 + f*x/2)**4/(21*a**2*c**4*f*tan(e/2 + f*x
/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x
/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*
x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f
*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*
x/2) - 21*a**2*c**4*f) - 152*tan(e/2 + f*x/2)**3/(21*a**2*c**4*f*tan(e/2 +
f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 +
f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 +
 f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2
+ f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 +
 f*x/2) - 21*a**2*c**4*f) + 48*tan(e/2 + f*x/2)**2/(21*a**2*c**4*f*tan(e/2
```

```
+ f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2
+ f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2
 + f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/
2 + f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2
 + f*x/2) - 21*a**2*c**4*f) + 6*tan(e/2 + f*x/2)/(21*a**2*c**4*f*tan(e/2 +
f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 +
f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 +
 f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2
+ f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 +
 f*x/2) - 21*a**2*c**4*f) - 12/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 84*a*
*2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 168*a*
*2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 294*a
**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 63*a
**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21*a**2*
c**4*f), Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**4), True))
```

**Giac [A]**
time = 0.47, size = 161, normalized size = 1.45

$$-\frac{\frac{7\left(9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8\right)}{a^2c^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{273\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-1155\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+2450\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2870\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2037\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-791\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+152}{a^2c^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^7}}{168\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*f*x + 1/2*e)^2 + 15*tan(1/2*f*x + 1/2*e) + 8)/(a^2*c^4
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (273*tan(1/2*f*x + 1/2*e)^6 - 1155*tan(1/2
*f*x + 1/2*e)^5 + 2450*tan(1/2*f*x + 1/2*e)^4 - 2870*tan(1/2*f*x + 1/2*e)^3
 + 2037*tan(1/2*f*x + 1/2*e)^2 - 791*tan(1/2*f*x + 1/2*e) + 152)/(a^2*c^4*(
tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

**Mupad [B]**
time = 6.98, size = 119, normalized size = 1.07

$$-\frac{\frac{\sin(e+f\,x)}{3}+\frac{4\,\cos(2\,e+2\,f\,x)}{21}+\frac{2\,\cos(4\,e+4\,f\,x)}{21}+\frac{\sin(3\,e+3\,f\,x)}{14}-\frac{\sin(5\,e+5\,f\,x)}{42}}{a^2\,c^4\,f\,\left(\frac{\cos(5\,e+5\,f\,x)}{16}-\frac{3\,\cos(3\,e+3\,f\,x)}{16}-\frac{7\,\cos(e+f\,x)}{8}+\frac{\sin(2\,e+2\,f\,x)}{2}+\frac{\sin(4\,e+4\,f\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -(sin(e + f*x)/3 + (4*cos(2*e + 2*f*x))/21 + (2*cos(4*e + 4*f*x))/21 + sin(
3*e + 3*f*x)/14 - sin(5*e + 5*f*x)/42)/(a^2*c^4*f*(cos(5*e + 5*f*x)/16 - (3
*cos(3*e + 3*f*x))/16 - (7*cos(e + f*x))/8 + sin(2*e + 2*f*x)/2 + sin(4*e +
 4*f*x)/4))
```

# 3.278 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=144

$$\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{2\sec^3(e+fx)}{21a^2f\,(c^5-c^5\sin(e+fx))}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{8\tan^3(e+fx)}{63a^2c^5f}$$

```
[Out] 1/9*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+2/21*sec(f*x+e)^3/a^2/c^3/f/(
c-c*sin(f*x+e))^2+2/21*sec(f*x+e)^3/a^2/f/(c^5-c^5*sin(f*x+e))+8/21*tan(f*x
+e)/a^2/c^5/f+8/63*tan(f*x+e)^3/a^2/c^5/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{8\tan^3(e+fx)}{63a^2c^5f}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{2\sec^3(e+fx)}{21a^2f\,(c^5-c^5\sin(e+fx))}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] Sec[e + f*x]^3/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + (2*Sec[e + f*x]^3)/(2
1*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + (2*Sec[e + f*x]^3)/(21*a^2*f*(c^5 - c
^5*Sin[e + f*x])) + (8*Tan[e + f*x])/(21*a^2*c^5*f) + (8*Tan[e + f*x]^3)/(6
3*a^2*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx &= \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2} \\
&= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3} \\
&= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+ \\
&= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+ \\
&= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+ \\
&= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+
\end{aligned}
$$

## Mathematica [A]

time = 0.78, size = 193, normalized size = 1.34

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-5580\cos(e+fx)+13824\cos(2(e+fx))-310\cos(3(e+fx))+6144\cos(4(e+fx))+930\cos(5(e+fx))-512\cos(6(e+fx))+18432\sin(e+fx)+4185\sin(2(e+fx))+1024\sin(3(e+fx))+1860\sin(4(e+fx))-3072\sin(5(e+fx))-155\sin(6(e+fx))\right)}{64512f(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-5580*Cos[e + f*x] + 13824*Cos[2*(e + f*x)] - 310*Cos[3*(e + f*x)] + 614
4*Cos[4*(e + f*x)] + 930*Cos[5*(e + f*x)] - 512*Cos[6*(e + f*x)] + 18432*Si
n[e + f*x] + 4185*Sin[2*(e + f*x)] + 1024*Sin[3*(e + f*x)] + 1860*Sin[4*(e
+ f*x)] - 3072*Sin[5*(e + f*x)] - 155*Sin[6*(e + f*x)]))/(64512*f*(a + a*Si
n[e + f*x])^2*(c - c*Sin[e + f*x])^5)
```

## Maple [A]

time = 0.44, size = 193, normalized size = 1.34

| method | result |
|---|---|
| risch | $-\frac{32\left(-6\,\mathrm{e}^{i(fx+e)}+2\,\mathrm{e}^{3i(fx+e)}+36\,\mathrm{e}^{5i(fx+e)}+i-27i\mathrm{e}^{4i(fx+e)}-12i\mathrm{e}^{2i(fx+e)}\right)}{63\left(\mathrm{e}^{i(fx+e)}-i\right)^{9}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}f\,c^{5}a^{2}}$ |

| | |
|---|---|
| derivativedivides | $\frac{-\frac{8}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{68}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{46}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{35}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{59}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{}$ |
| default | $\frac{-\frac{8}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{68}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{46}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{35}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{59}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{}$ |
| norman | $\frac{-\frac{28\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{38}{63acf}+\frac{6\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{12\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{26\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2\left(\tan^8\left(\frac{fx}{2}+\right.\right.}{acf}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3c^4\left(\tan\left(\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2/c^5*(-4/9/(tan(1/2*f*x+1/2*e)-1)^9-2/(tan(1/2*f*x+1/2*e)-1)^8-34/7/
(tan(1/2*f*x+1/2*e)-1)^7-23/3/(tan(1/2*f*x+1/2*e)-1)^6-35/4/(tan(1/2*f*x+1/
2*e)-1)^5-59/8/(tan(1/2*f*x+1/2*e)-1)^4-19/4/(tan(1/2*f*x+1/2*e)-1)^3-9/4/(
tan(1/2*f*x+1/2*e)-1)^2-57/64/(tan(1/2*f*x+1/2*e)-1)-1/48/(tan(1/2*f*x+1/2*
e)+1)^3+1/32/(tan(1/2*f*x+1/2*e)+1)^2-7/64/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(145) = 290$.

time = 0.34, size = 563, normalized size = 3.91

$$-\frac{2\left(\frac{51\sin(fx+e)}{\cos(fx+e)+1}-\frac{39\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{235\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{450\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{306\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{294\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{378\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{273\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{189\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{63\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-19\right)}{63\left(a^2c^5-\frac{6a^2c^5\sin(fx+e)}{\cos(fx+e)+1}+\frac{12a^2c^5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2a^2c^5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{27a^2c^5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{36a^2c^5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{36a^2c^5\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{27a^2c^5\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{2a^2c^5\sin(fx+e)^9}{(\cos(fx+e)+1)^9}-\frac{12a^2c^5\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{6a^2c^5\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^2c^5\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/63*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f*x
+ e) + 1)^11 - 19)/((a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36*a
^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2*c
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^5*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*f)
```

**Fricas** [A]

time = 0.32, size = 154, normalized size = 1.07

$$\frac{16\cos(fx+e)^6-72\cos(fx+e)^4+30\cos(fx+e)^2+2\left(24\cos(fx+e)^4-20\cos(fx+e)^2-7\right)\sin(fx+e)+7}{63\left(3a^2c^5f\cos(fx+e)^5-4a^2c^5f\cos(fx+e)^3-\left(a^2c^5f\cos(fx+e)^5-4a^2c^5f\cos(fx+e)^3\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(16*cos(f*x + e)^6 - 72*cos(f*x + e)^4 + 30*cos(f*x + e)^2 + 2*(24*cos
(f*x + e)^4 - 20*cos(f*x + e)^2 - 7)*sin(f*x + e) + 7)/(3*a^2*c^5*f*cos(f*x
 + e)^5 - 4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^
5*f*cos(f*x + e)^3)*sin(f*x + e))
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(131) = 262$.

time = 28.05, size = 3186, normalized size = 22.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 -
 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**1
0 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)
**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x
/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f
*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 +
f*x/2) - 63*a**2*c**5*f) + 378*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan(e/2
 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(
e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*t
an(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*
f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**
5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c*
*5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*tan(e/2 + f*x/2)**9/(63*a**2*
c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a*
*2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4
+ 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2
 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 126*tan(e/2 + f*x/2
)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2
)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*
x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 +
 f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/
2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e
/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 756*t
an(e/2 + f*x/2)**7/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*t
an(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*
f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c*
```

```
*5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**
2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c
**5*f) - 588*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378
*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 -
126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8
+ 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2
)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/
2) - 63*a**2*c**5*f) - 612*tan(e/2 + f*x/2)**5/(63*a**2*c**5*f*tan(e/2 + f*
x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 +
 f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/
2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan
(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*t
an(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*
tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 900*tan(e/2 + f*x/2)**4/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 470*tan(e/2 + f*x/2)**3/
(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11
 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)*
*9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/
2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f
*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 +
f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 78*tan(e/2
 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2
 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(
e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*
f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5
*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f)
 + 102*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**
5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*
c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a*
*2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**...
```

**Giac** [A]

time = 0.48, size = 189, normalized size = 1.31

$$-\frac{\frac{21\left(21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+36\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+19\right)}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}+\frac{3591\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-19656\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+56196\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-95760\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+107730\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-79464\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+38484\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-10944\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1615}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{9}}}{2016f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*f*x + 1/2*e)^2 + 36*tan(1/2*f*x + 1/2*e) + 19)/(a^2
*c^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*tan(1/2*f*x + 1/2*e)^8 - 19656*t
an(1/2*f*x + 1/2*e)^7 + 56196*tan(1/2*f*x + 1/2*e)^6 - 95760*tan(1/2*f*x +
1/2*e)^5 + 107730*tan(1/2*f*x + 1/2*e)^4 - 79464*tan(1/2*f*x + 1/2*e)^3 + 3
8484*tan(1/2*f*x + 1/2*e)^2 - 10944*tan(1/2*f*x + 1/2*e) + 1615)/(a^2*c^5*(
tan(1/2*f*x + 1/2*e) - 1)^9))/f
```

**Mupad** [B]
time = 9.29, size = 180, normalized size = 1.25

$$-\frac{2\left(63\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}-189\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+273\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}+63\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}-378\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}+294\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+306\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}-450\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+235\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+39\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-51\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+19\right)}{63\,a^2\,c^5\,f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^9\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)
```

```
[Out] -(2*(39*tan(e/2 + (f*x)/2)^2 - 51*tan(e/2 + (f*x)/2) + 235*tan(e/2 + (f*x)/
2)^3 - 450*tan(e/2 + (f*x)/2)^4 + 306*tan(e/2 + (f*x)/2)^5 + 294*tan(e/2 +
(f*x)/2)^6 - 378*tan(e/2 + (f*x)/2)^7 + 63*tan(e/2 + (f*x)/2)^8 + 273*tan(e
/2 + (f*x)/2)^9 - 189*tan(e/2 + (f*x)/2)^10 + 63*tan(e/2 + (f*x)/2)^11 + 19
))/(63*a^2*c^5*f*(tan(e/2 + (f*x)/2) - 1)^9*(tan(e/2 + (f*x)/2) + 1)^3)
```

# 3.279 $\int \frac{(c - c\sin(e+fx))^5}{(a + a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=161

$$-\frac{63c^5x}{2a^3}-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{}{2f\,(}$$

```
[Out] -63/2*c^5*x/a^3-63/2*c^5*cos(f*x+e)/a^3/f-2/5*a^4*c^5*cos(f*x+e)^9/f/(a+a*s
in(f*x+e))^7+6/5*a^2*c^5*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5-42/5*c^5*cos(f*x
+e)^5/f/(a+a*sin(f*x+e))^3-21/2*c^5*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

**Rubi** [A]
time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2758, 2761, 8}

$$-\frac{2a^4c^5\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{21c^5\cos^3(e+fx)}{2f\,(a^3\sin(e+fx)+a^3)}-\frac{63c^5x}{2a^3}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{42c^5\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-63*c^5*x)/(2*a^3) - (63*c^5*Cos[e + f*x])/(2*a^3*f) - (2*a^4*c^5*Cos[e +
f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (6*a^2*c^5*Cos[e + f*x]^7)/(5*f*(a +
 a*Sin[e + f*x])^5) - (42*c^5*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^3)
- (21*c^5*Cos[e + f*x]^3)/(2*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^8}\,dx \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}-\frac{1}{5}\left(9a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}+\frac{1}{5}\left(21ac^5\right)\int \frac{\cos^6(e+}{(a+a\sin(e+} \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3}- \\
&= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3}- \\
&= -\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{42}{5f(a} \\
&= -\frac{63c^5x}{2a^3}-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5}
\end{aligned}
$$

**Mathematica [A]**
time = 0.57, size = 303, normalized size = 1.88

$$
\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(256\sin\left(\frac{1}{2}(e+fx)\right)-128\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-896\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+448\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2304\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-630(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-160\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sin(2(e+fx))\right)}{20f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}(a+a\sin(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 896*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 448*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3 + 2304*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 - 630*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 160*Cos[e + f*x]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5*Sin[2*(e + f*x)]))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10
*(a + a*Sin[e + f*x])^3)
```

**Maple** [A]
time = 0.44, size = 156, normalized size = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{2c^5\left(-\frac{\frac{\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+8\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+8}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{63\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{128}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots}\cdots\right)}{f\,a^3}$ |
| default | $\frac{2c^5\left(-\frac{\frac{\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+8\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+8}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{63\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{128}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots}\cdots\right)}{f\,a^3}$ |
| risch | $-\frac{63c^5x}{2a^3}-\frac{ic^5e^{2i(fx+e)}}{8f\,a^3}-\frac{4c^5e^{i(fx+e)}}{a^3f}-\frac{4c^5e^{-i(fx+e)}}{a^3f}+\frac{ic^5e^{-2i(fx+e)}}{8f\,a^3}-\frac{32\left(-105c^5e^{2i(fx+e)}+75ic^5e^{3i(fx+\cdots}\cdots\right)}{5f\,a^3(e\cdots}$ |
| norman | $\frac{-\frac{8505c^5x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2a}-\frac{6363c^5x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2a}-\frac{63c^5x}{2a}-\frac{2036c^5\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2516c^5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{4412c^5\left(\tan\cdots\right)}{a\cdots}\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^5/a^3*(-(1/2*tan(1/2*f*x+1/2*e)^3+8*tan(1/2*f*x+1/2*e)^2-1/2*tan(1/2*
f*x+1/2*e)+8)/(1+tan(1/2*f*x+1/2*e)^2)^2-63/2*arctan(tan(1/2*f*x+1/2*e))-12
8/5/(tan(1/2*f*x+1/2*e)+1)^5+64/(tan(1/2*f*x+1/2*e)+1)^4-32/(tan(1/2*f*x+1/
2*e)+1)^3-16/(tan(1/2*f*x+1/2*e)+1)^2-32/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(158) = 316$.
time = 0.57, size = 1628, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
```

```
 + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2
275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*
a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 30*c^5*((105*s
in(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) + 20*c^5*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^3) + 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5) + 40*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5))/f
```

**Fricas** [A]
time = 0.33, size = 301, normalized size = 1.87

$$-\frac{5\,c^5\cos\left(fx+e\right)^5+70\,c^5\cos\left(fx+e\right)^4-1260\,c^5fx-64\,c^5+7\,(45\,c^5fx+113\,c^5)\cos\left(fx+e\right)^3+(945\,c^5fx-502\,c^5)\cos\left(fx+e\right)^2-2\,(315\,c^5fx+646\,c^5)\cos\left(fx+e\right)-\left(5\,c^5\cos\left(fx+e\right)^4-65\,c^5\cos\left(fx+e\right)^3+1260\,c^5fx-64\,c^5-3\,(105\,c^5fx-242\,c^5)\cos\left(fx+e\right)^2+2\,(315\,c^5fx+614\,c^5)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{10\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*c^5*cos(f*x + e)^5 + 70*c^5*cos(f*x + e)^4 - 1260*c^5*f*x - 64*c^5
 + 7*(45*c^5*f*x + 113*c^5)*cos(f*x + e)^3 + (945*c^5*f*x - 502*c^5)*cos(f*
x + e)^2 - 2*(315*c^5*f*x + 646*c^5)*cos(f*x + e) - (5*c^5*cos(f*x + e)^4 -
 65*c^5*cos(f*x + e)^3 + 1260*c^5*f*x - 64*c^5 - 3*(105*c^5*f*x - 242*c^5)*
cos(f*x + e)^2 + 2*(315*c^5*f*x + 614*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3
*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f
 + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3643 vs. $2(153) = 306$.
time = 29.23, size = 3643, normalized size = 22.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-315*c**5*f*x*tan(e/2 + f*x/2)**9/(10*a**3*f*tan(e/2 + f*x/2)**9
 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**
3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e
/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/
2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 1575*c**5*f*x*tan(e/2 + f
*x/2)**8/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 1
20*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f
*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2
+ f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) +
 10*a**3*f) - 3780*c**5*f*x*tan(e/2 + f*x/2)**7/(10*a**3*f*tan(e/2 + f*x/2)
**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*
a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*ta
n(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f
*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 6300*c**5*f*x*tan(e/2
+ f*x/2)**6/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8
+ 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**
3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e
/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2
) + 10*a**3*f) - 8190*c**5*f*x*tan(e/2 + f*x/2)**5/(10*a**3*f*tan(e/2 + f*x
/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 2
00*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f
*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2
+ f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 8190*c**5*f*x*tan(e
/2 + f*x/2)**4/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)*
*8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*
a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*ta
n(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*
```

```
x/2) + 10*a**3*f) - 6300*c**5*f*x*tan(e/2 + f*x/2)**3/(10*a**3*f*tan(e/2 +
f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7
+ 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**
3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e
/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 3780*c**5*f*x*ta
n(e/2 + f*x/2)**2/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/
2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 2
60*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f
*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 +
 f*x/2) + 10*a**3*f) - 1575*c**5*f*x*tan(e/2 + f*x/2)/(10*a**3*f*tan(e/2 +
f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7
+ 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**
3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e
/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 315*c**5*f*x/(10
*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*ta
n(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f
*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3
+ 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f)
- 650*c**5*tan(e/2 + f*x/2)**8/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*t
an(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 +
f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4
 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**
3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 3090*c**5*tan(e/2 + f*x/2)**7/(10*a**3*
f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2
+ f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)*
*5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*
a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 7610
*c**5*tan(e/2 + f*x/2)**6/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/
2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2
)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 20
0*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*t
an(e/2 + f*x/2) + 10*a**3*f) - 11090*c**5*tan(e/2 + f*x/2)**5/(10*a**3*f*ta
n(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*
x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 +
 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3
*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 14702*c*
*5*tan(e/2 + f*x/2)**4/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 +
 f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**
6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a
**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 ...
```

**Giac [A]**

time = 0.48, size = 186, normalized size = 1.16

$$-\frac{\frac{315\,(fx+e)c^5}{a^3}+\frac{10\left(c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+16\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+16\,c^5\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}+\frac{64\left(10\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+85\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+55\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,c^5\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/10*(315*(f*x + e)*c^5/a^3 + 10*(c^5*tan(1/2*f*x + 1/2*e)^3 + 16*c^5*tan(
1/2*f*x + 1/2*e)^2 - c^5*tan(1/2*f*x + 1/2*e) + 16*c^5)/((tan(1/2*f*x + 1/2
*e)^2 + 1)^2*a^3) + 64*(10*c^5*tan(1/2*f*x + 1/2*e)^4 + 45*c^5*tan(1/2*f*x
+ 1/2*e)^3 + 85*c^5*tan(1/2*f*x + 1/2*e)^2 + 55*c^5*tan(1/2*f*x + 1/2*e) +
13*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 11.14, size = 364, normalized size = 2.26

$$\frac{\frac{63\,c^5\,(e+f\,x)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{315\,c^5\,(e+f\,x)}{2}-\frac{c^5\,(1575\,e+1575\,f\,x+4310)}{10}\right)-\frac{c^5\,(315\,e+315\,f\,x+992)}{10}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{315\,c^5\,(e+f\,x)}{2}-\frac{c^5\,(1575\,e+1575\,f\,x+650)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(378\,c^5\,(e+f\,x)-\frac{c^5\,(3780\,e+3780\,f\,x+3090)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(378\,c^5\,(e+f\,x)-\frac{c^5\,(3780\,e+3780\,f\,x+8814)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(630\,c^5\,(e+f\,x)-\frac{c^5\,(6300\,e+6300\,f\,x+7610)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(630\,c^5\,(e+f\,x)-\frac{c^5\,(6300\,e+6300\,f\,x+12230)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(819\,c^5\,(e+f\,x)-\frac{c^5\,(8190\,e+8190\,f\,x+11090)}{10}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(819\,c^5\,(e+f\,x)-\frac{c^5\,(8190\,e+8190\,f\,x+14702)}{10}\right)}{a^3\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^5\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^2}-\frac{63\,c^5\,x}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^5/(a + a*sin(e + f*x))^3,x)
```

```
[Out] ((63*c^5*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((315*c^5*(e + f*x))/2 - (c^5*(1
575*e + 1575*f*x + 4310))/10) - (c^5*(315*e + 315*f*x + 992))/10 + tan(e/2
+ (f*x)/2)^8*((315*c^5*(e + f*x))/2 - (c^5*(1575*e + 1575*f*x + 650))/10) +
 tan(e/2 + (f*x)/2)^7*(378*c^5*(e + f*x) - (c^5*(3780*e + 3780*f*x + 3090))
/10) + tan(e/2 + (f*x)/2)^2*(378*c^5*(e + f*x) - (c^5*(3780*e + 3780*f*x +
8814))/10) + tan(e/2 + (f*x)/2)^6*(630*c^5*(e + f*x) - (c^5*(6300*e + 6300*
f*x + 7610))/10) + tan(e/2 + (f*x)/2)^3*(630*c^5*(e + f*x) - (c^5*(6300*e +
 6300*f*x + 12230))/10) + tan(e/2 + (f*x)/2)^5*(819*c^5*(e + f*x) - (c^5*(8
190*e + 8190*f*x + 11090))/10) + tan(e/2 + (f*x)/2)^4*(819*c^5*(e + f*x) -
(c^5*(8190*e + 8190*f*x + 14702))/10))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5*(t
an(e/2 + (f*x)/2)^2 + 1)^2) - (63*c^5*x)/(2*a^3)
```

# 3.280 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=124

$$-\frac{7c^4x}{a^3}-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{14c^4\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2}$$

```
[Out] -7*c^4*x/a^3-7*c^4*cos(f*x+e)/a^3/f-2/5*a^3*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x
+e))^6+14/15*a*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^4-14/3*c^4*cos(f*x+e)^3/
a/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2759, 2761, 8}

$$-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{7c^4x}{a^3}+\frac{14ac^4\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}-\frac{14c^4\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*c^4*x)/a^3 - (7*c^4*Cos[e + f*x])/(a^3*f) - (2*a^3*c^4*Cos[e + f*x]^7)/
(5*f*(a + a*Sin[e + f*x])^6) + (14*a*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e
 + f*x])^4) - (14*c^4*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^7}\,dx\\
&= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} + \frac{1}{3}\left(7c^4\right)\int \frac{\cos^4(e+}{(a+a\sin(e}\\
&= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\cos^3(e+fx)}{3af(a+a\sin(e+fx)}\\
&= -\frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{1}{3af}\\
&= -\frac{7c^4x}{a^3} - \frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))}\end{aligned}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(124) = 248.
time = 0.41, size = 270, normalized size = 2.18

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(96\sin\left(\frac{1}{2}(e+fx)\right)-48\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-256\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+128\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+464\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-105(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-15\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)(c-c\sin(e+fx))^4}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(a+a\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(96*Sin[(e + f*x)/2] - 48*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 256*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 + 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 464*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 105*(e + f*x)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5 - 15*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^5)*(c - c*Sin[e + f*x])^4)/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^8*(a + a*Sin[e + f*x])^3)
```

**Maple** [**A**]
time = 0.42, size = 102, normalized size = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{2c^4\left(-\frac{64}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{64}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-7\arctan\left(\tan\left(\frac{fx}{2}\right.\right.\right.}{f\,a^3}$ |
| default | $\frac{2c^4\left(-\frac{64}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{64}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{1}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-7\arctan\left(\tan\left(\frac{fx}{2}\right.\right.\right.}{f\,a^3}$ |
| risch | $-\frac{7c^4x}{a^3}-\frac{c^4\mathrm{e}^{i(fx+e)}}{2a^3f}-\frac{c^4\mathrm{e}^{-i(fx+e)}}{2a^3f}-\frac{16\left(120ic^4\mathrm{e}^{3i(fx+e)}+45c^4\mathrm{e}^{4i(fx+e)}-100ic^4\mathrm{e}^{i(fx+e)}-170c^4\mathrm{e}^{2i(fx+e)}+29c^4\right)}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+i\right)^5}$ |
| norman | $-\frac{334c^4}{15af}-\frac{7c^4x}{a}-\frac{35c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{35c^4x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{7c^4x\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{66c^4\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{16c^4\left(\tan^{12}\left(\frac{fx}{2}+\right.\right.}{af}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^4/a^3*(-64/5/(tan(1/2*f*x+1/2*e)+1)^5+32/(tan(1/2*f*x+1/2*e)+1)^4-64/
3/(tan(1/2*f*x+1/2*e)+1)^3-8/(tan(1/2*f*x+1/2*e)+1)-1/(1+tan(1/2*f*x+1/2*e)
^2)-7*arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(125) = 250$.
time = 0.58, size = 1192, normalized size = 9.61

$$-\frac{2\left(3c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{189\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{200\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{160\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{75\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+24}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{11a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{11a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{5a^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+4c^4\left(\frac{\frac{95\sin(fx+e)}{\cos(fx+e)+1}+\frac{145\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{75\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+22}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^4\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{12c^4\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{12c^4\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + 4*c^4*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(
```

```
f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(
f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^4*(5*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 12*c^4*(5*sin(f*x +
e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(125) = 250$.
time = 0.33, size = 270, normalized size = 2.18

$$-\frac{15\,c^4\cos\left(fx+e\right)^4-420\,c^4fx-48\,c^4+\left(105\,c^4fx+277\,c^4\right)\cos\left(fx+e\right)^3+\left(315\,c^4fx-134\,c^4\right)\cos\left(fx+e\right)^2-6\left(35\,c^4fx+74\,c^4\right)\cos\left(fx+e\right)+\left(15\,c^4\cos\left(fx+e\right)^3-420\,c^4fx+48\,c^4+\left(105\,c^4fx-262\,c^4\right)\cos\left(fx+e\right)^2-6\left(35\,c^4fx+66\,c^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^4*cos(f*x + e)^4 - 420*c^4*f*x - 48*c^4 + (105*c^4*f*x + 277*c^
4)*cos(f*x + e)^3 + (315*c^4*f*x - 134*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x
+ 74*c^4)*cos(f*x + e) + (15*c^4*cos(f*x + e)^3 - 420*c^4*f*x + 48*c^4 + (1
05*c^4*f*x - 262*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x + 66*c^4)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. $2(119) = 238$.
time = 17.26, size = 2314, normalized size = 18.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-105*c**4*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7
 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**
3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 525*c**4*f*x*tan
(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
```

)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 1155*c**4*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 1575*c**4*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a
**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan
(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1575*c**4*f*x*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3
*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
+ f*x/2) + 15*a**3*f) - 1155*c**4*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e
/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2
)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16
5*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52
5*c**4*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 105*c**4*f*x/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
+ 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 240*c*
*4*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 +
f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**
4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 990*c**4*tan(e/2 + f*x/2)**5/(15*a**3*
f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
+ f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 2470*c**4*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 2540*c**4*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) - 2684*c**4*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1430*c**4*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**
6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a
**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(

```
e/2 + f*x/2) + 15*a**3*f) - 334*c**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a*
*3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(
e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(-c*sin(e)
+ c)**4/(a*sin(e) + a)**3, True))
```

**Giac [A]**
time = 0.45, size = 135, normalized size = 1.09

$$-\frac{\frac{105\,(fx+e)c^4}{a^3}+\frac{30\,c^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}+\frac{16\,\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+60\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+130\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+80\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,c^4\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(105*(f*x + e)*c^4/a^3 + 30*c^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) +
16*(15*c^4*tan(1/2*f*x + 1/2*e)^4 + 60*c^4*tan(1/2*f*x + 1/2*e)^3 + 130*c^4
*tan(1/2*f*x + 1/2*e)^2 + 80*c^4*tan(1/2*f*x + 1/2*e) + 19*c^4)/(a^3*(tan(1
/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 10.90, size = 290, normalized size = 2.34

$$\frac{7\,c^4\,(e+f\,x)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(35\,c^4\,(e+f\,x)-\frac{c^4\,(525\,e+525\,f\,x+1430)}{15}\right)-\frac{c^4\,(105\,e+105\,f\,x+334)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(35\,c^4\,(e+f\,x)-\frac{c^4\,(525\,e+525\,f\,x+240)}{15}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(77\,c^4\,(e+f\,x)-\frac{c^4\,(1155\,e+1155\,f\,x+990)}{15}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(77\,c^4\,(e+f\,x)-\frac{c^4\,(1155\,e+1155\,f\,x+2684)}{15}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(105\,c^4\,(e+f\,x)-\frac{c^4\,(1575\,e+1575\,f\,x+2470)}{15}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(105\,c^4\,(e+f\,x)-\frac{c^4\,(1575\,e+1575\,f\,x+2540)}{15}\right)}{a^3\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^5\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}-\frac{7\,c^4\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (7*c^4*(e + f*x) + tan(e/2 + (f*x)/2)*(35*c^4*(e + f*x) - (c^4*(525*e + 525
*f*x + 1430))/15) - (c^4*(105*e + 105*f*x + 334))/15 + tan(e/2 + (f*x)/2)^6
*(35*c^4*(e + f*x) - (c^4*(525*e + 525*f*x + 240))/15) + tan(e/2 + (f*x)/2)
^5*(77*c^4*(e + f*x) - (c^4*(1155*e + 1155*f*x + 990))/15) + tan(e/2 + (f*x
)/2)^2*(77*c^4*(e + f*x) - (c^4*(1155*e + 1155*f*x + 2684))/15) + tan(e/2 +
 (f*x)/2)^4*(105*c^4*(e + f*x) - (c^4*(1575*e + 1575*f*x + 2470))/15) + tan
(e/2 + (f*x)/2)^3*(105*c^4*(e + f*x) - (c^4*(1575*e + 1575*f*x + 2540))/15)
)/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5*(tan(e/2 + (f*x)/2)^2 + 1)) - (7*c^4*x)
/a^3
```

# 3.281 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=103

$$-\frac{c^3x}{a^3}-\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}+\frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}-\frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -c^3*x/a^3-2/5*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5+2/3*c^3*cos(f*x+e)
^3/f/(a+a*sin(f*x+e))^3-2*c^3*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

**Rubi** [A]
time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2759, 8}

$$-\frac{2c^3\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}-\frac{c^3x}{a^3}-\frac{2a^2c^3\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}+\frac{2c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c^3*x)/a^3) - (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5) +
(2*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) - (2*c^3*Cos[e + f*x])/
(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^6}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\left(ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}+\frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{c^3\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{a}\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}+\frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}-\frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}\\ &= -\frac{c^3x}{a^3}-\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}+\frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}-\frac{2c^3\cos(e+f}{f\left(a^3+a^3\sin(e+\right.}\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(103) = 206.
time = 0.29, size = 239, normalized size = 2.32

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\frac{1}{2}(e+fx)\right)-24\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-88\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+44\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+92\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-15(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)(c-c\sin(e+fx))^3}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(a+a\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*Sin[(e + f*x)/2] - 24*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 88*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 + 44*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 92*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^5)*(c - c*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.38, size = 100, normalized size = 0.97

| method | result |
|---|---|
| risch | $-\frac{c^3x}{a^3}-\frac{4\left(90ic^3e^{3i(fx+e)}+45c^3e^{4i(fx+e)}-70ic^3e^{i(fx+e)}-140c^3e^{2i(fx+e)}+23c^3\right)}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$ |
| derivativedivides | $\frac{2c^3\left(-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{40}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{}{\tan(}\right.}{f\,a^3}$ |
| default | $\frac{2c^3\left(-\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{40}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{}{\tan(}\right.}{f\,a^3}$ |

| | |
|---|---|
| norman | $-\frac{4c^3\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8c^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{48c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{c^3x}{a}-\frac{52c^3}{15af}-\frac{5c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{13c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\ldots$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^3/a^3*(-arctan(tan(1/2*f*x+1/2*e))-32/5/(tan(1/2*f*x+1/2*e)+1)^5+16/(
tan(1/2*f*x+1/2*e)+1)^4-40/3/(tan(1/2*f*x+1/2*e)+1)^3+4/(tan(1/2*f*x+1/2*e)
+1)^2-2/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(105) = 210$.
time = 0.55, size = 849, normalized size = 8.24

$$-\frac{2\left(c^3\left(\frac{\frac{95\sin(fx+e)}{\cos(fx+e)+1}+\frac{145\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{75\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+22}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^3\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{6c^3\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{9c^3\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
+ c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) + 6*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
 + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5) - 9*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(105) = 210$.
time = 0.32, size = 245, normalized size = 2.38

$$\frac{60\,c^3fx-(15\,c^3fx+46\,c^3)\cos(fx+e)^3+24\,c^3-(45\,c^3fx-2\,c^3)\cos(fx+e)^2+6\,(5\,c^3fx+12\,c^3)\cos(fx+e)+\left(60\,c^3fx-24\,c^3-(15\,c^3fx-46\,c^3)\cos(fx+e)^2+6\,(5\,c^3fx+8\,c^3)\cos(fx+e)\right)\sin(fx+e)}{15\left(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f+\left(a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*c^3*f*x - (15*c^3*f*x + 46*c^3)*cos(f*x + e)^3 + 24*c^3 - (45*c^3*
f*x - 2*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 12*c^3)*cos(f*x + e) + (60*c^3
*f*x - 24*c^3 - (15*c^3*f*x - 46*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 8*c^3
)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^
2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(95) = 190$.
time = 9.38, size = 1284, normalized size = 12.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5
+ 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 150*c**3*f*x*tan(e/2 + f*x/2
)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a
**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(
e/2 + f*x/2) + 15*a**3*f) - 150*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x
/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
**3*f) - 75*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 15*c**3*f*x/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 60*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2
)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120
*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 400*c**3*tan(e/2 + f*x/2)*
*2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 200*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
```

```
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
52*c**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*sin(e) +
a)**3, True))
```

**Giac** [A]
time = 0.50, size = 111, normalized size = 1.08

$$-\frac{\frac{15\,(fx+e)c^3}{a^3}+\frac{4\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+50\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,c^3\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*c^3/a^3 + 4*(15*c^3*tan(1/2*f*x + 1/2*e)^4 + 30*c^3*tan
(1/2*f*x + 1/2*e)^3 + 100*c^3*tan(1/2*f*x + 1/2*e)^2 + 50*c^3*tan(1/2*f*x +
 1/2*e) + 13*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad** [B]
time = 8.94, size = 200, normalized size = 1.94

$$\frac{c^3\,\left(e+f\,x\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,c^3\,\left(e+f\,x\right)-\frac{c^3\,\left(75\,e+75\,f\,x+200\right)}{15}\right)-\frac{c^3\,\left(15\,e+15\,f\,x+52\right)}{15}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(5\,c^3\,\left(e+f\,x\right)-\frac{c^3\,\left(75\,e+75\,f\,x+60\right)}{15}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(10\,c^3\,\left(e+f\,x\right)-\frac{c^3\,\left(150\,e+150\,f\,x+120\right)}{15}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(10\,c^3\,\left(e+f\,x\right)-\frac{c^3\,\left(150\,e+150\,f\,x+400\right)}{15}\right)}{a^3\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^5}-\frac{c^3\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^3*(e + f*x) + tan(e/2 + (f*x)/2)*(5*c^3*(e + f*x) - (c^3*(75*e + 75*f*x
+ 200))/15) - (c^3*(15*e + 15*f*x + 52))/15 + tan(e/2 + (f*x)/2)^4*(5*c^3*(
e + f*x) - (c^3*(75*e + 75*f*x + 60))/15) + tan(e/2 + (f*x)/2)^3*(10*c^3*(e
 + f*x) - (c^3*(150*e + 150*f*x + 120))/15) + tan(e/2 + (f*x)/2)^2*(10*c^3*
(e + f*x) - (c^3*(150*e + 150*f*x + 400))/15))/(a^3*f*(tan(e/2 + (f*x)/2) +
 1)^5) - (c^3*x)/a^3
```

# 3.282 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3} dx$

**Optimal**. Leaf size=33

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}$$

```
[Out] -1/5*a^2*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5
```

**Rubi** [A]

time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2815, 2750}

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*(a^2*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\ &= -\frac{a^2c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.
time = 0.27, size = 81, normalized size = 2.45

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(10\sin\left(\frac{1}{2}(e+fx)\right)+5\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)\right)}{10a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(10*Sin[(e + f*x)/2] + 5*Sin[(3*
(e + f*x))/2] - Sin[(5*(e + f*x))/2]))/(10*a^3*f*(1 + Sin[e + f*x])^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.
time = 0.31, size = 88, normalized size = 2.67

| method | result | size |
|---|---|---|
| risch | $-\frac{2\left(5c^2 e^{4i(fx+e)}-10c^2 e^{2i(fx+e)}+c^2\right)}{5f\,a^3\left(e^{i(fx+e)}+i\right)^5}$ | 55 |
| derivativedivides | $\frac{2c^2\left(\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}\right)}{f\,a^3}$ | 88 |
| default | $\frac{2c^2\left(\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}\right)}{f\,a^3}$ | 88 |
| norman | $\frac{-\frac{2c^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c^2}{5af}-\frac{24c^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}-\frac{52c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}-\frac{8c^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2 a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$ | 133 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^2/a^3*(4/(tan(1/2*f*x+1/2*e)+1)^2+8/(tan(1/2*f*x+1/2*e)+1)^4-8/(tan(1
/2*f*x+1/2*e)+1)^3-1/(tan(1/2*f*x+1/2*e)+1)-16/5/(tan(1/2*f*x+1/2*e)+1)^5)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(33) = 66$.
time = 0.32, size = 602, normalized size = 18.24

$$-\frac{2\left(\frac{c^2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{6c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(33) = 66$.
time = 0.31, size = 180, normalized size = 5.45

$$-\frac{c^2\cos(fx+e)^3+3\,c^2\cos(fx+e)^2-2\,c^2\cos(fx+e)-4\,c^2-\left(c^2\cos(fx+e)^2-2\,c^2\cos(fx+e)-4\,c^2\right)\sin(fx+e)}{5\left(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f+\left(a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^
2 - (c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(31) = 62$.
time = 5.31, size = 354, normalized size = 10.73

$$\begin{cases} -\frac{10c^2\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5a^3f} - \frac{20c^2\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5a^3f} - \frac{2c^2}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5a^3f} & \text{for}\ f \neq 0 \\ \frac{x(-c\sin(e)+c)^2}{(a\sin(e)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*c**2*tan(e/2 + f*x/2)**4/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*
a**3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(
e/2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f) - 20*c**2*tan(e/2
+ f*x/2)**2/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a**3*f*tan(e/2 + f*x/2)**4 +
 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/2 + f*x/2)**2 + 25*a**3*f*
tan(e/2 + f*x/2) + 5*a**3*f) - 2*c**2/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a*
```

```
*3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/
2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f), Ne(f, 0)), (x*(-c*s
in(e) + c)**2/(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.44, size = 60, normalized size = 1.82

$$-\frac{2\left(5\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^2\right)}{5\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*c^2*tan(1/2*f*x + 1/2*e)^4 + 10*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

**Mupad** [B]
time = 7.21, size = 90, normalized size = 2.73

$$-\frac{2\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left({\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\right)}{5\,a^3\,f\,{\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*c^2*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^4 + 5*sin(e/2 + (f*x)/2)^4 +
 10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(5*a^3*f*(cos(e/2 + (f*x)/2
) + sin(e/2 + (f*x)/2))^5)
```

# 3.283 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=58

$$-\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}-\frac{c\cos^3(e+fx)}{15f(a+a\sin(e+fx))^3}$$

```
[Out] -1/5*a*c*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^4-1/15*c*cos(f*x+e)^3/f/(a+a*sin(f
*x+e))^3
```

**Rubi** [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2815, 2751, 2750}

$$-\frac{c\cos^3(e+fx)}{15f(a\sin(e+fx)+a)^3}-\frac{ac\cos^3(e+fx)}{5f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*(a*c*Cos[e + f*x]^3)/(f*(a + a*Sin[e + f*x])^4) - (c*Cos[e + f*x]^3)/(
15*f*(a + a*Sin[e + f*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}+\frac{1}{5}c\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}-\frac{c\cos^3(e+fx)}{15f(a+a\sin(e+fx))^3}\end{aligned}$$

**Mathematica** [A]
time = 0.23, size = 92, normalized size = 1.59

$$\frac{c\left(-15\cos\left(e+\frac{fx}{2}\right)+5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)+\sin\left(2e+\frac{5fx}{2}\right)\right)}{30a^3f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c*(-15*Cos[e + (f*x)/2] + 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e
+ (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5)
```

**Maple** [A]
time = 0.26, size = 86, normalized size = 1.48

| method | result |
|---|---|
| risch | $\frac{2ic\left(-5i\mathrm{e}^{2i(fx+e)}+15\,\mathrm{e}^{3i(fx+e)}-i-5\,\mathrm{e}^{i(fx+e)}\right)}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+i\right)^5}$ |
| derivativedivides | $\frac{2c\left(\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{14}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}\right)}{f\,a^3}$ |
| default | $\frac{2c\left(\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{14}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}\right)}{f\,a^3}$ |
| norman | $\frac{-\frac{2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8c}{15af}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3af}-\frac{16c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{8c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{58c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

[Out] 2/f*c/a^3*(3/(tan(1/2*f*x+1/2*e)+1)^2-1/(tan(1/2*f*x+1/2*e)+1)+4/(tan(1/2*f*x+1/2*e)+1)^4-14/3/(tan(1/2*f*x+1/2*e)+1)^3-8/5/(tan(1/2*f*x+1/2*e)+1)^5)

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(58) = 116$.
time = 0.32, size = 421, normalized size = 7.26

$$-\frac{2\left(\frac{c\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 3*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(58) = 116$.
time = 0.31, size = 166, normalized size = 2.86

$$\frac{c\cos\left(fx+e\right)^3-2\,c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)+6\,c\right)\sin\left(fx+e\right)+6\,c}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(c*cos(f*x + e)^3 - 2*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - (c*cos(f*x
 + e)^2 + 3*c*cos(f*x + e) + 6*c)*sin(f*x + e) + 6*c)/(a^3*f*cos(f*x + e)^3
 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x
 + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(53) = 106$.
time = 2.89, size = 573, normalized size = 9.88

$$\begin{cases} -\frac{30c\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f}-\frac{30c\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f}-\frac{50c\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f}-\frac{10c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f}-\frac{8c}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f} & \text{for}\: f \neq 0 \\ \frac{x\left(-c\sin\left(e\right)+c\right)}{\left(a\sin\left(e\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*c*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 50*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 10*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*c/(15*a**3*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
, Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**3, True))
```

**Giac** [**A**]
time = 0.44, size = 84, normalized size = 1.45

$$-\frac{2\left(15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,c\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 15*c*tan(1/2*f*x + 1/2*e)^3 + 25*c*tan
(1/2*f*x + 1/2*e)^2 + 5*c*tan(1/2*f*x + 1/2*e) + 4*c)/(a^3*f*(tan(1/2*f*x +
 1/2*e) + 1)^5)
```

**Mupad** [**B**]
time = 7.24, size = 134, normalized size = 2.31

$$-\frac{2\,c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}+5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+25\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+15\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+15\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\right)}{15\,a^{3}\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*c*cos(e/2 + (f*x)/2)*(4*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4
+ 15*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 + 5*cos(e/2 + (f*x)/2)^3*sin(e
/2 + (f*x)/2) + 25*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*a^3*f*(c
os(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^5)
```

# 3.284 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

**Optimal**. Leaf size=83

$$-\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)}+\frac{2\tan(e+fx)}{5a^3cf}$$

```
[Out] -1/5*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/5*sec(f*x+e)/c/f/(a^3+a^3*sin(f*
x+e))+2/5*tan(f*x+e)/a^3/c/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2751, 3852, 8}

$$\frac{2\tan(e+fx)}{5a^3cf}-\frac{\sec(e+fx)}{5cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/5*Sec[e + f*x]/(a*c*f*(a + a*Sin[e + f*x])^2) - Sec[e + f*x]/(5*c*f*(a^3
 + a^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{ac}\\ &= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}+\frac{3\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}\\ &= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)}+\frac{2}{\cdots}\\ &= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)}-\frac{2S}{\cdots}\\ &= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)}+\frac{2}{\cdots}\end{aligned}$$

## Mathematica [A]

time = 0.40, size = 111, normalized size = 1.34

$$\frac{-15+32\cos(e+fx)-12\cos(2(e+fx))+32\cos(3(e+fx))+3\cos(4(e+fx))-12\sin(e+fx)-32\sin(2(e+fx))-12\sin(3(e+fx))+8\sin(4(e+fx))}{160a^3cf(-1+\sin(e+fx))(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] (-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] + 3*Cos[
4*(e + f*x)] - 12*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 12*Sin[3*(e + f*x)]
+ 8*Sin[4*(e + f*x)])/(160*a^3*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3
)
```

## Maple [A]

time = 0.30, size = 101, normalized size = 1.22

| method | result |
|---|---|
| risch | $-\frac{4i\left(4i\,\mathrm{e}^{i(fx+e)}+5\,\mathrm{e}^{2i(fx+e)}-1\right)}{5\left(\mathrm{e}^{i(fx+e)}+i\right)^5\left(\mathrm{e}^{i(fx+e)}-i\right)a^3cf}$ |
| derivativedivides | $\frac{-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3cf}$ |
| default | $\frac{-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3cf}$ |

| | |
|---|---|
| norman | $\frac{-\frac{4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{4}{5acf}-\frac{2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5acf}}{a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3/c*(-1/8/(tan(1/2*f*x+1/2*e)-1)-2/5/(tan(1/2*f*x+1/2*e)+1)^5+1/(tan(
1/2*f*x+1/2*e)+1)^4-3/2/(tan(1/2*f*x+1/2*e)+1)^3+5/4/(tan(1/2*f*x+1/2*e)+1)
^2-7/8/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(82) = 164$.
time = 0.32, size = 229, normalized size = 2.76

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+2\right)}{5\left(a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 2)/((a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

**Fricas** [A]
time = 0.31, size = 89, normalized size = 1.07

$$\frac{4\cos(fx+e)^2+\left(2\cos(fx+e)^2-3\right)\sin(fx+e)-2}{5\left(a^3cf\cos(fx+e)^3-2a^3cf\cos(fx+e)\sin(fx+e)-2a^3cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(f*x + e)^2 + (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a^3*c*f*c
os(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e
))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(66) = 132$.
time = 2.80, size = 614, normalized size = 7.40

$$\begin{cases} -\frac{10\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf}-\frac{20\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf}-\frac{20\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf}+\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf}+\frac{4}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf} & \text{for}\ f \neq 0 \\ \frac{x}{(a\sin(e)+a)^3(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-10*tan(e/2 + f*x/2)**5/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**
3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*t
an(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) - 20*tan(e/
2 + f*x/2)**4/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2
)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 -
20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) - 20*tan(e/2 + f*x/2)**3/(5*a**3
*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*ta
n(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 +
 f*x/2) - 5*a**3*c*f) + 6*tan(e/2 + f*x/2)/(5*a**3*c*f*tan(e/2 + f*x/2)**6
+ 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a*
*3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) + 4
/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**
3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*t
an(e/2 + f*x/2) - 5*a**3*c*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(e)
+ c)), True))
```

**Giac** [A]
time = 0.45, size = 105, normalized size = 1.27

$$-\frac{\frac{5}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+120\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+70\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+21}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (35*tan(1/2*f*x + 1/2*e)^4 +
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 + 70*tan(1/2*f*x + 1
/2*e) + 21)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad** [B]
time = 7.38, size = 89, normalized size = 1.07

$$-\frac{2\,\left(5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-2\right)}{5\,a^3\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(10*tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + 10*tan(e/2 + (f*x)/2)
^4 + 5*tan(e/2 + (f*x)/2)^5 - 2))/(5*a^3*c*f*(tan(e/2 + (f*x)/2) - 1)*(tan(
e/2 + (f*x)/2) + 1)^5)
```

# 3.285 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=75

$$-\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)}+\frac{4\tan(e+fx)}{5a^3c^2f}+\frac{4\tan^3(e+fx)}{15a^3c^2f}$$

```
[Out] -1/5*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+4/5*tan(f*x+e)/a^3/c^2/f+4/15*
tan(f*x+e)^3/a^3/c^2/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{4\tan^3(e+fx)}{15a^3c^2f}+\frac{4\tan(e+fx)}{5a^3c^2f}-\frac{\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -1/5*Sec[e + f*x]^3/(c^2*f*(a^3 + a^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*
a^3*c^2*f) + (4*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))} + \frac{4\int \sec^4(e+fx)\,dx}{5a^3c^2}$$

$$= -\frac{\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))} - \frac{4\text{Subst}\left(\int (1+x^2)\,dx, x, -\text{t}\right.}{5a^3c^2f}$$

$$= -\frac{\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))} + \frac{4\tan(e+fx)}{5a^3c^2f} + \frac{4\tan^3(e+}{15a^3c^2f}$$

**Mathematica [A]**
time = 0.52, size = 131, normalized size = 1.75

$$\frac{54-128\cos(e+fx)+72\cos(2(e+fx))-192\cos(3(e+fx))+18\cos(4(e+fx))-64\cos(5(e+fx))+18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))}{1920a^3c^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (54 - 128*Cos[e + f*x] + 72*Cos[2*(e + f*x)] - 192*Cos[3*(e + f*x)] + 18*Co
s[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2*(e + f*x
)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x)])/(1920
*a^3*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.34, size = 133, normalized size = 1.77

| method | result |
|---|---|
| risch | $-\frac{16\left(6\,e^{3i(fx+e)}+2ie^{2i(fx+e)}+2\,e^{i(fx+e)}+i\right)}{15\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)^3 f\,c^2a^3}$ |
| derivativedivides | $\frac{-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{5}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}}{c^2f\,a^3}$ |
| default | $\frac{-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{5}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}}{c^2f\,a^3}$ |
| norman | $\frac{\frac{10\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{2}{5acf}-\frac{2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{14\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}-\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5acf}+\frac{2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{26(\text{t}}{}}{a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/c^2/a^3*(-1/12/(tan(1/2*f*x+1/2*e)-1)^3-1/8/(tan(1/2*f*x+1/2*e)-1)^2-5/
16/(tan(1/2*f*x+1/2*e)-1)-1/5/(tan(1/2*f*x+1/2*e)+1)^5+1/2/(tan(1/2*f*x+1/2
*e)+1)^4-5/6/(tan(1/2*f*x+1/2*e)+1)^3+3/4/(tan(1/2*f*x+1/2*e)+1)^2-11/16/(t
an(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(73) = 146$.
time = 0.33, size = 363, normalized size = 4.84

$$\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{15\left(a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/((a^3*c^2
+ 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

**Fricas** [A]
time = 0.32, size = 92, normalized size = 1.23

$$-\frac{8\cos(fx+e)^4-4\cos(fx+e)^2-4\left(2\cos(fx+e)^2+1\right)\sin(fx+e)-1}{15\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e
)^3)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(66) = 132$.
time = 6.55, size = 1418, normalized size = 18.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*
a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a
**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*
c**2*f) - 30*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a
**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a*
*3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**
3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c*
*2*f) + 10*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**
3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3
*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*
c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2
*f) + 50*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*
c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c
**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c*
*2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f
) - 26*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c*
*2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**
2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2
*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f)
- 42*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2
*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*
f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f
*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) -
18*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*ta
n(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan
(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(
e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 6/(15
*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*
a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a
**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a*
*3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x/((a*sin(e) + a)
**3*(-c*sin(e) + c)**2), True))
```

**Giac [A]**

time = 0.44, size = 133, normalized size = 1.77

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-24\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13\right)}{a^3c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}+\frac{165\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+650\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+400\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+113}{a^3c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 - 24*tan(1/2*f*x + 1/2*e) + 13)/(a^3*c
^2*(tan(1/2*f*x + 1/2*e) - 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 + 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 + 400*tan(1/2*f*x + 1/2*e) +
113)/(a^3*c^2*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 8.43, size = 128, normalized size = 1.71

$$-\frac{2\left(15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5-25\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+13\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3\right)}{15\,a^3\,c^2\,f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(9*tan(e/2 + (f*x)/2) + 21*tan(e/2 + (f*x)/2)^2 + 13*tan(e/2 + (f*x)/2)
^3 - 25*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^5 + 15*tan(e/2 + (f*x)/
2)^6 + 15*tan(e/2 + (f*x)/2)^7 - 3))/(15*a^3*c^2*f*(tan(e/2 + (f*x)/2) - 1)
^3*(tan(e/2 + (f*x)/2) + 1)^5)
```

# 3.286 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=59

$$\frac{\tan(e+fx)}{a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan^5(e+fx)}{5a^3c^3f}$$

```
[Out] tan(f*x+e)/a^3/c^3/f+2/3*tan(f*x+e)^3/a^3/c^3/f+1/5*tan(f*x+e)^5/a^3/c^3/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2815, 3852}

$$\frac{\tan^5(e+fx)}{5a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Tan[e + f*x]/(a^3*c^3*f) + (2*Tan[e + f*x]^3)/(3*a^3*c^3*f) + Tan[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)\,dx}{a^3c^3}\\ &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{a^3c^3f}\\ &= \frac{\tan(e+fx)}{a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan^5(e+fx)}{5a^3c^3f}\end{aligned}$$

**Mathematica [A]**
time = 0.09, size = 41, normalized size = 0.69

$$\frac{\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
[Out] (Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5)/(a^3*c^3*f)
```

**Maple [A]**
time = 0.36, size = 40, normalized size = 0.68

| method | result | size |
|---|---|---|
| default | $-\frac{\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)}{a^3c^3f}$ | 40 |
| risch | $\frac{16i\left(10\,e^{4i(fx+e)}+5\,e^{2i(fx+e)}+1\right)}{15\left(e^{i(fx+e)}-i\right)^5\left(e^{i(fx+e)}+i\right)^5 f\,a^3c^3}$ | 65 |
| norman | $\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{8\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{116\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}+\frac{8\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}}{a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$ | 143 |
| derivativedivides | `error in RationalFunction: argument is not a rational function\` | N/A |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] -1/a^3/c^3/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

**Maxima [A]**
time = 0.33, size = 43, normalized size = 0.73

$$\frac{3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)}{15\,a^3c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

**Fricas [A]**
time = 0.31, size = 51, normalized size = 0.86

$$\frac{\left(8\,\cos\left(fx+e\right)^4+4\,\cos\left(fx+e\right)^2+3\right)\sin\left(fx+e\right)}{15\,a^3c^3f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(f*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e)/(a^3*c^3*f*cos(
f*x + e)^5)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(53) = 106$.
time = 5.12, size = 687, normalized size = 11.64

$$\begin{cases} -\frac{30\tan^9\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3c^3f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^3c^3f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3c^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^3c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^3c^3f} + \frac{40\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3c^3f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^3c^3f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3c^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^3c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^3c^3f} - \frac{116\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3c^3f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^3c^3f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3c^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^3c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^3c^3f} + \frac{40\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3c^3f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^3c^3f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3c^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^3c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^3c^3f} - \frac{30\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3c^3f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^3c^3f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3c^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^3c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^3c^3f} & \text{for}\: f \neq 0 \\ \frac{x}{(a\sin(e)+a)^3(-c\sin(e)+c)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 7
5*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 1
50*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 1
5*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2)**1
0 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**
6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**
2 - 15*a**3*c**3*f) - 116*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2 + f*x
/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*
x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*
x/2)**2 - 15*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*tan(e/2
+ f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2
 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2
 + f*x/2)**2 - 15*a**3*c**3*f) - 30*tan(e/2 + f*x/2)/(15*a**3*c**3*f*tan(e/
2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e
/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e
/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(
e) + c)**3), True))
```

**Giac** [A]
time = 0.45, size = 43, normalized size = 0.73

$$\frac{3\tan(fx+e)^5 + 10\tan(fx+e)^3 + 15\tan(fx+e)}{15a^3c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

**Mupad [B]**
time = 8.34, size = 89, normalized size = 1.51

$$-\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+58\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+15\right)}{15\,a^3\,c^3\,f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2)*(58*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 -
 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 15))/(15*a^3*c^3*f*(ta
n(e/2 + (f*x)/2)^2 - 1)^5)
```

# 3.287 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=97

$$\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan^5(e+fx)}{35a^3c^4f}$$

```
[Out] 1/7*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+6/7*tan(f*x+e)/a^3/c^4/f+4/7*ta
n(f*x+e)^3/a^3/c^4/f+6/35*tan(f*x+e)^5/a^3/c^4/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{6\tan^5(e+fx)}{35a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^5/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + (6*Tan[e + f*x])/(7*a^3
*c^4*f) + (4*Tan[e + f*x]^3)/(7*a^3*c^4*f) + (6*Tan[e + f*x]^5)/(35*a^3*c^4
*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\int \sec^6(e+fx)\,dx}{7a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} - \frac{6\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, \right.}{7a^3c^4f}$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\tan(e+fx)}{7a^3c^4f} + \frac{4\tan^3(e+f}{7a^3c^4f}$$

**Mathematica** [**A**]
time = 0.74, size = 193, normalized size = 1.99

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-500\cos(e+fx)+1280\cos(2(e+fx))-250\cos(3(e+fx))+1024\cos(4(e+fx))-50\cos(5(e+fx))+256\cos(6(e+fx))+5120\sin(e+fx)+125\sin(2(e+fx))+2560\sin(3(e+fx))+100\sin(4(e+fx))+512\sin(5(e+fx))+25\sin(6(e+fx))\right)}{17920f(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-500*Cos[e + f*x] + 1280*Cos[2*(e + f*x)] - 250*Cos[3*(e + f*x)] + 1024*
Cos[4*(e + f*x)] - 50*Cos[5*(e + f*x)] + 256*Cos[6*(e + f*x)] + 5120*Sin[e
+ f*x] + 125*Sin[2*(e + f*x)] + 2560*Sin[3*(e + f*x)] + 100*Sin[4*(e + f*x)
] + 512*Sin[5*(e + f*x)] + 25*Sin[6*(e + f*x)]))/(17920*f*(a + a*Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^4)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(89) = 178$.
time = 0.41, size = 193, normalized size = 1.99

| method | result |
|---|---|
| risch | $\frac{\frac{64\,e^{i(fx+e)}}{35}-\frac{32i}{35}+\frac{64\,e^{3i(fx+e)}}{7}+\frac{128\,e^{5i(fx+e)}}{7}-\frac{32i e^{4i(fx+e)}}{7}-\frac{128i e^{2i(fx+e)}}{35}}{\left(e^{i(fx+e)}-i\right)^7\left(e^{i(fx+e)}+i\right)^5 f\,a^3c^4}$ |
| derivativedivides | $-\frac{1}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{11}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2}{7\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |
| default | $-\frac{1}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{11}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2}{7\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |

| | |
|---|---|
| norman | $\frac{-\frac{6\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{52\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}-\frac{2}{7acf}-\frac{2\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7acf}+}{a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5c^3\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3/c^4*(-1/20/(tan(1/2*f*x+1/2*e)+1)^5+1/8/(tan(1/2*f*x+1/2*e)+1)^4-1/
4/(tan(1/2*f*x+1/2*e)+1)^3+1/4/(tan(1/2*f*x+1/2*e)+1)^2-11/32/(tan(1/2*f*x+
1/2*e)+1)-1/7/(tan(1/2*f*x+1/2*e)-1)^7-1/2/(tan(1/2*f*x+1/2*e)-1)^6-21/20/(
tan(1/2*f*x+1/2*e)-1)^5-11/8/(tan(1/2*f*x+1/2*e)-1)^4-11/8/(tan(1/2*f*x+1/2
*e)-1)^3-15/16/(tan(1/2*f*x+1/2*e)-1)^2-21/32/(tan(1/2*f*x+1/2*e)-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(95) = 190$.
time = 0.34, size = 563, normalized size = 5.80

$$\frac{2\left(\frac{25\sin(fx+e)}{\cos(fx+e)+1}-\frac{55\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{130\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{26\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{182\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{126\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{105\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{35\sin(fx+e)^9}{(\cos(fx+e)+1)^9}-\frac{35\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{35\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+5\right)}{35\left(a^3c^4-\frac{2a^3c^4\sin(fx+e)}{\cos(fx+e)+1}-\frac{4a^3c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{20a^3c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{20a^3c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{5a^3c^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{10a^3c^4\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{4a^3c^4\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{2a^3c^4\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^3c^4\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/35*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
35*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 5)/((a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3
*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin
(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x
 + e)^12/(cos(f*x + e) + 1)^12)*f)
```

**Fricas** [A]
time = 0.32, size = 115, normalized size = 1.19

$$-\frac{16\cos(fx+e)^6-8\cos(fx+e)^4-2\cos(fx+e)^2+2\left(8\cos(fx+e)^4+4\cos(fx+e)^2+3\right)\sin(fx+e)-1}{35\left(a^3c^4f\cos(fx+e)^5\sin(fx+e)-a^3c^4f\cos(fx+e)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*cos(f*x + e)^6 - 8*cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 2*(8*cos(f
*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e) - 1)/(a^3*c^4*f*cos(f*x + e)
^5*sin(f*x + e) - a^3*c^4*f*cos(f*x + e)^5)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(87) = 174$.
time = 27.87, size = 3186, normalized size = 32.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*tan(e/2 + f*x/2)**11/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 -
70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10
+ 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8
 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**
5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)*
*3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2)
- 35*a**3*c**4*f) + 70*tan(e/2 + f*x/2)**10/(35*a**3*c**4*f*tan(e/2 + f*x/2
)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x
/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f
*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 +
f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 +
 f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 +
 f*x/2) - 35*a**3*c**4*f) + 70*tan(e/2 + f*x/2)**9/(35*a**3*c**4*f*tan(e/2
+ f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/
2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(
e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan
(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*ta
n(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*ta
n(e/2 + f*x/2) - 35*a**3*c**4*f) - 210*tan(e/2 + f*x/2)**8/(35*a**3*c**4*f*
tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*
f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**
4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c*
*4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c
**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c
**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 252*tan(e/2 + f*x/2)**7/(35*a**3
*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a*
*3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*
a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700
*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 35
0*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 7
0*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) + 364*tan(e/2 + f*x/2)**6/
(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11
```

```
- 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**
9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)*
*7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)
**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2
)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 52*tan(e/2 + f*x
/2)**5/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/
2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f
*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 +
f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 +
 f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2
+ f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 260*tan(e
/2 + f*x/2)**4/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/
2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan
(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*ta
n(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*t
an(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*
tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 3
0*tan(e/2 + f*x/2)**3/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f
*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**
4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c*
*4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c
**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*
c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4
*f) + 110*tan(e/2 + f*x/2)**2/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**
3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*
a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700
*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 17
5*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 1
40*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a
**3*c**4*f) - 50*tan(e/2 + f*x/2)/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70
*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 +
350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 -
 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5
- 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3
 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**...
```

**Giac** [A]

time = 0.45, size = 189, normalized size = 1.95

$$-\frac{\frac{7\left(55\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+180\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+250\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+160\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+43\right)}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{735\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-3360\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+7315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-8820\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+6321\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2492\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+461}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{7}}}{560f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/560*(7*(55*tan(1/2*f*x + 1/2*e)^4 + 180*tan(1/2*f*x + 1/2*e)^3 + 250*tan
(1/2*f*x + 1/2*e)^2 + 160*tan(1/2*f*x + 1/2*e) + 43)/(a^3*c^4*(tan(1/2*f*x
```

```
+ 1/2*e) + 1)^5) + (735*tan(1/2*f*x + 1/2*e)^6 - 3360*tan(1/2*f*x + 1/2*e)^
5 + 7315*tan(1/2*f*x + 1/2*e)^4 - 8820*tan(1/2*f*x + 1/2*e)^3 + 6321*tan(1/
2*f*x + 1/2*e)^2 - 2492*tan(1/2*f*x + 1/2*e) + 461)/(a^3*c^4*(tan(1/2*f*x +
 1/2*e) - 1)^7))/f
```

**Mupad [B]**
time = 9.42, size = 180, normalized size = 1.86

$$-\frac{2\left(35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}-35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}+105\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+126\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}-182\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+26\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}+130\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}-55\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+25\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5\right)}{35\,a^3\,c^4\,f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^7\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -(2*(25*tan(e/2 + (f*x)/2) - 55*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2
)^3 + 130*tan(e/2 + (f*x)/2)^4 + 26*tan(e/2 + (f*x)/2)^5 - 182*tan(e/2 + (f
*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^7 + 105*tan(e/2 + (f*x)/2)^8 - 35*tan(e/2
 + (f*x)/2)^9 - 35*tan(e/2 + (f*x)/2)^10 + 35*tan(e/2 + (f*x)/2)^11 + 5))/(
35*a^3*c^4*f*(tan(e/2 + (f*x)/2) - 1)^7*(tan(e/2 + (f*x)/2) + 1)^5)
```

# 3.288 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=131

$$\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}+\frac{\sec^5(e+fx)}{9a^3f\,(c^5-c^5\sin(e+fx))}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan^5(e+fx)}{15a^3c^5f}$$

```
[Out] 1/9*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/9*sec(f*x+e)^5/a^3/f/(c^5-c
^5*sin(f*x+e))+2/3*tan(f*x+e)/a^3/c^5/f+4/9*tan(f*x+e)^3/a^3/c^5/f+2/15*tan
(f*x+e)^5/a^3/c^5/f
```

**Rubi** [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{2\tan^5(e+fx)}{15a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{\sec^5(e+fx)}{9a^3f\,(c^5-c^5\sin(e+fx))}+\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] Sec[e + f*x]^5/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]^5/(9*a^3
*f*(c^5 - c^5*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^3*c^5*f) + (4*Tan[e +
f*x]^3)/(9*a^3*c^5*f) + (2*Tan[e + f*x]^5)/(15*a^3*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{7\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} +$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} -$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} +$$

## Mathematica [A]
time = 0.89, size = 213, normalized size = 1.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-7875\cos(e+fx)+20480\cos(2(e+fx))-3325\cos(3(e+fx))+16384\cos(4(e+fx))-175\cos(5(e+fx))+4096\cos(6(e+fx))+175\cos(7(e+fx))+46080\sin(e+fx)+3500\sin(2(e+fx))+19456\sin(3(e+fx))+2800\sin(4(e+fx))+1024\sin(5(e+fx))+700\sin(6(e+fx))-1024\sin(7(e+fx))\right)}{184320f(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-7875*Cos[e + f*x] + 20480*Cos[2*(e + f*x)] - 3325*Cos[3*(e + f*x)] + 16
384*Cos[4*(e + f*x)] - 175*Cos[5*(e + f*x)] + 4096*Cos[6*(e + f*x)] + 175*C
os[7*(e + f*x)] + 46080*Sin[e + f*x] + 3500*Sin[2*(e + f*x)] + 19456*Sin[3*
(e + f*x)] + 2800*Sin[4*(e + f*x)] + 1024*Sin[5*(e + f*x)] + 700*Sin[6*(e +
 f*x)] - 1024*Sin[7*(e + f*x)]))/(184320*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^5)
```

## Maple [A]
time = 0.54, size = 223, normalized size = 1.70

| method | result |
|---|---|
| risch | $-\frac{32i\left(-20i e^{5i(fx+e)}+45\,e^{6i(fx+e)}-16i e^{3i(fx+e)}+19\,e^{4i(fx+e)}-4i e^{i(fx+e)}+e^{2i(fx+e)}-1\right)}{45\left(e^{i(fx+e)}-i\right)^9\left(e^{i(fx+e)}+i\right)^5 f\,a^3c^5}$ |
| derivativedivides | $-\frac{4}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{49}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{49}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{35}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$ |

| | |
|---|---|
| default | $\frac{-\frac{4}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{49}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{49}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{35}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{}$ |
| norman | $\frac{\frac{4\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{344\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}-\frac{4}{9acf}-\frac{2\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{32\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{9acf}-\frac{4\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{9acf}-}{a^{2}\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3/c^5*(-2/9/(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)^8-5/2/(
tan(1/2*f*x+1/2*e)-1)^7-49/12/(tan(1/2*f*x+1/2*e)-1)^6-49/10/(tan(1/2*f*x+1
/2*e)-1)^5-35/8/(tan(1/2*f*x+1/2*e)-1)^4-49/16/(tan(1/2*f*x+1/2*e)-1)^3-51/
32/(tan(1/2*f*x+1/2*e)-1)^2-99/128/(tan(1/2*f*x+1/2*e)-1)-1/40/(tan(1/2*f*x
+1/2*e)+1)^5+1/16/(tan(1/2*f*x+1/2*e)+1)^4-13/96/(tan(1/2*f*x+1/2*e)+1)^3+9
/64/(tan(1/2*f*x+1/2*e)+1)^2-29/128/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(130) = 260$.

time = 0.33, size = 662, normalized size = 5.05

$$\frac{2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{80\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{190\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{50\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{269\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{96\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{516\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{354\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{69\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{240\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{30\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{90\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}+\frac{45\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}+10\right)}{45\left(a^{3}c^{5}-\frac{4a^{3}c^{5}\sin(fx+e)}{\cos(fx+e)+1}+\frac{a^{3}c^{5}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{16a^{3}c^{5}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{19a^{3}c^{5}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{20a^{3}c^{5}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{45a^{3}c^{5}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{45a^{3}c^{5}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{20a^{3}c^{5}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{19a^{3}c^{5}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{16a^{3}c^{5}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^{3}c^{5}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}+\frac{4a^{3}c^{5}\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}-\frac{a^{3}c^{5}\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 2/45*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 80*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 190*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 269*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 96*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 516*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - 69*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 2
40*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 30*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 - 90*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 10)/((a^3*c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e)
+ 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20
*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a
^3*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - a^3*c^5*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*f)
```

**Fricas** [A]

time = 0.35, size = 144, normalized size = 1.10

$$-\frac{32\cos(fx+e)^{6}-16\cos(fx+e)^{4}-4\cos(fx+e)^{2}-\left(16\cos(fx+e)^{6}-24\cos(fx+e)^{4}-10\cos(fx+e)^{2}-7\right)\sin(fx+e)-2}{45\left(a^{3}c^{5}f\cos(fx+e)^{7}+2a^{3}c^{5}f\cos(fx+e)^{5}\sin(fx+e)-2a^{3}c^{5}f\cos(fx+e)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/45*(32*cos(f*x + e)^6 - 16*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - (16*cos(f
*x + e)^6 - 24*cos(f*x + e)^4 - 10*cos(f*x + e)^2 - 7)*sin(f*x + e) - 2)/(a
^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(f*x + e) - 2*a^3*c
^5*f*cos(f*x + e)^5)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 4335 vs. $2(117) = 234$.
time = 50.44, size = 4335, normalized size = 33.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-90*tan(e/2 + f*x/2)**13/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 -
180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12
+ 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**
10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2
)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x
/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*
x/2) - 45*a**3*c**5*f) + 180*tan(e/2 + f*x/2)**12/(45*a**3*c**5*f*tan(e/2 +
 f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2
 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(
e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f
*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*
f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 60*tan(e/2 + f*x/2)**11/(45*a**3*c**
5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c
**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**
3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*
a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 90
0*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 7
20*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1
80*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 480*tan(e/2 + f*x/2)**1
0/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**
13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)
**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x
/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 +
f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 +
 f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 +
```

```
 f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) + 138*tan(e
/2 + f*x/2)**9/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e
/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan
(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*
tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5
*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**
5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**
5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f
) + 708*tan(e/2 + f*x/2)**8/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3
*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a*
*3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900
*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2
025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 +
 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3
- 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 4
5*a**3*c**5*f) - 1032*tan(e/2 + f*x/2)**7/(45*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*
x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 +
 f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2
 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/
2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/
2 + f*x/2) - 45*a**3*c**5*f) - 192*tan(e/2 + f*x/2)**6/(45*a**3*c**5*f*tan(
e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*ta
n(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f
*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**
5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c
**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*
c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*
c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) + 538*tan(e/2 + f*x/2)**5/(45*a**
3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a
**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 85
5*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 +
2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6
 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**
4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**
2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 100*tan(e/2 + f*x/
2)**4/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/
2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + ...
```

**Giac [A]**

time = 0.47, size = 217, normalized size = 1.66

$$-\frac{\frac{3\left(435\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+1470\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2060\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1330\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+353\right)}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{4455\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-26460\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+78120\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-137340\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+157374\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-118356\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+57744\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-16596\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2339}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{9}}}{2880\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] -1/2880*(3*(435*tan(1/2*f*x + 1/2*e)^4 + 1470*tan(1/2*f*x + 1/2*e)^3 + 2060*tan(1/2*f*x + 1/2*e)^2 + 1330*tan(1/2*f*x + 1/2*e) + 353)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) + 1)^5) + (4455*tan(1/2*f*x + 1/2*e)^8 - 26460*tan(1/2*f*x + 1/2*e)^7 + 78120*tan(1/2*f*x + 1/2*e)^6 - 137340*tan(1/2*f*x + 1/2*e)^5 + 157374*tan(1/2*f*x + 1/2*e)^4 - 118356*tan(1/2*f*x + 1/2*e)^3 + 57744*tan(1/2*f*x + 1/2*e)^2 - 16596*tan(1/2*f*x + 1/2*e) + 2339)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) - 1)^9))/f

**Mupad [B]**
time = 8.20, size = 190, normalized size = 1.45

$$-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{65\,\cos\left(\frac{5\,e}{2}+\frac{5\,f\,x}{2}\right)}{32}-\frac{225\,\cos\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)}{32}-5\,\cos\left(\frac{7\,e}{2}+\frac{7\,f\,x}{2}\right)+\cos\left(\frac{9\,e}{2}+\frac{9\,f\,x}{2}\right)-\frac{37\,\cos\left(\frac{11\,e}{2}+\frac{11\,f\,x}{2}\right)}{32}+\frac{5\,\cos\left(\frac{13\,e}{2}+\frac{13\,f\,x}{2}\right)}{32}-\frac{89\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}+11\,\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)-\frac{63\,\sin\left(\frac{5\,e}{2}+\frac{5\,f\,x}{2}\right)}{8}+\frac{25\,\sin\left(\frac{7\,e}{2}+\frac{7\,f\,x}{2}\right)}{8}-\frac{5\,\sin\left(\frac{9\,e}{2}+\frac{9\,f\,x}{2}\right)}{8}+\frac{3\,\sin\left(\frac{11\,e}{2}+\frac{11\,f\,x}{2}\right)}{8}+\frac{\sin\left(\frac{13\,e}{2}+\frac{13\,f\,x}{2}\right)}{4}\right)}{2880\,a^3\,c^5\,f\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)^5\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)

[Out] -(cos(e/2 + (f*x)/2)*((65*cos((5*e)/2 + (5*f*x)/2))/32 - (225*cos((3*e)/2 + (3*f*x)/2))/32 - 5*cos((7*e)/2 + (7*f*x)/2) + cos((9*e)/2 + (9*f*x)/2) - (37*cos((11*e)/2 + (11*f*x)/2))/32 + (5*cos((13*e)/2 + (13*f*x)/2))/32 - (89*sin(e/2 + (f*x)/2))/4 + 11*sin((3*e)/2 + (3*f*x)/2) - (63*sin((5*e)/2 + (5*f*x)/2))/8 + (25*sin((7*e)/2 + (7*f*x)/2))/8 - (5*sin((9*e)/2 + (9*f*x)/2))/8 + (3*sin((11*e)/2 + (11*f*x)/2))/8 + sin((13*e)/2 + (13*f*x)/2)/4))/(2880*a^3*c^5*f*cos(e/2 - pi/4 + (f*x)/2)^5*cos(e/2 + pi/4 + (f*x)/2)^9)

# 3.289 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

**Optimal**. Leaf size=167

$$\frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{32}{\cdots}$$

```
[Out] 1/11*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+8/99*sec(f*x+e)^5/a^3/f/(c^3
-c^3*sin(f*x+e))^2+8/99*sec(f*x+e)^5/a^3/f/(c^6-c^6*sin(f*x+e))+16/33*tan(f
*x+e)/a^3/c^6/f+32/99*tan(f*x+e)^3/a^3/c^6/f+16/165*tan(f*x+e)^5/a^3/c^6/f
```

**Rubi** [A]
time = 0.16, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2751, 3852}

$$\frac{16\tan^5(e+fx)}{165a^3c^6f}+\frac{32\tan^3(e+fx)}{99a^3c^6f}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] Sec[e + f*x]^5/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + (8*Sec[e + f*x]^5)/(
99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + (8*Sec[e + f*x]^5)/(99*a^3*f*(c^6 -
c^6*Sin[e + f*x])) + (16*Tan[e + f*x])/(33*a^3*c^6*f) + (32*Tan[e + f*x]^3)
/(99*a^3*c^6*f) + (16*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3} + \frac{8\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3} + \frac{8\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))}$$
$$= \frac{\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3} + \frac{8\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))}$$
$$= \frac{\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3} + \frac{8\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))}$$
$$= \frac{\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3} + \frac{8\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))}$$

**Mathematica [A]**
time = 1.05, size = 233, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-411950\cos(e+fx)+1081344\cos(2(e+fx))-127330\cos(3(e+fx))+819200\cos(4(e+fx))+37450\cos(5(e+fx))+163840\cos(6(e+fx))+22470\cos(7(e+fx))-16384\cos(8(e+fx))+1802240\sin(e+fx)+247170\sin(2(e+fx))+557056\sin(3(e+fx))+187250\sin(4(e+fx))-163840\sin(5(e+fx))+37450\sin(6(e+fx))-98304\sin(7(e+fx))-3745\sin(8(e+fx))\right)}{8110080f(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-411950*Cos[e + f*x] + 1081344*Cos[2*(e + f*x)] - 127330*Cos[3*(e + f*x)
] + 819200*Cos[4*(e + f*x)] + 37450*Cos[5*(e + f*x)] + 163840*Cos[6*(e + f*
x)] + 22470*Cos[7*(e + f*x)] - 16384*Cos[8*(e + f*x)] + 1802240*Sin[e + f*x
] + 247170*Sin[2*(e + f*x)] + 557056*Sin[3*(e + f*x)] + 187250*Sin[4*(e + f
*x)] - 163840*Sin[5*(e + f*x)] + 37450*Sin[6*(e + f*x)] - 98304*Sin[7*(e +
f*x)] - 3745*Sin[8*(e + f*x)]))/(8110080*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^6)
```

**Maple [A]**
time = 0.59, size = 253, normalized size = 1.51

| method | result |
|---|---|

| | |
|---|---|
| risch | $-\frac{256\left(-6\,{\mathrm e}^{i(fx+e)}+i-50i\,{\mathrm e}^{4i(fx+e)}-10i\,{\mathrm e}^{2i(fx+e)}+110\,{\mathrm e}^{7i(fx+e)}-10\,{\mathrm e}^{3i(fx+e)}+34\,{\mathrm e}^{5i(fx+e)}-66i\,{\mathrm e}^{6i(fx+e)}\right)}{495\left({\mathrm e}^{i(fx+e)}-i\right)^{11}\left({\mathrm e}^{i(fx+e)}+i\right)^{5}f\,c^{6}a^{3}}$ |
| derivativedivides | $\frac{-\frac{8}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{106}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{23}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{33}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{217}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{}$ |
| default | $\frac{-\frac{8}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{106}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{23}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{33}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{217}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{}$ |
| norman | $\frac{\frac{106\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{50}{99acf}-\frac{2\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{10\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{6\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{22\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{34\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{33acf}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3/c^6*(-4/11/(tan(1/2*f*x+1/2*e)-1)^11-2/(tan(1/2*f*x+1/2*e)-1)^10-53
/9/(tan(1/2*f*x+1/2*e)-1)^9-23/2/(tan(1/2*f*x+1/2*e)-1)^8-33/2/(tan(1/2*f*x
+1/2*e)-1)^7-217/12/(tan(1/2*f*x+1/2*e)-1)^6-623/40/(tan(1/2*f*x+1/2*e)-1)^
5-169/16/(tan(1/2*f*x+1/2*e)-1)^4-365/64/(tan(1/2*f*x+1/2*e)-1)^3-303/128/(
tan(1/2*f*x+1/2*e)-1)^2-219/256/(tan(1/2*f*x+1/2*e)-1)-1/80/(tan(1/2*f*x+1/
2*e)+1)^5+1/32/(tan(1/2*f*x+1/2*e)+1)^4-7/96/(tan(1/2*f*x+1/2*e)+1)^3+5/64/
(tan(1/2*f*x+1/2*e)+1)^2-37/256/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(167) = 334$.
time = 0.35, size = 763, normalized size = 4.57

$$-\frac{2\left(\frac{255\sin(fx+e)}{\cos(fx+e)+1}+\frac{235\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{3065\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3775\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{667\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{8217\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{2035\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{8745\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{11715\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{33\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{4917\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{2475\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-\frac{1815\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}+\frac{1485\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}-\frac{495\sin(fx+e)^{15}}{(\cos(fx+e)+1)^{15}}-125\right)}{495\left(a^{3}c^{6}-\frac{6a^{3}c^{6}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}c^{6}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}c^{6}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{50a^{3}c^{6}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{34a^{3}c^{6}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{66a^{3}c^{6}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{110a^{3}c^{6}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{110a^{3}c^{6}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{66a^{3}c^{6}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{34a^{3}c^{6}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{50a^{3}c^{6}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-\frac{10a^{3}c^{6}\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}-\frac{10a^{3}c^{6}\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}+\frac{6a^{3}c^{6}\sin(fx+e)^{15}}{(\cos(fx+e)+1)^{15}}-\frac{a^{3}c^{6}\sin(fx+e)^{16}}{(\cos(fx+e)+1)^{16}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/495*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
+ 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 181
5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x + e
) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/((a^3*c^6 - 6*
a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c^6
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6*s
```

```
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*c^
6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(f*
x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16)*f)
```

**Fricas** [A]
time = 0.36, size = 176, normalized size = 1.05

$$\frac{128 \cos(fx+e)^8 - 576 \cos(fx+e)^6 + 240 \cos(fx+e)^4 + 56 \cos(fx+e)^2 + 8\left(48 \cos(fx+e)^6 - 40 \cos(fx+e)^4 - 14 \cos(fx+e)^2 - 9\right)\sin(fx+e) + 27}{495\left(3\,a^3c^6f\cos(fx+e)^7 - 4\,a^3c^6f\cos(fx+e)^5 - \left(a^3c^6f\cos(fx+e)^7 - 4\,a^3c^6f\cos(fx+e)^5\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/495*(128*cos(f*x + e)^8 - 576*cos(f*x + e)^6 + 240*cos(f*x + e)^4 + 56*co
s(f*x + e)^2 + 8*(48*cos(f*x + e)^6 - 40*cos(f*x + e)^4 - 14*cos(f*x + e)^2
 - 9)*sin(f*x + e) + 27)/(3*a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x
+ e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x + e)^5)*sin(f*x +
e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 5661 vs. $2(151) = 302$.
time = 90.01, size = 5661, normalized size = 33.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-990*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16
- 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)
**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 +
f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan
(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6
*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3
*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 29
70*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*tan(e/2 + f*x/2)*
*14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/
2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 +
 f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*ta
n(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c*
*6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a*
*3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 247
```

50*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 3630*tan(e/2 + f*x/2)**13/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 4950*tan(e/2 + f*x/2)**12/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 9834*tan(e/2 + f*x/2)**11/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 66*tan(e/2 + f*x/2)**10/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 23430*tan(e/2 + f*x/2)**9/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f)

```
 + 17490*tan(e/2 + f*x/2)**8/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a
**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4
950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**
12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f
*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e
/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*
tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2...
```

**Giac [A]**
time = 0.47, size = 245, normalized size = 1.47

$$-\frac{\frac{33\left(555\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+1920\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2710\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1760\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+463\right)}{a^{3}c^{6}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{108405\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-784080\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+2901195\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-6652800\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+10407474\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-11435424\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+8949270\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-4899840\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+1816265\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-411664\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+47279}{a^{3}c^{6}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{11}}}{63360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -1/63360*(33*(555*tan(1/2*f*x + 1/2*e)^4 + 1920*tan(1/2*f*x + 1/2*e)^3 + 27
10*tan(1/2*f*x + 1/2*e)^2 + 1760*tan(1/2*f*x + 1/2*e) + 463)/(a^3*c^6*(tan(
1/2*f*x + 1/2*e) + 1)^5) + (108405*tan(1/2*f*x + 1/2*e)^10 - 784080*tan(1/2
*f*x + 1/2*e)^9 + 2901195*tan(1/2*f*x + 1/2*e)^8 - 6652800*tan(1/2*f*x + 1/
2*e)^7 + 10407474*tan(1/2*f*x + 1/2*e)^6 - 11435424*tan(1/2*f*x + 1/2*e)^5
+ 8949270*tan(1/2*f*x + 1/2*e)^4 - 4899840*tan(1/2*f*x + 1/2*e)^3 + 1816265
*tan(1/2*f*x + 1/2*e)^2 - 411664*tan(1/2*f*x + 1/2*e) + 47279)/(a^3*c^6*(ta
n(1/2*f*x + 1/2*e) - 1)^11))/f
```

**Mupad [B]**
time = 8.62, size = 185, normalized size = 1.11

$$-\frac{\frac{2\sin\left(e+f\,x\right)}{9}+\frac{2\cos\left(2\,e+2\,f\,x\right)}{15}+\frac{10\cos\left(4\,e+4\,f\,x\right)}{99}+\frac{2\cos\left(6\,e+6\,f\,x\right)}{99}-\frac{\cos\left(8\,e+8\,f\,x\right)}{495}+\frac{34\sin\left(3\,e+3\,f\,x\right)}{495}-\frac{2\sin\left(5\,e+5\,f\,x\right)}{99}-\frac{2\sin\left(7\,e+7\,f\,x\right)}{165}}{a^3\,c^6\,f\,\left(\frac{5\cos\left(5\,e+5\,f\,x\right)}{64}-\frac{17\cos\left(3\,e+3\,f\,x\right)}{64}-\frac{55\cos\left(e+f\,x\right)}{64}+\frac{3\cos\left(7\,e+7\,f\,x\right)}{64}+\frac{33\sin\left(2\,e+2\,f\,x\right)}{64}+\frac{25\sin\left(4\,e+4\,f\,x\right)}{64}+\frac{5\sin\left(6\,e+6\,f\,x\right)}{64}-\frac{\sin\left(8\,e+8\,f\,x\right)}{128}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)
```

```
[Out] -((2*sin(e + f*x))/9 + (2*cos(2*e + 2*f*x))/15 + (10*cos(4*e + 4*f*x))/99 +
 (2*cos(6*e + 6*f*x))/99 - cos(8*e + 8*f*x)/495 + (34*sin(3*e + 3*f*x))/495
 - (2*sin(5*e + 5*f*x))/99 - (2*sin(7*e + 7*f*x))/165)/(a^3*c^6*f*((5*cos(5
*e + 5*f*x))/64 - (17*cos(3*e + 3*f*x))/64 - (55*cos(e + f*x))/64 + (3*cos(
7*e + 7*f*x))/64 + (33*sin(2*e + 2*f*x))/64 + (25*sin(4*e + 4*f*x))/64 + (5
*sin(6*e + 6*f*x))/64 - sin(8*e + 8*f*x)/128))
```

# 3.290 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=137

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e-}{}$$

```
[Out] 256/315*a*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/9*a*c^2*cos(f*x+e)^3*
(c-c*sin(f*x+e))^(3/2)/f+64/105*a*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)
+8/21*a*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2753, 2752}

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*c^5*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (64*a*c^4*C
os[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (8*a*c^3*Cos[e + f*x]^3*S
qrt[c - c*Sin[e + f*x]])/(21*f) + (2*a*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*
x])^(3/2))/(9*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx$$
$$= \frac{2ac^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f} + \frac{1}{3}\left(4ac^2\right)\int \cos^2($$
$$= \frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} + \frac{2ac^2\cos^3(e+fx)}{}$$
$$= \frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(}}{21f}$$
$$= \frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}} + \frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} +$$

**Mathematica** [A]
time = 0.52, size = 104, normalized size = 0.76

$$\frac{ac^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\,(1606-330\cos(2(e+fx))-1389\sin(e+fx)+35\sin(3(e+fx)))}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
[Out] (a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(16
06 - 330*Cos[2*(e + f*x)] - 1389*Sin[e + f*x] + 35*Sin[3*(e + f*x)]))/(630*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 2.00, size = 79, normalized size = 0.58

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^2a(35(\sin^3(fx+e))-165(\sin^2(fx+e))+321\sin(fx+e)-319)}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 79 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*(35*sin(f*x+e)^3-165*sin(f*x+e)
^2+321*sin(f*x+e)-319)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [A]**
time = 0.36, size = 192, normalized size = 1.40

$$-\frac{2\left(35\,ac^3\cos\left(fx+e\right)^5-95\,ac^3\cos\left(fx+e\right)^4-226\,ac^3\cos\left(fx+e\right)^3+32\,ac^3\cos\left(fx+e\right)^2-128\,ac^3\cos\left(fx+e\right)-256\,ac^3+\left(35\,ac^3\cos\left(fx+e\right)^4+130\,ac^3\cos\left(fx+e\right)^3-96\,ac^3\cos\left(fx+e\right)^2-128\,ac^3\cos\left(fx+e\right)-256\,ac^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{315\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*c^3*cos(f*x + e)^5 - 95*a*c^3*cos(f*x + e)^4 - 226*a*c^3*cos(f
*x + e)^3 + 32*a*c^3*cos(f*x + e)^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3 +
(35*a*c^3*cos(f*x + e)^4 + 130*a*c^3*cos(f*x + e)^3 - 96*a*c^3*cos(f*x + e)
^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

**Giac [A]**
time = 0.53, size = 144, normalized size = 1.05

$$-\frac{\sqrt{2}\left(4410\,ac^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-504\,ac^3\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+225\,ac^3\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-35\,ac^3\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(4410*a*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - 504*a*c^3*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4
```

```
*pi + 1/2*f*x + 1/2*e)) + 225*a*c^3*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 35*a*c^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x)) (c - c \sin(e + f x))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x)
```

# 3.291 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=103

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] 64/105*a*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+16/35*a*c^3*cos(f*x+e)^3
/f/(c-c*sin(f*x+e))^(1/2)+2/7*a*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2753, 2752}

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*c^4*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (16*a*c^3*Co
s[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*c^2*Cos[e + f*x]^3*Sqr
t[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$
$$= \frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f} + \frac{1}{7}(8ac^2)\int \cos^2(e$$
$$= \frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} + \frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e}{7f}$$
$$= \frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}} + \frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} + \frac{2}{}$$

**Mathematica [A]**
time = 0.33, size = 94, normalized size = 0.91

$$-\frac{ac^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-157+15\cos(2(e+fx))+108\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/105*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-157 + 15*Cos[2*(e +
 f*x)] + 108*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.16, size = 69, normalized size = 0.67

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^2a(15(\sin^2(fx+e))-54\sin(fx+e)+71)}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 69 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(15*sin(f*x+e)^2-54*sin(f*x+e)
+71)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [A]**
time = 0.35, size = 163, normalized size = 1.58

$$\frac{2\left(15\,ac^{2}\cos\left(fx+e\right)^{4}+39\,ac^{2}\cos\left(fx+e\right)^{3}-8\,ac^{2}\cos\left(fx+e\right)^{2}+32\,ac^{2}\cos\left(fx+e\right)+64\,ac^{2}-\left(15\,ac^{2}\cos\left(fx+e\right)^{3}-24\,ac^{2}\cos\left(fx+e\right)^{2}-32\,ac^{2}\cos\left(fx+e\right)-64\,ac^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{105\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*c^2*cos(f*x + e)^4 + 39*a*c^2*cos(f*x + e)^3 - 8*a*c^2*cos(f*x
+ e)^2 + 32*a*c^2*cos(f*x + e) + 64*a*c^2 - (15*a*c^2*cos(f*x + e)^3 - 24*a
*c^2*cos(f*x + e)^2 - 32*a*c^2*cos(f*x + e) - 64*a*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)\right)dx+\int\left(-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx+\int c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-c**2*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x), x) + Integral(-c**2*sqrt(-c*sin(e + f*x) + c)*
sin(e + f*x)**2, x) + Integral(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)*
*3, x))
```

**Giac [A]**
time = 0.52, size = 144, normalized size = 1.40

$$-\frac{\sqrt{2}\left(525\,ac^{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+35\,ac^{2}\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-63\,ac^{2}\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15\,ac^{2}\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{c}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/420*sqrt(2)*(525*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 35*a*c^2*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi
```

```
 + 1/2*f*x + 1/2*e)) - 63*a*c^2*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + 15*a*c^2*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\,\sin\left(e + f\,x\right))\,\left(c - c\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x)
```

# 3.292 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=69

$$\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 8/15*a*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/5*a*c^2*cos(f*x+e)^3/f/(
c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2815, 2753, 2752}

$$\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*c^3*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*c^2*Cos[e
 + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx &= (ac)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\ &= \frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}\left(4ac^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\\ &= \frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.20, size = 82, normalized size = 1.19

$$-\frac{2ac\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-7+3\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
[Out] (-2*a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-7 + 3*Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.95, size = 59, normalized size = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^2a(3\sin(fx+e)-7)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 59 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(3*sin(f*x+e)-7)/cos(f*x+e)/(c-c
*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [A]**
time = 0.33, size = 118, normalized size = 1.71

$$\frac{2\left(3\,ac\cos\left(fx+e\right)^{3}-ac\cos\left(fx+e\right)^{2}+4\,ac\cos\left(fx+e\right)+8\,ac+\left(3\,ac\cos\left(fx+e\right)^{2}+4\,ac\cos\left(fx+e\right)+8\,ac\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*a*c*cos(f*x + e)^3 - a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*
c + (3*a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*c)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
[Out] a*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-c*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x)**2, x))
```

**Giac [A]**
time = 0.53, size = 105, normalized size = 1.52

$$-\frac{\sqrt{2}\left(30\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,ac\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,ac\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] -1/30*sqrt(2)*(30*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 5*a*c*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - 3*a*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2), x)
```

# 3.293 $\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=34

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

[Out] 2/3*a*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)

**Rubi** [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2815, 2752}

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]

[Out] (2*a*c^2*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2))

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}
\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(34) = 68.
time = 0.09, size = 71, normalized size = 2.09

$$\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])/(3*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.33, size = 47, normalized size = 1.38

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^2 a}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$ | 47 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(32) = 64.
time = 0.35, size = 86, normalized size = 2.53

$$-\frac{2\left(a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2\,a\right)\sin\left(fx+e\right)-2\,a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x +
e) - 2*a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt(-c*s
in(e + f*x) + c), x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.
time = 0.49, size = 71, normalized size = 2.09

$$-\frac{\sqrt{2}\left(3\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+a\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) + a*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))*sqrt(c)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x)
```

# 3.294 $\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=77

$$\frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/
f/c^(1/2)-2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2758, 2728, 212}

$$\frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(Sqrt[c]*f) - (2*a*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\ &= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{(4a)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.39, size = 135, normalized size = 1.75

$$-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{c}\,(1+\sin(e+fx))+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\,\sqrt{c}}\right)\sqrt{-c(1+\sin(e+fx))}\right)}{\sqrt{c}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Sqrt[c]*(1 + Sin[e + f*x]) + S
qrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1
+ Sin[e + f*x]))]))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.95, size = 94, normalized size = 1.22

| method | result |
| --- | --- |
| default | $-\frac{2(\sin(fx+e)-1)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a\left(\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{c\left(1+\sin\right.}}{c\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\, f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(1/2))/c/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(70) = 140$.
time = 0.35, size = 214, normalized size = 2.78

$$\frac{\frac{\sqrt{2}\,(ac\cos(fx+e)-ac\sin(fx+e)+ac)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-2\left(a\cos\left(fx+e\right)+a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*c*cos(f*x + e) - a*c*sin(f*x + e) + a*c)*log(-(cos(f*x + e)^2 +
 (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos
(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a*cos(
f*x + e) + a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e)
 - c*f*sin(f*x + e) + c*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\, dx+\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c
*sin(e + f*x) + c), x))
```

**Giac [A]**
time = 0.49, size = 126, normalized size = 1.64

$$\frac{\frac{\sqrt{2}\,a\log\left(-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}\right)}{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\sqrt{2}\,a}{\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*a*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*a
/(sqrt(c)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.295 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2759, 2728, 212}

$$\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(S
qrt[2]*c^(3/2)*f)) + (a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c}\\ &= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{a\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}\\ &= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

**Mathematica [A]**
time = 0.39, size = 107, normalized size = 1.41

$$\frac{a\sec(e+fx)\left(2\sqrt{c}\,(1+\sin(e+fx))-\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\,\sqrt{c}}\right)(-1+\sin(e+fx))\sqrt{-c(1+\sin(e+fx))}\right)}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(2*Sqrt[c]*(1 + Sin[e + f*x]) - Sqrt[2]*ArcTan[Sqrt[-(c*(1
+ Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(-1 + Sin[e + f*x])*Sqrt[-(c*(1 + Sin[
e + f*x]))]))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 1.73, size = 120, normalized size = 1.58

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{a\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c\sin(fx+e)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\cdots}{2c^{\frac{5}{2}}\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^(5/2)*a*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c*sin(f*x+e)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(69) = 138$.
time = 0.35, size = 277, normalized size = 3.64

$$\frac{\frac{\sqrt{2}\left(ac\cos(fx+e)^2-ac\cos(fx+e)-2\,ac+(ac\cos(fx+e)+2\,ac)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-4\left(a\cos\left(fx+e\right)+a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{4\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+2\,c^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*c*cos(f*x + e)^2 - a*c*cos(f*x + e) - 2*a*c + (a*c*cos(f*x
+ e) + 2*a*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e)
+ 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin
(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a*cos(f*x + e) + a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqr
t(-c*sin(e + f*x) + c)), x) + Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(69) = 138.
time = 0.53, size = 222, normalized size = 2.92

$$-\frac{\frac{2\sqrt{2}\,a\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\,a\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a\sqrt{c}+\frac{2a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c^{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*a*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt
(2)*a*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c^(3/2)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(a*sqrt(c) + 2
*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c^2*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.296 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=113

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*cos(f*x+e)/c/f/(c-c*sin(f*x
+e))^(3/2)-1/16*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(
1/2))/c^(5/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2729, 2728, 212}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/8*(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(Sqrt[2]*c^(5/2)*f) + (a*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^(5/2)) -
(a*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx=(ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$=\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c}$$
$$=\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}-\frac{a\int \frac{1}{\sqrt{c-c\sin(e+f}}}{16c^2}$$
$$=\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x\right.}{}$$
$$=-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a}{8cf(c-}$$

**Mathematica [A]**
time = 0.56, size = 176, normalized size = 1.56

$$\frac{a\left(-2\sqrt{c}\left(-7+\cos(2(e+fx))-8\sin(e+fx)\right)+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{-c(1+\sin(e+fx))}\right)}{32c^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(-2*Sqrt[c]*(-7 + Cos[2*(e + f*x)] - 8*Sin[e + f*x]) + 2*Sqrt[2]*ArcTan[
Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*Sqrt[-(c*(1 + Sin[e + f*x]))]))/(32*c^(5/2)*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*S
in[e + f*x]])
```

**Maple** [A]
time = 2.37, size = 189, normalized size = 1.67

| method | result |
|---|---|
| default | $-\frac{a\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2(fx+e))c^3+2(c(1+\sin(fx+e)))^{\frac{3}{2}}c^{\frac{3}{2}}+2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1}{}\right.\right.}{16c^{\frac{11}{2}}(\sin}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/c^(11/2)*a*(-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(
1/2))*sin(f*x+e)^2*c^3+2*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)+2*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3+4*(c*(1+sin(f
*x+e)))^(1/2)*c^(5/2)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f
*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(100) = 200.
time = 0.36, size = 364, normalized size = 3.22

$$\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^3+3\,a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)-4\,a\right)\sin\left(fx+e\right)-4\,a\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(a\cos\left(fx+e\right)^2-3\,a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+4\,a\right)\sin\left(fx+e\right)-4\,a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{32\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*(a*cos(f*x + e)^3 + 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - (
a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - 4*a)*sin(f*x + e) - 4*a)*sqrt(c)*log(
-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x +
 e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x
 + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)) + 4*(a*cos(f*x + e)^2 - 3*a*cos(f*x + e) - (a*cos(f*x + e) + 4*a)*
sin(f*x + e) - 4*a)*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2
 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin(e+fx)}{c^2\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{1}{c^2\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 -
2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f*x) +
 c)), x) + Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 - 2*c
**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f*x) + c)
), x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(100) = 200.
time = 0.55, size = 229, normalized size = 2.03

$$-\frac{\frac{2\sqrt{2}\,a\log\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c^{\frac{5}{2}}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\,a\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{c^{\frac{5}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(a\sqrt{c}-\frac{2a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{128\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/128*(2*sqrt(2)*a*log((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2)/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) -
 sqrt(2)*a*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(c^(5/2)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(a*sq
rt(c) - 2*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.297 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=145

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{32c^2f(c-\ldots}$$

```
[Out] 1/3*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*cos(f*x+e)/c/f/(c-c*sin(f*
x+e))^(5/2)-1/32*a*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)-1/64*a*arctanh(1
/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2815, 2759, 2729, 2728, 212}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/32*(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*c^(7/2)*f) + (a*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^(7/2)) -
 (a*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(5/2)) - (a*Cos[e + f*x])/(3
2*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx\\
&= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\int \frac{1}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}\\
&= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{16c^2}\\
&= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+f\\
&= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+f\\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a}{24cf(c-
\end{aligned}$$

**Mathematica [A]**
time = 0.70, size = 189, normalized size = 1.30

$$\frac{a\left(12\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{-c(1+\sin(e+fx))}+2\sqrt{c}\left(-14\cos(2(e+fx))+131\sin(e+fx)+3(38+\sin(3(e+fx)))\right)\right)}{768c^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*(12*Sqrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sqrt[-(c*(1 + Sin[e + f*x]))] + 2*Sqrt[
c]*(-14*Cos[2*(e + f*x)] + 131*Sin[e + f*x] + 3*(38 + Sin[3*(e + f*x)]))))/
(768*c^(7/2)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.57, size = 243, normalized size = 1.68

| method | result |
|---|---|
| default | $\frac{a\left(24\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, c^{\frac{9}{2}}+32(c(1+\sin(fx+e)))^{\frac{3}{2}} c^{\frac{7}{2}}-6(c(1+\sin(fx+e)))^{\frac{5}{2}} c^{\frac{5}{2}}+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(f\right.\right.}}{2\sqrt{}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*a*(24*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)+32*(c*(1+sin(f*x+e)))^(3/2)*c^
(7/2)-6*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)+3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5-9*2^(1/2)*arctanh(1/2*(c*(1+
sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^5+9*2^(1/2)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^5-3*2^(1/2)*arctanh(
1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)
/c^(17/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(130) = 260$.
time = 0.35, size = 442, normalized size = 3.05

$$\frac{3\sqrt{2}\left(a\cos\left(fx+e\right)^{4}-3\,a\cos\left(fx+e\right)^{3}-8\,a\cos\left(fx+e\right)^{2}+4\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{3}+4\,a\cos\left(fx+e\right)^{2}-4\,a\cos\left(fx+e\right)-8\,a\right)\sin\left(fx+e\right)+8\,a\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^{2}-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(3\,a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}+22\,a\cos\left(fx+e\right)+\left(3\,a\cos\left(fx+e\right)^{2}+10\,a\cos\left(fx+e\right)+32\,a\right)\sin\left(fx+e\right)+32\,a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{384\left(c^{4}f\cos\left(fx+e\right)^{4}-3\,c^{4}f\cos\left(fx+e\right)^{3}-8\,c^{4}f\cos\left(fx+e\right)^{2}+4\,c^{4}f\cos\left(fx+e\right)+8\,c^{4}f+\left(c^{4}f\cos\left(fx+e\right)^{3}+4\,c^{4}f\cos\left(fx+e\right)^{2}-4\,c^{4}f\cos\left(fx+e\right)-8\,c^{4}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*(a*cos(f*x + e)^4 - 3*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^
2 + 4*a*cos(f*x + e) + (a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - 4*a*cos(f*x
 + e) - 8*a)*sin(f*x + e) + 8*a)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)
*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*
cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
 (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*a*cos(f*x + e)
^3 - 7*a*cos(f*x + e)^2 + 22*a*cos(f*x + e) + (3*a*cos(f*x + e)^2 + 10*a*co
s(f*x + e) + 32*a)*sin(f*x + e) + 32*a)*sqrt(-c*sin(f*x + e) + c))/(c^4*f*c
os(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*c
os(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*
c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(130) = 260.
time = 0.61, size = 391, normalized size = 2.70

$$-\frac{\frac{12\sqrt{2}\,a\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a\sqrt{c}-\frac{3a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}+\frac{22a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}{c^{4}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\frac{3ac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3ac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}-\frac{ac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}\right)}{c^{12}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{1536\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/1536*(12*sqrt(2)*a*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) -
sqrt(2)*(a*sqrt(c) - 3*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 22*a*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*a*c^(17/2)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*a*c^(17/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^2 - a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^3)/(c^12*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a \sin(e + f x)}{(c - c \sin(e + f x))^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(7/2), x)
```

# 3.298 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=145

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-}}{11f}$$

```
[Out] 256/1155*a^2*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/231*a^2*c^5*cos(f
*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+8/33*a^2*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e
))^(1/2)+2/11*a^2*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a^2*c^6*Cos[e + f*x]^5)/(1155*f*(c - c*Sin[e + f*x])^(5/2)) + (64*a^2*
c^5*Cos[e + f*x]^5)/(231*f*(c - c*Sin[e + f*x])^(3/2)) + (8*a^2*c^4*Cos[e +
 f*x]^5)/(33*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*c^3*Cos[e + f*x]^5*Sqrt[c
 - c*Sin[e + f*x]])/(11*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$

$$= \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{11}\left(12a^2c^3\right)\int c$$

$$= \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(}}{11f}$$

$$= \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1105 vs. $2(145) = 290$.
time = 6.27, size = 1105, normalized size = 7.62

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (7*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4) - (Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^
(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4) + (11*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(7/2))/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x
])^2*(c - c*Sin[e + f*x])^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(9*(e + f*x))/2]*(a +
a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(11*(e + f
*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(176*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (
7*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f*
```

```
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4) + ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x)
)/2])/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) + (11*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Si
n[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*
x])^(7/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((a + a*Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e +
f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

**Maple** [**A**]
time = 1.77, size = 81, normalized size = 0.56

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^3a^2(105(\sin^3(fx+e))-455(\sin^2(fx+e))+755\sin(fx+e)-533)}{1155\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ | 81 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*(105*sin(f*x+e)^3-455*sin(f*
x+e)^2+755*sin(f*x+e)-533)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [**A**]
time = 0.34, size = 249, normalized size = 1.72

$$\frac{2\left(105\,a^2c^3\cos\left(fx+e\right)^6+245\,a^2c^3\cos\left(fx+e\right)^5-20\,a^2c^3\cos\left(fx+e\right)^4+32\,a^2c^3\cos\left(fx+e\right)^3-64\,a^2c^3\cos\left(fx+e\right)^2+256\,a^2c^3\cos\left(fx+e\right)+512\,a^2c^3-\left(105\,a^2c^3\cos\left(fx+e\right)^5-140\,a^2c^3\cos\left(fx+e\right)^4-160\,a^2c^3\cos\left(fx+e\right)^3-192\,a^2c^3\cos\left(fx+e\right)^2-256\,a^2c^3\cos\left(fx+e\right)-512\,a^2c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{1155\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^2*c^3*cos(f*x + e)^6 + 245*a^2*c^3*cos(f*x + e)^5 - 20*a^2*c^
3*cos(f*x + e)^4 + 32*a^2*c^3*cos(f*x + e)^3 - 64*a^2*c^3*cos(f*x + e)^2 +
```

```
256*a^2*c^3*cos(f*x + e) + 512*a^2*c^3 - (105*a^2*c^3*cos(f*x + e)^5 - 140*
a^2*c^3*cos(f*x + e)^4 - 160*a^2*c^3*cos(f*x + e)^3 - 192*a^2*c^3*cos(f*x +
 e)^2 - 256*a^2*c^3*cos(f*x + e) - 512*a^2*c^3)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

**Giac** [A]
time = 0.62, size = 222, normalized size = 1.53

$$-\frac{\sqrt{2}\left(16170\,a^{2}c^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2310\,a^{2}c^{3}\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2541\,a^{2}c^{3}\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+165\,a^{2}c^{3}\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+385\,a^{2}c^{3}\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-105\,a^{2}c^{3}\cos\left(-\frac{11}{4}\,\pi+\frac{11}{2}\,fx+\frac{11}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{18480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/18480*sqrt(2)*(16170*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + 2310*a^2*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) - 2541*a^2*c^3*cos(-5/4*pi + 5/2*f*x + 5/2*e
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 165*a^2*c^3*cos(-7/4*pi + 7/2*f*x +
 7/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 385*a^2*c^3*cos(-9/4*pi + 9/2
*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 105*a^2*c^3*cos(-11/4*p
i + 11/2*f*x + 11/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2), x)
```

# 3.299 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=109

$$\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 64/315*a^2*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/63*a^2*c^4*cos(f*x+
e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/9*a^2*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(
1/2)
```

**Rubi** [A]
time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {2815, 2753, 2752}

$$\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^2*c^5*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (16*a^2*c^
4*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^2*c^3*Cos[e + f*
x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{9}\left(8a^2c^3\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx}}$$
$$= \frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{63}$$
$$= \frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} +$$

**Mathematica [A]**
time = 3.35, size = 96, normalized size = 0.88

$$-\frac{a^2c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(-249+35\cos(2(e+fx))+220\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/315*(a^2*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-249 + 35*Cos[2*(e
 + f*x)] + 220*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 1.73, size = 71, normalized size = 0.65

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^3a^2\left(35\left(\sin^2(fx+e)\right)-110\sin(fx+e)+107\right)}{315\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ | 71 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(35*sin(f*x+e)^2-110*sin(f*x
+e)+107)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(103) = 206.
time = 0.34, size = 214, normalized size = 1.96

$$\frac{2\,\left(35\,a^{2}c^{2}\cos\left(fx+e\right)^{5}-5\,a^{2}c^{2}\cos\left(fx+e\right)^{4}+8\,a^{2}c^{2}\cos\left(fx+e\right)^{3}-16\,a^{2}c^{2}\cos\left(fx+e\right)^{2}+64\,a^{2}c^{2}\cos\left(fx+e\right)+128\,a^{2}c^{2}+\left(35\,a^{2}c^{2}\cos\left(fx+e\right)^{4}+40\,a^{2}c^{2}\cos\left(fx+e\right)^{3}+48\,a^{2}c^{2}\cos\left(fx+e\right)^{2}+64\,a^{2}c^{2}\cos\left(fx+e\right)+128\,a^{2}c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{315\,\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*c^2*cos(f*x + e)^5 - 5*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos
(f*x + e)^3 - 16*a^2*c^2*cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2
*c^2 + (35*a^2*c^2*cos(f*x + e)^4 + 40*a^2*c^2*cos(f*x + e)^3 + 48*a^2*c^2*
cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-2c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx+\int c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*c**2*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(c**2*sqrt(-c*sin(e + f*x
) + c)*sin(e + f*x)**4, x))
```

**Giac** [A]
time = 0.57, size = 187, normalized size = 1.72

$$-\frac{\sqrt{2}\,\left(1890\,a^{2}c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+420\,a^{2}c^{2}\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-252\,a^{2}c^{2}\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-45\,a^{2}c^{2}\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+35\,a^{2}c^{2}\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(1890*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 420*a^2*c^2*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 252*a^2*c^2*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 45*a^2*c^2*cos(-7/4*pi + 7/2*f*x + 7/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 35*a^2*c^2*cos(-9/4*pi + 9/2*f*x +
 9/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin(e + fx))^2 (c - c\sin(e + fx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2), x)
```

# 3.300 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=73

$$\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 8/35*a^2*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/7*a^2*c^3*cos(f*x+e)^5
/f/(c-c*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c^4*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^2*c^3*C
os[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{1}{7}\left(4a^2c^3\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx)}\\ &= \frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

**Mathematica** [A]
time = 0.85, size = 84, normalized size = 1.15

$$-\frac{2a^2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(-9+5\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
[Out] (-2*a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-9 + 5*Sin[e + f*x])*Sqr
t[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.94, size = 61, normalized size = 0.84

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^3a^2(5\sin(fx+e)-9)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/35*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(5*sin(f*x+e)-9)/cos(f*x+e)/(c
-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(69) = 138$.
time = 0.35, size = 163, normalized size = 2.23

$$-\frac{2\left(5 a^{2} c \cos\left(fx+e\right)^{4}-a^{2} c \cos\left(fx+e\right)^{3}+2 a^{2} c \cos\left(fx+e\right)^{2}-8 a^{2} c \cos\left(fx+e\right)-16 a^{2} c-\left(5 a^{2} c \cos\left(fx+e\right)^{3}+6 a^{2} c \cos\left(fx+e\right)^{2}+8 a^{2} c \cos\left(fx+e\right)+16 a^{2} c\right) \sin\left(fx+e\right)\right) \sqrt{-c \sin\left(fx+e\right)+c}}{35\left(f \cos\left(fx+e\right)-f \sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*a^2*c*cos(f*x + e)^4 - a^2*c*cos(f*x + e)^3 + 2*a^2*c*cos(f*x + e)
^2 - 8*a^2*c*cos(f*x + e) - 16*a^2*c - (5*a^2*c*cos(f*x + e)^3 + 6*a^2*c*co
s(f*x + e)^2 + 8*a^2*c*cos(f*x + e) + 16*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\, dx+\int c\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right) dx+\int\left(-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)\right) dx+\int\left(-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(c*sqrt(-c*sin(e +
 f*x) + c)*sin(e + f*x), x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e +
 f*x)**2, x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(69) = 138$.
time = 0.57, size = 144, normalized size = 1.97

$$-\frac{\sqrt{2}\left(105 a^{2} c \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+35 a^{2} c \cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7 a^{2} c \cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5 a^{2} c \cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{c}}{140 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/140*sqrt(2)*(105*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 35*a^2*c*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 7*a^2*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 5*a^2*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2), x)
```

# 3.301 $\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=36

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 2/5*a^2*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2815, 2752}

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\ &= \frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.
time = 0.16, size = 73, normalized size = 2.03

$$\frac{2a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])/(5
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.32, size = 49, normalized size = 1.36

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^3a^2}{5\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ | 49 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(34) = 68.
time = 0.34, size = 123, normalized size = 3.42

$$-\frac{2\left(a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2+\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^
2 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)\,dx+\int\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)\,dx+\int\sqrt{-c\sin(e+fx)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x) +
 c), x))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(34) = 68.
time = 0.51, size = 107, normalized size = 2.97

$$-\frac{\sqrt{2}\left(10\,a^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,a^2\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+a^2\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2)*(10*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 5*a^2*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + a^2*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*sqrt(c)/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2), x)
```

# 3.302 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=115

$$\frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*arctanh(1/2*cos(f*x+
e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*cos(f*x+
e)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2758, 2728, 212}

$$-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (4*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (4*a^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(2a^2c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+}\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{\left(8a^2\right)\operatorname{Subst}\left(\int \frac{1}{2c-x^2}\,dx,\right.}{}\\
&= \frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4}{f\sqrt{c}}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.29, size = 130, normalized size = 1.13

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((24+24i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+15\cos\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{3}{2}(e+fx)\right)+15\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)\right)}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/3*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((24 + 24*I)*(-1)^(1/4)*Arc
Tan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 15*Cos[(e + f*x)/2] -
Cos[(3*(e + f*x))/2] + 15*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(f*Sqrt
[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.29, size = 112, normalized size = 0.97

| method | result |
|---|---|
| default | $-\frac{2(\sin(fx+e)-1)\sqrt{c\,(1+\sin(fx+e))}\,a^2\left(6c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)-(c(1+\sin(fx+e$ |
| | $3c^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(6*c^(3/2)*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(3/2)-6*(
c*(1+sin(f*x+e)))^(1/2)*c)/c^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(106) = 212$.
time = 0.34, size = 258, normalized size = 2.24

$$\frac{2\left(\frac{3\sqrt{2}\left(a^2c\cos(fx+e)-a^2c\sin(fx+e)+a^2c\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}+\left(a^2\cos(fx+e)^2-7a^2\cos(fx+e)-8a^2-\left(a^2\cos(fx+e)+8a^2\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}\right)}{3\left(cf\cos(fx+e)-cf\sin(fx+e)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c*sin(f*x + e) + a^2*c)*log(-(cos(
f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c)
 + (a^2*cos(f*x + e)^2 - 7*a^2*cos(f*x + e) - 8*a^2 - (a^2*cos(f*x + e) + 8
*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{1}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c*sin(e + f*x)
 + c), x))
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(106) = 212.
time = 0.51, size = 220, normalized size = 1.91

$$\frac{2\left(\frac{3\sqrt{2}a^2\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{8\sqrt{2}\left(2a^2\sqrt{c}-\frac{3a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(2)*a^2*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 8*s
qrt(2)*(2*a^2*sqrt(c) - 3*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c*((cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^2}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.303 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=115

$$-\frac{3\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)-3*a^2*arctanh(1/2*cos(f*x+e)*c^
(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+3*a^2*cos(f*x+e)/c/
f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2758, 2728, 212}

$$-\frac{3\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(c^(3/2)*f) + (a^2*c*Cos[e + f*x]^3)/(f*(c - c*Sin[e + f*x])^(5/2)
) + (3*a^2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} - \frac{1}{2}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+f}}{c}\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{\left(6a^2\right)\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x\right.}{c}\\
&= -\frac{3\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{}{cf}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.42, size = 149, normalized size = 1.30

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{1}{2}(e+fx)\right)-(6+6i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)(-1+\sin(e+fx))-\sin\left(\frac{3}{2}(e+fx)\right)\right)}{cf(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] + Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] - (6 + 6*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1 + Sin[e + f*x]) - Sin[(3*(e + f*x))
/2]))/(c*f*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

**Maple [A]**
time = 2.07, size = 146, normalized size = 1.27

| method | result |
|---|---|
| default | $-\frac{a^{2}\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c\sin(fx+e)+2\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c}\,\sin(fx+e)+3}{c^{\frac{5}{2}}\cos(fx+e)\sqrt{c-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/c^(5/2)*(-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*c*sin(f*x+e)+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*sin(f*x+e)+3*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^
(1/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(108) = 216$.
time = 0.35, size = 323, normalized size = 2.81

$$\frac{\frac{3\sqrt{2}\left(a^{2}c\cos(fx+e)^{2}-a^{2}c\cos(fx+e)-2a^{2}c+\left(a^{2}c\cos(fx+e)+2a^{2}c\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^{2}+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-4\left(a^{2}\cos\left(fx+e\right)^{2}+2a^{2}\cos\left(fx+e\right)+a^{2}-\left(a^{2}\cos\left(fx+e\right)-a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2c^{2}f+\left(c^{2}f\cos\left(fx+e\right)+2c^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e)^2 - a^2*c*cos(f*x + e) - 2*a^2*c + (a^2*
c*cos(f*x + e) + 2*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + s
in(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x +
 e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a^2*cos(f*x + e)^2
+ 2*a^2*cos(f*x + e) + a^2 - (a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{2\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{\sin^{2}\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) +
c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(sin(e + f*x)**2/(-c*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(1
/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)),
 x))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(108) = 216.
time = 0.52, size = 312, normalized size = 2.71

$$-\frac{\frac{6\sqrt{2}\,a^{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\,a^{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a^{2}\sqrt{c}-\frac{14\,a^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3\,a^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)}{c^{2}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(6*sqrt(2)*a^2*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sq
rt(2)*a^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c^(3/2)*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(a^2*sqrt(
c) - 14*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) - 3*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - (cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\sin(e + fx))^2}{(c - c\sin(e + fx))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.304 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-3/4*a^2*cos(f*x+e)/c/f/(c-c
*sin(f*x+e))^(3/2)+3/8*a^2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(
f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2759, 2728, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) + (a^2*c*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x])^(7
/2)) - (3*a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{1}{4}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx \\
&= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e}}}{8c^2} \\
&= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}} - \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\right.}{} \\
&= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f} + \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{3}{4cf(c}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.62, size = 163, normalized size = 1.34

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\cos\left(\frac{1}{2}(e+fx)\right)-5\cos\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{1}{2}(e+fx)\right)+(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(-3+\cos(2(e+fx))+4\sin(e+fx)+5\sin\left(\frac{3}{2}(e+fx)\right)\right)\right)}{8c^2f(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] - 5*Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[2*(e + f*x)] + 4*Sin[e + f*x]
) + 5*Sin[(3*(e + f*x))/2]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[
e + f*x]])
```

**Maple [A]**
time = 2.19, size = 191, normalized size = 1.57

| method | result |
|---|---|
| default | $-\frac{a^{2}\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}-6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{}}{2\sqrt{c}}\right.\right.}{8c^{\frac{9}{2}}\left(\mathrm{s}\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^
(1/2))*sin(f*x+e)^2*c^2-6*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+10*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*(c*(1+sin(f*x+e
)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(109) = 218.
time = 0.35, size = 390, normalized size = 3.20

$$\frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{3}+3a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)-4a^{2}-\left(a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)-4a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos(fx+e)+\sin(fx+e)+1\right)+3c\cos(fx+e)+\left(c\cos(fx+e)-2c\right)\sin(fx+e)+2c}{\cos(fx+e)^{2}+\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(5a^{2}\cos\left(fx+e\right)^{2}+a^{2}\cos\left(fx+e\right)-4a^{2}-\left(5a^{2}\cos\left(fx+e\right)+4a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{16\left(c^{3}f\cos\left(fx+e\right)^{3}+3c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x
+ e) - 4*a^2 - (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + 4*(5*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - 4*a
^2 - (5*a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/
```

```
(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c
^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e)
)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin(e+fx)}{c^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{\sin^2(e+fx)}{c^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{1}{c^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)*
*2 - 2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f
*x) + c)), x) + Integral(sin(e + f*x)**2/(c**2*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**2 - 2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-
c*sin(e + f*x) + c)), x) + Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
 + f*x)**2 - 2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*s
in(e + f*x) + c)), x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(109) = 218.
time = 0.59, size = 336, normalized size = 2.75

$$\frac{\frac{12\sqrt{2}a^2\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a^2\sqrt{c}+\frac{8a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{18a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\frac{8\sqrt{2}a^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\sqrt{2}a^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}}{c^6}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(12*sqrt(2)*a^2*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - s
qrt(2)*(a^2*sqrt(c) + 8*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 18*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) + (8*sqrt(2)*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + sqrt(2)*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/c^
6)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.305 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=156

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/4*a^2*cos(f*x+e)/c/f/(c-c
*sin(f*x+e))^(5/2)+1/16*a^2*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)+1/32*a^
2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*
2^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2729, 2728, 212}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) + (a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(9/
2)) - (a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*Cos[e +
f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{1}{2}a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c^2}\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+f x}\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+f x}\\
&= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2 c}{4cf(c-}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.61, size = 307, normalized size = 1.97

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-28\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6+64\sin\left(\frac{1}{2}(e+fx)\right)-56\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+6\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{48f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 28*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 3*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*Sin[
(e + f*x)/2] - 56*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]
+ 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])
^(7/2))
```

**Maple [A]**
time = 2.47, size = 245, normalized size = 1.57

| method | result |
|---|---|
| default | $-\frac{a^{2}\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{3}\left(fx+e\right)\right)c^{4}-9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}}{2\sqrt{c}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
sin(f*x+e)^3*c^4-9*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(
1/2))*sin(f*x+e)^2*c^4-6*(c*(1+sin(f*x+e)))^(5/2)*c^(3/2)-32*(c*(1+sin(f*x+
e)))^(3/2)*c^(5/2)+24*(c*(1+sin(f*x+e)))^(1/2)*c^(7/2)+9*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^4-3*2^(1/2)*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/
2)/c^(15/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(141) = 282$.
time = 0.37, size = 474, normalized size = 3.04

$$\frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{4}-3a^{2}\cos\left(fx+e\right)^{3}-8a^{2}\cos\left(fx+e\right)^{2}+4a^{2}\cos\left(fx+e\right)+8a^{2}+\left(a^{2}\cos\left(fx+e\right)^{3}+4a^{2}\cos\left(fx+e\right)^{2}-4a^{2}\cos\left(fx+e\right)-8a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos(fx+e)+\sin(fx+e)+1\right)+3c\cos(fx+e)+\left(c\cos(fx+e)-2c\right)\sin(fx+e)+2c}{\cos(fx+e)^{2}+\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(3a^{2}\cos\left(fx+e\right)^{3}+25a^{2}\cos\left(fx+e\right)^{2}-10a^{2}\cos\left(fx+e\right)-32a^{2}+\left(3a^{2}\cos\left(fx+e\right)^{2}-22a^{2}\cos\left(fx+e\right)-32a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{192\left(c^{4}f\cos\left(fx+e\right)^{4}-3c^{4}f\cos\left(fx+e\right)^{3}-8c^{4}f\cos\left(fx+e\right)^{2}+4c^{4}f\cos\left(fx+e\right)+8c^{4}f+\left(c^{4}f\cos\left(fx+e\right)^{3}+4c^{4}f\cos\left(fx+e\right)^{2}-4c^{4}f\cos\left(fx+e\right)-8c^{4}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 - 8*a^2*cos(f*x
 + e)^2 + 4*a^2*cos(f*x + e) + 8*a^2 + (a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x
+ e)^2 - 4*a^2*cos(f*x + e) - 8*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(3*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 - 10*a^2*cos(f*x + e) - 32*a
^2 + (3*a^2*cos(f*x + e)^2 - 22*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8
*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e
)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e)
)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(141) = 282.
time = 0.64, size = 405, normalized size = 2.60

$$\frac{\frac{12\sqrt{2}\,a^2\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(a^2\sqrt{c}+\frac{3a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{22a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}{c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\frac{3a^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3a^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{a^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{c^{12}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] 1/768*(12*sqrt(2)*a^2*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) +
sqrt(2)*(a^2*sqrt(c) + 3*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 22*a^2*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*a^2*c^(17/2)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*
a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
```

```
+ 1/2*e) + 1)^2 - a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\sin(e + fx))^2}{(c - c\sin(e + fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(7/2), x)
```

# 3.306 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=190

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}}+\frac{a^2\cos}{64c^2f(c-c}$$

```
[Out] 1/4*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^2*cos(f*x+e)/c/f/(c-
c*sin(f*x+e))^(7/2)+1/64*a^2*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)+3/256*
a^2*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)+3/512*a^2*arctanh(1/2*cos(f*x+e
)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.21, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2729, 2728, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f}+\frac{3a^2\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(256*Sqrt[2]*c^(9/2)*f) + (a^2*c*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^
(11/2)) - (a^2*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(7/2)) + (a^2*Cos[
e + f*x])/(64*c^2*f*(c - c*Sin[e + f*x])^(5/2)) + (3*a^2*Cos[e + f*x])/(256
*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{5/2}}}{16c^2}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx}{64c^2f(c-c\sin(e+}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx}{64c^2f(c-c\sin(e+}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx}{64c^2f(c-c\sin(e+}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f} + \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{}{8cf(c}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.93, size = 371, normalized size = 1.95

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(128\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-96\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+4\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7-(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8+256\sin\left(\frac{1}{2}(e+fx)\right)-192\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)+6\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{256f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(128*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 96*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 4*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5 + 3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 - (3 +
 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 256*Sin[(e + f*x)/2] - 192*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 8*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4*Sin[(e + f*x)/2] + 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(256*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

**Maple** [A]
time = 2.68, size = 299, normalized size = 1.57

| method | result |
|---|---|
| default | $\frac{a^2\left(6(c(1+\sin(fx+e)))^{\frac{7}{2}}c^{\frac{5}{2}}-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^4(fx+e))c^6-44(c(1+\sin(fx+e)))^{\frac{5}{2}}c^{\frac{7}{2}}+12\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/c^(21/2)*a^2*(6*(c*(1+sin(f*x+e)))^(7/2)*c^(5/2)-3*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^6-44*(c*(1+sin(f
*x+e)))^(5/2)*c^(7/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^3*c^6-88*(c*(1+sin(f*x+e)))^(3/2)*c^(9/2)-18*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^6+48*
(c*(1+sin(f*x+e)))^(1/2)*c^(11/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^6-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+
e)))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/
cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(173) = 346$.
time = 0.35, size = 563, normalized size = 2.96

$$\frac{3\sqrt{2}\left(a^2\cos(fx+e)^5+5a^2\cos(fx+e)^4-8a^2\cos(fx+e)^3-20a^2\cos(fx+e)^2+8a^2\cos(fx+e)+16a^2-\left(a^2\cos(fx+e)^4-4a^2\cos(fx+e)^3-12a^2\cos(fx+e)^2+8a^2\cos(fx+e)+16a^2\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(3a^2\cos(fx+e)^4+13a^2\cos(fx+e)^3+86a^2\cos(fx+e)^2-52a^2\cos(fx+e)-128a^2-\left(3a^2\cos(fx+e)^3-10a^2\cos(fx+e)^2+76a^2\cos(fx+e)+128a^2\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{1024\left(c^5f\cos(fx+e)^5+5c^5f\cos(fx+e)^4-8c^5f\cos(fx+e)^3-20c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f-\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3-12c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*(a^2*cos(f*x + e)^5 + 5*a^2*cos(f*x + e)^4 - 8*a^2*cos(f*
x + e)^3 - 20*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + 16*a^2 - (a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 - 12*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x +
e) + 16*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*
x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*a^2*cos(f*x + e)^4 +
 13*a^2*cos(f*x + e)^3 + 86*a^2*cos(f*x + e)^2 - 52*a^2*cos(f*x + e) - 128*
a^2 - (3*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 + 76*a^2*cos(f*x + e) +
 128*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x + e)^5 +
5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 +
 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x
+ e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x
 + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

**Giac** [A]
time = 0.64, size = 343, normalized size = 1.81

$$\frac{\frac{12\sqrt{2}a^2\log\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c^{\frac{9}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a^2\sqrt{c}-\frac{8a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{18a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}{c^5\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\frac{8\sqrt{2}a^2c^{\frac{11}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{\sqrt{2}a^2c^{\frac{11}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{c^{10}}}{8192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] 1/8192*(12*sqrt(2)*a^2*log((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^(9/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - sqrt(2)*(a^2*sqrt(c) - 8*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 18*a^2*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (8*sqrt(2)*a^2*c^(11/2)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2 - sqrt(2)*a^2*c^(11/2)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^4)/c^10)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(9/2), x)
```

# 3.307 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=145

$$\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 256/3003*a^3*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/429*a^3*c^6*cos(f
*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+24/143*a^3*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x
+e))^(3/2)+2/13*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a^3*c^7*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x])^(7/2)) + (64*a^3*
c^6*Cos[e + f*x]^7)/(429*f*(c - c*Sin[e + f*x])^(5/2)) + (24*a^3*c^5*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*c^4*Cos[e + f*x]^7)/(
13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx &= \left(a^3c^3\right)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{1}{13}\left(12a^3c^4\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+}}\\ &= \frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{1}{14}\\ &= \frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\\ &= \frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

**Mathematica [A]**
time = 5.40, size = 112, normalized size = 0.77

$$\frac{a^3c^3\cos^6(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(5230-1890\cos(2(e+fx))-6377\sin(e+fx)+231\sin(3(e+fx)))}{6006f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Si
n[e + f*x]]*(5230 - 1890*Cos[2*(e + f*x)] - 6377*Sin[e + f*x] + 231*Sin[3*(
e + f*x)]))/(6006*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

**Maple [A]**
time = 2.32, size = 81, normalized size = 0.56

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^4a^3(231(\sin^3(fx+e))-945(\sin^2(fx+e))+1421\sin(fx+e)-835)}{3003\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 81 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

[Out] 2/3003*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*(231*sin(f*x+e)^3-945*sin(f*x+e)^2+1421*sin(f*x+e)-835)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(137) = 274$.
time = 0.34, size = 282, normalized size = 1.94

$$\frac{2\left(231\,a^3c^3\cos\left(fx+e\right)^7-21\,a^3c^3\cos\left(fx+e\right)^6+28\,a^3c^3\cos\left(fx+e\right)^5-40\,a^3c^3\cos\left(fx+e\right)^4+64\,a^3c^3\cos\left(fx+e\right)^3-128\,a^3c^3\cos\left(fx+e\right)^2+512\,a^3c^3\cos\left(fx+e\right)+1024\,a^3c^3+\left(231\,a^3c^3\cos\left(fx+e\right)^6+252\,a^3c^3\cos\left(fx+e\right)^5+280\,a^3c^3\cos\left(fx+e\right)^4+320\,a^3c^3\cos\left(fx+e\right)^3+384\,a^3c^3\cos\left(fx+e\right)^2+512\,a^3c^3\cos\left(fx+e\right)+1024\,a^3c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3003\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*a^3*c^3*cos(f*x + e)^7 - 21*a^3*c^3*cos(f*x + e)^6 + 28*a^3*c^3
*cos(f*x + e)^5 - 40*a^3*c^3*cos(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^3 - 1
28*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c^3*cos(f*x + e) + 1024*a^3*c^3 + (231*
a^3*c^3*cos(f*x + e)^6 + 252*a^3*c^3*cos(f*x + e)^5 + 280*a^3*c^3*cos(f*x +
 e)^4 + 320*a^3*c^3*cos(f*x + e)^3 + 384*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c
^3*cos(f*x + e) + 1024*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

**Giac [A]**
time = 0.60, size = 257, normalized size = 1.77

$$-\frac{\sqrt{2}\left(60060\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15015\,a^3c^3\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-9009\,a^3c^3\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2574\,a^3c^3\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2002\,a^3c^3\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+273\,a^3c^3\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-231\,a^3c^3\cos\left(-\frac{13}{4}\pi+\frac{13}{2}fx+\frac{13}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{c}}{96096\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/96096*sqrt(2)*(60060*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + 15015*a^3*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 9009*a^3*c^3*cos(-5/4*pi + 5/2*f*x + 5/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2574*a^3*c^3*cos(-7/4*pi + 7/2*f*x
 + 7/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2002*a^3*c^3*cos(-9/4*pi +
9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 273*a^3*c^3*cos(-11/
4*pi + 11/2*f*x + 11/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 231*a^3*c^3
*cos(-13/4*pi + 13/2*f*x + 13/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqr
t(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin(e + f\,x))^3\,(c - c\sin(e + f\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2), x)
```

# 3.308 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=109

$$\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 64/693*a^3*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/99*a^3*c^5*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/11*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
(3/2)
```

**Rubi** [A]
time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^3*c^6*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2)) + (16*a^3*c^
5*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^3*c^4*Cos[e + f*
x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{11}\left(8a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx)}\\ &= \frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{99}\\ &= \frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\end{aligned}$$

**Mathematica [A]**
time = 6.02, size = 96, normalized size = 0.88

$$-\frac{a^3c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(-365+63\cos(2(e+fx))+364\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{693f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/693*(a^3*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(-365 + 63*Cos[2*(e
 + f*x)] + 364*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.25, size = 71, normalized size = 0.65

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4a^3(63(\sin^2(fx+e))-182\sin(fx+e)+151)}{693\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 71 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(63*sin(f*x+e)^2-182*sin(f*x
+e)+151)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(103) = 206.
time = 0.34, size = 249, normalized size = 2.28

$$-\frac{2\,{\left(63\,a^{3} c^{2} \cos\left(f x + e\right)^{6} - 7\,a^{3} c^{2} \cos\left(f x + e\right)^{5} + 10\,a^{3} c^{2} \cos\left(f x + e\right)^{4} - 16\,a^{3} c^{2} \cos\left(f x + e\right)^{3} + 32\,a^{3} c^{2} \cos\left(f x + e\right)^{2} - 128\,a^{3} c^{2} \cos\left(f x + e\right) - 256\,a^{3} c^{2} - {\left(63\,a^{3} c^{2} \cos\left(f x + e\right)^{5} + 70\,a^{3} c^{2} \cos\left(f x + e\right)^{4} + 80\,a^{3} c^{2} \cos\left(f x + e\right)^{3} + 96\,a^{3} c^{2} \cos\left(f x + e\right)^{2} + 128\,a^{3} c^{2} \cos\left(f x + e\right) + 256\,a^{3} c^{2}\right)} \sin\left(f x + e\right)\right)} \sqrt{-c \sin\left(f x + e\right) + c}}{693\,{\left(f \cos\left(f x + e\right) - f \sin\left(f x + e\right) + f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^3*c^2*cos(f*x + e)^6 - 7*a^3*c^2*cos(f*x + e)^5 + 10*a^3*c^2*c
os(f*x + e)^4 - 16*a^3*c^2*cos(f*x + e)^3 + 32*a^3*c^2*cos(f*x + e)^2 - 128
*a^3*c^2*cos(f*x + e) - 256*a^3*c^2 - (63*a^3*c^2*cos(f*x + e)^5 + 70*a^3*c
^2*cos(f*x + e)^4 + 80*a^3*c^2*cos(f*x + e)^3 + 96*a^3*c^2*cos(f*x + e)^2 +
 128*a^3*c^2*cos(f*x + e) + 256*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3} \left(\int c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c}\, dx + \int c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c} \sin{\left(e + f x \right)}\, dx + \int \left(- 2 c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c} \sin^{2}{\left(e + f x \right)}\right)\, dx + \int \left(- 2 c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c} \sin^{3}{\left(e + f x \right)}\right)\, dx + \int c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c} \sin^{4}{\left(e + f x \right)}\, dx + \int c^{2} \sqrt{- c \sin{\left(e + f x \right)} + c} \sin^{5}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(c**2*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x), x) + Integral(-2*c**2*sqrt(-c*sin(e + f*x) +
 c)*sin(e + f*x)**2, x) + Integral(-2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x)**3, x) + Integral(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x)
 + Integral(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5, x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(103) = 206.
time = 0.62, size = 222, normalized size = 2.04

$$-\frac{\sqrt{2}\,{\left(6930\,a^{3} c^{2} \cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 2310\,a^{3} c^{2} \cos\left(-\frac{3}{4}\,\pi + \frac{3}{2}\,f x + \frac{3}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) - 693\,a^{3} c^{2} \cos\left(-\frac{5}{4}\,\pi + \frac{5}{2}\,f x + \frac{5}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) - 495\,a^{3} c^{2} \cos\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,f x + \frac{7}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 77\,a^{3} c^{2} \cos\left(-\frac{9}{4}\,\pi + \frac{9}{2}\,f x + \frac{9}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 63\,a^{3} c^{2} \cos\left(-\frac{11}{4}\,\pi + \frac{11}{2}\,f x + \frac{11}{2}\,e\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right)\right)} \sqrt{c}}{11088\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/11088*sqrt(2)*(6930*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) + 2310*a^3*c^2*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 693*a^3*c^2*cos(-5/4*pi + 5/2*f*x + 5/2*e)*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 495*a^3*c^2*cos(-7/4*pi + 7/2*f*x + 7
/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 77*a^3*c^2*cos(-9/4*pi + 9/2*f*
x + 9/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 63*a^3*c^2*cos(-11/4*pi +
11/2*f*x + 11/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sin(e+fx))^3 (c-c\sin(e+fx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2), x)
```

# 3.309 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=73

$$\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 8/63*a^3*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/9*a^3*c^4*cos(f*x+e)^7
/f/(c-c*sin(f*x+e))^(5/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^3*c^5*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*a^3*c^4*C
os[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{1}{9}\left(4a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))}\\ &= \frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

**Mathematica [A]**
time = 1.87, size = 84, normalized size = 1.15

$$-\frac{2a^3c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(-11+7\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
[Out] (-2*a^3*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(-11 + 7*Sin[e + f*x])*Sq
rt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.54, size = 61, normalized size = 0.84

| method | result | size |
|---|---|---|
| default | $\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^4a^3(7\sin(fx+e)-11)}{63\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/63*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(7*sin(f*x+e)-11)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(69) = 138$.
time = 0.36, size = 192, normalized size = 2.63

$$-\frac{2\,(7\,a^{3}c\cos\left(fx+e\right)^{5}+17\,a^{3}c\cos\left(fx+e\right)^{4}-2\,a^{3}c\cos\left(fx+e\right)^{3}+4\,a^{3}c\cos\left(fx+e\right)^{2}-16\,a^{3}c\cos\left(fx+e\right)-32\,a^{3}c+(7\,a^{3}c\cos\left(fx+e\right)^{4}-10\,a^{3}c\cos\left(fx+e\right)^{3}-12\,a^{3}c\cos\left(fx+e\right)^{2}-16\,a^{3}c\cos\left(fx+e\right)-32\,a^{3}c)\sin\left(fx+e\right))\sqrt{-c\sin\left(fx+e\right)+c}}{63\,(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^3*c*cos(f*x + e)^5 + 17*a^3*c*cos(f*x + e)^4 - 2*a^3*c*cos(f*x +
 e)^3 + 4*a^3*c*cos(f*x + e)^2 - 16*a^3*c*cos(f*x + e) - 32*a^3*c + (7*a^3*
c*cos(f*x + e)^4 - 10*a^3*c*cos(f*x + e)^3 - 12*a^3*c*cos(f*x + e)^2 - 16*a
^3*c*cos(f*x + e) - 32*a^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*co
s(f*x + e) - f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\left(-2c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)\right)dx+\int\left(-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{4}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*c*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(-2*c*sqrt(-c*sin(e + f*x) + c)*sin
(e + f*x)**3, x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x
))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(69) = 138$.
time = 0.58, size = 144, normalized size = 1.97

$$-\frac{\sqrt{2}\,(378\,a^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+168\,a^{3}c\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-27\,a^{3}c\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\,a^{3}c\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right))\sqrt{c}}{504\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/504*sqrt(2)*(378*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 168*a^3*c*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 27*a^3*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) - 7*a^3*c*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2), x)
```

# 3.310 $\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=36

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 2/7*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2815, 2752}

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*c^4*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.
time = 0.25, size = 73, normalized size = 2.03

$$\frac{2a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\sqrt{c-c\sin(e+fx)}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])/(7
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.69, size = 49, normalized size = 1.36

| method | result | size |
|---|---|---|
| default | $-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^4a^3}{7\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ | 49 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(34) = 68.
time = 0.33, size = 152, normalized size = 4.22

$$\frac{2\left(a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3-\left(a^3\cos\left(fx+e\right)^3+4\,a^3\cos\left(fx+e\right)^2-4\,a^3\cos\left(fx+e\right)-8\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{7\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
+ e) - f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int 3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x)
 + c)*sin(e + f*x)**3, x) + Integral(sqrt(-c*sin(e + f*x) + c), x))
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(34) = 68.
time = 0.56, size = 139, normalized size = 3.86

$$-\frac{\sqrt{2}\left(35\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+21\,a^{3}\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,a^{3}\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+a^{3}\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{28\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(2)*(35*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 21*a^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 7*a^3*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + a^3*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2), x)
```

# 3.311 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=151

$$\frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos}{f\sqrt{c-cs}}$$

```
[Out] -2/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*c*cos(f*x+e)^3/f
/(c-c*sin(f*x+e))^(3/2)+8*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/
2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2758, 2728, 212}

$$-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^
(5/2)) - (4*a^3*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) - (8*a^3
*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(2a^3c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(4a^3c\right)\int \frac{\cos^2(e}{(c-c\sin(e}$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{}{3f(}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.46, size = 156, normalized size = 1.03

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((480+480i)\sqrt[4]{-1}\,\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+330\cos\left(\frac{1}{2}(e+fx)\right)-35\cos\left(\frac{3}{2}(e+fx)\right)-3\cos\left(\frac{5}{2}(e+fx)\right)+330\sin\left(\frac{1}{2}(e+fx)\right)+35\sin\left(\frac{3}{2}(e+fx)\right)-3\sin\left(\frac{5}{2}(e+fx)\right)\right)}{30f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
[Out] -1/30*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((480 + 480*I)*(-1)^(1/4)*
ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 330*Cos[(e + f*x)/2
```

```
] - 35*Cos[(3*(e + f*x))/2] - 3*Cos[(5*(e + f*x))/2] + 330*Sin[(e + f*x)/2]
 + 35*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2]))/(f*Sqrt[c - c*Sin[e +
 f*x]])
```

**Maple [A]**
time = 2.70, size = 129, normalized size = 0.85

| method | result |
|---|---|
| default | $-\frac{2(\sin(fx+e)-1)\sqrt{c\,(1+\sin\,(fx+e))}\;a^3\left(60c^{\frac{5}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin\,(fx+e))}\;\sqrt{2}}{2\sqrt{c}}\right)-3(c(1+\sin(fx+e}{15c^3\cos(fx+e)\sqrt{c-c\sin\,(fx+e)}\;f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(60*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-3*(c*(1+sin(f*x+e)))^(5/2)
-10*c*(c*(1+sin(f*x+e)))^(3/2)-60*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(140) = 280.
time = 0.37, size = 287, normalized size = 1.90

$$\frac{2\left(\frac{30\sqrt{2}\left(a^3c\cos(fx+e)-a^3c\sin(fx+e)+a^3c\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin\,(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}+\left(3\,a^3\cos\,(fx+e)^3+19\,a^3\cos\,(fx+e)^2-76\,a^3\cos\,(fx+e)-92\,a^3+\left(3\,a^3\cos\,(fx+e)^2-16\,a^3\cos\,(fx+e)-92\,a^3\right)\sin\,(fx+e)\right)\sqrt{-c\sin\,(fx+e)+c}\right)}{15\,(cf\cos\,(fx+e)-cf\sin\,(fx+e)+cf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(30*sqrt(2)*(a^3*c*cos(f*x + e) - a^3*c*sin(f*x + e) + a^3*c)*log(-(co
s(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x
+ e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(
```

```
c) + (3*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 76*a^3*cos(f*x + e) -
92*a^3 + (3*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*x + e) - 92*a^3)*sin(f*x + e)
)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{3\sin{\left(e+fx\right)}}{\sqrt{-c\sin{\left(e+fx\right)}+c}}\,dx+\int \frac{3\sin^{2}{\left(e+fx\right)}}{\sqrt{-c\sin{\left(e+fx\right)}+c}}\,dx+\int \frac{\sin^{3}{\left(e+fx\right)}}{\sqrt{-c\sin{\left(e+fx\right)}+c}}\,dx+\int \frac{1}{\sqrt{-c\sin{\left(e+fx\right)}+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*si
n(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(sin(e + f*x)**3/sqrt
(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c*sin(e + f*x) + c), x))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(140) = 280.
time = 0.49, size = 304, normalized size = 2.01

$$\frac{4\left(\frac{15\sqrt{2}a^{3}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\sqrt{2}\left(23a^{3}\sqrt{c}-\frac{70a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{140a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}-\frac{90a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}+\frac{45a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{4}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{4}}\right)}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^{5}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*sqrt(2)*a^3*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 4
*sqrt(2)*(23*a^3*sqrt(c) - 70*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 140*a^3*sqrt(c)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*a^3*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^3 + 45*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^4)/(c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))
/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.312 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=150

$$-\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\,\cdots}{cf\sqrt{c-\cdots}}$$

```
[Out] a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+5/3*a^3*cos(f*x+e)^3/f/(c-c*s
in(f*x+e))^(3/2)-10*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x
+e))^(1/2))*2^(1/2)/c^(3/2)/f+10*a^3*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2758, 2728, 212}

$$-\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-10*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(c^(3/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^(7
/2)) + (5*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) + (10*a^3*Co
s[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} - \frac{1}{2}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \left(5a^3\right)\int \frac{\cos^2(e+}{(c-c\sin(e+} \\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} \\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} +
\end{aligned}
$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.50, size = 173, normalized size = 1.15

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(50\cos\left(\frac{1}{2}(e+fx)\right)+25\cos\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{5}{2}(e+fx)\right)+50\sin\left(\frac{1}{2}(e+fx)\right)-(120+120i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)(-1+\sin(e+fx))-25\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)\right)}{6cf(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(50*Cos[(e + f*x)/2] + 25*C
os[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] + 50*Sin[(e + f*x)/2] - (120 + 1
20*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1
+ Sin[e + f*x]) - 25*Sin[(3*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(c*f*(-1
 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 1.88, size = 189, normalized size = 1.26

| method | result |
|---|---|
| default | $\frac{2a^3\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin\,(fx+e))}\ \sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c^2-(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{c}\ \sin(fx+e)-12\sqrt{c\,(1+\mathrm{si}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*s
in(f*x+e)*c^2-(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)*sin(f*x+e)-12*(c*(1+sin(f*x+
e)))^(1/2)*c^(3/2)*sin(f*x+e)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)/c^(1/2))*c^2+(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+18*(c*(1+sin(f*x+e
)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(7/2)/cos(f*x+e)/(c-c*sin(f*x
+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(141) = 282$.
time = 0.36, size = 352, normalized size = 2.35

$$\frac{\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2-a^3c\cos(fx+e)-2a^3c+\left(a^3c\cos(fx+e)+2a^3c\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-2\left(a^3\cos(fx+e)^3+13a^3\cos(fx+e)^2+18a^3\cos(fx+e)+6a^3+\left(a^3\cos(fx+e)^2-12a^3\cos(fx+e)+6a^3\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{3\left(c^2f\cos(fx+e)^2-c^2f\cos(fx+e)-2c^2f+\left(c^2f\cos(fx+e)+2c^2f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e) - 2*a^3*c + (a^3
*c*cos(f*x + e) + 2*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x +
e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a^3*cos(f*x + e)^3
 + 13*a^3*cos(f*x + e)^2 + 18*a^3*cos(f*x + e) + 6*a^3 + (a^3*cos(f*x + e)^
2 - 12*a^3*cos(f*x + e) + 6*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(
c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) +
 2*c^2*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{3\sin{\left(e+fx\right)}}{-c\sqrt{-c\sin{\left(e+fx\right)}+c}\,\sin{\left(e+fx\right)}+c\sqrt{-c\sin{\left(e+fx\right)}+c}}\, dx+\int \frac{3\sin^{2}{\left(e+fx\right)}}{-c\sqrt{-c\sin{\left(e+fx\right)}+c}\,\sin{\left(e+fx\right)}+c\sqrt{-c\sin{\left(e+fx\right)}+c}}\, dx+\int \frac{\sin^{3}{\left(e+fx\right)}}{-c\sqrt{-c\sin{\left(e+fx\right)}+c}\,\sin{\left(e+fx\right)}+c\sqrt{-c\sin{\left(e+fx\right)}+c}}\, dx+\int \frac{1}{-c\sqrt{-c\sin{\left(e+fx\right)}+c}\,\sin{\left(e+fx\right)}+c\sqrt{-c\sin{\left(e+fx\right)}+c}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) +
c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(3*sin(e + f*x)**2/(-c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral
(sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin
(e + f*x) + c)), x) + Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + c*sqrt(-c*sin(e + f*x) + c)), x))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(141) = 282.
time = 0.57, size = 376, normalized size = 2.51

$$-\frac{\frac{30\sqrt{2}\,a^{3}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\,a^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(a^{3}+\frac{10a^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{16\sqrt{2}\left(7a^{3}\sqrt{c}-\frac{12a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)}{c^{2}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(30*sqrt(2)*a^3*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 3
*sqrt(2)*a^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c^(3/2)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(a^3
+ 10*a^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c^(3/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 16*sqrt(2)*(7*a^3*s
```

```
qrt(c) - 12*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 9*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^3}{(c - c\,\sin(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.313 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=157

$$\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}+\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}-\frac{15a^3\cos}{4c^2f\sqrt{c-c}}$$

```
[Out] 1/2*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-5/4*a^3*cos(f*x+e)^3/f/(c
-c*sin(f*x+e))^(5/2)+15/4*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-15/4*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+
e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2758, 2728, 212}

$$\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}+\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (15*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])
^(9/2)) - (5*a^3*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (15*a^3
*Cos[e + f*x])/(4*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(15a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}}{8c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f} + \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a}{4f(c-}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.63, size = 187, normalized size = 1.19

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-5\cos\left(\frac{1}{2}(e+fx)\right)-15\cos\left(\frac{3}{2}(e+fx)\right)+2\cos\left(\frac{5}{2}(e+fx)\right)-5\sin\left(\frac{1}{2}(e+fx)\right)+(15+15i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)(-3+\cos(2(e+fx))+4\sin(e+fx))+15\sin\left(\frac{3}{2}(e+fx)\right)+2\sin\left(\frac{5}{2}(e+fx)\right)\right)}{4c^2f(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-5*Cos[(e + f*x)/2] - 15*Cos[(3
*(e + f*x))/2] + 2*Cos[(5*(e + f*x))/2] - 5*Sin[(e + f*x)/2] + (15 + 15*I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[
2*(e + f*x)] + 4*Sin[e + f*x]) + 15*Sin[(3*(e + f*x))/2] + 2*Sin[(5*(e + f*
x))/2]))/(4*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 2.98, size = 239, normalized size = 1.52

| method | result |
|---|---|
| default | $-\frac{a^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2(fx+e)\right)c^2-8\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,c^{\frac{3}{2}}\left(\sin^2(fx+e)\right)\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^(9/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)^2*c^2-8*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*sin(f*x+e)^2-30
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^2+18*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+16*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*
sin(f*x+e)+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c^2-36*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x
+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(142) = 284.
time = 0.35, size = 418, normalized size = 2.66

$$\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^3+3\,a^3\cos\left(fx+e\right)^2-2\,a^3\cos\left(fx+e\right)-4\,a^3-\left(a^3\cos\left(fx+e\right)^2-2\,a^3\cos\left(fx+e\right)-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(4\,a^3\cos\left(fx+e\right)^3-13\,a^3\cos\left(fx+e\right)^2-13\,a^3\cos\left(fx+e\right)+4\,a^3+\left(4\,a^3\cos\left(fx+e\right)^2+17\,a^3\cos\left(fx+e\right)+4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{8\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^3 + 3*a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x
+ e) - 4*a^3 - (a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) - 4*a^3)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) - 4*(4*a^3*cos(f*x + e)^3 - 13*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3 + (4*a^3*cos(f*x + e)^2 + 17*a^3*cos(f*x + e)
 + 4*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 +
3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x +
e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(142) = 284.
time = 0.64, size = 400, normalized size = 2.55

$$\frac{\frac{60\sqrt{2}a^{3}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{128\sqrt{2}a^{3}}{c^{\frac{5}{2}}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(a^{3}\sqrt{c}+\frac{16a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{90a^{3}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}{c^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\frac{16\sqrt{2}a^{3}c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\sqrt{2}a^{3}c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}}{c^{6}}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(60*sqrt(2)*a^3*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 1
28*sqrt(2)*a^3/(c^(5/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)
*(a^3*sqrt(c) + 16*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))) + (16*sqrt(2)*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e
) + 1) + sqrt(2)*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.314 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=157

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}+\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}+\frac{5a^3 c}{8c^2f(c-c}$$

```
[Out] 1/3*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/12*a^3*cos(f*x+e)^3/f/
(c-c*sin(f*x+e))^(7/2)+5/8*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)-5/16
*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)
/f*2^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2759, 2728, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}+\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}+\frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(8*Sqrt[2]*c^(7/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])
^(11/2)) - (5*a^3*Cos[e + f*x]^3)/(12*f*(c - c*Sin[e + f*x])^(7/2)) + (5*a^
3*Cos[e + f*x])/(8*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
```

```
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{1}{6}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))}}{8c} \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx}{8c^2f(c-c\sin(e+f} \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx}{8c^2f(c-c\sin(e+f} \\
&= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{7/2}f} + \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{}{12f}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.00, size = 307, normalized size = 1.96

$$\frac{a^3\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(32\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-52\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3+33\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^5+(15+15i)\sqrt[4]{-1}\tan^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\tfrac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^6+64\sin\left(\tfrac{1}{2}(e+fx)\right)-104\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2\sin\left(\tfrac{1}{2}(e+fx)\right)+66\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4\sin\left(\tfrac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{24f\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 52*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 33*(Cos[(e + f*x)/
```

```
2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*S
in[(e + f*x)/2] - 104*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)
/2] + 66*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
f*x])^(7/2))
```

**Maple** [A]
time = 2.62, size = 245, normalized size = 1.56

| method | result |
|---|---|
| default | $\frac{a^3\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^3(fx+e)\right)c^3-45\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{}}{2\sqrt{c}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^3*c^3-45*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*2^(
1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3-1
60*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*c^3+120*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2))*(c*(1+s
in(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(142) = 284$.
time = 0.37, size = 474, normalized size = 3.02

$$\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3+\left(a^3\cos\left(fx+e\right)^3+4\,a^3\cos\left(fx+e\right)^2-4\,a^3\cos\left(fx+e\right)-8\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(33\,a^3\cos\left(fx+e\right)^3+19\,a^3\cos\left(fx+e\right)^2-46\,a^3\cos\left(fx+e\right)-32\,a^3+\left(33\,a^3\cos\left(fx+e\right)^2+14\,a^3\cos\left(fx+e\right)-32\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{96\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+e\right)^3+4\,c^4f\cos\left(fx+e\right)^2-4\,c^4f\cos\left(fx+e\right)-8\,c^4f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
 + e)^2 + 4*a^3*cos(f*x + e) + 8*a^3 + (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x
+ e)^2 - 4*a^3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(33*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 46*a^3*cos(f*x + e) - 32*
a^3 + (33*a^3*cos(f*x + e)^2 + 14*a^3*cos(f*x + e) - 32*a^3)*sin(f*x + e))*
sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 -
 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x +
 e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x +
e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^3}{(c-c\,\sin\left(e+f\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(7/2), x)
```

# 3.315 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,c^{9/2}f}+\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}-\frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}}+\frac{5a^3\,c}{32c^2f(c-}$$

```
[Out] 1/4*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-5/24*a^3*cos(f*x+e)^3/f/
(c-c*sin(f*x+e))^(9/2)+5/32*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)-5/1
28*a^3*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)-5/256*a^3*arctanh(1/2*cos(f*
x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2729, 2728, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,c^{9/2}f}-\frac{5a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}}-\frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(128*Sqrt[2]*c^(9/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x
])^(13/2)) - (5*a^3*Cos[e + f*x]^3)/(24*f*(c - c*Sin[e + f*x])^(9/2)) + (5*
a^3*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(5/2)) - (5*a^3*Cos[e + f*
x])/(128*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{1}{8}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))}}{16c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+f}{32c^2f(c-c\sin(e+}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+f}{32c^2f(c-c\sin(e+}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+f}{32c^2f(c-c\sin(e+}$$

$$= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{}{24f}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 1.61, size = 371, normalized size = 1.94

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(384\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-544\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+236\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-15\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7+(15+15i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8+768\sin\left(\frac{1}{2}(e+fx)\right)-1088\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+472\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)-30\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{384f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(384*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 544*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 236*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5 - 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 +
(15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 768*Sin[(e + f*x)/2] - 1088*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 472*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] - 30*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(384*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

**Maple [A]**
time = 3.56, size = 299, normalized size = 1.57

| method | result |
|---|---|
| default | $-\frac{a^3\left(-15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^4(fx+e))c^5+30(c(1+\sin(fx+e)))^{\frac{7}{2}}c^{\frac{3}{2}}+60\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/768/c^(19/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^4*c^5+30*(c*(1+sin(f*x+e)))^(7/2)*c^(3/2)+60*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5+292
*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)-90*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^5-440*(c*(1+sin(f*x+e)))^(3/2)*c^(7/
2)+60*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x
+e)*c^5+240*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)-15*2^(1/2)*arctanh(1/2*(c*(1+s
in(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e
)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(174) = 348$.
time = 0.38, size = 563, normalized size = 2.95

$$\frac{15\sqrt{2}\left(a^3\cos(fx+e)^5+5a^3\cos(fx+e)^4-8a^3\cos(fx+e)^3-20a^3\cos(fx+e)^2+8a^3\cos(fx+e)+16a^3-\left(a^3\cos(fx+e)^4-4a^3\cos(fx+e)^3-12a^3\cos(fx+e)^2+8a^3\cos(fx+e)+16a^3\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(15a^3\cos(fx+e)^4-191a^3\cos(fx+e)^3-338a^3\cos(fx+e)^2+252a^3\cos(fx+e)+384a^3-\left(15a^3\cos(fx+e)^3+206a^3\cos(fx+e)^2-132a^3\cos(fx+e)-384a^3\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{1536\left(c^5f\cos(fx+e)^5+5c^5f\cos(fx+e)^4-8c^5f\cos(fx+e)^3-20c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f-\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3-12c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(15*sqrt(2)*(a^3*cos(f*x + e)^5 + 5*a^3*cos(f*x + e)^4 - 8*a^3*cos(f
*x + e)^3 - 20*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) + 16*a^3 - (a^3*cos(
f*x + e)^4 - 4*a^3*cos(f*x + e)^3 - 12*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x +
 e) + 16*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt
(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f
*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^4
 - 191*a^3*cos(f*x + e)^3 - 338*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) +
 384*a^3 - (15*a^3*cos(f*x + e)^3 + 206*a^3*cos(f*x + e)^2 - 132*a^3*cos(f*
x + e) - 384*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x +
 e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*
cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)
*sin(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(174) = 348$.
time = 0.66, size = 539, normalized size = 2.82

$$-\frac{\frac{120\sqrt{2}a^3\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{9}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(3a^3\sqrt{c}+\frac{16a^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{24a^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{48a^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{250a^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}{c^5\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\frac{48\sqrt{2}a^3c^{\frac{31}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{24\sqrt{2}a^3c^{\frac{31}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{16\sqrt{2}a^3c^{\frac{31}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{3\sqrt{2}a^3c^{\frac{31}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{c^{20}}}{12288f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] -1/12288*(120*sqrt(2)*a^3*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(9/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) + sqrt(2)*(3*a^3*sqrt(c) + 16*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 24*a^3*sqrt(c)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 48*a^3*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^3 - 250*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
+ (48*sqrt(2)*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 24*sqrt(2)*
a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 16*sqrt(2)*a^3*c^(31/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3*sqrt(2)*a^3*c^(31/2)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^4)/c^20)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(9/2), x)
```

# 3.316 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx$

**Optimal**. Leaf size=225

$$-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f}+\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}}-\frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}}+\frac{a^3 \cdots}{16c^2f(c-\cdots}$$

```
[Out] 1/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(15/2)-1/8*a^3*cos(f*x+e)^3/f/(
c-c*sin(f*x+e))^(11/2)+1/16*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)-1/1
28*a^3*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(5/2)-3/512*a^3*cos(f*x+e)/c^4/f/(
c-c*sin(f*x+e))^(3/2)-3/1024*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-
c*sin(f*x+e))^(1/2))/c^(11/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2759, 2729, 2728, 212}

$$-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f}-\frac{3a^3\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}}+\frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}-\frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (-3*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(512*Sqrt[2]*c^(11/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*
x])^(15/2)) - (a^3*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e + f*x])^(11/2)) + (a^3
*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(7/2)) - (a^3*Cos[e + f*x])/(
128*c^3*f*(c - c*Sin[e + f*x])^(5/2)) - (3*a^3*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{1}{2}\left(a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{\left(3a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx)}}{16c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+f}{16c^2f(c-c\sin(e+}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+f}{16c^2f(c-c\sin(e+}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+f}{16c^2f(c-c\sin(e+}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+f}{16c^2f(c-c\sin(e+}$$

$$= -\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f} + \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{}{8f}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 2.68, size = 435, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(2048*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]) - 2688*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 992*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^5 - 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
- 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9 + (15 + 15*I)*(-1)^(1/4)*ArcTa
n[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^10 + 4096*Sin[(e + f*x)/2] - 5376*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^2*Sin[(e + f*x)/2] + 1984*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*
Sin[(e + f*x)/2] - 40*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2] - 30*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(2560*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e
+ f*x])^(11/2))
```

**Maple** [A]
time = 3.27, size = 353, normalized size = 1.57

| method | result |
|---|---|
| default | $\frac{a^3\left(480\sqrt{c(1+\sin(fx+e))}\, c^{\frac{13}{2}}-1120(c(1+\sin(fx+e)))^{\frac{3}{2}} c^{\frac{11}{2}}+1024(c(1+\sin(fx+e)))^{\frac{5}{2}} c^{\frac{9}{2}}+280(c(1+\sin(fx+e)))^{\frac{7}{2}} c^{\frac{7}{2}}-3\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/5120*a^3*(480*(c*(1+sin(f*x+e)))^(1/2)*c^(13/2)-1120*(c*(1+sin(f*x+e)))^(3/2)*c^(11/2)+1024*(c*(1+sin(f*x+e)))^(5/2)*c^(9/2)+280*(c*(1+sin(f*x+e)))^(7/2)*c^(7/2)-30*(c*(1+sin(f*x+e)))^(9/2)*c^(5/2)+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^5*c^7-75*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^7+150*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^7-150*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^7+75*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^7-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^7)*(c*(1+sin(f*x+e)))^(1/2)/c^(25/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(11/2), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(206) = 412.
time = 0.38, size = 646, normalized size = 2.87

$$\frac{15\sqrt{2}\left(a^3\cos(fx+e)^6-5a^3\cos(fx+e)^5-18a^3\cos(fx+e)^4+20a^3\cos(fx+e)^3+48a^3\cos(fx+e)^2-16a^3\cos(fx+e)-32a^3+\left(a^3\cos(fx+e)^5+6a^3\cos(fx+e)^4-12a^3\cos(fx+e)^3-32a^3\cos(fx+e)^2+16a^3\cos(fx+e)+32a^3\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(15a^3\cos(fx+e)^5-65a^3\cos(fx+e)^4+812a^3\cos(fx+e)^3+1796a^3\cos(fx+e)^2-1144a^3\cos(fx+e)-2048a^3+\left(15a^3\cos(fx+e)^4+80a^3\cos(fx+e)^3+892a^3\cos(fx+e)^2-904a^3\cos(fx+e)-2048a^3\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{10240\left(c^6f\cos(fx+e)^6-5c^6f\cos(fx+e)^5-18c^6f\cos(fx+e)^4+20c^6f\cos(fx+e)^3+48c^6f\cos(fx+e)^2-16c^6f\cos(fx+e)-32c^6f+\left(c^6f\cos(fx+e)^5+6c^6f\cos(fx+e)^4-12c^6f\cos(fx+e)^3-32c^6f\cos(fx+e)^2+16c^6f\cos(fx+e)+32c^6f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="fricas")

[Out] 1/10240*(15*sqrt(2)*(a^3*cos(f*x + e)^6 - 5*a^3*cos(f*x + e)^5 - 18*a^3*cos(f*x + e)^4 + 20*a^3*cos(f*x + e)^3 + 48*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*x

```
x + e) - 32*a^3 + (a^3*cos(f*x + e)^5 + 6*a^3*cos(f*x + e)^4 - 12*a^3*cos(f
*x + e)^3 - 32*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 32*a^3)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^5 - 65*a^3*cos(f*x + e)^
4 + 812*a^3*cos(f*x + e)^3 + 1796*a^3*cos(f*x + e)^2 - 1144*a^3*cos(f*x + e
) - 2048*a^3 + (15*a^3*cos(f*x + e)^4 + 80*a^3*cos(f*x + e)^3 + 892*a^3*cos
(f*x + e)^2 - 904*a^3*cos(f*x + e) - 2048*a^3)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(
f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*c
os(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 1
2*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) +
32*c^6*f)*sin(f*x + e))
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(11/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin(e + f\,x))^3}{(c - c\,\sin(e + f\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(11/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(11/2), x)
```

# 3.317 $\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=132

$$-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}+\frac{8c\sec(e+fx)(c-c\sin(e-}{5af}$$

```
[Out] 64/5*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f+8/5*c*sec(f*x+e)*(c-c*sin(f*
x+e))^(5/2)/a/f+2/5*sec(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f-256/5*c^3*sec(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

**Rubi** [A]
time = 0.25, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af}+\frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-256*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(5*a*f) + (64*c^2*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) + (8*c*Sec[e + f*x]*(c - c*Sin[e +
 f*x])^(5/2))/(5*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} + \frac{12\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}}{5a}$$
$$= \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} +$$
$$= \frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} + \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af}$$
$$= -\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af} + \frac{64c^2\sec(e+fx)(c-c\sin(e+fx}{5af}$$

**Mathematica** [A]
time = 1.30, size = 112, normalized size = 0.85

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(-350-14\cos(2(e+fx))-175\sin(e+fx)+\sin(3(e+fx)))}{10af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-350 -
 14*Cos[2*(e + f*x)] - 175*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*a*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

**Maple** [A]
time = 1.62, size = 69, normalized size = 0.52

| method | result | size |
|---|---|---|
| default | $\frac{2c^4(\sin(fx+e)-1)(\sin^3(fx+e)-7(\sin^2(fx+e))+43\sin(fx+e)+91)}{5a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 69 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/5*c^4/a*(sin(f*x+e)-1)*(sin(f*x+e)^3-7*sin(f*x+e)^2+43*sin(f*x+e)+91)/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(124) = 248$.

time = 0.60, size = 258, normalized size = 1.95

$$\frac{2\left(91\,c^{\frac{7}{2}}+\frac{86\,c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{336\,c^{\frac{7}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{266\,c^{\frac{7}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{490\,c^{\frac{7}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{266\,c^{\frac{7}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{336\,c^{\frac{7}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{86\,c^{\frac{7}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{91\,c^{\frac{7}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)}{5\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/5*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2)*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))
*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

**Fricas** [A]

time = 0.34, size = 79, normalized size = 0.60

$$-\frac{2\left(7\,c^{3}\cos\left(fx+e\right)^{2}+84\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-44\,c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/5*(7*c^3*cos(f*x + e)^2 + 84*c^3 - (c^3*cos(f*x + e)^2 - 44*c^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(124) = 248$.

time = 0.57, size = 343, normalized size = 2.60

$$\frac{16\sqrt{2}\left(\frac{5\,c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)}-\frac{11\,c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{50\,c^{3}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+\frac{80\,c^{3}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}-\frac{30\,c^{3}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{5\,c^{3}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{4}}}{a\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)^{5}}\right)\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 16/5*sqrt(2)*(5*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a*((cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (11*c^3*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 50*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) + 80*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 30*c^3*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^3 + 5*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(
a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) - 1)^5))*sqrt(c)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-c\,\sin(e+f\,x))^{7/2}}{a+a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x)), x)
```

# 3.318 $\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=98

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+f}{3af}$$

```
[Out] 16/3*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f+2/3*sec(f*x+e)*(c-c*sin(f*x+e)
)^(5/2)/a/f-64/3*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-64*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) + (16*c*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(3*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(3*a*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af} + \frac{8\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,}{3a}$$
$$= \frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}$$
$$= -\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} + \frac{16c\sec(e+fx)(c-c\sin(e+fx))}{3af}$$

**Mathematica** [A]
time = 0.44, size = 102, normalized size = 1.04

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(45+\cos(2(e+fx))+20\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
[Out] -1/3*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(45 + Cos[2*(e + f*x)] + 20
*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(1 + Sin[e + f*x]))
```

**Maple** [A]
time = 1.50, size = 59, normalized size = 0.60

| method | result | size |
|---|---|---|
| default | $-\frac{2c^3(\sin(fx+e)-1)(\sin^2(fx+e)-10\sin(fx+e)-23)}{3a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 59 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -2/3*c^3/a*(sin(f*x+e)-1)*(sin(f*x+e)^2-10*sin(f*x+e)-23)/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(92) = 184$.
time = 0.60, size = 208, normalized size = 2.12

$$\frac{2\left(23\,c^{\frac{5}{2}}+\frac{20\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{40\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{65\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{20\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{23\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}\right)}{3\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

**Fricas** [A]
time = 0.36, size = 62, normalized size = 0.63

$$-\frac{2\left(c^{2}\cos\left(fx+e\right)^{2}+10\,c^{2}\sin\left(fx+e\right)+22\,c^{2}\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(c^2*cos(f*x + e)^2 + 10*c^2*sin(f*x + e) + 22*c^2)*sqrt(-c*sin(f*x +
e) + c)/(a*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^{2}\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{2c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\right)dx+\int\frac{c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(
-2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Int
egral(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1), x)
)/a
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(92) = 184.

time = 0.57, size = 237, normalized size = 2.42

$$\frac{8\,\sqrt{2}\,\sqrt{c}\left(\frac{3\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)}-\frac{5\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{12\,c^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+\frac{3\,c^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}}{a\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 8/3*sqrt(2)*sqrt(c)*(3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a*((cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (
5*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 12*c^2*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 3*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a*((cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3))/f
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)), x)
```

# 3.319 $\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=60

$$-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}$$

```
[Out] 2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-8*c*sec(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-8*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f) + (2*Sec[e + f*x]*(c - c
*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{ac}\\ &= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}+\frac{4\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a}\\ &= -\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}\end{aligned}$$

**Mathematica [A]**
time = 0.18, size = 88, normalized size = 1.47

$$-\frac{2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(3+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3 + Sin[e + f*x])*Sqrt[c - c*S
in[e + f*x]])/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])
)
```

**Maple [A]**
time = 1.76, size = 49, normalized size = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{2c^2(\sin(fx+e)-1)(3+\sin(fx+e))}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 49 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2*c^2/a*(sin(f*x+e)-1)*(3+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(60) = 120.
time = 0.51, size = 158, normalized size = 2.63

$$\frac{2\left(3c^{\frac{3}{2}}+\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a + a*sin(f*x + e)/(c
os(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

**Fricas** [**A**]
time = 0.37, size = 44, normalized size = 0.73

$$-\frac{2\left(c\sin\left(fx+e\right)+3\,c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(c*sin(f*x + e) + 3*c)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\right)dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(-c*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

**Giac** [**A**]
time = 0.50, size = 67, normalized size = 1.12

$$-\frac{8\sqrt{2}\,c^{\frac{3}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{af\left(\frac{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a*f*((cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1))
```

**Mupad** [B]

time = 7.29, size = 90, normalized size = 1.50

$$-\frac{2\,c\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(22\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+4\,\sin\left(2\,e+2\,f\,x\right)-12\right)}{a\,f\,\left(4\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(2*c*(-c*(sin(e + f*x) - 1))^(1/2)*(4*sin(2*e + 2*f*x) + 22*sin(e/2 + (f*x
)/2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2 - 12))/(a*f*(sin(e + f*x) + sin(3*e +
 3*f*x) + 4*sin(e + f*x)^2 - 4))
```

# 3.320 $\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=29

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] -2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2815, 2752}

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}\end{aligned}$$

**Mathematica** [A]
time = 0.07, size = 29, normalized size = 1.00

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

**Maple** [A]
time = 1.24, size = 39, normalized size = 1.34

| method | result | size |
|---|---|---|
| default | $\frac{2c(\sin(fx+e)-1)}{a\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\, f}$ | 39 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2*c/a*(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(29) = 58.
time = 0.49, size = 82, normalized size = 2.83

$$\frac{2\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\sqrt{\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] 2*(sqrt(c) + sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x +
 e)/(cos(f*x + e) + 1))*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

**Fricas** [A]
time = 0.34, size = 31, normalized size = 1.07

$$-\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x)/a
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.
time = 0.46, size = 65, normalized size = 2.24

$$\frac{2\sqrt{2}\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{af\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a*f*((cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1))
```

**Mupad** [B]
time = 0.20, size = 40, normalized size = 1.38

$$-\frac{4\cos(e+f\,x)\,\sqrt{-c\,(\sin(e+f\,x)-1)}}{a\,f\,(\cos(2\,e+2\,f\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(4*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a*f*(cos(2*e + 2*f*x) + 1)
)
```

# 3.321 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] 1/2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/f*2^(1
/2)/c^(1/2)-sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2754, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]
*a*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
```

```
m + 1/2, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} + \frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}}{2a}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} - \frac{\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\right.}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f} - \frac{\sec(e+fx)\sqrt{c-c\,\mathrm{s}}}{acf}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.21, size = 97, normalized size = 1.17

$$-\frac{\cos(e+fx)\left(1+(1+i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af(1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(1 + (1 + I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e
+ f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

**Maple** [A]
time = 2.10, size = 85, normalized size = 1.02

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| default | $-\frac{(\sin(fx+e)-1)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c\sqrt{c(1+\sin(fx+e))}-2c^{\frac{3}{2}}\right)}{2a\,c^{\frac{3}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 85 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)-2*c^(3/2))/c^(3/2)/cos(f*x+e)/(c-c*sin
(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.
time = 0.35, size = 168, normalized size = 2.02

$$\frac{\sqrt{2}\sqrt{c}\cos(fx+e)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\sqrt{-c\sin(fx+e)+c}}{4\,acf\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)
*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x
 + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) +
2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c))/(a*c*f*
cos(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) +
 c)), x)/a
```

**Giac [A]**
time = 0.51, size = 131, normalized size = 1.58

$$\frac{\frac{\sqrt{2}\,\log\left(-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}\right)}{a\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{4\,\sqrt{2}}{a\sqrt{c}\,\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1))/(a*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*sqrt(
2)/(a*sqrt(c)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.322 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=117

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+3/8*arctanh(1/2*cos(f*x+e)*c^(1/2
)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-sec(f*x+e)/a/c/f/(c-c
*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2815, 2766, 2729, 2728, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) + (3*Cos[e + f*x])/(4*a*f*(c - c*Sin[e + f*x])^(3/2)) -
 Sec[e + f*x]/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}\\
&= -\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}+\frac{3\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}\\
&= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}+\frac{3\int}{}\\
&= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}-\frac{3S}{}\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{3\cos(e+fx}{4af(c-c\sin(e+}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.38, size = 125, normalized size = 1.07

$$-\frac{\sec(e+fx)\left(1+(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\sin(e+fx)\right)}{4acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/4*(Sec[e + f*x]*(1 + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*
(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) - 3*Sin[e + f*x]))/(a*c*f*Sqrt[c - c*Sin[e + f*x
]])
```

**Maple [A]**
time = 2.14, size = 134, normalized size = 1.15

| method | result |
|---|---|
| default | $-\frac{3\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1}{}\right.}{8c^{\frac{5}{2}}\,a\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^(5/2)/a*(3*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-3*2^(1/2)*arctanh(1/2*(c*(1+sin(
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)-6*c^(3/2)*sin(f*
x+e)+2*c^(3/2))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(106) = 212.
time = 0.34, size = 226, normalized size = 1.93

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\sqrt{-c\sin\left(fx+e\right)+c}\,(3\sin\left(fx+e\right)-1)}{16\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
```

```
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) - 1))/(a*c^2*f*cos(f*x +
 e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{-c\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
+ f*x) + c)), x)/a
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(106) = 212.
time = 0.53, size = 304, normalized size = 2.60

$$\frac{\frac{6\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^{\frac{3}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{ac^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\sqrt{c}+\frac{14\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{ac^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(6*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1))/(a*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt
(2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(a*c^(3/2)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(sqrt(c) + 14*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e
) + 1) - 3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^2)/(a*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + (cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)), x)
```

# 3.323 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=156

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}-\frac{5}{8ac^2f\sqrt{}}$$

```
[Out] 15/32*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*sec(f*x+e)/a/c/f/(c-c*sin
(f*x+e))^(3/2)+15/64*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e)
)^(1/2))/a/c^(5/2)/f*2^(1/2)-5/8*sec(f*x+e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2815, 2760, 2766, 2729, 2728, 212}

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (15*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) + (15*Cos[e + f*x])/(32*a*c*f*(c - c*Sin[e + f*x])^(3
/2)) + Sec[e + f*x]/(4*a*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*Sec[e + f*x])
/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{15\cos(e}{32acf(c-c\sin}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.50, size = 162, normalized size = 1.04

$$\frac{\left(\frac{1}{128}+\frac{i}{128}\right)\cos(e+fx)\left(-60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1-i)(-9+15\cos(2(e+fx))+40\sin(e+fx))\right)}{ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((1/128 + I/128)*Cos[e + f*x]*(-60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)
*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + (1 - I)*(-9 + 15*Cos[2*(e + f*x)] + 40*Sin[e
+ f*x])))/(a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])
```

**Maple** [A]
time = 2.77, size = 210, normalized size = 1.35

| method | result |
|---|---|
| default | $-\frac{15\sqrt{c(1+\sin(fx+e))}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2-30c^{\frac{5}{2}}(\sin^2(fx+e))-3}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/c^(9/2)/a*(15*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-30*c^(5/2)*sin(f*x+e)^2-30
*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+40*c^(5/2)*sin(f*x+e)+15*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+6*c^(5
/2))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas [A]**
time = 0.37, size = 263, normalized size = 1.69

$$\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{3}+2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(15\cos\left(fx+e\right)^{2}+20\sin\left(fx+e\right)-12\right)\sqrt{-c\sin\left(fx+e\right)+c}}{128\left(ac^{3}f\cos\left(fx+e\right)^{3}+2ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2ac^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 + 20*sin(f*x + e) - 12)*
sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)
*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+c^{2}\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3 - c**2*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**2 - c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x) + c**2*sqrt(-c*sin(e + f*x) + c)), x)/a
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(141) = 282.
time = 0.58, size = 394, normalized size = 2.53

$$\frac{\frac{60\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\sqrt{c}-\frac{16\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{90\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}{ac^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{128\sqrt{2}}{ac^{\frac{5}{2}}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\frac{16\sqrt{2}ac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\sqrt{2}ac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}}{a^{2}c^{6}}}{512\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/512*(60*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1))/(a*c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sq
rt(2)*(sqrt(c) - 16*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1) + 90*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2/(a*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) + 128*sqrt(2)/(a*c^(5/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) - (16*sqrt(2)*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - sqrt(2
)*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^6))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.324 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=176

$$\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{128c\sec^3(e+fx)(c-}{5a^2f}$$

```
[Out] 4096/15*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-1024/5*c^2*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(5/2)/a^2/f+128/5*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)
/a^2/f+32/15*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^2/f+2/5*sec(f*x+e)^3*(c-
c*sin(f*x+e))^(11/2)/a^2/c/f
```

**Rubi** [A]

time = 0.28, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf}+\frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f}+\frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (4096*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (1024*c^2
*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^2*f) + (128*c*Sec[e + f*x]
^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^2*f) + (32*Sec[e + f*x]^3*(c - c*Sin[e
+ f*x])^(9/2))/(15*a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/
(5*a^2*c*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf} + \frac{16\int \sec^4(e+fx)(c-c\sin(e+fx))^{1}}{5a^2c}$$
$$= \frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf}$$
$$= \frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f} + \frac{32\sec^3(e+fx)(c-c\sin(e+fx))}{15a^2f}$$
$$= -\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f} + \frac{128c\sec^3(e+fx)(c-c\sin(e}{5a^2f}$$
$$= \frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{1024c^2\sec^3(e+fx)(c-c\sin(e}{5a^2f}$$

## Mathematica [A]

time = 1.91, size = 124, normalized size = 0.70

$$\frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(6825-1044\cos(2(e+fx))+3\cos(4(e+fx))+8568\sin(e+fx)+56\sin(3(e+fx)))}{60a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(6825 -
 1044*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8568*Sin[e + f*x] + 56*Sin[3*
(e + f*x)]))/(60*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f
*x])^2)
```

## Maple [A]

time = 2.24, size = 91, normalized size = 0.52

| method | result | size |
|---|---|---|
| default | $-\frac{2c^5(\sin(fx+e)-1)(3(\sin^4(fx+e))-28(\sin^3(fx+e))+258(\sin^2(fx+e))+1092\sin(fx+e)+723)}{15a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 91 |

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)^4-28*sin(f*x+e)^3
+258*sin(f*x+e)^2+1092*sin(f*x+e)+723)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(166) = 332$.
time = 0.51, size = 412, normalized size = 2.34

$$-\frac{2\left(723\,c^{\frac{9}{2}}+\frac{2184\,c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{5370\,c^{\frac{9}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10696\,c^{\frac{9}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{15021\,c^{\frac{9}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{21168\,c^{\frac{9}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{20748\,c^{\frac{9}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{21168\,c^{\frac{9}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{15021\,c^{\frac{9}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{10696\,c^{\frac{9}{2}}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{5370\,c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{2184\,c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{723\,c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)}{15\left(a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)f\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/15*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5370*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 21
168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(9/2)
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

**Fricas** [A]
time = 0.38, size = 112, normalized size = 0.64

$$\frac{2\left(3\,c^{4}\cos\left(fx+e\right)^{4}-264\,c^{4}\cos\left(fx+e\right)^{2}+984\,c^{4}+28\left(c^{4}\cos\left(fx+e\right)^{2}+38\,c^{4}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^4*cos(f*x + e)^4 - 264*c^4*cos(f*x + e)^2 + 984*c^4 + 28*(c^4*cos
(f*x + e)^2 + 38*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*
x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(166) = 332.
time = 0.61, size = 450, normalized size = 2.56

$$-\frac{16\sqrt{2}\sqrt{c}\left(\frac{5\left(11\,c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{24\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3}-\frac{73\,c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{320\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{490\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{240\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45\,c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{a^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^5}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -16/15*sqrt(2)*sqrt(c)*(5*(11*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*
c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) + 1)^3) - (73*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 32
0*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 490*c^4*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2 - 240*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^2, x)
```

# 3.325 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=136

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{8\sec^3(e+fx)(c-c\sin(e+}{a^2f}$$

```
[Out] 256/3*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-64*c*sec(f*x+e)^3*(c-c*
sin(f*x+e))^(5/2)/a^2/f+8*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^2/f+2/3*sec
(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^2/c/f
```

**Rubi** [A]
time = 0.24, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf}+\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f}-\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (256*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (64*c*Sec[e
 + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (8*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(7/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/
(3*a^2*c*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2} \\
&= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}}{a^2c} \\
&= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \\
&= -\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7}}{a^2f} \\
&= \frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{64c\sec^3(e+fx)(c-c\sin(e+fx}{a^2f}
\end{aligned}$$

**Mathematica** [A]
time = 0.75, size = 112, normalized size = 0.82

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(210-30\cos(2(e+fx))+273\sin(e+fx)+\sin(3(e+fx)))}{6a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(210 -
30*Cos[2*(e + f*x)] + 273*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*a^2*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 2.29, size = 79, normalized size = 0.58

| method | result | size |
|---|---|---|
| default | $\frac{2c^4(\sin(fx+e)-1)(\sin^3(fx+e)-15(\sin^2(fx+e))-69\sin(fx+e)-45)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 79 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(sin(f*x+e)^3-15*sin(f*x+e)^2-69*
sin(f*x+e)-45)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(132) = 264$.
time = 0.50, size = 362, normalized size = 2.66

$$-\frac{2\left(45\,c^{\frac{7}{2}}+\frac{138\,c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{285\,c^{\frac{7}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{544\,c^{\frac{7}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{630\,c^{\frac{7}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{812\,c^{\frac{7}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{630\,c^{\frac{7}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{544\,c^{\frac{7}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{285\,c^{\frac{7}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{138\,c^{\frac{7}{2}}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{45\,c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)}{3\left(a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)f\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^(7/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^(7/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(7/2))
```

**Fricas** [A]
time = 0.34, size = 98, normalized size = 0.72

$$-\frac{2\left(15\,c^{3}\cos\left(fx+e\right)^{2}-60\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}+68\,c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(15*c^3*cos(f*x + e)^2 - 60*c^3 - (c^3*cos(f*x + e)^2 + 68*c^3)*sin(f*
x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*
cos(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac** [A]
time = 0.58, size = 125, normalized size = 0.92

$$\frac{128\sqrt{2}\left(c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{3c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\sqrt{c}}{3a^2f\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 128/3*sqrt(2)*(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*c^3*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2)*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1)^3)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^2, x)
```

# 3.326 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2} dx$

**Optimal**. Leaf size=100

$$\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+f$$

```
[Out] 64/3*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-16*sec(f*x+e)^3*(c-c*sin(f
*x+e))^(5/2)/a^2/f+2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^2/c/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (64*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (16*Sec[e + f*
x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e +
 f*x])^(7/2))/(a^2*c*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf} + \frac{8\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2c}$$
$$= -\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/}}{a^2cf}$$
$$= \frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/}}{a^2f}$$

**Mathematica** [A]
time = 0.49, size = 104, normalized size = 1.04

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(25-3\cos(2(e+fx))+36\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(25 - 3*Cos[2*(e + f*x)] + 36*Si
n[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])*(1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 2.26, size = 71, normalized size = 0.71

| method | result | size |
|---|---|---|
| default | $-\frac{2c^3(\sin(fx+e)-1)(3(\sin^2(fx+e))+18\sin(fx+e)+11)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 71 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)^2+18*sin(f*x+e)+11
)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(98) = 196$.
time = 0.51, size = 312, normalized size = 3.12

$$-\frac{2\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{90\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{11\,c^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)}{3\left(a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

**Fricas** [**A**]
time = 0.34, size = 82, normalized size = 0.82

$$-\frac{2\left(3\,c^2\cos\left(fx+e\right)^2-18\,c^2\sin\left(fx+e\right)-14\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*c^2*cos(f*x + e)^2 - 18*c^2*sin(f*x + e) - 14*c^2)*sqrt(-c*sin(f*x
+ e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(98) = 196$.
time = 0.57, size = 237, normalized size = 2.37

$$\frac{4\sqrt{2}\sqrt{c}\left(\frac{3\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)}-\frac{5\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{12\,c^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+\frac{3\,c^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}}{a^2\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*sqrt(c)*(3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^2*((cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)) -
 (5*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*c^2*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 3*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*((cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3))/
f
```

**Mupad [B]**
time = 11.68, size = 360, normalized size = 3.60

$$\frac{\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(\frac{2\,c^2}{a^2\,f}-\frac{c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{a^2\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}}+\frac{16\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}-\frac{c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,32\mathrm{i}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^2}-\frac{32\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*c^2)/(a^2*f) - (c^2*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i)
- 1i) + (16*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^2*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i
 + f*x*1i) + 1i)) - (c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1
i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*32i)/(3*a^2*f*(exp(e*1i + f*x*1i)
- 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (32*c^2*exp(e*1i + f*x*1i)*(c - c*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(3*a^2*f*(exp
(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3)
```

# 3.327 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2} dx$

**Optimal**. Leaf size=68

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

```
[Out] 8/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-2*sec(f*x+e)^3*(c-c*sin(f*x+e
))^(5/2)/a^2/c/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (8*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (2*Sec[e + f*x]^3
*(c - c*Sin[e + f*x])^(5/2))/(a^2*c*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}$$
$$= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf} - \frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^5}{a^2c}$$
$$= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

**Mathematica** [A]
time = 0.22, size = 92, normalized size = 1.35

$$\frac{2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+3\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + 3*Sin[e + f*x])*Sqrt[c - c*
Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)
```

**Maple** [A]
time = 2.00, size = 61, normalized size = 0.90

| method | result | size |
|---|---|---|
| default | $-\frac{2c^2(\sin(fx+e)-1)(3\sin(fx+e)+1)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)+1)/cos(f*x+e)/(c-c
*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(66) = 132.
time = 0.51, size = 259, normalized size = 3.81

$$-\frac{2\left(c^{\frac{3}{2}}+\frac{6c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((
a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(3/2))
```

**Fricas** [A]
time = 0.32, size = 62, normalized size = 0.91

$$\frac{2\left(3\,c\sin\left(fx+e\right)+c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*sin(f*x + e) + c)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*si
n(f*x + e) + a^2*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\right)dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
), x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2
 + 2*sin(e + f*x) + 1), x))/a**2
```

**Giac** [A]
time = 0.57, size = 117, normalized size = 1.72

$$-\frac{4\sqrt{2}\left(c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{3c\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\sqrt{c}}{3a^{2}f\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -4/3*sqrt(2)*(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*c*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1))*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3)
```

**Mupad [B]**
time = 10.32, size = 120, normalized size = 1.76

$$-\frac{4\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(4*c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(2*exp(e*1i + f*x*1i) - exp(e*2i + f*x*2i)*3i + 3i))/
(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i + f*x*1i)*1i + 1))
```

# 3.328 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=36

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

```
[Out] -2/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2815, 2752}

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}\\ &= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.
time = 0.09, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [A]
time = 1.46, size = 49, normalized size = 1.36

| method | result | size |
|---|---|---|
| default | $\frac{2c(\sin(fx+e)-1)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$ | 49 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(34) = 68.
time = 0.52, size = 161, normalized size = 4.47

$$\frac{2\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{3\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(c)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

**Fricas** [A]
time = 0.34, size = 50, normalized size = 1.39

$$-\frac{2\sqrt{-c\sin(fx+e)+c}}{3\left(a^2f\cos(fx+e)\sin(fx+e)+a^2f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos
(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sin{(e+fx)}+c}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
x)/a**2
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(34) = 68.
time = 0.50, size = 116, normalized size = 3.22

$$\frac{\sqrt{2}\,\sqrt{c}\left(\frac{3\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}+\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)}{3\,a^2f\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(c)*(3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3)
```

**Mupad [B]**
time = 9.35, size = 227, normalized size = 6.31

$$-\frac{4\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\sin\left(2\,e+2\,f\,x\right)-4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-\sin\left(e+f\,x\right)^2\,2{}\mathrm{i}+2+2{}\mathrm{i}\right)}{3\,a^2\,f\,\left(-4\,\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+4\right)}+\frac{4\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(-\sin\left(e+f\,x\right)^2\,4{}\mathrm{i}+\sin\left(e+f\,x\right)\,1{}\mathrm{i}-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\,1{}\mathrm{i}+4{}\mathrm{i}\right)}{3\,a^2\,f\,\left(-8\,\sin\left(e+f\,x\right)^2+4\,\sin\left(e+f\,x\right)+2\,\sin\left(2\,e+2\,f\,x\right)^2+4\,\sin\left(3\,e+3\,f\,x\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (4*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(e + f*x)*1i + 2*sin(2*e + 2*f*x) + si
n(3*e + 3*f*x)*1i - 2*sin(e/2 + (f*x)/2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2 -
 sin(e + f*x)^2*4i + 4i))/(3*a^2*f*(4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2
*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2 + 8)) - (4*(-c*(sin(e + f*x) - 1))^(
1/2)*(sin(2*e + 2*f*x) - 4*sin(e/2 + (f*x)/2)^2 - sin(e + f*x)^2*2i + (2 +
2i)))/(3*a^2*f*(sin(e + f*x) + sin(3*e + 3*f*x) - 4*sin(e + f*x)^2 + 4))
```

# 3.329 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=124

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,a^2\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}$$

```
[Out] -1/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*arctanh(1/2*cos(f*x+
e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1/2)-1/2*sec(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

**Rubi** [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2754, 2728, 212}

$$-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,a^2\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(2*a^2*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
```

```
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{\int \sec^2(e+fx)\sqrt{c-\cdots}}{2a^{\cdots}}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin\cdots}{3a^2c^2f}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin\cdots}{3a^2c^2f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,a^2\sqrt{c}\,f} - \frac{\sec(e+fx)\sqrt{c-c\cdots}}{2a^2cf}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.33, size = 109, normalized size = 0.88

$$\frac{\cos(e+fx)\left(-5-(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3\sin(e+fx)\right)}{6a^2f(1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*Sin[e + f*x
]))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.43, size = 109, normalized size = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{(\sin(fx+e)-1)\left(6c^{\frac{7}{2}}\sin(fx+e)+10c^{\frac{7}{2}}-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}(c(1+\sin(fx+e)))^{\frac{3}{2}}\right)}{12a^{2}c^{\frac{7}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 109 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(sin(f*x+e)-1)*(6*c^(7/2)*sin(f*x+e)+10*c^(7/2)-3*2^(1/2)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2*(c*(1+sin(f*x+e)))^(3/2))/a^2
/c^(7/2)/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(111) = 222$.
time = 0.35, size = 223, normalized size = 1.80

$$\frac{3\sqrt{2}\left(\cos(fx+e)\sin(fx+e)+\cos(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}\left(\cos(fx+e)+\sin(fx+e)+1\right)+3c\cos(fx+e)+\left(c\cos(fx+e)-2c\right)\sin(fx+e)+2c}{\cos(fx+e)^{2}+\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)-4\sqrt{-c\sin(fx+e)+c}\left(3\sin(fx+e)+5\right)}{24\left(a^{2}cf\cos(fx+e)\sin(fx+e)+a^{2}cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/24*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) + 5))/(a^2*c*f*cos(f*x +
 e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin(e+fx)+c}\sin^{2}(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x)/a**2
```

**Giac [A]**
time = 0.52, size = 211, normalized size = 1.70

$$\frac{\frac{3\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{8\left(2\sqrt{2}+\frac{3\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1))/(a^2*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 8*
(2*sqrt(2) + 3*sqrt(2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) + 3*sqrt(2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*sqrt(c)*((cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.330 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=155

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}-\sec^3(e+f$$

```
[Out] 5/8*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+5/16*arctanh(1/2*cos(f*x+e)*c^(
1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2)-5/6*sec(f*x+e)/a
^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^2/c
^2/f
```

**Rubi** [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2815, 2754, 2766, 2729, 2728, 212}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f}-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) + (5*Cos[e + f*x])/(8*a^2*f*(c - c*Sin[e + f*x])^(3/2
)) - (5*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^
3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+}}}{6a^2c}$$

$$= -\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e}}{3a^2c^2f}$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{5\cos(e+}{8a^2f(c-c\sin(e}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.49, size = 164, normalized size = 1.06

$$\frac{\left(\frac{1}{96}+\frac{i}{96}\right)\cos(e+fx)\left(60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(1-i)(11+15\cos(2(e+fx))-20\sin(e+fx))\right)}{a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/96 + I/96)*Cos[e + f*x]*(60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(11 + 15*Cos[2*(e + f*x)] - 20*Sin[e +
 f*x])))/(a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e
 + f*x]])
```

**Maple [A]**
time = 2.19, size = 157, normalized size = 1.01

| method | result |
|---|---|
| default | $-\frac{15(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-15(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}(}{48c^{\frac{7}{2}}a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(f}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/c^(7/2)/a^2*(15*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+si
n(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-15*(c*(1+sin(f*x+e)))^(3/2)*
2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-20*c^(5/2)*
sin(f*x+e)-30*c^(5/2)*sin(f*x+e)^2+26*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c
-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [A]
time = 0.38, size = 202, normalized size = 1.30

$$\frac{15\,\sqrt{2}\,\sqrt{c}\,\cos\left(fx+e\right)^{3}\log\left(-\frac{c\cos(fx+e)^{2}+2\,\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(15\,\cos\left(fx+e\right)^{2}-10\,\sin\left(fx+e\right)-2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{96\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*c
os(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2
 - 10*sin(f*x + e) - 2)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^
3)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

[Out] Timed out

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(140) = 280.
time = 0.54, size = 368, normalized size = 2.37

$$-\frac{\frac{3\sqrt{2}\left(\frac{10\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{30\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^{\frac{3}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{a^2c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{16\sqrt{2}\left(7\sqrt{c}+\frac{12\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2c^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/192*(3*sqrt(2)*(10*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) - 1)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*c^(3/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
- 30*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1))/(a^2*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 3*sqr
t(2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(a^2*c^(3/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 16*sqrt(2)*(7*sqrt(
c) + 12*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 9*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\sin(e+fx))^2\,(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)), x)
```

# 3.331 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=192

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}+\frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}-\frac{\quad}{48a^2c^2}$$

```
[Out] 35/64*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+7/24*sec(f*x+e)/a^2/c/f/(c-
c*sin(f*x+e))^(3/2)+35/128*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(
f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-35/48*sec(f*x+e)/a^2/c^2/f/(c-c*sin(f*
x+e))^(1/2)-1/3*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2815, 2766, 2760, 2729, 2728, 212}

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}+\frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (35*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) + (35*Cos[e + f*x])/(64*a^2*c*f*(c - c*Sin[e + f*x]
)^(3/2)) + (7*Sec[e + f*x])/(24*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) - (35*S
ec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - Sec[e + f*x]^3/(3*a^
2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{7\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} + \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.73, size = 156, normalized size = 0.81

$$-\frac{\left(\frac{1}{1536}+\frac{i}{1536}\right)\sec^3(e+fx)\left(840\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(1-i)(102+70\cos(2(e+fx))-329\sin(e+fx)-105\sin(3(e+fx)))\right)}{a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/1536 - I/1536)*Sec[e + f*x]^3*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^
(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(102 + 70*Cos[2*(e + f*x)] - 3
29*Sin[e + f*x] - 105*Sin[3*(e + f*x)])))/(a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

**Maple** [A]
time = 3.06, size = 233, normalized size = 1.21

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{105(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin\,(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2+70c^{\frac{7}{2}}(\sin^2(fx+e))-210c^{\frac{7}{2}}(\sin^3}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/c^(11/2)/a^2*(105*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1
+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+70*c^(7/2)*sin(f*x+e)
^2-210*c^(7/2)*sin(f*x+e)^3-210*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+322*c^(7/2)*sin(
f*x+e)+105*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^
2*(c*(1+sin(f*x+e)))^(3/2)-86*c^(7/2))/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas [A]**
time = 0.35, size = 262, normalized size = 1.36

$$\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3\sin\left(fx+e\right)-\cos\left(fx+e\right)^3\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(35\cos\left(fx+e\right)^2-7\left(15\cos\left(fx+e\right)^2+8\right)\sin\left(fx+e\right)+8\right)\sqrt{-c\sin\left(fx+e\right)+c}}{768\left(a^2c^3f\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/768*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) + 4*(35*cos(f*x + e)^2 - 7*(15*cos(f*x + e)^2 + 8)*sin(f*x + e)
 + 8)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sqrt{-c\sin(e+fx)+c}\,\sin^4(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4 - 2*c**2*sqrt(-c
*sin(e + f*x) + c)*sin(e + f*x)**2 + c**2*sqrt(-c*sin(e + f*x) + c)), x)/a*
*2
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(175) = 350.

time = 0.54, size = 480, normalized size = 2.50

$$\frac{\frac{420\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(\sqrt{c}-\frac{24\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{210\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{a^2c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{256\sqrt{2}\left(5\sqrt{c}+\frac{9\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{6\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2c^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\left(\frac{24\sqrt{2}a^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\sqrt{2}a^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^4c^6}}{3072\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3072*(420*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1))/(a^2*c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
- 3*sqrt(2)*(sqrt(c) - 24*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 210*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2/(a^2*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(5*sqrt(c) + 9*sqrt(c)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 6*sqrt(c)*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
)/(a^2*c^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*(24*sqrt(2)*a^2*
c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - sqrt(2)*a^2*c^(7/2)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^4*c^6))/f
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.332 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=174

$$-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f}-\frac{128\sec^5(e+fx)(c}{a^3}$$

```
[Out] -4096/15*c^2*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+1024/3*c*sec(f*x+e)^
5*(c-c*sin(f*x+e))^(7/2)/a^3/f-128*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/
f+32/3*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c/f+2/3*sec(f*x+e)^5*(c-c*s
in(f*x+e))^(13/2)/a^3/c^2/f
```

**Rubi** [A]

time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf}-\frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f}+\frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-4096*c^2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (1024*c*
Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*f) - (128*Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(9/2))/(a^3*f) + (32*Sec[e + f*x]^5*(c - c*Sin[e + f*x
])^(11/2))/(3*a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(3*
a^3*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f} + \frac{16\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/}}{3a^3c^2}$$
$$= \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}$$
$$= -\frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f} + \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{1}}{3a^3cf}$$
$$= \frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f} - \frac{128\sec^5(e+fx)(c-c\sin(e+fx)}{a^3f}$$
$$= -\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{1024c\sec^5(e+fx)(c-c\sin(e+}{3a^3f}$$

**Mathematica [A]**
time = 1.90, size = 124, normalized size = 0.71

$$\frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(-5649+2740\cos(2(e+fx))+5\cos(4(e+fx))-7800\sin(e+fx)+200\sin(3(e+fx)))}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-5649
+ 2740*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 7800*Sin[e + f*x] + 200*Sin[
3*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
 f*x])^3)
```

**Maple [A]**
time = 2.11, size = 91, normalized size = 0.52

| method | result | size |
|---|---|---|
| default | $-\frac{2c^5(\sin(fx+e)-1)(5(\sin^4(fx+e))-100(\sin^3(fx+e))-690(\sin^2(fx+e))-900\sin(fx+e)-363)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 91 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)^4-100*sin(f*x+e
)^3-690*sin(f*x+e)^2-900*sin(f*x+e)-363)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(166) = 332$.
time = 0.52, size = 512, normalized size = 2.94

$$\frac{2\left(363\,c^{\frac{9}{2}} + \frac{1800\,c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{5301\,c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{11600\,c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{21343\,c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{30200\,c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{40065\,c^{\frac{9}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{40800\,c^{\frac{9}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7} + \frac{40065\,c^{\frac{9}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} + \frac{30200\,c^{\frac{9}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9} + \frac{21343\,c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}} + \frac{11600\,c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}} + \frac{5301\,c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}} + \frac{1800\,c^{\frac{9}{2}}\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}} + \frac{363\,c^{\frac{9}{2}}\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}\right)}{15\left(a^3 + \frac{5\,a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10\,a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10\,a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5\,a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^(
9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 302
00*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9/
2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)/((a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

**Fricas** [A]
time = 0.36, size = 128, normalized size = 0.74

$$-\frac{2\left(5\,c^4\cos(fx+e)^4 + 680\,c^4\cos(fx+e)^2 - 1048\,c^4 + 100\left(c^4\cos(fx+e)^2 - 10\,c^4\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{15\left(a^3f\cos(fx+e)^3 - 2\,a^3f\cos(fx+e)\sin(fx+e) - 2\,a^3f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c^4*cos(f*x + e)^4 + 680*c^4*cos(f*x + e)^2 - 1048*c^4 + 100*(c^4*
cos(f*x + e)^2 - 10*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos
(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(166) = 332.
time = 0.61, size = 450, normalized size = 2.59

$$-\frac{8\sqrt{2}\sqrt{c}\left(\frac{5\left(11c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{24c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3}-\frac{73c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{320c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{490c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{240c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -8/15*sqrt(2)*sqrt(c)*(5*(11*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 24*c
^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) - 1)^3) - (73*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 320
*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 490*c^4*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2 + 240*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*c^4*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^3, x)
```

# 3.333 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=134

$$-\frac{256c\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}-\frac{24\sec^5(e+fx)(c-c\sin(}{a^3cf}$$

```
[Out] -256/5*c*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+64*sec(f*x+e)^5*(c-c*sin
(f*x+e))^(7/2)/a^3/f-24*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/c/f+2*sec(f
*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c^2/f
```

**Rubi** [A]
time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}-\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}-\frac{256c\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-256*c*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) + (64*Sec[e +
f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(a^3*f) - (24*Sec[e + f*x]^5*(c - c*Sin[
e + f*x])^(9/2))/(a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))
/(a^3*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f} + \frac{12\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/}}{a^3c^2}$$
$$= -\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}$$
$$= \frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f} - \frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}$$
$$= -\frac{256c\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} + \frac{64\sec^5(e+fx)(c-c\sin(e+fx))}{a^3f}$$

**Mathematica [A]**
time = 0.79, size = 114, normalized size = 0.85

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(-182+90\cos(2(e+fx))-235\sin(e+fx)+5\sin(3(e+fx)))}{10a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-182 +
 90*Cos[2*(e + f*x)] - 235*Sin[e + f*x] + 5*Sin[3*(e + f*x)]))/(10*a^3*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

**Maple [A]**
time = 2.68, size = 81, normalized size = 0.60

| method | result | size |
|---|---|---|
| default | $\frac{2c^4(\sin(fx+e)-1)(5(\sin^3(fx+e))+45(\sin^2(fx+e))+55\sin(fx+e)+23)}{5a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 81 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 2/5*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)^3+45*sin(f*x+e)^2
+55*sin(f*x+e)+23)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(132) = 264$.

time = 0.52, size = 462, normalized size = 3.45

$$\frac{2\left(23\,c^{\frac{7}{2}}+\frac{110\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{318\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{590\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{1065\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{1220\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{1540\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{1220\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}+\frac{1065\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}+\frac{590\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{9}}{\left(\cos\left(fx+e\right)+1\right)^{9}}+\frac{318\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}+\frac{110\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{11}}{\left(\cos\left(fx+e\right)+1\right)^{11}}+\frac{23\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{12}}{\left(\cos\left(fx+e\right)+1\right)^{12}}\right)}{5\left(a^{3}+\frac{5\,a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\,a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5\,a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(7/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c^(7/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1065*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 110*
c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)^12/
(cos(f*x + e) + 1)^12)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

**Fricas** [A]

time = 0.35, size = 114, normalized size = 0.85

$$-\frac{2\left(45\,c^{3}\cos\left(fx+e\right)^{2}-68\,c^{3}+5\left(c^{3}\cos\left(fx+e\right)^{2}-12\,c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(45*c^3*cos(f*x + e)^2 - 68*c^3 + 5*(c^3*cos(f*x + e)^2 - 12*c^3)*sin(
f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(132) = 264$.

time = 0.63, size = 343, normalized size = 2.56

$$-\frac{4\sqrt{2}\sqrt{c}\left(\frac{5c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)}-\frac{11c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{50c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{80c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{30c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{5c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5}\right)}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -4/5*sqrt(2)*sqrt(c)*(5*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^3*((cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1))
- (11*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 50*c^3*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 80*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 30*c^3*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 5*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 1)^5))/f
```

**Mupad** [B]

time = 13.87, size = 542, normalized size = 4.04

$$-\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2c^3}{a^3f}-\frac{c^3e^{e\,1i+f\,x\,1i}\,2i}{a^3f}\right)}{e^{e\,1i+f\,x\,1i}-1i}-\frac{24c^3e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)}+\frac{c^3e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,32i}{a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^2}+\frac{288c^3e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^3}-\frac{c^3e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,256i}{5a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^4}-\frac{128c^3e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*32i)/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*
1i) + 1i)^2) - (24*c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)) - ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2))^(1/2)*((2*c^3)/(a^3*f) - (c^3*exp(e*1i + f*x*1i)*2i)/(a^
3*f)))/(exp(e*1i + f*x*1i) - 1i) + (288*c^3*exp(e*1i + f*x*1i)*(c - c*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*a^3*f*(exp(
e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (c^3*exp(e*1i + f*x*1i)
*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*25
6i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (128*
c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2))/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
 + 1i)^5)
```

# 3.334 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=104

$$-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}$$

```
[Out] -64/15*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+16/3*sec(f*x+e)^5*(c-c*sin
(f*x+e))^(7/2)/a^3/c/f-2*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/c^2/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-64*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (16*Sec[e + f*
x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c*f) - (2*Sec[e + f*x]^5*(c - c*Sin
[e + f*x])^(9/2))/(a^3*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}\\ &= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} - \frac{8\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2}\\ &= \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf} - \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} +\\ &= -\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/}}{3a^3cf}\end{aligned}$$

**Mathematica** [A]
time = 0.53, size = 104, normalized size = 1.00

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-29+15\cos(2(e+fx))-20\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-29 + 15*Cos[2*(e + f*x)] - 20*
Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

**Maple** [A]
time = 2.12, size = 71, normalized size = 0.68

| method | result | size |
|---|---|---|
| default | $\frac{2c^3(\sin(fx+e)-1)(15(\sin^2(fx+e))+10\sin(fx+e)+7)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 71 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*sin(f*x+e)^2+10*sin(f*x+e)
+7)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(100) = 200$.
time = 0.51, size = 412, normalized size = 3.96

$$\frac{2\left(7\,c^{\frac{5}{2}} + \frac{20\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{95\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}} + \frac{80\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}} + \frac{250\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}} + \frac{120\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}} + \frac{250\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}} + \frac{80\,c^{\frac{5}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}} + \frac{95\,c^{\frac{5}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}} + \frac{20\,c^{\frac{5}{2}}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}} + \frac{7\,c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)}{15\left(a^{3} + \frac{5\,a^{3}\sin(fx+e)}{\cos(fx+e)+1} + \frac{10\,a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}} + \frac{10\,a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}} + \frac{5\,a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}} + \frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)f\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}} + 1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^3 + 5*a^3*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 1)^(5/2))
```

**Fricas** [A]
time = 0.34, size = 98, normalized size = 0.94

$$-\frac{2\left(15\,c^{2}\cos\left(fx+e\right)^{2} - 10\,c^{2}\sin\left(fx+e\right) - 22\,c^{2}\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3} - 2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*c^2*cos(f*x + e)^2 - 10*c^2*sin(f*x + e) - 22*c^2)*sqrt(-c*sin(f*
x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a
^3*f*cos(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.55, size = 174, normalized size = 1.67

$$\frac{16\sqrt{2}\left(c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{5c^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{10c^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\sqrt{c}}{15a^3f\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 16/15*sqrt(2)*(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*c^2*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 10*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sqr
t(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 1)^5)
```

**Mupad [B]**
time = 11.95, size = 453, normalized size = 4.36

$$-\frac{4c^2e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)}+\frac{c^2e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,32i}{3a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^2}+\frac{352c^2e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{15a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^3}-\frac{c^2e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,128i}{5a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^4}-\frac{64c^2e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5a^3f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*32i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*
x*1i) + 1i)^2) - (4*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(
exp(e*1i + f*x*1i) + 1i)) + (352*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(15*a^3*f*(exp(e*1i +
 f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (c^2*exp(e*1i + f*x*1i)*(c -
c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*128i)/(5
*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (64*c^2*exp
(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1
i)/2))^(1/2))/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^
5)
```

# 3.335 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3} dx$

**Optimal**. Leaf size=73

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

```
[Out] 8/15*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-2/3*sec(f*x+e)^5*(c-c*sin(
f*x+e))^(7/2)/a^3/c^2/f
```

**Rubi** [A]
time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2815, 2753, 2752}

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (8*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*c*f) - (2*Sec[e + f*x
]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}\\
&= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f} - \frac{4\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2}\\
&= \frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf} - \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}
\end{aligned}
$$

**Mathematica [A]**
time = 0.23, size = 92, normalized size = 1.26

$$\frac{2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+5\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + 5*Sin[e + f*x])*Sqrt[c - c
*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x])^3)
```

**Maple [A]**
time = 1.86, size = 61, normalized size = 0.84

| method | result | size |
|---|---|---|
| default | $-\frac{2c^2(\sin(fx+e)-1)(5\sin(fx+e)-1)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] -2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)-1)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(69) = 138$.
time = 0.53, size = 359, normalized size = 4.92

$$\frac{2\left(c^{\frac{3}{2}}-\frac{10c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{4c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{30c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{30c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{4c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{10c^{\frac{3}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{c^{\frac{3}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)}{15\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

**Fricas** [**A**]
time = 0.47, size = 80, normalized size = 1.10

$$-\frac{2\,(5\,c\sin\left(fx+e\right)-c)\sqrt{-c\sin\left(fx+e\right)+c}}{15\,\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c*sin(f*x + e) - c)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(69) = 138.
time = 0.52, size = 219, normalized size = 3.00

$$-\frac{2\,\sqrt{2}\left(c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{5\,c\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-\frac{5\,c\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}+\frac{15\,c\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}\right)\sqrt{c}}{15\,a^{3}f\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*sqrt(2)*(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*c*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) - 5*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*c*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5)
```

**Mupad** [B]
time = 10.91, size = 355, normalized size = 4.86

$$\frac{c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,8{}\mathrm{i}}{3\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-\mathrm{i}\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}^2}+\frac{136\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-\mathrm{i}\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}^3}-\frac{c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,64{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-\mathrm{i}\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}^4}-\frac{32\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-\mathrm{i}\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*8i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1
i) + 1i)^2) + (136*c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(15*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(
exp(e*1i + f*x*1i) + 1i)^3) - (c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i +
f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (32*c*exp(e*1i + f*x*1i)*(c -
c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*a^3*
f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5)
```

## 3.336 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=36

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

```
[Out] -2/5*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^2/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2815, 2752}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}\\ &= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.
time = 0.10, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{5a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

**Maple** [A]
time = 1.96, size = 49, normalized size = 1.36

| method | result | size |
|---|---|---|
| default | $\frac{2c(\sin(fx+e)-1)}{5a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ | 49 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)
/f
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(34) = 68.
time = 0.61, size = 236, normalized size = 6.56

$$\frac{2\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{5\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) +
1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

**Fricas** [A]
time = 0.35, size = 66, normalized size = 1.83

$$\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*
sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^3\left(e+fx\right)+3\sin^2\left(e+fx\right)+3\sin\left(e+fx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3
*sin(e + f*x) + 1), x)/a**3
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(34) = 68.
time = 0.47, size = 166, normalized size = 4.61

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{10\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{5\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}+\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{10\,a^3f\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2)*sqrt(c)*(10*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 5*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3
*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 1)^5)
```

**Mupad** [B]
time = 9.88, size = 90, normalized size = 2.50

$$\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,16\mathrm{i}}{5\,a^3\,f\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^5\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (exp(e*3i + f*x*3i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) + 1i)^5*(exp(e*1i + f*x*
1i)*1i + 1))
```

# 3.337 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=160

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-$$

```
[Out] -1/6*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*sec(f*x+e)^5*(c-c*si
n(f*x+e))^(5/2)/a^3/c^3/f+1/8*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a^3/c/f
```

**Rubi** [A]

time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2815, 2754, 2728, 212}

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(4*a^3*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3*c^2*f) - (Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}\\
&= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}+\frac{\int \sec^4(e+fx)(c-}{2a^}\\
&= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec^5(e+fx)(c-cs}{5a^3c^3}\\
&= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}-\frac{\sec^3(e+fx)(c-c\sin}{6a^3c^2f}\\
&= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}-\frac{\sec^3(e+fx)(c-c\sin}{6a^3c^2f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c}}{4a^3cf}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.40, size = 189, normalized size = 1.18

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-12-10\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-15\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-(15+15i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)}{60a^3f(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12 - 10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(Cos[(e + f*x)/2]
```

```
+ Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(60*a^3
*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.28, size = 122, normalized size = 0.76

| method | result |
|---|---|
| default | $-\frac{(\sin(fx+e)-1)\left(-74c^{\frac{11}{2}}-80c^{\frac{11}{2}}\sin(fx+e)-30c^{\frac{11}{2}}(\sin^2(fx+e))+15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^3(c}{120a^3c^{\frac{11}{2}}(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/a^3*(sin(f*x+e)-1)/c^(11/2)/(1+sin(f*x+e))^2*(-74*c^(11/2)-80*c^(11/
2)*sin(f*x+e)-30*c^(11/2)*sin(f*x+e)^2+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x
+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3*(c*(1+sin(f*x+e)))^(5/2))/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas [A]**
time = 0.36, size = 263, normalized size = 1.64

$$\frac{15\sqrt{2}\left(\cos(fx+e)^3-2\cos(fx+e)\sin(fx+e)-2\cos(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(15\cos(fx+e)^2-40\sin(fx+e)-52\right)\sqrt{-c\sin(fx+e)+c}}{240\left(a^3cf\cos(fx+e)^3-2a^3cf\cos(fx+e)\sin(fx+e)-2a^3cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/240*(15*sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 - 40*sin(f*x + e) - 52)*
```

```
sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)
*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)+3\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+3\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3 + 3*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x)**2 + 3*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(
-c*sin(e + f*x) + c)), x)/a**3
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(145) = 290.
time = 0.57, size = 292, normalized size = 1.82

$$\frac{\frac{15\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\sqrt{2}\left(23\sqrt{c}+\frac{70\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{140\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{90\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{a^3c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*(15*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1))/(a^3*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) +
4*sqrt(2)*(23*sqrt(c) + 70*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 140*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^4)/(a^3*c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) + 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.338 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=191

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}+\frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}}-\frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}-\frac{7\sec^3(e$$

[Out] 7/16*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*sec(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+7/32*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-7/12*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-7/30*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^3/c^2/f

**Rubi** [A]
time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2815, 2754, 2766, 2729, 2728, 212}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}}-\frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]

[Out] (7*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + (7*Cos[e + f*x])/(16*a^3*f*(c - c*Sin[e + f*x])^(3/2)) - (7*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(30*a^3*c^2*f) - (Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c^3*f)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{7\int \sec^4(e+fx)\,\cdots}{1\cdots}$$

$$= -\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{\sec^5(e+fx)(c-\cdots}{5a^3\cdots}$$

$$= -\frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec^3(e+fx)\sqrt{c-c\,\mathrm{si}\cdots}}{30a^3c^2f}$$

$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx\cdots}}$$

$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx\cdots}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f} + \frac{7\cos(e+\cdots}{16a^3f(c-c\sin(\cdots}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.74, size = 174, normalized size = 0.91

$$\frac{\left(\frac{1}{1920}+\frac{i}{1920}\right)\cos(e+fx)\left(840\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+(1-i)(206+350\cos(2(e+fx))-231\sin(e+fx)+105\sin(3(e+fx)))\right)}{a^3cf(-1+\sin(e+fx))(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/1920 + I/1920)*Cos[e + f*x]*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5 + (1 - I)*(206 + 350*Cos[2*(e + f*x)] - 231
*Sin[e + f*x] + 105*Sin[3*(e + f*x)])))/(a^3*c*f*(-1 + Sin[e + f*x])*(1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [A]**
time = 2.36, size = 170, normalized size = 0.89

| method | result |
|---|---|

| default | $-\frac{105(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-105(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}}{480c^{\frac{9}{2}}a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-}}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/c^(9/2)/a^3*(105*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+
sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-105*(c*(1+sin(f*x+e)))^(5/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+42*c^(7/
2)*sin(f*x+e)-350*c^(7/2)*sin(f*x+e)^2-210*c^(7/2)*sin(f*x+e)^3+278*c^(7/2)
)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**A**]
time = 0.34, size = 259, normalized size = 1.36

$$\frac{105\sqrt{2}\left(\cos(fx+e)^3\sin(fx+e)+\cos(fx+e)^3\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(175\cos(fx+e)^2+21\left(5\cos(fx+e)^2-4\right)\sin(fx+e)-36\right)\sqrt{-c\sin(fx+e)+c}}{960\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/960*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) - 4*(175*cos(f*x + e)^2 + 21*(5*cos(f*x + e)^2 - 4)*sin(f*x + e
) - 36)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) +
 a^3*c^2*f*cos(f*x + e)^3)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(174) = 348.

time = 0.55, size = 446, normalized size = 2.34

$$-\frac{\frac{15\sqrt{2}\left(\frac{14\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{210\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c^{\frac{3}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{15\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{a^3c^{\frac{3}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{32\sqrt{2}\left(29\sqrt{c}+\frac{100\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{170\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{120\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{a^3c^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{1920f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/1920*(15*sqrt(2)*(14*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) - 1)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*c^(3/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 210*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1))/(a^3*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 15
*sqrt(2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(a^3*c^(3/2)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 32*sqrt(2)*(29*
sqrt(c) + 100*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 170*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 120*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^
3*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) + 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)), x)
```

# 3.339 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=228

$$\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}+\frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}}+\frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}}-\frac{}{32a^3c}$$

```
[Out] 63/128*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+21/80*sec(f*x+e)/a^3/c/f/(
c-c*sin(f*x+e))^(3/2)+63/256*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*si
n(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-21/32*sec(f*x+e)/a^3/c^2/f/(c-c*sin(
f*x+e))^(1/2)-3/10*sec(f*x+e)^3/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/5*sec(f*
x+e)^5*(c-c*sin(f*x+e))^(1/2)/a^3/c^3/f
```

**Rubi** [A]
time = 0.28, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2815, 2754, 2766, 2760, 2729, 2728, 212}

$$\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}}+\frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (63*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) + (63*Cos[e + f*x])/(128*a^3*c*f*(c - c*Sin[e + f*
x])^(3/2)) + (21*Sec[e + f*x])/(80*a^3*c*f*(c - c*Sin[e + f*x])^(3/2)) - (2
1*Sec[e + f*x])/(32*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (3*Sec[e + f*x]^3
)/(10*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^5*Sqrt[c - c*Sin[
e + f*x]])/(5*a^3*c^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2754

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m + 1/2, 2*p]

Rule 2760

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2766

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2815

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{9\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+}}}{10a^3c^2}$$

$$= -\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\sec^5(e+fx)\sqrt{c-c\sin(}}{5a^3c^3f}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx)}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx)}$$

$$= \frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} + \frac{63\cos(e}{128a^3cf(c-c\sin}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.96, size = 443, normalized size = 1.94

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-240\cos^4(e+fx)-32\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-80\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+20\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+75\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-(315+315i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+40\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+150\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)}{640a^3f(1+\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*Cos[e + f*x]^4 - 32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 80*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^2 + 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 + 75*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5 - (315 + 315*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5 + 40*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
```

```
in[(e + f*x)/2])^5 + 150*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(640*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

**Maple [A]**
time = 3.02, size = 246, normalized size = 1.08

| method | result |
|---|---|
| default | $-\frac{315(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2+1176c^{\frac{9}{2}}(\sin^2(fx+e))-420c^{\frac{9}{2}}(\mathrm{si}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280/c^(13/2)/a^3*(315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+1176*c^(9/2)*sin(f*x
+e)^2-420*c^(9/2)*sin(f*x+e)^3-630*c^(9/2)*sin(f*x+e)^4-630*(c*(1+sin(f*x+e
)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin
(f*x+e)*c^2+708*c^(9/2)*sin(f*x+e)+315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-514*c^(9/2))/(1+sin(
f*x+e))^2/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

**Maxima [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas [A]**
time = 0.35, size = 226, normalized size = 0.99

$$\frac{315\sqrt{2}\sqrt{c}\cos(fx+e)^5\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(315\cos(fx+e)^4-42\cos(fx+e)^2-6\left(35\cos(fx+e)^2+24\right)\sin(fx+e)-16\right)\sqrt{-c\sin(fx+e)+c}}{2560a^3c^3f\cos(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*
```

```
c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(315*cos(f*x +
e)^4 - 42*cos(f*x + e)^2 - 6*(35*cos(f*x + e)^2 + 24)*sin(f*x + e) - 16)*sq
rt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{c^2\sqrt{-c\sin(e+fx)+c}\,\sin^5(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}\,\sin^4(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\,\sin^3(e+fx)-2c^2\sqrt{-c\sin(e+fx)+c}\,\sin^2(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c^2\sqrt{-c\sin(e+fx)+c}}}{a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5 + c**2*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**4 - 2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e +
 f*x)**3 - 2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c**2*sqrt(-c*
sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f*x) + c)), x)/a**3
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(209) = 418.
time = 0.64, size = 558, normalized size = 2.45

$$\frac{\frac{1260\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{5\sqrt{2}\left(\sqrt{c}-\frac{32\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{378\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{a^3c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{256\sqrt{2}\left(18\sqrt{c}+\frac{65\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{105\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{75\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{25\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{a^3c^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{5\left(\frac{32\sqrt{2}a^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\sqrt{2}a^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^6c^6}}{10240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*sqrt(2)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1))/(a^3*c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 5*sqrt(2)*(sqrt(c) - 32*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 378*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)^2/(a^3*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(18*sqrt(c) + 65*sqrt(c)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 105*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^2 + 75*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)^3 + 25*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c^3*((cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) - 5*(32*sqrt(2)*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - sqrt(2)*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^6*c^
6))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\sin(e + fx))^3 (c - c\sin(e + fx))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.340 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/4*a*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/4*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.25, size = 83, normalized size = 1.93

$$-\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(-28\cos(2(e+fx))+\cos(4(e+fx))+8(-7\sin(e+fx)+\sin(3(e+fx))))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/32*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(-28*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 8*(-7*Sin[e + f*x] + Sin[3*(e +
 f*x)])))/f
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(37) = 74$.
time = 16.28, size = 103, normalized size = 2.40

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}} \sin(fx+e) \sqrt{a(1+\sin(fx+e))} (\cos^6(fx+e)+\sin(fx+e)(\cos^4(fx+e))+\sin(fx+e)(\cos^2(fx+e))-(}{4f\cos(fx+e)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f*
x+e)^6+sin(f*x+e)*cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^2-cos(f*x+e)^2+4*sin(f
*x+e)+4)/cos(f*x+e)^7
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(40) = 80$.
time = 0.32, size = 102, normalized size = 2.37

$$-\frac{\left(c^{3} \cos\left(fx+e\right)^{4} - 8 c^{3} \cos\left(fx+e\right)^{2} + 7 c^{3} + 4 \left(c^{3} \cos\left(fx+e\right)^{2} - 2 c^{3}\right) \sin\left(fx+e\right)\right) \sqrt{a \sin\left(fx+e\right) + a} \sqrt{-c \sin\left(fx+e\right) + c}}{4 f \cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/4*(c^3*cos(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 + 7*c^3 + 4*(c^3*cos(f*x +
e)^2 - 2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.49, size = 54, normalized size = 1.26

$$\frac{4\sqrt{a}\,c^{\frac{7}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] 4*sqrt(a)*c^(7/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8/f
```

**Mupad** [B]
time = 8.25, size = 99, normalized size = 2.30

$$\frac{c^3\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)+48\,\sin\left(2\,e+2\,f\,x\right)-8\,\sin\left(4\,e+4\,f\,x\right)\right)}{32\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e +
 f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) + 48*sin(2*e + 2*f*x) - 8*si
n(4*e + 4*f*x)))/(32*f*(cos(2*e + 2*f*x) + 1))
```

# 3.341 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/3*a*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{2817\}$

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.18, size = 74, normalized size = 1.72

$$\frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(6\cos(2(e+fx))+15\sin(e+fx)-\sin(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(6*Cos[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f)

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(37) = 74$.
time = 16.48, size = 78, normalized size = 1.81

| method | result | size |
|---|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}} \sin(fx+e) \sqrt{a\left(1+\sin\left(fx+e\right)\right)}\, (\cos^4(fx+e)+\sin(fx+e)(\cos^2(fx+e))+2+2\sin(fx+e))}{3f\cos(fx+e)^5}$ | 78 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^2+2+2*sin(f*x+e))/cos(f*x+e)^5

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(40) = 80$.
time = 0.34, size = 89, normalized size = 2.07

$$\frac{\left(3\,c^2\cos\left(fx+e\right)^2-3\,c^2-\left(c^2\cos\left(fx+e\right)^2-4\,c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2 - (c^2*cos(f*x + e)^2 - 4*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac** [A]
time = 0.49, size = 54, normalized size = 1.26

$$\frac{8\,\sqrt{a}\,c^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 8/3*sqrt(a)*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6/f
```

**Mupad** [B]
time = 7.71, size = 88, normalized size = 2.05

$$\frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e +
f*x) + 6*cos(3*e + 3*f*x) + 14*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(12*f*
(cos(2*e + 2*f*x) + 1))
```

# 3.342 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*a*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.14, size = 60, normalized size = 1.40

$$\frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,(\cos(2(e+fx))+4\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

**Maple [A]**
time = 15.67, size = 61, normalized size = 1.42

| method | result | size |
|---|---|---|
| default | $\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(\cos^{2}\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{2f\cos\left(fx+e\right)^{3}}$ | 61 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f*
x+e)^2+sin(f*x+e)+1)/cos(f*x+e)^3
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [A]**
time = 0.34, size = 66, normalized size = 1.53

$$\frac{\left(c\cos\left(fx+e\right)^{2}+2\,c\sin\left(fx+e\right)-c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(c*cos(f*x + e)^2 + 2*c*sin(f*x + e) - c)*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac** [A]
time = 0.48, size = 54, normalized size = 1.26

$$\frac{2\sqrt{a}\,c^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 2*sqrt(a)*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4/f
```

**Mupad** [B]
time = 0.84, size = 71, normalized size = 1.65

$$\frac{c\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)+4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x)
 + cos(3*e + 3*f*x) + 4*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1))
```

# 3.343 $\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{2817\}$

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.06, size = 39, normalized size = 0.95

$$\frac{\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*Tan[e + f*x])/f
```

**Maple [A]**
time = 16.09, size = 44, normalized size = 1.07

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{f\cos\left(fx+e\right)}$ | 44 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/cos(f*x+e
)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [A]**
time = 0.35, size = 47, normalized size = 1.15

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x
+ e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac** [A]
time = 0.48, size = 54, normalized size = 1.32

$$\frac{2\sqrt{a}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 2*sqrt(a)*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2/f
```

**Mupad** [B]
time = 7.01, size = 47, normalized size = 1.15

$$\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{2\,f\,{\cos\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] (sin(2*e + 2*f*x)*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2
))/(2*f*cos(e + f*x)^2)
```

# 3.344 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=52

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2816, 2746, 31}

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{(a\cos(e+fx))\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.63, size = 119, normalized size = 2.29

$$
-\frac{\sqrt{2}\left(-i+e^{i(e+fx)}\right)\left(fx+2i\log\left(i-e^{i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{\sqrt{ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}\left(i+e^{i(e+fx)}\right)f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(-I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I - E^(I*(e + f*x))])*Sq
rt[a*(1 + Sin[e + f*x])])/(Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*
x))]*(I + E^(I*(e + f*x)))*f))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.
time = 8.93, size = 105, normalized size = 2.02

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(-1+\cos(fx+e)+\sin(fx+e))\left(2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)}{f(-1+\cos(fx+e)-\sin(fx+e))\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$ | 105 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e)+sin(f*x+e))*(2*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1)))/(-1+cos(f*x+e)-sin(f*x+e))
/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima [A]**
time = 0.50, size = 67, normalized size = 1.29

$$
\frac{\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e
) - c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac** [A]
time = 0.49, size = 56, normalized size = 1.08

$$\frac{2\sqrt{a}\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{c}\,f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 2*sqrt(a)*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))/(sqrt(c)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.345 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=40

$$\frac{a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$\frac{a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(40) = 80.
time = 0.12, size = 84, normalized size = 2.10

$$\frac{\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}{c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^2*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 9.18, size = 69, normalized size = 1.72

| method | result | size |
|---|---|---|
| default | $-\frac{(-1+\cos(fx+e)+\sin(fx+e))\sin(fx+e)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{f(-1+\cos(fx+e)-\sin(fx+e))(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$ | 69 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-1+cos
(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [A]
time = 0.35, size = 64, normalized size = 1.60

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-c^{2}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*f*cos(f*x + e)*sin
(f*x + e) - c^2*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac** [A]
time = 0.52, size = 56, normalized size = 1.40

$$-\frac{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{2\,c^{\frac{3}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/2*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(3/2)*f*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.346 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=43

$$\frac{a\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{2817\}$

$$\frac{a\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.
time = 0.13, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(2*c^3*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.
time = 8.98, size = 95, normalized size = 2.21

| method | result | size |
|---|---|---|
| default | $-\frac{\left(\cos^{2}\left(fx+e\right)-\cos\left(fx+e\right)\sin\left(fx+e\right)+2\cos\left(fx+e\right)+3\sin\left(fx+e\right)-3\right)\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{2f\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}}$ | 95 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+2*cos(f*x+e)+3*sin(f*x+e)-3)*sin
(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)
-1))^(5/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [A]
time = 0.34, size = 79, normalized size = 1.84

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,\left(c^{3}f\cos\left(fx+e\right)^{3}+2\,c^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^3*f*cos(f*x + e)
^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(-c*(sin(e + f*x) - 1))**(5/2), x)
```

**Giac** [A]
time = 0.48, size = 56, normalized size = 1.30

$$-\frac{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{8\,c^{\frac{5}{2}}\,f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/8*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(5/2)*f*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)
```

**Mupad** [B]
time = 8.89, size = 142, normalized size = 3.30

$$\frac{4\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(10\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+4\,\sin\left(2\,e+2\,f\,x\right)-4\right)}{c^3\,f\,\left(30\sin\left(e+f\,x\right)^2+48\,\sin\left(e+f\,x\right)-52\sin\left(2\,e+2\,f\,x\right)^2+2\sin\left(3\,e+3\,f\,x\right)^2+40\,\sin\left(3\,e+3\,f\,x\right)-8\,\sin\left(5\,e+5\,f\,x\right)-32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (4*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(4*sin(2*e +
2*f*x) + 10*sin(e/2 + (f*x)/2)^2 - 2*sin((3*e)/2 + (3*f*x)/2)^2 - 4))/(c^3*
f*(48*sin(e + f*x) + 40*sin(3*e + 3*f*x) - 8*sin(5*e + 5*f*x) - 52*sin(2*e
+ 2*f*x)^2 + 2*sin(3*e + 3*f*x)^2 + 30*sin(e + f*x)^2 - 32))
```

# 3.347 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=43

$$\frac{a\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}$$

[Out] 1/3*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$\frac{a\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]

[Out] (a*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.
time = 0.17, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}{3c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(3*c^4*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(37) = 74$.
time = 8.43, size = 120, normalized size = 2.79

| method | result |
|---|---|
| default | $\frac{(\cos^3(fx+e)+\sin(fx+e)(\cos^2(fx+e))-4(\cos^2(fx+e))+3\cos(fx+e)\sin(fx+e)-4\cos(fx+e)-7\sin(fx+e)+7)\sin(fx+e)\sqrt{a\,(1}}{3f(-1+\cos(fx+e)-\sin(fx+e))(-c(\sin(fx+e)-1))^{\frac{7}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^2-4*cos(f*x+e)^2+3*cos(f*x+e)*sin
(f*x+e)-4*cos(f*x+e)-7*sin(f*x+e)+7)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-
1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(7/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.
time = 0.32, size = 97, normalized size = 2.26

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x +
e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))
*sin(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

**Giac** [A]
time = 0.48, size = 56, normalized size = 1.30

$$-\frac{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{24\,c^{\frac{7}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/24*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*f*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

**Mupad** [B]
time = 10.98, size = 190, normalized size = 4.42

$$-\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,16\mathrm{i}}{3\,c^4\,f\,\left(1+14\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}-\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}-14\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6\mathrm{i}-\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,14\mathrm{i}-\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,14\mathrm{i}+\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,6\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] -(exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*16i)/(3*c^4*f*(exp(e*1i + f*x*1i)*6i - 14*exp(e*2i + f*x*2i)
 - exp(e*3i + f*x*3i)*14i - exp(e*5i + f*x*5i)*14i + 14*exp(e*6i + f*x*6i)
+ exp(e*7i + f*x*7i)*6i - exp(e*8i + f*x*8i) + 1))
```

# 3.348 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{5f}$$

```
[Out] -1/10*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin[e + f
*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)
)/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}(c - c\sin(e + fx))^{7/2}\,dx = -\frac{a\cos(e + fx)\sqrt{a + a\sin(e + fx)}\,(c - c\sin(e + fx))^{7/2}}{5f}$$

$$= -\frac{a^2\cos(e + fx)(c - c\sin(e + fx))^{7/2}}{10f\sqrt{a + a\sin(e + fx)}} - \frac{a\cos(e + fx)\sqrt{}}{}$$

**Mathematica [A]**
time = 0.64, size = 146, normalized size = 1.64

$$-\frac{c^3(-1 + \sin(e + fx))^3(a(1 + \sin(e + fx)))^{3/2}\sqrt{c - c\sin(e + fx)}\,(20\cos(2(e + fx)) + 5\cos(4(e + fx)) + 70\sin(e + fx) + 5\sin(3(e + fx)) - \sin(5(e + fx)))}{80f\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/80*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Si
n[e + f*x]]*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5
*Sin[3*(e + f*x)] - Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple [A]**
time = 15.48, size = 106, normalized size = 1.19

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(2(\cos^6(fx+e))+\sin(fx+e)(\cos^4(fx+e))+2(\cos^4(fx+e))+3\sin(fx+e)(\cos^2(fx\right.}{10f\cos(fx+e)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(2*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4+3*sin(f*x+e)*cos(f*x+e)^2+
6*sin(f*x+e)+6)/cos(f*x+e)^7
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [A]
time = 0.34, size = 108, normalized size = 1.21

$$\frac{\left(5\,ac^3\cos\left(fx+e\right)^4-5\,ac^3-2\left(ac^3\cos\left(fx+e\right)^4-2\,ac^3\cos\left(fx+e\right)^2-4\,ac^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/10*(5*a*c^3*cos(f*x + e)^4 - 5*a*c^3 - 2*(a*c^3*cos(f*x + e)^4 - 2*a*c^3*
cos(f*x + e)^2 - 4*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.53, size = 110, normalized size = 1.24

$$-\frac{8\left(4\,ac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-5\,ac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8\right)\sqrt{a}\,\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -8/5*(4*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*a*c^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

**Mupad** [B]
time = 8.92, size = 111, normalized size = 1.25

$$\frac{a\,c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)+75\,\sin\left(2\,e+2\,f\,x\right)+4\,\sin\left(4\,e+4\,f\,x\right)-\sin\left(6\,e+6\,f\,x\right)\right)}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (a*c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e
 + f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) + 75*sin(2*e + 2*f*x) +
4*sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

# 3.349 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{4f}$$

```
[Out] -1/6*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/4*a*c
os(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[e + f*
x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
/(4*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}(c - c\sin(e + fx))^{5/2}\,dx = -\frac{a\cos(e + fx)\sqrt{a + a\sin(e + fx)}\,(c - c\sin(e + fx))^{5}}{4f}$$
$$= -\frac{a^2\cos(e + fx)(c - c\sin(e + fx))^{5/2}}{6f\sqrt{a + a\sin(e + fx)}} - \frac{a\cos(e + fx)\,\sqrt{}}{}$$

**Mathematica [A]**
time = 0.40, size = 137, normalized size = 1.54

$$\frac{c^2(-1 + \sin(e + fx))^2(a(1 + \sin(e + fx)))^{3/2}\sqrt{c - c\sin(e + fx)}\,(12\cos(2(e + fx)) + 3\cos(4(e + fx)) + 8(9\sin(e + fx) + \sin(3(e + fx))))}{96f\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(12*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3
*(e + f*x)])))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

**Maple [A]**
time = 15.81, size = 90, normalized size = 1.01

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}(3(\cos^4(fx+e))+\sin(fx+e)(\cos^2(fx+e))+4(\cos^2(fx+e))+5\sin(fx+e)+5)}{12f\cos(fx+e)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(3*cos
(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^2+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e)
^5
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [A]**
time = 0.34, size = 93, normalized size = 1.04

$$\frac{\left(3\,ac^2\cos\left(fx+e\right)^4-3\,ac^2+4\,\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] 1/12*(3*a*c^2*cos(f*x + e)^4 - 3*a*c^2 + 4*(a*c^2*cos(f*x + e)^2 + 2*a*c^2)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e))
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac [A]**
time = 0.49, size = 159, normalized size = 1.79

$$-\frac{4\left(3\,ac^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-8\,ac^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\,ac^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] -4/3*(3*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*a*c^2*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) + 6*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 1.75, size = 100, normalized size = 1.12

$$\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)+80\,\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2),x)
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e
 + f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) + 80*sin(2*e + 2*f*x) +
8*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

# 3.350 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}{3f}$$

```
[Out] -1/3*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*c
os(f*x+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e + f*
x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
/(3*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}{3f}$$

$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{}}{}$$

**Mathematica [A]**
time = 0.26, size = 70, normalized size = 0.79

$$-\frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(9\sin(e+fx)+\sin(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/12*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sq
rt[c - c*Sin[e + f*x]]*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))/f
```

**Maple [A]**
time = 15.54, size = 55, normalized size = 0.62

| method | result | size |
|---|---|---|
| default | $\frac{(\cos^2(fx+e)+2)(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}}{3f\cos(fx+e)^3}$ | 55 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^2+2)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(3/2)/cos(f*x+e)^3
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [A]**
time = 0.34, size = 65, normalized size = 0.73

$$\frac{\left(ac\cos\left(fx+e\right)^{2}+2\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 + 2*a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac [A]**
time = 0.51, size = 106, normalized size = 1.19

$$\frac{4\left(2\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 4/3*(2*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a*c*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 0.89, size = 66, normalized size = 0.74

$$\frac{a\,c\,\left(10\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (a*c*(10*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*
(-c*(sin(e + f*x) - 1))^(1/2))/(12*f*(cos(2*e + 2*f*x) + 1))
```

# 3.351 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=43

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2817}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.15, size = 60, normalized size = 1.40

$$-\frac{a\sec(e+fx)(\cos(2(e+fx))-4\sin(e+fx))\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/4*(a*Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
+ f*x])]*Sqrt[c - c*Sin[e + f*x]])/f
```

**Maple [A]**
time = 16.77, size = 63, normalized size = 1.47

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(\cos^{2}(fx+e)-\sin(fx+e)+1\right)}{2f\cos(fx+e)^{3}}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(cos(f*
x+e)^2-sin(f*x+e)+1)/cos(f*x+e)^3
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [A]**
time = 0.34, size = 66, normalized size = 1.53

$$-\frac{\left(a\cos\left(fx+e\right)^{2}-2\,a\sin\left(fx+e\right)-a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - a)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac** [A]
time = 0.50, size = 54, normalized size = 1.26

$$-\frac{2\,a^{\frac{3}{2}}\sqrt{c}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*a^(3/2)*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [B]
time = 7.35, size = 71, normalized size = 1.65

$$-\frac{a\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x
) + cos(3*e + 3*f*x) - 4*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1))
```

# 3.352 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=96

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*a^2*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2819, 2816, 2746, 31}

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+\frac{\left(2a^2c\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}\\ &= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(2a^2\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx,x,-\right.}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}\\ &= -\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+}}{f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.20, size = 113, normalized size = 1.18

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Sin[e + f*x]))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(88) = 176.
time = 9.32, size = 250, normalized size = 2.60

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{\left(\cos^2(fx+e)-\cos(fx+e)\sin(fx+e)-4\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\cos(fx+e)+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)\cos(fx+e)-4\ln\left(-\frac{-1+\cos}{}\right.\right.}{f(\cos^2(fx+e)+\cos(fx+e)\sin(fx+e)+}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))*cos(f*x+e)-4*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)+sin(
f*x+e)+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-
1)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-
2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x +
 e) - c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac [A]**
time = 0.47, size = 101, normalized size = 1.05

$$\frac{2\,a^{\frac{3}{2}}\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 2*a^(3/2)*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))) + log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.353 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=97

$$\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*cos(f*x+e)
*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2818, 2816, 2746, 31}

$$\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e + f*x])^(3/2)) +
(a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(a^2\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{(a^2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin\ldots\right.}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e\ldots}}$$

$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+\ldots}}$$

**Mathematica** [**A**]
time = 0.29, size = 153, normalized size = 1.58

$$\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-1-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)\right)}{cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-1 -
 Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]]*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(89) = 178$.
time = 9.22, size = 375, normalized size = 3.87

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)(\cos^2(fx+e))-2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)\cos(fx+e)+2\ln\left(-\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))*cos(f*x+e)^2-ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)^2-2*c
os(f*x+e)*sin(f*x+e)+ln(2/(cos(f*x+e)+1))*cos(f*x+e)-2*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-4*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+2*sin(f*x+e)-2*ln(2/(cos(f
*x+e)+1))+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2)*(a*(1+sin(f*x+e))
)^(3/2)/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*
(sin(f*x+e)-1))^(3/2)
```

**Maxima [A]**
time = 0.53, size = 147, normalized size = 1.52

$$-\frac{\frac{2\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{4\,a^{\frac{3}{2}}\sqrt{c}\,\sin(fx+e)}{\left(c^2-\frac{2\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - a^(3/2)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4*a^(3/2)*sqrt(c)*sin(f*
x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + c^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x
 + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac** [A]
time = 0.48, size = 91, normalized size = 0.94

$$-\frac{\left(2\,a\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)\sqrt{a}}{c^{\frac{3}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -(2*a*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/sin(-1/4*pi + 1/2*f*x + 1/2
*e)^2)*sqrt(a)/(c^(3/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.354 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(42) = 84.
time = 0.29, size = 99, normalized size = 2.36

$$\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)\sqrt{a(1+\sin(e+fx))}}{c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f
*x])])/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*S
qrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.
time = 9.81, size = 90, normalized size = 2.14

| method | result | size |
|---|---|---|
| default | $-\frac{(-1+\cos(fx+e)+\sin(fx+e))(a(1+\sin(fx+e)))^{\frac{3}{2}}\sin(fx+e)}{f(-c(\sin(fx+e)-1))^{\frac{5}{2}}(\cos^2(fx+e)+\cos(fx+e)\sin(fx+e)+\cos(fx+e)-2\sin(fx+e)-2)}$ | 90 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)/(-c*(si
n(f*x+e)-1))^(5/2)/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-2*sin(f*x
+e)-2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.
time = 0.33, size = 87, normalized size = 2.07

$$-\frac{\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,a\sin(fx+e)}{c^3f\cos(fx+e)^3+2\,c^3f\cos(fx+e)\sin(fx+e)-2\,c^3f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*a*sin(f*x + e)/(c^3*f*c
os(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(-c*(sin(e + f*x) - 1))**(5/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(39) = 78.
time = 0.50, size = 98, normalized size = 2.33

$$\frac{\left(2\,a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{4\,c^{3}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/4*(2*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^2 - a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^3*f*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.355 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=88

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*cos(f*x
+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(5
/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

**Mathematica** [A]
time = 0.36, size = 106, normalized size = 1.20

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(1+3\sin(e+fx))}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1
 + 3*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 8.92, size = 141, normalized size = 1.60

| method | result |
|---|---|
| default | $\frac{(a(1+\sin(fx+e)))^{\frac{3}{2}}\sin(fx+e)\left(\sin(fx+e)\left(\cos^2(fx+e)\right)+\cos^3(fx+e)+3\cos(fx+e)\sin(fx+e)-4\left(\cos^2(fx+e)\right)-10\sin(fx+e)-7\,\mathrm{co}}{6f(-c(\sin(fx+e)-1))^{\frac{7}{2}}\left(\cos^2(fx+e)+\cos(fx+e)\sin(fx+e)+\cos(fx+e)-2\sin(fx+e)-2\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)^2+cos(f*x+
e)^3+3*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)^2-10*sin(f*x+e)-7*cos(f*x+e)+10)/
(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-2*
sin(f*x+e)-2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [A]**
time = 0.33, size = 109, normalized size = 1.24

$$-\frac{(3\,a\sin\left(fx+e\right)+a)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\,\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3
- 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

**Giac [A]**
time = 0.51, size = 92, normalized size = 1.05

$$\frac{\left(3\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{24\,c^{\frac{7}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] 1/24*(3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(7/2)*f*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

**Mupad [B]**
time = 10.65, size = 124, normalized size = 1.41

$$-\frac{a\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}+3\,a\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\frac{9\,c^4\,f\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{21\,c^4\,f\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{3\,c^4\,f\,\sin\left(4\,e+4\,f\,x\right)}{4}-\frac{21\,c^4\,f\,\cos\left(e+f\,x\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] -(a*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*a*sin(e + f*x
)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e
+ 3*f*x))/2 + (21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4
- (21*c^4*f*cos(e + f*x))/2)
```

# 3.356 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}$$

```
[Out] -1/12*a^2*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+1/4*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(9/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2818, 2817}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*(c - c*Sin[e + f*x])^(9/2))
- (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(7/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx}{4c}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{12cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+}$$

**Mathematica [A]**
time = 0.66, size = 106, normalized size = 1.15

$$\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(1+2\sin(e+fx))}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1 + 2*
Sin[e + f*x]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
 f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(80) = 160$.
time = 9.30, size = 168, normalized size = 1.83

| method | result |
|---|---|
| default | $\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(\cos^4(fx+e)-(\cos^3(fx+e))\sin(fx+e)+4(\cos^3(fx+e))+5\sin(fx+e)(\cos^2(fx+e))-12(\cos^2(fx+e)\right.}{6f(-c(\sin(fx+e)-1))^{\frac{9}{2}}\left(\cos^2(fx+e)+\cos(fx+e)\sin(fx+e)+\cos(fx+e)-2\sin(f\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*
x+e)+4*cos(f*x+e)^3+5*sin(f*x+e)*cos(f*x+e)^2-12*cos(f*x+e)^2+7*cos(f*x+e)*
sin(f*x+e)-10*cos(f*x+e)-17*sin(f*x+e)+17)/(-c*(sin(f*x+e)-1))^(9/2)/(cos(f
*x+e)^2+cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas [A]**
time = 0.34, size = 123, normalized size = 1.34

$$\frac{(2\,a\sin\left(fx+e\right)+a)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x + e) +
4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.47, size = 98, normalized size = 1.07

$$\frac{\left(4\,a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-3\,a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{96\,c^{5}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] 1/96*(4*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - 3*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^5
*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)
```

**Mupad [B]**
time = 11.81, size = 195, normalized size = 2.12

$$\frac{\left(\frac{16\,a\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,a\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{84\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)+16\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (((16*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2))/(3*c^5*f) + (32*a*ex
p(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^5*f))*(c - c
*sin(e + f*x))^(1/2))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*exp(e*5i + f
*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) - 96*exp(e*
5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4*f*x))
```

# 3.357 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

**Optimal**. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}$$

```
[Out] -1/20*a^2*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/2)+1/5*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(11/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2818, 2817}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*f*(c - c*Sin[e + f*x])^(11/2))
 - (a^2*Cos[e + f*x])/(20*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(9/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{20cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e-$$

**Mathematica [A]**
time = 0.95, size = 106, normalized size = 1.15

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(3+5\sin(e+fx))}{20c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -1/20*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
3 + 5*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(80) = 160$.
time = 17.52, size = 196, normalized size = 2.13

| method | result |
|---|---|
| default | $-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}(3(\cos^5(fx+e))+3\sin(fx+e)(\cos^4(fx+e))-18(\cos^4(fx+e))+15(\cos^3(fx+e))\sin(fx+e)-36(\cos}{20f(-c(\sin(fx+e)-1))^{\frac{11}{2}}(\cos^2(fx+e)+\cos(fx+e)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(3*cos(f*x+e)^5+3*sin(f*x+e)*co
s(f*x+e)^4-18*cos(f*x+e)^4+15*cos(f*x+e)^3*sin(f*x+e)-36*cos(f*x+e)^3-51*si
n(f*x+e)*cos(f*x+e)^2+96*cos(f*x+e)^2-45*cos(f*x+e)*sin(f*x+e)+53*cos(f*x+e
)+98*sin(f*x+e)-98)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^2+cos(f*x+e)*sin
(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

**Fricas** [A]
time = 0.34, size = 141, normalized size = 1.53

$$\frac{(5\,a\sin\left(fx+e\right)+3\,a)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{20\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*a*sin(f*x + e) + 3*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x
+ e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x +
 e))*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.49, size = 92, normalized size = 1.00

$$\frac{\left(5\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{320\,c^{\frac{11}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] 1/320*(5*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - 4*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(11/2)*f*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)
```

**Mupad** [B]
time = 12.59, size = 225, normalized size = 2.45

$$\frac{\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,48\mathrm{i}}{5\,c^6\,f}+\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16\mathrm{i}}{c^6\,f}\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220\mathrm{i}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330\mathrm{i}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] (((a*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*48i)/(5*c^6*f) + (a*exp(
e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^6*f))*(c - c
*sin(e + f*x))^(1/2))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*
x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp
(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)
*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

# 3.358 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=134

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a\cos(e+\cdots}{}$$

```
[Out] -1/6*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/15*a^3*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/15*a^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/15*(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin[e + f
*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(15*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(7/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))}{6f}$$
$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx}{15f}$$
$$= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx}{}$$

**Mathematica [A]**
time = 0.86, size = 156, normalized size = 1.16

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(75\cos(2(e+fx))+30\cos(4(e+fx))+5\cos(6(e+fx))+600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx)))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*S
in[e + f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)
] + 600*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
)
```

**Maple [A]**
time = 16.49, size = 116, normalized size = 0.87

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(5\left(\cos^{6}(fx+e)\right)+\sin(fx+e)\left(\cos^{4}(fx+e)\right)+6\left(\cos^{4}(fx+e)\right)+3\sin(fx+e)\left(\cos^{2}(\right.\right.}{30f\cos(fx+e)^{7}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(5*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4+3*sin(f*x+e)*cos(f*x+e)^2+
8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^7
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [A]
time = 0.36, size = 119, normalized size = 0.89

$$\frac{\left(5\,a^2c^3\cos\left(fx+e\right)^6-5\,a^2c^3+2\left(3\,a^2c^3\cos\left(fx+e\right)^4+4\,a^2c^3\cos\left(fx+e\right)^2+8\,a^2c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/30*(5*a^2*c^3*cos(f*x + e)^6 - 5*a^2*c^3 + 2*(3*a^2*c^3*cos(f*x + e)^4 +
4*a^2*c^3*cos(f*x + e)^2 + 8*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.51, size = 216, normalized size = 1.61

$$\frac{16\left(10\,a^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-36\,a^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+45\,a^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-20\,a^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] 16/15*(10*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 36*a^2*c^3*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 45*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*a^2*c^3*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

**Mupad** [B]
time = 9.94, size = 124, normalized size = 0.93

$$\frac{a^2\,c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\,f\,x\right)+5\,\cos\left(7\,e+7\,f\,x\right)+700\,\sin\left(2\,e+2\,f\,x\right)+112\,\sin\left(4\,e+4\,f\,x\right)+12\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (a^2*c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos
(e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x)
 + 700*sin(2*e + 2*f*x) + 112*sin(4*e + 4*f*x) + 12*sin(6*e + 6*f*x)))/(960
*f*(cos(2*e + 2*f*x) + 1))
```

# 3.359 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=134

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+$$

```
[Out] -1/5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))}{5f}$$
$$= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))}{5f}$$
$$= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx}{}$$

**Mathematica** [A]
time = 0.32, size = 77, normalized size = 0.57

$$\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(e + f*x)]))/(240*f)
```

**Maple** [A]
time = 16.94, size = 67, normalized size = 0.50

| method | result | size |
|---|---|---|
| default | $\frac{\left(3\left(\cos^4(fx+e)\right)+4\left(\cos^2(fx+e)\right)+8\right)\left(-c(\sin(fx+e)-1)\right)^{\frac{5}{2}}\sin(fx+e)\left(a(1+\sin(fx+e))\right)^{\frac{5}{2}}}{15f\cos(fx+e)^5}$ | 67 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+
e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [A]
time = 0.35, size = 91, normalized size = 0.68

$$\frac{\left(3\,a^2c^2\cos\left(fx+e\right)^4+4\,a^2c^2\cos\left(fx+e\right)^2+8\,a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/15*(3*a^2*c^2*cos(f*x + e)^4 + 4*a^2*c^2*cos(f*x + e)^2 + 8*a^2*c^2)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e)
)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.51, size = 165, normalized size = 1.23

$$-\frac{16\left(6\,a^2c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-15\,a^2c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+10\,a^2c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -16/15*(6*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*a^2*c^2*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 10*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c
)/f
```

**Mupad** [B]
time = 1.50, size = 83, normalized size = 0.62

$$\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(175\,\sin\left(2\,e+2\,f\,x\right)+28\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)\right)}{240\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(175*si
n(2*e + 2*f*x) + 28*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x)))/(240*f*(cos(2*e
 + 2*f*x) + 1))
```

# 3.360 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=89

$$\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

```
[Out] 1/6*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4*c*co
s(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[c - c*Sin[e + f*x]]
) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4
*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f\sqrt{c-c\sin(e+fx)}} + \frac{c\cos(e+fx)(a\cdots}{\cdots}$$

**Mathematica** [A]
time = 0.41, size = 133, normalized size = 1.49

$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(-12\cos(2(e+fx))-3\cos(4(e+fx))+8(9\sin(e+fx)+\sin(3(e+fx))))}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/96*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(-12*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Si
n[3*(e + f*x)])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^5)
```

**Maple** [A]
time = 17.41, size = 91, normalized size = 1.02

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(3\left(\cos^4(fx+e)\right)-\sin(fx+e)\left(\cos^2(fx+e)\right)+4\left(\cos^2(fx+e)\right)-5\sin(fx+e)+5\right)}{12f\cos(fx+e)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(3*cos
(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^2+4*cos(f*x+e)^2-5*sin(f*x+e)+5)/cos(f*x+e)
^5
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [A]**
time = 0.35, size = 93, normalized size = 1.04

$$-\frac{\left(3\,a^{2}c\cos\left(fx+e\right)^{4}-3\,a^{2}c-4\,\left(a^{2}c\cos\left(fx+e\right)^{2}+2\,a^{2}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
[Out] -1/12*(3*a^2*c*cos(f*x + e)^4 - 3*a^2*c - 4*(a^2*c*cos(f*x + e)^2 + 2*a^2*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac [A]**
time = 0.49, size = 110, normalized size = 1.24

$$\frac{4\left(3\,a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] 4/3*(3*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*a^2*c*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 8.27, size = 100, normalized size = 1.12

$$-\frac{a^{2}\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)-80\,\sin\left(2\,e+2\,f\,x\right)-8\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2),x)
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(
e + f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) - 80*sin(2*e + 2*f*x) -
 8*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

# 3.361 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=43

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.18, size = 72, normalized size = 1.67

$$-\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(6\cos(2(e+fx))-15\sin(e+fx)+\sin(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

[Out] -1/12*(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(6*Cos[2*(e + f*x)] - 15*Sin[e + f*x] + Sin[3*(e + f*x)]))/f

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 17.67, size = 79, normalized size = 1.84

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(\cos^4(fx+e)-\sin(fx+e)\left(\cos^2(fx+e)\right)+2-2\sin(fx+e)\right)}{3f\cos(fx+e)^5}$ | 79 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^2+2-2*sin(f*x+e))/cos(f*x+e)^5

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(40) = 80$.
time = 0.35, size = 88, normalized size = 2.05

$$-\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{2}-3\,a^{2}+\left(a^{2}\cos\left(fx+e\right)^{2}-4\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] -1/3*(3*a^2*cos(f*x + e)^2 - 3*a^2 + (a^2*cos(f*x + e)^2 - 4*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac** [A]
time = 0.46, size = 54, normalized size = 1.26

$$-\frac{8\,a^{\frac{5}{2}}\sqrt{c}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -8/3*a^(5/2)*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [B]
time = 7.75, size = 86, normalized size = 2.00

$$-\frac{a^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e +
 f*x) + 6*cos(3*e + 3*f*x) - 14*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(12*f
*(cos(2*e + 2*f*x) + 1))
```

# 3.362 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*cos
(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*
a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2819, 2816, 2746, 31}

$$-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2f\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2f\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2f\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e}}{f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.35, size = 127, normalized size = 0.90

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+12\sin(e+fx)\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-
Cos[2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e +
 f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]]
)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(127) = 254$.
time = 16.86, size = 315, normalized size = 2.23

| method | result |
|---|---|
| default | $-\frac{\left(-\left(\cos^3(fx+e)\right)-\sin(fx+e)\left(\cos^2(fx+e)\right)+16\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\cos(fx+e)+16\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(f}{2f(\cos^3(fx+e)-\sin(fx+e)(\cos}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2+16*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))*cos(f*x+e)+16*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*s
in(f*x+e)-5*cos(f*x+e)^2+6*cos(f*x+e)*sin(f*x+e)-8*ln(2/(cos(f*x+e)+1))*cos
(f*x+e)-8*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-16*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))+cos(f*x+e)-5*sin(f*x+e)+8*ln(2/(cos(f*x+e)+1))+5)*(a*(1+sin(f*
x+e)))^(5/2)/(cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2-2*cos(f*x
+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**A**]
time = 0.48, size = 135, normalized size = 0.96

$$\frac{2\,a^{\frac{5}{2}}\sqrt{c}\left(\frac{c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{2}}+\frac{2\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 2*a^(5/2)*sqrt(c)*((c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))/c^2 + 2*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.363 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=144

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}}+\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+4*a^3*cos(f*x+
e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a^2
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \cdots$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \cdots$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \cdots$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3\cos(e+fx)\log(1-\sin(e+f\cdots}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+\cdots}}$$

## Mathematica [A]

time = 0.51, size = 169, normalized size = 1.17

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(7+\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(2-16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(7 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 -
16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(132) = 264$.
time = 15.75, size = 439, normalized size = 3.05

| method | result |
|---|---|
| default | $-\frac{\left(4\ln\left(\frac{2}{\cos(fx+e)+1}\right)(\cos^2(fx+e))-4\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)\cos(fx+e)-8\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+8\ln\left(\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(4*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2-4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)
*cos(f*x+e)-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+8*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^3-
sin(f*x+e)*cos(f*x+e)^2+4*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+8*ln(2/(cos(f*x+e
)+1))*sin(f*x+e)-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)-16
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+6*cos(f*x+e)^2-5*cos
(f*x+e)*sin(f*x+e)-8*ln(2/(cos(f*x+e)+1))+16*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))+cos(f*x+e)+6*sin(f*x+e)-6)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e
)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*
x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(3/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e)
 - 2*c^2), x)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**A**]
time = 0.52, size = 142, normalized size = 0.99

$$-\frac{2\,a^{\frac{5}{2}}\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{1}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -2*a^(5/2)*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + 2*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 1/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.364 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=147

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)\log(1-\sin(e-}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}$$

[Out] 1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

**Rubi** [A]
time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2818, 2816, 2746, 31}

$$-\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} -$$

**Mathematica** [A]
time = 0.71, size = 190, normalized size = 1.29

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-2-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-2 -
 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(133) = 266$.

time = 16.66, size = 553, normalized size = 3.76

| method | result |
|---|---|
| default | $-\frac{\left(\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\left(\cos^2(fx+e)\right)\sin(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2(\mathrm{co}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f
*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^
3+2*sin(f*x+e)*cos(f*x+e)^2-3*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2+6*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(2/(cos(f*x+e)+1))*sin(f
*x+e)*cos(f*x+e)-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*co
s(f*x+e)-2*cos(f*x+e)^2-2*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+4*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)-4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)+8*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-2*cos(f*x+e)-2*sin(f*
x+e)+4*ln(2/(cos(f*x+e)+1))-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2)
*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e
)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1)
)^(5/2)
```

**Maxima** [A]

time = 0.49, size = 198, normalized size = 1.35

$$-\frac{\frac{8\,a^{\frac{5}{2}}\sqrt{c}\,\sin(fx+e)^2}{\left(c^3-\frac{4\,c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6\,c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4\,c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2}-\frac{2\,a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}+\frac{a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -(8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))/f
```

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos
(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**A**]
time = 0.52, size = 132, normalized size = 0.90

$$\frac{\left(4\,a^2\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{4\,a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}\right)\sqrt{a}}{2\,c^{\frac{5}{2}}f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/2*(4*a^2*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + (4*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^2 - a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4)*sqrt(a)/(c^(5/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.365 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=42

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. 2(42) = 84.
time = 0.60, size = 110, normalized size = 2.62

$$\frac{a^2(-5+3\cos(2(e+fx)))\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a
*(1 + Sin[e + f*x])])/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(36) = 72$.
time = 17.67, size = 129, normalized size = 3.07

| method | result |
|---|---|
| default | $-\frac{(-1+\cos(fx+e)+\sin(fx+e))\left(\cos^2(fx+e)-4\right)\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}}{3f(\cos^3(fx+e)-\sin(fx+e)(\cos^2(fx+e))-3(\cos^2(fx+e))-2\cos(fx+e)\sin(fx+e)-2\cos(fx+e)+4\sin(fx+e)+4)(-c(\sin(fx+e)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(-1+cos(f*x+e)+sin(f*x+e))*(cos(f*x+e)^2-4)*sin(f*x+e)*(a*(1+sin(f*x
+e)))^(5/2)/(cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2-2*cos(f*x+
e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(7/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(39) = 78$.
time = 0.37, size = 117, normalized size = 2.79

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^2-4\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 - 4*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x
+ e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(39) = 78.
time = 0.50, size = 138, normalized size = 3.29

$$-\frac{\left(3\,a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-3\,a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{6\,c^{4}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/6*(3*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^4 - 3*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^2 + a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*
sqrt(a)/(c^4*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^6)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(7/2), x)
```

# 3.366 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}} dx$

**Optimal**. Leaf size=88

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*cos(f*x
+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c - c*Sin[e + f*x])^(7
/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

**Mathematica** [A]
time = 1.38, size = 118, normalized size = 1.34

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(5-3\cos(2(e+fx))+4\sin(e+fx))}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5 -
3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.
time = 17.25, size = 197, normalized size = 2.24

| method | result |
|---|---|
| default | $-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(\cos^4(fx+e)-\left(\cos^3(fx+e)\right)\sin(fx+e)+4\left(\cos^3(fx+e)\right)+5\sin(fx+e)\left(\cos^2(fx+e)\right)-9\left(\cos^2(fx+e)\right)\right.}{6f(-c(\sin(fx+e)-1))^{\frac{9}{2}}\left(\cos^3(fx+e)-\sin(fx+e)(\cos^2(fx+e))-3(\cos^2(fx+e))-2\cos(fx+e)\sin(fx+e\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f
*x+e)+4*cos(f*x+e)^3+5*sin(f*x+e)*cos(f*x+e)^2-9*cos(f*x+e)^2+4*cos(f*x+e)*
sin(f*x+e)-10*cos(f*x+e)-14*sin(f*x+e)+14)/(-c*(sin(f*x+e)-1))^(9/2)/(cos(f
*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*co
s(f*x+e)+4*sin(f*x+e)+4)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [A]
time = 0.35, size = 143, normalized size = 1.62

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-4\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 4*a^2)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x +
e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e
))*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.49, size = 130, normalized size = 1.48

$$-\frac{\left(6\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-8\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{48\,c^{\frac{9}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] -1/48*(6*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^4 - 8*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^2 + 3*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(9/2)*f*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)
```

**Mupad** [B]
time = 11.57, size = 242, normalized size = 2.75

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{40\,a^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,a^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}-\frac{8\,a^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^5\,f}\right)}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)+16\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((40*a^2*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x)
)^(1/2))/(3*c^5*f) + (32*a^2*exp(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e +
 f*x))^(1/2))/(3*c^5*f) - (8*a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*ex
p(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) -
 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4
*f*x))
```

# 3.367 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

**Optimal**. Leaf size=133

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240*cos(f*x+e)*(a
+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^
(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(240*c^2*f*(c - c*Sin[e
+ f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(}{(}}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\cos}{2}$$

**Mathematica** [A]
time = 2.12, size = 118, normalized size = 0.89

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(9-5\cos(2(e+fx))+10\sin(e+fx))}{30c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -1/30*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(9 - 5*Cos[2*(e + f*x)] + 10*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 17.88, size = 225, normalized size = 1.69

| method | result |
|---|---|
| default | $\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(2(\cos^5(fx+e))+2\sin(fx+e)(\cos^4(fx+e))-12(\cos^4(fx+e))+10(\cos^3(fx+e))\sin(fx+e)-24(\cos^3(f\right.}{15f(-c(\sin(fx+e)-1))^{\frac{11}{2}}\left(\cos^3(fx+e)-\sin(fx+e)(\cos^2(fx+e))-3(\cos^2(fx+e\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(2*cos(f*x+e)^5+2*sin(f*x+e)*cos
(f*x+e)^4-12*cos(f*x+e)^4+10*cos(f*x+e)^3*sin(f*x+e)-24*cos(f*x+e)^3-34*sin
(f*x+e)*cos(f*x+e)^2+59*cos(f*x+e)^2-25*cos(f*x+e)*sin(f*x+e)+37*cos(f*x+e)
+62*sin(f*x+e)-62)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^3-sin(f*x+e)*cos(
f*x+e)^2-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4
)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

**Fricas** [A]
time = 0.35, size = 159, normalized size = 1.20

$$-\frac{\left(5\,a^{2}\cos\left(fx+e\right)^{2}-5\,a^{2}\sin\left(fx+e\right)-7\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(5*a^2*cos(f*x + e)^2 - 5*a^2*sin(f*x + e) - 7*a^2)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*
x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x
 + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.49, size = 139, normalized size = 1.05

$$-\frac{\left(10\,a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+6\,a^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{240\,c^{6}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] -1/240*(10*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 - 15*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 + 6*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)))*sqrt(a)/(c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^10)
```

**Mupad [B]**
time = 12.08, size = 273, normalized size = 2.05

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,96{}\mathrm{i}}{5\,c^6\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,64{}\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{3\,c^6\,f}\right)}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*96i)/(5*c^6*f) + (a^2*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e + f
*x))^(1/2)*64i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2)*32i)/(3*c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i
 - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e +
5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*
sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

# 3.368 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

**Optimal**. Leaf size=140

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a+a\sin(e+fx)}\,(c-c$$

```
[Out] 1/6*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60*a^3*
cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/2)-1/15*a^2*cos
(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(11/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2818, 2817}

$$\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(13/2
)) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*c*f*(c - c*Sin[e + f*x
])^(11/2)) + (a^3*Cos[e + f*x])/(60*c^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(9/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{3c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} +$$

**Mathematica** [A]
time = 3.00, size = 118, normalized size = 0.84

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(29-15\cos(2(e+fx))+36\sin(e+fx))}{120c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^6\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29 -
 15*Cos[2*(e + f*x)] + 36*Sin[e + f*x]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(122) = 244$.
time = 17.43, size = 251, normalized size = 1.79

| method | result |
|---|---|
| default | $\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(7(\cos^6(fx+e))-7(\cos^5(fx+e))\sin(fx+e)+42(\cos^5(fx+e))+49\sin(fx+e)(\cos^4(fx+e))-168(\cos^4(f\right.}{60f(-c(\sin(fx+e)-1))^{\frac{13}{2}}(\cos^3(fx+e)-\sin(fx+e}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x,method=_RETURNVERBOSE)
[Out] 1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(7*cos(f*x+e)^6-7*cos(f*x+e)^5*s
in(f*x+e)+42*cos(f*x+e)^5+49*sin(f*x+e)*cos(f*x+e)^4-168*cos(f*x+e)^4+119*c
os(f*x+e)^3*sin(f*x+e)-224*cos(f*x+e)^3-343*sin(f*x+e)*cos(f*x+e)^2+545*cos
(f*x+e)^2-202*cos(f*x+e)*sin(f*x+e)+242*cos(f*x+e)+444*sin(f*x+e)-444)/(-c*
(sin(f*x+e)-1))^(13/2)/(cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2
-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)
```

**Maple**

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

**Fricas** [A]
time = 0.35, size = 174, normalized size = 1.24

$$\frac{\left(15\,a^2\cos\left(fx+e\right)^2-18\,a^2\sin\left(fx+e\right)-22\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{60\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5-16\,c^7f\cos\left(fx+e\right)^3+16\,c^7f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*a^2*cos(f*x + e)^2 - 18*a^2*sin(f*x + e) - 22*a^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*
x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos
(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e)
)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.51, size = 130, normalized size = 0.93

$$-\frac{\left(15\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-24\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{960\,c^{\frac{13}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] -1/960*(15*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^4 - 24*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 + 10*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(13/2)
*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12)
```

**Mupad [B]**
time = 12.24, size = 287, normalized size = 2.05

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{464\,a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,c^7\,f}+\frac{192\,a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}-\frac{16\,a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^7\,f}\right)}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+858\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(7\,e+7\,f\,x\right)+1144\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)-416\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)+24\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((464*a^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(15*c^7*f) + (192*a^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin
(e + f*x))^(1/2))/(5*c^7*f) - (16*a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(
a + a*sin(e + f*x))^(1/2))/(c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*
x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e +
 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*s
in(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f
*x*7i)*sin(6*e + 6*f*x))
```

# 3.369 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$

**Optimal**. Leaf size=179

$$-\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a+a\sin(e+fx)}}-\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{14f}-\frac{3a^2\cos(e$$

```
[Out] -3/28*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/f-1/8*a*
cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/f-1/35*a^4*cos(f*x
+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/14*a^3*cos(f*x+e)*(c-
c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]

time = 0.25, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{14f}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{28f}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9/2}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] -1/35*(a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(f*Sqrt[a + a*Sin[e + f
*x]]) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/
2))/(14*f) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(9/2))/(28*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(9/2))/(8*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/}}{8f}$$
$$= -\frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx)}{28f}$$
$$= -\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/}}{14f}$$
$$= -\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{a^3\cos(e+fx)\sqrt{}}{}$$

**Mathematica [A]**
time = 3.34, size = 127, normalized size = 0.71

$$\frac{a^3c^4\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(1960\cos(2(e+fx))+980\cos(4(e+fx))+280\cos(6(e+fx))+35\cos(8(e+fx))+19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+fx))+80\sin(7(e+fx)))}{35840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e + f*x)] + 35*Co
s[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
+ f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

**Maple [A]**
time = 16.28, size = 143, normalized size = 0.80

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{9}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(35(\cos^8(fx+e))+5(\cos^6(fx+e))\sin(fx+e)+40(\cos^6(fx+e))+13\sin(fx+e)(\text{co}}{280f\cos(fx+e)^9}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/280/f*(-c*(sin(f*x+e)-1))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(35*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*x
+e)^4+48*cos(f*x+e)^4+29*sin(f*x+e)*cos(f*x+e)^2+64*cos(f*x+e)^2+93*sin(f*x
+e)+93)/cos(f*x+e)^9
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [**A**]
time = 0.36, size = 136, normalized size = 0.76

$$\frac{\left(35 a^{3} c^{4} \cos \left(f x+e\right)^{8}-35 a^{3} c^{4}+8 \left(5 a^{3} c^{4} \cos \left(f x+e\right)^{6}+6 a^{3} c^{4} \cos \left(f x+e\right)^{4}+8 a^{3} c^{4} \cos \left(f x+e\right)^{2}+16 a^{3} c^{4}\right) \sin \left(f x+e\right)\right) \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c}}{280 f \cos \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/280*(35*a^3*c^4*cos(f*x + e)^8 - 35*a^3*c^4 + 8*(5*a^3*c^4*cos(f*x + e)^6
 + 6*a^3*c^4*cos(f*x + e)^4 + 8*a^3*c^4*cos(f*x + e)^2 + 16*a^3*c^4)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.56, size = 267, normalized size = 1.49

$$-\frac{32 \left(35 a^{3} c^{4} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{16} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)-160 a^{3} c^{4} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{14} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)+280 a^{3} c^{4} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{12} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)-224 a^{3} c^{4} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{10} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)+70 a^{3} c^{4} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{8} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)\right) \sqrt{a} \sqrt{c}}{35 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] -32/35*(35*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 160*a^3*c^4*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 280*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 224*a^3*
c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 70*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2
```

```
*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 11.30, size = 376, normalized size = 2.10

$$\frac{\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{128\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(8\,e+8\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{512\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^3*c^4*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^3*c^4
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) +
(a^3*c^4*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) + (a^3*c^4*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^3*c^4*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
 a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^4*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

# 3.370 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

**Optimal**. Leaf size=179

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos}{}$$

```
[Out] -1/7*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*cos(f*x+
e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*cos(f*x+e)*(c-c
*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.26, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{7f}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(35*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2))/(7*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(7/2))/(7*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/}}{7f}$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))}{7f}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))}{35f}$$
$$= -\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos(e+fx}{}$$

**Mathematica [A]**
time = 0.61, size = 87, normalized size = 0.49

$$\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

**Maple [A]**
time = 17.50, size = 77, normalized size = 0.43

| method | result | size |
|---|---|---|
| default | $\frac{(5(\cos^6(fx+e))+6(\cos^4(fx+e))+8(\cos^2(fx+e))+16)(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}}{35f\cos(fx+e)^7}$ | 77 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(sin(f*x+e)-1)
)^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e)^7
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [A]**
time = 0.35, size = 108, normalized size = 0.60

$$\frac{\left(5\,a^3c^3\cos\left(fx+e\right)^6+6\,a^3c^3\cos\left(fx+e\right)^4+8\,a^3c^3\cos\left(fx+e\right)^2+16\,a^3c^3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/35*(5*a^3*c^3*cos(f*x + e)^6 + 6*a^3*c^3*cos(f*x + e)^4 + 8*a^3*c^3*cos(f
*x + e)^2 + 16*a^3*c^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.50, size = 216, normalized size = 1.21

$$\frac{32\left(20\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-70\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+84\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-35\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] 32/35*(20*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 70*a^3*c^3*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 84*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 35*a^3*c^3*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 10.65, size = 179, normalized size = 1.00

$$\frac{\frac{1225\,a^3\,c^3\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{245\,a^3\,c^3\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{49\,a^3\,c^3\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{5\,a^3\,c^3\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}}{70\,f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] ((1225*a^3*c^3*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2))/32 + (245*a^3*c^3*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*(c -
c*sin(e + f*x))^(1/2))/32 + (49*a^3*c^3*sin(5*e + 5*f*x)*(a + a*sin(e + f*x
))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32 + (5*a^3*c^3*sin(7*e + 7*f*x)*(a +
a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32)/(70*f*cos(e + f*x))
```

# 3.371 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=134

$$\frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c\cos(e+$$

```
[Out] 1/6*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/15*c^3*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/15*c^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]

time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*f*Sqrt[c - c*Sin[e + f*x]
]) + (2*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*f) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$$= \frac{2c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}$$

$$= \frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15f\sqrt{c-c\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(a}{}$$

**Mathematica [A]**
time = 0.77, size = 107, normalized size = 0.80

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-75\cos(2(e+fx))-30\cos(4(e+fx))-5\cos(6(e+fx))+600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 600*Sin[e
 + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f)
```

**Maple [A]**
time = 18.65, size = 117, normalized size = 0.87

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(5(\cos^6(fx+e))-\sin(fx+e)(\cos^4(fx+e))+6(\cos^4(fx+e))-3\sin(fx+e)(\cos^2(fx}{30f\cos(fx+e)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(5*cos
(f*x+e)^6-sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4-3*sin(f*x+e)*cos(f*x+e)^2+
8*cos(f*x+e)^2-11*sin(f*x+e)+11)/cos(f*x+e)^7
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [A]**
time = 0.35, size = 119, normalized size = 0.89

$$-\frac{\left(5\,a^3c^2\cos\left(fx+e\right)^6-5\,a^3c^2-2\left(3\,a^3c^2\cos\left(fx+e\right)^4+4\,a^3c^2\cos\left(fx+e\right)^2+8\,a^3c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/30*(5*a^3*c^2*cos(f*x + e)^6 - 5*a^3*c^2 - 2*(3*a^3*c^2*cos(f*x + e)^4 +
 4*a^3*c^2*cos(f*x + e)^2 + 8*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.51, size = 165, normalized size = 1.23

$$-\frac{16\left(10\,a^3c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-24\,a^3c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+15\,a^3c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -16/15*(10*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 24*a^3*c^2*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) + 15*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt
(c)/f
```

**Mupad [B]**
time = 10.29, size = 124, normalized size = 0.93

$$-\frac{a^3\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\,f\,x\right)+5\,\cos\left(7\,e+7\,f\,x\right)-700\,\sin\left(2\,e+2\,f\,x\right)-112\,\sin\left(4\,e+4\,f\,x\right)-12\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] -(a^3*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*co
s(e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x
) - 700*sin(2*e + 2*f*x) - 112*sin(4*e + 4*f*x) - 12*sin(6*e + 6*f*x)))/(96
0*f*(cos(2*e + 2*f*x) + 1))
```

# 3.372 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=89

$$\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

```
[Out] 1/10*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5*c*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2819, 2817}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt[c - c*Sin[e + f*x]
]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(
5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5f} +$$

$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f\sqrt{c-c\sin(e+fx)}} + \frac{c\cos(e+fx)(a+$$

**Mathematica [A]**
time = 0.61, size = 93, normalized size = 1.04

$$-\frac{a^3c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(20\cos(2(e+fx))+5\cos(4(e+fx))-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx)))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/80*(a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x
]]*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e
 + f*x)] + Sin[5*(e + f*x)]))/f
```

**Maple [A]**
time = 16.94, size = 107, normalized size = 1.20

| method | result |
|---|---|
| default | $\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(2(\cos^6(fx+e))-\sin(fx+e)(\cos^4(fx+e))+2(\cos^4(fx+e))-3\sin(fx+e)(\cos^2(fx}{10f\cos(fx+e)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(2*cos
(f*x+e)^6-sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4-3*sin(f*x+e)*cos(f*x+e)^2-
6*sin(f*x+e)+6)/cos(f*x+e)^7
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [A]**
time = 0.34, size = 108, normalized size = 1.21

$$-\frac{\left(5\,a^{3}c\cos\left(fx+e\right)^{4}-5\,a^{3}c+2\,\left(a^{3}c\cos\left(fx+e\right)^{4}-2\,a^{3}c\cos\left(fx+e\right)^{2}-4\,a^{3}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/10*(5*a^3*c*cos(f*x + e)^4 - 5*a^3*c + 2*(a^3*c*cos(f*x + e)^4 - 2*a^3*c
*cos(f*x + e)^2 - 4*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.50, size = 110, normalized size = 1.24

$$\frac{8\,\left(4\,a^{3}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-5\,a^{3}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 8/5*(4*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*a^3*c*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)))*sqrt(a)*sqrt(c)/f
```

**Mupad [B]**
time = 9.19, size = 109, normalized size = 1.22

$$-\frac{a^{3}\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)-75\,\sin\left(2\,e+2\,f\,x\right)-4\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(6\,e+6\,f\,x\right)\right)}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(a^3*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(
e + f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) - 75*sin(2*e + 2*f*x) -
 4*sin(4*e + 4*f*x) + sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

# 3.373 $\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=43

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.23, size = 82, normalized size = 1.91

$$\frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(-28\cos(2(e+fx))+\cos(4(e+fx))+56\sin(e+fx)-8\sin(3(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-28*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)])
)/(32*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(37) = 74$.
time = 18.07, size = 105, normalized size = 2.44

| method | result |
|---|---|
| default | $\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(\cos^{6}(fx+e)-\sin(fx+e)\left(\cos^{4}(fx+e)\right)-\sin(fx+e)\left(\cos^{2}(fx+e)\right)-\right.}{4f\cos(fx+e)^{7}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(cos(f*
x+e)^6-sin(f*x+e)*cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^2-cos(f*x+e)^2-4*sin(f
*x+e)+4)/cos(f*x+e)^7
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(40) = 80$.
time = 0.34, size = 102, normalized size = 2.37

$$\frac{\left(a^{3}\cos\left(fx+e\right)^{4}-8\,a^{3}\cos\left(fx+e\right)^{2}+7\,a^{3}-4\,\left(a^{3}\cos\left(fx+e\right)^{2}-2\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{4\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(a^3*cos(f*x + e)^4 - 8*a^3*cos(f*x + e)^2 + 7*a^3 - 4*(a^3*cos(f*x + e
)^2 - 2*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.49, size = 54, normalized size = 1.26

$$-\frac{4\,a^{\frac{7}{2}}\sqrt{c}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -4*a^(7/2)*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [B]**
time = 8.29, size = 99, normalized size = 2.30

$$-\frac{a^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)-48\,\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(4\,e+4\,f\,x\right)\right)}{32\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e
+ f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) - 48*sin(2*e + 2*f*x) + 8*s
in(4*e + 4*f*x)))/(32*f*(cos(2*e + 2*f*x) + 1))
```

# 3.374 $\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=184

$$-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a+a\,\mathrm{s}}{f\sqrt{c-c\sin(e}$$

```
[Out] -a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*a*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*cos(f*x+e)*ln(1
-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^3*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2819, 2816, 2746, 31}

$$-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

**Mathematica [A]**
time = 0.65, size = 150, normalized size = 0.82

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\left(-12\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+87\sin(e+fx)-\sin(3(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/12*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[
a*(1 + Sin[e + f*x])]*(-12*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Si
```

```
n[(e + f*x)/2]] + 87*Sin[e + f*x] - Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(166) = 332$.
time = 16.55, size = 366, normalized size = 1.99

| method | result |
|---|---|
| default | $-\frac{\left(\cos^4(fx+e)-(\cos^3(fx+e))\sin(fx+e)-6(\cos^3(fx+e))-5\sin(fx+e)(\cos^2(fx+e))+48\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\cos(fx+}{3f(\cos^4(fx+e)+(\cos^3(fx}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*x+e)-6*cos(f*x+e)^3-5*sin(f*x+e)*co
s(f*x+e)^2+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)+48*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-17*cos(f*x+e)^2+22*cos(f*
x+e)*sin(f*x+e)-24*ln(2/(cos(f*x+e)+1))*cos(f*x+e)-24*ln(2/(cos(f*x+e)+1))*
sin(f*x+e)-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)-16*si
n(f*x+e)+24*ln(2/(cos(f*x+e)+1))+16)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4
+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x
+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-
1))^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e)
 - c), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.54, size = 148, normalized size = 0.80

$$\frac{4\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+3\,c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+6\,c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 4/3*a^(7/2)*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6 +
3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 6*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.375 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=192

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}}+\frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a^3\cos(e+fx)\sqrt{a+a}}{cf\sqrt{c-c\sin(e+}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+3/2*a^2*cos(f*
x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+12*a^4*cos(f*x+e)*ln
(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6*a^3*cos(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$\frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (6*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(c*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{12a^4\cos(e+fx)\log(1-\sin(e+fx}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}$$

**Mathematica [A]**
time = 1.02, size = 179, normalized size = 0.93

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(44+18\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(39-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{8cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/8*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(44 + 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] +
(39 - 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e
 + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*S
qrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(174) = 348$.
time = 17.42, size = 490, normalized size = 2.55

| method | result |
|---|---|
| default | $-\frac{\left(\cos^4(fx+e)-(\cos^3(fx+e))\sin(fx+e)+24\ln\left(\frac{2}{\cos(fx+e)+1}\right)(\cos^2(fx+e))-24\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)\cos(fx+e)-48\ln\left(-\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*x+e)+24*ln(2/(cos(f*x+e)+1))*cos(f*
x+e)^2-24*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))*sin(f*x+e)*cos(f*x+e)-9*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)^2+24*l
n(2/(cos(f*x+e)+1))*cos(f*x+e)+48*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-48*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)-96*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))*sin(f*x+e)+33*cos(f*x+e)^2-25*cos(f*x+e)*sin(f*x+e)-48*l
n(2/(cos(f*x+e)+1))+96*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9*cos(f*x
+e)+34*sin(f*x+e)-34)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*s
in(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x
+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(3/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x +
e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.52, size = 179, normalized size = 0.93

$$-\frac{2\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+4\,c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{4}}-\frac{2}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -2*a^(7/2)*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))) + (c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^4 - 2/((cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.376 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=195

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}-\frac{6a^4\cos(e+fx)\log(1-\sin}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

```
[Out] 1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-3/2*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-6*a^4*cos(f*x+e)
*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-3*a^3
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$-\frac{6a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int\frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$

**Mathematica [A]**
time = 1.21, size = 207, normalized size = 1.06

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-28-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\cos(2(e+fx))\left(-1+6\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\left(41+96\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{4c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-28
- 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(-1 + 6*
Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^2*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(175) = 350$.
time = 18.15, size = 618, normalized size = 3.17

| method | result |
|---|---|
| default | $-\frac{\left(\cos^4(fx+e)-\left(\cos^3(fx+e)\right)\sin(fx+e)+6\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-12\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+6(c\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*x+e)+6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+
1))-12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)
^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-12*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+10*cos(f*x+e)^3+11*sin(f*x+e)*cos(f*x+e)^2-18
*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2+36*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))*cos(f*x+e)^2+12*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-24*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-17*cos(f*x+e)^2+6
*cos(f*x+e)*sin(f*x+e)-12*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+24*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)-24*ln(2/(cos(f*x+e)+1))*sin(f*x+e)+
48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-10*cos(f*x+e)-16*s
in(f*x+e)+24*ln(2/(cos(f*x+e)+1))-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+16)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*c
os(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e
)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x
+ e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.51, size = 161, normalized size = 0.83

$$\frac{a^{\frac{7}{2}}\sqrt{c}\left(\frac{2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{6\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] a^(7/2)*sqrt(c)*(2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^3*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) + 6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (6*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 5)
/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.377 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=193

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+fx}}{c^2f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^3*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^4*cos(f*x+e)*ln(1-sin(
f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2818, 2816, 2746, 31}

$$\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin[e + f*x])^(7/2)
) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

**Mathematica** [A]
time = 1.47, size = 232, normalized size = 1.20

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-34-30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+18\cos(2(e+fx))\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+9\left(4+5\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-34
- 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 18*Cos[2*(e + f*x)]*(1 + Lo
```

```
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + 9*(4 + 5*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]])*Sin[e + f*x] - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*Sin[3*(e + f*x)]))/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(173) = 346$.
time = 18.72, size = 748, normalized size = 3.88

| method | result |
|---|---|
| default | $-\frac{\left(-20+3\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)(\cos^3(fx+e))\sin(fx+e)+20\sin(fx+e)+6\cos(fx+e)+28(\cos^2(fx+e))-6\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(-20-6*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3*sin(f
*x+e)-14*sin(f*x+e)*cos(f*x+e)^2+20*sin(f*x+e)+6*cos(f*x+e)+28*cos(f*x+e)^2
+8*cos(f*x+e)^3*sin(f*x+e)-8*cos(f*x+e)^4+6*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))*cos(f*x+e)^4-3*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^4-12*ln(2/(cos(f
*x+e)+1))*sin(f*x+e)*cos(f*x+e)+3*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3*sin(f*x
+e)-6*cos(f*x+e)^3-14*cos(f*x+e)*sin(f*x+e)-24*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))*cos(f*x+e)+12*ln(2/(cos(f*x+e)+1))*cos(f*x+e)-48*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+24*ln(2/(cos(f*x+e)+1))*sin(f*x+
e)+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*ln(2/(cos(f*x+e)+1))+24
*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2+24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))*sin(f*x+e)*cos(f*x+e)-9*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+18*cos(f*x+
e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*cos(f*x+e)^2*sin(f*x+e)*
ln(2/(cos(f*x+e)+1))+24*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2)
*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e
)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(
f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)
```

**Maxima** [A]
time = 0.52, size = 362, normalized size = 1.88

$$-\frac{\frac{6\,a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{7}{2}}}-\frac{3\,a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{7}{2}}}+\frac{4\left(\frac{3\,a^{\frac{7}{2}}\sqrt{c}\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{6\,a^{\frac{7}{2}}\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{22\,a^{\frac{7}{2}}\sqrt{c}\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6\,a^{\frac{7}{2}}\sqrt{c}\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{3\,a^{\frac{7}{2}}\sqrt{c}\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{c^4-\frac{6\,c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{15\,c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{20\,c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\,c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{6\,c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(6*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(
c)*sin(f*x + e)/(cos(f*x + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 22*a^(7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
6*a^(7/2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e)
 + 1) + 15*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6))/f
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x +
 e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f
*x + e)), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.48, size = 152, normalized size = 0.79

$$-\frac{\sqrt{2}\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{6\,\sqrt{2}\,\log\left(-2\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\right)}{c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\sqrt{2}\,\left(18\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-27\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+11\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/12*sqrt(2)*a^(7/2)*sqrt(c)*(6*sqrt(2)*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^2 + 2)/(c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(18*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^4 - 27*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 11)/((cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(7/2), x)
```

# 3.378 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}} \, dx$

**Optimal**. Leaf size=42

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}} \, dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(42) = 84.
time = 2.74, size = 115, normalized size = 2.74

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(-7\sin(e+fx)+\sin(3(e+fx)))}{4c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] -1/4*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(-7*Sin[e + f*x] + Sin[3*(e + f*x)]))/(c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(36) = 72$.
time = 18.54, size = 154, normalized size = 3.67

| method | result |
|---|---|
| default | $-\frac{(-1+\cos(fx+e)+\sin(fx+e))\left(\cos^2(fx+e)-2\right)\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}}{f(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4\sin(fx+e)(\cos^2(fx+e))-8(\cos^2(fx+e))-4\cos(fx+e)\sin(fx+e)-4\cos}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*(cos(f*x+e)^2-2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)
*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x
+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(39) = 78$.
time = 0.34, size = 137, normalized size = 3.26

$$-\frac{\left(a^3\cos\left(fx+e\right)^2-2\,a^3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -(a^3*cos(f*x + e)^2 - 2*a^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f
```

```
*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e
))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(39) = 78.
time = 0.53, size = 164, normalized size = 3.90

$$\frac{\left(4\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-6\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+4\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{8\,c^{\frac{9}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] 1/8*(4*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^6 - 6*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^4 + 4*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^2 - a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(9/2)*f*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(9/2), x)
```

# 3.379 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

**Optimal**. Leaf size=88

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^
(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

**Mathematica [A]**
time = 4.26, size = 128, normalized size = 1.45

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(-14+10\cos(2(e+fx))-35\sin(e+fx)+5\sin(3(e+fx)))}{40c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-14
+ 10*Cos[2*(e + f*x)] - 35*Sin[e + f*x] + 5*Sin[3*(e + f*x)]))/(40*c^5*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e
+ f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(76) = 152$.
time = 17.35, size = 247, normalized size = 2.81

| method | result |
|---|---|
| default | $\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(\cos^5(fx+e)+\sin(fx+e)(\cos^4(fx+e))-6(\cos^4(fx+e))+5(\cos^3(fx+e))\sin(fx+e)-17(\cos^3(fx+e))}{10f(-c(\sin(fx+e)-1))^{\frac{11}{2}}(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4\sin(fx+e)(\cos^2(f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(cos(f*x+e)^5+sin(f*x+e)*cos(f*x
+e)^4-6*cos(f*x+e)^4+5*cos(f*x+e)^3*sin(f*x+e)-17*cos(f*x+e)^3-22*sin(f*x+e
)*cos(f*x+e)^2+32*cos(f*x+e)^2-10*cos(f*x+e)*sin(f*x+e)+26*cos(f*x+e)+36*si
n(f*x+e)-36)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+
e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin
(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(82) = 164$.
time = 0.36, size = 175, normalized size = 1.99

$$-\frac{\left(5\,a^{3}\cos\left(fx+e\right)^{2}-6\,a^{3}+5\,\left(a^{3}\cos\left(fx+e\right)^{2}-2\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/10*(5*a^3*cos(f*x + e)^2 - 6*a^3 + 5*(a^3*cos(f*x + e)^2 - 2*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x
 + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x
+ e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.48, size = 176, normalized size = 2.00

$$\frac{\left(10\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-20\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-4\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{80\,c^{6}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] 1/80*(10*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^6 - 20*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^4 + 15*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 4*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
```

```
f*x + 1/2*e)))*sqrt(a)/(c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^10)
```

**Mupad [B]**
time = 12.45, size = 317, normalized size = 3.60

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,112{}\mathrm{i}}{5\,c^6\,f}+\frac{a^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,56{}\mathrm{i}}{c^6\,f}-\frac{a^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16{}\mathrm{i}}{c^6\,f}-\frac{a^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{c^6\,f}\right)}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^3*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*112i)/(5*c^6*f) + (a^3*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e +
f*x))^(1/2)*56i)/(c^6*f) - (a^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a*
sin(e + f*x))^(1/2)*16i)/(c^6*f) - (a^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(a + a*sin(e + f*x))^(1/2)*8i)/(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*
264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*
e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*
6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

# 3.380 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

**Optimal**. Leaf size=133

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/12*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/480*cos(f*x+e)*(
a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

**Rubi** [A]

time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2822, 2821\}$

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c - c*Sin[e + f*x])^
(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(480*c^2*f*(c - c*Sin[e
 + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{6c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \int$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \mathrm{co}$$

**Mathematica** [**A**]
time = 6.18, size = 128, normalized size = 0.96

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(21-15\cos(2(e+fx))+39\sin(e+fx)-5\sin(3(e+fx)))}{60c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^6\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(21 -
 15*Cos[2*(e + f*x)] + 39*Sin[e + f*x] - 5*Sin[3*(e + f*x)]))/(60*c^6*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e +
 f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(115) = 230$.
time = 17.08, size = 276, normalized size = 2.08

| method | result |
|---|---|
| default | $\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(3(\cos^6(fx+e))-3(\cos^5(fx+e))\sin(fx+e)+18(\cos^5(fx+e))+21\sin(fx+e)(\cos^4(fx+e))-72(\cos^4(\right.}{30f(-c(\sin(fx+e)-1))^{\frac{13}{2}}(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3*cos(f*x+e)^6-3*cos(f*x+e)^5*s
in(f*x+e)+18*cos(f*x+e)^5+21*sin(f*x+e)*cos(f*x+e)^4-72*cos(f*x+e)^4+51*cos
(f*x+e)^3*sin(f*x+e)-106*cos(f*x+e)^3-157*sin(f*x+e)*cos(f*x+e)^2+235*cos(f
*x+e)^2-78*cos(f*x+e)*sin(f*x+e)+118*cos(f*x+e)+196*sin(f*x+e)-196)/(-c*(si
n(f*x+e)-1))^(13/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*
sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)
+8*sin(f*x+e)+8)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

**Fricas [A]**
time = 0.35, size = 191, normalized size = 1.44

$$\frac{\left(15\,a^{3}\cos\left(fx+e\right)^{2}-18\,a^{3}+2\,\left(5\,a^{3}\cos\left(fx+e\right)^{2}-11\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,\left(c^{7}f\cos\left(fx+e\right)^{7}-18\,c^{7}f\cos\left(fx+e\right)^{5}+48\,c^{7}f\cos\left(fx+e\right)^{3}-32\,c^{7}f\cos\left(fx+e\right)+2\,\left(3\,c^{7}f\cos\left(fx+e\right)^{5}-16\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*a^3*cos(f*x + e)^2 - 18*a^3 + 2*(5*a^3*cos(f*x + e)^2 - 11*a^3)*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f
*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*co
s(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f
*cos(f*x + e))*sin(f*x + e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.52, size = 164, normalized size = 1.23

$$\frac{\left(20\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-45\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+36\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-10\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{480\,c^{\frac{13}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] 1/480*(20*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^6 - 45*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^4 + 36*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^2 - 10*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(13/
2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12)
```

**Mupad [B]**

time = 12.06, size = 330, normalized size = 2.48

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{224\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}+\frac{416\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}-\frac{32\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^7\,f}-\frac{32\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^7\,f}\right)}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+858\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(7\,e+7\,f\,x\right)+1144\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)-416\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)+24\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((224*a^3*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(5*c^7*f) + (416*a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2))/(5*c^7*f) - (32*a^3*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(a
 + a*sin(e + f*x))^(1/2))/(c^7*f) - (32*a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*
f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*
e + 3*f*x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*c
os(5*e + 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f
*x*7i)*sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(
e*7i + f*x*7i)*sin(6*e + 6*f*x))
```

# 3.381 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

**Optimal**. Leaf size=178

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{c}{-}$$

```
[Out] 1/14*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/56*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+1/280*cos(f*x+e)*(
a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(11/2)+1/2240*cos(f*x+e)*(a+a*
sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(9/2)
```

**Rubi** [A]

time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2240c^3f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x])^(15/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c - c*Sin[e + f*x])^
(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(280*c^2*f*(c - c*Sin[e
 + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2240*c^3*f*(c
 - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{3\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{14c}\\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\int
\\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\text{co}
\\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\text{co}
\end{aligned}
$$

**Mathematica** [A]
time = 6.43, size = 333, normalized size = 1.87

$$
\frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}+\frac{6\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (2*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (6*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - ((Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

**Maple** [A]
time = 18.59, size = 302, normalized size = 1.70

| method | result |
|---|---|
| default | $-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(13(\cos^7(fx+e))+13(\cos^6(fx+e))\sin(fx+e)-104(\cos^6(fx+e))+91(\cos^5(fx+e))\sin(fx+e)-31\right.}{140f(-c(\sin(fx+e)-1))^{\frac{15}{2}}(\cos^4(fx+}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/140/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(13*cos(f*x+e)^7+13*cos(f*x+e)
^6*sin(f*x+e)-104*cos(f*x+e)^6+91*cos(f*x+e)^5*sin(f*x+e)-312*cos(f*x+e)^5-
```

```
403*sin(f*x+e)*cos(f*x+e)^4+1040*cos(f*x+e)^4-637*cos(f*x+e)^3*sin(f*x+e)+1
075*cos(f*x+e)^3+1712*sin(f*x+e)*cos(f*x+e)^2-2468*cos(f*x+e)^2+756*cos(f*x
+e)*sin(f*x+e)-916*cos(f*x+e)-1672*sin(f*x+e)+1672)/(-c*(sin(f*x+e)-1))^(15
/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f
*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(15/2), x)
```

**Fricas** [**A**]
time = 0.44, size = 206, normalized size = 1.16

$$\frac{\left(63\,a^{3}\cos\left(fx+e\right)^{2}-76\,a^{3}+7\,\left(5\,a^{3}\cos\left(fx+e\right)^{2}-12\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{140\,\left(7\,c^{8}f\cos\left(fx+e\right)^{7}-56\,c^{8}f\cos\left(fx+e\right)^{5}+112\,c^{8}f\cos\left(fx+e\right)^{3}-64\,c^{8}f\cos\left(fx+e\right)-\left(c^{8}f\cos\left(fx+e\right)^{7}-24\,c^{8}f\cos\left(fx+e\right)^{5}+80\,c^{8}f\cos\left(fx+e\right)^{3}-64\,c^{8}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="fric
as")
```

```
[Out] 1/140*(63*a^3*cos(f*x + e)^2 - 76*a^3 + 7*(5*a^3*cos(f*x + e)^2 - 12*a^3)*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*co
s(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^8*
f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8*f
*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.49, size = 176, normalized size = 0.99

$$\frac{\left(35\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-84\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+70\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-20\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{2240\,c^{8}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac
")
```

```
[Out] 1/2240*(35*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^6 - 84*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^4 + 70*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 20*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sqrt(a)/(c^8*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^14)
```

**Mupad [B]**
time = 13.32, size = 647, normalized size = 3.63

$$\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(-\frac{8\,a^3\,e^{e\,5i+f\,x\,5i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{c^8\,f}+\frac{a^3\,e^{e\,6i+f\,x\,6i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,144i}{5\,c^8\,f}+\frac{344\,a^3\,e^{e\,7i+f\,x\,7i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5\,c^8\,f}-\frac{a^3\,e^{e\,8i+f\,x\,8i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,2848i}{35\,c^8\,f}-\frac{344\,a^3\,e^{e\,9i+f\,x\,9i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5\,c^8\,f}+\frac{a^3\,e^{e\,10i+f\,x\,10i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,144i}{5\,c^8\,f}+\frac{8\,a^3\,e^{e\,11i+f\,x\,11i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{c^8\,f}\right)}{1+910\,e^{e\,4i+f\,x\,4i}-2002\,e^{e\,6i+f\,x\,6i}+2002\,e^{e\,10i+f\,x\,10i}-910\,e^{e\,12i+f\,x\,12i}+90\,e^{e\,14i+f\,x\,14i}-e^{e\,16i+f\,x\,16i}-90\,e^{e\,2i+f\,x\,2i}+e^{e\,1i+f\,x\,1i}\,14i-e^{e\,3i+f\,x\,3i}\,350i+e^{e\,5i+f\,x\,5i}\,1638i-e^{e\,7i+f\,x\,7i}\,1430i-e^{e\,9i+f\,x\,9i}\,1430i+e^{e\,11i+f\,x\,11i}\,1638i-e^{e\,13i+f\,x\,13i}\,350i+e^{e\,15i+f\,x\,15i}\,14i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(15/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*144i)/(5*c^8*f) - (8*a^3*exp(e*5i + f*x*5i)*(a + a*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^8*f) + (34
4*a^3*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2))/(5*c^8*f) - (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e
*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*2848i)/(35*c^8*f) -
 (344*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2))/(5*c^8*f) + (a^3*exp(e*10i + f*x*10i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*144i)/(5*c^8*
f) + (8*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp
(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^8*f)))/(exp(e*1i + f*x*1i)*14i - 90*exp(e
*2i + f*x*2i) - exp(e*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*4i) + exp(e*5i
 + f*x*5i)*1638i - 2002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*1430i - exp
(e*9i + f*x*9i)*1430i + 2002*exp(e*10i + f*x*10i) + exp(e*11i + f*x*11i)*16
38i - 910*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*350i + 90*exp(e*14i +
 f*x*14i) + exp(e*15i + f*x*15i)*14i - exp(e*16i + f*x*16i) + 1)
```

# 3.382 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

**Optimal**. Leaf size=188

$$\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}}-\frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+f}}{56c^2f(c-c\sin(e+fx))^{13/2}}$$

```
[Out] 1/8*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(17/2)-3/56*a^2*
cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(15/2)-1/280*a^4*cos
(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(11/2)/(a+a*sin(f*x+e))^(1/2)+1/56*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2818, 2817}

$$-\frac{a^4\cos(e+fx)}{280c^3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{56c^2f(c-c\sin(e+fx))^{13/2}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(17/2
)) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(56*c*f*(c - c*Sin[e +
 f*x])^(15/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(56*c^2*f*(c -
 c*Sin[e + f*x])^(13/2)) - (a^4*Cos[e + f*x])/(280*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(11/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{15/2}}\,dx}{8c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}}$$

**Mathematica** [A]
time = 6.46, size = 329, normalized size = 1.75

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} - \frac{12\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} + \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} - \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) - (12*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) - ((Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
```

**Maple** [A]
time = 19.44, size = 328, normalized size = 1.74

| method | result |
|---|---|
| default | $-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(3(\cos^8(fx+e))-3(\cos^7(fx+e))\sin(fx+e)+24(\cos^7(fx+e))+27(\cos^6(fx+e))\sin(fx+e)-120(\cos}{35f(-c(\sin(fx+e)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3*cos(f*x+e)^8-3*cos(f*x+e)^7*
sin(f*x+e)+24*cos(f*x+e)^7+27*cos(f*x+e)^6*sin(f*x+e)-120*cos(f*x+e)^6+93*c
```

```
os(f*x+e)^5*sin(f*x+e)-240*cos(f*x+e)^5-333*sin(f*x+e)*cos(f*x+e)^4+720*cos
(f*x+e)^4-387*cos(f*x+e)^3*sin(f*x+e)+583*cos(f*x+e)^3+970*sin(f*x+e)*cos(f
*x+e)^2-1337*cos(f*x+e)^2+367*cos(f*x+e)*sin(f*x+e)-402*cos(f*x+e)-769*sin(
f*x+e)+769)/(-c*(sin(f*x+e)-1))^(17/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e
)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(
f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(17/2), x)
```

**Fricas [A]**
time = 0.38, size = 219, normalized size = 1.16

$$-\frac{\left(14\,a^3\cos\left(fx+e\right)^2-17\,a^3+\left(7\,a^3\cos\left(fx+e\right)^2-18\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{35\,\left(c^9f\cos\left(fx+e\right)^9-32\,c^9f\cos\left(fx+e\right)^7+160\,c^9f\cos\left(fx+e\right)^5-256\,c^9f\cos\left(fx+e\right)^3+128\,c^9f\cos\left(fx+e\right)+8\,\left(c^9f\cos\left(fx+e\right)^7-10\,c^9f\cos\left(fx+e\right)^5+24\,c^9f\cos\left(fx+e\right)^3-16\,c^9f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="fric
as")
```

```
[Out] -1/35*(14*a^3*cos(f*x + e)^2 - 17*a^3 + (7*a^3*cos(f*x + e)^2 - 18*a^3)*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*
x + e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*c
os(f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f
*cos(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x
+ e))
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.55, size = 164, normalized size = 0.87

$$\frac{\left(56\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-140\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+120\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-35\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{8960\,c^{\frac{17}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac
")
```

```
[Out] 1/8960*(56*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^6 - 140*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^4 + 120*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 35*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(
17/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
16)
```

**Mupad [B]**
time = 13.98, size = 673, normalized size = 3.58

$$-\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{a^3\,e^{e\,6i+f\,x\,6i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,64i}{5\,c^9\,f}+\frac{256\,a^3\,e^{e\,7i+f\,x\,7i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5\,c^9\,f}-\frac{a^3\,e^{e\,8i+f\,x\,8i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,832i}{7\,c^9\,f}-\frac{1024\,a^3\,e^{e\,9i+f\,x\,9i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{7\,c^9\,f}+\frac{a^3\,e^{e\,10i+f\,x\,10i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,832i}{7\,c^9\,f}+\frac{256\,a^3\,e^{e\,11i+f\,x\,11i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{5\,c^9\,f}-\frac{a^3\,e^{e\,12i+f\,x\,12i}\,\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,64i}{5\,c^9\,f}\right)}{1+1700\,e^{e\,4i+f\,x\,4i}-6188\,e^{e\,6i+f\,x\,6i}+4862\,e^{e\,8i+f\,x\,8i}+4862\,e^{e\,10i+f\,x\,10i}-6188\,e^{e\,12i+f\,x\,12i}+1700\,e^{e\,14i+f\,x\,14i}-119\,e^{e\,16i+f\,x\,16i}+e^{e\,18i+f\,x\,18i}-119\,e^{e\,2i+f\,x\,2i}+e^{e\,1i+f\,x\,1i}\,16i-e^{e\,3i+f\,x\,3i}\,544i+e^{e\,5i+f\,x\,5i}\,3808i-e^{e\,7i+f\,x\,7i}\,7072i+e^{e\,11i+f\,x\,11i}\,7072i-e^{e\,13i+f\,x\,13i}\,3808i+e^{e\,15i+f\,x\,15i}\,544i-e^{e\,17i+f\,x\,17i}\,16i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(17/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*64i)/(5*c^9*f) + (256*a^3*exp(e*7i + f*x*7i)*(a + a*((
exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c^9*f) -
 (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*832i)/(7*c^9*f) - (1024*a^3*exp(e*9i + f*x*9i)*(a + a
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(7*c^9*f
) + (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*832i)/(7*c^9*f) + (256*a^3*exp(e*11i + f*x*11i)*
(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5
*c^9*f) - (a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*64i)/(5*c^9*f)))/(exp(e*1i + f*x*1i)*16i -
 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e*4i + f*x*4i)
 + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*
7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) + exp(e*11i + f
*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*3808i + 17
00*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(e*16i + f*x*1
6i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)
```

# 3.383 $\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=139

$$\frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/2*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+4*c^3*cos(
f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*c
^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2819, 2816, 2746, 31}

$$\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+(2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+ \\
&= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+ \\
&= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+ \\
&= \frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+}}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

**Mathematica [A]**
time = 0.34, size = 136, normalized size = 0.98

$$
-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\left(\cos(2(e+fx))-32\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+12\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{a(1+\sin(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[
2*(e + f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqr
t[a*(1 + Sin[e + f*x])])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(125) = 250.
time = 18.58, size = 319, normalized size = 2.29

| method | result |
|---|---|
| default | $\frac{\left(-\left(\cos^3(fx+e)\right)+\sin(fx+e)\left(\cos^2(fx+e)\right)+16\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-16\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+}{2f(\cos^3(fx+e)+\sin(fx+e)(\cos}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^2+16*cos(f*x+e)*ln(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))-16*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*si
n(f*x+e)-5*cos(f*x+e)^2-6*cos(f*x+e)*sin(f*x+e)-8*ln(2/(cos(f*x+e)+1))*cos(
f*x+e)+8*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-16*ln(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))+cos(f*x+e)+5*sin(f*x+e)+8*ln(2/(cos(f*x+e)+1))+5)*(-c*(sin(f*x+
e)-1))^(5/2)/(cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2+2*cos(f*x
+e)*sin(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(-c*sin(f*x
 + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac [A]**
time = 0.53, size = 135, normalized size = 0.97

$$-\frac{2\,\sqrt{a}\,c^{\frac{5}{2}}\left(\frac{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+2\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{2}}+\frac{2\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*sqrt(a)*c^(5/2)*((a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 + 2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^2)/a^2 + 2*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.384 $\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=93

$$\frac{2c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2819, 2816, 2746, 31}

$$\frac{2c^2\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + (2c)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(2ac^2\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(2c^2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{2c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [**A**]
time = 0.18, size = 119, normalized size = 1.28

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\left(-4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*(-4*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(85) = 170$.
time = 19.59, size = 259, normalized size = 2.78

| method | result |
|---|---|

| default | $-\frac{\left(4\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-4\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-(\cos^2(fx+e))-\cos(fx+e)\sin(fx+e)-}{f(\cos^2(fx+e)-\cos(fx+e)\sin(fx+e)}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*ln(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-cos(f*x+e)^2-cos(f*x+e)*sin(f*x+
e)-2*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-4*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+sin(f*x+e)+2*ln(2/(cos(f*x+e)+1))+
1)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)
+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c(\sin(e+fx)-1)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac [A]**
time = 0.51, size = 101, normalized size = 1.09

$$-\frac{2\sqrt{a}\,c^{\frac{3}{2}}\left(\frac{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*sqrt(a)*c^(3/2)*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2/(a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))) + log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-c\,\sin\left(e+f\,x\right))^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.385 $\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=49

$$\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2816, 2746, 31}

$$\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{(c\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.67, size = 118, normalized size = 2.41

$$-\frac{\sqrt{2}\left(i+e^{i(e+fx)}\right)\left(fx+2i\log\left(i+e^{i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{\left(-i+e^{i(e+fx)}\right)\sqrt{-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
[Out] -((Sqrt[2]*(I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I + E^(I*(e + f*x))])*Sqr
t[c - c*Sin[e + f*x]])/((-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x))]*f))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(45) = 90.
time = 19.70, size = 105, normalized size = 2.14

| method | result | size |
|---|---|---|
| default | $\frac{(-1+\cos(fx+e)-\sin(fx+e))\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(-1+\cos(fx+e)+\sin(fx+e))}$ | 105 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1+cos(f*x+e)-sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(ln(2/(cos(f*x+e)+
1))-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))/(a*(1+sin(f*x+e)))^(1/2)/
(-1+cos(f*x+e)+sin(f*x+e))
```

**Maxima** [A]
time = 0.52, size = 68, normalized size = 1.39

$$-\frac{\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a))/f
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(49) = 98.
time = 0.58, size = 231, normalized size = 4.71

$$\frac{\sqrt{2}\,\sqrt{c}\left(\frac{\sqrt{2}\,\log\left(\left|-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+2\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\sqrt{2}\,\log\left(\left|-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}-\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-2\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/2*sqrt(2)*sqrt(c)*(sqrt(2)*log(abs(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1) - (cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 2))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*log(abs(-(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1) - (cos
```

```
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 2))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c - c \sin(e + f x)}}{\sqrt{a + a \sin(e + f x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2),x)
[Out] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.386 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=46

$$\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.14, size = 89, normalized size = 1.93

$$-\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
+ f*x]]))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.
time = 10.66, size = 92, normalized size = 2.00

| method | result | size |
|---|---|---|
| default | $-\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$ | 92 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(
1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas [A]**
time = 0.39, size = 170, normalized size = 3.70

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)}{2\,acf}, -\frac{\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)}{acf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x +
e)^3)/(a*c*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a*c*f)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.387 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=95

$$\frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(
sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{a+}}}{}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{co}}{2c\sqrt{a+}}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{tan}}{2cf\sqrt{a+}}$$

**Mathematica** [A]
time = 0.25, size = 161, normalized size = 1.69

$$-\frac{\cos(e+fx)\left(1-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2cf(-1+\sin(e+fx))\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*(Cos[e + f*x]*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(-1 + Sin
[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 18.80, size = 165, normalized size = 1.74

| method | result |
|---|---|
| default | $\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)+\sin(fx+e)-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+}{2f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+sin(f*x+e)-ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(
a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [A]**
time = 0.39, size = 337, normalized size = 3.55

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)-2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{4\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}, -\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)), x)
```

**Giac [A]**
time = 0.52, size = 177, normalized size = 1.86

$$-\frac{\sqrt{c}\left(\frac{\log\left(-64\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+64\right)}{\sqrt{a}\,c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{2\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{\sqrt{a}\,c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{1}{\sqrt{a}\,c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] -1/4*sqrt(c)*(log(-64*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 64)/(sqrt(a)*c^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2
```

```
*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sqrt(a)*c^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 1/(sqrt(a)*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4
*pi + 1/2*f*x + 1/2*e)^2))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\sin\left(e+f\,x\right)}\,{\left(c-c\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

# 3.388 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=140

$$\frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{\tanh^{-}}{4c^2f\sqrt{a+a}}$$

```
[Out] 1/4*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*cos(f*x+
e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*arctanh(sin(f*x+e)
)*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + Co
s[e + f*x]/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\int \overline{\quad}}{\sqrt{a+}}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{4cf\sqrt{a+}}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{4cf\sqrt{a+}}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{4cf\sqrt{a+}}$$

**Mathematica** [A]
time = 0.40, size = 224, normalized size = 1.60

$$\frac{\cos(e+fx)\left(4-3\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)+3\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+\left(-2+4\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-4\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{8c^2f(-1+\sin(e+fx))^2\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[e + f*x]*(4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 + 4*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]])*Sin[e + f*x]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(122) = 244.
time = 10.46, size = 249, normalized size = 1.78

| method | result |
|---|---|
| default | $\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+2(\cos^2(fx+e))+2\ln\left(-\frac{-1+\cos(fx+e)+\sin(}{\sin(fx+e)}\right.}{4f\sqrt{a\,(1+\mathrm{si}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*cos(f*x+e)^2+2*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))*sin(f*x+e)+3*sin(f*x+e)-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2)*cos(f*x+e)/(a*(1+sin(f*x+e)))
^(1/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas [A]**
time = 0.40, size = 408, normalized size = 2.91

$$\left[\frac{\left(\cos\left(fx+e\right)^{3}+2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(\sin\left(fx+e\right)-2\right)}{8\,\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)},\,-\frac{\left(\cos\left(fx+e\right)^{3}+2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)-\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(\sin\left(fx+e\right)-2\right)}{4\,\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c
^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*
f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(5/2)), x)
```

**Giac** [A]
time = 0.53, size = 197, normalized size = 1.41

$$-\frac{\sqrt{c}\left(\frac{2\,\log\left(-256\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+256\right)}{\sqrt{a}\,c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{\sqrt{a}\,c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1}{\sqrt{a}\,c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}\right)}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/16*sqrt(c)*(2*log(-256*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 256)/(sqrt(a)*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 4*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sqrt(a)*c^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^2 + 1)/(sqrt(a)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.389 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{12c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}-\frac{3c^2\cos(e+fx)(c-c}{2af\sqrt{a+a\sin}}$$

```
[Out] -c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-3/2*c^2*cos(f
*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-12*c^4*cos(f*x+e)*l
n(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6*c^3*cos
(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$-\frac{12c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-12*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (6*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3
/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{12c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e}{af\sqrt{a+a\sin(e+fx)}}$$

**Mathematica [A]**
time = 1.02, size = 162, normalized size = 0.85

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(-44-18\cos(2(e+fx))-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(39-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-44 -
18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39 -
192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x
)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(173) = 346$.
time = 17.26, size = 500, normalized size = 2.62

| method | result |
|---|---|
| default | $\frac{\left(\cos^4(fx+e)+\left(\cos^3(fx+e)\right)\sin(fx+e)+24\ln\left(\frac{2}{\cos(fx+e)+1}\right)\left(\cos^2(fx+e)\right)+24\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)\cos(fx+e)-48\ln\left(-\frac{-1}{}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+24*ln(2/(cos(f*x+e)+1))*cos(f*x
+e)^2+24*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))*cos(f*x+e)*sin(f*x+e)-9*cos(f*x+e)^3+8*sin(f*x+e)*cos(f*x+e)^2+24*ln
(2/(cos(f*x+e)+1))*cos(f*x+e)-48*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-48*cos(f*x
+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+96*ln(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))*sin(f*x+e)+33*cos(f*x+e)^2+25*cos(f*x+e)*sin(f*x+e)-48*ln
(2/(cos(f*x+e)+1))+96*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+9*cos(f*x+
e)-34*sin(f*x+e)-34)*(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^4-cos(f*x+e)^3*s
in(f*x+e)+3*cos(f*x+e)^3+4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2+4*cos(f*x
+e)*sin(f*x+e)-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(3/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x +
e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.50, size = 179, normalized size = 0.94

$$\frac{2\,\sqrt{a}\,c^{\frac{7}{2}}\left(\frac{6\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+4\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{4}}-\frac{2}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 2*sqrt(a)*c^(7/2)*(6*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + (a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 4*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^4 - 2/((sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.390 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=143

$$-\frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}-\frac{c\cos(e+fx)(c-c\,\mathrm{si}}{f(a+a\sin(e+}$$

```
[Out] -c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-4*c^3*cos(f*x
+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*c^
2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$-\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} -$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} -$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} -$$

$$= -\frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e}{af\sqrt{a+a\sin(e+fx)}}$$

**Mathematica [A]**
time = 0.48, size = 153, normalized size = 1.07

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(7+\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2\left(-1+8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(7
 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 +
 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(131) = 262$.
time = 19.31, size = 448, normalized size = 3.13

| method | result |
|---|---|
| default | $-\frac{(\cos^3(fx+e)-\sin(fx+e)(\cos^2(fx+e))-4\ln(\frac{2}{\cos(fx+e)+1})(\cos^2(fx+e))+8\ln(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)})(\cos^2(fx+e))-4\ln(}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^2-4*ln(2/(cos(f*x+e)+1))*cos(f*x+e
)^2+8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*ln(2/(cos(f
*x+e)+1))*sin(f*x+e)*cos(f*x+e)+8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)*cos(f*x+e)*sin(f*x+e)-6*cos(f*x+e)^2-5*cos(f*x+e)*sin(f*x+e)-4*ln(2/(cos(
f*x+e)+1))*cos(f*x+e)+8*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))+8*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-16*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))*sin(f*x+e)-cos(f*x+e)+6*sin(f*x+e)+8*ln(2/(cos(f*x+e)+1))-16*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+6)*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+
e)^3+sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2+2*cos(f*x+e)*sin(f*x+e)-2*cos(f
*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [A]
time = 0.52, size = 138, normalized size = 0.97

$$-\frac{2\left(\sqrt{a}\,c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,\sqrt{a}\,c^2\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right)\sqrt{c}}{a^2f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -2*(sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 4*sqrt(a)*c^2*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.391 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=97

$$-\frac{c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2818, 2816, 2746, 31}

$$-\frac{c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\ &= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{(c^2\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\ &= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{(c^2\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin\ldots\right)}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e\ldots}}\\ &= -\frac{c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+\ldots}}{f(a+a\sin(e+fx))^{3/2}}\end{aligned}$$

**Mathematica [A]**
time = 0.27, size = 134, normalized size = 1.38

$$-\frac{2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(1 + L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin
[e + f*x]))^(3/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(89) = 178$.
time = 10.06, size = 388, normalized size = 4.00

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\left(2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\cos(fx+e)\sin(fx+e)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)(\cos^2(fx$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-ln(2/(cos(f*x+e)+1))
*cos(f*x+e)^2-ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))*sin(f*x+e)-2*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-ln(2/(cos(f*x+e)+
1))*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-4*ln(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))+2*sin(f*x+e)+2*ln(2/(cos(f*x+e)+1))+2)*(-c*(sin(f*x+e)-1
))^(3/2)/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*
(1+sin(f*x+e)))^(3/2)
```

**Maxima [A]**
time = 0.50, size = 146, normalized size = 1.51

$$\frac{\frac{2c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{4\sqrt{a}\,c^{\frac{3}{2}}\sin(fx+e)}{\left(a^2+\frac{2a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - c^(3/2)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4*sqrt(a)*c^(3/2)*sin(f*x
 + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)/(a^2*cos(f*x
 + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [A]
time = 0.55, size = 115, normalized size = 1.19

$$\frac{\sqrt{2}\,\sqrt{a}\,c^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\log\left(-2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\sqrt{2}}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*c^(3/2)*(sqrt(2)*log(-2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
2 + 2)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)/((sin(-1/4*pi +
1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-c\,\sin(e+f\,x))^{3/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.392 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{2817\}$

$$-\frac{c\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.
time = 0.13, size = 85, normalized size = 2.07

$$-\frac{\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a^2*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))
```

**Maple** [A]
time = 18.19, size = 69, normalized size = 1.68

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(-1+\cos(fx+e)-\sin(fx+e))}{f(-1+\cos(fx+e)+\sin(fx+e))(a(1+\sin(fx+e)))^{\frac{3}{2}}}$ | 69 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(-1+cos(f*x+e)-sin(f*x+e))/(-1+co
s(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [A]
time = 0.37, size = 63, normalized size = 1.54

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin
(f*x + e) + a^2*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac [A]**
time = 0.45, size = 56, normalized size = 1.37

$$\frac{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{2\,a^{\frac{3}{2}}f\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/2*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^(3/2)*f*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad [B]**
time = 7.52, size = 52, normalized size = 1.27

$$-\frac{2\,\cos\left(e+f\,x\right)\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{a\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] -(2*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a*f*(cos(2*e + 2*f*x) + 1)
*(a*(sin(e + f*x) + 1))^(1/2))
```

# 3.393 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=95

$$-\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*arctanh
(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) +
 (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\int \frac{\cdots}{\sqrt{a\cdots}}}{\cdots}\\ &= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\cdots}{2a\sqrt{a\cdots}}\\ &= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\cdots}{2af\sqrt{\cdots}}\end{aligned}$$

**Mathematica** [A]
time = 0.26, size = 148, normalized size = 1.56

$$-\frac{\cos(e+fx)\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2f(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -1/2*(Cos[e + f*x]*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(a*(1 + Sin
[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 10.82, size = 165, normalized size = 1.74

| method | result |
|---|---|
| default | $\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)+\sin(fx+e)+\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\cdots\right.}{2f(a(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*(ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+sin(f*x+e)+ln(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(
a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas [A]**
time = 0.39, size = 331, normalized size = 3.48

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)-2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{4\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf\cos\left(fx+e\right)\right)},-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

**Giac [A]**
time = 0.53, size = 180, normalized size = 1.89

$$\frac{\sqrt{a}\,\sqrt{c}\left(\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{2\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{2}c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{1}{a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/4*sqrt(a)*sqrt(c)*(log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*c*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*lo
```

```
g(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 1/(a^2*c*cos(-1/4*pi + 1/2*f*
x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))))/f
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.394 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=143

$$-\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{\tanh}{2acf\sqrt{a+}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*cos(f*x
+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(sin(f*x+e
))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2acf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))
 + Cos[e + f*x]/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
 + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} +$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} +$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} +$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} +$$

**Mathematica** [A]
time = 0.40, size = 170, normalized size = 1.19

$$\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-2\sin(e+fx)\right)}{4cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*
(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]) - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 2*Sin[e + f*x]))/(4
*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*
x]])
```

**Maple** [A]
time = 11.61, size = 114, normalized size = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{\left(\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)+\sin(fx+e)\right)\cos(fx+e)}{2f(a(1+\sin(fx+e)))^{\frac{3}{2}}(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$ | 114 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*(ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+sin(f*x+e))*cos(f*x+e)/(a*(1+sin(
f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [A]**
time = 0.40, size = 282, normalized size = 1.97

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos(fx+e)^{3}-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin(fx+e)}{\cos(fx+e)^{3}}\right)+2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{4\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}},-\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)\cos\left(fx+e\right)^{3}-\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{2\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
+ e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a
*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*si
n(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) - 1))**(3/2)),
x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

## 3.395 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{3\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{8acf\sqrt{a+}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*cos(f*x
+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*cos(f*x+e)/a/c/f/
(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f
*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)}{8af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))
 + (3*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2)) + (3*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*c^2*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int}{}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af}$$

**Mathematica [A]**
time = 0.48, size = 287, normalized size = 1.50

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2\cos^2(e+fx)-\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)}{8f(a(1+\sin(e+fx)))^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + (Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^2 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e
+ f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/2))
```

**Maple [A]**
time = 11.03, size = 227, normalized size = 1.19

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\left(3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)\sin(fx+e)-3\ln\left(-\frac{-1+\cos}{}\right.\right.}{8f(a(1+\sin(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/8/f*(3*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-3*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+3*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)^2+cos(f*x+e)^2+
3*sin(f*x+e)-1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(5/
2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas [A]**
time = 0.40, size = 407, normalized size = 2.13

$$\left[\frac{3\left(\cos(fx+e)^3\sin(fx+e)-\cos(fx+e)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)-2\left(3\cos(fx+e)^2+3\sin(fx+e)-1\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\left(a^2c^3f\cos(fx+e)^3\sin(fx+e)-a^2c^3f\cos(fx+e)^3\right)}, -\frac{3\left(\cos(fx+e)^3\sin(fx+e)-\cos(fx+e)^3\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\left(3\cos(fx+e)^2+3\sin(fx+e)-1\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8\left(a^2c^3f\cos(fx+e)^3\sin(fx+e)-a^2c^3f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac** [**A**]
time = 0.56, size = 236, normalized size = 1.24

$$\frac{\sqrt{a}\,\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{12\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{2}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{6\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-9\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}a^{2}c^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/32*sqrt(a)*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*c^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) -
 12*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*c^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (6*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^4 - 9*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2)/((cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 1)^2*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.396 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=237

$$\frac{24c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}+\frac{3c^3\cos(e+fx)(c-c\sin}{a^2f\sqrt{a+a\sin(e}$$

```
[Out] 2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/2*c*co
s(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3*c^3*cos(f*x+e)*(
c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+24*c^5*cos(f*x+e)*ln(1+s
in(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+12*c^4*cos(f
*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.35, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$\frac{24c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (24*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (12*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[
e + f*x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(7/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} +$$

$$= \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{24c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e}{a^2f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [A]
time = 3.24, size = 202, normalized size = 0.85

$$\frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(273+\cos(4(e+fx))+\cos(2(e+fx))\left(106-384\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+1152\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+320\sin(e+fx)+1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+24\sin(3(e+fx))\right)}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(273 +
Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 320*Sin[e +
 f*x] + 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 24*Sin
[3*(e + f*x)]))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e +
 f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(215) = 430$.
time = 17.45, size = 684, normalized size = 2.89

| method | result |
|---|---|

| default | $-\frac{\left(132+132\sin(fx+e)-74\cos(fx+e)-143\left(\cos^2(fx+e)\right)-96\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-96\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin\right.}{}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(132+96*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(
f*x+e)-85*sin(f*x+e)*cos(f*x+e)^2+132*sin(f*x+e)-74*cos(f*x+e)-143*cos(f*x+
e)^2+12*cos(f*x+e)^3*sin(f*x+e)+11*cos(f*x+e)^4-96*ln(2/(cos(f*x+e)+1))*sin
(f*x+e)*cos(f*x+e)-96*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))-sin(f*x+e)*cos(f*x+e)^4-384*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+7
3*cos(f*x+e)^3-58*cos(f*x+e)*sin(f*x+e)-96*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+
192*ln(2/(cos(f*x+e)+1))*sin(f*x+e)+192*ln(2/(cos(f*x+e)+1))-144*ln(2/(cos(
f*x+e)+1))*cos(f*x+e)^2+48*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+192*ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-48*cos(f*x+e)^2*sin(
f*x+e)*ln(2/(cos(f*x+e)+1))+192*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))-384*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+cos(f*
x+e)^5+288*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2)*(-c*(si
n(f*x+e)-1))^(9/2)/(cos(f*x+e)^5+sin(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^4+4*c
os(f*x+e)^3*sin(f*x+e)-8*cos(f*x+e)^3-12*sin(f*x+e)*cos(f*x+e)^2+20*cos(f*x
+e)^2-8*cos(f*x+e)*sin(f*x+e)+8*cos(f*x+e)+16*sin(f*x+e)-16)/(a*(1+sin(f*x+
e)))^(5/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f
*x + e)^2 - 2*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
```

```
 e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e)), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.53, size = 198, normalized size = 0.84

$$-\frac{2\sqrt{a}\,c^{\frac{9}{2}}\left(\frac{12\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+6a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^6}-\frac{8\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-7}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -2*sqrt(a)*c^(9/2)*(12*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) + (a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 6*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^6 - (8*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 7)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.397 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=193

$$\frac{6c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}+\frac{3c^2\cos(e+fx)(c-c\sin}{2af(a+a\sin(e-}$$

```
[Out] 3/2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/2*c*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+6*c^4*cos(f*x+e)
*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+3*c^3
*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2818, 2819, 2816, 2746, 31}

$$\frac{6c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (6*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e +
f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} +$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} -$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} -$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} -$$

$$= \frac{6c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e-}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica [A]**
time = 1.28, size = 187, normalized size = 0.97

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(28+\cos(2(e+fx))\left(4-24\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(41+96\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(28 + C
os[2*(e + f*x)]*(4 - 24*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 72*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (41 + 96*Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(173) = 346$.
time = 18.73, size = 631, normalized size = 3.27

| method | result |
|---|---|
| default | $\frac{\left(\cos^4(fx+e)+\left(\cos^3(fx+e)\right)\sin(fx+e)-12\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+6\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+12\ln\left(\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)-12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+12*ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-6*cos(f*x+e)^2*sin
(f*x+e)*ln(2/(cos(f*x+e)+1))+10*cos(f*x+e)^3-11*sin(f*x+e)*cos(f*x+e)^2+36*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-18*ln(2/(cos(f*x+e)
+1))*cos(f*x+e)^2+24*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)*
sin(f*x+e)-12*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-17*cos(f*x+e)^2-6*
cos(f*x+e)*sin(f*x+e)+24*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))-12*ln(2/(cos(f*x+e)+1))*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))*sin(f*x+e)+24*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-10*cos(f*x+e)+16*si
n(f*x+e)-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+e)+
1))+16)*(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*x+e)+3*c
os(f*x+e)^3+4*sin(f*x+e)*cos(f*x+e)^2-8*cos(f*x+e)^2+4*cos(f*x+e)*sin(f*x+e
)-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(5/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x
+ e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.50, size = 162, normalized size = 0.84

$$-\frac{\sqrt{a}\,c^{\frac{7}{2}}\left(\frac{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{6\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{6\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-5}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -sqrt(a)*c^(7/2)*(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2/(a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) + 6*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (6*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 5
)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.398 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}} \, dx$

**Optimal**. Leaf size=143

$$\frac{c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -1/2*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)+c^3*cos(f
*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+
c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2818, 2816, 2746, 31}

$$\frac{c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{c\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \cdots$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \cdots$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \cdots$$

$$= \frac{c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e \cdots}}{af(a+a\sin(e+fx))^{3/ \cdots}}$$

**Mathematica [A]**
time = 0.75, size = 172, normalized size = 1.20

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(2+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2 + 3*
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - Cos[2*(e + f*x)]*Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f
*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(129) = 258$.

time = 18.65, size = 566, normalized size = 3.96

| method | result |
|---|---|
| default | $\frac{\left((\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2(\cos^3(fx+e))\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-(\cos^2(fx+e))\sin(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+2\ln\left(-\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))-cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+2*cos(f*x+e)^3
-2*sin(f*x+e)*cos(f*x+e)^2-3*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2+6*ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(2/(cos(f*x+e)+1))*sin(f*
x+e)*cos(f*x+e)+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)*sin
(f*x+e)-2*cos(f*x+e)^2-2*ln(2/(cos(f*x+e)+1))*cos(f*x+e)+4*cos(f*x+e)*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-8*l
n(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-2*cos(f*x+e)+2*sin(f*x
+e)+4*ln(2/(cos(f*x+e)+1))-8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2)*
(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e
)^2+2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))
^(5/2)
```

**Maxima** [A]

time = 0.51, size = 197, normalized size = 1.38

$$\frac{\frac{8\sqrt{a}\,c^{\frac{5}{2}}\sin(fx+e)^2}{\left(a^3+\frac{4a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2}-\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{5}{2}}}+\frac{c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
+ 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))/f
```

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos
(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac [A]**
time = 0.47, size = 135, normalized size = 0.94

$$-\frac{\sqrt{2}\,\sqrt{a}\,c^{\frac{5}{2}}\left(\frac{2\,\sqrt{2}\,\log\left(-2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\right)}{a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\sqrt{2}\,\left(4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-3\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*sqrt(2)*sqrt(a)*c^(5/2)*(2*sqrt(2)*log(-2*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 + 2)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(4*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2 - 3)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.399 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=42

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -1/4*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2821}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f*x])^(5/2
))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(42) = 84.
time = 0.28, size = 86, normalized size = 2.05

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)\sqrt{c-c\sin(e+fx)}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[c - c*Sin[e + f*
x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.
time = 16.62, size = 93, normalized size = 2.21

| method | result | size |
|---|---|---|
| default | $-\frac{\sin(fx+e)\left(-c(\sin(fx+e)-1)\right)^{\frac{3}{2}}\left(-1+\cos(fx+e)-\sin(fx+e)\right)}{f\left(\cos^2(fx+e)-\cos(fx+e)\sin(fx+e)+\cos(fx+e)+2\sin(fx+e)-2\right)\left(a(1+\sin(fx+e))\right)^{\frac{5}{2}}}$ | 93 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(3/2)*(-1+cos(f*x+e)-sin(f*x+e))/(cos(f
*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))
^(5/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.
time = 0.34, size = 87, normalized size = 2.07

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,c\sin\left(fx+e\right)}{a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*c*sin(f*x + e)/(a^3*f*c
os(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(39) = 78$.
time = 0.53, size = 98, normalized size = 2.33

$$-\frac{\left(2\,\sqrt{a}\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\sqrt{a}\,c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{4\,a^{3}f\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(2*sqrt(a)*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^3*f
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad** [B]
time = 8.21, size = 118, normalized size = 2.81

$$\frac{2\,c\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(-2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^{2}+2\,\sin\left(2\,e+2\,f\,x\right)\right)}{a^{2}\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-8\sin\left(e+f\,x\right)^{2}+4\,\sin\left(e+f\,x\right)+2\sin\left(2\,e+2\,f\,x\right)^{2}+4\,\sin\left(3\,e+3\,f\,x\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] (2*c*(-c*(sin(e + f*x) - 1))^(1/2)*(2*sin(2*e + 2*f*x) - 2*sin(e/2 + (f*x)/
2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(
4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)
^2 + 8))
```

## 3.400 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=43

$$-\frac{c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2817}

$$-\frac{c\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.
time = 0.15, size = 87, normalized size = 2.02

$$-\frac{\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a^3*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(37) = 74$.
time = 19.16, size = 92, normalized size = 2.14

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)\left(\cos^{2}(fx+e)+\cos(fx+e)\sin(fx+e)+2\cos(fx+e)-3\sin(fx+e)-3\right)}{2f\left(a(1+\sin(fx+e))\right)^{\frac{5}{2}}\left(-1+\cos(fx+e)+\sin(fx+e)\right)}$ | 92 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(cos(f*x+e)^2+cos(f*x+e)*sin(f*
x+e)+2*cos(f*x+e)-3*sin(f*x+e)-3)/(a*(1+sin(f*x+e)))^(5/2)/(-1+cos(f*x+e)+s
in(f*x+e))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [A]
time = 0.34, size = 79, normalized size = 1.84

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [A]
time = 0.50, size = 56, normalized size = 1.30

$$\frac{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{8\,a^{\frac{5}{2}}f\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/8*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^(5/2)*f*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

**Mupad** [B]
time = 7.70, size = 103, normalized size = 2.40

$$-\frac{2\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(-4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\sin\left(2\,e+2\,f\,x\right)+2\right)}{a^2\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-8\sin\left(e+f\,x\right)^2+4\,\sin\left(e+f\,x\right)+2\sin\left(2\,e+2\,f\,x\right)^2+4\,\sin\left(3\,e+3\,f\,x\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(2*e + 2*f*x) - 4*sin(e/2 + (f*x)/2)^
2 + 2))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(4*sin(e + f*x) + 4*sin(3*e + 3
*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2 + 8))
```

## 3.401 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=140

$$-\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\tanh}{4a^2f\sqrt{a+}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*arctanh(sin(f*x+e
))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -1/4*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]) -
 Cos[e + f*x]/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) +
 (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{}{(a+a}}{}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{4af(a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{4af(a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{4af(a}$$

**Mathematica** [A]
time = 0.42, size = 211, normalized size = 1.51

$$\frac{\cos(e+fx)\left(-4-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(-2-4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{8f(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
 f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 - 4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]])*Sin[e + f*x]))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(122) = 244$.
time = 10.94, size = 252, normalized size = 1.80

| method | result |
|---|---|
| default | $-\frac{\left(-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)+\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)+2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right.}{4f(a(1}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))*sin(f*x+e)-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin
(f*x+e)+2*cos(f*x+e)^2-3*sin(f*x+e)+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*cos(f*x+e)/(a*(1+sin(
f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

**Fricas [A]**
time = 0.39, size = 408, normalized size = 2.91

$$\left[\frac{\left(\cos\left(fx+e\right)^{3}-2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos(fx+e)^{3}-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin(fx+e)}{\cos(fx+e)^{3}}\right)+2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(\sin\left(fx+e\right)+2\right)}{8\,\left(a^{3}cf\cos\left(fx+e\right)^{3}-2\,a^{3}cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}cf\cos\left(fx+e\right)\right)},\,-\frac{\left(\cos\left(fx+e\right)^{3}-2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)-\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(\sin\left(fx+e\right)+2\right)}{4\,\left(a^{3}cf\cos\left(fx+e\right)^{3}-2\,a^{3}cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}cf\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3
*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*
f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

**Giac [A]**
time = 0.53, size = 197, normalized size = 1.41

$$\frac{\sqrt{c}\left(\frac{2\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{\frac{5}{2}}c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{\frac{5}{2}}c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1}{a^{\frac{5}{2}}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/16*sqrt(c)*(2*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^(5/2)*c*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*log
(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(5/2)*c*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^2 + 1)/(a^(5/2)*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

# 3.402 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=188

$$-\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{3\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{}{8a^2f\sqrt{a}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-3/8*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+3/8*cos(f*x+e)/a^2/f/
(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f
*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3\cos(e+fx)}{8af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/4*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2))
 - (3*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2)) + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int}{}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af}$$

**Mathematica** [A]
time = 0.46, size = 287, normalized size = 1.53

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\cos^2(e+fx)-\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{8f(a(1+\sin(e+fx)))^{5/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e
 + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/2))
```

**Maple** [A]
time = 11.04, size = 227, normalized size = 1.21

| method | result |
|---|---|

| default | $-\frac{\left(3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+3\ln\left(-\frac{-1+}{}\right.\right.}{8f(a(1+\sin}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/8/f*(3*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+3*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-3*ln(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)^2+cos(f*x+e)^2
-3*sin(f*x+e)-1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(3
/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [A]**
time = 0.39, size = 401, normalized size = 2.13

$$\left[\frac{3\left(\cos(fx+e)^3\sin(fx+e)+\cos(fx+e)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)-2\left(3\cos(fx+e)^2-3\sin(fx+e)-1\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}, -\frac{3\left(\cos(fx+e)^3\sin(fx+e)+\cos(fx+e)^3\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\left(3\cos(fx+e)^2-3\sin(fx+e)-1\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a
^3*c^2*f*cos(f*x + e)^3)]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac** [**A**]
time = 0.57, size = 233, normalized size = 1.24

$$\frac{\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{\frac{5}{2}}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{12\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{\frac{5}{2}}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{6\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-3\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{\frac{5}{2}}c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/32*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^(5/2)*c^2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 12*
log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(5/2)*c^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (6*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^4 - 3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)/((cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 1)*a^(5/2)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

## 3.403 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=236

$$-\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{}{8a^2f\sqrt{a}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*cos(f*x+e)/a^2/f/
(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*cos(f*x+e)/a^2/c/f/(c-c*s
in(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f*x+e)/
a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2822, 2820, 3855}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{\cos(e+fx)}{2af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/4*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))
 - Cos[e + f*x]/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2
)) + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(5/2)) + (3*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[
e + f*x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{}{(a}}{}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

**Mathematica** [A]
time = 0.63, size = 237, normalized size = 1.00

$$\frac{\sec^3(e+fx)\left(-9\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-3\cos(4(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+9\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+22\sin(e+fx)+6\sin(3(e+fx))\right)}{64a^2c^2f\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(-9*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*Cos[2*(e
+ f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]]) - 3*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]] + 22*Sin[e + f*x] + 6*Sin[3*(e + f*x)]))/(64*a^2*c^2*f
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [A]
time = 10.94, size = 134, normalized size = 0.57

| method | result |
|---|---|
| default | $\frac{\left(3\left(\cos^4(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-3\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^4(fx+e)\right)+3\sin(fx+e)\left(\cos^2(fx+e)\right)+2\sin(fx}{8f(a(1+\sin(fx+e)))^{\frac{5}{2}}(-c(\sin(fx+e)-1))^{\frac{5}{2}}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/8/f*(3*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4+3*sin(f*x+e)*cos(f*x+e)^2+2
*sin(f*x+e))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas [A]**
time = 0.41, size = 310, normalized size = 1.31

$$\left[\frac{3\sqrt{ac}\cos(fx+e)^5\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)+2\left(3\cos(fx+e)^2+2\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{16a^3c^3f\cos(fx+e)^5}, -\frac{3\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^5-\left(3\cos(fx+e)^2+2\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{8a^3c^3f\cos(fx+e)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*sqrt(a*c)*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x
+ e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f
*x + e))/cos(f*x + e)^3) + 2*(3*cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^5), -1/8*
(3*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (3*cos(f*x + e)^2
 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3
*c^3*f*cos(f*x + e)^5)]
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\,\sin\left(e+f\,x\right))^{5/2}\,(c-c\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

# 3.404 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=110

$$\frac{2^{\frac{1}{2}+n}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}-n}(a+a\sin(}{f(1+2m)}$$

```
[Out] 2^(1/2+n)*c*cos(f*x+e)*hypergeom([1/2+m, 1/2-n],[3/2+m],1/2+1/2*sin(f*x+e))
*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/(1+2*m
)
```

**Rubi** [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2824, 2768, 72, 71}

$$\frac{c2^{n+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+f\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n);\frac{1}{2}(3+2m);\frac{1}{2}\right.}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.97, size = 365, normalized size = 3.32

$$\frac{4(3+2n)F_1\left(\frac{1}{2}+n;-2m,1+2(m+n);\frac{3}{2}+n;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\cos^3\left(\frac{1}{8}(2e-\pi+2fx)\right)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n\sin\left(\frac{1}{8}(2e-\pi+2fx)\right)}{f(1+2n)\left((3+2n)F_1\left(\frac{1}{2}+n;-2m,1+2(m+n);\frac{3}{2}+n;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\cos^2\left(\frac{1}{8}(2e-\pi+2fx)\right)-2\left(2mF_1\left(\frac{3}{2}+n;1-2m,1+2(m+n);\frac{5}{2}+n;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)+(1+2m+2n)F_1\left(\frac{3}{2}+n;-2m,2(1+m+n);\frac{5}{2}+n;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sin^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (4*(3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^3*(
a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/8])/(
f*(1 + 2*n)*((3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[
(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f
*x)/8]^2 - 2*(2*m*AppellF1[3/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, Tan[(-
2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (1 + 2*m + 2*n)*App
```

```
ellF1[3/2 + n, -2*m, 2*(1 + m + n), 5/2 + n, Tan[(-2*e + Pi - 2*f*x)/8]^2,
-Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

**Maple** [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^m (c - c\sin(fx + e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin(e + f x))^m (c - c\sin(e + f x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

# 3.405 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=86

$$-\frac{2^{\frac{1}{2}+m}a^4c^3\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{-4+m}}{7f}$$

```
[Out] -1/7*2^(1/2+m)*a^4*c^3*cos(f*x+e)^7*hypergeom([7/2, 1/2-m],[9/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2768, 72, 71}

$$-\frac{a^4c^32^{m+\frac{1}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -1/7*(2^(1/2 + m)*a^4*c^3*Cos[e + f*x]^7*Hypergeometric2F1[7/2, 1/2 - m, 9/
2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^
(-4 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$
$$= \frac{\left(a^5c^3\cos^7(e+fx)\right)\text{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{-\frac{1}{2}+m}\,dx, x\right.}{f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+fx))^{7/}}$$
$$= \frac{\left(2^{-\frac{1}{2}+m}a^5c^3\cos^7(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+a\sin(}{a}\right.\right.}{f(a-a}$$
$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx\right.}{7f}$$

**Mathematica [F]**
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
[Out] $Aborted
```

**Maple [F]**
time = 1.33, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m(c-c\sin(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(
f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int 3(a\sin{(e+fx)}+a)^{m}\sin{(e+fx)}\,dx+\int\left(-3(a\sin{(e+fx)}+a)^{m}\sin^{2}{(e+fx)}\right)\,dx+\int(a\sin{(e+fx)}+a)^{m}\sin^{3}{(e+fx)}\,dx+\int\left(-(a\sin{(e+fx)}+a)^{m}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(3*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-3*(a
*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral((a*sin(e + f*x) + a)**
m*sin(e + f*x)**3, x) + Integral(-(a*sin(e + f*x) + a)**m, x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin (e + f\,x))^m\,(c - c\sin (e + f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

# 3.406 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=86

$$-\frac{2^{\frac{1}{2}+m}a^3c^2\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{-3+m}}{5f}$$

```
[Out] -1/5*2^(1/2+m)*a^3*c^2*cos(f*x+e)^5*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2768, 72, 71}

$$-\frac{a^3c^22^{m+\frac{1}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/5*(2^(1/2 + m)*a^3*c^2*Cos[e + f*x]^5*Hypergeometric2F1[5/2, 1/2 - m, 7/
2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^
(-3 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$

$$= \frac{\left(a^4c^2\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{-\frac{1}{2}+m}\,dx, x\right.}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/}}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a^4c^2\cos^5(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+a\sin(}{a}\right.\right.}{f(a-a}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx\right.}{5f}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 139.29, size = 88512, normalized size = 1029.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m(c-c\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*(a*sin(f*x + e)
 + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2(a\sin(e+fx)+a)^m\sin(e+fx)\right)dx+\int(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx+\int(a\sin(e+fx)+a)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
[Out] c**2*(Integral(-2*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral((a*si
n(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral((a*sin(e + f*x) + a)**m,
x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sin(e+fx))^m (c-c\sin(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

# 3.407 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

**Optimal**. Leaf size=84

$$-\frac{2^{\frac{1}{2}+m}a^2c\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{-2+m}}{3f}$$

```
[Out] -1/3*2^(1/2+m)*a^2*c*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f
```

**Rubi** [A]

time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2815, 2768, 72, 71}

$$-\frac{a^2c2^{m+\frac{1}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/3*(2^(1/2 + m)*a^2*c*Cos[e + f*x]^3*Hypergeometric2F1[3/2, 1/2 - m, 5/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-
2 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$

$$= \frac{(a^3c\cos^3(e+fx))\,\text{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{-\frac{1}{2}+m}\,dx, x, \text{s}\right.}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a^3c\cos^3(e+fx)(a+a\sin(e+fx))^{-2+m}\left(\frac{a+a\sin(e+}{a}\right.\right.}{f(a-a\,\text{si}}$$

$$= -\frac{2^{\frac{1}{2}+m}a^2c\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right.}{3f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.08, size = 285, normalized size = 3.39

$$-\frac{i2^{-1-2m}ce^{-i(e+fx)}\left(1+ie^{-i(e+fx)}\right)^{-2m}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\right)^{2m}\left(e^{2i(e+fx)}(-1+m)m\,{}_2F_1\left(-1-m,-2m;-m;-ie^{-i(e+fx)}\right)+(1+m)\left(m\,{}_2F_1\left(1-m,-2m;2-m;-ie^{-i(e+fx)}\right)-2e^{i(e+fx)}(-1+m)\,{}_2F_1\left(-2m,-m;1-m;-ie^{-i(e+fx)}\right)\right)\right)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^m\sin^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f(-1+m)m(1+m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] ((-I)*2^(-1 - 2*m)*c*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*(e + f*
x))))^(2*m)*(E^((2*I)*(e + f*x))*(-1 + m)*m*Hypergeometric2F1[-1 - m, -2*m,
 -m, (-I)/E^(I*(e + f*x))] + (1 + m)*(m*Hypergeometric2F1[1 - m, -2*m, 2 -
m, (-I)/E^(I*(e + f*x))] - 2*E^(I*(e + f*x))*(-1 + m)*Hypergeometric2F1[-2*
m, -m, 1 - m, (-I)/E^(I*(e + f*x))]))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f
*x]))^m)/(E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2*m)*f*(-1 + m)*m*(1 + m
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m))
```

**Maple** [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m\,(c-c\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int (a\sin(e+fx)+a)^m\sin(e+fx)\,dx+\int\left(-(a\sin(e+fx)+a)^m\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral((a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-(a*sin(e
+ f*x) + a)**m, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x))^m (c - c \sin(e + f x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

# 3.408 $\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=76

$$\frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^m}{cf}$$

```
[Out] 2^(1/2+m)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+s
in(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2768, 72, 71}

$$\frac{2^{m+\frac{1}{2}}\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Se
c[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}\,dx}{ac} \\
&= \frac{\left(a\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+\cdots}}{(a-ax)^{3/2}}\cdots\right.}{cf} \\
&= \frac{\left(2^{-\frac{1}{2}+m}a\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)\cdots\right.}{cf} \\
&= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-\cdots}}{cf}
\end{aligned}
$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.21, size = 3844, normalized size = 50.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/2*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*(a + a*Sin[e + f*x])^m*(-(AppellF1[-1/2, -2*
m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[
(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2,
 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
```

+ Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/(f*(c - c*Sin[e + f*x])*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^2*(-1/2*(m*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Csc[(-e + Pi/2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(m*AppellF1[1/2, 1 - 2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*AppellF1[1/2, -2*m, 1 + 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(3*Ap

```
pellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]
^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/
4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/
4]^2*Tan[(-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 -
f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f...
```

**Maple** [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m}{c - c\sin(fx + e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)`

[Out] `int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)`

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out] `-integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)`

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out] `integral(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)`

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a\sin(e+fx)+a)^m}{\sin(e+fx)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
[Out] -Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x)/c
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+f\,x))^m}{c-c\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x)), x)
```

# 3.409 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=86

$$\frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{1+m}}{3ac^2f}$$

```
[Out] 1/3*2^(1/2+m)*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)
^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f
```

**Rubi** [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2768, 72, 71}

$$\frac{2^{m+\frac{1}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*c^2*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$
$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{5/2}}\right.}{c^2f}$$
$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{c^2f}$$
$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-}}{3ac^2f}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.25, size = 5391, normalized size = 62.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) -
 2*c^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a\sin{(e+fx)}+a)^m}{\sin^2{(e+fx)}-2\sin{(e+fx)}+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1), x)
/c**2
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^m}{(c - c \sin(e + f x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^2, x)
```

# 3.410 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=86

$$\frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+fx))^{2+m}}{5a^2c^3f}$$

```
[Out] 1/5*2^(1/2+m)*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)
^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2815, 2768, 72, 71}

$$\frac{2^{m+\frac{1}{2}}\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-5/2, 1/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*c^3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$
$$= \frac{\left(\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{7/2}}\right.}{ac^3f}$$
$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx}{a}\right.\right.}{ac^3f}$$
$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-}}{5a^2c^3f}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.39, size = 6906, normalized size = 80.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

**Maple [F]**
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f
*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^3, x)
```

# 3.411 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=160

$$\frac{64c^3\cos(e+fx)(a+a\sin(e+fx))^m}{f(5+2m)\,(3+8m+4m^2)\,\sqrt{c-c\sin(e+fx)}}+\frac{16c^2\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f\,(15+16m+4m^2)}$$

```
[Out] 2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2)/f/(5+2*m)+64*c^3*c
os(f*x+e)*(a+a*sin(f*x+e))^m/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)
+16*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(4*m^2+16*m+
15)
```

**Rubi** [A]
time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2819, 2817}

$$\frac{64c^3\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\,(4m^2+8m+3)\,\sqrt{c-c\sin(e+fx)}}+\frac{16c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f\,(4m^2+16m+15)}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(5 + 2*m)*(3 + 8*m + 4*m^2)
*Sqrt[c - c*Sin[e + f*x]]) + (16*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c - c*Sin[e + f*x]])/(f*(15 + 16*m + 4*m^2)) + (2*c*Cos[e + f*x]*(a + a*
Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)} -$$
$$= \frac{16c^2\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(15+16m+4m^2)}$$
$$= \frac{64c^3\cos(e+fx)(a+a\sin(e+fx))^m}{f(15+46m+36m^2+8m^3)\sqrt{c-c\sin(e+fx)}} + \frac{16c^2}{}$$

**Mathematica [A]**
time = 1.58, size = 149, normalized size = 0.93

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\,(-89-56m-12m^2+(3+8m+4m^2)\cos(2(e+fx))+4(7+16m+4m^2)\sin(e+fx))}{f(1+2m)(3+2m)(5+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-89 - 56*m - 12*m^2 + (3 + 8*m + 4*m^2)*Cos[2*(e + f*x
)] + 4*(7 + 16*m + 4*m^2)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

**Maple [F]**
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

**Maxima [A]**
time = 0.51, size = 306, normalized size = 1.91

$$-\frac{2\left((4m^2+24m+43)a^m c^{\frac{5}{2}}-\frac{(12m^2+40m-15)a^m c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2(4m^2+8m+35)a^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2(4m^2+8m+35)a^m c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{(12m^2+40m-15)a^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{(4m^2+24m+43)a^m c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)e^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{(8m^3+36m^2+46m+15)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(
f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
```

```
^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

**Fricas** [A]
time = 0.37, size = 277, normalized size = 1.73

$$-\frac{2\left(\left(4\,c^2m^2+8\,c^2m+3\,c^2\right)\cos\left(fx+e\right)^3-\left(4\,c^2m^2+24\,c^2m+11\,c^2\right)\cos\left(fx+e\right)^2-32\,c^2-2\left(4\,c^2m^2+16\,c^2m+23\,c^2\right)\cos\left(fx+e\right)+\left(\left(4\,c^2m^2+8\,c^2m+3\,c^2\right)\cos\left(fx+e\right)^2-32\,c^2+2\left(4\,c^2m^2+16\,c^2m+7\,c^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{8\,fm^3+36\,fm^2+46\,fm+\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)\cos\left(fx+e\right)-\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)\sin\left(fx+e\right)+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^3 - (4*c^2*m^2 + 24*c^2*m +
11*c^2)*cos(f*x + e)^2 - 32*c^2 - 2*(4*c^2*m^2 + 16*c^2*m + 23*c^2)*cos(f*x
 + e) + ((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^2 - 32*c^2 + 2*(4*c^2*m
^2 + 16*c^2*m + 7*c^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c
)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2
 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f
*x + e) + 15*f)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad** [B]
time = 10.05, size = 163, normalized size = 1.02

$$\frac{c^2\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,\cos\left(3\,e+3\,f\,x\right)-175\,\cos\left(e+f\,x\right)+28\,\sin\left(2\,e+2\,f\,x\right)+16\,m^2\,\sin\left(2\,e+2\,f\,x\right)-104\,m\,\cos\left(e+f\,x\right)+8\,m\,\cos\left(3\,e+3\,f\,x\right)-20\,m^2\,\cos\left(e+f\,x\right)+64\,m\,\sin\left(2\,e+2\,f\,x\right)+4\,m^2\,\cos\left(3\,e+3\,f\,x\right)\right)}{2\,f\,\left(\sin\left(e+f\,x\right)-1\right)\,\left(8\,m^3+36\,m^2+46\,m+15\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(3*cos(3*e + 3*
f*x) - 175*cos(e + f*x) + 28*sin(2*e + 2*f*x) + 16*m^2*sin(2*e + 2*f*x) - 1
04*m*cos(e + f*x) + 8*m*cos(3*e + 3*f*x) - 20*m^2*cos(e + f*x) + 64*m*sin(2
*e + 2*f*x) + 4*m^2*cos(3*e + 3*f*x)))/(2*f*(sin(e + f*x) - 1)*(46*m + 36*m
^2 + 8*m^3 + 15))
```

# 3.412 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=100

$$\frac{8c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(3+8m+4m^2)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)}$$

```
[Out] 8*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)+
2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(3+2*m)
```

**Rubi** [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2819, 2817}

$$\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f(4m^2+8m+3)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(3 + 8*m + 4*m^2)*Sqrt[c - c
*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e
 + f*x]])/(f*(3 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)} +$$
$$= \frac{8c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(3+8m+4m^2)\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)(a}{}$$

**Mathematica** [A]
time = 0.30, size = 110, normalized size = 1.10

$$-\frac{2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\,(-5-2m+(1+2m)\sin(e+fx))}{f(1+2m)(3+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-5 - 2*m + (1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(102) = 204.
time = 0.50, size = 205, normalized size = 2.05

$$-\frac{2\left(a^m c^{\frac{3}{2}}(2m+5)-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)}{\cos(fx+e)+1}-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^m c^{\frac{3}{2}}(2m+5)\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)e^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{(4m^2+8m+3)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x + e
) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c^
```

```
(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*e^(2*m*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

**Fricas [A]**
time = 0.36, size = 153, normalized size = 1.53

$$\frac{2\left(\left(2\,cm+c\right)\cos\left(fx+e\right)^{2}+\left(2\,cm+5\,c\right)\cos\left(fx+e\right)-\left(\left(2\,cm+c\right)\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)+4\,c\right)\sqrt{-c\sin\left(fx+e\right)+c}\,\left(a\sin\left(fx+e\right)+a\right)^{m}}{4\,fm^{2}+8\,fm+\left(4\,fm^{2}+8\,fm+3\,f\right)\cos\left(fx+e\right)-\left(4\,fm^{2}+8\,fm+3\,f\right)\sin\left(fx+e\right)+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((2*c*m + c)*cos(f*x + e)^2 + (2*c*m + 5*c)*cos(f*x + e) - ((2*c*m + c)*c
os(f*x + e) - 4*c)*sin(f*x + e) + 4*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(4*f*m^2 + 8*f*m + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f
*m^2 + 8*f*m + 3*f)*sin(f*x + e) + 3*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [B]**
time = 1.19, size = 94, normalized size = 0.94

$$-\frac{c\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(10\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)+4\,m\,\cos\left(e+f\,x\right)-2\,m\,\sin\left(2\,e+2\,f\,x\right)\right)}{f\,\left(\sin\left(e+f\,x\right)-1\right)\,\left(4\,m^2+8\,m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(10*cos(e + f*x)
 - sin(2*e + 2*f*x) + 4*m*cos(e + f*x) - 2*m*sin(2*e + 2*f*x)))/(f*(sin(e +
 f*x) - 1)*(8*m + 4*m^2 + 3))
```

# 3.413 $\int (a+a\sin(e+fx))^m \sqrt{c-c\sin(e+fx)}\,dx$

**Optimal**. Leaf size=46

$$\frac{2c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2817}

$$\frac{2c\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.12, size = 85, normalized size = 1.85

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}}{f(1+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]])/(f*(1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

**Maple** [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a + a \sin (fx + e))^m \sqrt{c - c \sin (fx + e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(47) = 94.
time = 0.50, size = 124, normalized size = 2.70

$$-\frac{2 \left(a^m \sqrt{c} + \frac{a^m \sqrt{c} \sin(fx+e)}{\cos(fx+e)+1}\right) e^{\left(2 m \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right) - m \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{f(2 m + 1) \sqrt{\frac{\sin (fx+e)^2}{(\cos (fx+e)+1)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1))*e^(2*m*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

**Fricas** [A]
time = 0.37, size = 82, normalized size = 1.78

$$\frac{2 \sqrt{-c \sin (fx+e) + c}\, (a \sin (fx+e) + a)^m (\cos (fx+e) + \sin (fx+e) + 1)}{2\, fm + (2\, fm + f) \cos (fx+e) - (2\, fm + f) \sin (fx+e) + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) + sin(f*x
+ e) + 1)/(2*f*m + (2*f*m + f)*cos(f*x + e) - (2*f*m + f)*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e + fx) + 1))^m \sqrt{-c (\sin (e + fx) - 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [B]**
time = 0.45, size = 53, normalized size = 1.15

$$-\frac{2\,\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{f\,\left(2\,m+1\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(2*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2))/(f
*(2*m + 1)*(sin(e + f*x) - 1))
```

# 3.414 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=68

$$\frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} \\
&= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.
time = 0.98, size = 157, normalized size = 2.31

$$
\frac{2^{-\frac{3}{2}-2m}\left(4^m\,{}_2F_1\left(1,2m;1+2m;\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)-{}_2F_1\left(2m,2m;1+2m;\frac{1}{2}\left(1-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\frac{1}{8}(2e-\pi+2fx)\right)^{2m}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{fm\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$
\int \frac{(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.415 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=74

$$\frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{2cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*
x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{2cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 15.36, size = 3006, normalized size = 40.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/8*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Ta
n[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)
/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e
 + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m
, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 +
 Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2
*m)))/(Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]
 - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos
[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -
2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[
(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*
m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e
 + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2
- f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*A
ppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - T
an[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e +
```

```
 Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^
2)^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 -
f*x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 -
f*x)/4]^3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(
-1/2*(m*AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (
m*AppellF1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*
AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x
)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 -
Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) +
 m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 -
f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1
- 2*m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]
^2)^(2*m) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
 + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^
(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 -
f*x)/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2
)^(2*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(
-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)
/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
+ Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2)
*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(
2*m) + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e +
Pi/2 - f*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*
m)*Sec[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(1
 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m,
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^
4)^(2*m))/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2 +
2*m, 2*m, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/
2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 + 2*
m) - (m*(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/
2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^
2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2
)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*Appell
F1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-
e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3
*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 + 2
*m))/(1 + 2*m)))/(8*Sqrt[2])))
```

**Maple [F]**

time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m}{(c - c\sin(fx + e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e
)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m}{(-c(\sin(e + fx) - 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin\left(e + f\,x\right))^m}{(c - c\,\sin\left(e + f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(3/2), x)
```

# 3.416 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=74

$$\frac{\cos(e+fx)\,{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{4c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

[Out] 1/4*cos(f*x+e)*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)

**Rubi** [A]
time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]

[Out] (Cos[e + f*x]*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)\int \sec^5(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^3}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{4c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 19.73, size = 5136, normalized size = 69.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

**Maple** [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x +
 e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin (e + fx) + 1))^m}{(-c (\sin (e + fx) - 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/(-c*(sin(e + f*x) - 1))**(5/2), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin \left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin \left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(5/2), x)
```

# 3.417 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=68

$$\frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.
time = 0.27, size = 157, normalized size = 2.31

$$\frac{2^{-\frac{3}{2}-2m}\left(4^m\,{}_2F_1\left(1,2m;1+2m;\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)-{}_2F_1\left(2m,2m;1+2m;\frac{1}{2}\left(1-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\frac{1}{8}(2e-\pi+2fx)\right)^{2m}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{fm\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin (e + fx) + 1))^m}{\sqrt{-c (\sin (e + fx) - 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin (e + f\,x))^m}{\sqrt{c - c\,\sin (e + f\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2), x)
```

# 3.418 $\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=68

$$\frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e)
)^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2824, 2746, 70}

$$\frac{\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\ &= \frac{(c\cos(e+fx))\mathrm{Subst}\left(\int \frac{(c+x)^{-\frac{1}{2}+m}}{c-x}\,dx, x, c\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.
time = 0.24, size = 157, normalized size = 2.31

$$\frac{2^{-\frac{3}{2}-2m}\left(4^m\,{}_2F_1\left(1,2m;1+2m;\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)-{}_2F_1\left(2m,2m;1+2m;\frac{1}{2}\left(1-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\frac{1}{8}(2e-\pi+2fx)\right)^{2m}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^m}{fm\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[a - a*Sin[e + f*x]])
```

**Maple** [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(c+c\sin(fx+e))^m}{\sqrt{a-a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(a*sin(f*x + e)
- a), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin{(e+fx)}+1))^m}{\sqrt{-a\,(\sin{(e+fx)}-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m/sqrt(-a*(sin(e + f*x) - 1)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{a-a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + c*sin(e + f*x))^m/(a - a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + c*sin(e + f*x))^m/(a - a*sin(e + f*x))^(1/2), x)
```

# 3.419 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

**Optimal**. Leaf size=164

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}+\frac{2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+f\ldots}{cf\,(15+16m+4m^2)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+2*cos(f*x+e
)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m^2+16*m+15)+2*cos(f*x+
e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

**Rubi** [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2822, 2821\}$

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\,(4m^2+8m+3)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{cf\,(4m^2+16m+15)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 +
 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m))/(c*f*(15 + 16*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c -
 c*Sin[e + f*x])^(-1 - m))/(c^2*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-3-m}\, dx = \frac{\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-3-}}{f(5 + 2m)}$$
$$= \frac{\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-3-}}{f(5 + 2m)}$$
$$= \frac{\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-3-}}{f(5 + 2m)}$$

**Mathematica** [A]
time = 7.29, size = 174, normalized size = 1.06

$$\frac{2^{-2-m}\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-5-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-3-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\left(4(2+3m+m^2)+\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)-2(3+2m)\sin(e+fx)\right)}{f(1+2m)(3+2m)(5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(4*(2 + 3*m + m^2) + Cos
[2*(-e + Pi/2 - f*x)] - 2*(3 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(
5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

**Maple** [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(c - c\sin\left(fx + e\right)\right)^{-3-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

**Fricas** [A]
time = 0.36, size = 107, normalized size = 0.65

$$-\frac{\left(2\,\cos\left(fx+e\right)^{3}+2\,\left(2\,m+3\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(4\,m^{2}+12\,m+9\right)\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}}{8\,fm^{3}+36\,fm^{2}+46\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(2*m + 3)*cos(f*x + e)*sin(f*x + e) - (4*m^2 + 12*m
+ 9)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8
*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

**Mupad** [B]
time = 8.25, size = 149, normalized size = 0.91

$$-\frac{2\,\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\left(15\,\cos\left(e+f\,x\right)-\cos\left(3\,e+3\,f\,x\right)-6\,\sin\left(2\,e+2\,f\,x\right)+24\,m\,\cos\left(e+f\,x\right)+8\,m^{2}\,\cos\left(e+f\,x\right)-4\,m\,\sin\left(2\,e+2\,f\,x\right)\right)}{c^{3}\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^{m}\,\left(8\,m^{3}+36\,m^{2}+46\,m+15\right)\,\left(15\,\sin\left(e+f\,x\right)+6\,\cos\left(2\,e+2\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 3),x)
```

```
[Out] -(2*(a*(sin(e + f*x) + 1))^m*(15*cos(e + f*x) - cos(3*e + 3*f*x) - 6*sin(2*
e + 2*f*x) + 24*m*cos(e + f*x) + 8*m^2*cos(e + f*x) - 4*m*sin(2*e + 2*f*x))
)/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 36*m^2 + 8*m^3 + 15)*(15*sin(e +
 f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x) - 10))
```

# 3.420 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

**Optimal**. Leaf size=101

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}+\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+f}{cf\,(3+8m+4m^2)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+cos(f*x+e)*
(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2+8*m+3)
```

**Rubi** [A]

time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf\,(4m^2+8m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 +
 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(c*f*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}$$

**Mathematica** [A]
time = 1.76, size = 136, normalized size = 1.35

$$-\frac{2^{-m}\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-3-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-2-m)}(-2(1+m)+\sin(e+fx))(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f\left(6+16m+8m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] -((Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-3 - 2*m)*(-2*(1 + m)
 + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(2^m
*f*(6 + 16*m + 8*m^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

**Maple** [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

**Fricas** [A]
time = 0.36, size = 77, normalized size = 0.76

$$\frac{\left(2\,(m+1)\cos\left(fx+e\right)-\cos\left(fx+e\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}}{4\,fm^{2}+8\,fm+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="fricas")
```

```
[Out] (2*(m + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(a*sin(f*x + e) + a)^m
*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m + 3*f)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

**Mupad** [**B**]
time = 0.88, size = 111, normalized size = 1.10

$$-\frac{\left(a\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+4\,m\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)-4\right)}{c^2\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(4\,m^2+8\,m+3\right)\,\left(2\,\sin\left(e+f\,x\right)^2-4\,\sin\left(e+f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(sin(2*e + 2*f*x) + 8*sin(e/2 + (f*x)/2)^2 + 4*m
*(2*sin(e/2 + (f*x)/2)^2 - 1) - 4))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m +
 4*m^2 + 3)*(2*sin(e + f*x)^2 - 4*sin(e + f*x) + 2))
```

# 3.421 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=46

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 +
 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.
time = 0.88, size = 107, normalized size = 2.33

$$\frac{2^{-m}\cos^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-m}\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{cf(1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*c*f*(1 + 2*
m)*(c - c*Sin[e + f*x])^m)
```

**Maple [F]**
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(c-c\sin \left(fx+e\right)\right)^{-1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

**Fricas [A]**
time = 0.35, size = 47, normalized size = 1.02

$$\frac{(a\sin\left(fx+e\right)+a)^{m}(-c\sin\left(fx+e\right)+c)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

[Out] Timed out

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

**Mupad** [B]
time = 0.41, size = 58, normalized size = 1.26

$$-\frac{\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{c\,f\,\left(2\,m+1\right)\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 1),x)
```

```
[Out] -(cos(e + f*x)*(a*(sin(e + f*x) + 1))^m)/(c*f*(2*m + 1)*(-c*(sin(e + f*x) -
 1))^m*(sin(e + f*x) - 1))
```

# 3.422 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

**Optimal**. Leaf size=112

$$\frac{2^{\frac{1}{2}-m}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+a\sin}{f(1+2m)}$$

```
[Out] 2^(1/2-m)*c*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m
)
```

**Rubi** [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2824, 2768, 72, 71}

$$\frac{c2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 + 2*m)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right.$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\right.}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.86, size = 388, normalized size = 3.46

$$\frac{2^{1-m}(-3+2m)F_1\left(\frac{1}{2}-m;-2m,1;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\cos^{1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-m}\sin^2\left(\frac{1}{8}(2e+3\pi+2fx)\right)}{f(-1+2m)\left((-3+2m)F_1\left(\frac{1}{2}-m;-2m,1;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\cos^2\left(\frac{1}{8}(2e-\pi+2fx)\right)+2\left(2mF_1\left(\frac{3}{2}-m;1-2m,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)+F_1\left(\frac{3}{2}-m;-2m,2;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sin^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1 - m)*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi -
2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e + Pi + 2*f*x)/4]^(1 - 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a*(1 + Sin[e + f*x]))^m*Si
n[(2*e + 3*Pi + 2*f*x)/8]^2)/(f*(-1 + 2*m)*(c - c*Sin[e + f*x])^m*((-3 + 2*
m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(
2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^2 + 2*(2*m*AppellF1[3/2 -
 m, 1 - 2*m, 1, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f
```

```
*x)/8]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^
2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
[Out] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (-c(\sin(e+fx)-1))^{-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/(-c*(sin(e + f*x) - 1))**m, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\sin(e + f x))^m}{(c - c\sin(e + f x))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^m, x)
```

# 3.423 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

**Optimal**. Leaf size=114

$$\frac{2^{\frac{3}{2}-m}c^2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+}{f(1+2m)}$$

```
[Out] 2^(3/2-m)*c^2*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+
2*m)
```

**Rubi** [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2824, 2768, 72, 71}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{m}\right.$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(}\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2\right.}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.93, size = 602, normalized size = 5.28

$$-\frac{2^{2-m}c(-3+2m)\left(F_1\left(\frac{1}{2}-m;-2m,2;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)-F_1\left(\frac{1}{2}-m;-2m,3;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\cos^{3-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2(-1+m)}(-1+\sin(e+fx))(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-m}}{f(-1+2m)\left((-3+2m)F_1\left(\frac{1}{2}-m;-2m,2;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)+(3-2m)F_1\left(\frac{1}{2}-m;-2m,3;\frac{3}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)+2\left(2mF_1\left(\frac{3}{2}-m;1-2m,2;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)-2mF_1\left(\frac{3}{2}-m;1-2m,3;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)+2F_1\left(\frac{3}{2}-m;-2m,3;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)-3F_1\left(\frac{3}{2}-m;-2m,4;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right),-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(2 - m)*c*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e +
Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Cos
[(2*e + Pi + 2*f*x)/4]^(3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(
-1 + m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + 2*m)*(c - c
*Sin[e + f*x])^m*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e
+ Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (3 - 2*m)*AppellF1[1/2
- m, -2*m, 3, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x
```

```
)/8]^2] + 2*(2*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*
f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 2*m*AppellF1[3/2 - m, 1 - 2*m, 3
, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 2*
AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e
 - Pi + 2*f*x)/8]^2] - 3*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Tan[(2*e - Pi + 2*f*x)/8]^2)
))
```

**Maple** [**F**]
time = 0.38, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^m (c - c\sin(fx + e))^{1-m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin\left(e+f\,x\right))^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m),x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m), x)
```

# 3.424 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

**Optimal**. Leaf size=114

$$\frac{2^{\frac{5}{2}-m}c^3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+}{f(1+2m)}$$

```
[Out] 2^(5/2-m)*c^3*cos(f*x+e)*hypergeom([-3/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+
2*m)
```

**Rubi** [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2824, 2768, 72, 71}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right.$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}c^4\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2\right.}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 9.04, size = 1201, normalized size = 10.54

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(4 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 2*AppellF1[1/2 - m, -2*m, 4, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1
/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 -
 f*x)/2]^(4 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(f*
(-1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 - m))*((-3 + 2*m)*Ap
pellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
```

```
/2 - f*x)/4]^2] + (6 - 4*m)*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 3*AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 2*m*A
ppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - 8*AppellF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 8*Appel
lF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 4*m*Appe
llF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - 8*m*AppellF1[3/2 - m, 1 - 2*m
, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[
(-e + Pi/2 - f*x)/4]^2 + 4*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^
2 + 6*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 10*AppellF1[3/2 - m, -
2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*T
an[(-e + Pi/2 - f*x)/4]^2))
```

**Maple** [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin\left(e+f\,x\right))^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m),x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m), x)
```

## 3.425 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=227

$$\frac{1}{8}a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)x-\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\cos(e+fx)}{30f}-\frac{ad(24$$

```
[Out] 1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*x-1/30*a*(12*c^4+95*c^3*d+
112*c^2*d^2+80*c*d^3+16*d^4)*cos(f*x+e)/f-1/120*a*d*(24*c^3+130*c^2*d+116*c
*d^2+45*d^3)*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(12*c^2+35*c*d+16*d^2)*cos(f*x+
e)*(c+d*sin(f*x+e))^2/f-1/20*a*(4*c+5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f-1/
5*a*cos(f*x+e)*(c+d*sin(f*x+e))^4/f
```

**Rubi** [A]
time = 0.20, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{a(12c^2+35cd+16d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sin(e+fx)\cos(e+fx)}{120f}-\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\cos(e+fx)}{30f}+\frac{1}{8}ax\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^4}{5f}-\frac{a(4c+5d)\cos(e+fx)(c+d\sin(e+fx))^3}{20f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*x)/8 - (a*(12*c^4 + 9
5*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Cos[e + f*x])/(30*f) - (a*d*(24*
c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Cos[e + f*x]*Sin[e + f*x])/(120*f) -
(a*(12*c^2 + 35*c*d + 16*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*f) -
 (a*(4*c + 5*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*f) - (a*Cos[e + f*
x]*(c + d*Sin[e + f*x])^4)/(5*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^4}{5f}+\frac{1}{5}\int (c+d\sin(e+fx)$$
$$= -\frac{a(4c+5d)\cos(e+fx)(c+d\sin(e+fx))^3}{20f}-\frac{a\cos(e+fx)}{}$$
$$= -\frac{a(12c^2+35cd+16d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{a}{}$$
$$= \frac{1}{8}a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)x-\frac{a(12c^4+95c^3d}{}$$

**Mathematica [A]**
time = 0.87, size = 207, normalized size = 0.91

$$\frac{a(1+\sin(e+fx))\left(-60(8c^4+32c^3d+36c^2d^2+24cd^3+5d^4)\cos(e+fx)+10d^2(24c^2+16cd+5d^2)\cos(3(e+fx))-6d^4\cos(5(e+fx))+15(4(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)fx-8d(4c^3+6c^2d+4cd^2+d^3)\sin(2(e+fx))+d^3(4c+d)\sin(4(e+fx)))\right)}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(8*c^4 + 32*c^3*d + 36*c^2*d^2 + 24*c*d^3 + 5*d^
4)*Cos[e + f*x] + 10*d^2*(24*c^2 + 16*c*d + 5*d^2)*Cos[3*(e + f*x)] - 6*d^4
*Cos[5*(e + f*x)] + 15*(4*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4
)*f*x - 8*d*(4*c^3 + 6*c^2*d + 4*c*d^2 + d^3)*Sin[2*(e + f*x)] + d^3*(4*c +
 d)*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

**Maple [A]**
time = 0.38, size = 259, normalized size = 1.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^4*cos(f*x+e)+4*a*c^3*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)
-2*a*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+4*a*c*d^3*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a*d^4*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+a*c^4*(f*x+e)-4*a*c^3*d*cos(f*x+e)+6*a*c^2*d^2*(-1/2*co
s(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-4/3*a*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+
a*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

**Maxima [A]**
time = 0.29, size = 270, normalized size = 1.19

$$\frac{480\,(fx+e)ac^4+480\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ac^3d+960\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)ac^2d^2+720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ac^2d^2+640\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)acd^3+60\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))acd^3-32\,\left(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e)\right)ad^4+15\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))ad^4-480\,ac^4\cos(fx+e)-1920\,ac^3d\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a*c^4 + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^3*d +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2*d^2 + 720*(2*f*x + 2*e - sin(2
*f*x + 2*e))*a*c^2*d^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^3 + 60
*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c*d^3 - 32*(3*co
s(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*d^4 + 15*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^4 - 480*a*c^4*cos(f*x + e)
- 1920*a*c^3*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.36, size = 210, normalized size = 0.93

$$-\frac{24\,ad^4\cos\left(fx+e\right)^5-80\left(3\,ac^2d^2+2\,acd^3+ad^4\right)\cos\left(fx+e\right)^3-15\left(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4\right)fx+120\left(ac^4+4\,ac^3d+6\,ac^2d^2+4\,acd^3+ad^4\right)\cos\left(fx+e\right)-15\left(2\left(4\,acd^3+ad^4\right)\cos\left(fx+e\right)^3-\left(16\,ac^3d+24\,ac^2d^2+20\,acd^3+5\,ad^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*d^4*cos(f*x + e)^5 - 80*(3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(
f*x + e)^3 - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4
)*f*x + 120*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x +
 e) - 15*(2*(4*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (16*a*c^3*d + 24*a*c^2*d^2
 + 20*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $\overline{2(218)} = 436$.
time = 0.39, size = 580, normalized size = 2.56

$$\begin{cases} ac^4x - \frac{ac^4\cos(e+fx)}{f} + 2ac^3dx\sin^2(e+fx) + 2ac^3dx\cos^2(e+fx) - \frac{2ac^3d\sin(e+fx)\cos(e+fx)}{f} - \frac{4ac^3d\cos(e+fx)}{f} + 3ac^2d^2x\sin^2(e+fx) + 3ac^2d^2x\cos^2(e+fx) - \frac{6ac^2d^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3ac^2d^2\sin(e+fx)\cos(e+fx)}{f} - \frac{4ac^2d^2\cos^3(e+fx)}{f} + \frac{3acd^3x\sin^4(e+fx)}{2} + 3acd^3x\sin^2(e+fx)\cos^2(e+fx) + \frac{3acd^3x\cos^4(e+fx)}{2} - \frac{5acd^3\sin^3(e+fx)\cos(e+fx)}{2f} - \frac{4acd^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3acd^3\sin(e+fx)\cos^3(e+fx)}{2f} - \frac{8acd^3\cos^3(e+fx)}{3f} + \frac{3ad^4x\sin^4(e+fx)}{8} + \frac{3ad^4x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3ad^4x\cos^4(e+fx)}{8} - \frac{ad^4\sin^4(e+fx)\cos(e+fx)}{f} - \frac{5ad^4\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{4ad^4\sin^2(e+fx)\cos^3(e+fx)}{3f} - \frac{3ad^4\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{8ad^4\cos^5(e+fx)}{15f} & \text{for}\: f \neq 0 \\ x(c+d\sin(e))^4(a\sin(e)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a*c**4*x - a*c**4*cos(e + f*x)/f + 2*a*c**3*d*x*sin(e + f*x)**2
+ 2*a*c**3*d*x*cos(e + f*x)**2 - 2*a*c**3*d*sin(e + f*x)*cos(e + f*x)/f - 4
*a*c**3*d*cos(e + f*x)/f + 3*a*c**2*d**2*x*sin(e + f*x)**2 + 3*a*c**2*d**2*
x*cos(e + f*x)**2 - 6*a*c**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c**2
*d**2*sin(e + f*x)*cos(e + f*x)/f - 4*a*c**2*d**2*cos(e + f*x)**3/f + 3*a*c
*d**3*x*sin(e + f*x)**4/2 + 3*a*c*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2 +
3*a*c*d**3*x*cos(e + f*x)**4/2 - 5*a*c*d**3*sin(e + f*x)**3*cos(e + f*x)/(2
*f) - 4*a*c*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c*d**3*sin(e + f*x)*c
os(e + f*x)**3/(2*f) - 8*a*c*d**3*cos(e + f*x)**3/(3*f) + 3*a*d**4*x*sin(e
+ f*x)**4/8 + 3*a*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a*d**4*x*cos
(e + f*x)**4/8 - a*d**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*a*d**4*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 4*a*d**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f)
- 3*a*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*a*d**4*cos(e + f*x)**5/(1
5*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a), True))
```

**Giac [A]**
time = 0.50, size = 272, normalized size = 1.20

$$-\frac{ad^4\cos(5fx+5e)}{80f}+\frac{acd^3\cos(3fx+3e)}{3f}+\frac{acd^3\sin(4fx+4e)}{8f}+\frac{ad^4\sin(4fx+4e)}{32f}+\frac{1}{8}\left(8ac^4+24ac^2d^2+3ad^4\right)x+\frac{1}{2}\left(4ac^3d+3acd^3\right)x+\frac{(24ac^2d^2+5ad^4)\cos(3fx+3e)}{48f}-\frac{(8ac^4+36ac^2d^2+5ad^4)\cos(fx+e)}{8f}-\frac{(4ac^3d+3acd^3)\cos(fx+e)}{f}-\frac{(ac^3d+acd^3)\sin(2fx+2e)}{f}-\frac{(6ac^2d^2+ad^4)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/80*a*d^4*cos(5*f*x + 5*e)/f + 1/3*a*c*d^3*cos(3*f*x + 3*e)/f + 1/8*a*c*d
^3*sin(4*f*x + 4*e)/f + 1/32*a*d^4*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^4 + 24*a
*c^2*d^2 + 3*a*d^4)*x + 1/2*(4*a*c^3*d + 3*a*c*d^3)*x + 1/48*(24*a*c^2*d^2
+ 5*a*d^4)*cos(3*f*x + 3*e)/f - 1/8*(8*a*c^4 + 36*a*c^2*d^2 + 5*a*d^4)*cos(
f*x + e)/f - (4*a*c^3*d + 3*a*c*d^3)*cos(f*x + e)/f - (a*c^3*d + a*c*d^3)*s
in(2*f*x + 2*e)/f - 1/4*(6*a*c^2*d^2 + a*d^4)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 9.94, size = 559, normalized size = 2.46

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)}{4\left(2ac^4+4ac^3d+6ac^2d^2+3acd^3+\frac{3ad^4}{4}\right)}\right)(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)}{4f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(8ac^4+32ac^3d+40ac^2d^2+\frac{80acd^3}{3}+\frac{16ad^4}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(12ac^4+48ac^3d+56ac^2d^2+\frac{112acd^3}{3}+\frac{32ad^4}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(4ac^3d+6ac^2d^2+3acd^3+\frac{3ad^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(2ac^4+8adc^3)+2ac^4+\frac{16ad^4}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(8ac^4+32ac^3d+24ac^2d^2+16acd^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(4ac^3d+6ac^2d^2+3acd^3+\frac{3ad^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(8ac^3d+12ac^2d^2+14acd^3+\frac{7ad^4}{2}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(8ac^3d+12ac^2d^2+14acd^3+\frac{7ad^4}{2}\right)+8ac^2d^2+\frac{16acd^3}{3}+8ac^3d}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2
*d^2))/(4*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d)))*(
12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)
/2)^2*(8*a*c^4 + (16*a*d^4)/3 + 40*a*c^2*d^2 + (80*a*c*d^3)/3 + 32*a*c^3*d)
 + tan(e/2 + (f*x)/2)^4*(12*a*c^4 + (32*a*d^4)/3 + 56*a*c^2*d^2 + (112*a*c*
d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*
c*d^3 + 4*a*c^3*d) + tan(e/2 + (f*x)/2)^8*(2*a*c^4 + 8*a*c^3*d) + 2*a*c^4 +
 (16*a*d^4)/15 + tan(e/2 + (f*x)/2)^6*(8*a*c^4 + 24*a*c^2*d^2 + 16*a*c*d^3
+ 32*a*c^3*d) - tan(e/2 + (f*x)/2)^9*((3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3
 + 4*a*c^3*d) + tan(e/2 + (f*x)/2)^3*((7*a*d^4)/2 + 12*a*c^2*d^2 + 14*a*c*d
^3 + 8*a*c^3*d) - tan(e/2 + (f*x)/2)^7*((7*a*d^4)/2 + 12*a*c^2*d^2 + 14*a*c
*d^3 + 8*a*c^3*d) + 8*a*c^2*d^2 + (16*a*c*d^3)/3 + 8*a*c^3*d)/(f*(5*tan(e/2
 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e
/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

# 3.426 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=162

$$\frac{1}{8}a\left(8c^3+12c^2d+12cd^2+3d^3\right)x-\frac{a(3c^3+16c^2d+12cd^2+4d^3)\cos(e+fx)}{6f}-\frac{ad(6c^2+20cd+9d^2)\cos(e-}{24f}$$

```
[Out] 1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*x-1/6*a*(3*c^3+16*c^2*d+12*c*d^2+4*d^
3)*cos(f*x+e)/f-1/24*a*d*(6*c^2+20*c*d+9*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/12*
a*(3*c+4*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f-1/4*a*cos(f*x+e)*(c+d*sin(f*x+e
))^3/f
```

**Rubi** [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2832, 2813}

$$-\frac{ad(6c^2+20cd+9d^2)\sin(e+fx)\cos(e+fx)}{24f}-\frac{a(3c^3+16c^2d+12cd^2+4d^3)\cos(e+fx)}{6f}+\frac{1}{8}ax\left(8c^3+12c^2d+12cd^2+3d^3\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^3}{4f}-\frac{a(3c+4d)\cos(e+fx)(c+d\sin(e+fx))^2}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*x)/8 - (a*(3*c^3 + 16*c^2*d + 12*c
*d^2 + 4*d^3)*Cos[e + f*x])/(6*f) - (a*d*(6*c^2 + 20*c*d + 9*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(24*f) - (a*(3*c + 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*f) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a + a\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^3}{4f} + \frac{1}{4}\int (c+d\sin(e+fx))$$
$$= -\frac{a(3c+4d)\cos(e+fx)(c+d\sin(e+fx))^2}{12f} - \frac{a\cos(e+fx)}{}$$
$$= \frac{1}{8}a\left(8c^3+12c^2d+12cd^2+3d^3\right)x - \frac{a(3c^3+16c^2d+12cd^2+}{6f}$$

**Mathematica** [A]
time = 0.53, size = 124, normalized size = 0.77

$$\frac{a\left(-24\left(4c^3+12c^2d+9cd^2+3d^3\right)\cos(e+fx)+8d^2(3c+d)\cos(3(e+fx))+3\left(4\left(8c^3+12c^2d+12cd^2+3d^3\right)fx-8d\left(3c^2+3cd+d^2\right)\sin(2(e+fx))+d^3\sin(4(e+fx))\right)\right)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(-24*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3)*Cos[e + f*x] + 8*d^2*(3*c + d)
*Cos[3*(e + f*x)] + 3*(4*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*f*x - 8*d*(3
*c^2 + 3*c*d + d^2)*Sin[2*(e + f*x)] + d^3*Sin[4*(e + f*x)])))/(96*f)
```

**Maple** [A]
time = 0.33, size = 182, normalized size = 1.12

| method | result |
|---|---|
| derivativedivides | $\frac{-a\,c^3\cos(fx+e)+3a\,c^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ac\,d^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)+a\,d^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(f}{2}\right.}{4}\right.}{}$ |
| default | $\frac{-a\,c^3\cos(fx+e)+3a\,c^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ac\,d^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)+a\,d^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(f}{2}\right.}{4}\right.}{}$ |
| risch | $a\,c^3x+\frac{3a\,c^2dx}{2}+\frac{3ac\,d^2x}{2}+\frac{3a\,d^3x}{8}-\frac{a\cos(fx+e)c^3}{f}-\frac{3a\cos(fx+e)c^2d}{f}-\frac{9a\cos(fx+e)c\,d^2}{4f}-\frac{3a\cos(fx+}{4f}$ |
| norman | $\frac{\left(a\,c^3+\frac{3}{2}a\,c^2d+\frac{3}{2}ac\,d^2+\frac{3}{8}a\,d^3\right)x+\left(a\,c^3+\frac{3}{2}a\,c^2d+\frac{3}{2}ac\,d^2+\frac{3}{8}a\,d^3\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(4a\,c^3+6a\,c^2d+6ac\,d^2+\frac{3}{2}a\,d^3\right)x\left(\tan\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^3*cos(f*x+e)+3*a*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)
-a*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)+a*c^3*(f*x+e)-3*a*c^2*d*cos(f*x+e)+3*a*c*d^2*
```

```
(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*a*d^3*(2+sin(f*x+e)^2)*cos(f
*x+e))
```

**Maxima [A]**
time = 0.28, size = 189, normalized size = 1.17

$$\frac{96\,(fx+e)ac^3+72\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ac^2d+96\,(\cos(fx+e)^3-3\,\cos(fx+e))acd^2+72\,(2\,fx+2\,e-\sin(2\,fx+2\,e))acd^2+32\,(\cos(fx+e)^3-3\,\cos(fx+e))ad^3+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))ad^3-96\,ac^3\cos(fx+e)-288\,ac^2d\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^2*d + 96
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^2 + 72*(2*f*x + 2*e - sin(2*f*x +
2*e))*a*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^3 - 96*a*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.36, size = 150, normalized size = 0.93

$$\frac{8\,(3\,acd^2+ad^3)\cos(fx+e)^3+3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)fx-24\,(ac^3+3\,ac^2d+3\,acd^2+ad^3)\cos(fx+e)+3\,\left(2\,ad^3\cos(fx+e)^3-(12\,ac^2d+12\,acd^2+5\,ad^3)\cos(fx+e)\right)\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*a*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*a*c^2*d + 12*a
*c*d^2 + 3*a*d^3)*f*x - 24*(a*c^3 + 3*a*c^2*d + 3*a*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*a*d^3*cos(f*x + e)^3 - (12*a*c^2*d + 12*a*c*d^2 + 5*a*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(153) = 306$.
time = 0.27, size = 386, normalized size = 2.38

$$\begin{cases} ac^3x-\frac{ac^3\cos(e+fx)}{f}+\frac{3ac^2dx\sin^2(e+fx)}{2}+\frac{3ac^2dx\cos^2(e+fx)}{2}-\frac{3ac^2d\sin(e+fx)\cos(e+fx)}{2f}-\frac{3ac^2d\cos(e+fx)}{f}+\frac{3acd^2x\sin^2(e+fx)}{2}+\frac{3acd^2x\cos^2(e+fx)}{2}-\frac{3acd^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3acd^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2acd^2\cos^3(e+fx)}{f}+\frac{3ad^3x\sin^4(e+fx)}{8}+\frac{3ad^3x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{3ad^3x\cos^4(e+fx)}{8}-\frac{5ad^3\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{ad^3\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3ad^3\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{2ad^3\cos^3(e+fx)}{3f} & \text{for}\ f\neq 0\\ x(c+d\sin(e))^3\,(a\sin(e)+a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x*sin(e + f*x)**2/
2 + 3*a*c**2*d*x*cos(e + f*x)**2/2 - 3*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(
2*f) - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)**2/2 + 3*a*c*d
**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c
*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a*c*d**2*cos(e + f*x)**3/f + 3*a*
d**3*x*sin(e + f*x)**4/8 + 3*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3
*a*d**3*x*cos(e + f*x)**4/8 - 5*a*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
```

```
 a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*d**3*sin(e + f*x)*cos(e + f*x)
**3/(8*f) - 2*a*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3
*(a*sin(e) + a), True))
```

**Giac** [A]
time = 0.49, size = 191, normalized size = 1.18

$$\frac{acd^2\cos(3fx+3e)}{4f}+\frac{ad^3\cos(3fx+3e)}{12f}+\frac{ad^3\sin(4fx+4e)}{32f}-\frac{3acd^2\sin(2fx+2e)}{4f}+\frac{1}{2}(2ac^3+3acd^2)x+\frac{3}{8}(4ac^2d+ad^3)x-\frac{(4ac^3+9acd^2)\cos(fx+e)}{4f}-\frac{3(4ac^2d+ad^3)\cos(fx+e)}{4f}-\frac{(3ac^2d+ad^3)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*c*d^2*cos(3*f*x + 3*e)/f + 1/12*a*d^3*cos(3*f*x + 3*e)/f + 1/32*a*d^3
*sin(4*f*x + 4*e)/f - 3/4*a*c*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^3 + 3*a*c
*d^2)*x + 3/8*(4*a*c^2*d + a*d^3)*x - 1/4*(4*a*c^3 + 9*a*c*d^2)*cos(f*x + e
)/f - 3/4*(4*a*c^2*d + a*d^3)*cos(f*x + e)/f - 1/4*(3*a*c^2*d + a*d^3)*sin(
2*f*x + 2*e)/f
```

**Mupad** [B]
time = 8.17, size = 460, normalized size = 2.84

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8c^3+12c^2d+12cd^2+3d^3)}{4\left(2ac^3+3ac^2d+3acd^2+\frac{3ad^3}{4}\right)}\right)(8c^3+12c^2d+12cd^2+3d^3)}{4f}-\frac{a\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(8c^3+12c^2d+12cd^2+3d^3)}{4f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3ac^2d+3acd^2+\frac{11ad^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3ac^2d+3acd^2+\frac{3ad^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(3ac^2d+3acd^2+\frac{11ad^3}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(2ac^3+6adc^2)+2ac^3+\frac{4ad^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6ac^3+18ac^2d+12acd^2+4ad^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(6ac^3+18ac^2d+16acd^2+\frac{16ad^3}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3ac^2d+3acd^2+\frac{3ad^3}{4}\right)+4acd^2+6ac^2d}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(4*(2*
a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d)))*(12*c*d^2 + 12*c^2*d + 8*c^3
 + 3*d^3))/(4*f) - (a*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*c*d^2 + 12*c
^2*d + 8*c^3 + 3*d^3))/(4*f) - (tan(e/2 + (f*x)/2)^3*((11*a*d^3)/4 + 3*a*c*
d^2 + 3*a*c^2*d) - tan(e/2 + (f*x)/2)^7*((3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*
d) - tan(e/2 + (f*x)/2)^5*((11*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) + tan(e/2
+ (f*x)/2)^6*(2*a*c^3 + 6*a*c^2*d) + 2*a*c^3 + (4*a*d^3)/3 + tan(e/2 + (f*x
)/2)^4*(6*a*c^3 + 4*a*d^3 + 12*a*c*d^2 + 18*a*c^2*d) + tan(e/2 + (f*x)/2)^2
*(6*a*c^3 + (16*a*d^3)/3 + 16*a*c*d^2 + 18*a*c^2*d) + tan(e/2 + (f*x)/2)*((
3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) + 4*a*c*d^2 + 6*a*c^2*d)/(f*(4*tan(e/2
+ (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 +
(f*x)/2)^8 + 1))
```

# 3.427 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=99

$$\frac{1}{2}a\left(2c^2+2cd+d^2\right)x-\frac{2a(c^2+3cd+d^2)\cos(e+fx)}{3f}-\frac{ad(2c+3d)\cos(e+fx)\sin(e+fx)}{6f}-\frac{a\cos(e+fx)}{}$$

```
[Out] 1/2*a*(2*c^2+2*c*d+d^2)*x-2/3*a*(c^2+3*c*d+d^2)*cos(f*x+e)/f-1/6*a*d*(2*c+3
*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^2/f
```

**Rubi** [A]

time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{2a(c^2+3cd+d^2)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2c^2+2cd+d^2\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}-\frac{ad(2c+3d)\sin(e+fx)\cos(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*x)/2 - (2*a*(c^2 + 3*c*d + d^2)*Cos[e + f*x])/(3*f
) - (a*d*(2*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (a*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^2)/(3*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= -\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}+\frac{1}{3}\int (c+d\sin(e+fx\\ &= \frac{1}{2}a\left(2c^2+2cd+d^2\right)x-\frac{2a(c^2+3cd+d^2)\cos(e+fx)}{3f}-\frac{ad}{}\end{aligned}$$

**Mathematica [A]**
time = 0.26, size = 89, normalized size = 0.90

$$\frac{a(12c^2fx+12cdfx+6d^2fx-3(4c^2+8cd+3d^2)\cos(e+fx)+d^2\cos(3(e+fx))-6cd\sin(2(e+fx))-3d^2\sin(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(12*c^2*f*x + 12*c*d*f*x + 6*d^2*f*x - 3*(4*c^2 + 8*c*d + 3*d^2)*Cos[e +
 f*x] + d^2*Cos[3*(e + f*x)] - 6*c*d*Sin[2*(e + f*x)] - 3*d^2*Sin[2*(e + f*
x)]))/(12*f)
```

**Maple [A]**
time = 0.24, size = 115, normalized size = 1.16

| method | result |
|---|---|
| derivativedivides | $\frac{-a\,c^2\cos(fx+e)+2acd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a\,c^2(fx+e)-2acd\cos(fx+e)+d^2}{f}$ |
| default | $\frac{-a\,c^2\cos(fx+e)+2acd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a\,c^2(fx+e)-2acd\cos(fx+e)+d^2}{f}$ |
| risch | $a\,c^2x+acdx+\frac{a\,d^2x}{2}-\frac{a\cos(fx+e)c^2}{f}-\frac{2a\cos(fx+e)cd}{f}-\frac{3a\cos(fx+e)d^2}{4f}+\frac{a\,d^2\cos(3fx+3e)}{12f}-\frac{\sin(2fx}{}$ |
| norman | $\frac{(a\,c^2+acd+\frac{1}{2}d^2a)x+(a\,c^2+acd+\frac{1}{2}d^2a)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a\,c^2+3acd+\frac{3}{2}d^2a)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a\,c^2+3acd+\frac{3}{2}d^2a)}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^2*cos(f*x+e)+2*a*c*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1
/3*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+a*c^2*(f*x+e)-2*a*c*d*cos(f*x+e)+d^2*a
*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

**Maxima [A]**
time = 0.28, size = 121, normalized size = 1.22

$$\frac{12\,(fx+e)ac^2+6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))acd+4\,(\cos(fx+e)^3-3\,\cos(fx+e))ad^2+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ad^2-12\,ac^2\cos(fx+e)-24\,acd\cos(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a*d^2 + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
*d^2 - 12*a*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.35, size = 94, normalized size = 0.95

$$\frac{2\,ad^2\cos\left(fx+e\right)^3+3\left(2\,ac^2+2\,acd+ad^2\right)fx-3\left(2\,acd+ad^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(ac^2+2\,acd+ad^2\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*a*c*d + a*d^2)*f*x - 3*(2*a*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(a*c^2 + 2*a*c*d + a*d^2)*cos(f*x
+ e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(94) = 188.
time = 0.15, size = 199, normalized size = 2.01

$$\begin{cases} ac^2x-\frac{ac^2\cos\left(e+fx\right)}{f}+acdx\sin^2\left(e+fx\right)+acdx\cos^2\left(e+fx\right)-\frac{acd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2acd\cos\left(e+fx\right)}{f}+\frac{ad^2x\sin^2\left(e+fx\right)}{2}+\frac{ad^2x\cos^2\left(e+fx\right)}{2}-\frac{ad^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{ad^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ad^2\cos^3\left(e+fx\right)}{3f} & \text{for } f\neq 0\\ x(c+d\sin\left(e\right))^2\left(a\sin\left(e\right)+a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x*sin(e + f*x)**2 + a*c
*d*x*cos(e + f*x)**2 - a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*c*d*cos(e +
f*x)/f + a*d**2*x*sin(e + f*x)**2/2 + a*d**2*x*cos(e + f*x)**2/2 - a*d**2*s
in(e + f*x)**2*cos(e + f*x)/f - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*
a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a
), True))
```

**Giac [A]**
time = 0.59, size = 117, normalized size = 1.18

$$acdx+\frac{ad^2\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{2\,acd\cos\left(fx+e\right)}{f}-\frac{acd\sin\left(2\,fx+2\,e\right)}{2\,f}-\frac{ad^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,ac^2+ad^2\right)x-\frac{\left(4\,ac^2+3\,ad^2\right)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*c*d*x + 1/12*a*d^2*cos(3*f*x + 3*e)/f - 2*a*c*d*cos(f*x + e)/f - 1/2*a*c*
d*sin(2*f*x + 2*e)/f - 1/4*a*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^2 + a*d^2)
*x - 1/4*(4*a*c^2 + 3*a*d^2)*cos(f*x + e)/f
```

**Mupad [B]**
time = 6.99, size = 108, normalized size = 1.09

$$-\frac{\frac{3\,a\,d^2\,\sin(2\,e+2\,f\,x)}{2}-\frac{a\,d^2\,\cos(3\,e+3\,f\,x)}{2}+6\,a\,c^2\,\cos\left(e+f\,x\right)+\frac{9\,a\,d^2\,\cos(e+f\,x)}{2}+3\,a\,c\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a\,c^2\,f\,x-3\,a\,d^2\,f\,x+12\,a\,c\,d\,\cos\left(e+f\,x\right)-6\,a\,c\,d\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)

[Out] -((3*a*d^2*sin(2*e + 2*f*x))/2 - (a*d^2*cos(3*e + 3*f*x))/2 + 6*a*c^2*cos(e
 + f*x) + (9*a*d^2*cos(e + f*x))/2 + 3*a*c*d*sin(2*e + 2*f*x) - 6*a*c^2*f*x
 - 3*a*d^2*f*x + 12*a*c*d*cos(e + f*x) - 6*a*c*d*f*x)/(6*f)
```

# 3.428 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=48

$$\frac{1}{2}a(2c+d)x-\frac{a(c+d)\cos(e+fx)}{f}-\frac{ad\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*c+d)*x-a*(c+d)*cos(f*x+e)/f-1/2*a*d*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2813}

$$-\frac{a(c+d)\cos(e+fx)}{f}+\frac{1}{2}ax(2c+d)-\frac{ad\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*x)/2 - (a*(c + d)*Cos[e + f*x])/f - (a*d*Cos[e + f*x]*Sin[e + 
f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}a(2c+d)x-\frac{a(c+d)\cos(e+fx)}{f}-\frac{ad\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.08, size = 45, normalized size = 0.94

$$\frac{a(2de+4cfx+2dfx-4(c+d)\cos(e+fx)-d\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*d*e + 4*c*f*x + 2*d*f*x - 4*(c + d)*Cos[e + f*x] - d*Sin[2*(e + f*x)]
))/(4*f)
```

**Maple [A]**
time = 0.18, size = 59, normalized size = 1.23

| method | result |
|---|---|
| risch | $acx+\frac{axd}{2}-\frac{a\cos(fx+e)c}{f}-\frac{a\cos(fx+e)d}{f}-\frac{da\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{da\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ca\cos(fx+e)-da\cos(fx+e)+ca(fx+e)}{f}$ |
| default | $\frac{da\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ca\cos(fx+e)-da\cos(fx+e)+ca(fx+e)}{f}$ |
| norman | $\frac{(ca+\frac{1}{2}da)x+(ca+\frac{1}{2}da)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2ca+da)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(2ca+2da)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{da\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\cdots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-c*a*cos(f*x+e)-d*a*cos(
f*x+e)+c*a*(f*x+e))
```

**Maxima [A]**
time = 0.29, size = 62, normalized size = 1.29

$$\frac{4\,(fx+e)ac+(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))ad-4\,ac\cos\left(fx+e\right)-4\,ad\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*a*d - 4*a*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.35, size = 51, normalized size = 1.06

$$-\frac{ad\cos\left(fx+e\right)\sin\left(fx+e\right)-(2\,ac+ad)fx+2\,(ac+ad)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + a*d)*f*x + 2*(a*c + a*d)*cos
(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.09, size = 94, normalized size = 1.96

$$\begin{cases} acx - \frac{ac\cos(e+fx)}{f} + \frac{adx\sin^2(e+fx)}{2} + \frac{adx\cos^2(e+fx)}{2} - \frac{ad\sin(e+fx)\cos(e+fx)}{2f} - \frac{ad\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(c+d\sin(e))(a\sin(e)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x*sin(e + f*x)**2/2 + a*d*x*cos
(e + f*x)**2/2 - a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - a*d*cos(e + f*x)/f,
Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a), True))
```

**Giac** [A]
time = 0.56, size = 55, normalized size = 1.15

$$acx + \frac{1}{2}\,adx - \frac{ac\cos(fx+e)}{f} - \frac{ad\cos(fx+e)}{f} - \frac{ad\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*a*d*x - a*c*cos(f*x + e)/f - a*d*cos(f*x + e)/f - 1/4*a*d*sin(2
*f*x + 2*e)/f
```

**Mupad** [B]
time = 6.99, size = 100, normalized size = 2.08

$$a\,c\,x + \frac{a\,d\,x}{2} - \frac{-a\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 + (2\,a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + a\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + 2\,a\,c + 2\,a\,d}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] a*c*x + (a*d*x)/2 - (2*a*c + 2*a*d + tan(e/2 + (f*x)/2)^2*(2*a*c + 2*a*d) -
 a*d*tan(e/2 + (f*x)/2)^3 + a*d*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2
)^2 + tan(e/2 + (f*x)/2)^4 + 1))
```

# 3.429 $\int (a + a\sin(e + fx))\,dx$

**Optimal**. Leaf size=16

$$ax - \frac{a\cos(e + fx)}{f}$$

```
[Out] a*x-a*cos(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2718}

$$ax - \frac{a\cos(e + fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sin[e + f*x],x]
[Out] a*x - (a*Cos[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\sin(e + fx))\,dx &= ax + a\int \sin(e + fx)\,dx \\ &= ax - \frac{a\cos(e + fx)}{f}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 27, normalized size = 1.69

$$ax - \frac{a\cos(e)\cos(fx)}{f} + \frac{a\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sin[e + f*x],x]
[Out] a*x - (a*Cos[e]*Cos[f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

**Maple** [A]
time = 0.05, size = 17, normalized size = 1.06

| method | result | size |
|---|---|---|
| default | $ax-\frac{a\cos(fx+e)}{f}$ | 17 |
| risch | $ax-\frac{a\cos(fx+e)}{f}$ | 17 |
| derivativedivides | $\frac{a(fx+e)-\cos(fx+e)a}{f}$ | 22 |
| norman | $\frac{ax+ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{2a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ | 52 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sin(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-a*cos(f*x+e)/f
```

**Maxima** [A]
time = 0.29, size = 17, normalized size = 1.06

$$ax-\frac{a\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] a*x - a*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.35, size = 19, normalized size = 1.19

$$\frac{afx-a\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (a*f*x - a*cos(f*x + e))/f
```

**Sympy** [A]
time = 0.04, size = 19, normalized size = 1.19

$$ax+a\left(\begin{cases}-\frac{\cos(e+fx)}{f} & \text{for } f\neq 0\\ x\sin(e) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x)
```

```
[Out] a*x + a*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

**Giac** [A]
time = 0.46, size = 17, normalized size = 1.06

$$ax - \frac{a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - a*cos(f*x + e)/f
```

**Mupad** [B]
time = 6.72, size = 25, normalized size = 1.56

$$a\,x-\frac{2\,a}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*sin(e + f*x),x)
```

```
[Out] a*x - (2*a)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

# 3.430 $\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=63

$$\frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}$$

```
[Out] a*x/d-2*a*(c-d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d/f/(c^2-d
^2)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2814, 2739, 632, 210}

$$\frac{ax}{d}-\frac{2a(c-d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*x)/d - (2*a*(c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d
*Sqrt[c^2 - d^2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx &= \frac{ax}{d}-\frac{(a(c-d))\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}\\ &= \frac{ax}{d}-\frac{(2a(c-d))\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\ &= \frac{ax}{d}+\frac{(4a(c-d))\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\ &= \frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.21, size = 182, normalized size = 2.89

$$\frac{a\left(-2(c-d)\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))+\sqrt{c^2-d^2}\,fx\sqrt{(\cos(e)-i\sin(e))^2}\right)(1+\sin(e+fx))}{d\sqrt{c^2-d^2}\,f\sqrt{(\cos(e)-i\sin(e))^2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(-2*(c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2]
 + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e]
- I*Sin[e]) + Sqrt[c^2 - d^2]*f*x*Sqrt[(Cos[e] - I*Sin[e])^2])*(1 + Sin[e +
 f*x]))/(d*Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^2)
```

**Maple [A]**
time = 0.26, size = 72, normalized size = 1.14

| method | result |
|---|---|
| derivativedivides | $\frac{2a\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d}+\frac{(-c+d)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}}\right)}{f}$ |

| | |
|---|---|
| default | $\frac{2a\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d}+\frac{(-c+d)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}}\right)}{f}$ |
| risch | $\frac{ax}{d}+\frac{\sqrt{-(c+d)(c-d)}\,a\ln\left(e^{i(fx+e)}-\frac{-ic+\sqrt{-(c+d)(c-d)}}{d}\right)}{(c+d)fd}-\frac{\sqrt{-(c+d)(c-d)}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a*(1/d*arctan(tan(1/2*f*x+1/2*e))+(-c+d)/d/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.37, size = 237, normalized size = 3.76

$$\left[\frac{2\,afx+a\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2+2\left((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)}{2\,df},\ \frac{afx+a\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)}{df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*f*x + a*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2
- 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e)
 + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 -
2*c*d*sin(f*x + e) - c^2 - d^2)))/(d*f), (a*f*x + a*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))))/
(d*f)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(49) = 98$.
time = 40.18, size = 537, normalized size = 8.52

$$\begin{cases} \frac{\tilde{\infty} x(a\sin(e)+a)}{\sin(e)} & \text{for } c=0 \wedge d=0 \wedge f=0 \\ \frac{x(a\sin(e)+a)}{c+d\sin(e)} & \text{for } f=0 \\ \frac{ax-\frac{a\cos(e+fx)}{f}}{c} & \text{for } d=0 \\ \frac{ax+\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}}{d} & \text{for } c=0 \\ \frac{ad^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{2ad^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}-\frac{adfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}} & \text{for } c=-\sqrt{d^2} \\ \frac{ad^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{2ad^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{adfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}-\frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}} & \text{for } c=\sqrt{d^2} \\ -\frac{ac\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{df\sqrt{-c^2+d^2}}+\frac{ac\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{df\sqrt{-c^2+d^2}}+\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}-\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}+\frac{ax}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (x
*(a*sin(e) + a)/(c + d*sin(e)), Eq(f, 0)), ((a*x - a*cos(e + f*x)/f)/c, Eq(
d, 0)), ((a*x + a*log(tan(e/2 + f*x/2))/f)/d, Eq(c, 0)), (a*d**2*f*x*tan(e/
2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + 2*a*d**2/(d**3*f*t
an(e/2 + f*x/2) - f*(d**2)**(3/2)) - a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f
*x/2) - f*(d**2)**(3/2)) + 2*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) - f*(d
**2)**(3/2)), Eq(c, -sqrt(d**2))), (a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan
(e/2 + f*x/2) + f*(d**2)**(3/2)) + 2*a*d**2/(d**3*f*tan(e/2 + f*x/2) + f*(d
**2)**(3/2)) + a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2
)) - 2*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)), Eq(c, sq
rt(d**2))), (-a*c*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d*f*s
qrt(-c**2 + d**2)) + a*c*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)
/(d*f*sqrt(-c**2 + d**2)) + a*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**
2)/c)/(f*sqrt(-c**2 + d**2)) - a*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 +
d**2)/c)/(f*sqrt(-c**2 + d**2)) + a*x/d, True))
```

**Giac** [A]
time = 0.48, size = 86, normalized size = 1.37

$$\frac{\frac{(fx+e)a}{d}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-ad)}{\sqrt{c^2-d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(a*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

## Mupad [B]

time = 7.39, size = 449, normalized size = 7.13

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)}-\frac{2\,a\,\mathrm{atanh}\left(\frac{3\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(d^2-c^2\right)}^{3/2}-2\,c^4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}-2\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(d^2-c^2\right)}^{3/2}+d^4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}+2\,c^2\,d^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}+3\,c^2\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}+c\,d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(d^2-c^2\right)}^{3/2}+c\,d^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}+c^3\,d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}+4\,c\,d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}-2\,c^3\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{2\,\left(d^2+c\,d\right)\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3+2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^2-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^3\right)}\right)\,\sqrt{d^2-c^2}}{d\,f\,\left(c+d\right)}+\frac{2\,a\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f\,\left(c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (2*a*atanh(
(3*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2) - 2*c^4*sin(e/2 + (f*x)/2)*(d^2
 - c^2)^(1/2) - 2*c^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2) + d^4*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2) + 2*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)
 + 3*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + c*d*cos(e/2 + (f*x)/2)*
(d^2 - c^2)^(3/2) + c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + c^3*d*cos(
e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + 4*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(
1/2) - 2*c^3*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/(2*(c*d + d^2)*(c^3*co
s(e/2 + (f*x)/2) - 2*d^3*sin(e/2 + (f*x)/2) - c*d^2*cos(e/2 + (f*x)/2) + 2*
c^2*d*sin(e/2 + (f*x)/2))))*(d^2 - c^2)^(1/2))/(d*f*(c + d)) + (2*a*c*atan(
sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c + d))
```

# 3.431 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}\,f}-\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}$$

```
[Out] -a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))+2*a*arctan((d+c*tan(1/2*f*x+1/2*e))/
(c^2-d^2)^(1/2))/(c+d)/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\sqrt{c^2-d^2}}-\frac{a\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 -
d^2]*f) - (a*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{\int \frac{a(c-d)}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{a\int \frac{1}{c+d\sin(e+fx)}\,dx}{c+d}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)f}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}\right.\right.}{(c+d)f}\\
&= \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}\,f}-\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}
\end{aligned}
$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.40, size = 220, normalized size = 2.65

$$
\frac{a(1+\sin(e+fx))\left(2\sqrt{c^2-d^2}\,\csc(e)\sqrt{(\cos(e)-i\sin(e))^2}\,(c\cos(e)+d\sin(fx))+4d\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))(c+d\sin(e+fx))\right)}{2d(c+d)\sqrt{c^2-d^2}\,f\sqrt{(\cos(e)-i\sin(e))^2}\,\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2(c+d\sin(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(2*Sqrt[c^2 - d^2]*Csc[e]*Sqrt[(Cos[e] - I*Sin[e])^2]
*(c*Cos[e] + d*Sin[f*x]) + 4*d*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*
Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2])]*(Cos[e] - I*Sin[e])*(c + d*Sin[e + f*x])))/(2*d*(c + d)*Sqrt[c^2 -
```

```
d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*
(c + d*Sin[e + f*x]))
```

**Maple [A]**
time = 0.36, size = 113, normalized size = 1.36

| method | result |
|---|---|
| derivativedivides | $\frac{2a\left(\frac{-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{1}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{f}$ |
| default | $\frac{2a\left(\frac{-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{1}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{f}$ |
| risch | $-\frac{2a\left(id+c\,e^{i(fx+e)}\right)}{d(c+d)f\left(d\,e^{2i(fx+e)}-d+2ic\,e^{i(fx+e)}\right)}-\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)}{\sqrt{-c^2+d^2}\,(c+d)f}+\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2}}{\sqrt{-c}}\right.}{\sqrt{-c^2+d^2}\,(}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a*((-d/(c+d)/c*tan(1/2*f*x+1/2*e)-1/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*
tan(1/2*f*x+1/2*e)+c)+1/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.36, size = 377, normalized size = 4.54

$$\left[-\frac{(ad\sin(fx+e)+ac)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2(ac^2-ad^2)\cos(fx+e)}{2\left((c^3d+c^2d^2-cd^3-d^4)f\sin(fx+e)+(c^4+c^3d-c^2d^2-cd^3)f\right)}, -\frac{(ad\sin(fx+e)+ac)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+(ac^2-ad^2)\cos(fx+e)}{(c^3d+c^2d^2-cd^3-d^4)f\sin(fx+e)+(c^4+c^3d-c^2d^2-cd^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*d*sin(f*x + e) + a*c)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
 + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
 d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2)) + 2*(a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3
- d^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f), -((a*d*sin(f*x
+ e) + a*c)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*c
os(f*x + e))) + (a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3 - d
^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.46, size = 129, normalized size = 1.55

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a}{\sqrt{c^2-d^2}\,(c+d)}-\frac{ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ac}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)(c^2+cd)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a/(sqrt(c^2 - d^2)*(c + d)) - (a*d*tan(1/2*f*x + 1
/2*e) + a*c)/((c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)*(c^
2 + c*d)))/f
```

**Mupad** [**B**]
time = 6.95, size = 140, normalized size = 1.69

$$\frac{2\,a\,\mathrm{atan}\left(\frac{(c+d)\left(\frac{2\,a\,\left(d^2+c\,d\right)}{(c+d)^{5/2}\,\sqrt{c-d}}+\frac{2\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{(c+d)^{3/2}\,\sqrt{c-d}}\right)}{2\,a}\right)}{f\,(c+d)^{3/2}\,\sqrt{c-d}}-\frac{\frac{2\,a}{c+d}+\frac{2\,a\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{c\,(c+d)}}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] (2*a*atan(((c + d)*((2*a*(c*d + d^2))/((c + d)^(5/2)*(c - d)^(1/2)) + (2*a*
c*tan(e/2 + (f*x)/2))/((c + d)^(3/2)*(c - d)^(1/2))))/(2*a)))/(f*(c + d)^(3
/2)*(c - d)^(1/2)) - ((2*a)/(c + d) + (2*a*d*tan(e/2 + (f*x)/2))/(c*(c + d)
))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2))
```

# 3.432 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=134

$$\frac{a(2c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{3/2}f}-\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] a*(2*c-d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)/(c^2-d^2)^
(3/2)/f-1/2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a*(c-2*d)*cos(f*x+e
)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]

time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{a(2c-d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a(c-2d)\cos(e+fx)}{2f(c-d)(c+d)^2(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*(c^
2 - d^2)^(3/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*f*(c + d*Sin[e + f*x])^2) -
 (a*(c - 2*d)*Cos[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx &= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{\int \frac{-2a(c-d)-a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}\\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))}+\ldots\\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))}+\ldots\\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))}+\ldots\\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))}-\ldots\\
&= \frac{a(2c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}\,f}-\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2}-\ldots
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.79, size = 242, normalized size = 1.81

$$\frac{a(1+\sin(e+fx))\left(\frac{4(2c-d)\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{(c-d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{2(c+d)\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d\sin(e+fx))^2}+\frac{(-4c+2d)\cot(e)+2(c-2d)\csc(e)\sin(fx)}{(c-d)(c+d\sin(e+fx))}\right)}{4(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e]
)*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*
Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c - d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (2*(c + d)*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d*Sin[e +
 f*x])^2) + ((-4*c + 2*d)*Cot[e] + 2*(c - 2*d)*Csc[e]*Sin[f*x])/((c - d)*(c
 + d*Sin[e + f*x]))))/(4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(125) = 250$.

time = 0.54, size = 325, normalized size = 2.43

| method | result |
|---|---|
| derivativedivides | $\frac{2a\left(\frac{-\frac{d\left(3c^2-2cd-2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^3+c^2d-c\,d^2-d^3\right)c}-\frac{\left(2c^4-2c^3d+3c^2d^2-4d^3c-2d^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^3+c^2d-c\,d^2-d^3\right)c^2}-\frac{d\left(5c^2-6cd-2d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c\left(c^3+c^2d-c\,d^2-d^3\right)}-\frac{}{2(c}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{f}$ |
| default | $\frac{2a\left(\frac{-\frac{d\left(3c^2-2cd-2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^3+c^2d-c\,d^2-d^3\right)c}-\frac{\left(2c^4-2c^3d+3c^2d^2-4d^3c-2d^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^3+c^2d-c\,d^2-d^3\right)c^2}-\frac{d\left(5c^2-6cd-2d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c\left(c^3+c^2d-c\,d^2-d^3\right)}-\frac{}{2(c}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{f}$ |
| risch | $\frac{ia\left(2ic\,d^2e^{3i(fx+e)}-id^3e^{3i(fx+e)}+4ic^2d\,e^{i(fx+e)}-6ic\,d^2e^{i(fx+e)}-id^3e^{i(fx+e)}+2c^3e^{2i(fx+e)}-4d\,c^2e^{2i(fx+e)}+d^2e^{2i(fx}\right.}{\left(-id\,e^{2i(fx+e)}+id+2c\,e^{i(fx+e)}\right)^2(c+d)^2(c-d)fd}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a*((-1/2*d*(3*c^2-2*c*d-2*d^2)/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*
e)^3-1/2*(2*c^4-2*c^3*d+3*c^2*d^2-4*c*d^3-2*d^4)/(c^3+c^2*d-c*d^2-d^3)/c^2*
tan(1/2*f*x+1/2*e)^2-1/2*d*(5*c^2-6*c*d-2*d^2)/c/(c^3+c^2*d-c*d^2-d^3)*tan(
1/2*f*x+1/2*e)-1/2*(2*c^2-2*c*d-d^2)/(c^3+c^2*d-c*d^2-d^3))/(c*tan(1/2*f*x+
1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/(c^2
-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(130) = 260.
time = 0.38, size = 826, normalized size = 6.16

$$\left[\frac{2\left(ac^{3}d-2ac^{2}d^{2}-acd^{3}+2ad^{4}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2ac^{3}-ac^{2}d+2acd^{2}-ad^{3}-\left(2acd^{2}-ad^{3}\right)\cos\left(fx+e\right)^{2}+2\left(2ac^{2}d-acd^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c^{2}+d^{2}}\log\left(\frac{\left(2c^{2}-d^{2}\right)\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}+2\left(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e)\right)\sqrt{-c^{2}+d^{2}}}{d^{2}\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}}\right)+2\left(2ac^{4}-2ac^{3}d-3ac^{2}d^{2}+2acd^{3}+ad^{4}\right)\cos\left(fx+e\right)}{4\left(\left(c^{5}d^{2}+c^{4}d^{3}-2c^{3}d^{4}-2c^{2}d^{5}+cd^{6}+d^{7}\right)f\cos\left(fx+e\right)^{2}-2\left(c^{6}d+c^{5}d^{2}-2c^{4}d^{3}-2c^{3}d^{4}+c^{2}d^{5}+cd^{6}\right)f\sin\left(fx+e\right)-\left(c^{7}+c^{6}d-c^{5}d^{2}-c^{4}d^{3}-c^{3}d^{4}-c^{2}d^{5}+cd^{6}+d^{7}\right)f\right)},\frac{\left(ac^{3}d-2ac^{2}d^{2}-acd^{3}+2ad^{4}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2ac^{3}-ac^{2}d+2acd^{2}-ad^{3}-\left(2acd^{2}-ad^{3}\right)\cos\left(fx+e\right)^{2}+2\left(2ac^{2}d-acd^{2}\right)\sin\left(fx+e\right)\right)\sqrt{c^{2}-d^{2}}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^{2}-d^{2}}\cos(fx+e)}\right)+\left(2ac^{4}-2ac^{3}d-3ac^{2}d^{2}+2acd^{3}+ad^{4}\right)\cos\left(fx+e\right)}{2\left(\left(c^{5}d^{2}+c^{4}d^{3}-2c^{3}d^{4}-2c^{2}d^{5}+cd^{6}+d^{7}\right)f\cos\left(fx+e\right)^{2}-2\left(c^{6}d+c^{5}d^{2}-2c^{4}d^{3}-2c^{3}d^{4}+c^{2}d^{5}+cd^{6}\right)f\sin\left(fx+e\right)-\left(c^{7}+c^{6}d-c^{5}d^{2}-c^{4}d^{3}-c^{3}d^{4}-c^{2}d^{5}+cd^{6}+d^{7}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4)*cos(f*x + e)*sin(f*x +
e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 - (2*a*c*d^2 - a*d^3)*cos(f*x +
 e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2
- d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*
sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2)) + 2*(2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c
*d^3 + a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c
*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 +
 c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d
^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a
*d^4)*cos(f*x + e)*sin(f*x + e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 -
(2*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*
sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))
) + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))/(
(c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos(f*x + e)^2
- 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x +
 e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f
)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(130) = 260.
time = 0.53, size = 384, normalized size = 2.87

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)(2ac-ad)}{\left(c^{3}+c^{2}d-cd^{2}-d^{3}\right)\sqrt{c^{2}-d^{2}}}-\frac{3ac^{3}d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-2ac^{2}d^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-2acd^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2ac^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2ac^{3}d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3ac^{2}d^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-4acd^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2ad^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+5ac^{3}d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6ac^{2}d^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2acd^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2ac^{4}-2ac^{3}d-ac^{2}d^{2}}{\left(c^{5}+c^{4}d-c^{3}d^{2}-c^{2}d^{3}\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e)
+ d)/sqrt(c^2 - d^2)))*(2*a*c - a*d)/((c^3 + c^2*d - c*d^2 - d^3)*sqrt(c^2
- d^2)) - (3*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*d^2*tan(1/2*f*x + 1/2
*e)^3 - 2*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a*c^4*tan(1/2*f*x + 1/2*e)^2 -
 2*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*
a*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 2*a*d^4*tan(1/2*f*x + 1/2*e)^2 + 5*a*c^3*d
*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*a*c*d^3*tan(1/
2*f*x + 1/2*e) + 2*a*c^4 - 2*a*c^3*d - a*c^2*d^2)/((c^5 + c^4*d - c^3*d^2 -
 c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

**Mupad [B]**
time = 8.97, size = 445, normalized size = 3.32

$$-\frac{\frac{-2\,a\,c^2+2\,a\,c\,d+a\,d^2}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(c^2+2\,d^2\right)\,\left(-2\,c^2+2\,c\,d+d^2\right)}{c^2\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(-5\,c^2+6\,c\,d+2\,d^2\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-3\,c^2+2\,c\,d+2\,d^2\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+c^2+4\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,\left(2\,c-d\right)\,\left(-2\,c^3\,d-2\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{2\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-d\right)}{{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}{2\,a\,c-a\,d}\right)\,\left(2\,c-d\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((a*d^2 - 2*a*c^2 + 2*a*c*d)/(c*d^2 - c^2*d - c^3 + d^3) + (a*tan(e/2 + (
f*x)/2)^2*(c^2 + 2*d^2)*(2*c*d - 2*c^2 + d^2))/(c^2*(c*d^2 - c^2*d - c^3 +
d^3)) + (a*d*tan(e/2 + (f*x)/2)*(6*c*d - 5*c^2 + 2*d^2))/(c*(c*d^2 - c^2*d
- c^3 + d^3)) + (a*d*tan(e/2 + (f*x)/2)^3*(2*c*d - 3*c^2 + 2*d^2))/(c*(c*d^
2 - c^2*d - c^3 + d^3)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan
(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x
)/2))) - (a*atan((((a*(2*c - d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2
*(c + d)^(5/2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 +
(f*x)/2)*(2*c - d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c^3 + d
^3))/(2*a*c - a*d))*(2*c - d))/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

# 3.433 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=192

$$\frac{a(2c^2-2cd+d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{5/2}f}-\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3}-\frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

[Out] a*(2*c^2-2*c*d+d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)/(c^2-d^2)^(5/2)/f-1/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^3-1/6*a*(2*c-3*d)*cos(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a*(c-4*d)*(2*c-d)*cos(f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sin(f*x+e))

**Rubi** [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{a(2c^2-2cd+d^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{5/2}}-\frac{a(c-4d)(2c-d)\cos(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sin(e+fx))}-\frac{a(2c-3d)\cos(e+fx)}{6f(c-d)(c+d)^2(c+d\sin(e+fx))^2}-\frac{a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]

[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*(c^2 - d^2)^(5/2)*f) - (a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e + f*x])^3) - (a*(2*c - 3*d)*Cos[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x])^2) - (a*(c - 4*d)*(2*c - d)*Cos[e + f*x])/(6*(c - d)^2*(c + d)^3*f*(c + d*Sin[e + f*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx = -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{\int \frac{-3a(c-d)-2a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{a(2c^2-2cd+d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)^2(c+d)^3\sqrt{c^2-d^2}\,f} - \frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 1.79, size = 428, normalized size = 2.23

$$\frac{a(1+\sin(e+fx))\left(\frac{24(2c^2-2cd+d^2)\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{2c(4c^4-18c^3d+14c^2d^2-27cd^3+12d^4)\cot(e)-d\csc(e)(3d(4c^3-16c^2d+6cd^2+d^3)\cos(e+2fx)-3d^2(2c^2-2cd+d^2)\cos(3e+2fx)-24c^4\sin(fx)+78c^3d\sin(fx)-24c^2d^2\sin(fx)+12cd^3\sin(fx)-12d^4\sin(fx)+30c^3d\sin(2e+fx)-30c^2d^2\sin(2e+fx)+15cd^3\sin(2e+fx)+2c^2d^2\sin(2e+3fx)-9cd^3\sin(2e+3fx)+4d^4\sin(2e+3fx))}{d(c+d\sin(e+fx))^3}\right)}{24(c-d)^2(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((24*(2*c^2 - 2*c*d + d^2)*ArcTan[(Sec[(f*x)/2]*(Cos[
e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt
[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e
] - I*Sin[e])^2]) + (2*c*(4*c^4 - 18*c^3*d + 14*c^2*d^2 - 27*c*d^3 + 12*d^4
)*Cot[e] - d*Csc[e]*(3*d*(4*c^3 - 16*c^2*d + 6*c*d^2 + d^3)*Cos[e + 2*f*x]
- 3*d^2*(2*c^2 - 2*c*d + d^2)*Cos[3*e + 2*f*x] - 24*c^4*Sin[f*x] + 78*c^3*d
*Sin[f*x] - 24*c^2*d^2*Sin[f*x] + 12*c*d^3*Sin[f*x] - 12*d^4*Sin[f*x] + 30*
c^3*d*Sin[2*e + f*x] - 30*c^2*d^2*Sin[2*e + f*x] + 15*c*d^3*Sin[2*e + f*x]
+ 2*c^2*d^2*Sin[2*e + 3*f*x] - 9*c*d^3*Sin[2*e + 3*f*x] + 4*d^4*Sin[2*e + 3
*f*x]))/(d*(c + d*Sin[e + f*x])^3)))/(24*(c - d)^2*(c + d)^3*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(181) = 362$.
time = 0.74, size = 641, normalized size = 3.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a*((-1/2*d*(4*c^4-5*c^3*d-4*c^2*d^2+2*c*d^3+2*d^4)/c/(c^5+c^4*d-2*c^3*d
^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5-1/2*(2*c^6-4*c^5*d+10*c^4*d^2-
17*c^3*d^3-6*c^2*d^4+6*c*d^5+4*d^6)/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^
5)/c^2*tan(1/2*f*x+1/2*e)^4-1/3/c^3*d*(18*c^6-36*c^5*d+6*c^4*d^2-15*c^3*d^3
+2*c^2*d^4+6*c*d^5+4*d^6)/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2
*f*x+1/2*e)^3-(2*c^6-4*c^5*d+6*c^4*d^2-14*c^3*d^3+3*c*d^5+2*d^6)/(c^5+c^4*d
-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c^2*tan(1/2*f*x+1/2*e)^2-1/2*d*(8*c^4-19*c^
3*d+4*c*d^3+2*d^4)/c/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+
1/2*e)-1/6*(6*c^4-12*c^3*d-2*c^2*d^2+3*c*d^3+2*d^4)/(c^5+c^4*d-2*c^3*d^2-2*
c^2*d^3+c*d^4+d^5))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^3+1/2
*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(188) = 376$.
time = 0.40, size = 1373, normalized size = 7.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6)*c
os(f*x + e)^3 - 6*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a*d^
6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*a*c
^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e)^2
 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2*d^
3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log((
(2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*
x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(a*c^6 - 2*a*c^5*d - a*c^4*d^2 + a
*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/(3*(c^8*d^2 + c^7*d^3
 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f
*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*c^4*d^6 + 8*c^3*d^7 -
 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c
^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 - (3*c^9*d + 3*c^8*d^2
- 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^9 - d^10)*f)*sin(f*x
+ e)), -1/6*((2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6
)*cos(f*x + e)^3 - 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a
*d^6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*
a*c^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e
)^2 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2
*d^3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(a*c^6 - 2*a*
c^5*d - a*c^4*d^2 + a*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/
(3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2
*d^8 - c*d^9)*f*cos(f*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*
c^4*d^6 + 8*c^3*d^7 - 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*
d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 -
(3*c^9*d + 3*c^8*d^2 - 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^
9 - d^10)*f)*sin(f*x + e))]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(188) = 376.
time = 0.50, size = 808, normalized size = 4.21

$$\frac{\frac{3\,(2\,ac^2-2\,acd+ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^5+c^4d-2\,c^3d^2-2\,c^2d^3+cd^4+d^5)\sqrt{c^2-d^2}}-\frac{12\,ac^6d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-15\,ac^5d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,ac^4d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ac^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ac^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ac^7\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-12\,ac^6d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,ac^5d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-51\,ac^4d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-18\,ac^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+18\,ac^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+12\,acd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+36\,ac^6d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-72\,ac^5d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,ac^4d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ac^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+4\,ac^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,acd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,ad^7\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,ac^7\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-24\,ac^6d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,ac^5d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-84\,ac^4d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+18\,ac^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,acd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,ac^6d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-57\,ac^5d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,ac^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,ac^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,ac^7-12\,ac^6d-2\,ac^5d^2+3\,ac^4d^3+2\,ac^3d^4}{(c^8+c^7d-2\,c^6d^2-2\,c^5d^3+c^4d^4+c^3d^5)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + c^4*d - 2*
c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(c^2 - d^2)) - (12*a*c^6*d*tan(1/2*f
*x + 1/2*e)^5 - 15*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*d^3*tan(1/2*
f*x + 1/2*e)^5 + 6*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*c^2*d^5*tan(1/2*f
*x + 1/2*e)^5 + 6*a*c^7*tan(1/2*f*x + 1/2*e)^4 - 12*a*c^6*d*tan(1/2*f*x + 1
/2*e)^4 + 30*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^4 - 51*a*c^4*d^3*tan(1/2*f*x +
1/2*e)^4 - 18*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^4 + 18*a*c^2*d^5*tan(1/2*f*x +
 1/2*e)^4 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^4 + 36*a*c^6*d*tan(1/2*f*x + 1/
2*e)^3 - 72*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 12*a*c^4*d^3*tan(1/2*f*x + 1
/2*e)^3 - 30*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a*c^2*d^5*tan(1/2*f*x + 1
/2*e)^3 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*d^7*tan(1/2*f*x + 1/2*e)^
3 + 12*a*c^7*tan(1/2*f*x + 1/2*e)^2 - 24*a*c^6*d*tan(1/2*f*x + 1/2*e)^2 + 3
6*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 84*a*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 +
18*a*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2
4*a*c^6*d*tan(1/2*f*x + 1/2*e) - 57*a*c^5*d^2*tan(1/2*f*x + 1/2*e) + 12*a*c
^3*d^4*tan(1/2*f*x + 1/2*e) + 6*a*c^2*d^5*tan(1/2*f*x + 1/2*e) + 6*a*c^7 -
12*a*c^6*d - 2*a*c^5*d^2 + 3*a*c^4*d^3 + 2*a*c^3*d^4)/((c^8 + c^7*d - 2*c^6
*d^2 - 2*c^5*d^3 + c^4*d^4 + c^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1
/2*f*x + 1/2*e) + c)^3))/f
```

**Mupad** [B]
time = 9.71, size = 877, normalized size = 4.57

$$\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{5/2}}+\frac{a\,\left(2\,c^2-2\,c\,d+d^2\right)\,\left(2\,c^5\,d+2\,c^4\,d^2-4\,c^3\,d^3-4\,c^2\,d^4+2\,c\,d^5+2\,d^6\right)}{2\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{5/2}\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}\right)\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}{2\,a\,c^2-2\,a\,c\,d+a\,d^2}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{5/2}}-\frac{\frac{6\,a\,c^4-12\,a\,c^3\,d-2\,a\,c^2\,d^2+3\,a\,c\,d^3+2\,a\,d^4}{3\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}+\frac{a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,c^6-4\,c^5\,d+10\,c^4\,d^2-17\,c^3\,d^3-6\,c^2\,d^4+6\,c\,d^5+4\,d^6\right)}{c^2\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}+\frac{2\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^6-4\,c^5\,d+6\,c^4\,d^2-14\,c^3\,d^3+3\,c\,d^5+2\,d^6\right)}{c^2\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}+\frac{a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^4-19\,c^3\,d+4\,c\,d^3+2\,d^4\right)}{c\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}+\frac{a\,d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(4\,c^4-5\,c^3\,d-4\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{c\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}+\frac{2\,a\,d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(3\,c^2+2\,d^2\right)\,\left(6\,c^4-12\,c^3\,d-2\,c^2\,d^2+3\,c\,d^3+2\,d^4\right)}{3\,c^3\,\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}}{f\,\left(c^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(3\,c^3+12\,c\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(3\,c^3+12\,c\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(12\,c^2\,d+8\,d^3\right)+c^3+6\,c^2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+6\,c^2\,d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a*atan((((a*c*tan(e/2 + (f*x)/2)*(2*c^2 - 2*c*d + d^2))/((c + d)^(7/2)*(c
- d)^(5/2)) + (a*(2*c^2 - 2*c*d + d^2)*(2*c*d^5 + 2*c^5*d + 2*d^6 - 4*c^2*d
```

```
^4 - 4*c^3*d^3 + 2*c^4*d^2))/(2*(c + d)^(7/2)*(c - d)^(5/2)*(c*d^4 + c^4*d
+ c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)))*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d
^3 - 2*c^3*d^2))/(2*a*c^2 + a*d^2 - 2*a*c*d))*(2*c^2 - 2*c*d + d^2))/(f*(c
+ d)^(7/2)*(c - d)^(5/2)) - ((6*a*c^4 + 2*a*d^4 - 2*a*c^2*d^2 + 3*a*c*d^3 -
 12*a*c^3*d)/(3*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (a*t
an(e/2 + (f*x)/2)^4*(6*c*d^5 - 4*c^5*d + 2*c^6 + 4*d^6 - 6*c^2*d^4 - 17*c^3
*d^3 + 10*c^4*d^2))/(c^2*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2
)) + (2*a*tan(e/2 + (f*x)/2)^2*(3*c*d^5 - 4*c^5*d + 2*c^6 + 2*d^6 - 14*c^3*
d^3 + 6*c^4*d^2))/(c^2*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2))
 + (a*d*tan(e/2 + (f*x)/2)*(4*c*d^3 - 19*c^3*d + 8*c^4 + 2*d^4))/(c*(c*d^4
+ c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (a*d*tan(e/2 + (f*x)/2)^5*(
2*c*d^3 - 5*c^3*d + 4*c^4 + 2*d^4 - 4*c^2*d^2))/(c*(c*d^4 + c^4*d + c^5 + d
^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (2*a*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*
(3*c*d^3 - 12*c^3*d + 6*c^4 + 2*d^4 - 2*c^2*d^2))/(3*c^3*(c*d^4 + c^4*d + c
^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2
+ (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) +
 tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2)
 + 6*c^2*d*tan(e/2 + (f*x)/2)^5))
```

# 3.434 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx$

**Optimal**. Leaf size=318

$$\frac{1}{16}a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)x+\frac{a^2(4c^5-48c^4d-311c^3d^2-448c^2d^3-288cd^4-64d^5)\cos(e}{60df}$$

```
[Out] 1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*x+1/60*a^2*(4*c^5-48*
c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*cos(f*x+e)/d/f+1/240*a^2*(8
*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*cos(f*x+e)*sin(f*x+e)/f+1/120*
a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/1
20*a^2*(4*c^2-48*c*d-55*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f+1/30*a^2*(c-
12*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f-1/6*a^2*cos(f*x+e)*(c+d*sin(f*x+e))
^5/d/f
```

**Rubi** [A]
time = 0.32, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2842, 2832, 2813}

$$\frac{a^2(4c^2-48cd-55d^2)\cos(e+fx)(c+d\sin(e+fx))^3}{120df}+\frac{a^2(4c^3-48c^2d-123cd^2-64d^3)\cos(e+fx)(c+d\sin(e+fx))^2}{120df}+\frac{a^2(8c^4-96c^3d-438c^2d^2-464cd^3-165d^4)\sin(e+fx)\cos(e+fx)}{240f}+\frac{1}{16}a^2x\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)+\frac{a^2(4c^5-48c^4d-311c^3d^2-448c^2d^3-288cd^4-64d^5)\cos(e+fx)}{60df}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^5}{6df}+\frac{a^2(c-12d)\cos(e+fx)(c+d\sin(e+fx))^4}{30df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*x)/16 + (a^2*(4*c
^5 - 48*c^4*d - 311*c^3*d^2 - 448*c^2*d^3 - 288*c*d^4 - 64*d^5)*Cos[e + f*x
])/(60*d*f) + (a^2*(8*c^4 - 96*c^3*d - 438*c^2*d^2 - 464*c*d^3 - 165*d^4)*C
os[e + f*x]*Sin[e + f*x])/(240*f) + (a^2*(4*c^3 - 48*c^2*d - 123*c*d^2 - 64
*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d*f) + (a^2*(4*c^2 - 48*c*d
 - 55*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d*f) + (a^2*(c - 12*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^5)/(6*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
```

```
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^5}{6df} + \frac{\int (11a^2d - a^2(c-1$$

$$= \frac{a^2(c-12d)\cos(e+fx)(c+d\sin(e+fx))^4}{30df} - \frac{a^2\cos(e+f$$

$$= \frac{a^2(4c^2-48cd-55d^2)\cos(e+fx)(c+d\sin(e+fx))^3}{120df} + \frac{a$$

$$= \frac{a^2(4c^3-48c^2d-123cd^2-64d^3)\cos(e+fx)(c+d\sin(e+$$

$$= \frac{1}{16}a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)x + \frac{a^2(4c^5-$$

**Mathematica [A]**
time = 0.88, size = 262, normalized size = 0.82

$$-\frac{a^2\cos(e+fx)\left(30(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(32(15c^4+50c^3d+60c^2d^2+36cd^3+8d^4)+15(8c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\sin(e+fx)+64d(5c^3+15c^2d+9cd^2+2d^3)\sin^2(e+fx)+10d^2(36c^2+48cd+11d^2)\sin^3(e+fx)+96d^3(2c+d)\sin^4(e+fx)+40d^4\sin^5(e+fx))\right)}{240f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -1/240*(a^2*Cos[e + f*x]*(30*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 1
1*d^4)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(32*(1
5*c^4 + 50*c^3*d + 60*c^2*d^2 + 36*c*d^3 + 8*d^4) + 15*(8*c^4 + 64*c^3*d +
84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Sin[e + f*x] + 64*d*(5*c^3 + 15*c^2*d + 9*c
```

```
*d^2 + 2*d^3)*Sin[e + f*x]^2 + 10*d^2*(36*c^2 + 48*c*d + 11*d^2)*Sin[e + f*
x]^3 + 96*d^3*(2*c + d)*Sin[e + f*x]^4 + 40*d^4*Sin[e + f*x]^5)))/(f*Sqrt[C
os[e + f*x]^2])
```

**Maple** [A]
time = 0.52, size = 462, normalized size = 1.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^4*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-4/3*a^2*c^3*d*(2+si
n(f*x+e)^2)*cos(f*x+e)+6*a^2*c^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-4/5*a^2*c*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)+a^2*d^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos
(f*x+e)+5/16*f*x+5/16*e)-2*a^2*c^4*cos(f*x+e)+8*a^2*c^3*d*(-1/2*cos(f*x+e)*
sin(f*x+e)+1/2*f*x+1/2*e)-4*a^2*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+8*a^2*c
*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*a^2*
d^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*c^4*(f*x+e)-4*a^2*c^
3*d*cos(f*x+e)+6*a^2*c^2*d^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-4/3
*a^2*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

**Maxima** [A]
time = 0.29, size = 485, normalized size = 1.53

$$\frac{240\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^4+960\,(fx+e)a^2c^4+1280\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2c^3d+1920\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^3d+3840\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2c^2d^2+180\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2c^2d^2+1440\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^2d^2-256\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))a^2cd^3+1280\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2cd^3+240\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2cd^3-128\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))a^2d^4+5\,(4\,\sin(2\,fx+2\,e)^3+60\,fx+60\,e+9\,\sin(4\,fx+4\,e)-48\,\sin(2\,fx+2\,e))a^2d^4+30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2d^4-1920\,a^2c^4\cos(fx+e)-3840\,a^2c^3d\cos(fx+e)}{960\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3*d + 1920*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^2*c^3*d + 3840*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2
*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2*
d^2 + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2*d^2 - 256*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c*d^3 + 1280*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a^2*c*d^3 + 240*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^2*c*d^3 - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^2*d^4 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*
f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*d^4 + 30*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^4 - 1920*a^2*c^4*cos(f*x + e) - 3840*a^2*
c^3*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.38, size = 306, normalized size = 0.96

$$-\frac{96\,(2\,a^2cd^3+a^2d^4)\cos(fx+e)^5-320\,(a^2c^3d+3\,a^2c^2d^2+3\,a^2cd^3+a^2d^4)\cos(fx+e)^3-15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)fx+480\,(a^2c^4+4\,a^2c^3d+6\,a^2c^2d^2+4\,a^2cd^3+a^2d^4)\cos(fx+e)+5\,(8\,a^2d^4\cos(fx+e)^5-2\,(36\,a^2c^2d^2+48\,a^2cd^3+19\,a^2d^4)\cos(fx+e)^3+3\,(8\,a^2c^4+64\,a^2c^3d+108\,a^2c^2d^2+80\,a^2cd^3+21\,a^2d^4)\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/240*(96*(2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^5 - 320*(a^2*c^3*d + 3*a^2*
c^2*d^2 + 3*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - 15*(24*a^2*c^4 + 64*a^2*c
^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*f*x + 480*(a^2*c^4 + 4*a
^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + 5*(8*a^2*d
^4*cos(f*x + e)^5 - 2*(36*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x
+ e)^3 + 3*(8*a^2*c^4 + 64*a^2*c^3*d + 108*a^2*c^2*d^2 + 80*a^2*c*d^3 + 21*
a^2*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(299) = 598$.
time = 0.67, size = 1136, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**2*c**4*x*sin(e + f*x)**2/2 + a**2*c**4*x*cos(e + f*x)**2/2 +
a**2*c**4*x - a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**4*cos(e
 + f*x)/f + 4*a**2*c**3*d*x*sin(e + f*x)**2 + 4*a**2*c**3*d*x*cos(e + f*x)*
*2 - 4*a**2*c**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*a**2*c**3*d*sin(e + f
*x)*cos(e + f*x)/f - 8*a**2*c**3*d*cos(e + f*x)**3/(3*f) - 4*a**2*c**3*d*co
s(e + f*x)/f + 9*a**2*c**2*d**2*x*sin(e + f*x)**4/4 + 9*a**2*c**2*d**2*x*si
n(e + f*x)**2*cos(e + f*x)**2/2 + 3*a**2*c**2*d**2*x*sin(e + f*x)**2 + 9*a*
*2*c**2*d**2*x*cos(e + f*x)**4/4 + 3*a**2*c**2*d**2*x*cos(e + f*x)**2 - 15*
a**2*c**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 12*a**2*c**2*d**2*sin(e
 + f*x)**2*cos(e + f*x)/f - 9*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(
4*f) - 3*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)/f - 8*a**2*c**2*d**2*cos(
e + f*x)**3/f + 3*a**2*c*d**3*x*sin(e + f*x)**4 + 6*a**2*c*d**3*x*sin(e + f
*x)**2*cos(e + f*x)**2 + 3*a**2*c*d**3*x*cos(e + f*x)**4 - 4*a**2*c*d**3*si
n(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c*d**3*sin(e + f*x)**3*cos(e + f*x)/f
 - 16*a**2*c*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c*d**3*sin
(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c*d**3*sin(e + f*x)*cos(e + f*x)**3/f
- 32*a**2*c*d**3*cos(e + f*x)**5/(15*f) - 8*a**2*c*d**3*cos(e + f*x)**3/(3*
f) + 5*a**2*d**4*x*sin(e + f*x)**6/16 + 15*a**2*d**4*x*sin(e + f*x)**4*cos(
e + f*x)**2/16 + 3*a**2*d**4*x*sin(e + f*x)**4/8 + 15*a**2*d**4*x*sin(e + f
*x)**2*cos(e + f*x)**4/16 + 3*a**2*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4
 + 5*a**2*d**4*x*cos(e + f*x)**6/16 + 3*a**2*d**4*x*cos(e + f*x)**4/8 - 11*
a**2*d**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*a**2*d**4*sin(e + f*x)**4
*cos(e + f*x)/f - 5*a**2*d**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*a**
2*d**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 8*a**2*d**4*sin(e + f*x)**2*cos
(e + f*x)**3/(3*f) - 5*a**2*d**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*a*
```

```
*2*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 16*a**2*d**4*cos(e + f*x)**5/(
15*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a)**2, True))
```

**Giac** [A]
time = 0.47, size = 458, normalized size = 1.44

$$\frac{a^{2} c d^{3} \cos\left(3 f x + 3 e\right)}{3 f} - \frac{a^{2} d^{4} \sin\left(6 f x + 6 e\right)}{192 f} + \frac{a^{2} d^{4} \sin\left(4 f x + 4 e\right)}{32 f} + \frac{1}{16} \left(8 a^{2} c^{4} + 64 a^{2} c^{3} d + 36 a^{2} c^{2} d^{2} + 48 a^{2} c d^{3} + 5 a^{2} d^{4}\right) x + \frac{1}{8} \left(8 a^{2} c^{4} + 24 a^{2} c^{2} d^{2} + 3 a^{2} d^{4}\right) x - \frac{\left(2 a^{2} c d^{3} + a^{2} d^{4}\right) \cos\left(5 f x + 5 e\right)}{40 f} + \frac{\left(8 a^{2} c^{3} d + 24 a^{2} c^{2} d^{2} + 10 a^{2} c d^{3} + 5 a^{2} d^{4}\right) \cos\left(3 f x + 3 e\right)}{24 f} - \frac{\left(8 a^{2} c^{4} + 12 a^{2} c^{3} d + 36 a^{2} c^{2} d^{2} + 10 a^{2} c d^{3} + 5 a^{2} d^{4}\right) \cos\left(f x + e\right)}{4 f} - \frac{\left(4 a^{2} c^{3} d + 3 a^{2} c d^{3}\right) \cos\left(f x + e\right)}{f} + \frac{\left(12 a^{2} c^{2} d^{2} + 16 a^{2} c d^{3} + 3 a^{2} d^{4}\right) \sin\left(4 f x + 4 e\right)}{64 f} - \frac{\left(16 a^{2} c^{4} + 128 a^{2} c^{3} d + 96 a^{2} c^{2} d^{2} + 128 a^{2} c d^{3} + 15 a^{2} d^{4}\right) \sin\left(2 f x + 2 e\right)}{64 f} - \frac{\left(6 a^{2} c^{2} d^{2} + a^{2} d^{4}\right) \sin\left(2 f x + 2 e\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c*d^3*cos(3*f*x + 3*e)/f - 1/192*a^2*d^4*sin(6*f*x + 6*e)/f + 1/32*
a^2*d^4*sin(4*f*x + 4*e)/f + 1/16*(8*a^2*c^4 + 64*a^2*c^3*d + 36*a^2*c^2*d^
2 + 48*a^2*c*d^3 + 5*a^2*d^4)*x + 1/8*(8*a^2*c^4 + 24*a^2*c^2*d^2 + 3*a^2*d
^4)*x - 1/40*(2*a^2*c*d^3 + a^2*d^4)*cos(5*f*x + 5*e)/f + 1/24*(8*a^2*c^3*d
 + 24*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(3*f*x + 3*e)/f - 1/4*(8*a
^2*c^4 + 12*a^2*c^3*d + 36*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(f*x
+ e)/f - (4*a^2*c^3*d + 3*a^2*c*d^3)*cos(f*x + e)/f + 1/64*(12*a^2*c^2*d^2
+ 16*a^2*c*d^3 + 3*a^2*d^4)*sin(4*f*x + 4*e)/f - 1/64*(16*a^2*c^4 + 128*a^2
*c^3*d + 96*a^2*c^2*d^2 + 128*a^2*c*d^3 + 15*a^2*d^4)*sin(2*f*x + 2*e)/f -
1/4*(6*a^2*c^2*d^2 + a^2*d^4)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 9.92, size = 865, normalized size = 2.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 +
84*c^2*d^2))/(8*(3*a^2*c^4 + (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (
21*a^2*c^2*d^2)/2)))*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/
(8*f) - (tan(e/2 + (f*x)/2)^8*(20*a^2*c^4 + 16*a^2*c*d^3 + 56*a^2*c^3*d + 4
8*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^10*(4*a^2*c^4 + 8*a^2*c^3*d) + tan(e/2
+ (f*x)/2)*(a^2*c^4 + (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*
c^2*d^2)/2) + 4*a^2*c^4 + (32*a^2*d^4)/15 - tan(e/2 + (f*x)/2)^11*(a^2*c^4
+ (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/
2 + (f*x)/2)^5*(2*a^2*c^4 + (47*a^2*d^4)/4 + 28*a^2*c*d^3 + 16*a^2*c^3*d +
33*a^2*c^2*d^2) - tan(e/2 + (f*x)/2)^7*(2*a^2*c^4 + (47*a^2*d^4)/4 + 28*a^2
*c*d^3 + 16*a^2*c^3*d + 33*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(3*a^2*c^4 +
 (187*a^2*d^4)/24 + 34*a^2*c*d^3 + 24*a^2*c^3*d + (87*a^2*c^2*d^2)/2) - tan
(e/2 + (f*x)/2)^9*(3*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + 24*a^2*c^3
*d + (87*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)^4*(40*a^2*c^4 + 32*a^2*d^4 +
128*a^2*c*d^3 + 144*a^2*c^3*d + 192*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(20
*a^2*c^4 + (64*a^2*d^4)/5 + (288*a^2*c*d^3)/5 + 72*a^2*c^3*d + 96*a^2*c^2*d
```

```
^2) + tan(e/2 + (f*x)/2)^6*(40*a^2*c^4 + (64*a^2*d^4)/3 + 96*a^2*c*d^3 + (4
00*a^2*c^3*d)/3 + 160*a^2*c^2*d^2) + (48*a^2*c*d^3)/5 + (40*a^2*c^3*d)/3 +
16*a^2*c^2*d^2)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*t
an(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + t
an(e/2 + (f*x)/2)^12 + 1))
```

# 3.435 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=233

$$\frac{3}{8}a^2(2c+d)\left(2c^2+3cd+2d^2\right)x+\frac{a^2(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)\cos(e+fx)}{10df}+\frac{a^2(2c^3-20c^2d-57c}{}$$

```
[Out] 3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*x+1/10*a^2*(c^4-10*c^3*d-44*c^2*d^2-40*
c*d^3-12*d^4)*cos(f*x+e)/d/f+1/40*a^2*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)*cos(
f*x+e)*sin(f*x+e)/f+1/20*a^2*(c^2-10*c*d-12*d^2)*cos(f*x+e)*(c+d*sin(f*x+e)
)^2/d/f+1/20*a^2*(c-10*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^4/d/f
```

**Rubi** [A]

time = 0.22, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2842, 2832, 2813}

$$\frac{a^2(c^2-10cd-12d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{20df}+\frac{3}{8}a^2x(2c+d)\left(2c^2+3cd+2d^2\right)+\frac{a^2(2c^3-20c^2d-57cd^2-30d^3)\sin(e+fx)\cos(e+fx)}{40f}+\frac{a^2(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)\cos(e+fx)}{10df}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df}+\frac{a^2(c-10d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*x)/8 + (a^2*(c^4 - 10*c^3*d - 44*c
^2*d^2 - 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(10*d*f) + (a^2*(2*c^3 - 20*c^2*d
 - 57*c*d^2 - 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(40*f) + (a^2*(c^2 - 10*c*
d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(20*d*f) + (a^2*(c - 10*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx &= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df}+\frac{\int (9a^2d-a^2(c-10\\ &= \frac{a^2(c-10d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df}-\frac{a^2\cos(e+f\\ &= \frac{a^2(c^2-10cd-12d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{20df}+\frac{a^2\\ &= \frac{3}{8}a^2(2c+d)\left(2c^2+3cd+2d^2\right)x+\frac{a^2(c^4-10c^3d-44c^2d^2\end{aligned}$$

**Mathematica [A]**
time = 0.58, size = 188, normalized size = 0.81

$$\frac{(a+a\sin(e+fx))^2\left(60(2c+d)\left(2c^2+3cd+2d^2\right)(e+fx)-20\left(16c^3+42c^2d+36cd^2+11d^3\right)\cos(e+fx)+10d(2c+d)(2c+3d)\cos(3(e+fx))-2d^3\cos(5(e+fx))-40\left(c^3+6c^2d+6cd^2+2d^3\right)\sin(2(e+fx))+5d^2(3c+2d)\sin(4(e+fx))\right)}{160f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(60*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(e + f*x) - 2
0*(16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3)*Cos[e + f*x] + 10*d*(2*c + d)*(2*
c + 3*d)*Cos[3*(e + f*x)] - 2*d^3*Cos[5*(e + f*x)] - 40*(c^3 + 6*c^2*d + 6*
c*d^2 + 2*d^3)*Sin[2*(e + f*x)] + 5*d^2*(3*c + 2*d)*Sin[4*(e + f*x)]))/(160
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

**Maple [A]**
time = 0.47, size = 329, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^2*c^2*d*(2+sin(f*
x+e)^2)*cos(f*x+e)+3*a^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-1/5*a^2*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e
)-2*a^2*c^3*cos(f*x+e)+6*a^2*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*
e)-2*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*d^3*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x
+e)+3*a^2*c*d^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+s
in(f*x+e)^2)*cos(f*x+e))
```

**Maxima** [A]
time = 0.29, size = 342, normalized size = 1.47

$$\frac{120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^3+480\,(fx+e)a^2c^3+480\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2c^2d+720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^2d+960\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2cd^2+45\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2cd^2+360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2cd^2-32\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))a^2d^3+160\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2d^3+30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2d^3-960\,a^2c^3\cos(fx+e)-1440\,a^2c^2d\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3
 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2*d + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*c^2*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d^2
+ 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c*d^2 + 36
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 - 32*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*a^2*d^3 + 160*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a
^2*d^3 - 960*a^2*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.37, size = 223, normalized size = 0.96

$$-\frac{8\,a^2d^3\cos(fx+e)^5-40\,(a^2c^2d+2\,a^2cd^2+a^2d^3)\cos(fx+e)^3-15\,(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3)fx+80\,(a^2c^3+3\,a^2c^2d+3\,a^2cd^2+a^2d^3)\cos(fx+e)-5\,(2\,(3\,a^2cd^2+2\,a^2d^3)\cos(fx+e)^3-(4\,a^2c^3+24\,a^2c^2d+27\,a^2cd^2+10\,a^2d^3)\cos(fx+e))\sin(fx+e)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^2*d^3*cos(f*x + e)^5 - 40*(a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*co
s(f*x + e)^3 - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*f*x +
 80*(a^2*c^3 + 3*a^2*c^2*d + 3*a^2*c*d^2 + a^2*d^3)*cos(f*x + e) - 5*(2*(3*
a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^3 - (4*a^2*c^3 + 24*a^2*c^2*d + 27*a^2*
c*d^2 + 10*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(218) = 436.
time = 0.42, size = 729, normalized size = 3.13

$$\begin{cases} \frac{a^2c^3x\sin^2(e+fx)}{2}+\frac{a^2c^3x\cos^2(e+fx)}{2}+a^2c^3x-\frac{a^2c^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{2a^2c^3\cos(e+fx)}{f}+3a^2c^2dx\sin^2(e+fx)+3a^2c^2dx\cos^2(e+fx)-\frac{3a^2c^2d\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3a^2c^2d\sin(e+fx)\cos(e+fx)}{f}-\frac{2a^2c^2d\cos^3(e+fx)}{f}-\frac{3a^2c^2d\cos(e+fx)}{f}+\frac{9a^2cd^2x\sin^4(e+fx)}{8}+\frac{9a^2cd^2x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{3a^2cd^2x\sin^2(e+fx)}{2}+\frac{9a^2cd^2x\cos^4(e+fx)}{8}+\frac{3a^2cd^2x\cos^2(e+fx)}{2}-\frac{15a^2cd^2\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{6a^2cd^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{9a^2cd^2\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{3a^2cd^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{4a^2cd^2\cos^3(e+fx)}{f}+\frac{3a^2d^3x\sin^4(e+fx)}{4}+\frac{3a^2d^3x\sin^2(e+fx)\cos^2(e+fx)}{2}+\frac{3a^2d^3x\cos^4(e+fx)}{4}-\frac{a^2d^3\sin^4(e+fx)\cos(e+fx)}{f}-\frac{5a^2d^3\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{4a^2d^3\sin^2(e+fx)\cos^3(e+fx)}{3f}-\frac{a^2d^3\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3a^2d^3\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{8a^2d^3\cos^5(e+fx)}{15f}-\frac{2a^2d^3\cos^3(e+fx)}{3f} & \text{for}\ f\neq 0 \\ x(c+d\sin(e))^3(a\sin(e)+a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)

[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 + a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos(e + f*x)/f + 3*a**2*c**2*d*x*sin(e + f*x)**2 + 3*a**2*c**2*d*x*cos(e + f*x)**2 - 3*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*a**2*c**2*d*cos(e + f*x)**3/f - 3*a**2*c**2*d*cos(e + f*x)/f + 9*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a**2*c*d**2*x*sin(e + f*x)**2/2 + 9*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 15*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*c*d**2*cos(e + f*x)**3/f + 3*a**2*d**3*x*sin(e + f*x)**4/4 + 3*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*a**2*d**3*x*cos(e + f*x)**4/4 - a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**2*d**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*a**2*d**3*cos(e + f*x)**5/(15*f) - 2*a**2*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))

**Giac** [A]
time = 0.47, size = 324, normalized size = 1.39

$$-\frac{a^2 d^3 \cos(5fx+5e)}{80f}+\frac{a^2 d^3 \cos(3fx+3e)}{12f}-\frac{3a^2 c d^2 \sin(2fx+2e)}{4f}+\frac{1}{8}\left(4a^2c^3+24a^2c^2d+9a^2cd^2+6a^2d^3\right)x+\frac{1}{2}\left(2a^2c^3+3a^2cd^2\right)x+\frac{(12a^2c^2d+24a^2cd^2+5a^2d^3)\cos(3fx+3e)}{48f}-\frac{(16a^2c^3+18a^2c^2d+36a^2cd^2+5a^2d^3)\cos(fx+e)}{8f}-\frac{3(4a^2c^2d+a^2d^3)\cos(fx+e)}{4f}+\frac{(3a^2cd^2+2a^2d^3)\sin(4fx+4e)}{32f}-\frac{(a^2c^3+6a^2c^2d+3a^2cd^2+2a^2d^3)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] -1/80*a^2*d^3*cos(5*f*x + 5*e)/f + 1/12*a^2*d^3*cos(3*f*x + 3*e)/f - 3/4*a^2*c*d^2*sin(2*f*x + 2*e)/f + 1/8*(4*a^2*c^3 + 24*a^2*c^2*d + 9*a^2*c*d^2 + 6*a^2*d^3)*x + 1/2*(2*a^2*c^3 + 3*a^2*c*d^2)*x + 1/48*(12*a^2*c^2*d + 24*a^2*c*d^2 + 5*a^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a^2*c^3 + 18*a^2*c^2*d + 36*a^2*c*d^2 + 5*a^2*d^3)*cos(f*x + e)/f - 3/4*(4*a^2*c^2*d + a^2*d^3)*cos(f*x + e)/f + 1/32*(3*a^2*c*d^2 + 2*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/4*(a^2*c^3 + 6*a^2*c^2*d + 3*a^2*c*d^2 + 2*a^2*d^3)*sin(2*f*x + 2*e)/f

**Mupad** [B]
time = 8.42, size = 611, normalized size = 2.62

$$\frac{3a^2\,\mathrm{atan}\left(\frac{3a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c+d)(2c^2+3cd+2d^2)}{4\left(3a^2c^3+6a^2c^2d+\frac{21a^2cd^2}{4}+\frac{3a^2d^3}{2}\right)}\right)(2c+d)(2c^2+3cd+2d^2)}{4f}-\frac{3a^2\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(4c^3+8c^2d+7cd^2+2d^3)}{4f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(4a^2c^3+6da^2c^2)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(a^2c^3+6a^2c^2d+\frac{21a^2cd^2}{4}+\frac{3a^2d^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(2a^2c^3+12a^2c^2d+\frac{33a^2cd^2}{2}+7a^2d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(2a^2c^3+12a^2c^2d+\frac{33a^2cd^2}{2}+7a^2d^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(16a^2c^3+36a^2c^2d+24a^2cd^2+4a^2d^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(16a^2c^3+44a^2c^2d+40a^2cd^2+12a^2d^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(24a^2c^3+64a^2c^2d+56a^2cd^2+20a^2d^3)+4a^2c^3+\frac{12a^2d^3}{5}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a^2c^3+6a^2c^2d+\frac{21a^2cd^2}{4}+\frac{3a^2d^3}{2}\right)+8a^2cd^2+10a^2c^2d}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)

[Out] (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(4*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)/4 + 6*a^2*c^2*d)))*(2*c + d)*(

```
3*c*d + 2*c^2 + 2*d^2))/(4*f) - (3*a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
*(7*c*d^2 + 8*c^2*d + 4*c^3 + 2*d^3))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*a^2*
c^3 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^9*(a^2*c^3 + (3*a^2*d^3)/2 + (21*a^
2*c*d^2)/4 + 6*a^2*c^2*d) + tan(e/2 + (f*x)/2)^3*(2*a^2*c^3 + 7*a^2*d^3 + (
33*a^2*c*d^2)/2 + 12*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(2*a^2*c^3 + 7*a^2*d
^3 + (33*a^2*c*d^2)/2 + 12*a^2*c^2*d) + tan(e/2 + (f*x)/2)^6*(16*a^2*c^3 +
4*a^2*d^3 + 24*a^2*c*d^2 + 36*a^2*c^2*d) + tan(e/2 + (f*x)/2)^2*(16*a^2*c^3
 + 12*a^2*d^3 + 40*a^2*c*d^2 + 44*a^2*c^2*d) + tan(e/2 + (f*x)/2)^4*(24*a^2
*c^3 + 20*a^2*d^3 + 56*a^2*c*d^2 + 64*a^2*c^2*d) + 4*a^2*c^3 + (12*a^2*d^3)
/5 + tan(e/2 + (f*x)/2)*(a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)/4 + 6*a^2
*c^2*d) + 8*a^2*c*d^2 + 10*a^2*c^2*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e
/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/
2 + (f*x)/2)^10 + 1))
```

# 3.436 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=156

$$\frac{1}{8}a^2\left(12c^2+16cd+7d^2\right)x-\frac{a^2(12c^2+16cd+7d^2)\cos(e+fx)}{6f}-\frac{a^2(12c^2+16cd+7d^2)\cos(e+fx)\sin(e+}{24f}$$

```
[Out] 1/8*a^2*(12*c^2+16*c*d+7*d^2)*x-1/6*a^2*(12*c^2+16*c*d+7*d^2)*cos(f*x+e)/f-
1/24*a^2*(12*c^2+16*c*d+7*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/12*(8*c-d)*d*cos(f
*x+e)*(a+a*sin(f*x+e))^2/f-1/4*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^3/a/f
```

**Rubi** [A]
time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2840, 2830, 2723}

$$-\frac{a^2(12c^2+16cd+7d^2)\cos(e+fx)}{6f}-\frac{a^2(12c^2+16cd+7d^2)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12c^2+16cd+7d^2)-\frac{d(8c-d)\cos(e+fx)(a\sin(e+fx)+a)^2}{12f}-\frac{d^2\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*x)/8 - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e
+ f*x])/(6*f) - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e + f*x]*Sin[e + f*x])/(
24*f) - ((8*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(12*f) - (d^2*Cos
[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^3}{4af} + \frac{\int (a+a\sin(e+fx))}{}$$

$$= -\frac{(8c-d)d\cos(e+fx)(a+a\sin(e+fx))^2}{12f} - \frac{d^2\cos(e+fx}{}$$

$$= \frac{1}{8}a^2\left(12c^2+16cd+7d^2\right)x - \frac{a^2(12c^2+16cd+7d^2)\cos(e+}{6f}$$

**Mathematica [A]**
time = 0.36, size = 148, normalized size = 0.95

$$-\frac{a^2\cos(e+fx)\left(6(12c^2+16cd+7d^2)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(16(3c^2+5cd+2d^2)+3(4c^2+16cd+7d^2)\sin(e+fx)+16d(c+d)\sin^2(e+fx)+6d^2\sin^3(e+fx))\right)}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*(12*c^2 + 16*c*d + 7*d^2)*ArcSin[Sqrt[1 - Sin[e
+ f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(16*(3*c^2 + 5*c*d + 2*d^2) + 3*(4*
c^2 + 16*c*d + 7*d^2)*Sin[e + f*x] + 16*d*(c + d)*Sin[e + f*x]^2 + 6*d^2*Si
n[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

**Maple [A]**
time = 0.27, size = 219, normalized size = 1.40

| method | result |
|---|---|
| risch | $\frac{3a^2c^2x}{2}+2a^2cdx+\frac{7a^2d^2x}{8}-\frac{2a^2\cos(fx+e)c^2}{f}-\frac{7a^2\cos(fx+e)cd}{2f}-\frac{3a^2\cos(fx+e)d^2}{2f}+\frac{d^2a^2\sin(4fx+4e)}{32f}$ |
| derivativedivides | $\frac{a^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+d^2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}\right.}{}$ |
| default | $\frac{a^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+d^2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}\right.}{}$ |
| norman | $\frac{(\frac{3}{2}a^2c^2+2a^2cd+\frac{7}{8}d^2a^2)x+(6a^2c^2+8a^2cd+\frac{7}{2}d^2a^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6a^2c^2+8a^2cd+\frac{7}{2}d^2a^2)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(9a^2c^2}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*c*d*(2+sin(
f*x+e)^2)*cos(f*x+e)+d^2*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)-2*a^2*c^2*cos(f*x+e)+4*a^2*c*d*(-1/2*cos(f*x+e)*sin(f*x+e)+
1/2*f*x+1/2*e)-2/3*d^2*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*c^2*(f*x+e)-2*a^
2*c*d*cos(f*x+e)+d^2*a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

**Maxima [A]**
time = 0.29, size = 227, normalized size = 1.46

$$\frac{24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c^2+96\,(fx+e)a^2c^2+64\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2cd+96\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2cd+64\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2d^2+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))a^2d^2+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2d^2-192\,a^2c^2\cos(fx+e)-192\,a^2cd\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 96*(f*x + e)*a^2*c^2 +
64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d + 96*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*a^2*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*d^2 + 3*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^2 + 24*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*a^2*d^2 - 192*a^2*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

**Fricas [A]**
time = 0.35, size = 150, normalized size = 0.96

$$\frac{16\,(a^2cd+a^2d^2)\cos(fx+e)^3+3\,(12\,a^2c^2+16\,a^2cd+7\,a^2d^2)fx-48\,(a^2c^2+2\,a^2cd+a^2d^2)\cos(fx+e)+3\,(2\,a^2d^2\cos(fx+e)^3-(4\,a^2c^2+16\,a^2cd+9\,a^2d^2)\cos(fx+e))\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(a^2*c*d + a^2*d^2)*cos(f*x + e)^3 + 3*(12*a^2*c^2 + 16*a^2*c*d +
7*a^2*d^2)*f*x - 48*(a^2*c^2 + 2*a^2*c*d + a^2*d^2)*cos(f*x + e) + 3*(2*a^2
*d^2*cos(f*x + e)^3 - (4*a^2*c^2 + 16*a^2*c*d + 9*a^2*d^2)*cos(f*x + e))*si
n(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(143) = 286$.
time = 0.26, size = 459, normalized size = 2.94

$$\begin{cases} \frac{a^2c^2x\sin^2(e+fx)}{2}+\frac{a^2c^2x\cos^2(e+fx)}{2}+a^2c^2x-\frac{a^2c^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2a^2c^2\cos(e+fx)}{f}+2a^2cdx\sin^2(e+fx)+2a^2cdx\cos^2(e+fx)-\frac{2a^2cd\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2a^2cd\sin(e+fx)\cos(e+fx)}{f}-\frac{4a^2cd\cos^3(e+fx)}{3f}-\frac{2a^2cd\cos(e+fx)}{f}+\frac{3a^2d^2x\sin^4(e+fx)}{8}+\frac{3a^2d^2x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{a^2d^2x\sin^2(e+fx)}{2}+\frac{3a^2d^2x\cos^4(e+fx)}{8}+\frac{a^2d^2x\cos^2(e+fx)}{2}-\frac{5a^2d^2\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{2a^2d^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{3a^2d^2\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{4a^2d^2\cos^3(e+fx)}{3f} & \text{for}\ f\neq 0 \\ x(c+d\sin(e))^2(a\sin(e)+a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos(e
```

+ f*x)/f + 2*a**2*c*d*x*sin(e + f*x)**2 + 2*a**2*c*d*x*cos(e + f*x)**2 - 2
*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*c*d*sin(e + f*x)*cos(e +
f*x)/f - 4*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*a**2*c*d*cos(e + f*x)/f + 3*a
**2*d**2*x*sin(e + f*x)**4/8 + 3*a**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**
2/4 + a**2*d**2*x*sin(e + f*x)**2/2 + 3*a**2*d**2*x*cos(e + f*x)**4/8 + a**
2*d**2*x*cos(e + f*x)**2/2 - 5*a**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f)
 - 2*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*d**2*sin(e + f*x)*co
s(e + f*x)**3/(8*f) - a**2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*d*
*2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**2
, True))

**Giac** [A]
time = 0.48, size = 208, normalized size = 1.33

$$-\frac{2\,a^2cd\cos(fx+e)}{f}+\frac{a^2d^2\sin(4\,fx+4\,e)}{32\,f}-\frac{a^2d^2\sin(2\,fx+2\,e)}{4\,f}+\frac{1}{8}\left(4\,a^2c^2+16\,a^2cd+3\,a^2d^2\right)x+\frac{1}{2}\left(2\,a^2c^2+a^2d^2\right)x+\frac{(a^2cd+a^2d^2)\cos(3\,fx+3\,e)}{6\,f}-\frac{(4\,a^2c^2+3\,a^2cd+3\,a^2d^2)\cos(fx+e)}{2\,f}-\frac{(a^2c^2+4\,a^2cd+a^2d^2)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] -2*a^2*c*d*cos(f*x + e)/f + 1/32*a^2*d^2*sin(4*f*x + 4*e)/f - 1/4*a^2*d^2*s
in(2*f*x + 2*e)/f + 1/8*(4*a^2*c^2 + 16*a^2*c*d + 3*a^2*d^2)*x + 1/2*(2*a^2
*c^2 + a^2*d^2)*x + 1/6*(a^2*c*d + a^2*d^2)*cos(3*f*x + 3*e)/f - 1/2*(4*a^2
*c^2 + 3*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)/f - 1/4*(a^2*c^2 + 4*a^2*c*d + a
^2*d^2)*sin(2*f*x + 2*e)/f

**Mupad** [B]
time = 8.34, size = 440, normalized size = 2.82

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(12c^2+16cd+7d^2)}{4\left(3a^2c^2+4a^2cd+\frac{7a^2d^2}{4}\right)}\right)(12c^2+16cd+7d^2)}{4f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a^2c^2+4a^2cd+\frac{7a^2d^2}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(a^2c^2+4a^2cd+\frac{7a^2d^2}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(a^2c^2+4a^2cd+\frac{15a^2d^2}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(a^2c^2+4a^2cd+\frac{15a^2d^2}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(12a^2c^2+20a^2cd+8a^2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(12a^2c^2+\frac{68a^2cd}{3}+\frac{32a^2d^2}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4a^2c^2+4da^2c)+4a^2c^2+\frac{8a^2d^2}{3}+\frac{20a^2cd}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}-\frac{a^2\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(12c^2+16cd+7d^2)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)

[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(4*(3*a^2*c^2
+ (7*a^2*d^2)/4 + 4*a^2*c*d)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f) - (tan(e/2
+ (f*x)/2)*(a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) - tan(e/2 + (f*x)/2)^7*(a^
2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^3*(a^2*c^2 + (15*a^
2*d^2)/4 + 4*a^2*c*d) - tan(e/2 + (f*x)/2)^5*(a^2*c^2 + (15*a^2*d^2)/4 + 4*
a^2*c*d) + tan(e/2 + (f*x)/2)^4*(12*a^2*c^2 + 8*a^2*d^2 + 20*a^2*c*d) + tan
(e/2 + (f*x)/2)^2*(12*a^2*c^2 + (32*a^2*d^2)/3 + (68*a^2*c*d)/3) + tan(e/2
+ (f*x)/2)^6*(4*a^2*c^2 + 4*a^2*c*d) + 4*a^2*c^2 + (8*a^2*d^2)/3 + (20*a^2*
c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (
f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (
f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(4*f)

# 3.437 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=94

$$\frac{1}{2}a^2(3c+2d)x-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\cos(e+fx)\sin(e+fx)}{6f}-\frac{d\cos(e+fx)(a+a\sin(e+$$

```
[Out] 1/2*a^2*(3*c+2*d)*x-2/3*a^2*(3*c+2*d)*cos(f*x+e)/f-1/6*a^2*(3*c+2*d)*cos(f*
x+e)*sin(f*x+e)/f-1/3*d*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2830, 2723}

$$-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3c+2d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*x)/2 - (2*a^2*(3*c + 2*d)*Cos[e + f*x])/(3*f) - (a^2*(3*c
+ 2*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx=-\frac{d\cos(e+fx)(a+a\sin(e+fx))^2}{3f}+\frac{1}{3}(3c+2d)\int (a+a\,\mathrm{s}$$

$$=\frac{1}{2}a^2(3c+2d)x-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\,\mathrm{co}$$

## Mathematica [A]

time = 0.22, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e+fx)\left(6(3c+2d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(2(6c+5d)+3(c+2d)\sin(e+fx)+2d\sin^2(e+fx))\right)}{6f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(6*(3*c + 2*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(2*(6*c + 5*d) + 3*(c + 2*d)*Sin[e + f*x] + 2*d*Si
n[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

## Maple [A]

time = 0.23, size = 117, normalized size = 1.24

| method | result |
|---|---|
| risch | $\frac{3a^2cx}{2}+a^2xd-\frac{2a^2\cos(fx+e)c}{f}-\frac{7a^2\cos(fx+e)d}{4f}+\frac{a^2d\cos(3fx+3e)}{12f}-\frac{\sin(2fx+2e)a^2c}{4f}-\frac{\sin(2fx+2e)a^2d}{2f}$ |
| derivativedivides | $\frac{a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2a^2c\cos(fx+e)+2a^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| default | $\frac{a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2a^2c\cos(fx+e)+2a^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| norman | $\frac{(\frac{3}{2}a^2c+a^2d)x+(\frac{3}{2}a^2c+a^2d)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}a^2c+3a^2d)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}a^2c+3a^2d)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(c+2}{}}{\left(1+\tan^2\left(\frac{fx}{2}\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d*(2+sin(f*x+
e)^2)*cos(f*x+e)-2*a^2*c*cos(f*x+e)+2*a^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)+a^2*c*(f*x+e)-a^2*d*cos(f*x+e))
```

## Maxima [A]

time = 0.32, size = 123, normalized size = 1.31

$$\frac{3\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))a^2c+12\,(fx+e)a^2c+4\,\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2d+6\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))a^2d-24\,a^2c\cos\left(fx+e\right)-12\,a^2d\cos\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 12*(f*x + e)*a^2*c + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a^2*d + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
^2*d - 24*a^2*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.36, size = 86, normalized size = 0.91

$$\frac{2\,a^{2}d\cos\left(fx+e\right)^{3}+3\left(3\,a^{2}c+2\,a^{2}d\right)fx-3\left(a^{2}c+2\,a^{2}d\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-12\left(a^{2}c+a^{2}d\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*d*cos(f*x + e)^3 + 3*(3*a^2*c + 2*a^2*d)*f*x - 3*(a^2*c + 2*a^2*
d)*cos(f*x + e)*sin(f*x + e) - 12*(a^2*c + a^2*d)*cos(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.
time = 0.15, size = 199, normalized size = 2.12

$$\begin{cases} \frac{a^{2}cx\sin^{2}\left(e+fx\right)}{2}+\frac{a^{2}cx\cos^{2}\left(e+fx\right)}{2}+a^{2}cx-\frac{a^{2}c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^{2}c\cos\left(e+fx\right)}{f}+a^{2}dx\sin^{2}\left(e+fx\right)+a^{2}dx\cos^{2}\left(e+fx\right)-\frac{a^{2}d\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{a^{2}d\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a^{2}d\cos^{3}\left(e+fx\right)}{3f}-\frac{a^{2}d\cos\left(e+fx\right)}{f} & \text{for}\: f\neq 0\\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a**
2*d*x*sin(e + f*x)**2 + a**2*d*x*cos(e + f*x)**2 - a**2*d*sin(e + f*x)**2*c
os(e + f*x)/f - a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*a**2*d*cos(e + f*x)*
*3/(3*f) - a**2*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) +
a)**2, True))
```

**Giac** [A]
time = 0.44, size = 109, normalized size = 1.16

$$a^{2}cx+\frac{a^{2}d\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^{2}d\cos\left(fx+e\right)}{f}+\frac{1}{2}\left(a^{2}c+2\,a^{2}d\right)x-\frac{\left(8\,a^{2}c+3\,a^{2}d\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(a^{2}c+2\,a^{2}d\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^2*c*x + 1/12*a^2*d*cos(3*f*x + 3*e)/f - a^2*d*cos(f*x + e)/f + 1/2*(a^2*c
 + 2*a^2*d)*x - 1/4*(8*a^2*c + 3*a^2*d)*cos(f*x + e)/f - 1/4*(a^2*c + 2*a^2
*d)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 6.92, size = 91, normalized size = 0.97

$$-\frac{\frac{3\,a^2\,c\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{a^2\,d\,\cos\left(3\,e+3\,f\,x\right)}{2}+3\,a^2\,d\,\sin\left(2\,e+2\,f\,x\right)+12\,a^2\,c\,\cos\left(e+f\,x\right)+\frac{21\,a^2\,d\,\cos\left(e+f\,x\right)}{2}-9\,a^2\,c\,f\,x-6\,a^2\,d\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*a^2*c*sin(2*e + 2*f*x))/2 - (a^2*d*cos(3*e + 3*f*x))/2 + 3*a^2*d*sin(2
*e + 2*f*x) + 12*a^2*c*cos(e + f*x) + (21*a^2*d*cos(e + f*x))/2 - 9*a^2*c*f
*x - 6*a^2*d*f*x)/(6*f)
```

# 3.438 $\int (a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=45

$$\frac{3a^2x}{2}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 3/2*a^2*x-2*a^2*cos(f*x+e)/f-1/2*a^2*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2723}

$$-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f
)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.12, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(e+fx)+8\cos(e+fx)+\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(a^2*(-6*(e + f*x) + 8*Cos[e + f*x] + Sin[2*(e + f*x)]))/f
```

**Maple [A]**
time = 0.12, size = 52, normalized size = 1.16

| method | result |
|---|---|
| risch | $\frac{3a^2x}{2} - \frac{2a^2\cos(fx+e)}{f} - \frac{a^2\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\cos(fx+e)a^2+a^2(fx+e)}{f}$ |
| default | $\frac{a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\cos(fx+e)a^2+a^2(fx+e)}{f}$ |
| norman | $\frac{\frac{a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2x}{2}-\frac{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+3a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{3a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{4a^2\left(\tan^2\left(\frac{f}{}\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*cos(f*x+e)*a^2+a^2*(f
*x+e))
```

**Maxima [A]**
time = 0.28, size = 50, normalized size = 1.11

$$a^2x + \frac{(2\,fx + 2\,e - \sin(2\,fx + 2\,e))a^2}{4\,f} - \frac{2\,a^2\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2/f - 2*a^2*cos(f*x + e)/f
```

**Fricas [A]**
time = 0.35, size = 44, normalized size = 0.98

$$\frac{3\,a^2fx - a^2\cos(fx + e)\sin(fx + e) - 4\,a^2\cos(fx + e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*f*x - a^2*cos(f*x + e)*sin(f*x + e) - 4*a^2*cos(f*x + e))/f
```

**Sympy [A]**
time = 0.09, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^2(e+fx)}{2} + \frac{a^2x\cos^2(e+fx)}{2} + a^2x - \frac{a^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^2\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sin(e) + a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*x - a
**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**2, True))
```

**Giac** [A]
time = 0.46, size = 40, normalized size = 0.89

$$\frac{3}{2}\,a^2x-\frac{2\,a^2\cos\left(fx+e\right)}{f}-\frac{a^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x - 2*a^2*cos(f*x + e)/f - 1/4*a^2*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 6.87, size = 123, normalized size = 2.73

$$\frac{3\,a^2\,x}{2}-\frac{a^2\,\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)-a^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-a^2\,\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}-4\right)+a^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^2\,\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)-a^2\,\left(3\,e+3\,f\,x-4\right)\right)}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2,x)
```

```
[Out] (3*a^2*x)/2 - (a^2*((3*e)/2 + (3*f*x)/2) - a^2*tan(e/2 + (f*x)/2)^3 - a^2*(
(3*e)/2 + (3*f*x)/2 - 4) + a^2*tan(e/2 + (f*x)/2) + tan(e/2 + (f*x)/2)^2*(2
*a^2*((3*e)/2 + (3*f*x)/2) - a^2*(3*e + 3*f*x - 4)))/(f*(tan(e/2 + (f*x)/2)
^2 + 1)^2)
```

# 3.439 $\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=92

$$-\frac{a^2(c-2d)x}{d^2}+\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{a^2\cos(e+fx)}{df}$$

```
[Out] -a^2*(c-2*d)*x/d^2-a^2*cos(f*x+e)/d/f+2*a^2*(c-d)^2*arctan((d+c*tan(1/2*f*x
+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2825, 2814, 2739, 632, 210}

$$\frac{2a^2(c-d)^2\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{a^2x(c-2d)}{d^2}-\frac{a^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(c - 2*d)*x)/d^2) + (2*a^2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a^2*Cos[e + f*x])/(d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)}{df}+\frac{\int \frac{a^2d-a^2(c-2d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{(a^2(c-d)^2)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{(2a^2(c-d)^2)\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\right.\right.}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}-\frac{(4a^2(c-d)^2)\operatorname{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d\right.}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}+\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{a^2\cos(e+fx)}{df}$$

**Mathematica** [A]
time = 0.27, size = 130, normalized size = 1.41

$$-\frac{a^2\left(-2(c-d)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)+\sqrt{c^2-d^2}\left((c-2d)(e+fx)+d\cos(e+fx)\right)\right)(1+\sin(e+fx))^2}{d^2\sqrt{c^2-d^2}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]] + Sqr
t[c^2 - d^2]*((c - 2*d)*(e + f*x) + d*Cos[e + f*x]))*(1 + Sin[e + f*x])^2)/
(d^2*Sqrt[c^2 - d^2]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
```

**Maple** [A]
time = 0.33, size = 105, normalized size = 1.14

| method | result |
|---|---|
| derivativedivides | $\frac{2a^2\left(\frac{(c^2-2cd+d^2)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}-\frac{\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(c-2d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$ |
| default | $\frac{2a^2\left(\frac{(c^2-2cd+d^2)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}-\frac{\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(c-2d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$ |
| risch | $-\frac{a^2xc}{d^2}+\frac{2a^2x}{d}-\frac{a^2e^{i(fx+e)}}{2df}-\frac{a^2e^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}+\frac{ic+\sqrt{-(c+d)(}}{d}\right.}{(c+d)f\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2*((c^2-2*c*d+d^2)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))-1/d^2*(d/(1+tan(1/2*f*x+1/2*e)^2)+(c-2*d)*arctan
(tan(1/2*f*x+1/2*e))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**

time = 0.38, size = 307, normalized size = 3.34

$$\left[-\frac{2a^2d\cos(fx+e)+2(a^2c-2a^2d)fx+(a^2c-a^2d)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2\left((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)}{2d^2f},\ -\frac{a^2d\cos(fx+e)+(a^2c-2a^2d)fx+(a^2c-a^2d)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)}{d^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^2*d*cos(f*x + e) + 2*(a^2*c - 2*a^2*d)*f*x + (a^2*c - a^2*d)*sqr
t(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*
x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)))/(d^2*f), -(a^2*d*cos(f*x + e) + (a^2*c - 2*a^2*d)*f*x + (a^2*c
 - a^2*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(
c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3709 vs. $2(76) = 152$.
time = 180.78, size = 3709, normalized size = 40.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a*sin(e) + a)**2/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (2*a**2*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*
tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) -
 a**2*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan
(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*
a**2*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2
 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - a**2*d
**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(
3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 4*a**2*d**2*tan(e/2 + f*x/2)*
*2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*
tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - 2*a**2*d**2*tan(e/2 + f*x/2)/(d**3
*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2
+ f*x/2)**2 - f*(d**2)**(3/2)) + 4*a**2*d**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(
3/2)) + a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)*
*3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) - 2*a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 +
f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 -
 f*(d**2)**(3/2)) + a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2
+ f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
 - f*(d**2)**(3/2)) - 2*a**2*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 +
 d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**
(3/2)) + 4*a**2*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**
3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2
)**(3/2)) + 6*a**2*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/
2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)), Eq(c,
-sqrt(d**2))), (2*a**2*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2
)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d
```

```
**2)**(3/2)) - a**2*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**
3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2
)**(3/2)) + 2*a**2*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(
3/2)) - a**2*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2)
 + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 4*a**2*d**2*tan
(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*
(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 2*a**2*d**2*tan(e/2
+ f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**
(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 4*a**2*d**2/(d**3*f*tan(e/2
+ f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
 + f*(d**2)**(3/2)) - a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**3/(d**3*f*tan
(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/
2)**2 + f*(d**2)**(3/2)) + 2*a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**
3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2
 + f*x/2)**2 + f*(d**2)**(3/2)) - a**2*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)/(d
**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e
/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*a**2*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2
 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**
2 + f*(d**2)**(3/2)) - 4*a**2*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(
e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2
)**2 + f*(d**2)**(3/2)) - 6*a**2*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 +
 d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**
(3/2)), Eq(c, sqrt(d**2))), ((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x
)**2/2 + a**2*x - a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*cos(e + f*x
)/f)/c, Eq(d, 0)), (x*(a*sin(e) + a)**2/(c + d*sin(e)), Eq(f, 0)), ((2*a**2
*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + 2*a**2*f*x/(f*tan(e/
2 + f*x/2)**2 + f) + a**2*log(tan(e/2 + f*x/2))*tan(e/2 + f*x/2)**2/(f*tan(
e/2 + f*x/2)**2 + f) + a**2*log(tan(e/2 + f*x/2))/(f*tan(e/2 + f*x/2)**2 +
f) - 2*a**2/(f*tan(e/2 + f*x/2)**2 + f))/d, Eq(c, 0)), (a**2*c**2*log(tan(e
/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(
-c**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) + a**2*c**2*
log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**
2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - a**2*c**2*log(tan(e/2
 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c
**2 + d**2)*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - a**2*c**2*lo
g(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*f*sqrt(-c**2 + d**2)
*tan(e/2 + f*x/2)**2 + d**2*f*sqrt(-c**2 + d**2)) - 2*a**2*c*d*log(tan(e/2
+ f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(e/2 ...
```

**Giac [A]**

time = 0.46, size = 136, normalized size = 1.48

$$-\frac{\frac{(a^2c-2\,a^2d)(fx+e)}{d^2}+\frac{2\,a^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\,(a^2c^2-2\,a^2cd+a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((a^2*c - 2*a^2*d)*(f*x + e)/d^2 + 2*a^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

**Mupad** [B]
time = 7.68, size = 940, normalized size = 10.22

$$\frac{4\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,(c+d)}-\frac{a^2\cos(e+f\,x)}{f\,(c+d)}+\frac{2\,a^2\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f\,(c+d)}-\frac{2\,a^2\,c^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d^2\,f\,(c+d)}-\frac{a^2\,c\cos(e+f\,x)}{d\,f\,(c+d)}+\text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x)),x)
```

```
[Out] (4*a^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (a^2*cos(
e + f*x))/(f*(c + d)) + (2*a^2*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)
))/(d*f*(c + d)) - (2*a^2*c^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/
(d^2*f*(c + d)) + (a^2*atan(((3*d^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 -
 c^4 + d^4)^(3/2) - 2*c^6*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4
)^(1/2) - 2*c^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(3/2) +
7*d^6*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 10*c*d^5*s
in(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 4*c^5*d*sin(e/2 +
 (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 4*c^2*d^4*cos(e/2 + (f*x)
/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) - 3*c^3*d^3*cos(e/2 + (f*x)/2)*(2
*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) - 2*c^4*d^2*cos(e/2 + (f*x)/2)*(2*c^3*d
 - 2*c*d^3 - c^4 + d^4)^(1/2) - 9*c^2*d^4*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c
*d^3 - c^4 + d^4)^(1/2) - 12*c^3*d^3*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3
- c^4 + d^4)^(1/2) + 6*c^4*d^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4
+ d^4)^(1/2) + c*d*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(3/2)
 + 4*c*d^5*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + c^5*d
*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2))*1i)/(d*(c + d)*(
3*c^5*d*cos(e/2 + (f*x)/2) - 5*c*d^5*cos(e/2 + (f*x)/2) - 10*d^6*sin(e/2 +
(f*x)/2) + 16*c*d^5*sin(e/2 + (f*x)/2) + 8*c^2*d^4*cos(e/2 + (f*x)/2) + 2*c
^3*d^3*cos(e/2 + (f*x)/2) - 8*c^4*d^2*cos(e/2 + (f*x)/2) + 4*c^2*d^4*sin(e/
2 + (f*x)/2) - 16*c^3*d^3*sin(e/2 + (f*x)/2) + 6*c^4*d^2*sin(e/2 + (f*x)/2)
)))*(-(c + d)*(c - d)^3)^(1/2)*2i)/(d^2*f*(c + d)) - (a^2*c*cos(e + f*x))/(
d*f*(c + d))
```

# 3.440 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=112

$$\frac{a^2x}{d^2}-\frac{2a^2(c-d)^2(c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] a^2*x/d^2-2*a^2*(c-d)^2*(c+2*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(
1/2))/d^2/(c^2-d^2)^(3/2)/f+a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2841, 2814, 2739, 632, 210}

$$-\frac{2a^2(c-d)(c+2d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(c-d)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{a^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/d^2 - (2*a^2*(c - d)*(c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/(d^2*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/
(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx &= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}-\frac{a\int\frac{-2ad-a(c+d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d(c+d)}\\
&= \frac{a^2x}{d^2}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}-\frac{\left(a^2(c-d)(c+2d)\right)\int\frac{1}{c+d\sin(e+fx)}\,dx}{d^2(c+d)}\\
&= \frac{a^2x}{d^2}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}-\frac{\left(2a^2(c-d)(c+2d)\right)\text{Subst}\left(\int\frac{1}{c+2dx+cx^2}\right.}{d^2(c+d)f}\\
&= \frac{a^2x}{d^2}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}+\frac{\left(4a^2(c-d)(c+2d)\right)\text{Subst}\left(\int\frac{1}{-4(c^2-d^2)-}\right.}{d^2(c+}\\
&= \frac{a^2x}{d^2}-\frac{2a^2(c-d)(c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+}
\end{aligned}
$$

**Mathematica [A]**
time = 0.33, size = 139, normalized size = 1.24

$$
\frac{a^2(1+\sin(e+fx))^2\left(e+fx-\frac{2\left(c^2+cd-2d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}+\frac{(c-d)d\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}\right)}{d^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(e + f*x - (2*(c^2 + c*d - 2*d^2)*ArcTan[(d + c*T
an[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) + ((c - d)*d*C
os[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^2*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4)
```

**Maple [A]**
time = 0.47, size = 160, normalized size = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{2a^2\left(-\frac{\frac{-\frac{d^2(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(c-d)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(c^2+cd-2d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$ |
| default | $\frac{2a^2\left(-\frac{\frac{-\frac{d^2(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(c-d)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(c^2+cd-2d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$ |
| risch | $\frac{a^2x}{d^2}-\frac{2ia^2(c-d)\left(id+c\,e^{i(fx+e)}\right)}{d^2(c+d)f\left(-id\,e^{2i(fx+e)}+id+2c\,e^{i(fx+e)}\right)}+\frac{\sqrt{-(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}+\frac{ic-\sqrt{-(c+d)}}{d}\right.}{(c+d)^2fd^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2*(-1/d^2*((-d^2*(c-d)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(c-d)/(c+d))/(c*t
an(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(c^2+c*d-2*d^2)/(c+d)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))+1/d^2*ar
ctan(tan(1/2*f*x+1/2*e)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.37, size = 493, normalized size = 4.40

$$\left[\frac{2\,(a^2cd+a^2d^2)fx\sin(fx+e)+2\,(a^2c^2+a^2cd)fx+(a^2c^2+2\,a^2cd+(a^2cd+2\,a^2d^2)\sin(fx+e))\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2+2\,((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e))\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)+2\,(a^2cd-a^2d^2)\cos(fx+e)}{2\,((cd^3+d^4)f\sin(fx+e)+(c^2d^2+cd^3)f)}, \frac{(a^2cd+a^2d^2)fx\sin(fx+e)+(a^2c^2+a^2cd)fx+(a^2c^2+2\,a^2cd+(a^2cd+2\,a^2d^2)\sin(fx+e))\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)+(a^2cd-a^2d^2)\cos(fx+e)}{(cd^3+d^4)f\sin(fx+e)+(c^2d^2+cd^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + 2*(a^2*c^2 + a^2*c*d)*f*x +
(a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x + e) + (c^2*d
^2 + c*d^3)*f), ((a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + (a^2*c^2 + a^2*c*d)
*f*x + (a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt((c -
 d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*co
s(f*x + e))) + (a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x +
 e) + (c^2*d^2 + c*d^3)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.45, size = 205, normalized size = 1.83

$$\frac{\frac{(fx+e)a^2}{d^2}-\frac{2\,(a^2c^2+a^2cd-2\,a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}}+\frac{2\,(a^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2c^2-a^2cd)}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/d^2 - 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x +
 e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2))
)/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(a^2*c*d*tan(1/2*f*x + 1/2*e) - a^2*d
^2*tan(1/2*f*x + 1/2*e) + a^2*c^2 - a^2*c*d)/((c^2*d + c*d^2)*(c*tan(1/2*f*
x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

**Mupad [B]**
time = 11.24, size = 2836, normalized size = 25.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*c - a^2*d))/(d*(c + d)) + (2*a^2*tan(e/2 + (f*x)/2)*(c - d))/(c*(c
 + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (2*a^2*
atan((192*a^6*c^3*d*tan(e/2 + (f*x)/2))/((128*a^6*c^3*d^5)/(2*c*d^3 + d^4 +
 c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*d^3 + d^4 + c^2*d^2) - (320*a^6*c*d^7)/(
2*c*d^3 + d^4 + c^2*d^2) + (512*a^6*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^2) + (1
92*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^2*d^2)) - (320*a^6*c*d^3*tan(e/2 + (f*x)
/2))/((128*a^6*c^3*d^5)/(2*c*d^3 + d^4 + c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*
d^3 + d^4 + c^2*d^2) - (320*a^6*c*d^7)/(2*c*d^3 + d^4 + c^2*d^2) + (512*a^6
*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^2) + (192*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^
2*d^2)) + (128*a^6*c^2*d^2*tan(e/2 + (f*x)/2))/((128*a^6*c^3*d^5)/(2*c*d^3
+ d^4 + c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*d^3 + d^4 + c^2*d^2) - (320*a^6*c
*d^7)/(2*c*d^3 + d^4 + c^2*d^2) + (512*a^6*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^
2) + (192*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^2*d^2))))/(d^2*f) + (a^2*atan(((a
^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a
^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^
5 + 2*a^4*c^5*d - 8*a^4*c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4
+ d^5 + c^2*d^3) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*tan(e/2 +
 (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) - (32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c
^2*d^2) + (a^2*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d
^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2
*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))
/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d
^2))*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*(-(c + d)^3*(c - d))^
(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c*d^3 +
d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d - 8*a^4*
c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*
(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d
^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 -
 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*(c^2
*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x
```

)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))/((64*(2*a^6*c^3 - 4*a^6*c*d^2 + 2*a^6*c^2*d))/(2*c*d^3 + d^4 + c^2*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^6*c^4 - 4*a^6*c*d^3 + 4*a^6*c^3*d - 2*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d - 8*a^4*c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*tan(e/2 + (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c^2*d^2) + (a^2*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d - 8*a^4*c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*2i)/(f*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))

# 3.441 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=138

$$\frac{3a^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}\,f}+\frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))}$$

```
[Out] 1/2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a^2*(c+4*d)*cos(f
*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))+3*a^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c
^2-d^2)^(1/2))/(c+d)^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2841, 2833, 12, 2739, 632, 210}

$$\frac{3a^2\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(c+4d)\cos(e+fx)}{2df(c+d)^2(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^
2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x]
)^2) - (a^2*(c + 4*d)*Cos[e + f*x])/(2*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{-4ad-a(c+3d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{a\int \frac{3a(c-d}{c+d\sin(e}}{2(c-d)d(}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{(3a^2)\int \frac{}{c+d}}{2(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{(3a^2)\,\text{Subst}}{}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} - \frac{(6a^2)\,\text{Subst}}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d}{2d(c+d)^2f(c}$$

**Mathematica [A]**
time = 0.40, size = 140, normalized size = 1.01

$$\frac{a^2\cos(e+fx)\left(-\frac{6\tanh^{-1}\left(\frac{\sqrt{c-d}\,\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\,\sqrt{1+\sin(e+fx)}}\right)}{(-c-d)^{3/2}\sqrt{c-d}\,\sqrt{\cos^2(e+fx)}} - \frac{4c+d+(c+4d)\sin(e+fx)}{(c+d)(c+d\sin(e+fx))^2}\right)}{2(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*Cos[e + f*x]*((-6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(3/2)*Sqrt[c - d]*Sqrt[Cos[e + f
*x]^2]) - (4*c + d + (c + 4*d)*Sin[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^
2)))/(2*(c + d)*f)
```

**Maple [A]**
time = 0.58, size = 250, normalized size = 1.81

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2a^2\left(\frac{\frac{(c^2-4cd-2d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c^2+2cd+d^2)c}-\frac{(4c^3+c^2d+8c\,d^2+2d^3)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c^2+2cd+d^2)c^2}-\frac{(c^2+12cd+2d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c(c^2+2cd+d^2)}-\frac{4c+d}{2(c^2+2cd+d^2)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{f}$ |
| default | $\frac{2a^2\left(\frac{\frac{(c^2-4cd-2d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c^2+2cd+d^2)c}-\frac{(4c^3+c^2d+8c\,d^2+2d^3)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c^2+2cd+d^2)c^2}-\frac{(c^2+12cd+2d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c(c^2+2cd+d^2)}-\frac{4c+d}{2(c^2+2cd+d^2)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{f}$ |
| risch | $\frac{ia^2\left(-2ic^2d\,e^{3i(fx+e)}-4ic\,d^2e^{3i(fx+e)}+id^3e^{3i(fx+e)}+2ic^2d\,e^{i(fx+e)}+12ic\,d^2e^{i(fx+e)}+id^3e^{i(fx+e)}+2c^3e^{2i(fx+e)}+8d\right.}{(c+d)^2\left(-id\,e^{2i(fx+e)}+id+2c\,e^{i(fx+e)}\right)^2f\,d^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2*((1/2*(c^2-4*c*d-2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*
(4*c^3+c^2*d+8*c*d^2+2*d^3)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2-1/2*(c
^2+12*c*d+2*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-1/2*(4*c+d)/(c^2+2*c*
d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+3/2/(c^2+2*c*d+
d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(134) = 268$.
time = 0.36, size = 702, normalized size = 5.09

$$\left[\frac{2\,(a^2c^3+4\,a^2c^2d-a^2cd^2-4\,a^2d^3)\cos(fx+e)\sin(fx+e)-3\,(a^2d^2\cos(fx+e)^2-2\,a^2cd\sin(fx+e)-a^2c^2-a^2d^2)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2\,(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2\,(4\,a^2c^3+a^2c^2d-4\,a^2cd^2-a^2d^3)\cos(fx+e)}{4\,((c^4d^2+2\,c^3d^3-2\,cd^5-d^6)f\cos(fx+e)^2-2\,(c^5d+2\,c^4d^2-2\,c^2d^4-cd^5)f\sin(fx+e)-(c^6+2\,c^5d+c^4d^2-c^2d^4-2\,cd^5-d^6)f)},\ \frac{(a^2c^3+4\,a^2c^2d-a^2cd^2-4\,a^2d^3)\cos(fx+e)\sin(fx+e)-3\,(a^2d^2\cos(fx+e)^2-2\,a^2cd\sin(fx+e)-a^2c^2-a^2d^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+(4\,a^2c^3+a^2c^2d-4\,a^2cd^2-a^2d^3)\cos(fx+e)}{2\,((c^4d^2+2\,c^3d^3-2\,cd^5-d^6)f\cos(fx+e)^2-2\,(c^5d+2\,c^4d^2-2\,c^2d^4-cd^5)f\sin(fx+e)-(c^6+2\,c^5d+c^4d^2-c^2d^4-2\,cd^5-d^6)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2
 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a^2*
c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c^4*d^2 + 2*c^3*d^
3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2 - 2*c^2*d^4 - c*
d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 - 2*c*d^5 - d^6)*f
), 1/2*((a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c
^4*d^2 + 2*c^3*d^3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2
 - 2*c^2*d^4 - c*d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 -
 2*c*d^5 - d^6)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(134) = 268.
time = 0.51, size = 348, normalized size = 2.52

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a^2}{(c^2+2cd+d^2)\sqrt{c^2-d^2}}+\frac{a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-8a^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-12a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2a^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4a^2c^3-a^2c^2d}{(c^4+2c^3d+c^2d^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a^2/((c^2 + 2*c*d + d^2)*sqrt(c^2 - d^2)) + (a^2*c
^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^3*tan(1/2*f*x + 1/2*e)^2 - a^2*c^2*d*tan
(1/2*f*x + 1/2*e)^2 - 8*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^3*tan(1/
2*f*x + 1/2*e)^2 - a^2*c^3*tan(1/2*f*x + 1/2*e) - 12*a^2*c^2*d*tan(1/2*f*x
+ 1/2*e) - 2*a^2*c*d^2*tan(1/2*f*x + 1/2*e) - 4*a^2*c^3 - a^2*c^2*d)/((c^4
+ 2*c^3*d + c^2*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

**Mupad [B]**
time = 9.43, size = 362, normalized size = 2.62

$$\frac{3\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{3\,a^2\,\left(2\,c^2\,d+4\,c\,d^2+2\,d^3\right)}{2\,{\left(c+d\right)}^{5/2}\,\sqrt{c-d}\,\left(c^2+2\,c\,d+d^2\right)}+\frac{3\,a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{{\left(c+d\right)}^{5/2}\,\sqrt{c-d}}\right)\,\left(c^2+2\,c\,d+d^2\right)}{3\,a^2}\right)}{f\,{\left(c+d\right)}^{5/2}\,\sqrt{c-d}}-\frac{\frac{4\,a^2\,c+a^2\,d}{c^2+2\,c\,d+d^2}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^2+12\,a^2\,c\,d+2\,a^2\,d^2\right)}{c\,\left(c^2+2\,c\,d+d^2\right)}+\frac{a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(-c^2+4\,c\,d+2\,d^2\right)}{c\,\left(c^2+2\,c\,d+d^2\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(c^2+2\,d^2\right)\,\left(4\,a^2\,c+a^2\,d\right)}{c^2\,\left(c^2+2\,c\,d+d^2\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2+4\,d^2\right)+c^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+c^2+4\,c\,d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (3*a^2*atan((((3*a^2*(4*c*d^2 + 2*c^2*d + 2*d^3))/(2*(c + d)^(5/2)*(c - d)^
(1/2)*(2*c*d + c^2 + d^2)) + (3*a^2*c*tan(e/2 + (f*x)/2))/((c + d)^(5/2)*(c
 - d)^(1/2)))*(2*c*d + c^2 + d^2))/(3*a^2)))/(f*(c + d)^(5/2)*(c - d)^(1/2)
) - ((4*a^2*c + a^2*d)/(2*c*d + c^2 + d^2) + (tan(e/2 + (f*x)/2)*(a^2*c^2 +
 2*a^2*d^2 + 12*a^2*c*d))/(c*(2*c*d + c^2 + d^2)) + (a^2*tan(e/2 + (f*x)/2)
^3*(4*c*d - c^2 + 2*d^2))/(c*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/2)^2*(
c^2 + 2*d^2)*(4*a^2*c + a^2*d))/(c^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f
*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 +
 (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

# 3.442 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=207

$$\frac{a^2(3c-2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^3\sqrt{c^2-d^2}\,f}+\frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3}-\frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2}-\frac{a}{6(c}$$

```
[Out] 1/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^3-1/6*a^2*(c+6*d)*cos(f
*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a^2*(c^2+6*c*d-10*d^2)*cos(f*x+e)/
(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))+a^2*(3*c-2*d)*arctan((d+c*tan(1/2*f*x+1/
2*e))/(c^2-d^2)^(1/2))/(c-d)/(c+d)^3/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.23, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2841, 2833, 12, 2739, 632, 210}

$$\frac{a^2(3c-2d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c-d)(c+d)^3\sqrt{c^2-d^2}}-\frac{a^2(c^2+6cd-10d^2)\cos(e+fx)}{6df(c-d)(c+d)^3(c+d\sin(e+fx))}-\frac{a^2(c+6d)\cos(e+fx)}{6df(c+d)^2(c+d\sin(e+fx))^2}+\frac{a^2(c-d)\cos(e+fx)}{3df(c+d)(c+d\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(3*c - 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c - d)
*(c + d)^3*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(
c + d*Sin[e + f*x])^3) - (a^2*(c + 6*d)*Cos[e + f*x])/(6*d*(c + d)^2*f*(c +
 d*Sin[e + f*x])^2) - (a^2*(c^2 + 6*c*d - 10*d^2)*Cos[e + f*x])/(6*(c - d)*
d*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx = \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{-6ad-a(c+5d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} + \frac{a\int \frac{10a(c-d}{}}{6(}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2(c^2+}{6(c-d)d(}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2(c^2+}{6(c-d)d(}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2(c^2+}{6(c-d)d(}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2(c^2+}{6(c-d)d(}$$

$$= \frac{a^2(3c-2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^3\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a}{6d(c-}$$

**Mathematica** [A]
time = 1.64, size = 196, normalized size = 0.95

$$\frac{a^2\cos(e+fx)\left(-\frac{d(1+\sin(e+fx))^2}{(c+d\sin(e+fx))^3}-\frac{(3c-2d)\left(\frac{6\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)}{\sqrt{-c-d}\sqrt{c-d}}-\frac{\sqrt{\cos^2(e+fx)}(4c+d+(c+4d)\sin(e+fx))}{(c+d\sin(e+fx))^2}\right)}{2(c+d)^2\sqrt{\cos^2(e+fx)}}\right)}{3(-c+d)(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^2)/(c + d*Sin[e + f*x])^3) - ((3
*c - 2*d)*((6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*Sq
rt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*Sqrt[c - d]) - (Sqrt[Cos[e + f*x]^2]*
(4*c + d + (c + 4*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2))/(2*(c + d)^2*S
qrt[Cos[e + f*x]^2])))/(3*(-c + d)*(c + d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(196) = 392$.
time = 0.86, size = 523, normalized size = 2.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2*((1/2*(c^4-6*c^3*d+4*c*d^3+2*d^4)/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1
/2*f*x+1/2*e)^5-1/2*(4*c^5-3*c^4*d+18*c^3*d^2-8*c^2*d^3-12*c*d^4-4*d^5)/c^2
/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4-1/3/c^3*d*(36*c^5-21*c^4*d+
6*c^3*d^2-20*c^2*d^3-12*c*d^4-4*d^5)/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+
1/2*e)^3-(4*c^5-2*c^4*d+12*c^3*d^2-11*c^2*d^3-6*c*d^4-2*d^5)/c^2/(c^4+2*c^3
*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2-1/2*(c^4+18*c^3*d-14*c^2*d^2-8*c*d^3-2
*d^4)/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)-1/6*(12*c^3-7*c^2*d-6*
c*d^2-2*d^3)/(c^4+2*c^3*d-2*c*d^3-d^4))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2
*f*x+1/2*e)+c)^3+1/2*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(203) = 406.
time = 0.40, size = 1395, normalized size = 6.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^
2*d^5)*cos(f*x + e)^3 - 6*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^
2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4 -
 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3
)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4 -
 (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*l
og(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*co
s(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(2*a^2*c^5 - a^2*c^4*d - 2*a^2
```

```
*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c^7*d^2 + 2*c^6*d^3 -
 c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*cos(f*x + e)^2 - (c^9
 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4*d^5 - 2*c^3*d^6 + 6
*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6 - c^2*d
^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7*d^2 - 2*c^6*d^3 - 1
0*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8 + d^9)*f)*sin(f*x +
 e)), -1/6*((a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*
a^2*d^5)*cos(f*x + e)^3 - 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*
c^2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4
 - 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d
^3)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4
 - (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c
^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*c
os(f*x + e)^2 - (c^9 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4
*d^5 - 2*c^3*d^6 + 6*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5
 - 4*c^3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7
*d^2 - 2*c^6*d^3 - 10*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8
 + d^9)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(203) = 406.
time = 0.51, size = 781, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 + 2*c^3*d - 2*c*d^3
- d^4)*sqrt(c^2 - d^2)) + (3*a^2*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^5*d*
tan(1/2*f*x + 1/2*e)^5 + 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^5 + 6*a^2*c^2*
d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a^2*c^6*tan(1/2*f*x + 1/2*e)^4 + 9*a^2*c^5*
d*tan(1/2*f*x + 1/2*e)^4 - 54*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a^2*c
```

^3*d^3*tan(1/2*f*x + 1/2*e)^4 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 + 12*
a^2*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 72*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 42
*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 + 40*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^5*tan(1/2*f*x + 1/2*e
)^3 + 8*a^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 24*a^2*c^6*tan(1/2*f*x + 1/2*e)^2
+ 12*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)
^2 + 66*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1
/2*e)^2 + 12*a^2*c*d^5*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*c^6*tan(1/2*f*x + 1/2
*e) - 54*a^2*c^5*d*tan(1/2*f*x + 1/2*e) + 42*a^2*c^4*d^2*tan(1/2*f*x + 1/2*
e) + 24*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^4*tan(1/2*f*x + 1/2*
e) - 12*a^2*c^6 + 7*a^2*c^5*d + 6*a^2*c^4*d^2 + 2*a^2*c^3*d^3)/((c^7 + 2*c^
6*d - 2*c^4*d^3 - c^3*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/
2*e) + c)^3))/f

**Mupad [B]**
time = 10.32, size = 735, normalized size = 3.55

$$-\frac{\frac{-12\,a^2\,c^3+7\,a^2\,c^2\,d+6\,a^2\,c\,d^2+2\,a^2\,d^3}{3\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(c^4-6\,c^3\,d+4\,c\,d^3+2\,d^4\right)}{c\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-4\,c^5+2\,c^4\,d-12\,c^3\,d^2+11\,c^2\,d^3+6\,c\,d^4+2\,d^5\right)}{c^2\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(-4\,c^5+3\,c^4\,d-18\,c^3\,d^2+8\,c^2\,d^3+12\,c\,d^4+4\,d^5\right)}{c^2\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(-c^4-18\,c^3\,d+14\,c^2\,d^2+8\,c\,d^3+2\,d^4\right)}{c\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{2\,a^2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,c^2+2\,d^2\right)\,\left(-12\,c^3+7\,c^2\,d+6\,c\,d^2+2\,d^3\right)}{3\,c^3\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}}{f\,\left(c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(3\,c^3+12\,c\,d^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(3\,c^3+12\,c\,d^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(12\,c^2\,d+8\,d^3\right)+c^3+6\,c^2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+6\,c^2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right)}-\frac{a^2\,\mathrm{atan}\left(\frac{\left(\frac{a^2\,\left(3\,c-2\,d\right)\,\left(-2\,c^4\,d-4\,c^3\,d^2+4\,c\,d^4+2\,d^5\right)}{2\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{3/2}\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,c-2\,d\right)}{{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{3/2}}\right)\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}{3\,a^2\,c-2\,a^2\,d}\right)\,\left(3\,c-2\,d\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^4,x)

[Out] - ((2*a^2*d^3 - 12*a^2*c^3 + 6*a^2*c*d^2 + 7*a^2*c^2*d)/(3*(2*c*d^3 - 2*c^3
*d - c^4 + d^4)) + (a^2*tan(e/2 + (f*x)/2)^5*(4*c*d^3 - 6*c^3*d + c^4 + 2*d
^4))/(c*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) + (2*a^2*tan(e/2 + (f*x)/2)^2*(6*c
*d^4 + 2*c^4*d - 4*c^5 + 2*d^5 + 11*c^2*d^3 - 12*c^3*d^2))/(c^2*(2*c*d^3 -
2*c^3*d - c^4 + d^4)) + (a^2*tan(e/2 + (f*x)/2)^4*(12*c*d^4 + 3*c^4*d - 4*c
^5 + 4*d^5 + 8*c^2*d^3 - 18*c^3*d^2))/(c^2*(2*c*d^3 - 2*c^3*d - c^4 + d^4))
 + (a^2*tan(e/2 + (f*x)/2)*(8*c*d^3 - 18*c^3*d - c^4 + 2*d^4 + 14*c^2*d^2))
/(c*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) + (2*a^2*d*tan(e/2 + (f*x)/2)^3*(3*c^2
 + 2*d^2)*(6*c*d^2 + 7*c^2*d - 12*c^3 + 2*d^3))/(3*c^3*(2*c*d^3 - 2*c^3*d -
 c^4 + d^4)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2
 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*
(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (
f*x)/2)^5)) - (a^2*atan((((a^2*(3*c - 2*d)*(4*c*d^4 - 2*c^4*d + 2*d^5 - 4*c
^3*d^2))/(2*(c + d)^(7/2)*(c - d)^(3/2)*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) +
(a^2*c*tan(e/2 + (f*x)/2)*(3*c - 2*d))/((c + d)^(7/2)*(c - d)^(3/2)))*(2*c*
d^3 - 2*c^3*d - c^4 + d^4))/(3*a^2*c - 2*a^2*d))*(3*c - 2*d))/(f*(c + d)^(7
/2)*(c - d)^(3/2))

## 3.443 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=286

$$\frac{a^2(12c^2-16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)^2(c+d)^4\sqrt{c^2-d^2}\,f}+\frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4}-\frac{a^2(c+8d)\cos(e+fx}{12d(c+d)^2f(c+d\sin(e+}$$

```
[Out] 1/4*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^4-1/12*a^2*(c+8*d)*cos(
f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^3-1/24*a^2*(2*c^2+16*c*d-21*d^2)*cos(f*
x+e)/(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))^2-1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2
+32*d^3)*cos(f*x+e)/(c-d)^2/d/(c+d)^4/f/(c+d*sin(f*x+e))+1/4*a^2*(12*c^2-16
*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c-d)^2/(c+d)^
4/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.36, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2841, 2833, 12, 2739, 632, 210}

$$\frac{a^2(12c^2-16cd+7d^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)^2(c+d)^4\sqrt{c^2-d^2}}-\frac{a^2(2c^2+16cd-21d^2)\cos(e+fx)}{24df(c-d)(c+d)^3(c+d\sin(e+fx))^2}-\frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\cos(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sin(e+fx))}-\frac{a^2(c+8d)\cos(e+fx)}{12df(c+d)^2(c+d\sin(e+fx))^3}+\frac{a^2(c-d)\cos(e+fx)}{4df(c+d)(c+d\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(4*(c - d)^2*(c + d)^4*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x]
)/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) - (a^2*(c + 8*d)*Cos[e + f*x])/(12
*d*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^2*(2*c^2 + 16*c*d - 21*d^2)*Cos
[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sin[e + f*x])^2) - (a^2*(2*c^3
+ 16*c^2*d - 59*c*d^2 + 32*d^3)*Cos[e + f*x])/(24*(c - d)^2*d*(c + d)^4*f*(
c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx = \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{-8ad-a(c+7d)\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx}{4d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} + \frac{a\int \frac{21a(c-}{}}{1}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2(2c^2}{24(c-d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2(2c^2}{24(c-d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2(2c^2}{24(c-d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2(2c^2}{24(c-d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2(2c^2}{24(c-d}$$

$$= \frac{a^2(12c^2-16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)^2(c+d)^4\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx)}$$

**Mathematica** [A]
time = 3.09, size = 269, normalized size = 0.94

$$-\frac{a^2\cos(e+fx)\left(6d(1+\sin(e+fx))^2+\frac{2(5c-2d)d(1+\sin(e+fx))^2(c+d\sin(e+fx))}{(c-d)(c+d)}+\frac{(12c^2-16cd+7d^2)(c+d\sin(e+fx))^2\left(-6\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)(c+d\sin(e+fx))^2+\sqrt{-c-d}\sqrt{c-d}\sqrt{\cos^2(e+fx)}(4c+d+(c+4d)\sin(e+fx))\right)}{(-c-d)^{7/2}(c-d)^{3/2}\sqrt{\cos^2(e+fx)}}\right)}{24(-c+d)(c+d)f(c+d\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*d*(1 + Sin[e + f*x])^2 + (2*(5*c - 2*d)*d*(1 + S
in[e + f*x])^2*(c + d*Sin[e + f*x]))/((c - d)*(c + d)) + ((12*c^2 - 16*c*d
+ 7*d^2)*(c + d*Sin[e + f*x])^2*(-6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f
*x]])/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])]*(c + d*Sin[e + f*x])^2 + Sqrt[
-c - d]*Sqrt[c - d]*Sqrt[Cos[e + f*x]^2]*(4*c + d + (c + 4*d)*Sin[e + f*x])
))/((-c - d)^(7/2)*(c - d)^(3/2)*Sqrt[Cos[e + f*x]^2])))/((-c + d)*(c + d)*
f*(c + d*Sin[e + f*x])^4)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(271) = 542$.
time = 1.47, size = 1056, normalized size = 3.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2*((1/8*(4*c^6-32*c^5*d+15*c^4*d^2+32*c^3*d^3+8*c^2*d^4-16*c*d^5-8*d^
6)/c/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6)*tan(1/2*f*x+1/2*e)
^7-1/8*(16*c^7-28*c^6*d+144*c^5*d^2-137*c^4*d^3-112*c^3*d^4+8*c^2*d^5+64*c*
d^6+24*d^7)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6)/c^2*tan(1/2
*f*x+1/2*e)^6+1/24/c^3*(12*c^8-480*c^7*d+597*c^6*d^2-480*c^5*d^3+836*c^4*d^
4+208*c^3*d^5-152*c^2*d^6-256*c*d^7-96*d^8)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-
c^2*d^4+2*c*d^5+d^6)*tan(1/2*f*x+1/2*e)^5-1/24/c^4*(144*c^9-204*c^8*d+1104*
c^7*d^2-1617*c^6*d^3+48*c^5*d^4-406*c^4*d^5+256*c^3*d^6+184*c^2*d^7+128*c*d
^8+48*d^9)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6)*tan(1/2*f*x+
1/2*e)^4-1/24/c^3*(12*c^8+672*c^7*d-1035*c^6*d^2+672*c^5*d^3-1220*c^4*d^4+8
0*c^3*d^5+152*c^2*d^6+256*c*d^7+96*d^8)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*
d^4+2*c*d^5+d^6)*tan(1/2*f*x+1/2*e)^3-1/24*(144*c^7-188*c^6*d+656*c^5*d^2-1
201*c^4*d^3+16*c^3*d^4+120*c^2*d^5+192*c*d^6+72*d^7)/(c^6+2*c^5*d-c^4*d^2-4
*c^3*d^3-c^2*d^4+2*c*d^5+d^6)/c^2*tan(1/2*f*x+1/2*e)^2-1/24*(12*c^6+288*c^5
*d-499*c^4*d^2-32*c^3*d^3+64*c^2*d^4+80*c*d^5+24*d^6)/c/(c^6+2*c^5*d-c^4*d^
2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6)*tan(1/2*f*x+1/2*e)-1/24*(48*c^5-68*c^4*d-1
6*c^3*d^2+5*c^2*d^3+16*c*d^4+6*d^5)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+
2*c*d^5+d^6))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^4+1/8*(12*c
^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6)/(c^2-d
^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(280) = 560$.
time = 0.46, size = 2186, normalized size = 7.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

[Out] [1/48*(2*(8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^6 - 16*a^2*c^5*d + 79*a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^4 - 2*(36*a^2*c^4*d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + 2*((2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2*c^5*d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6*c^9*d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2 + 6*c^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9 - 4*c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 - (c^11*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4*d^8 + c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e)), 1/24*((8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^6 - 16*a^2*c^5*d + 79*a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^4 - 2*(36*a^2*c^4*d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 3*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + ((2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2*c^5*d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6*c^9*d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2 + 6*c^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9 - 4*c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 - (c^11*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4*d^8

```
+ c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. 2(280) = 560.
time = 0.58, size = 1555, normalized size = 5.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 +
 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2))
+ (12*a^2*c^9*tan(1/2*f*x + 1/2*e)^7 - 96*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^7
+ 45*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 96*a^2*c^6*d^3*tan(1/2*f*x + 1/2*
e)^7 + 24*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^4*d^5*tan(1/2*f*x +
 1/2*e)^7 - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^9*tan(1/2*f*x
+ 1/2*e)^6 + 84*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^6 - 432*a^2*c^7*d^2*tan(1/2*
f*x + 1/2*e)^6 + 411*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 + 336*a^2*c^5*d^4*t
an(1/2*f*x + 1/2*e)^6 - 24*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 192*a^2*c^3
*d^6*tan(1/2*f*x + 1/2*e)^6 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 12*a^
2*c^9*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^5 + 597*a
^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^5
+ 836*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 208*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e)^5 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 256*a^2*c^2*d^7*tan(1/2*f
*x + 1/2*e)^5 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 144*a^2*c^9*tan(1/2*f
*x + 1/2*e)^4 + 204*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^4 - 1104*a^2*c^7*d^2*tan
(1/2*f*x + 1/2*e)^4 + 1617*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*c^5*
d^4*tan(1/2*f*x + 1/2*e)^4 + 406*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 - 256*a
^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 - 184*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^4
- 128*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*d^9*tan(1/2*f*x + 1/2*e)^4
- 12*a^2*c^9*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^3
+ 1035*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^6*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1220*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 80*a^2*c^4*d^5*tan(1/2*
f*x + 1/2*e)^3 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 - 256*a^2*c^2*d^7*t
```

```
an(1/2*f*x + 1/2*e)^3 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^3 - 144*a^2*c^9*t
an(1/2*f*x + 1/2*e)^2 + 188*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^2 - 656*a^2*c^7*
d^2*tan(1/2*f*x + 1/2*e)^2 + 1201*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 16*a
^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 - 120*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^2
- 192*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2
*e)^2 - 12*a^2*c^9*tan(1/2*f*x + 1/2*e) - 288*a^2*c^8*d*tan(1/2*f*x + 1/2*e
) + 499*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e) + 32*a^2*c^6*d^3*tan(1/2*f*x + 1/2
*e) - 64*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e) - 80*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e) - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e) - 48*a^2*c^9 + 68*a^2*c^8*d + 16
*a^2*c^7*d^2 - 5*a^2*c^6*d^3 - 16*a^2*c^5*d^4 - 6*a^2*c^4*d^5)/((c^10 + 2*c
^9*d - c^8*d^2 - 4*c^7*d^3 - c^6*d^4 + 2*c^5*d^5 + c^4*d^6)*(c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

**Mupad [B]**
time = 10.35, size = 1411, normalized size = 4.93

$$\frac{a^2\,\mathrm{atan}\left(\frac{4\left(\frac{a^2(12c^2-16cd+7d^2)(8c^6d+16c^5d^2-8c^4d^3-32c^3d^4-8c^2d^5+16cd^6+8d^7)}{32(c+d)^{9/2}(c-d)^{5/2}(c^6+2c^5d-c^4d^2-4c^3d^3-c^2d^4+2cd^5+d^6)}+\frac{a^2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(12c^2-16cd+7d^2)}{4(c+d)^{9/2}(c-d)^{5/2}}\right)(c^6+2c^5d-c^4d^2-4c^3d^3-c^2d^4+2cd^5+d^6)}{12a^2c^2-16a^2cd+7a^2d^2}\right)(12c^2-16cd+7d^2)}{4f(c+d)^{9/2}(c-d)^{5/2}} - \ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^5,x)
```

```
[Out] (a^2*atan((4*((a^2*(12*c^2 - 16*c*d + 7*d^2)*(16*c*d^6 + 8*c^6*d + 8*d^7 -
8*c^2*d^5 - 32*c^3*d^4 - 8*c^4*d^3 + 16*c^5*d^2))/(32*(c + d)^(9/2)*(c - d)
^(5/2)*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (
a^2*c*tan(e/2 + (f*x)/2)*(12*c^2 - 16*c*d + 7*d^2))/(4*(c + d)^(9/2)*(c - d
)^(5/2)))*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2))/
(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))*(12*c^2 - 16*c*d + 7*d^2))/(4*f*(c +
 d)^(9/2)*(c - d)^(5/2)) - ((48*a^2*c^5 + 6*a^2*d^5 + 16*a^2*c*d^4 - 68*a^2
*c^4*d + 5*a^2*c^2*d^3 - 16*a^2*c^3*d^2)/(12*(2*c*d^5 + 2*c^5*d + c^6 + d^6
 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)*(80*c*d^5 + 28
8*c^5*d + 12*c^6 + 24*d^6 + 64*c^2*d^4 - 32*c^3*d^3 - 499*c^4*d^2))/(12*c*(
2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(
e/2 + (f*x)/2)^5*(256*c*d^7 + 480*c^7*d - 12*c^8 + 96*d^8 + 152*c^2*d^6 - 2
08*c^3*d^5 - 836*c^4*d^4 + 480*c^5*d^3 - 597*c^6*d^2))/(12*c^3*(2*c*d^5 + 2
*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)
/2)^3*(256*c*d^7 + 672*c^7*d + 12*c^8 + 96*d^8 + 152*c^2*d^6 + 80*c^3*d^5 -
 1220*c^4*d^4 + 672*c^5*d^3 - 1035*c^6*d^2))/(12*c^3*(2*c*d^5 + 2*c^5*d + c
^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^6*(64*
c*d^6 - 28*c^6*d + 16*c^7 + 24*d^7 + 8*c^2*d^5 - 112*c^3*d^4 - 137*c^4*d^3
+ 144*c^5*d^2))/(4*c^2*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3
 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^2*(192*c*d^6 - 188*c^6*d + 144*c^7 +
 72*d^7 + 120*c^2*d^5 + 16*c^3*d^4 - 1201*c^4*d^3 + 656*c^5*d^2))/(12*c^2*(
2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) - (a^2*tan(
e/2 + (f*x)/2)^7*(4*c^6 - 32*c^5*d - 16*c*d^5 - 8*d^6 + 8*c^2*d^4 + 32*c^3*
d^3 + 15*c^4*d^2))/(4*c*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^
```

```
3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^4*(3*c^4 + 8*d^4 + 24*c^2*d^2)*(16*
c*d^4 - 68*c^4*d + 48*c^5 + 6*d^5 + 5*c^2*d^3 - 16*c^3*d^2))/(12*c^4*(2*c*d
^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)))/(f*(tan(e/2 + (
f*x)/2)^4*(6*c^4 + 16*d^4 + 48*c^2*d^2) + c^4*tan(e/2 + (f*x)/2)^8 + c^4 +
tan(e/2 + (f*x)/2)^2*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (f*x)/2)^6*(4*c^4 + 2
4*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(32*c*d^3 + 24*c^3*d) + tan(e/2 + (f*x)/2
)^5*(32*c*d^3 + 24*c^3*d) + 8*c^3*d*tan(e/2 + (f*x)/2) + 8*c^3*d*tan(e/2 +
(f*x)/2)^7))
```

# 3.444 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=215

$$\frac{1}{16}a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)x-\frac{4a^3(c+d)^3\cos(e+fx)}{f}+\frac{a^3(c+d)^2(c+7d)\cos^3(e+fx)}{3f}-\frac{3a^3d^2(c+$$

```
[Out] 1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*x-4*a^3*(c+d)^3*cos(f*x+e)/f+1/3
*a^3*(c+d)^2*(c+7*d)*cos(f*x+e)^3/f-3/5*a^3*d^2*(c+d)*cos(f*x+e)^5/f-1/16*a
^3*(24*c^3+90*c^2*d+78*c*d^2+23*d^3)*cos(f*x+e)*sin(f*x+e)/f-1/24*a^3*d*(18
*c^2+54*c*d+23*d^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*a^3*d^3*cos(f*x+e)*sin(f*
x+e)^5/f
```

**Rubi** [A]
time = 0.38, antiderivative size = 326, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 3047, 3102, 2832, 2813}

$$-\frac{a^3(2c^2-18cd+115d^2)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}-\frac{a^3(2c^3-18c^2d+111cd^2+136d^3)\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}+\frac{1}{16}a^3x\left(40c^3+90c^2d+78cd^2+23d^3\right)-\frac{a^3(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)\sin(e+fx)\cos(e+fx)}{240df}-\frac{a^3(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)\cos(e+fx)}{60d^2f}+\frac{a^3(2c-13d)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}-\frac{\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^4}{6df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*x)/16 - (a^3*(2*c^5 - 18*c^4*d
 + 107*c^3*d^2 + 472*c^2*d^3 + 456*c*d^4 + 136*d^5)*Cos[e + f*x])/(60*d^2*f
) - (a^3*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4)*Cos[e + f*x
]*Sin[e + f*x])/(240*d*f) - (a^3*(2*c^3 - 18*c^2*d + 111*c*d^2 + 136*d^3)*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) - (a^3*(2*c^2 - 18*c*d + 11
5*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^3*(2*c - 13*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (Cos[e + f*x]*(a^3 + a^3
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \int$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \int$$

$$= \frac{a^3(2c-13d)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} - \frac{\cos(e+fx)}{}$$

$$= -\frac{a^3(2c^2-18cd+115d^2)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f} +$$

$$= -\frac{a^3(2c^3-18c^2d+111cd^2+136d^3)\cos(e+fx)(c+d\sin(e}{120d^2f}$$

$$= \frac{1}{16}a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)x - \frac{a^3(2c^5-18c^4d+10}{}$$

**Mathematica** [A]
time = 0.84, size = 215, normalized size = 1.00

$$\frac{(a+a\sin(e+fx))^3\left(60(40c^3+90c^2d+78cd^2+23d^3)(e+fx)-360(c+d)(10c^2+16cd+7d^2)\cos(e+fx)+20(c+d)(4c^2+32cd+19d^2)\cos(3(e+fx))-36d^2(c+d)\cos(5(e+fx))-45(16c^3+64c^2d+64cd^2+21d^3)\sin(2(e+fx))+45d(2c^2+6cd+3d^2)\sin(4(e+fx))-5d^3\sin(6(e+fx))\right)}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(60*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(e + f*
x) - 360*(c + d)*(10*c^2 + 16*c*d + 7*d^2)*Cos[e + f*x] + 20*(c + d)*(4*c^2
 + 32*c*d + 19*d^2)*Cos[3*(e + f*x)] - 36*d^2*(c + d)*Cos[5*(e + f*x)] - 45
*(16*c^3 + 64*c^2*d + 64*c*d^2 + 21*d^3)*Sin[2*(e + f*x)] + 45*d*(2*c^2 + 6
*c*d + 3*d^2)*Sin[4*(e + f*x)] - 5*d^3*Sin[6*(e + f*x)]))/(960*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^6)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(203) = 406.
time = 0.53, size = 481, normalized size = 2.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*c^2*d*(-1/4*(sin(f*x+e)
^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*c*d^2*(8/3+sin(f*x+e)^
4+4/3*sin(f*x+e)^2)*cos(f*x+e)+d^3*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*c^3*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a^3*c*d^2
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*d^3*a^3*
```

```
(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c^3*cos(f*x+e)+9*a^3*c
^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c*d^2*(2+sin(f*x+e)^2
)*cos(f*x+e)+3*d^3*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+a^3*c^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*cos(f*x+
e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*d^3*a^3*(2+sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(212) = 424$.
time = 0.32, size = 505, normalized size = 2.35

$$\frac{320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c^{3}+720\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^{3}c^{3}+960\left(fx+e\right)a^{3}c^{3}+2880\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c^{2}d+90\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^{3}c^{2}d+2160\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^{3}c^{2}d-192\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{3}cd^{2}+2880\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}cd^{2}+270\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^{3}cd^{2}+720\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^{3}cd^{2}-192\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{3}d^{3}+320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}d^{3}+5\left(4\sin\left(2fx+2e\right)^{3}+60fx+60e+9\sin\left(4fx+4e\right)-48\sin\left(2fx+2e\right)\right)a^{3}d^{3}+90\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^{3}d^{3}-2880a^{3}c^{3}\cos\left(fx+e\right)-2880a^{3}c^{2}d\cos\left(fx+e\right)}{960f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^3 + 720*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^3 + 960*(f*x + e)*a^3*c^3 + 2880*(cos(f*x + e)^3 - 3
*cos(f*x + e))*a^3*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f
*x + 2*e))*a^3*c^2*d + 2160*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^2*d - 19
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c*d^2 + 2880
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c*d^2 + 270*(12*f*x + 12*e + sin(4*f
*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c*d^2 + 720*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^3*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
e))*a^3*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 + 5*(4*sin(2*f*
x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*
d^3 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d^3 -
2880*a^3*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.37, size = 268, normalized size = 1.25

$$-\frac{144\left(a^{3}cd^{2}+a^{3}d^{3}\right)\cos\left(fx+e\right)^{5}-80\left(a^{3}c^{3}+9a^{3}c^{2}d+15a^{3}cd^{2}+7a^{3}d^{3}\right)\cos\left(fx+e\right)^{3}-15\left(40a^{3}c^{3}+90a^{3}c^{2}d+78a^{3}cd^{2}+23a^{3}d^{3}\right)fx+960\left(a^{3}c^{3}+3a^{3}c^{2}d+3a^{3}cd^{2}+a^{3}d^{3}\right)\cos\left(fx+e\right)+5\left(8a^{3}d^{3}\cos\left(fx+e\right)^{5}-2\left(18a^{3}c^{2}d+54a^{3}cd^{2}+31a^{3}d^{3}\right)\cos\left(fx+e\right)^{3}+3\left(24a^{3}c^{3}+102a^{3}c^{2}d+114a^{3}cd^{2}+41a^{3}d^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(a^3*c*d^2 + a^3*d^3)*cos(f*x + e)^5 - 80*(a^3*c^3 + 9*a^3*c^2*
d + 15*a^3*c*d^2 + 7*a^3*d^3)*cos(f*x + e)^3 - 15*(40*a^3*c^3 + 90*a^3*c^2*
d + 78*a^3*c*d^2 + 23*a^3*d^3)*f*x + 960*(a^3*c^3 + 3*a^3*c^2*d + 3*a^3*c*d
^2 + a^3*d^3)*cos(f*x + e) + 5*(8*a^3*d^3*cos(f*x + e)^5 - 2*(18*a^3*c^2*d
+ 54*a^3*c*d^2 + 31*a^3*d^3)*cos(f*x + e)^3 + 3*(24*a^3*c^3 + 102*a^3*c^2*d
 + 114*a^3*c*d^2 + 41*a^3*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(206) = 412$.
time = 0.60, size = 1176, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*c**3*x*sin(e + f*x)**2/2 + 3*a**3*c**3*x*cos(e + f*x)**2/
2 + a**3*c**3*x - a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**3*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**3*cos(e + f*x)**3/(3*f) - 3*a**3*
c**3*cos(e + f*x)/f + 9*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9*a**3*c**2*d*x*s
in(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**3*c**2*d*x*sin(e + f*x)**2/2 + 9*a*
*3*c**2*d*x*cos(e + f*x)**4/8 + 9*a**3*c**2*d*x*cos(e + f*x)**2/2 - 15*a**3
*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*a**3*c**2*d*sin(e + f*x)**2*
cos(e + f*x)/f - 9*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 9*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**3*c**2*d*cos(e + f*x)**3/f -
3*a**3*c**2*d*cos(e + f*x)/f + 27*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*a**3
*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a**3*c*d**2*x*sin(e + f*x)*
*2/2 + 27*a**3*c*d**2*x*cos(e + f*x)**4/8 + 3*a**3*c*d**2*x*cos(e + f*x)**2
/2 - 3*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*a**3*c*d**2*sin(e +
f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 9*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*a**3*c*d**2*sin(e + f
*x)*cos(e + f*x)**3/(8*f) - 3*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 8*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*a**3*c*d**2*cos(e + f*x)**3/f + 5*
a**3*d**3*x*sin(e + f*x)**6/16 + 15*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x
)**2/16 + 9*a**3*d**3*x*sin(e + f*x)**4/8 + 15*a**3*d**3*x*sin(e + f*x)**2*
cos(e + f*x)**4/16 + 9*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a*
*3*d**3*x*cos(e + f*x)**6/16 + 9*a**3*d**3*x*cos(e + f*x)**4/8 - 11*a**3*d*
*3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*a**3*d**3*sin(e + f*x)**4*cos(e
+ f*x)/f - 5*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*d**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**3*sin(e + f*x)**2*cos(e + f
*x)**3/f - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*a**3*d**3*sin(e + f
*x)*cos(e + f*x)**5/(16*f) - 9*a**3*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f)
 - 8*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*a**3*d**3*cos(e + f*x)**3/(3*f), N
e(f, 0)), (x*(c + d*sin(e))**3*(a*sin(e) + a)**3, True))
```

**<u>Giac</u> [A]**
time = 0.48, size = 373, normalized size = 1.73

$$\frac{a^3d^3\cos(3fx+3e)}{12f}-\frac{a^3d^3\sin(6fx+6e)}{192f}-\frac{3a^3cd^2\sin(2fx+2e)}{4f}+\frac{1}{16}\left(24a^3c^3+90a^3c^2d+54a^3cd^2+23a^3d^3\right)x+\frac{1}{2}\left(2a^3c^3+3a^3cd^2\right)x-\frac{3\left(a^3cd^2+a^3d^3\right)\cos(5fx+5e)}{80f}+\frac{\left(4a^3c^3+36a^3c^2d+51a^3cd^2+15a^3d^3\right)\cos(3fx+3e)}{48f}-\frac{3\left(10a^3c^3+18a^3c^2d+23a^3cd^2+5a^3d^3\right)\cos(fx+e)}{8f}-\frac{3\left(4a^3c^2d+a^3d^3\right)\cos(fx+e)}{4f}+\frac{3\left(2a^3c^2d+6a^3cd^2+3a^3d^3\right)\sin(4fx+4e)}{64f}-\frac{3\left(16a^3c^3+64a^3c^2d+48a^3cd^2+21a^3d^3\right)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*a^3*d^3*cos(3*f*x + 3*e)/f - 1/192*a^3*d^3*sin(6*f*x + 6*e)/f - 3/4*a^
3*c*d^2*sin(2*f*x + 2*e)/f + 1/16*(24*a^3*c^3 + 90*a^3*c^2*d + 54*a^3*c*d^2
 + 23*a^3*d^3)*x + 1/2*(2*a^3*c^3 + 3*a^3*c*d^2)*x - 3/80*(a^3*c*d^2 + a^3*
d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*a^3*c^3 + 36*a^3*c^2*d + 51*a^3*c*d^2 + 1
5*a^3*d^3)*cos(3*f*x + 3*e)/f - 3/8*(10*a^3*c^3 + 18*a^3*c^2*d + 23*a^3*c*d
```

```
^2 + 5*a^3*d^3)*cos(f*x + e)/f - 3/4*(4*a^3*c^2*d + a^3*d^3)*cos(f*x + e)/f
 + 3/64*(2*a^3*c^2*d + 6*a^3*c*d^2 + 3*a^3*d^3)*sin(4*f*x + 4*e)/f - 3/64*(
16*a^3*c^3 + 64*a^3*c^2*d + 48*a^3*c*d^2 + 21*a^3*d^3)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 8.48, size = 773, normalized size = 3.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/
(8*(5*a^3*c^3 + (23*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4)))*(78
*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(8*f) - (a^3*(atan(tan(e/2 + (f*x)/2)
) - (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f
*x)/2)^10*(6*a^3*c^3 + 6*a^3*c^2*d) - tan(e/2 + (f*x)/2)^11*(3*a^3*c^3 + (2
3*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*
(34*a^3*c^3 + 4*a^3*d^3 + 36*a^3*c*d^2 + 66*a^3*c^2*d) + tan(e/2 + (f*x)/2)
^5*(6*a^3*c^3 + (75*a^3*d^3)/4 + (75*a^3*c*d^2)/2 + (57*a^3*c^2*d)/2) - tan
(e/2 + (f*x)/2)^7*(6*a^3*c^3 + (75*a^3*d^3)/4 + (75*a^3*c*d^2)/2 + (57*a^3*
c^2*d)/2) + tan(e/2 + (f*x)/2)^4*(76*a^3*c^3 + 64*a^3*d^3 + 192*a^3*c*d^2 +
 204*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((220*a^3*c^3)/3 + (136*a^3*d^3)/3 +
 152*a^3*c*d^2 + 180*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*(38*a^3*c^3 + (136*a
^3*d^3)/5 + (456*a^3*c*d^2)/5 + 102*a^3*c^2*d) + tan(e/2 + (f*x)/2)^3*(9*a^
3*c^3 + (391*a^3*d^3)/24 + (189*a^3*c*d^2)/4 + (159*a^3*c^2*d)/4) - tan(e/2
 + (f*x)/2)^9*(9*a^3*c^3 + (391*a^3*d^3)/24 + (189*a^3*c*d^2)/4 + (159*a^3*
c^2*d)/4) + (22*a^3*c^3)/3 + (68*a^3*d^3)/15 + tan(e/2 + (f*x)/2)*(3*a^3*c^
3 + (23*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) + (76*a^3*c*d^2)/
5 + 18*a^3*c^2*d)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20
*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 +
 tan(e/2 + (f*x)/2)^12 + 1))
```

# 3.445 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=164

$$\frac{1}{8}a^3\left(20c^2+30cd+13d^2\right)x-\frac{4a^3(c+d)^2\cos(e+fx)}{f}+\frac{a^3(c^2+6cd+5d^2)\cos^3(e+fx)}{3f}-\frac{a^3d^2\cos^5(e+fx)}{5f}-$$

```
[Out] 1/8*a^3*(20*c^2+30*c*d+13*d^2)*x-4*a^3*(c+d)^2*cos(f*x+e)/f+1/3*a^3*(c^2+6*
c*d+5*d^2)*cos(f*x+e)^3/f-1/5*a^3*d^2*cos(f*x+e)^5/f-1/8*a^3*(12*c^2+30*c*d
+13*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/4*a^3*d*(2*c+3*d)*cos(f*x+e)*sin(f*x+e)^
3/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2840, 2830, 2724, 2718, 2715, 8, 2713}

$$\frac{a^3(20c^2+30cd+13d^2)\cos^3(e+fx)}{60f}-\frac{a^3(20c^2+30cd+13d^2)\cos(e+fx)}{5f}-\frac{3a^3(20c^2+30cd+13d^2)\sin(e+fx)\cos(e+fx)}{40f}+\frac{1}{8}a^3x\left(20c^2+30cd+13d^2\right)-\frac{d(10c-d)\cos(e+fx)(a\sin(e+fx)+a)^3}{20f}-\frac{d^2\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*x)/8 - (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[
e + f*x])/(5*f) + (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]^3)/(60*f) -
(3*a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sin[e + f*x])/(40*f) - ((10*
c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(20*f) - (d^2*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx &= -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^4}{5af}+\frac{\int(a+a\sin(e+fx\\ &= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f}-\frac{d^2\cos(e+\\ &= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f}-\frac{d^2\cos(e+\\ &= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x-\frac{(10c-d)d\cos(e+fx)(a+\\ &= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x-\frac{3a^3(20c^2+30cd+13d^2)\cos\\ &= \frac{1}{8}a^3\left(20c^2+30cd+13d^2\right)x-\frac{a^3(20c^2+30cd+13d^2)\cos(e\end{aligned}$$

**Mathematica** [A]

time = 0.48, size = 177, normalized size = 1.08

$$-\frac{a^3\cos(e+fx)\left(30(20c^2+30cd+13d^2)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(8(55c^2+90cd+38d^2)+15(12c^2+30cd+13d^2)\sin(e+fx)+8(5c^2+30cd+19d^2)\sin^2(e+fx)+30d(2c+3d)\sin^3(e+fx)+24d^2\sin^4(e+fx))\right)}{120f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/120*(a^3*Cos[e + f*x]*(30*(20*c^2 + 30*c*d + 13*d^2)*ArcSin[Sqrt[1 - Sin
[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(55*c^2 + 90*c*d + 38*d^2) +
15*(12*c^2 + 30*c*d + 13*d^2)*Sin[e + f*x] + 8*(5*c^2 + 30*c*d + 19*d^2)*Si
n[e + f*x]^2 + 30*d*(2*c + 3*d)*Sin[e + f*x]^3 + 24*d^2*Sin[e + f*x]^4)))/(
f*Sqrt[Cos[e + f*x]^2])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(154) = 308$.
time = 0.40, size = 319, normalized size = 1.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*c*d*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*d^2*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e)+3*a^3*c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/
2*e)-2*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*c^2*cos(f*x+e)+6*a^3*c*d*(-1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+
a^3*c^2*(f*x+e)-2*a^3*c*d*cos(f*x+e)+a^3*d^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/
2*f*x+1/2*e))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(161) = 322$.
time = 0.33, size = 332, normalized size = 2.02

$$\frac{160\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3c^2+360\left(2fx+2e-\sin(2fx+2e)\right)a^3c^2+480(fx+e)a^3c^2+960\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3cd+30\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^3cd+720\left(2fx+2e-\sin(2fx+2e)\right)a^3cd-32\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^3d^2+480\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3d^2+45\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^3d^2+120\left(2fx+2e-\sin(2fx+2e)\right)a^3d^2-1440a^3c^2\cos(fx+e)-960a^3cd\cos(fx+e)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^2 + 480*(f*x + e)*a^3*c^2 + 960*(cos(f*x + e)^3 - 3*
cos(f*x + e))*a^3*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*a^3*c*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d - 32*(3*cos(
f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*d^2 + 480*(cos(f*x +
e)^3 - 3*cos(f*x + e))*a^3*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^3*d^2 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 - 1
440*a^3*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.37, size = 186, normalized size = 1.13

$$-\frac{24\,a^3d^2\cos\left(fx+e\right)^5-40\left(a^3c^2+6\,a^3cd+5\,a^3d^2\right)\cos\left(fx+e\right)^3-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)fx+480\left(a^3c^2+2\,a^3cd+a^3d^2\right)\cos\left(fx+e\right)-15\left(2\left(2\,a^3cd+3\,a^3d^2\right)\cos\left(fx+e\right)^3-\left(12\,a^3c^2+34\,a^3cd+19\,a^3d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a^3*d^2*cos(f*x + e)^5 - 40*(a^3*c^2 + 6*a^3*c*d + 5*a^3*d^2)*co
s(f*x + e)^3 - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*f*x + 480*(a^3*c^2
 + 2*a^3*c*d + a^3*d^2)*cos(f*x + e) - 15*(2*(2*a^3*c*d + 3*a^3*d^2)*cos(f*
x + e)^3 - (12*a^3*c^2 + 34*a^3*c*d + 19*a^3*d^2)*cos(f*x + e))*sin(f*x + e
))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(153) = 306$.
time = 0.44, size = 702, normalized size = 4.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**2/2 + 3*a**3*c**2*x*cos(e + f*x)**2/
2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**2*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**2*cos(e + f*x)**3/(3*f) - 3*a**3*
c**2*cos(e + f*x)/f + 3*a**3*c*d*x*sin(e + f*x)**4/4 + 3*a**3*c*d*x*sin(e +
 f*x)**2*cos(e + f*x)**2/2 + 3*a**3*c*d*x*sin(e + f*x)**2 + 3*a**3*c*d*x*co
s(e + f*x)**4/4 + 3*a**3*c*d*x*cos(e + f*x)**2 - 5*a**3*c*d*sin(e + f*x)**3
*cos(e + f*x)/(4*f) - 6*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*
d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*a**3*c*d*sin(e + f*x)*cos(e + f*x)
/f - 4*a**3*c*d*cos(e + f*x)**3/f - 2*a**3*c*d*cos(e + f*x)/f + 9*a**3*d**2
*x*sin(e + f*x)**4/8 + 9*a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*
*3*d**2*x*sin(e + f*x)**2/2 + 9*a**3*d**2*x*cos(e + f*x)**4/8 + a**3*d**2*x
*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*a**3*d**
2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/(3*f) - 3*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*a**3*d**2*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f
) - 8*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*a**3*d**2*cos(e + f*x)**3/f, Ne(
f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.47, size = 251, normalized size = 1.53

$$-\frac{a^3d^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{2\,a^3cd\cos\left(fx+e\right)}{f}-\frac{a^3d^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{3}{8}\left(4\,a^3c^2+10\,a^3cd+3\,a^3d^2\right)x+\frac{1}{2}\left(2\,a^3c^2+a^3d^2\right)x+\frac{\left(4\,a^3c^2+24\,a^3cd+17\,a^3d^2\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(30\,a^3c^2+36\,a^3cd+23\,a^3d^2\right)\cos\left(fx+e\right)}{8\,f}+\frac{\left(2\,a^3cd+3\,a^3d^2\right)\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{\left(3\,a^3c^2+8\,a^3cd+3\,a^3d^2\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*a^3*d^2*cos(5*f*x + 5*e)/f - 2*a^3*c*d*cos(f*x + e)/f - 1/4*a^3*d^2*s
in(2*f*x + 2*e)/f + 3/8*(4*a^3*c^2 + 10*a^3*c*d + 3*a^3*d^2)*x + 1/2*(2*a^3
*c^2 + a^3*d^2)*x + 1/48*(4*a^3*c^2 + 24*a^3*c*d + 17*a^3*d^2)*cos(3*f*x +
3*e)/f - 1/8*(30*a^3*c^2 + 36*a^3*c*d + 23*a^3*d^2)*cos(f*x + e)/f + 1/32*(
2*a^3*c*d + 3*a^3*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a^3*c^2 + 8*a^3*c*d + 3*
a^3*d^2)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.36, size = 493, normalized size = 3.01

$$\frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9\,\left(3\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(6\,a^3\,c^2+19\,a^3\,c\,d+\frac{25\,a^3\,d^2}{2}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(6\,a^3\,c^2+19\,a^3\,c\,d+\frac{25\,a^3\,d^2}{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(28\,a^3\,c^2+40\,a^3\,c\,d+12\,a^3\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{92\,a^3\,c^2}{3}+56\,a^3\,c\,d+\frac{76\,a^3\,d^2}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{136\,a^3\,c^2}{3}+80\,a^3\,c\,d+\frac{116\,a^3\,d^2}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(6\,a^3\,c^2+4\,d\,a^3\,c\right)+\frac{22\,a^3\,c^2}{3}+\frac{76\,a^3\,d^2}{15}+12\,a^3\,c\,d}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(4*(5*a^3*c^2
 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (
tan(e/2 + (f*x)/2)*(3*a^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2
+ (f*x)/2)^9*(3*a^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) + tan(e/2 + (f*x
)/2)^3*(6*a^3*c^2 + (25*a^3*d^2)/2 + 19*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*
a^3*c^2 + (25*a^3*d^2)/2 + 19*a^3*c*d) + tan(e/2 + (f*x)/2)^6*(28*a^3*c^2 +
 12*a^3*d^2 + 40*a^3*c*d) + tan(e/2 + (f*x)/2)^2*((92*a^3*c^2)/3 + (76*a^3*
d^2)/3 + 56*a^3*c*d) + tan(e/2 + (f*x)/2)^4*((136*a^3*c^2)/3 + (116*a^3*d^2
)/3 + 80*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(6*a^3*c^2 + 4*a^3*c*d) + (22*a^3*
c^2)/3 + (76*a^3*d^2)/15 + 12*a^3*c*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(
e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e
/2 + (f*x)/2)^10 + 1)) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(30*c*d
+ 20*c^2 + 13*d^2))/(4*f)
```

# 3.446 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=110

$$\frac{5}{8}a^3(4c+3d)x-\frac{4a^3(c+d)\cos(e+fx)}{f}+\frac{a^3(c+3d)\cos^3(e+fx)}{3f}-\frac{3a^3(4c+5d)\cos(e+fx)\sin(e+fx)}{8f}-\,a^3$$

```
[Out] 5/8*a^3*(4*c+3*d)*x-4*a^3*(c+d)*cos(f*x+e)/f+1/3*a^3*(c+3*d)*cos(f*x+e)^3/f
-3/8*a^3*(4*c+5*d)*cos(f*x+e)*sin(f*x+e)/f-1/4*a^3*d*cos(f*x+e)*sin(f*x+e)^
3/f
```

**Rubi** [A]

time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2830, 2724, 2718, 2715, 8, 2713}

$$\frac{a^3(4c+3d)\cos^3(e+fx)}{12f}-\frac{a^3(4c+3d)\cos(e+fx)}{f}-\frac{3a^3(4c+3d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4c+3d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*x)/8 - (a^3*(4*c + 3*d)*Cos[e + f*x])/f + (a^3*(4*c + 3*
d)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx &= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int (a+a\sin \\ &= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int \left(a^3+3a\right. \\ &= \frac{1}{4}a^3(4c+3d)x-\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}\left(a^3(4\right. \\ &= \frac{1}{4}a^3(4c+3d)x-\frac{3a^3(4c+3d)\cos(e+fx)}{4f}-\frac{3a^3(4c+3d)\,\mathrm{co}}{} \\ &= \frac{5}{8}a^3(4c+3d)x-\frac{a^3(4c+3d)\cos(e+fx)}{f}+\frac{a^3(4c+3d)\cos^3(}{12f}\end{aligned}$$

**Mathematica [A]**
time = 0.33, size = 120, normalized size = 1.09

$$-\frac{a^3\cos(e+fx)\left(30(4c+3d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(88c+72d+9(4c+5d)\sin(e+fx)+8(c+3d)\sin^2(e+fx)+6d\sin^3(e+fx))\right)}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*c + 3*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(88*c + 72*d + 9*(4*c + 5*d)*Sin[e + f*x] + 8*(c
 + 3*d)*Sin[e + f*x]^2 + 6*d*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

**Maple [A]**
time = 0.31, size = 178, normalized size = 1.62

| method | result |
|---|---|
| risch | $\frac{5a^3cx}{2}+\frac{15a^3xd}{8}-\frac{15a^3\cos(fx+e)c}{4f}-\frac{13a^3\cos(fx+e)d}{4f}+\frac{a^3d\sin(4fx+4e)}{32f}+\frac{a^3\cos(3fx+3e)c}{12f}+\frac{a^3\cos(3fx}{4f}$ |
| derivativedivides | $-\frac{a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^3d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^3c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.$ |
| default | $-\frac{a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^3d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^3c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.$ |
| norman | $(\frac{5}{2}a^3c+\frac{15}{8}a^3d)x+(10a^3c+\frac{15}{2}a^3d)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10a^3c+\frac{15}{2}a^3d)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(15a^3c+\frac{45}{4}a^3d)x\left(\tan^4\left(\frac{fx}{2}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*d*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/
2*f*x+1/2*e)-a^3*d*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c*cos(f*x+e)+3*a^3*d*(
-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^3*c*(f*x+e)-a^3*d*cos(f*x+e))
```

## Maxima [A]

time = 0.35, size = 185, normalized size = 1.68

$$\frac{32\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3c+72\left(2fx+2e-\sin(2fx+2e)\right)a^3c+96(fx+e)a^3c+96\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3d+3\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^3d+72\left(2fx+2e-\sin(2fx+2e)\right)a^3d-288a^3c\cos(fx+e)-96a^3d\cos(fx+e)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*a^3*c + 96*(f*x + e)*a^3*c + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*a^3*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d - 288*a^3*c*cos(f*x + e) - 96*a
^3*d*cos(f*x + e))/f
```

## Fricas [A]

time = 0.37, size = 113, normalized size = 1.03

$$\frac{8\left(a^3c+3a^3d\right)\cos(fx+e)^3+15\left(4a^3c+3a^3d\right)fx-96\left(a^3c+a^3d\right)\cos(fx+e)+3\left(2a^3d\cos(fx+e)^3-\left(12a^3c+17a^3d\right)\cos(fx+e)\right)\sin(fx+e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(a^3*c + 3*a^3*d)*cos(f*x + e)^3 + 15*(4*a^3*c + 3*a^3*d)*f*x - 96*
(a^3*c + a^3*d)*cos(f*x + e) + 3*(2*a^3*d*cos(f*x + e)^3 - (12*a^3*c + 17*a
^3*d)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(104) = 208$.
time = 0.24, size = 371, normalized size = 3.37

$$\begin{cases} \frac{3a^3cx\sin^2(e+fx)}{2} + \frac{3a^3cx\cos^2(e+fx)}{2} + a^3cx - \frac{a^3c\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3a^3c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^3c\cos^3(e+fx)}{3f} - \frac{3a^3c\cos(e+fx)}{f} + \frac{3a^3dx\sin^4(e+fx)}{8} + \frac{3a^3dx\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3a^3dx\sin^2(e+fx)}{2} + \frac{3a^3dx\cos^4(e+fx)}{8} + \frac{3a^3dx\cos^2(e+fx)}{2} - \frac{5a^3d\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3a^3d\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3a^3d\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{3a^3d\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^3d\cos^3(e+fx)}{f} - \frac{a^3d\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(c+d\sin(e))(a\sin(e)+a)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*a**3*c*x*sin(e + f*x)**2/2 + 3*a**3*c*x*cos(e + f*x)**2/2 + a*
*3*c*x - a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*a**3*c*cos(e + f*x)**3/(3*f) - 3*a**3*c*cos(e + f*x)/f +
 3*a**3*d*x*sin(e + f*x)**4/8 + 3*a**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a**3*d*x*sin(e + f*x)**2/2 + 3*a**3*d*x*cos(e + f*x)**4/8 + 3*a**3*d*
x*cos(e + f*x)**2/2 - 5*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*a**3*
d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*a**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*d*cos(e + f*x)**3/f
 - a**3*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)**3, T
rue))
```

**Giac** [A]
time = 0.49, size = 138, normalized size = 1.25

$$a^3cx - \frac{a^3d\cos(fx+e)}{f} + \frac{a^3d\sin(4fx+4e)}{32f} + \frac{3}{8}(4a^3c+5a^3d)x + \frac{(a^3c+3a^3d)\cos(3fx+3e)}{12f} - \frac{3(5a^3c+3a^3d)\cos(fx+e)}{4f} - \frac{(3a^3c+4a^3d)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^3*c*x - a^3*d*cos(f*x + e)/f + 1/32*a^3*d*sin(4*f*x + 4*e)/f + 3/8*(4*a^3
*c + 5*a^3*d)*x + 1/12*(a^3*c + 3*a^3*d)*cos(3*f*x + 3*e)/f - 3/4*(5*a^3*c
+ 3*a^3*d)*cos(f*x + e)/f - 1/4*(3*a^3*c + 4*a^3*d)*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 8.07, size = 330, normalized size = 3.00

$$\frac{5a^3\,\mathrm{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c+3d)}{4\left(5a^3c+\frac{15a^3d}{4}\right)}\right)(4c+3d)}{4f} - \frac{5a^3\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(4c+3d)}{4f} - \frac{\frac{22a^3c}{3}+6a^3d+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3a^3c+\frac{15a^3d}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(6a^3c+2a^3d)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3a^3c+\frac{15a^3d}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3a^3c+\frac{23a^3d}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(3a^3c+\frac{23a^3d}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(22a^3c+18a^3d)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{70a^3c}{3}+22a^3d\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(4*(5*a^3*c + (15*a^3*d)
/4)))*(4*c + 3*d))/(4*f) - (5*a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(4*c
 + 3*d))/(4*f) - ((22*a^3*c)/3 + 6*a^3*d + tan(e/2 + (f*x)/2)*(3*a^3*c + (1
5*a^3*d)/4) + tan(e/2 + (f*x)/2)^6*(6*a^3*c + 2*a^3*d) - tan(e/2 + (f*x)/2)
^7*(3*a^3*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^3*(3*a^3*c + (23*a^3*d)/4)
 - tan(e/2 + (f*x)/2)^5*(3*a^3*c + (23*a^3*d)/4) + tan(e/2 + (f*x)/2)^4*(22
*a^3*c + 18*a^3*d) + tan(e/2 + (f*x)/2)^2*((70*a^3*c)/3 + 22*a^3*d))/(f*(4*
tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + ta
n(e/2 + (f*x)/2)^8 + 1))
```

# 3.447 $\int (a+a\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=63

$$\frac{5a^3x}{2}-\frac{4a^3\cos(e+fx)}{f}+\frac{a^3\cos^3(e+fx)}{3f}-\frac{3a^3\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 5/2*a^3*x-4*a^3*cos(f*x+e)/f+1/3*a^3*cos(f*x+e)^3/f-3/2*a^3*cos(f*x+e)*sin(
f*x+e)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2724, 2718, 2715, 8, 2713}

$$\frac{a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{3a^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*
Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3\,dx &= \int \left(a^3+3a^3\sin(e+fx)+3a^3\sin^2(e+fx)+a^3\sin^3(e+fx)\right)\,dx\\ &= a^3x+a^3\int \sin^3(e+fx)\,dx+\left(3a^3\right)\int \sin(e+fx)\,dx+\left(3a^3\right)\int \sin^2(e+fx)\,dx\\ &= a^3x-\frac{3a^3\cos(e+fx)}{f}-\frac{3a^3\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\text{Sub}}{}\\ &= \frac{5a^3x}{2}-\frac{4a^3\cos(e+fx)}{f}+\frac{a^3\cos^3(e+fx)}{3f}-\frac{3a^3\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

**Mathematica [A]**
time = 0.22, size = 44, normalized size = 0.70

$$\frac{a^3(30e+30fx-45\cos(e+fx)+\cos(3(e+fx))-9\sin(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3,x]
[Out] (a^3*(30*e + 30*f*x - 45*Cos[e + f*x] + Cos[3*(e + f*x)] - 9*Sin[2*(e + f*x
)]))/(12*f)
```

**Maple [A]**
time = 0.23, size = 74, normalized size = 1.17

| method | result |
|---|---|
| risch | $\frac{5a^3x}{2}-\frac{15a^3\cos(fx+e)}{4f}+\frac{a^3\cos(3fx+3e)}{12f}-\frac{3a^3\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{-\frac{a^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^3\cos(fx+e)+a^3(fx+e)}{f}$ |
| default | $\frac{-\frac{a^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^3\cos(fx+e)+a^3(fx+e)}{f}$ |
| norman | $\frac{\frac{5a^3x}{2}-\frac{22a^3}{3f}-\frac{3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{3a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{15a^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{15a^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{5a^3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-6}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*(-1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-3*a^3*cos(f*x+e)+a^3*(f*x+e))
```

**Maxima** [A]
time = 0.35, size = 77, normalized size = 1.22

$$a^3x + \frac{(\cos(fx+e)^3 - 3\cos(fx+e))a^3}{3f} + \frac{3(2fx + 2e - \sin(2fx + 2e))a^3}{4f} - \frac{3a^3\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3/f + 3/4*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^3/f - 3*a^3*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.34, size = 58, normalized size = 0.92

$$\frac{2a^3\cos(fx+e)^3 + 15a^3fx - 9a^3\cos(fx+e)\sin(fx+e) - 24a^3\cos(fx+e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(f*x + e)^3 + 15*a^3*f*x - 9*a^3*cos(f*x + e)*sin(f*x + e) -
24*a^3*cos(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(58) = 116$.
time = 0.15, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2(e+fx)}{2} + \frac{3a^3x\cos^2(e+fx)}{2} + a^3x - \frac{a^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sin(e) + a)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(e + f*x)**2/2 + 3*a**3*x*cos(e + f*x)**2/2 + a**3*x
 - a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*sin(e + f*x)*cos(e + f*x)/(
2*f) - 2*a**3*cos(e + f*x)**3/(3*f) - 3*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.44, size = 58, normalized size = 0.92

$$\frac{5}{2}a^3x + \frac{a^3\cos(3fx+3e)}{12f} - \frac{15a^3\cos(fx+e)}{4f} - \frac{3a^3\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*f*x + 3*e)/f - 15/4*a^3*cos(f*x + e)/f - 3/4*a^3
*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 9.12, size = 156, normalized size = 2.48

$$\frac{5a^3x}{2} - \frac{\frac{5a^3(e+fx)}{2} - 3a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5 - \frac{a^3(15e+15fx-44)}{6} + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{15a^3(e+fx)}{2} - \frac{a^3(45e+45fx-36)}{6}\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{15a^3(e+fx)}{2} - \frac{a^3(45e+45fx-96)}{6}\right) + 3a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*x)/2 - ((5*a^3*(e + f*x))/2 - 3*a^3*tan(e/2 + (f*x)/2)^5 - (a^3*(15*
e + 15*f*x - 44))/6 + tan(e/2 + (f*x)/2)^4*((15*a^3*(e + f*x))/2 - (a^3*(45
*e + 45*f*x - 36))/6) + tan(e/2 + (f*x)/2)^2*((15*a^3*(e + f*x))/2 - (a^3*(
45*e + 45*f*x - 96))/6) + 3*a^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^
2 + 1)^3)
```

# 3.448 $\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=143

$$\frac{a^3(2c^2-6cd+7d^2)\,x}{2d^3}-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\,(a^3+a$$

```
[Out] 1/2*a^3*(2*c^2-6*c*d+7*d^2)*x/d^3+1/2*a^3*(2*c-5*d)*cos(f*x+e)/d^2/f-1/2*co
s(f*x+e)*(a^3+a^3*sin(f*x+e))/d/f-2*a^3*(c-d)^3*arctan((d+c*tan(1/2*f*x+1/2
*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2842, 3047, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^3(c-d)^3\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{a^3x(2c^2-6cd+7d^2)}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\,(a^3\sin(e+fx)+a^3)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(2*c^2 - 6*c*d + 7*d^2)*x)/(2*d^3) - (2*a^3*(c - d)^3*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f) + (a^3*(2*c - 5*d
)*Cos[e + f*x])/(2*d^2*f) - (Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df} + \frac{\int \frac{(a+a\sin(e+fx))\left(a^2(c+2d)-a^2(2c-5d)\sin(e+fx)\right)}{c+d\sin(e+fx)}}{2d}$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df} + \frac{\int \frac{a^3(c+2d)+\left(-a^3(2c-5d)+a^3(c+2d)\right)\sin(e+fx)-a}{c+d\sin(e+fx)}}{2d}$$

$$= \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f} - \frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df} + \frac{\int \frac{a^3d(c+2d)+a^3(}{c+}}{}$$

$$= \frac{a^3(2c^2-6cd+7d^2)\,x}{2d^3} + \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f} - \frac{\cos(e+fx)\left(a^3+a^3\sin(e\right.}{2df}$$

$$= \frac{a^3(2c^2-6cd+7d^2)\,x}{2d^3} + \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f} - \frac{\cos(e+fx)\left(a^3+a^3\sin(e\right.}{2df}$$

$$= \frac{a^3(2c^2-6cd+7d^2)\,x}{2d^3} + \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f} - \frac{\cos(e+fx)\left(a^3+a^3\sin(e\right.}{2df}$$

$$= \frac{a^3(2c^2-6cd+7d^2)\,x}{2d^3} - \frac{2a^3(c-d)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} + \frac{a^3(2c-5d)\,\mathrm{co}}{2d^2 f}$$

**Mathematica** [A]
time = 0.43, size = 162, normalized size = 1.13

$$\frac{a^3(1+\sin(e+fx))^3\left(-8(c-d)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)+\sqrt{c^2-d^2}\left(2\left(2c^2-6cd+7d^2\right)(e+fx)+4(c-3d)d\cos(e+fx)-d^2\sin(2(e+fx))\right)\right)}{4d^3\sqrt{c^2-d^2}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-8*(c - d)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]] + Sqrt[c^2 - d^2]*(2*(2*c^2 - 6*c*d + 7*d^2)*(e + f*x) + 4*(c
 - 3*d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)])))/(4*d^3*Sqrt[c^2 - d^2]*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

**Maple** [A]
time = 0.38, size = 184, normalized size = 1.29

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2a^3\left(\frac{\frac{\frac{d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(cd-3d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+cd-3d^2}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2c^2-6cd+7d^2\right)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{d^3}+\frac{(-c^3+3}{}\right.}{f}$ |
| default | $\frac{2a^3\left(\frac{\frac{\frac{d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(cd-3d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+cd-3d^2}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2c^2-6cd+7d^2\right)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{d^3}+\frac{(-c^3+3}{}\right.}{f}$ |
| risch | $\frac{a^3x\,c^2}{d^3}-\frac{3a^3xc}{d^2}+\frac{7a^3x}{2d}+\frac{a^3\mathrm{e}^{i(fx+e)}c}{2d^2f}-\frac{3a^3\mathrm{e}^{i(fx+e)}}{2df}+\frac{a^3\mathrm{e}^{-i(fx+e)}c}{2d^2f}-\frac{3a^3\mathrm{e}^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c+d)\,(c}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3*(1/d^3*((1/2*d^2*tan(1/2*f*x+1/2*e)^3+(c*d-3*d^2)*tan(1/2*f*x+1/2*e
)^2-1/2*d^2*tan(1/2*f*x+1/2*e)+c*d-3*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(2
*c^2-6*c*d+7*d^2)*arctan(tan(1/2*f*x+1/2*e)))+(-c^3+3*c^2*d-3*c*d^2+d^3)/d^
3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.39, size = 419, normalized size = 2.93

$$\left[-\frac{a^3d^2\cos(fx+e)\sin(fx+e)-(2a^3c^2-6a^3cd+7a^3d^2)fx-(a^3c^2-2a^3cd+a^3d^2)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2\left((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)-2(a^3cd-3a^3d^2)\cos(fx+e)}{2d^3f},\ -\frac{a^3d^2\cos(fx+e)\sin(fx+e)-(2a^3c^2-6a^3cd+7a^3d^2)fx-2(a^3c^2-2a^3cd+a^3d^2)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)-2(a^3cd-3a^3d^2)\cos(fx+e)}{2d^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d
^2)*f*x - (a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*log(((2*c^
2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*c
os(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d))
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(a^3*c*d - 3*a
^3*d^2)*cos(f*x + e))/(d^3*f), -1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2
*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*f*x - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*s
qrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c
 - d)*cos(f*x + e))) - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))/(d^3*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.49, size = 239, normalized size = 1.67

$$\frac{\frac{(2a^3c^2-6a^3cd+7a^3d^2)(fx+e)}{d^3}-\frac{4(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(a^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6a^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2a^3c-6a^3d\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2d^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*(f*x + e)/d^3 - 4*(a^3*c^3 - 3*a^3
*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) +
arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3)
 + 2*(a^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^3
*d*tan(1/2*f*x + 1/2*e)^2 - a^3*d*tan(1/2*f*x + 1/2*e) + 2*a^3*c - 6*a^3*d)
/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

**Mupad** [**B**]
time = 9.22, size = 2500, normalized size = 17.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x)),x)
```

```
[Out] ((2*(a^3*c - 3*a^3*d))/d^2 + (a^3*tan(e/2 + (f*x)/2)^3)/d - (a^3*tan(e/2 +
(f*x)/2))/d + (2*tan(e/2 + (f*x)/2)^2*(a^3*c - 3*a^3*d))/d^2)/(f*(2*tan(e/2
```

+ (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (2*a^3*atan(((a^3*((8*(49*a^6*
c^2*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2)
)/d^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*
d^6 + 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/
d^6 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^
3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3
*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^
8))/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d
^3)*(c^2 - 3*c*d + (7*d^2)/2))/d^3 + (a^3*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
- 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*((8*(14*a^
3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a
^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 + (a^3*(32
*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(c^2 - 3*c*d
+ (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7
*d^2)/2))/d^3)/((16*(2*a^9*c^7 - 14*a^9*c*d^6 - 8*a^9*c^6*d + 47*a^9*c^2*d^
5 - 55*a^9*c^3*d^4 + 21*a^9*c^4*d^3 + 7*a^9*c^5*d^2))/d^5 + (16*tan(e/2 + (
f*x)/2)*(8*a^9*c^8 - 98*a^9*c*d^7 - 72*a^9*c^7*d + 462*a^9*c^2*d^6 - 926*a^
9*c^3*d^5 + 1034*a^9*c^4*d^4 - 704*a^9*c^5*d^3 + 296*a^9*c^6*d^2))/d^6 - (a
^3*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 +
4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^
7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8
*a^6*c^7*d^2))/d^6 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*
d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*
d^7 + 2*a^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*
d^10 - 8*c^3*d^8))/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (
7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3 + (a^3*((8*(49*a^6*c^2
*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d
^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6
+ 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6
+ (a^3*((8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^6))/d^5 - (8*tan(e
/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^
6))/d^6 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))
/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)
*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3))*(c^2 - 3*c*d + (7*d^2)/2))/(d^3*f) + (
a^3*atan(((a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
- 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(-(c + d)*
(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*
a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a
^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*
c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c + d))*1
i)/(d^3*(c + d)) + (a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84

*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*t
an(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^
6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(
-(c + d)*(c - d)^5)^(1/2)*((8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^
6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*
d^7 - 8*a^3*c^4*d^6))/d^6 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*
d^10 - 8*c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c
 + d)))*1i)/(d^3*(c + d)))/((16*(2*a^9*c^7 - 14*a^9*c*d^6 - 8*a^9*c^6*d + 4
7*a^9*c^2*d^5 - 55*a^9*c^3*d^4 + 21*a^9*c^4*d^3 + 7*a^9*c^5*d^2))/d^5 + (16
*tan(e/2 + (f*x)/2)*(8*a^9*c^8 - 98*a^9*c*d^7 - 72*a^9*c^7*d + 462*a^9*c^2*
d^6 - 926*a^9*c^3*d^5 + 1034*a^9*c^4*d^4 - 704*a^9*c^5*d^3 + 296*a^9*c^6*d^
2))/d^6 - (a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(-(c + d)*
(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*
a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a
^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*
c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c + d))))/
(d^3*(c + d)) + (a^3*(-(c + d)*(c - d)^5)^(1/2)...

# 3.449 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=161

$$-\frac{a^3(2c-3d)x}{d^3}+\frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}\,f}-\frac{2a^3c\cos(e+fx)}{d^2(c+d)f}+\frac{(c-d)\cos(e+fx)\,(a^3+\ldots}{d(c+d)f(c+d\sin\ldots}$$

[Out] -a^3*(2*c-3*d)*x/d^3-2*a^3*c*cos(f*x+e)/d^2/(c+d)/f+(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(2*c+3*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)

**Rubi** [A]
time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2841, 3047, 3102, 2814, 2739, 632, 210}

$$\frac{2a^3(c-d)^2(2c+3d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{a^3x(2c-3d)}{d^3}-\frac{2a^3c\cos(e+fx)}{d^2f(c+d)}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]

[Out] -((a^3*(2*c - 3*d)*x)/d^3) + (2*a^3*(c - d)^2*(2*c + 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)*Sqrt[c^2 - d^2]*f) - (2*a^3*c*Cos[e + f*x])/(d^2*(c + d)*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-3d)-2ac\sin(e+fx))}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)+(-2a^2c+a^2(c-3d))\sin(e+fx)-}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= -\frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)d+}{}}{}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{a^3(2c-3d)x}{d^3} + \frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}\,f} - \frac{2a^3c\cos(e+}{d^2(c+d)}$$

**Mathematica [A]**
time = 0.47, size = 162, normalized size = 1.01

$$\frac{a^3(1+\sin(e+fx))^3\left((-2c+3d)(e+fx)+\frac{2(c-d)^2(2c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}-d\cos(e+fx)-\frac{(c-d)^2d\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}\right)}{d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((-2*c + 3*d)*(e + f*x) + (2*(c - d)^2*(2*c + 3*d
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2
]) - d*Cos[e + f*x] - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*
x]))))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

**Maple [A]**
time = 0.54, size = 209, normalized size = 1.30

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2a^3\left(-\frac{\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2c-3d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{-\frac{d^2\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(c^2-2cd+d^2\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2c^3-c^2d-4c\,d^2+\cdots\right.}{(c\cdots}}{d^3}\right.}{f}$ |
| default | $\frac{2a^3\left(-\frac{\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2c-3d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{-\frac{d^2\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(c^2-2cd+d^2\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2c^3-c^2d-4c\,d^2+\cdots\right.}{(c\cdots}}{d^3}\right.}{f}$ |
| risch | $-\frac{2a^3xc}{d^3}+\frac{3a^3x}{d^2}-\frac{a^3\mathrm{e}^{i(fx+e)}}{2d^2f}-\frac{a^3\mathrm{e}^{-i(fx+e)}}{2d^2f}+\frac{2ia^3\left(c^2-2cd+d^2\right)\left(id+c\,\mathrm{e}^{i(fx+e)}\right)}{d^3(c+d)f\left(-id\,\mathrm{e}^{2i(fx+e)}+id+2c\,\mathrm{e}^{i(fx+e)}\right)}+\frac{2\sqrt{-(c+d)\cdots}}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3*(-1/d^3*(d/(1+tan(1/2*f*x+1/2*e)^2)+(2*c-3*d)*arctan(tan(1/2*f*x+1/
2*e)))+1/d^3*((-d^2*(c^2-2*c*d+d^2)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(c^2-2*c*d
+d^2)/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(2*c^3-c^2*d
-4*c*d^2+3*d^3)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.40, size = 664, normalized size = 4.12

$$\left[-\frac{2\left(2a^3c^3-a^3c^2d-3a^3cd^2\right)fx+\left(2a^3c^3+a^3c^2d-3a^3cd^2+\left(2a^3c^2d+a^3cd^2-3a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(\left(c^2+cd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right)+2\left(2a^3c^2d-a^3cd^2+a^3d^3\right)\cos\left(fx+e\right)+2\left(\left(2a^3c^2d-a^3cd^2-3a^3d^3\right)fx+\left(a^3cd^2+a^3d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{2\left(\left(cd^4+d^5\right)f\sin\left(fx+e\right)+\left(c^2d^3+cd^4\right)f\right)},-\frac{\left(2a^3c^3-a^3c^2d-3a^3cd^2\right)fx+\left(2a^3c^3+a^3c^2d-3a^3cd^2+\left(2a^3c^2d+a^3cd^2-3a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{\left(c\sin\left(fx+e\right)+d\right)\sqrt{\frac{c-d}{c+d}}}{\left(c-d\right)\cos\left(fx+e\right)}\right)+\left(2a^3c^2d-a^3cd^2+a^3d^3\right)\cos\left(fx+e\right)+\left(\left(2a^3c^2d-a^3cd^2-3a^3d^3\right)fx+\left(a^3cd^2+a^3d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{\left(cd^4+d^5\right)f\sin\left(fx+e\right)+\left(c^2d^3+cd^4\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d
 - 3*a^3*c*d^2 + (2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt(-
(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x +
 e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 2*(2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + 2*((2*a^3*c
^2*d - a^3*c*d^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin
(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*a^3*c
^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2 +
(2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) +
 (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + ((2*a^3*c^2*d - a^3*c*d
^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c
*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(161) = 322.
time = 0.46, size = 395, normalized size = 2.45

$$\frac{\frac{2\,{\left(2\,a^{3}c^{3}-a^{3}c^{2}d-4\,a^{3}cd^{2}+3\,a^{3}d^{3}\right)}\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)}{{\left(cd^{3}+d^{4}\right)}\sqrt{c^{2}-d^{2}}}-\frac{2\,{\left(a^{3}c^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,a^{3}cd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+a^{3}d^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,a^{3}c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a^{3}c^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a^{3}cd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,a^{3}c^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^{3}d^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,a^{3}c^{3}-a^{3}c^{2}d+a^{3}cd^{2}\right)}}{{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}{\left(c^{2}d^{2}+cd^{3}\right)}}-\frac{{\left(2\,a^{3}c-3\,a^{3}d\right)}{\left(fx+e\right)}}{d^{3}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x + e
)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/
((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(a^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a
^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^
3*tan(1/2*f*x + 1/2*e)^2 - a^3*c^2*d*tan(1/2*f*x + 1/2*e)^2 + a^3*c*d^2*tan
(1/2*f*x + 1/2*e)^2 + 3*a^3*c^2*d*tan(1/2*f*x + 1/2*e) + a^3*d^3*tan(1/2*f*
x + 1/2*e) + 2*a^3*c^3 - a^3*c^2*d + a^3*c*d^2)/((c*tan(1/2*f*x + 1/2*e)^4
+ 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*a^3*c - 3*a^3*d)*(f*x + e)/d^3)/f
```

**Mupad [B]**
time = 12.95, size = 2500, normalized size = 15.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(2*a^3*c^2 + a^3*d^2 - a^3*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2
)^2*(2*a^3*c^2 + a^3*d^2 - a^3*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2)*
(3*a^3*c^2 + a^3*d^2))/(c*d*(c + d)) + (2*tan(e/2 + (f*x)/2)^3*(a^3*c^2 + a
^3*d^2 - 2*a^3*c*d))/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*ta
n(e/2 + (f*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) -
(2*a^3*atan(((a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*
c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*t
an(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c
^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 +
c^2*d^6) + (a^3*(2*c - 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c
^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 +
c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(
2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c
*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3
*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d)*1i)/
d^3)*1i)/d^3))/d^3 + (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 -
 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5)
 + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 -
34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7
+ d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3
*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)
*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5
*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^
8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^
11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3
*d)*1i)/d^3)*1i)/d^3))/d^3)/((64*(4*a^9*c^6 + 27*a^9*c*d^5 - 20*a^9*c^5*d -
 63*a^9*c^2*d^4 + 33*a^9*c^3*d^3 + 19*a^9*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^
5) - (64*tan(e/2 + (f*x)/2)*(40*a^9*c^6*d - 54*a^9*c*d^6 - 16*a^9*c^7 + 90*
a^9*c^2*d^5 + 42*a^9*c^3*d^4 - 130*a^9*c^4*d^3 + 28*a^9*c^5*d^2))/(2*c*d^7
+ d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 1
1*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) +
 (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34
*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 +
d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2
*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 +
d^8 + c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d
^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)
```

)/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d)*1i)/d^3)*1i)/d^3)*1i)/d^3 - (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d)*1i)/d^3)*1i)/d^3)*1i)/d^3))*(2*c - 3*d))/(d^3*f) - (a^3*atan((a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c...

# 3.450 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=187

$$\frac{a^3x}{d^3}-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f}+\frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}+\frac{a^3(c}{2d^2(}$$

```
[Out] a^3*x/d^3+1/2*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+
e))^2+1/2*a^3*(c-d)*(2*c+5*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3
*(c-d)*(2*c^2+6*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))
/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2841, 3047, 3100, 2814, 2739, 632, 210}

$$-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df(c+d)(c+d\sin(e+fx))^2}+\frac{a^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*x)/d^3 - (a^3*(c - d)*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[
e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) +
 (a^3*(c - d)*(2*c + 5*d)*Cos[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f
*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-5d)-2a(c+d)\sin(e}{(c+d\sin(e+fx))^2}}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{a^2(c-5d)+\left(a^2(c-5d)-2a^2(c+d)\right)\sin(e+}{(c+d\sin(e+fx)}}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} +$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+}{2d^2(c+d)^2f(c+d\sin(e+}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+}{2d^2(c+d)^2f(c+d\sin(e+}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+}{2d^2(c+d)^2f(c+d\sin(e+}$$

$$= \frac{a^3x}{d^3} - \frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+}{2d(c+d)f}$$

**Mathematica** [A]
time = 0.63, size = 196, normalized size = 1.05

$$\frac{a^3(1+\sin(e+fx))^3\left(2(e+fx)-\frac{2\left(2c^3+4c^2d+cd^2-7d^3\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}}-\frac{(c-d)^2d\cos(e+fx)}{(c+d)(c+d\sin(e+fx))^2}+\frac{3d\left(c^2+cd-2d^2\right)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))}\right)}{2d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(e + f*x) - (2*(2*c^3 + 4*c^2*d + c*d^2 - 7*d^
3)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^2 -
d^2]) - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^2) + (3*d*
(c^2 + c*d - 2*d^2)*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

**Maple** [A]
time = 0.73, size = 353, normalized size = 1.89

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2a^3\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(c^3+5c^2d-4c\,d^2-2d^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(2c^5+4c^4d-c^3d^2+7c^2d^3-10c\,d^4-2d^5\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c^2}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}{f}$ |
| default | $\frac{2a^3\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(c^3+5c^2d-4c\,d^2-2d^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(2c^5+4c^4d-c^3d^2+7c^2d^3-10c\,d^4-2d^5\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c^2}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}{f}$ |
| risch | $\frac{a^3x}{d^3}-\frac{ia^3\left(-id^4\mathrm{e}^{i(fx+e)}-17ic\,d^3\mathrm{e}^{i(fx+e)}-id^4\mathrm{e}^{3i(fx+e)}-4ic^3d\,\mathrm{e}^{3i(fx+e)}+10ic^2d^2\mathrm{e}^{i(fx+e)}+7ic\,d^3\mathrm{e}^{3i(fx+e)}+8ic^3d\,\mathrm{e}^{i(f}}{\left(-id\,\mathrm{e}^{2i(fx+e)}+i\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3*(1/d^3*arctan(tan(1/2*f*x+1/2*e))-1/d^3*((-1/2*d^2*(c^3+5*c^2*d-4*c
*d^2-2*d^3)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(2*c^5+4*c^4*d-c^3
*d^2+7*c^2*d^3-10*c*d^4-2*d^5)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2-1/2
*d^2*(7*c^3+11*c^2*d-16*c*d^2-2*d^3)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-1
/2*d*(2*c^3+4*c^2*d-5*c*d^2-d^3)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2
*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(2*c^3+4*c^2*d+c*d^2-7*d^3)/(c^2+2*c*d+d^2)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(184) = 368$.

time = 0.42, size = 1089, normalized size = 5.82

$$\left[\frac{4\,{\left(a^{3} c^{2} d^{2} + 2\,a^{3} c d^{3} + a^{3} d^{4}\right)} f x \cos\left(f x + e\right)^{2} - 4\,{\left(a^{3} c^{4} + 2\,a^{3} c^{3} d + 2\,a^{3} c^{2} d^{2} + 2\,a^{3} c d^{3} + a^{3} d^{4}\right)} f x - {\left(2\,a^{3} c^{4} + 6\,a^{3} c^{3} d + 9\,a^{3} c^{2} d^{2} + 6\,a^{3} c d^{3} + 7\,a^{3} d^{4} - {\left(2\,a^{3} c^{2} d^{2} + 6\,a^{3} c d^{3} + 7\,a^{3} d^{4}\right)} \cos\left(f x + e\right)^{2} + 2\,{\left(2\,a^{3} c^{3} d + 6\,a^{3} c^{2} d^{2} + 7\,a^{3} c d^{3}\right)} \sin\left(f x + e\right)\right)} \sqrt{-\frac{c - d}{c + d}} \log\left(\frac{{\left(2\,c^{2} - d^{2}\right)} \cos\left(f x + e\right)^{2} - 2\,c d \sin\left(f x + e\right) - c^{2} - d^{2} + 2\,{\left({\left(c^{2} + c d\right)} \cos\left(f x + e\right) \sin\left(f x + e\right) + {\left(c d + d^{2}\right)} \cos\left(f x + e\right)\right)} \sqrt{-\frac{c - d}{c + d}}}{d^{2} \cos\left(f x + e\right)^{2} - 2\,c d \sin\left(f x + e\right) - c^{2} - d^{2}}\right) - 2\,{\left(2\,a^{3} c^{3} d + 4\,a^{3} c^{2} d^{2} - 5\,a^{3} c d^{3} - a^{3} d^{4}\right)} \cos\left(f x + e\right) - 2\,{\left(4\,{\left(a^{3} c^{3} d + 2\,a^{3} c^{2} d^{2} + a^{3} c d^{3}\right)} f x + 3\,{\left(a^{3} c^{2} d^{2} + a^{3} c d^{3} - 2\,a^{3} d^{4}\right)} \cos\left(f x + e\right)\right)} \sin\left(f x + e\right)}{4\,{\left({\left(c^{2} d^{5} + 2\,c d^{6} + d^{7}\right)} f \cos\left(f x + e\right)^{2} - 2\,{\left(c^{3} d^{4} + 2\,c^{2} d^{5} + c d^{6}\right)} f \sin\left(f x + e\right) - {\left(c^{4} d^{3} + 2\,c^{3} d^{4} + 2\,c^{2} d^{5} + 2\,c d^{6} + d^{7}\right)} f\right)}}, \frac{2\,{\left(a^{3} c^{2} d^{2} + 2\,a^{3} c d^{3} + a^{3} d^{4}\right)} f x \cos\left(f x + e\right)^{2} - 2\,{\left(a^{3} c^{4} + 2\,a^{3} c^{3} d + 2\,a^{3} c^{2} d^{2} + 2\,a^{3} c d^{3} + a^{3} d^{4}\right)} f x - {\left(2\,a^{3} c^{4} + 6\,a^{3} c^{3} d + 9\,a^{3} c^{2} d^{2} + 6\,a^{3} c d^{3} + 7\,a^{3} d^{4} - {\left(2\,a^{3} c^{2} d^{2} + 6\,a^{3} c d^{3} + 7\,a^{3} d^{4}\right)} \cos\left(f x + e\right)^{2} + 2\,{\left(2\,a^{3} c^{3} d + 6\,a^{3} c^{2} d^{2} + 7\,a^{3} c d^{3}\right)} \sin\left(f x + e\right)\right)} \sqrt{\frac{c - d}{c + d}} \arctan\left(-\frac{{\left(c \sin\left(f x + e\right) + d\right)} \sqrt{\frac{c - d}{c + d}}}{{\left(c - d\right)} \cos\left(f x + e\right)}\right) - {\left(2\,a^{3} c^{3} d + 4\,a^{3} c^{2} d^{2} - 5\,a^{3} c d^{3} - a^{3} d^{4}\right)} \cos\left(f x + e\right) - {\left(4\,{\left(a^{3} c^{3} d + 2\,a^{3} c^{2} d^{2} + a^{3} c d^{3}\right)} f x + 3\,{\left(a^{3} c^{2} d^{2} + a^{3} c d^{3} - 2\,a^{3} d^{4}\right)} \cos\left(f x + e\right)\right)} \sin\left(f x + e\right)}{2\,{\left({\left(c^{2} d^{5} + 2\,c d^{6} + d^{7}\right)} f \cos\left(f x + e\right)^{2} - 2\,{\left(c^{3} d^{4} + 2\,c^{2} d^{5} + c d^{6}\right)} f \sin\left(f x + e\right) - {\left(c^{4} d^{3} + 2\,c^{3} d^{4} + 2\,c^{2} d^{5} + 2\,c d^{6} + d^{7}\right)} f\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 4*(a^3*c
^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a^3*c^4
+ 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2*d^2 +
6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 +
7*a^3*c*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*
x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin
(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(2*a^3*c^3*d + 4*a^3*c^2*d^2
- 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - 2*(4*(a^3*c^3*d + 2*a^3*c^2*d^2 + a
^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e))*sin(f
*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d
^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d
^7)*f), 1/2*(2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 2
*(a^3*c^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a
^3*c^4 + 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2
*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2
*d^2 + 7*a^3*c*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x
+ e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (2*a^3*c^3*d + 4*
a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - (4*(a^3*c^3*d + 2*a^3*c
^2*d^2 + a^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x +
 e))*sin(f*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4
 + 2*c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2
*c*d^6 + d^7)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(184) = 368.
time = 0.53, size = 522, normalized size = 2.79

$$\frac{\frac{(fx+e)a^3}{d^3} - \frac{(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3+2cd^4+d^5)\sqrt{c^2-d^2}} + \frac{a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-10a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+11a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-16a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2a^3c^5+4a^3c^4d-5a^3c^3d^2-a^3c^2d^3}{(c^4d^2+2c^3d^3+c^2d^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/d^3 - (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c^2 - d^2)) + (a^3*c^4*d*
tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^3*c^2*d
^3*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^5*
tan(1/2*f*x + 1/2*e)^2 + 4*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^2 - a^3*c^3*d^2*t
an(1/2*f*x + 1/2*e)^2 + 7*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*c*d^4
*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*a^3*c^4*d*ta
n(1/2*f*x + 1/2*e) + 11*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 16*a^3*c^2*d^3*t
an(1/2*f*x + 1/2*e) - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*c^5 + 4*a^3*
c^4*d - 5*a^3*c^3*d^2 - a^3*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2*d^4)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

**Mupad [B]**
time = 14.27, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^3,x)
```

```
[Out] ((2*a^3*c^3 - a^3*d^3 - 5*a^3*c*d^2 + 4*a^3*c^2*d)/(d^2*(2*c*d + c^2 + d^2)
) + (tan(e/2 + (f*x)/2)^3*(a^3*c^3 - 2*a^3*d^3 - 4*a^3*c*d^2 + 5*a^3*c^2*d)
)/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/2)*(7*a^3*c^3 - 2*a^3*d^3 -
16*a^3*c*d^2 + 11*a^3*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/
2)^2*(c^2 + 2*d^2)*(2*a^3*c^3 - a^3*d^3 - 5*a^3*c*d^2 + 4*a^3*c^2*d))/(c^2*
d^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*ta
n(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*
x)/2))) - (2*a^3*atan(-(((((((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 1
6*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) +
 (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11
 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c
^3*d^7 + c^4*d^6))*1i)/d^3 - (8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d
^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan
(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*
d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d
^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*
c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*
c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*
c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))/d^3 + ((((8*(
4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 +
 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4
```

*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))/d^3)/((16*(18*c^4*d - 49*c*d^4 + 2*c^5 + 29*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (((((((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (((((8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (16*tan(e/2 + (f*x)/2)*(28*c*d^5 - 32*c^5*d - 8*c^6 + 52*c^2*d^4 + 4*c^3*d^3 - 44*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))))/(d^3*f) + (a^3*atan(((a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^6*c^2*d^6 + 16*a^6*c^3*d^5 + 24*a^6*c^4*d^4 + 16*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*a^6*c*d^8 - 46*a^6*c^2*d^7 - 99*a^6*c^3*d^6 - 36*a^6*c^4*d^5 + 36*a^6*c^5*d^4 + 32*a^6*c^6*d^3 + 8*a^6*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*tan(e/2 + (f*x)/2)*(28*a^3*c*d^11 + 52*a^3*c^2*d^10 + 4*a^3*c^3*d^9 - 44*a^3*c^4*d^8 - 32*a^3*c^5*d^7 - 8*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*a^3*c*d^10 + 2*a^

```
3*c^2*d^9 - 6*a^3*c^3*d^8 - 2*a^3*c^4*d^7 + 2*a...
```

# 3.451 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=207

$$\frac{5a^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^3\sqrt{c^2-d^2}\,f}+\frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}-$$

```
[Out] 1/3*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^3+1/6*
a^3*(c-d)*(2*c+7*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a^3*(2*
c^2+9*c*d+22*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))+5*a^3*arctan((d
+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)^3/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2841, 3047, 3100, 2833, 12, 2739, 632, 210}

$$\frac{5a^3\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^3\sqrt{c^2-d^2}}-\frac{a^3(2c^2+9cd+22d^2)\cos(e+fx)}{6d^2f(c+d)^3(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2f(c+d)^2(c+d\sin(e+fx))^2}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{3df(c+d)(c+d\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^3*Sqrt[c^
2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*
f*(c + d*Sin[e + f*x])^3) + (a^3*(c - d)*(2*c + 7*d)*Cos[e + f*x])/(6*d^2*(
c + d)^2*f*(c + d*Sin[e + f*x])^2) - (a^3*(2*c^2 + 9*c*d + 22*d^2)*Cos[e +
f*x])/(6*d^2*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-7d)-2a(c+2d)\sin}{(c+d\sin(e+fx))^3}}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{a^2(c-7d)+\left(a^2(c-7d)-2a^2(c+2d)\right)\sin(e}{(c+d\sin(e+fx}}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2}$$

$$= \frac{5a^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^3\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3}{6d}$$

**Mathematica** [A]
time = 1.50, size = 178, normalized size = 0.86

$$\frac{a^3\cos(e+fx)\left(\frac{15\tanh^{-1}\left(\frac{\sqrt{c-d}\,\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\,\sqrt{1+\sin(e+fx)}}\right)}{(-c-d)^{5/2}\sqrt{c-d}\,\sqrt{\cos^2(e+fx)}} - \frac{(1+\sin(e+fx))^2}{(c+d\sin(e+fx))^3} - \frac{5(1+\sin(e+fx))}{2(c+d)(c+d\sin(e+fx))^2} - \frac{15}{2(c+d)^2(c+d\sin(e+fx))}\right)}{3(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
[Out] (a^3*Cos[e + f*x]*((15*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(5/2)*Sqrt[c - d]*Sqrt[Cos[e + f
*x]^2]) - (1 + Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3 - (5*(1 + Sin[e + f*x
]))/(2*(c + d)*(c + d*Sin[e + f*x])^2) - 15/(2*(c + d)^2*(c + d*Sin[e + f*x
]))))/(3*(c + d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(196) = 392$.
time = 0.89, size = 466, normalized size = 2.25

| method | result |
|---|---|
| derivativedivides | $\frac{2a^3\left(\frac{\frac{\left(3c^3-6c^2d-6c\,d^2-2d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{\left(6c^4-3c^3d+30c^2d^2+18d^3c+4d^4\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{d\left(66c^4+27c^3d+50c^2d^2+18d^3c}{3c^3\left(c^3+3c^2d+3\right.}}{\left(c\left(\tan^2\left(\frac{fx}{2}\right.\right.\right.}\right.}{}$ |
| default | $\frac{2a^3\left(\frac{\frac{\left(3c^3-6c^2d-6c\,d^2-2d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{\left(6c^4-3c^3d+30c^2d^2+18d^3c+4d^4\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{d\left(66c^4+27c^3d+50c^2d^2+18d^3c}{3c^3\left(c^3+3c^2d+3\right.}}{\left(c\left(\tan^2\left(\frac{fx}{2}\right.\right.\right.}\right.}{}$ |
| risch | $\frac{ia^3\left(8c^5e^{3i(fx+e)}-22id^5+9d^5e^{5i(fx+e)}-9d^5e^{i(fx+e)}-54ic^3d^2e^{4i(fx+e)}-90ic^2d^3e^{4i(fx+e)}+9ic\,d^4e^{4i(fx+e)}+12ic^4d\,e^{2i(\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3*((1/2*(3*c^3-6*c^2*d-6*c*d^2-2*d^3)/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan
(1/2*f*x+1/2*e)^5-1/2*(6*c^4-3*c^3*d+30*c^2*d^2+18*c*d^3+4*d^4)/c^2/(c^3+3*
c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4-1/3/c^3*d*(66*c^4+27*c^3*d+50*c^2*d
^2+18*c*d^3+4*d^4)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-(8*c^4+6*
c^3*d+30*c^2*d^2+9*c*d^3+2*d^4)/c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1
/2*e)^2-1/2*(3*c^3+38*c^2*d+12*c*d^2+2*d^3)/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan
(1/2*f*x+1/2*e)-1/6*(22*c^2+9*c*d+2*d^2)/(c^3+3*c^2*d+3*c*d^2+d^3))/(c*tan(
1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^3+5/2/(c^3+3*c^2*d+3*c*d^2+d^3)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(204) = 408$.
time = 0.41, size = 1135, normalized size = 5.48

$$\left[-\frac{2\left(2a^3c^4+9a^3c^3d+20a^3c^2d^2-9a^3cd^3-22a^3d^4\right)\cos\left(fx+e\right)^3-6\left(3a^3c^4+16a^3c^3d-16a^3cd^3-3a^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+15\left(3a^3cd^2\cos\left(fx+e\right)^2-a^3c^3-3a^3cd^2+\left(a^3d^3\cos\left(fx+e\right)^2-3a^3c^2d-a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)-12\left(4a^3c^4+3a^3c^3d-3a^3cd^3-4a^3d^4\right)\cos\left(fx+e\right)}{12\left(3\left(c^6d^2+3c^5d^3+2c^4d^4-2c^3d^5-3c^2d^6-cd^7\right)f\cos\left(fx+e\right)^2-\left(c^8+3c^7d+5c^6d^2+7c^5d^3+3c^4d^4-7c^3d^5-9c^2d^6-3cd^7\right)f+\left(\left(c^5d^3+3c^4d^4+2c^3d^5-2c^2d^6-3cd^7-d^8\right)f\cos\left(fx+e\right)^2-\left(3c^7d+9c^6d^2+7c^5d^3-3c^4d^4-7c^3d^5-5c^2d^6-3cd^7-d^8\right)f\right)\sin\left(fx+e\right)\right)}, -\frac{\left(2a^3c^4+9a^3c^3d+20a^3c^2d^2-9a^3cd^3-22a^3d^4\right)\cos\left(fx+e\right)^3-3\left(3a^3c^4+16a^3c^3d-16a^3cd^3-3a^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+15\left(3a^3cd^2\cos\left(fx+e\right)^2-a^3c^3-3a^3cd^2+\left(a^3d^3\cos\left(fx+e\right)^2-3a^3c^2d-a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin\left(fx+e\right)+d}{\sqrt{c^2-d^2}\cos\left(fx+e\right)}\right)-6\left(4a^3c^4+3a^3c^3d-3a^3cd^3-4a^3d^4\right)\cos\left(fx+e\right)}{6\left(3\left(c^6d^2+3c^5d^3+2c^4d^4-2c^3d^5-3c^2d^6-cd^7\right)f\cos\left(fx+e\right)^2-\left(c^8+3c^7d+5c^6d^2+7c^5d^3+3c^4d^4-7c^3d^5-9c^2d^6-3cd^7\right)f+\left(\left(c^5d^3+3c^4d^4+2c^3d^5-2c^2d^6-3cd^7-d^8\right)f\cos\left(fx+e\right)^2-\left(3c^7d+9c^6d^2+7c^5d^3-3c^4d^4-7c^3d^5-5c^2d^6-3cd^7-d^8\right)f\right)\sin\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*
d^4)*cos(f*x + e)^3 - 6*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^
4)*cos(f*x + e)*sin(f*x + e) + 15*(3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3
*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(4*a^3*c^4
+ 3*a^3*c^3*d - 3*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*
d^3 + 2*c^4*d^4 - 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 +
3*c^7*d + 5*c^6*d^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d
^7)*f + ((c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*co
s(f*x + e)^2 - (3*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5
*c^2*d^6 - 3*c*d^7 - d^8)*f)*sin(f*x + e)), -1/6*((2*a^3*c^4 + 9*a^3*c^3*d
+ 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4)*cos(f*x + e)^3 - 3*(3*a^3*c^4
+ 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e)*sin(f*x + e) + 15*(
3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^
2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(4*a^3*c^4 + 3*a^3*c^3*d - 3
*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 -
 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 + 3*c^7*d + 5*c^6*d
^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d^7)*f + ((c^5*d^3
 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*cos(f*x + e)^2 - (3
*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5*c^2*d^6 - 3*c*d^
7 - d^8)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(204) =$ 408.

time = 0.54, size = 667, normalized size = 3.22

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a^3}{(c^3+3c^2d+3cd^2+d^3)\sqrt{c^2-d^2}}+\frac{9a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-6a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+9a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-90a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-54a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-12a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-132a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-54a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-100a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-36a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-8a^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-48a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-36a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-180a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-54a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12a^3cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-9a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-114a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-36a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6a^3c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-22a^3c^5-9a^3c^4d-2a^3c^3d^2}{(c^6+3c^5d+3c^4d^2+c^3d^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x +
1/2*e) + d)/sqrt(c^2 - d^2)))*a^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sqrt(c^2
 - d^2)) + (9*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d*tan(1/2*f*x + 1
/2*e)^5 - 18*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^3*tan(1/2*f*x
 + 1/2*e)^5 - 18*a^3*c^5*tan(1/2*f*x + 1/2*e)^4 + 9*a^3*c^4*d*tan(1/2*f*x +
 1/2*e)^4 - 90*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 - 54*a^3*c^2*d^3*tan(1/2*
f*x + 1/2*e)^4 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 132*a^3*c^4*d*tan(1/
2*f*x + 1/2*e)^3 - 54*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 100*a^3*c^2*d^3*
tan(1/2*f*x + 1/2*e)^3 - 36*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*d^5*ta
n(1/2*f*x + 1/2*e)^3 - 48*a^3*c^5*tan(1/2*f*x + 1/2*e)^2 - 36*a^3*c^4*d*tan
(1/2*f*x + 1/2*e)^2 - 180*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 54*a^3*c^2*d
^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*c^5
*tan(1/2*f*x + 1/2*e) - 114*a^3*c^4*d*tan(1/2*f*x + 1/2*e) - 36*a^3*c^3*d^2
*tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 22*a^3*c^5 - 9
*a^3*c^4*d - 2*a^3*c^3*d^2)/((c^6 + 3*c^5*d + 3*c^4*d^2 + c^3*d^3)*(c*tan(1
/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f
```

**Mupad [B]**
time = 10.36, size = 649, normalized size = 3.14

$$\frac{5a^3\,\mathrm{atan}\left(\frac{\left(\frac{5a^3\,(2c^3d+6c^2d^2+6cd^3+2d^4)}{2\,(c+d)^{7/2}\sqrt{c-d}\,(c^3+3c^2d+3cd^2+d^3)}+\frac{5a^3c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c+d)^{7/2}\sqrt{c-d}}\right)(c^3+3c^2d+3cd^2+d^3)}{5a^3}\right)}{f\,(c+d)^{7/2}\sqrt{c-d}}-\frac{\frac{22a^3c^2+9a^3cd+2a^3d^2}{3\,(c^3+3c^2d+3cd^2+d^3)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(-3c^3+6c^2d+6cd^2+2d^3)}{c\,(c^3+3c^2d+3cd^2+d^3)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3c^3+38c^2d+12cd^2+2d^3)}{c\,(c^3+3c^2d+3cd^2+d^3)}+\frac{2a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(8c^4+6c^3d+30c^2d^2+9cd^3+2d^4)}{c^2\,(c^3+3c^2d+3cd^2+d^3)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6c^4-3c^3d+30c^2d^2+18cd^3+4d^4)}{c^2\,(c^3+3c^2d+3cd^2+d^3)}+\frac{2a^3d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(3c^2+2d^2)\,(22c^2+9cd+2d^2)}{3c^3\,(c^3+3c^2d+3cd^2+d^3)}}{f\left(c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(3c^3+12cd^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(3c^3+12cd^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(12c^2d+8d^3)+c^3+6c^2d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+6c^2d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^4,x)
```

```
[Out] (5*a^3*atan((((5*a^3*(6*c*d^3 + 2*c^3*d + 2*d^4 + 6*c^2*d^2))/(2*(c + d)^(7
/2)*(c - d)^(1/2)*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (5*a^3*c*tan(e/2 + (f*
x)/2))/((c + d)^(7/2)*(c - d)^(1/2)))*(3*c*d^2 + 3*c^2*d + c^3 + d^3))/(5*a
^3)))/(f*(c + d)^(7/2)*(c - d)^(1/2)) - ((22*a^3*c^2 + 2*a^3*d^2 + 9*a^3*c*
d)/(3*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan(e/2 + (f*x)/2)^5*(6*c*d^2
 + 6*c^2*d - 3*c^3 + 2*d^3))/(c*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan
(e/2 + (f*x)/2)*(12*c*d^2 + 38*c^2*d + 3*c^3 + 2*d^3))/(c*(3*c*d^2 + 3*c^2*
d + c^3 + d^3)) + (2*a^3*tan(e/2 + (f*x)/2)^2*(9*c*d^3 + 6*c^3*d + 8*c^4 +
2*d^4 + 30*c^2*d^2))/(c^2*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan(e/2 +
 (f*x)/2)^4*(18*c*d^3 - 3*c^3*d + 6*c^4 + 4*d^4 + 30*c^2*d^2))/(c^2*(3*c*d^
2 + 3*c^2*d + c^3 + d^3)) + (2*a^3*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(
9*c*d + 22*c^2 + 2*d^2))/(3*c^3*(3*c*d^2 + 3*c^2*d + c^3 + d^3)))/(f*(c^3*t
an(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (
f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^
3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (f*x)/2)^5))
```

# 3.452 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx$

**Optimal**. Leaf size=289

$$\frac{5a^3(4c-3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)(c+d)^4\sqrt{c^2-d^2}\,f}+\frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4}+\frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3}$$

```
[Out] 1/4*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^4+1/12
*a^3*(c-d)*(2*c+9*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^3-1/24*a^3*(
2*c^2+12*c*d+45*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^2-1/24*a^3*(
2*c^3+12*c^2*d+43*c*d^2-72*d^3)*cos(f*x+e)/(c-d)/d^2/(c+d)^4/f/(c+d*sin(f*x
+e))+5/4*a^3*(4*c-3*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c-
d)/(c+d)^4/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.48, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2841, 3047, 3100, 2833, 12, 2739, 632, 210}

$$\frac{5a^3(4c-3d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)(c+d)^4\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{24d^2f(c+d)^3(c+d\sin(e+fx))^2}-\frac{a^3\left(2c^3+12c^2d+43cd^2-72d^3\right)\cos(e+fx)}{24d^2f(c-d)(c+d)^4(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2f(c+d)^2(c+d\sin(e+fx))^3}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{4df(c+d)(c+d\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (5*a^3*(4*c - 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(4*(c
- d)*(c + d)^4*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e
+ f*x]))/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) + (a^3*(c - d)*(2*c + 9*d)*
Cos[e + f*x])/(12*d^2*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^3*(2*c^2 + 1
2*c*d + 45*d^2)*Cos[e + f*x])/(24*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^2) -
 (a^3*(2*c^3 + 12*c^2*d + 43*c*d^2 - 72*d^3)*Cos[e + f*x])/(24*(c - d)*d^2*
(c + d)^4*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
```

```
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx &= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-9d)-2a(c+3d)\sin}{(c+d\sin(e+fx))^4}}{4d(c+d)} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{a^2(c-9d)+\left(a^2(c-9d)-2a^2(c+3d)\right)\sin(e}{(c+d\sin(e+fx}}{4d(c+d)} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} \\
&= \frac{5a^3(4c-3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)(c+d)^4\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+f\right.}{4d(c+d)f(c+d\sin(e+fx))^4}
\end{aligned}$$

**Mathematica** [A]
time = 2.01, size = 240, normalized size = 0.83

$$\frac{a^3\cos(e+fx)\left(-\frac{d(1+\sin(e+fx))^3}{(c+d\sin(e+fx))^4}-\frac{(4c-3d)\left(-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)}{(-c-d)^{7/2}\sqrt{c-d}}-\frac{\sqrt{\cos^2(e+fx)}\left(22c^2+9cd+2d^2+\left(9c^2+48cd+9d^2\right)\sin(e+fx)+\left(2c^2+9cd+22d^2\right)\sin^2(e+fx)\right)}{6(c+d)^3(c+d\sin(e+fx))^3}\right)}{\sqrt{\cos^2(e+fx)}}\right)}{4(-c+d)(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^3)/(c + d*Sin[e + f*x])^4) - ((4
*c - 3*d)*((-5*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*S
qrt[1 + Sin[e + f*x]])])/((-c - d)^(7/2)*Sqrt[c - d]) - (Sqrt[Cos[e + f*x]^
2]*(22*c^2 + 9*c*d + 2*d^2 + (9*c^2 + 48*c*d + 9*d^2)*Sin[e + f*x] + (2*c^2
 + 9*c*d + 22*d^2)*Sin[e + f*x]^2))/(6*(c + d)^3*(c + d*Sin[e + f*x])^3)))/
Sqrt[Cos[e + f*x]^2]))/(4*(-c + d)*(c + d)*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(274) = 548$.
time = 1.40, size = 922, normalized size = 3.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3*((1/8*(12*c^5-39*c^4*d-16*c^3*d^2+16*c^2*d^3+24*c*d^4+8*d^5)/c/(c^5
+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)*tan(1/2*f*x+1/2*e)^7-1/8*(24*c^6-
44*c^5*d+225*c^4*d^2-120*c^2*d^4-96*c*d^5-24*d^6)/(c^5+3*c^4*d+2*c^3*d^2-2*
c^2*d^3-3*c*d^4-d^5)/c^2*tan(1/2*f*x+1/2*e)^6+1/24/c^3*(36*c^7-813*c^6*d+28
8*c^5*d^2-892*c^4*d^3+552*c^3*d^4+664*c^2*d^5+384*c*d^6+96*d^7)/(c^5+3*c^4*
d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)*tan(1/2*f*x+1/2*e)^5-1/24/c^4*(264*c^8-1
08*c^7*d+2001*c^6*d^2-936*c^5*d^3-202*c^4*d^4-864*c^3*d^5-440*c^2*d^6-192*c
*d^7-48*d^8)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)*tan(1/2*f*x+1/2*
e)^4-1/24/c^3*(36*c^7+1299*c^6*d-576*c^5*d^2+1036*c^4*d^3-1176*c^3*d^4-664*
c^2*d^5-384*c*d^6-96*d^7)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)*tan
(1/2*f*x+1/2*e)^3-1/24*(280*c^6-12*c^5*d+1289*c^4*d^2-960*c^3*d^3-552*c^2*d
^4-288*c*d^5-72*d^6)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)/c^2*tan(
1/2*f*x+1/2*e)^2-1/24*(36*c^5+587*c^4*d-336*c^3*d^2-248*c^2*d^3-120*c*d^4-2
4*d^5)/c/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)*tan(1/2*f*x+1/2*e)-1
/24*(88*c^4-36*c^3*d-37*c^2*d^2-24*c*d^3-6*d^4)/(c^5+3*c^4*d+2*c^3*d^2-2*c^
2*d^3-3*c*d^4-d^5))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^4+5/8
*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-3*c*d^4-d^5)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(284) = 568$.
time = 0.44, size = 2044, normalized size = 7.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 21
7*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e)^3 - 15*(4*a^3*c^5
- 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5 +
 (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3*c^3*d^2 - 9*a^3*c^2*d
^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d
^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x +
e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) - 6*(32*a^3*c^6 + 4*a^3*c^5*d + 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3
*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d^6)*cos(f*x + e) + 2*((2*a^3*c^5*d + 12*a
^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*c
os(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*
d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5 + 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*
d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c
^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*
f*cos(f*x + e)^2 + (c^11 + 3*c^10*d + 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 -
26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7 + 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d
^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^
8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3
- 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6 - 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f)*sin(f*x + e)), 1/24*((8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2
- 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x +
 e)^3 - 15*(4*a^3*c^5 - 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a
^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3
*c^3*d^2 - 9*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a
^3*c^4*d - 3*a^3*c^3*d^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3
*a^3*c*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*
x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 3*(32*a^3*c^6 + 4*a^3*c^5*d +
 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d
^6)*cos(f*x + e) + ((2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3
*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*cos(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a
^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5
+ 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 -
 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3
```

```
*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*
c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^2 + (c^11 + 3*c^10*d
+ 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 - 26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7
+ 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c
^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x +
e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3 - 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6
- 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c*d^10)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(284) = 568$.
time = 0.53, size = 1338, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + 3*c^4*d + 2*c^3*
d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(c^2 - d^2)) + (36*a^3*c^8*tan(1/2*f*x
 + 1/2*e)^7 - 117*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^7 - 48*a^3*c^6*d^2*tan(1/2
*f*x + 1/2*e)^7 + 48*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^7 + 72*a^3*c^4*d^4*ta
n(1/2*f*x + 1/2*e)^7 + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^7 - 72*a^3*c^8*t
an(1/2*f*x + 1/2*e)^6 + 132*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^6 - 675*a^3*c^6*
d^2*tan(1/2*f*x + 1/2*e)^6 + 360*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^6 + 288*a
^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^6 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^6 +
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 813*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 +
 288*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 892*a^3*c^5*d^3*tan(1/2*f*x + 1/2
*e)^5 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 664*a^3*c^3*d^5*tan(1/2*f*
x + 1/2*e)^5 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 + 96*a^3*c*d^7*tan(1/
2*f*x + 1/2*e)^5 - 264*a^3*c^8*tan(1/2*f*x + 1/2*e)^4 + 108*a^3*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 2001*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 936*a^3*c^5*d
^3*tan(1/2*f*x + 1/2*e)^4 + 202*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 + 864*a^
3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 440*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 +
 192*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 + 48*a^3*d^8*tan(1/2*f*x + 1/2*e)^4 -
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^3 - 1299*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^3
```

```
+ 576*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 1036*a^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1176*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 664*a^3*c^3*d^5*tan(1/2
*f*x + 1/2*e)^3 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 96*a^3*c*d^7*tan
(1/2*f*x + 1/2*e)^3 - 280*a^3*c^8*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c^7*d*tan
(1/2*f*x + 1/2*e)^2 - 1289*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 960*a^3*c^5
*d^3*tan(1/2*f*x + 1/2*e)^2 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 288*
a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^2
- 36*a^3*c^8*tan(1/2*f*x + 1/2*e) - 587*a^3*c^7*d*tan(1/2*f*x + 1/2*e) + 33
6*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 248*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) +
 120*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)
 - 88*a^3*c^8 + 36*a^3*c^7*d + 37*a^3*c^6*d^2 + 24*a^3*c^5*d^3 + 6*a^3*c^4*
d^4)/((c^9 + 3*c^8*d + 2*c^7*d^2 - 2*c^6*d^3 - 3*c^5*d^4 - c^4*d^5)*(c*tan(
1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

**Mupad** [B]
time = 10.10, size = 1231, normalized size = 4.26

$$-\frac{\frac{-88a^3c^4+36a^3c^3d+37a^3c^2d^2+24a^3cd^3+6a^3d^4}{12(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7(12c^5-39c^4d-16c^3d^2+16c^2d^3+24cd^4+8d^5)}{4c(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(-24c^6+44c^5d-225c^4d^2+120c^2d^4+96cd^5+24d^6)}{4c^2(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(-36c^5-587c^4d+336c^3d^2+248c^2d^3+120cd^4+24d^5)}{12c(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(36c^7-813c^6d+288c^5d^2-892c^4d^3+552c^3d^4+664c^2d^5+384cd^6+96d^7)}{12c^3(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(-36c^7-1299c^6d+576c^5d^2-1036c^4d^3+1176c^3d^4+664c^2d^5+384cd^6+96d^7)}{12c^3(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-280c^6+12c^5d-1289c^4d^2+960c^3d^3+552c^2d^4+288cd^5+72d^6)}{12c^2(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(3c^4+24c^2d^2+8d^4)(-88c^4+36c^3d+37c^2d^2+24cd^3+6d^4)}{12c^4(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6c^4+48c^2d^2+16d^4)+c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+c^4+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(4c^4+24c^2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4c^4+24c^2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(24c^3d+32cd^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(24c^3d+32cd^3)+8c^3d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+8c^3d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\right)}-\frac{5a^3\,\mathrm{atan}\left(\frac{4\left(\frac{5a^3(4c-3d)(-8c^5d-24c^4d^2-16c^3d^3+16c^2d^4+24cd^5+8d^6)}{32(c+d)^{9/2}(c-d)^{3/2}(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}+\frac{5a^3c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c-3d)}{4(c+d)^{9/2}(c-d)^{3/2}}\right)(-c^5-3c^4d-2c^3d^2+2c^2d^3+3cd^4+d^5)}{20a^3c-15a^3d}\right)(4c-3d)}{4f(c+d)^{9/2}(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^5,x)
```

```
[Out] - ((6*a^3*d^4 - 88*a^3*c^4 + 24*a^3*c*d^3 + 36*a^3*c^3*d + 37*a^3*c^2*d^2)/
(12*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2
 + (f*x)/2)^7*(24*c*d^4 - 39*c^4*d + 12*c^5 + 8*d^5 + 16*c^2*d^3 - 16*c^3*d
^2))/(4*c*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*t
an(e/2 + (f*x)/2)^6*(96*c*d^5 + 44*c^5*d - 24*c^6 + 24*d^6 + 120*c^2*d^4 -
225*c^4*d^2))/(4*c^2*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2
)) + (a^3*tan(e/2 + (f*x)/2)*(120*c*d^4 - 587*c^4*d - 36*c^5 + 24*d^5 + 248
*c^2*d^3 + 336*c^3*d^2))/(12*c*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 -
 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x)/2)^5*(384*c*d^6 - 813*c^6*d + 36*c^7 +
96*d^7 + 664*c^2*d^5 + 552*c^3*d^4 - 892*c^4*d^3 + 288*c^5*d^2))/(12*c^3*(3
*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*
x)/2)^3*(384*c*d^6 - 1299*c^6*d - 36*c^7 + 96*d^7 + 664*c^2*d^5 + 1176*c^3*
d^4 - 1036*c^4*d^3 + 576*c^5*d^2))/(12*c^3*(3*c*d^4 - 3*c^4*d - c^5 + d^5 +
 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x)/2)^2*(288*c*d^5 + 12*c^5*d
- 280*c^6 + 72*d^6 + 552*c^2*d^4 + 960*c^3*d^3 - 1289*c^4*d^2))/(12*c^2*(3*
c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x
)/2)^4*(3*c^4 + 8*d^4 + 24*c^2*d^2)*(24*c*d^3 + 36*c^3*d - 88*c^4 + 6*d^4 +
 37*c^2*d^2))/(12*c^4*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^
2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 16*d^4 + 48*c^2*d^2) + c^4*tan(e/2 +
 (f*x)/2)^8 + c^4 + tan(e/2 + (f*x)/2)^2*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (
f*x)/2)^6*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(32*c*d^3 + 24*c^3*d)
 + tan(e/2 + (f*x)/2)^5*(32*c*d^3 + 24*c^3*d) + 8*c^3*d*tan(e/2 + (f*x)/2)
+ 8*c^3*d*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((4*((5*a^3*(4*c - 3*d)*(24*c
```

```
*d^5 - 8*c^5*d + 8*d^6 + 16*c^2*d^4 - 16*c^3*d^3 - 24*c^4*d^2))/(32*(c + d)
^(9/2)*(c - d)^(3/2)*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2
)) + (5*a^3*c*tan(e/2 + (f*x)/2)*(4*c - 3*d))/(4*(c + d)^(9/2)*(c - d)^(3/2
)))*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2))/(20*a^3*c - 15
*a^3*d))*(4*c - 3*d))/(4*f*(c + d)^(9/2)*(c - d)^(3/2))
```

# 3.453 $\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=189

$$\frac{d(8c^3-12c^2d+12cd^2-3d^3)\,x}{2a}+\frac{2d(3c^3-16c^2d+12cd^2-4d^3)\cos(e+fx)}{3af}+\frac{d^2(6c^2-20cd+9d^2)\cos(e+}{6af}$$

```
[Out] 1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*x/a+2/3*d*(3*c^3-16*c^2*d+12*c*d^2-4*
d^3)*cos(f*x+e)/a/f+1/6*d^2*(6*c^2-20*c*d+9*d^2)*cos(f*x+e)*sin(f*x+e)/a/f+
1/3*(3*c-4*d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f-(c-d)*cos(f*x+e)*(c+d*sin
(f*x+e))^3/f/(a+a*sin(f*x+e))
```

**Rubi** [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2846, 2832, 2813}

$$\frac{d^2(6c^2-20cd+9d^2)\sin(e+fx)\cos(e+fx)}{6af}+\frac{2d(3c^3-16c^2d+12cd^2-4d^3)\cos(e+fx)}{3af}+\frac{dx(8c^3-12c^2d+12cd^2-3d^3)}{2a}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}+\frac{d(3c-4d)\cos(e+fx)(c+d\sin(e+fx))^2}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*x)/(2*a) + (2*d*(3*c^3 - 16*c^2*d
 + 12*c*d^2 - 4*d^3)*Cos[e + f*x])/(3*a*f) + (d^2*(6*c^2 - 20*c*d + 9*d^2)*C
os[e + f*x]*Sin[e + f*x])/(6*a*f) + ((3*c - 4*d)*d*Cos[e + f*x]*(c + d*Sin[
e + f*x])^2)/(3*a*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a
 + a*Sin[e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} - \frac{d\int(-a(4c-3d)+a(3c-4d)\sin(e}{a^2}$$
$$= \frac{(3c-4d)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{f(a+a\sin(e+fx))}$$
$$= \frac{d(8c^3-12c^2d+12cd^2-3d^3)\,x}{2a} + \frac{2d(3c^3-16c^2d+12cd^2-4d^3)\cos(e+fx)}{3af} +$$

**Mathematica** [A]
time = 0.28, size = 234, normalized size = 1.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(c-d)^4\sin\left(\frac{1}{2}(e+fx)\right)-6d(-8c^3+12c^2d-12cd^2+3d^3)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-3d^2(24c^2-16cd+7d^2)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+d^4\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-3(4c-d)d^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(2(e+fx))\right)}{12af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(c - d)^4*Sin[(e + f*x)/2] - 6*d
*(-8*c^3 + 12*c^2*d - 12*c*d^2 + 3*d^3)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 3*d^2*(24*c^2 - 16*c*d + 7*d^2)*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) + d^4*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 3*(4*c - d)*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e +
 f*x)]))/(12*a*f*(1 + Sin[e + f*x]))
```

**Maple** [A]
time = 0.39, size = 230, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(d*(((2*c*d^2-1/2*d^3)*tan(1/2*f*x+1/2*e)^5+(-6*c^2*d+4*c*d^2-d^3)*ta
n(1/2*f*x+1/2*e)^4+(-12*c^2*d+8*c*d^2-4*d^3)*tan(1/2*f*x+1/2*e)^2+(-2*c*d^2
+1/2*d^3)*tan(1/2*f*x+1/2*e)-6*c^2*d+4*c*d^2-5/3*d^3)/(1+tan(1/2*f*x+1/2*e)
^2)^3+1/2*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*arctan(tan(1/2*f*x+1/2*e)))-(c^4-
4*c^3*d+6*c^2*d^2-4*c*d^3+d^4)/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(189) = 378$.
time $= 0.60$, size $= 791$, normalized size $= 4.19$

$$-\frac{d^{4}\left(\frac{\frac{7\sin(fx+e)}{\cos(fx+e)+1}+\frac{39\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{24\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{24\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{9\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{9\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+16}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{3a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{3a\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{a\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{a\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}}+\frac{9\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-12cd^{3}\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{3\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{2a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+36c^{2}d^{2}\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-24c^{3}d\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{6c^{4}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(d^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 12*c*d^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^2*d^2*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 24*c^3*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [**A**]
time $= 0.38$, size $= 359$, normalized size $= 1.90$

$$\frac{2d^{4}\cos(fx+e)^{4}-6c^{4}+24c^{3}d-36c^{2}d^{2}+24cd^{3}-6d^{4}+(12cd^{3}-d^{4})\cos(fx+e)^{3}+3(8c^{3}d-12c^{2}d^{2}+12cd^{3}-3d^{4})fx-12(3c^{2}d^{2}-2cd^{3}+d^{4})\cos(fx+e)^{2}-3(2c^{4}-8c^{3}d+24c^{2}d^{2}-12cd^{3}+5d^{4}-(8c^{3}d-12c^{2}d^{2}+12cd^{3}-3d^{4})fx)\cos(fx+e)+(2d^{4}\cos(fx+e)^{3}+6c^{4}-24c^{3}d+36c^{2}d^{2}-24cd^{3}+6d^{4}+3(8c^{3}d-12c^{2}d^{2}+12cd^{3}-3d^{4})fx-3(4cd^{3}-d^{4})\cos(fx+e)^{2}-3(12c^{2}d^{2}-4cd^{3}+3d^{4})\cos(fx+e))\sin(fx+e)}{6(af\cos(fx+e)+af\sin(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^4*cos(f*x + e)^4 - 6*c^4 + 24*c^3*d - 36*c^2*d^2 + 24*c*d^3 - 6*d^
4 + (12*c*d^3 - d^4)*cos(f*x + e)^3 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 -
3*d^4)*f*x - 12*(3*c^2*d^2 - 2*c*d^3 + d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 8*c
^3*d + 24*c^2*d^2 - 12*c*d^3 + 5*d^4 - (8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3
*d^4)*f*x)*cos(f*x + e) + (2*d^4*cos(f*x + e)^3 + 6*c^4 - 24*c^3*d + 36*c^2
*d^2 - 24*c*d^3 + 6*d^4 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*f*x -
 3*(4*c*d^3 - d^4)*cos(f*x + e)^2 - 3*(12*c^2*d^2 - 4*c*d^3 + 3*d^4)*cos(f*
x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 8605 vs. $2(170) = 340$.
time = 5.19, size = 8605, normalized size = 45.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-12*c**4*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) - 36*c**4*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2
+ f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a
*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*c**4/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2 + f*x/2)**7/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2 + f*x
/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f
*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)*
*6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e
/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*
f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**2/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2
+ f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d
*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
```

```
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 48*c**3*d*tan
(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 1
44*c**3*d*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) + 144*c**3*d*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/
2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan
(e/2 + f*x/2) + 6*a*f) + 48*c**3*d/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e
/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f
*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2 + f*x/2)**6/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*ta
n(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) -
108*c**2*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c**...
```

**Giac** [A]
time = 0.47, size = 306, normalized size = 1.62

$$\frac{\frac{3\,(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4)(fx+e)}{a}-\frac{12\,(c^4-4\,c^3d+6\,c^2d^2-4\,cd^3+d^4)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(12\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-36\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+24\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-6\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-72\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+48\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-24\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,c^2d^2+24\,cd^3-10\,d^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*(f*x + e)/a - 12*(c^4 - 4*
c^3*d + 6*c^2*d^2 - 4*c*d^3 + d^4)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(12*c
*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*d^4*tan(1/2*f*x + 1/2*e)^5 - 36*c^2*d^2*tan
(1/2*f*x + 1/2*e)^4 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^4 - 6*d^4*tan(1/2*f*x +
 1/2*e)^4 - 72*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 48*c*d^3*tan(1/2*f*x + 1/2*
```

```
e)^2 - 24*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 3*d^
4*tan(1/2*f*x + 1/2*e) - 36*c^2*d^2 + 24*c*d^3 - 10*d^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

**Mupad [B]**
time = 9.79, size = 451, normalized size = 2.39

$$\frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3\right)}{8\,c^3\,d-12\,c^2\,d^2+12\,c\,d^3-3\,d^4}\right)\,\left(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3\right)}{a\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,c^2\,d^2-4\,c\,d^3+\frac{7\,d^4}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,c^4-8\,c^3\,d+12\,c^2\,d^2-12\,c\,d^3+3\,d^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(6\,c^4-24\,c^3\,d+48\,c^2\,d^2-32\,c\,d^3+8\,d^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(6\,c^4-24\,c^3\,d+60\,c^2\,d^2-36\,c\,d^3+13\,d^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(12\,c^2\,d^2-12\,c\,d^3+3\,d^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(24\,c^2\,d^2-16\,c\,d^3+8\,d^4\right)-16\,c\,d^3-8\,c^3\,d+2\,c^4+\frac{16\,d^4}{3}+24\,c^2\,d^2}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x)),x)
```

```
[Out] (d*atan((d*tan(e/2 + (f*x)/2)*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(12*c*
d^3 + 8*c^3*d - 3*d^4 - 12*c^2*d^2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))
/(a*f) - (tan(e/2 + (f*x)/2)*((7*d^4)/3 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 +
 (f*x)/2)^6*(2*c^4 - 8*c^3*d - 12*c*d^3 + 3*d^4 + 12*c^2*d^2) + tan(e/2 + (
f*x)/2)^4*(6*c^4 - 24*c^3*d - 32*c*d^3 + 8*d^4 + 48*c^2*d^2) + tan(e/2 + (f
*x)/2)^2*(6*c^4 - 24*c^3*d - 36*c*d^3 + 13*d^4 + 60*c^2*d^2) + tan(e/2 + (f
*x)/2)^5*(3*d^4 - 12*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(8*d^4 - 16
*c*d^3 + 24*c^2*d^2) - 16*c*d^3 - 8*c^3*d + 2*c^4 + (16*d^4)/3 + 24*c^2*d^2
)/(f*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (
f*x)/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e/2
 + (f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7))
```

# 3.454 $\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=121

$$\frac{3d(2c^2-2cd+d^2)\,x}{2a}+\frac{2d(c^2-3cd+d^2)\cos(e+fx)}{af}+\frac{(2c-3d)d^2\cos(e+fx)\sin(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}$$

```
[Out] 3/2*d*(2*c^2-2*c*d+d^2)*x/a+2*d*(c^2-3*c*d+d^2)*cos(f*x+e)/a/f+1/2*(2*c-3*d
)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*
sin(f*x+e))
```

**Rubi** [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2846, 2813}

$$\frac{2d(c^2-3cd+d^2)\cos(e+fx)}{af}+\frac{3dx(2c^2-2cd+d^2)}{2a}+\frac{d^2(2c-3d)\sin(e+fx)\cos(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*x)/(2*a) + (2*d*(c^2 - 3*c*d + d^2)*Cos[e + f*x]
)/(a*f) + ((2*c - 3*d)*d^2*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} - \frac{d\int(-a(3c-2d)+a(2c-3d)\sin(e}{a^2}$$
$$= \frac{3d(2c^2-2cd+d^2)\,x}{2a} + \frac{2d(c^2-3cd+d^2)\cos(e+fx)}{af} + \frac{(2c-3d)d^2\cos(e+fx}{2af}$$

## Mathematica [A]

time = 0.39, size = 192, normalized size = 1.59

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(6(2c^2-2cd+d^2)(e+fx)-4(3c-d)d\cos(e+fx)-d^2\sin(2(e+fx))\right)+\sin\left(\frac{1}{2}(e+fx)\right)\left(2(4c^3+6c^2d(-2+e+fx)-6cd^2(-2+e+fx)+d^3(-4+3e+3fx))-4(3c-d)d^2\cos(e+fx)-d^3\sin(2(e+fx))\right)\right)}{4af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(6*(2*c^2 - 2*c*
d + d^2)*(e + f*x) - 4*(3*c - d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)]) + S
in[(e + f*x)/2]*(2*(4*c^3 + 6*c^2*d*(-2 + e + f*x) - 6*c*d^2*(-2 + e + f*x)
 + d^3*(-4 + 3*e + 3*f*x)) - 4*(3*c - d)*d^2*Cos[e + f*x] - d^3*Sin[2*(e +
f*x)])))/(4*a*f*(1 + Sin[e + f*x]))
```

## Maple [A]

time = 0.32, size = 148, normalized size = 1.22

| method | result |
|---|---|
| derivativedivides | $\frac{2d\left(\frac{\frac{d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-3cd+d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-3cd+d^2}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{3\left(2c^2-2cd+d^2\right)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)-\frac{2(c^3}{\text{ta}}}{af}$ |
| default | $\frac{2d\left(\frac{\frac{d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-3cd+d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-3cd+d^2}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{3\left(2c^2-2cd+d^2\right)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)-\frac{2(c^3}{\text{ta}}}{af}$ |
| risch | $\frac{3dx\,c^2}{a}-\frac{3d^2xc}{a}+\frac{3d^3x}{2a}-\frac{3d^2e^{i(fx+e)}c}{2af}+\frac{d^3e^{i(fx+e)}}{2af}-\frac{3d^2e^{-i(fx+e)}c}{2af}+\frac{d^3e^{-i(fx+e)}}{2af}-\frac{2c^3}{fa\left(e^{i(fx+e)}+i\right)}+$ |
| norman | $\frac{\frac{-6c\,d^2+d^3}{af}+\frac{\left(-6c\,d^2-d^3\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(2c^3-6c^2d-2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(2c^3-6c^2d+6c\,d^2-3d^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{(6c^3-18c}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(d*((1/2*d^2*tan(1/2*f*x+1/2*e)^3+(-3*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-1
/2*d^2*tan(1/2*f*x+1/2*e)-3*c*d+d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+3/2*(2*c^2-
```

```
2*c*d+d^2)*arctan(tan(1/2*f*x+1/2*e)))-(c^3-3*c^2*d+3*c*d^2-d^3)/(tan(1/2*f
*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(123) = 246$.

time = 0.53, size = 463, normalized size = 3.83

$$\frac{d^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-6cd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+6c^2d\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{2c^3}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (d^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*c*d^2*((sin(f*x + e)/(cos(f*x +
 e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^2*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) - 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [A]

time = 0.34, size = 244, normalized size = 2.02

$$\frac{d^3\cos(fx+e)^3-2c^3+6c^2d-6cd^2+2d^3+3(2c^2d-2cd^2+d^3)fx-2(3cd^2-d^3)\cos(fx+e)^2-(2c^3-6c^2d+12cd^2-3d^3-3(2c^2d-2cd^2+d^3)fx)\cos(fx+e)-(d^3\cos(fx+e)^2-2c^3+6c^2d-6cd^2+2d^3-3(2c^2d-2cd^2+d^3)fx+(6cd^2-d^3)\cos(fx+e))\sin(fx+e)}{2(af\cos(fx+e)+af\sin(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^3*cos(f*x + e)^3 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 + 3*(2*c^2*d -
2*c*d^2 + d^3)*f*x - 2*(3*c*d^2 - d^3)*cos(f*x + e)^2 - (2*c^3 - 6*c^2*d +
12*c*d^2 - 3*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^3)*f*x)*cos(f*x + e) - (d^3*cos
(f*x + e)^2 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^
3)*f*x + (6*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*
f*sin(f*x + e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3602 vs. $2(107) = 214$.

time = 2.50, size = 3602, normalized size = 29.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)

[Out] Piecewise((-4*c**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*c**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e
/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*
a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*c**3/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*
x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**
4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
 f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d*f*x*tan(e/2 + f*
x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(
e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*
f) + 12*c**2*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x
/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f
*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12
*c**2*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) + 24*c**2*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan
(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d**2*f*x*tan(e/2 + f*x/2)*
*3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
 f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) -
12*c*d**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/
2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a
*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2
 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x/(2*a
*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d*
*2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
```

```
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) - 12*c*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 36*c*d**2*tan(e/2 + f*x/2)**
2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 1
2*c*d**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) - 24*c*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2
 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*
f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2
 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/
2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 6*d**3*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*
f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)*
*2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*d**3*f*x*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f
*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 3*d**3*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan...
```

**Giac** [A]
time = 0.47, size = 172, normalized size = 1.42

$$\frac{\frac{3\,(2\,c^2d-2\,cd^2+d^3)(fx+e)}{a}-\frac{4\,(c^3-3\,c^2d+3\,cd^2-d^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\,\left(d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,cd^2+2\,d^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*(f*x + e)/a - 4*(c^3 - 3*c^2*d + 3*c*d^2 -
 d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(d^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*
d^2*tan(1/2*f*x + 1/2*e)^2 + 2*d^3*tan(1/2*f*x + 1/2*e)^2 - d^3*tan(1/2*f*x
 + 1/2*e) - 6*c*d^2 + 2*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

**Mupad** [B]
time = 9.43, size = 282, normalized size = 2.33

$$\frac{3\,d\,\mathrm{atan}\left(\frac{3\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c^2-2\,c\,d+d^2)}{6\,c^2\,d-6\,c\,d^2+3\,d^3}\right)\,(2\,c^2-2\,c\,d+d^2)}{a\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(6\,c\,d^2-d^3)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,c^3-6\,c^2\,d+6\,c\,d^2-3\,d^3)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(4\,c^3-12\,c^2\,d+18\,c\,d^2-5\,d^3)+12\,c\,d^2-6\,c^2\,d+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(6\,c\,d^2-3\,d^3)+2\,c^3-4\,d^3}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x)),x)

[Out] (3*d*atan((3*d*tan(e/2 + (f*x)/2)*(2*c^2 - 2*c*d + d^2))/(6*c^2*d - 6*c*d^2
 + 3*d^3))*(2*c^2 - 2*c*d + d^2))/(a*f) - (tan(e/2 + (f*x)/2)*(6*c*d^2 - d^
3) + tan(e/2 + (f*x)/2)^4*(6*c*d^2 - 6*c^2*d + 2*c^3 - 3*d^3) + tan(e/2 + (
f*x)/2)^2*(18*c*d^2 - 12*c^2*d + 4*c^3 - 5*d^3) + 12*c*d^2 - 6*c^2*d + tan(
e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^3) + 2*c^3 - 4*d^3)/(f*(a + a*tan(e/2 + (f*
x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (
f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^5))
```

# 3.455 $\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=62

$$\frac{(2c-d)dx}{a}-\frac{d^2\cos(e+fx)}{af}-\frac{(c-d)^2\cos(e+fx)}{af(1+\sin(e+fx))}$$

```
[Out] (2*c-d)*d*x/a-d^2*cos(f*x+e)/a/f-(c-d)^2*cos(f*x+e)/a/f/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2825, 2814, 2727}

$$-\frac{(c-d)^2\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{dx(2c-d)}{a}-\frac{d^2\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*c - d)*d*x)/a - (d^2*Cos[e + f*x])/(a*f) - ((c - d)^2*Cos[e + f*x])/(a*
f*(1 + Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)}{af}+\frac{\int \frac{ac^2+a(2c-d)d\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}$$
$$= \frac{(2c-d)dx}{a}-\frac{d^2\cos(e+fx)}{af}+(c-d)^2\int \frac{1}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(2c-d)dx}{a}-\frac{d^2\cos(e+fx)}{af}-\frac{(c-d)^2\cos(e+fx)}{f(a+a\sin(e+fx))}$$

**Mathematica [A]**
time = 0.31, size = 122, normalized size = 1.97

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(-((2c-d)(e+fx))+d\cos(e+fx)\right)+\left(-2c^2-2cd(-2+e+fx)+d^2(-2+e+fx)+d^2\cos(e+fx)\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(-((2*c - d)*(
e + f*x)) + d*Cos[e + f*x]) + (-2*c^2 - 2*c*d*(-2 + e + f*x) + d^2*(-2 + e
+ f*x) + d^2*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])))
```

**Maple [A]**
time = 0.31, size = 75, normalized size = 1.21

| method | result |
|---|---|
| derivativedivides | $\frac{2d\left(-\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2c-d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)-\frac{2\left(c^2-2cd+d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$ |
| default | $\frac{2d\left(-\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2c-d)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)-\frac{2\left(c^2-2cd+d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$ |
| risch | $\frac{2dxc}{a}-\frac{d^2x}{a}-\frac{d^2e^{i(fx+e)}}{2af}-\frac{d^2e^{-i(fx+e)}}{2af}-\frac{2c^2}{fa\left(e^{i(fx+e)}+i\right)}+\frac{4cd}{fa\left(e^{i(fx+e)}+i\right)}-\frac{2d^2}{fa\left(e^{i(fx+e)}+i\right)}$ |
| norman | $\frac{\frac{-2c^2+4cd-4d^2}{af}+\frac{(2c-d)dx}{a}-\frac{2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(-2c^2+4cd-2d^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{(2c-d)dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{(2c-d)dx\left(\tan^4\left(\frac{fx}{2}+\right.\right.}{a}}{\left(1+\tan^2\left(\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(d*(-d/(1+tan(1/2*f*x+1/2*e)^2)+(2*c-d)*arctan(tan(1/2*f*x+1/2*e)))-(
c^2-2*c*d+d^2)/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(65) = 130$.

time = 0.51, size = 227, normalized size = 3.66

$$-\frac{2\left(d^{2}\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2\,cd\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c^{2}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - 2*c*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(65) = 130$.

time = 0.35, size = 148, normalized size = 2.39

$$-\frac{d^{2}\cos(fx+e)^{2}-(2\,cd-d^{2})fx+c^{2}-2\,cd+d^{2}-((2\,cd-d^{2})fx-c^{2}+2\,cd-2\,d^{2})\cos(fx+e)-((2\,cd-d^{2})fx-d^{2}\cos(fx+e)+c^{2}-2\,cd+d^{2})\sin(fx+e)}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*cos(f*x + e)^2 - (2*c*d - d^2)*f*x + c^2 - 2*c*d + d^2 - ((2*c*d - d^
2)*f*x - c^2 + 2*c*d - 2*d^2)*cos(f*x + e) - ((2*c*d - d^2)*f*x - d^2*cos(f
*x + e) + c^2 - 2*c*d + d^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(46) = 92$.

time = 1.39, size = 940, normalized size = 15.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c**2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*c**2/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan
(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*t
an(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2
```

```
)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*ta
n(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) + 2*c*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f
*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 4*c*d*tan(e/2 + f*x/2)**2/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
4*c*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2
)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x
/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x/(a*f*tan(e/2 + f*x/
2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*d**2*tan(
e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) - 2*d**2*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 +
a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*d**2/(a*f*tan(e/2
 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(f,
 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a), True))
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(65) = 130$.

time = 0.41, size = 143, normalized size = 2.31

$$\frac{\frac{(2\,cd-d^2)(fx+e)}{a} - \frac{2\left(c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2-2\,cd+2\,d^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*c*d - d^2)*(f*x + e)/a - 2*(c^2*tan(1/2*f*x + 1/2*e)^2 - 2*c*d*tan(1/2*
f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e) + c^
2 - 2*c*d + 2*d^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(
1/2*f*x + 1/2*e) + 1)*a))/f
```

**Mupad** [**B**]
time = 7.38, size = 124, normalized size = 2.00

$$-\frac{d^2\,f\,x-2\,c\,d\,f\,x}{a\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2-4\,c\,d+2\,d^2\right)-4\,c\,d+2\,c^2+4\,d^2+2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x)),x)
```

```
[Out] - (d^2*f*x - 2*c*d*f*x)/(a*f) - (tan(e/2 + (f*x)/2)^2*(2*c^2 - 4*c*d + 2*d^
2) - 4*c*d + 2*c^2 + 4*d^2 + 2*d^2*tan(e/2 + (f*x)/2))/(f*(a + a*tan(e/2 +
(f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

# 3.456 $\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=35

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] d*x/a-(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

**Rubi** [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2814, 2727}

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
[Out] (d*x)/a - ((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= \frac{dx}{a}-(-c+d)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.
time = 0.11, size = 79, normalized size = 2.26

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2c+d(-2+e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*(e + f*x)*Cos[(e + f*x)/2] + (2*c
 + d*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

**Maple [A]**
time = 0.25, size = 42, normalized size = 1.20

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{2(c-d)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+2d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$ | 42 |
| default | $\frac{-\frac{2(c-d)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+2d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$ | 42 |
| risch | $\frac{dx}{a}-\frac{2c}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2d}{fa\left(e^{i(fx+e)}+i\right)}$ | 54 |
| norman | $\frac{\frac{dx}{a}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{dx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(2c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{(2c-2d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ | 134 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/f/a*(-(c-d)/(tan(1/2*f*x+1/2*e)+1)+d*arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.
time = 0.49, size = 84, normalized size = 2.40

$$\frac{2\left(d\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{c}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] 2*(d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

**Fricas [A]**
time = 0.34, size = 70, normalized size = 2.00

$$\frac{dfx+(dfx-c+d)\cos(fx+e)+(dfx+c-d)\sin(fx+e)-c+d}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d*f*x + (d*f*x - c + d)*cos(f*x + e) + (d*f*x + c - d)*sin(f*x + e) - c +
d)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.63, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{dfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{2d}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f \neq 0 \\ \frac{x(c+d\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c/(a*f*tan(e/2 + f*x/2) + a*f) + d*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + d*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*d/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a), True))
```

**Giac** [A]
time = 0.55, size = 40, normalized size = 1.14

$$\frac{\frac{(fx+e)d}{a} - \frac{2(c-d)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/a - 2*(c - d)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

**Mupad** [B]
time = 6.83, size = 35, normalized size = 1.00

$$\frac{d\,x}{a} - \frac{2\,c - 2\,d}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

```
[Out] (d*x)/a - (2*c - 2*d)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

# 3.457 $\int \frac{1}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=23

$$-\frac{\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] -cos(f*x+e)/f/(a+a*sin(f*x+e))
```

**Rubi** [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2727}

$$-\frac{\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] -(Cos[e + f*x]/(f*(a + a*Sin[e + f*x])))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(e+fx)}\,dx = -\frac{\cos(e+fx)}{f(a+a\sin(e+fx))}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.
time = 0.03, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f(a+a\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*(a + a*Sin[e
+ f*x]))
```

**Maple** [A]
time = 0.18, size = 22, normalized size = 0.96

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ | 22 |
| default | $-\frac{2}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ | 22 |
| risch | $-\frac{2}{fa\left(e^{i(fx+e)}+i\right)}$ | 23 |
| norman | $\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ | 31 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/a/(tan(1/2*f*x+1/2*e)+1)
```

**Maxima** [A]
time = 0.29, size = 29, normalized size = 1.26

$$-\frac{2}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

**Fricas** [A]
time = 0.35, size = 46, normalized size = 2.00

$$-\frac{\cos\left(fx+e\right)-\sin\left(fx+e\right)+1}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(cos(f*x + e) - sin(f*x + e) + 1)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a
*f)
```

**Sympy** [A]
time = 0.39, size = 27, normalized size = 1.17

$$\begin{cases}-\frac{2}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\: f\neq 0\\\frac{x}{a\sin\left(e\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x/(a*sin(e) + a), T
rue))
```

**Giac [A]**
time = 0.42, size = 22, normalized size = 0.96

$$-\frac{2}{af\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/(a*f*(tan(1/2*f*x + 1/2*e) + 1))
```

**Mupad [B]**
time = 6.97, size = 21, normalized size = 0.91

$$-\frac{2}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x)),x)
```

```
[Out] -2/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

# 3.458 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=89

$$-\frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

```
[Out] -cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))-2*d*arctan((d+c*tan(1/2*f*x+1/2*e))/(c
^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2826, 2727, 2739, 632, 210}

$$-\frac{2d\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*Sqrt[c^2
 - d^2]*f) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx &= \frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{c-d} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)} \\
&= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} - \frac{(2d)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x,\right.}{a(c-d)f} \\
&= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} + \frac{(4d)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx\right.}{a(c-} \\
&= -\frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}
\end{aligned}
$$

**Mathematica [A]**
time = 0.23, size = 114, normalized size = 1.28

$$
\frac{\cos(e+fx)\left(\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)}{\sqrt{-c-d}\sqrt{c-d}\sqrt{\cos^2(e+fx)}} + \frac{1}{1+\sin(e+fx)}\right)}{a(-c+d)f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((2*d*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c -
 d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*Sqrt[c - d]*Sqrt[Cos[e + f*x]^2
]) + (1 + Sin[e + f*x])^(-1)))/(a*(-c + d)*f)
```

**Maple [A]**
time = 0.44, size = 83, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)\sqrt{c^{2}-d^{2}}}}{af}$ |
| default | $\frac{-\frac{2}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)\sqrt{c^{2}-d^{2}}}}{af}$ |
| risch | $-\frac{2}{f(c-d)a\left(e^{i(fx+e)}+i\right)}-\frac{d\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^{2}+d^{2}}+c^{2}-d^{2}}{\sqrt{-c^{2}+d^{2}}\,d}\right)}{\sqrt{-c^{2}+d^{2}}\,(c-d)fa}+\frac{d\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^{2}+d^{2}}-c^{2}+d^{2}}{\sqrt{-c^{2}+d^{2}}\,d}\right)}{\sqrt{-c^{2}+d^{2}}\,(c-d)fa}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(-1/(c-d)/(tan(1/2*f*x+1/2*e)+1)-d/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(87) = 174$.
time = 0.39, size = 510, normalized size = 5.73

$$\left[\frac{\sqrt{-c^{2}+d^{2}}\,(d\cos(fx+e)+d\sin(fx+e)+d)\log\left(\frac{(2c^{2}-d^{2})\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^{2}+d^{2}}}{d^{2}\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}}\right)-2c^{2}+2d^{2}-2(c^{2}-d^{2})\cos(fx+e)+2(c^{2}-d^{2})\sin(fx+e)}{2\left((ac^{3}-ac^{2}d-acd^{2}+ad^{3})f\cos(fx+e)+(ac^{3}-ac^{2}d-acd^{2}+ad^{3})f\sin(fx+e)+(ac^{3}-ac^{2}d-acd^{2}+ad^{3})f\right)},\ \frac{\sqrt{c^{2}-d^{2}}\,(d\cos(fx+e)+d\sin(fx+e)+d)\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^{2}-d^{2}}\cos(fx+e)}\right)-c^{2}+d^{2}-(c^{2}-d^{2})\cos(fx+e)+(c^{2}-d^{2})\sin(fx+e)}{(ac^{3}-ac^{2}d-acd^{2}+ad^{3})f\cos(fx+e)+(ac^{3}-ac^{2}d-acd^{2}+ad^{3})f\sin(fx+e)+(ac^{3}-ac^{2}d-acd^{2}+ad^{3})f}\right]$$

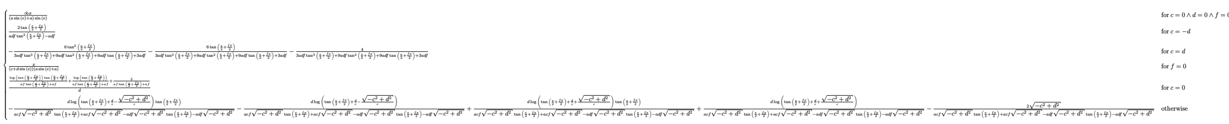
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-c^2 + d^2)*(d*cos(f*x + e) + d*sin(f*x + e) + d)*log(((2*c^2 -
d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*si
```

```
n(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) - 2*c^2 + 2*d^2 - 2*(c^2 - d^2)*cos(f*x + e) + 2
*(c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x +
 e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d
 - a*c*d^2 + a*d^3)*f), (sqrt(c^2 - d^2)*(d*cos(f*x + e) + d*sin(f*x + e) +
 d)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - c^2 + d^
2 - (c^2 - d^2)*cos(f*x + e) + (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d
- a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*s
in(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(68) = 136$.
time = 119.14, size = 916, normalized size = 10.29



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x/((a*sin(e) + a)*sin(e)), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
(2*tan(e/2 + f*x/2)/(a*d*f*tan(e/2 + f*x/2)**2 - a*d*f), Eq(c, -d)), (-6*ta
n(e/2 + f*x/2)**2/(3*a*d*f*tan(e/2 + f*x/2)**3 + 9*a*d*f*tan(e/2 + f*x/2)**
2 + 9*a*d*f*tan(e/2 + f*x/2) + 3*a*d*f) - 6*tan(e/2 + f*x/2)/(3*a*d*f*tan(e
/2 + f*x/2)**3 + 9*a*d*f*tan(e/2 + f*x/2)**2 + 9*a*d*f*tan(e/2 + f*x/2) + 3
*a*d*f) - 4/(3*a*d*f*tan(e/2 + f*x/2)**3 + 9*a*d*f*tan(e/2 + f*x/2)**2 + 9*
a*d*f*tan(e/2 + f*x/2) + 3*a*d*f), Eq(c, d)), (x/((c + d*sin(e))*(a*sin(e)
+ a)), Eq(f, 0)), ((log(tan(e/2 + f*x/2))*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f
*x/2) + a*f) + log(tan(e/2 + f*x/2))/(a*f*tan(e/2 + f*x/2) + a*f) + 2/(a*f*
tan(e/2 + f*x/2) + a*f))/d, Eq(c, 0)), (-d*log(tan(e/2 + f*x/2) + d/c - sqr
t(-c**2 + d**2)/c)*tan(e/2 + f*x/2)/(a*c*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x
/2) + a*c*f*sqrt(-c**2 + d**2) - a*d*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2)
- a*d*f*sqrt(-c**2 + d**2)) - d*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d
**2)/c)/(a*c*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2) + a*c*f*sqrt(-c**2 + d**
2) - a*d*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2) - a*d*f*sqrt(-c**2 + d**2))
+ d*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(e/2 + f*x/2)/(a*
c*f*sqrt(-c**2 + d**2)*tan(e/2 + f*x/2) + a*c*f*sqrt(-c**2 + d**2) - a*d*f*
sqrt(-c**2 + d**2)*tan(e/2 + f*x/2) - a*d*f*sqrt(-c**2 + d**2)) + d*log(tan
(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(a*c*f*sqrt(-c**2 + d**2)*tan(e
/2 + f*x/2) + a*c*f*sqrt(-c**2 + d**2) - a*d*f*sqrt(-c**2 + d**2)*tan(e/2 +
 f*x/2) - a*d*f*sqrt(-c**2 + d**2)) - 2*sqrt(-c**2 + d**2)/(a*c*f*sqrt(-c**
2 + d**2)*tan(e/2 + f*x/2) + a*c*f*sqrt(-c**2 + d**2) - a*d*f*sqrt(-c**2 +
d**2)*tan(e/2 + f*x/2) - a*d*f*sqrt(-c**2 + d**2)), True))
```

**Giac** [**A**]

time = 0.46, size = 100, normalized size = 1.12

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d}{(ac-ad)\sqrt{c^2-d^2}}+\frac{1}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d/((a*c - a*d)*sqrt(c^2 - d^2)) + 1/((a*c - a*d)*
(tan(1/2*f*x + 1/2*e) + 1)))/f
```

**Mupad [B]**
time = 6.99, size = 121, normalized size = 1.36

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}-\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c-a\,d\right)}{a\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}}{2\,d}\right)}{a\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}-\frac{2}{f\,\left(a+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(c-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)) - (2*c*d
*tan(e/2 + (f*x)/2)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)))/(2*d)))/(
a*f*(c + d)^(1/2)*(c - d)^(3/2)) - 2/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

# 3.459 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=150

$$-\frac{2d(2c+d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{3/2}f}-\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

```
[Out] -2*d*(2*c+d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-
d^2)^(3/2)/f-d*(c+2*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))-cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2847, 2833, 12, 2739, 632, 210}

$$-\frac{2d(2c+d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}-\frac{d(c+2d)\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d*(2*c + d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d
)*(c^2 - d^2)^(3/2)*f) - (d*(c + 2*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*
(c + d*Sin[e + f*x])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d
*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} + \frac{d\int \frac{-2a}{(c+}}{a^2}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{}{(c-d)f(a+a\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{}{(c-d)f(a+a\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{}{(c-d)f(a+a\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{}{(c-d)f(a+a\sin}$$

$$= -\frac{2d(2c+d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}\,f} - \frac{d(c+2d)\,\mathrm{c}}{a(c-d)^2(c+d)f(}$$

## Mathematica [A]

time = 0.43, size = 162, normalized size = 1.08

$$\frac{\cos(e+fx)\left(\frac{2d(2c+d)\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)}{\sqrt{-c-d}\,(c-d)^{3/2}\sqrt{\cos^2(e+fx)}}+\frac{c+2d}{(c-d)(1+\sin(e+fx))}-\frac{d}{(1+\sin(e+fx))(c+d\sin(e+fx))}\right)}{a(-c+d)(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (Cos[e + f*x]*((2*d*(2*c + d)*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/
(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*(c - d)^(3/2)*Sqrt[Co
s[e + f*x]^2]) + (c + 2*d)/((c - d)*(1 + Sin[e + f*x])) - d/((1 + Sin[e + f
*x])*(c + d*Sin[e + f*x]))))/(a*(-c + d)*(c + d)*f)
```

## **Maple** [A]
time = 0.61, size = 154, normalized size = 1.03

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2d\left(\frac{\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(2c+d)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^2}-\frac{2}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{af}$ |
| default | $\frac{-\frac{2d\left(\frac{\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(2c+d)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^2}-\frac{2}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{af}$ |
| risch | $-\frac{2\left(-2icd\,e^{2i(fx+e)}-id^2e^{2i(fx+e)}+icd+2id^2+2c^2e^{i(fx+e)}+3cd\,e^{i(fx+e)}+d^2e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+i\right)(c+d)\left(-id\,e^{2i(fx+e)}+id+2c\,e^{i(fx+e)}\right)f(c-d)^2a}-\frac{2d\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2-}}{\sqrt{-c^2}}\right.}{\sqrt{-c^2+d^2}\,(c+d}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(-d/(c-d)^2*((d^2/(c+d)/c*tan(1/2*f*x+1/2*e)+d/(c+d))/(c*tan(1/2*f*x+
1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(2*c+d)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/(c-d)^2/(tan(1/2*f*x+1/2*
e)+1))
```

## **Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(151) = 302$.
time = 0.42, size = 1153, normalized size = 7.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*c^4 - 4*c^2*d^2 + 2*d^4 + 2*(c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos
(f*x + e)^2 + (2*c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (
2*c^2*d + c*d^2)*cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*
cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)) + 2*(c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - 2*(c^4 - 2*c^2*d
^2 + d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/
((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(
f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c
*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*
c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
+ e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (c^4 -
 2*c^2*d^2 + d^4 + (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e)^2 - (2*
c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (2*c^2*d + c*d^2)*
cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*cos(f*x + e))*sin
(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*co
s(f*x + e))) + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - (c^4 - 2*c^2*d^2
+ d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/((a
*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
 + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*c^5
*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e
) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(151) = 302.
time = 0.51, size = 311, normalized size = 2.07

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(2cd+d^2)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}}+\frac{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c^3+c^2d+cd^2}{(ac^4-ac^3d-ac^2d^2+acd^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3
)*sqrt(c^2 - d^2)) + (c^3*tan(1/2*f*x + 1/2*e)^2 + c^2*d*tan(1/2*f*x + 1/2*
e)^2 + d^3*tan(1/2*f*x + 1/2*e)^2 + 2*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*
tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x + 1/2*e) + c^3 + c^2*d + c*d^2)/((a*
c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1/2*
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2*d*
tan(1/2*f*x + 1/2*e) + c)))/f
```

**Mupad** [B]
time = 8.57, size = 309, normalized size = 2.06

$$-\frac{\frac{2\,(c^2+c\,d+d^2)}{(c+d)\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(d^2+2\,c\,d)}{c\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(c^3+c^2\,d+d^3)}{c\,(c+d)\,(c-d)^2}}{f\,\left(a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}-\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(2\,c+d)\,(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4)}{a\,(c+d)^{3/2}\,(c-d)^{5/2}}+\frac{2\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(2\,c+d)\,(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3)}{a\,(c+d)^{3/2}\,(c-d)^{5/2}}}{2\,d^2+4\,c\,d}\right)(2\,c+d)}{a\,f\,(c+d)^{3/2}\,(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] - ((2*(c*d + c^2 + d^2))/((c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(2*c*d
 + d^2))/(c*(c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(c^2*d + c^3 + d^3))/(c*(c
 + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c + 2*a*d) + tan(e/2 +
(f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3)) - (2*d*atan(((d*(2*c +
d)*(2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d))/(a*(c + d)^(3/2)*(c - d
)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(2*c + d)*(a*c^3 + a*d^3 - a*c*d^2 - a
*c^2*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(4*c*d + 2*d^2))*(2*c + d))/(a*f*
(c + d)^(3/2)*(c - d)^(5/2))
```

# 3.460 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=213

$$-\frac{3d(2c^2+2cd+d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{5/2}f}-\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2}-\frac{c}{(c-d)f(a+a\sin(}$$

```
[Out] -3*d*(2*c^2+2*c*d+d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(
c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*c+3*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*si
n(f*x+e))^2-cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2
*c+d)*(c+4*d)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2847, 2833, 12, 2739, 632, 210}

$$-\frac{3d(2c^2+2cd+d^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2c+d)(c+4d)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2c+3d)\cos(e+fx)}{2af(c-d)^2(c+d)(c+d\sin(e+fx))^2}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (-3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/(a*(c - d)*(c^2 - d^2)^(5/2)*f) - (d*(2*c + 3*d)*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*(c + d*Sin[e + f*x])^2) - Cos[e + f*x]/((c - d)*f*(a + a*Si
n[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*c + d)*(c + 4*d)*Cos[e + f*x])/
(2*a*(c - d)^3*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} + \frac{d\int \frac{-3}{(c\cdots}}{\cdots}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{(c-d)f(a+a\cdots}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{(c-d)f(a+a\cdots}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{(c-d)f(a+a\cdots}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{(c-d)f(a+a\cdots}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{(c-d)f(a+a\cdots}$$

$$= -\frac{3d(2c^2+2cd+d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(\cdots}{2a(c-d)^2\cdots}$$

**Mathematica** [A]
time = 1.27, size = 230, normalized size = 1.08

$$\frac{\cos(e+fx)\left(-\frac{6d(2c^2+2cd+d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{1+\sin(e+fx)}}\right)}{(-c-d)^{3/2}(c-d)^{5/2}\sqrt{\cos^2(e+fx)}} + \frac{2c^2+9cd+4d^2}{(c-d)^2(c+d)(1+\sin(e+fx))} - \frac{d}{(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{d(4c+d)}{(c-d)(c+d)(1+\sin(e+fx))(c+d\sin(e+fx))}\right)}{2a(-c+d)(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (Cos[e + f*x]*((-6*d*(2*c^2 + 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Si
n[e + f*x]])/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(3/2)*(c - d
)^(5/2)*Sqrt[Cos[e + f*x]^2]) + (2*c^2 + 9*c*d + 4*d^2)/((c - d)^2*(c + d)*
(1 + Sin[e + f*x])) - d/((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(4
*c + d))/((c - d)*(c + d)*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x]))))/(2*a*(
-c + d)*(c + d)*f)
```

**Maple** [A]
time = 0.87, size = 327, normalized size = 1.54

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $-\frac{2d\left(\frac{\frac{d^2(7c^2+2cd-2d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c(c^2+2cd+d^2)}+\frac{d(6c^4+2c^3d+11c^2d^2+4d^3c-2d^4)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2(c^2+2cd+d^2)}+\frac{d^2(17c^2+6cd-2d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c^2+2cd+d^2)c}+}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{(c-d)^3}}{af}$ |
| default | $-\frac{2d\left(\frac{\frac{d^2(7c^2+2cd-2d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c(c^2+2cd+d^2)}+\frac{d(6c^4+2c^3d+11c^2d^2+4d^3c-2d^4)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2(c^2+2cd+d^2)}+\frac{d^2(17c^2+6cd-2d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c^2+2cd+d^2)c}+}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}\right.}{(c-d)^3}}{af}$ |
| risch | $-\frac{8ic^3d\,e^{i(fx+e)}-3id^4e^{3i(fx+e)}+19icd^3e^{i(fx+e)}+id^4e^{i(fx+e)}-6c^2d^2e^{4i(fx+e)}-6cd^3e^{4i(fx+e)}-3d^4e^{4i(fx+e)}+32ic^2d^2e}{(e^{i(fx+e}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(-d/(c-d)^3*((1/2*d^2*(7*c^2+2*c*d-2*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f
*x+1/2*e)^3+1/2*d*(6*c^4+2*c^3*d+11*c^2*d^2+4*c*d^3-2*d^4)/c^2/(c^2+2*c*d+d
^2)*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(17*c^2+6*c*d-2*d^2)/(c^2+2*c*d+d^2)/c*tan
(1/2*f*x+1/2*e)+1/2*d*(6*c^2+2*c*d-d^2)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2
*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+3/2*(2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/(c^
2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/(c
-d)^3/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(212) = 424$.
time = 0.44, size = 2410, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(4*c^6 - 12*c^4*d^2 + 12*c^2*d^4 - 4*d^6 - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + 2*(4*c^5*d + 12*c^4*d^2 - 2*c^3*d^3 - 15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^3 - (4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c^3*d^2 + 3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(2*c^3*d^2 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^6 + 4*c^5*d + 8*c^4*d^2 + 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - 2*(2*c^6 - 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e)), 1/2*(2*c^6 - 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + (4*c^5*d + 12*c^4*d^2 - 2*c^3*d^3 - 15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^3 - (4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c^3*d^2 + 3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(2*c^3*d^2 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^6 + 4*c^5*d + 8*c^4*d^2 + 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - (2*c^6 - 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4

```
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(212) = 424.
time = 0.46, size = 482, normalized size = 2.26

$$-\frac{\frac{3\,(2\,c^2d+2\,cd^2+d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^5-ac^4d-2\,ac^3d^2+2\,ac^2d^3+acd^4-ad^5)\sqrt{c^2-d^2}}+\frac{7\,c^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,c^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,cd^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,c^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,c^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+11\,c^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,cd^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+17\,c^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,c^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,cd^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,c^4d^2+2\,c^3d^3-c^2d^4}{(ac^7-ac^6d-2\,ac^5d^2+2\,ac^4d^3+ac^3d^4-ac^2d^5)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)^2}+\frac{2}{(ac^3-3\,ac^2d+3\,acd^2-ad^3)\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a
*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) + (7*c^3*d^3*tan
(1/2*f*x + 1/2*e)^3 + 2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^5*tan(1/2*f*
x + 1/2*e)^3 + 6*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*c^3*d^3*tan(1/2*f*x + 1
/2*e)^2 + 11*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*c*d^5*tan(1/2*f*x + 1/2*e)^
2 - 2*d^6*tan(1/2*f*x + 1/2*e)^2 + 17*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*c^2*
d^4*tan(1/2*f*x + 1/2*e) - 2*c*d^5*tan(1/2*f*x + 1/2*e) + 6*c^4*d^2 + 2*c^3
*d^3 - c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 -
 a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) +
2/((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

**Mupad** [**B**]
time = 10.20, size = 753, normalized size = 3.54

$$\frac{3\,d\,\mathrm{atan}\left(\frac{\frac{3\,d\,\left(2\,c^2+2\,c\,d+d^2\right)\,\left(-2\,a\,c^5\,d+2\,a\,c^4\,d^2+4\,a\,c^3\,d^3-4\,a\,c^2\,d^4-2\,a\,c\,d^5+2\,a\,d^6\right)}{2\,a\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}-\frac{3\,c\,d\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)\,\left(a\,c^5-a\,c^4\,d-2\,a\,c^3\,d^2+2\,a\,c^2\,d^3+a\,c\,d^4-a\,d^5\right)}{a\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}}{6\,c^2\,d+6\,c\,d^2+3\,d^3}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{a\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}-\frac{\frac{2\,c^4+4\,c^3\,d+8\,c^2\,d^2+2\,c\,d^3-d^4}{\left(c+d\right)\,\left(c^2-d^2\right)\,\left(c^2-2\,c\,d+d^2\right)}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(8\,c^5\,d+22\,c^4\,d^2+17\,c^3\,d^3+13\,c^2\,d^4+2\,c\,d^5-2\,d^6\right)}{c^2\,\left(c^2-2\,c\,d+d^2\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(4\,c^5+4\,c^4\,d+14\,c^3\,d^2+21\,c^2\,d^3+4\,c\,d^4-2\,d^5\right)}{c^2\,\left(c^2-d^2\right)\,\left(c^2-2\,c\,d+d^2\right)}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,c^5+4\,c^4\,d+2\,c^3\,d^2+7\,c^2\,d^3+2\,c\,d^4-2\,d^5\right)}{c\,\left(c^2-2\,c\,d+d^2\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^4\,d+22\,c^3\,d^2+27\,c^2\,d^3+5\,c\,d^4-2\,d^5\right)}{c\,\left(c+d\right)\,\left(c^2-d^2\right)\,\left(c^2-2\,c\,d+d^2\right)}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a\,c^2+4\,a\,c\,d+4\,a\,d^2\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,a\,c^2+4\,a\,c\,d+4\,a\,d^2\right)+a\,c^2+\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^2+4\,a\,d\,c\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(a\,c^2+4\,a\,d\,c\right)+a\,c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)

[Out] (3*d*atan(((3*d*(2*c*d + 2*c^2 + d^2)*(2*a*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d))/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (3*c*d*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2)*(a*c^5 - a*d^5 + 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d))/(a*(c + d)^(5/2)*(c - d)^(7/2)))/(6*c*d^2 + 6*c^2*d + 3*d^3))*(2*c*d + 2*c^2 + d^2))/(a*f*(c + d)^(5/2)*(c - d)^(7/2)) - ((2*c*d^3 + 4*c^3*d + 2*c^4 - d^4 + 8*c^2*d^2)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*c*d^5 + 8*c^5*d - 2*d^6 + 13*c^2*d^4 + 17*c^3*d^3 + 22*c^4*d^2))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)^2*(4*c*d^4 + 4*c^4*d + 4*c^5 - 2*d^5 + 21*c^2*d^3 + 14*c^3*d^2))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^4*(2*c*d^4 + 4*c^4*d + 2*c^5 - 2*d^5 + 7*c^2*d^3 + 2*c^3*d^2))/(c*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(5*c*d^4 + 8*c^4*d - 2*d^5 + 27*c^2*d^3 + 22*c^3*d^2))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5))

# 3.461 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=260

$$\frac{5(2c-d)d^2(2c^2-3cd+2d^2)\,x}{2a^2}+\frac{2d(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)\cos(e+fx)}{3a^2f}+\frac{d^2(2c^3+20c^2d-57c$$

```
[Out] 5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*x/a^2+2/3*d*(c^4+10*c^3*d-44*c^2*d^2+40
*c*d^3-12*d^4)*cos(f*x+e)/a^2/f+1/6*d^2*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*co
s(f*x+e)*sin(f*x+e)/a^2/f+1/3*d*(c^2+10*c*d-12*d^2)*cos(f*x+e)*(c+d*sin(f*x
+e))^2/a^2/f-1/3*(c-d)*(c+10*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/a^2/f/(1+sin(
f*x+e))-1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]

time = 0.33, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2844, 3056, 2832, 2813}

$$\frac{d(c^2+10cd-12d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f}+\frac{5d^2x(2c-d)\,(2c^2-3cd+2d^2)}{2a^2}+\frac{d^2(2c^3+20c^2d-57cd^2+30d^3)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{2d(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)\cos(e+fx)}{3a^2f}-\frac{(c-d)(c+10d)\cos(e+fx)(c+d\sin(e+fx))^3}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*x)/(2*a^2) + (2*d*(c^4 + 10*c^3*d
 - 44*c^2*d^2 + 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^3 + 2
0*c^2*d - 57*c*d^2 + 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f) + (d*(c^2
 + 10*c*d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a^2*f) - ((c -
d)*(c + 10*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^3(-a(c^2+6cd-4d^2)+3a(c \cdots}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+10d)\cos(e+fx)(c+d\sin(e+fx))^3}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+ \cdots}{3f(a+a\sin(e+ \cdots}$$

$$= \frac{d(c^2+10cd-12d^2)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f} - \frac{(c-d)(c+10d)\cos(e \cdots}{3a^2f(1+ \cdots}$$

$$= \frac{5(2c-d)d^2(2c^2-3cd+2d^2)\,x}{2a^2} + \frac{2d(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)\cos \cdots}{3a^2f}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 837 vs. 2(260) = 520.
time = 1.12, size = 837, normalized size = 3.22

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(80*c^4 + 80*c^3*d*(-4 + 3*e +
3*f*x) - 80*c^2*d^2*(-5 + 6*e + 6*f*x) + 35*c*d^3*(-7 + 12*e + 12*f*x) - 4*
d^4*(-13 + 30*e + 30*f*x))*Cos[(e + f*x)/2] - (16*c^5 + 160*c^4*d + 80*c^3*
d^2*(-10 + 3*e + 3*f*x) - 40*c^2*d^3*(-41 + 12*e + 12*f*x) - 6*d^5*(-57 + 2
0*e + 20*f*x) + 5*c*d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] + 120*
c^2*d^3*Cos[(5*(e + f*x))/2] - 75*c*d^4*Cos[(5*(e + f*x))/2] + 30*d^5*Cos[(
5*(e + f*x))/2] + 15*c*d^4*Cos[(7*(e + f*x))/2] - 3*d^5*Cos[(7*(e + f*x))/2
] - d^5*Cos[(9*(e + f*x))/2] + 48*c^5*Sin[(e + f*x)/2] + 240*c^4*d*Sin[(e +
 f*x)/2] - 1440*c^3*d^2*Sin[(e + f*x)/2] + 2640*c^2*d^3*Sin[(e + f*x)/2] -
1905*c*d^4*Sin[(e + f*x)/2] + 516*d^5*Sin[(e + f*x)/2] + 720*c^3*d^2*e*Sin[
(e + f*x)/2] - 1440*c^2*d^3*e*Sin[(e + f*x)/2] + 1260*c*d^4*e*Sin[(e + f*x)
/2] - 360*d^5*e*Sin[(e + f*x)/2] + 720*c^3*d^2*f*x*Sin[(e + f*x)/2] - 1440*
c^2*d^3*f*x*Sin[(e + f*x)/2] + 1260*c*d^4*f*x*Sin[(e + f*x)/2] - 360*d^5*f*
x*Sin[(e + f*x)/2] - 360*c^2*d^3*Sin[(3*(e + f*x))/2] + 315*c*d^4*Sin[(3*(e
 + f*x))/2] - 118*d^5*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*e*Sin[(3*(e + f*x)
)/2] - 480*c^2*d^3*e*Sin[(3*(e + f*x))/2] + 420*c*d^4*e*Sin[(3*(e + f*x))/2
] - 120*d^5*e*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*f*x*Sin[(3*(e + f*x))/2] -
 480*c^2*d^3*f*x*Sin[(3*(e + f*x))/2] + 420*c*d^4*f*x*Sin[(3*(e + f*x))/2]
- 120*d^5*f*x*Sin[(3*(e + f*x))/2] - 120*c^2*d^3*Sin[(5*(e + f*x))/2] + 75*
c*d^4*Sin[(5*(e + f*x))/2] - 30*d^5*Sin[(5*(e + f*x))/2] + 15*c*d^4*Sin[(7*
(e + f*x))/2] - 3*d^5*Sin[(7*(e + f*x))/2] + d^5*Sin[(9*(e + f*x))/2]))/(48
*a^2*f*(1 + Sin[e + f*x])^2)
```

**Maple** [**A**]
time = 0.50, size = 342, normalized size = 1.32

| method | result |
|---|---|
| derivativedivides | $2d^2\left(\frac{\left(\frac{5}{2}c\,d^2-d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-10c^2d+10c\,d^2-3d^3\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-20c^2d+20c\,d^2-8d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{5}{2}c\,d^2+d^3\right)}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.$ |
| default | $2d^2\left(\frac{\left(\frac{5}{2}c\,d^2-d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-10c^2d+10c\,d^2-3d^3\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-20c^2d+20c\,d^2-8d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{5}{2}c\,d^2+d^3\right)}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.$ |
| risch | $\frac{10d^2x\,c^3}{a^2}-\frac{20d^3x\,c^2}{a^2}+\frac{35d^4xc}{2a^2}-\frac{5d^5x}{a^2}+\frac{5id^4e^{2i(fx+e)}c}{8f\,a^2}-\frac{2i(-50ic^3d^2+ic^5+10ic^4d+80ic^2d^3+60ic^3d^2e^{2i(fx+e}}{}$ |
| norman | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(d^2*(((5/2*c*d^2-d^3)*tan(1/2*f*x+1/2*e)^5+(-10*c^2*d+10*c*d^2-3*d
^3)*tan(1/2*f*x+1/2*e)^4+(-20*c^2*d+20*c*d^2-8*d^3)*tan(1/2*f*x+1/2*e)^2+(-
```

```
5/2*c*d^2+d^3)*tan(1/2*f*x+1/2*e)-10*c^2*d+10*c*d^2-11/3*d^3)/(1+tan(1/2*f*
x+1/2*e)^2)^3+5/2*(4*c^3-8*c^2*d+7*c*d^2-2*d^3)*arctan(tan(1/2*f*x+1/2*e)))
-(c^5-10*c^3*d^2+20*c^2*d^3-15*c*d^4+4*d^5)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*
c^5+10*c^4*d-20*c^3*d^2+20*c^2*d^3-10*c*d^4+2*d^5)/(tan(1/2*f*x+1/2*e)+1)^2
-1/3*(2*c^5-10*c^4*d+20*c^3*d^2-20*c^2*d^3+10*c*d^4-2*d^5)/(tan(1/2*f*x+1/2
*e)+1)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(259) = 518$.
time = 0.52, size = 1426, normalized size = 5.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c*d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a^2) - 2*d^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*s
in(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 40*c^2*d^3*(
(12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^3*d^2*((9*si
n(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x
 + e)/(cos(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
```

```
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) - 10*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(259) = 518$.
time = 0.35, size = 592, normalized size = 2.28

$$\frac{2 d^5 \cos(fx+e)^5 + 2c^5 - 10c^4 d + 20c^3 d^2 - 20c^2 d^3 + 10cd^4 - 2d^5 - (15cd^4 - 4d^5)\cos(fx+e)^4 - 2(30c^2 d^3 - 15cd^4 + 8d^5)\cos(fx+e)^3 - 30(4c^3 d^2 - 8c^2 d^3 + 7cd^4 - 2d^5)fx + (2c^5 + 20c^4 d - 100c^3 d^2 + 220c^2 d^3 - 155cd^4 + 46d^5 + 15(4c^3 d^2 - 8c^2 d^3 + 7cd^4 - 2d^5)fx)\cos(fx+e)^2 + (4c^5 + 10c^4 d - 80c^3 d^2 + 260c^2 d^3 - 190cd^4 + 62d^5 - 15(4c^3 d^2 - 8c^2 d^3 + 7cd^4 - 2d^5)fx)\cos(fx+e) - (2d^5\cos(fx+e)^4 + 2c^5 - 10c^4 d + 20c^3 d^2 - 20c^2 d^3 + 10cd^4 - 2d^5 + (15cd^4 - 2d^5)\cos(fx+e)^3 + 30(4c^3 d^2 - 8c^2 d^3 + 7cd^4 - 2d^5)fx - 3(20c^2 d^3 - 15cd^4 + 6d^5)\cos(fx+e)^2 - (2c^5 + 20c^4 d - 100c^3 d^2 + 280c^2 d^3 - 200cd^4 + 64d^5 - 15(4c^3 d^2 - 8c^2 d^3 + 7cd^4 - 2d^5)fx)\cos(fx+e))\sin(fx+e)}{6(a^2 f\cos(fx+e)^2 - a^2 f\cos(fx+e) - 2a^2 f - (a^2 f\cos(fx+e) + 2a^2 f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^5*cos(f*x + e)^5 + 2*c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10
*c*d^4 - 2*d^5 - (15*c*d^4 - 4*d^5)*cos(f*x + e)^4 - 2*(30*c^2*d^3 - 15*c*d
^4 + 8*d^5)*cos(f*x + e)^3 - 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f
*x + (2*c^5 + 20*c^4*d - 100*c^3*d^2 + 220*c^2*d^3 - 155*c*d^4 + 46*d^5 + 1
5*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e)^2 + (4*c^5 +
10*c^4*d - 80*c^3*d^2 + 260*c^2*d^3 - 190*c*d^4 + 62*d^5 - 15*(4*c^3*d^2 -
8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e) - (2*d^5*cos(f*x + e)^4 + 2*
c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10*c*d^4 - 2*d^5 + (15*c*d^4 - 2
*d^5)*cos(f*x + e)^3 + 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x - 3
*(20*c^2*d^3 - 15*c*d^4 + 6*d^5)*cos(f*x + e)^2 - (2*c^5 + 20*c^4*d - 100*c
^3*d^2 + 280*c^2*d^3 - 200*c*d^4 + 64*d^5 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c
*d^4 - 2*d^5)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*
f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 17899 vs. $2(250) = 500$.
time = 17.40, size = 17899, normalized size = 68.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-12*c**5*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*
a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(
e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2
)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**5*tan(e/2 + f*x/2)**7/(6*a**2*f*
tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f
*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*t
```

an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 44*c**5*tan(e
/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 36*c**5*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*
a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(
e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2
)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 60*c**5*tan(e/2 + f*x/2)**4/(6*a**2*f*
tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f
*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*t
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*c**5*tan(e
/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 36*c**5*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*
a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(
e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2
)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**5*tan(e/2 + f*x/2)/(6*a**2*f*tan
(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/
2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*
a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 8*c**5/(6*a**2*f
*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 60*c**4*d*ta
n(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2
)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a
**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e
/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) - 20*c**4*d*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9
+ 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f
*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +
f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 +
18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 180*c**4*d*tan(e/2 + f*x/2)**5/(6
*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan
(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/
2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 60*c*
*4*d*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2

```
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*
f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
 f*x/2) + 6*a**2*f) - 180*c**4*d*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*
x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 6
0*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*ta
n(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x
/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 60*c**4*d*tan(e/2 + f*x/2
)**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**
2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**
3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f)
- 60*c**4*d*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(
e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7...
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. 2(259) = 518.
time = 0.45, size = 977, normalized size = 3.76

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*(f*x + e)/a^2 - 2*(6*c^5*
tan(1/2*f*x + 1/2*e)^8 - 60*c^3*d^2*tan(1/2*f*x + 1/2*e)^8 + 120*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^8 - 105*c*d^4*tan(1/2*f*x + 1/2*e)^8 + 30*d^5*tan(1/2*f*
x + 1/2*e)^8 + 6*c^5*tan(1/2*f*x + 1/2*e)^7 + 30*c^4*d*tan(1/2*f*x + 1/2*e)
^7 - 180*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 360*c^2*d^3*tan(1/2*f*x + 1/2*e)^
7 - 315*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 90*d^5*tan(1/2*f*x + 1/2*e)^7 + 22*c
^5*tan(1/2*f*x + 1/2*e)^6 + 10*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 260*c^3*d^2*t
an(1/2*f*x + 1/2*e)^6 + 680*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 595*c*d^4*tan(
1/2*f*x + 1/2*e)^6 + 170*d^5*tan(1/2*f*x + 1/2*e)^6 + 18*c^5*tan(1/2*f*x +
1/2*e)^5 + 90*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 540*c^3*d^2*tan(1/2*f*x + 1/2*
e)^5 + 1200*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 945*c*d^4*tan(1/2*f*x + 1/2*e)
^5 + 270*d^5*tan(1/2*f*x + 1/2*e)^5 + 30*c^5*tan(1/2*f*x + 1/2*e)^4 + 30*c^
4*d*tan(1/2*f*x + 1/2*e)^4 - 420*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 1200*c^2*
d^3*tan(1/2*f*x + 1/2*e)^4 - 975*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 306*d^5*tan
(1/2*f*x + 1/2*e)^4 + 18*c^5*tan(1/2*f*x + 1/2*e)^3 + 90*c^4*d*tan(1/2*f*x
+ 1/2*e)^3 - 540*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 1320*c^2*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 1005*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 310*d^5*tan(1/2*f*x + 1/2*
e)^3 + 18*c^5*tan(1/2*f*x + 1/2*e)^2 + 30*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 30
0*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 840*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 645
*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 198*d^5*tan(1/2*f*x + 1/2*e)^2 + 6*c^5*tan(
1/2*f*x + 1/2*e) + 30*c^4*d*tan(1/2*f*x + 1/2*e) - 180*c^3*d^2*tan(1/2*f*x
```

```
+ 1/2*e) + 480*c^2*d^3*tan(1/2*f*x + 1/2*e) - 375*c*d^4*tan(1/2*f*x + 1/2*e
) + 114*d^5*tan(1/2*f*x + 1/2*e) + 4*c^5 + 10*c^4*d - 80*c^3*d^2 + 200*c^2*
d^3 - 160*c*d^4 + 48*d^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2
 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

**Mupad [B]**
time = 9.43, size = 692, normalized size = 2.66

$$\frac{5\,d^2\,\mathrm{atan}\left(\frac{5\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{20\,c^3\,d^2-40\,c^2\,d^3+35\,c\,d^4-10\,d^5}\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(2\,c^5+10\,c^4\,d-60\,c^3\,d^2+120\,c^2\,d^3-105\,c\,d^4+30\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(6\,c^5+10\,c^4\,d-100\,c^3\,d^2+280\,c^2\,d^3-215\,c\,d^4+66\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(10\,c^5+10\,c^4\,d-140\,c^3\,d^2+400\,c^2\,d^3-325\,c\,d^4+102\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(6\,c^5+30\,c^4\,d-180\,c^3\,d^2+400\,c^2\,d^3-315\,c\,d^4+90\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(6\,c^5+30\,c^4\,d-180\,c^3\,d^2+440\,c^2\,d^3-335\,c\,d^4+\frac{310\,d^5}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{22\,c^5}{3}+\frac{10\,c^4\,d}{3}-\frac{260\,c^3\,d^2}{3}+\frac{680\,c^2\,d^3}{3}-\frac{595\,c\,d^4}{3}+\frac{170\,d^5}{3}\right)-\frac{160\,c\,d^4}{3}+\frac{10\,c^4\,d}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(2\,c^5-20\,c^3\,d^2+40\,c^2\,d^3-35\,c\,d^4+10\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^5+10\,c^4\,d-60\,c^3\,d^2+160\,c^2\,d^3-125\,c\,d^4+38\,d^5\right)+\frac{4\,c^5}{3}+16\,d^5+\frac{200\,c^2\,d^3}{3}-\frac{80\,c^3\,d^2}{3}}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+6\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+10\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+12\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+12\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+6\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (5*d^2*atan((5*d^2*tan(e/2 + (f*x)/2)*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(3
5*c*d^4 - 10*d^5 - 40*c^2*d^3 + 20*c^3*d^2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d
^2))/(a^2*f) - (tan(e/2 + (f*x)/2)^7*(10*c^4*d - 105*c*d^4 + 2*c^5 + 30*d^5
 + 120*c^2*d^3 - 60*c^3*d^2) + tan(e/2 + (f*x)/2)^2*(10*c^4*d - 215*c*d^4 +
 6*c^5 + 66*d^5 + 280*c^2*d^3 - 100*c^3*d^2) + tan(e/2 + (f*x)/2)^4*(10*c^4
*d - 325*c*d^4 + 10*c^5 + 102*d^5 + 400*c^2*d^3 - 140*c^3*d^2) + tan(e/2 +
(f*x)/2)^5*(30*c^4*d - 315*c*d^4 + 6*c^5 + 90*d^5 + 400*c^2*d^3 - 180*c^3*d
^2) + tan(e/2 + (f*x)/2)^3*(30*c^4*d - 335*c*d^4 + 6*c^5 + (310*d^5)/3 + 44
0*c^2*d^3 - 180*c^3*d^2) + tan(e/2 + (f*x)/2)^6*((10*c^4*d)/3 - (595*c*d^4)
/3 + (22*c^5)/3 + (170*d^5)/3 + (680*c^2*d^3)/3 - (260*c^3*d^2)/3) - (160*c
*d^4)/3 + (10*c^4*d)/3 + tan(e/2 + (f*x)/2)^8*(2*c^5 - 35*c*d^4 + 10*d^5 +
40*c^2*d^3 - 20*c^3*d^2) + tan(e/2 + (f*x)/2)*(10*c^4*d - 125*c*d^4 + 2*c^5
 + 38*d^5 + 160*c^2*d^3 - 60*c^3*d^2) + (4*c^5)/3 + 16*d^5 + (200*c^2*d^3)/
3 - (80*c^3*d^2)/3)/(f*(6*a^2*tan(e/2 + (f*x)/2)^2 + 10*a^2*tan(e/2 + (f*x)
/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(e/2 + (f*x)/2)^5 + 10*a^2*
tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^7 + 3*a^2*tan(e/2 + (f*x)/2
)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

# 3.462 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=195

$$\frac{d^2(12c^2-16cd+7d^2)\,x}{2a^2}+\frac{2d(c^3+8c^2d-20cd^2+8d^3)\cos(e+fx)}{3a^2f}+\frac{d^2(2c^2+16cd-21d^2)\cos(e+fx)\sin(e}{6a^2f}$$

```
[Out] 1/2*d^2*(12*c^2-16*c*d+7*d^2)*x/a^2+2/3*d*(c^3+8*c^2*d-20*c*d^2+8*d^3)*cos(
f*x+e)/a^2/f+1/6*d^2*(2*c^2+16*c*d-21*d^2)*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*
(c-d)*(c+8*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*
cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]

time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2844, 3056, 2813}

$$\frac{d^2(2c^2+16cd-21d^2)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{d^2x(12c^2-16cd+7d^2)}{2a^2}+\frac{2d(c^3+8c^2d-20cd^2+8d^3)\cos(e+fx)}{3a^2f}-\frac{(c-d)(c+8d)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*x)/(2*a^2) + (2*d*(c^3 + 8*c^2*d - 20*c*d^2
+ 8*d^3)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^2 + 16*c*d - 21*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(6*a^2*f) - ((c - d)*(c + 8*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^2)/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[
e + f*x])^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(-a\left(c^2+5cd-3d^2\right)+a(2c \ldots\right.}{a+a\sin(e+fx)}}{3a^2} \ldots$$

$$= -\frac{(c-d)(c+8d)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d \ldots}{3f(a+a\sin(e+ \ldots} \ldots$$

$$= \frac{d^2\left(12c^2-16cd+7d^2\right)x}{2a^2} + \frac{2d\left(c^3+8c^2d-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f} + \frac{d^2(2c^2 \ldots}{\ldots} \ldots$$

**Mathematica** [A]
time = 1.25, size = 378, normalized size = 1.94

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3d\left(64c^3+48c^2d(-4+3e+3fx)-32cd^2(-5+6e+6fx)+7d^3(-7+12e+12fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)-\left(16c^4+128c^3d+48c^2d^2(-10+3e+3fx)-16cd^3(-41+12e+12fx)+d^4(-239+84e+84fx)\right)\cos\left(\frac{3}{2}(e+fx)\right)+3\left((16c-5d)d^3\cos\left(\frac{5}{2}(e+fx)\right)+d^4\cos\left(\frac{7}{2}(e+fx)\right)+2\left(8c^4+32c^3d-144c^2d^2+144cd^3-50d^4+96c^2d^2e-128cd^3e+56d^4e+96c^2d^2fx-128cd^3fx+56d^4fx+d^2\left(48c^2(e+fx)-64cd(1+e+fx)+d^2(27+28e+28fx)\right)\cos(e+fx)-2(8c-3d)d^3\cos(2(e+fx))+d^4\cos(3(e+fx))\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{48a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(64*c^3 + 48*c^2*d*(-4 + 3*e +
3*f*x) - 32*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x))*Cos[(e +
 f*x)/2] - (16*c^4 + 128*c^3*d + 48*c^2*d^2*(-10 + 3*e + 3*f*x) - 16*c*d^3*
(-41 + 12*e + 12*f*x) + d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] +
3*((16*c - 5*d)*d^3*Cos[(5*(e + f*x))/2] + d^4*Cos[(7*(e + f*x))/2] + 2*(8*
c^4 + 32*c^3*d - 144*c^2*d^2 + 144*c*d^3 - 50*d^4 + 96*c^2*d^2*e - 128*c*d^
3*e + 56*d^4*e + 96*c^2*d^2*f*x - 128*c*d^3*f*x + 56*d^4*f*x + d^2*(48*c^2*
(e + f*x) - 64*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x))*Cos[e + f*x] -
 2*(8*c - 3*d)*d^3*Cos[2*(e + f*x)] + d^4*Cos[3*(e + f*x)])*Sin[(e + f*x)/2
])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.46, size = 250, normalized size = 1.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-(c^4-6*c^2*d^2+8*c*d^3-3*d^4)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*c^4+
8*c^3*d-12*c^2*d^2+8*c*d^3-2*d^4)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*c^4-8*c^3
*d+12*c^2*d^2-8*c*d^3+2*d^4)/(tan(1/2*f*x+1/2*e)+1)^3+d^2*((1/2*d^2*tan(1/2
*f*x+1/2*e)^3+(-4*c*d+2*d^2)*tan(1/2*f*x+1/2*e)^2-1/2*d^2*tan(1/2*f*x+1/2*e
)-4*c*d+2*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(12*c^2-16*c*d+7*d^2)*arctan(
tan(1/2*f*x+1/2*e))))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(194) = 388$.

time = 0.53, size = 986, normalized size = 5.06

$$\frac{d^4\left(\frac{\frac{75\sin(fx+e)}{\cos(fx+e)+1}+\frac{97\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{126\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{98\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{63\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{21\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+32}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{3a^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{21\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-16cd^3\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+12c^2d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{2c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{8c^3d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) - 16*c*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) + 12*c^2*d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 8*c^3*d*(3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(194) = 388$.

time = 0.38, size = 452, normalized size = 2.32

$$-\frac{3\,d^4\cos\left(fx+e\right)^4-2\,c^4+8\,c^3d-12\,c^2d^2+8\,cd^3-2\,d^4+6\,(4\,cd^3-d^4)\cos\left(fx+e\right)^3+6\,(12\,c^2d^2-16\,cd^3+7\,d^4)fx-(2\,c^4+16\,c^3d-60\,c^2d^2+88\,cd^3-31\,d^4+3\,(12\,c^2d^2-16\,cd^3+7\,d^4)fx)\cos\left(fx+e\right)^2-(4\,c^4+8\,c^3d-48\,c^2d^2+104\,cd^3-38\,d^4-3\,(12\,c^2d^2-16\,cd^3+7\,d^4)fx)\cos\left(fx+e\right)+\left(3\,d^4\cos\left(fx+e\right)^3+2\,c^4-8\,c^3d+12\,c^2d^2-8\,cd^3+2\,d^4+6\,(12\,c^2d^2-16\,cd^3+7\,d^4)fx-3\,(8\,cd^3-3\,d^4)\cos\left(fx+e\right)^2-(2\,c^4+16\,c^3d-60\,c^2d^2+112\,cd^3-40\,d^4-3\,(12\,c^2d^2-16\,cd^3+7\,d^4)fx)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*d^4*cos(f*x + e)^4 - 2*c^4 + 8*c^3*d - 12*c^2*d^2 + 8*c*d^3 - 2*d^4
 + 6*(4*c*d^3 - d^4)*cos(f*x + e)^3 + 6*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x
 - (2*c^4 + 16*c^3*d - 60*c^2*d^2 + 88*c*d^3 - 31*d^4 + 3*(12*c^2*d^2 - 16*
c*d^3 + 7*d^4)*f*x)*cos(f*x + e)^2 - (4*c^4 + 8*c^3*d - 48*c^2*d^2 + 104*c*
d^3 - 38*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x)*cos(f*x + e) + (3*d^4
*cos(f*x + e)^3 + 2*c^4 - 8*c^3*d + 12*c^2*d^2 - 8*c*d^3 + 2*d^4 + 6*(12*c^
2*d^2 - 16*c*d^3 + 7*d^4)*f*x - 3*(8*c*d^3 - 3*d^4)*cos(f*x + e)^2 - (2*c^4
 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 40*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 +
7*d^4)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f
*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 8950 vs. $2(185) = 370$.
time = 9.53, size = 8950, normalized size = 45.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-12*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**4*tan(e/2 + f*x/2)**5
/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*
tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f
*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a
**2*f) - 32*c**4*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**
2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*c**4*tan(e/2 + f*x/2)**3/(6
*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan
(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/
2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) - 28*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f
*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 +
f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**4*tan(e/2 + f*x/2)/(6*a**2*
```

f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
+ 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
8*c**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) - 48*c**3*d*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 16*c**3*d*tan(e/2 + f*
x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*
a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2
) + 6*a**2*f) - 96*c**3*d*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*
f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 32*c**3*d*tan(e/2 + f
*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30
*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan
(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) - 48*c**3*d*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 16*c**3*d/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 36*c*
*2*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 108*c**2*d**2*f*x*tan(e/2 + f*x/2)*
*6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 180*c**2*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)
**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a*
*2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/
2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 252*c**2*d**2*f*x*
tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 252*c**2*d**2*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1

```
80*c**2*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**
2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 10...
```

**Giac** [A]
time = 0.44, size = 338, normalized size = 1.73

$$\frac{\frac{3\,(12\,c^2d^2-16\,cd^3+7\,d^4)(fx+e)}{a^2}+\frac{6\left(d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,cd^3+4\,d^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}-\frac{4\left(3\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-18\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-54\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+60\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-21\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c^4+4\,c^3d-24\,c^2d^2+28\,cd^3-10\,d^4\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*(f*x + e)/a^2 + 6*(d^4*tan(1/2*f*x +
 1/2*e)^3 - 8*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*d^4*tan(1/2*f*x + 1/2*e)^2 -
 d^4*tan(1/2*f*x + 1/2*e) - 8*c*d^3 + 4*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^
2*a^2) - 4*(3*c^4*tan(1/2*f*x + 1/2*e)^2 - 18*c^2*d^2*tan(1/2*f*x + 1/2*e)^
2 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*d^4*tan(1/2*f*x + 1/2*e)^2 + 3*c^4*
tan(1/2*f*x + 1/2*e) + 12*c^3*d*tan(1/2*f*x + 1/2*e) - 54*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 60*c*d^3*tan(1/2*f*x + 1/2*e) - 21*d^4*tan(1/2*f*x + 1/2*e) +
 2*c^4 + 4*c^3*d - 24*c^2*d^2 + 28*c*d^3 - 10*d^4)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

**Mupad** [B]
time = 9.21, size = 478, normalized size = 2.45

$$\frac{d^2\,\mathrm{atan}\left(\frac{d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(12\,c^2-16\,c\,d+7\,d^2)}{12\,c^2\,d^2-16\,c\,d^3+7\,d^4}\right)\,(12\,c^2-16\,c\,d+7\,d^2)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(2\,c^4+8\,c^3\,d-36\,c^2\,d^2+48\,c\,d^3-21\,d^4)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(4\,c^4+16\,c^3\,d-72\,c^2\,d^2+112\,c\,d^3-42\,d^4)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{16\,c^4}{3}+\frac{8\,c^3\,d}{3}-40\,c^2\,d^2+\frac{224\,c\,d^3}{3}-\frac{98\,d^4}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{14\,c^4}{3}+\frac{16\,c^3\,d}{3}-44\,c^2\,d^2+\frac{256\,c\,d^3}{3}-\frac{97\,d^4}{3}\right)+\frac{80\,c\,d^3}{3}+\frac{8\,c^3\,d}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(2\,c^4-12\,c^2\,d^2+16\,c\,d^3-7\,d^4)+\frac{4\,c^4}{3}-\frac{32\,d^4}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c^4+8\,c^3\,d-36\,c^2\,d^2+64\,c\,d^3-25\,d^4)-16\,c^2\,d^2}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+5\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+7\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+7\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+5\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d^2*atan((d^2*tan(e/2 + (f*x)/2)*(12*c^2 - 16*c*d + 7*d^2))/(7*d^4 - 16*c*
d^3 + 12*c^2*d^2))*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f) - (tan(e/2 + (f*x)/2)
^5*(48*c*d^3 + 8*c^3*d + 2*c^4 - 21*d^4 - 36*c^2*d^2) + tan(e/2 + (f*x)/2)^
3*(112*c*d^3 + 16*c^3*d + 4*c^4 - 42*d^4 - 72*c^2*d^2) + tan(e/2 + (f*x)/2)
^4*((224*c*d^3)/3 + (8*c^3*d)/3 + (16*c^4)/3 - (98*d^4)/3 - 40*c^2*d^2) + t
an(e/2 + (f*x)/2)^2*((256*c*d^3)/3 + (16*c^3*d)/3 + (14*c^4)/3 - (97*d^4)/3
 - 44*c^2*d^2) + (80*c*d^3)/3 + (8*c^3*d)/3 + tan(e/2 + (f*x)/2)^6*(16*c*d^
3 + 2*c^4 - 7*d^4 - 12*c^2*d^2) + (4*c^4)/3 - (32*d^4)/3 + tan(e/2 + (f*x)/
2)*(64*c*d^3 + 8*c^3*d + 2*c^4 - 25*d^4 - 36*c^2*d^2) - 16*c^2*d^2)/(f*(5*a
^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x
)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(
e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

# 3.463 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=120

$$\frac{(3c-2d)d^2x}{a^2}+\frac{(c-4d)d^2\cos(e+fx)}{3a^2f}-\frac{(c-d)^2(c+6d)\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}$$

```
[Out] (3*c-2*d)*d^2*x/a^2+1/3*(c-4*d)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)^2*(c+6*d)*co
s(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+
a*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2844, 3047, 3102, 2814, 2727}

$$\frac{d^2(c-4d)\cos(e+fx)}{3a^2f}+\frac{d^2x(3c-2d)}{a^2}-\frac{(c+6d)(c-d)^2\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((3*c - 2*d)*d^2*x)/a^2 + ((c - 4*d)*d^2*Cos[e + f*x])/(3*a^2*f) - ((c - d)
^2*(c + 6*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e +
f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))(-a(c^2+4cd-2d^2)+a(c-\ldots}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \int \frac{-ac(c^2+4cd-2d^2)+(ac(c-4d)d-ad(c^2+4\ldots}{a+a\sin\ldots}$$

$$= \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \int \frac{-a^2c(c^2+\ldots}{\ldots}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+f\ldots}{3f(a+a\sin(e+fx))^2}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)^2(c+6d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)} - \frac{(c-\ldots}{\ldots}$$

**Mathematica [A]**
time = 0.24, size = 212, normalized size = 1.77

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)^3\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c-d)^2(c+8d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3(3c-2d)d^2(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3d^3\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{3a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^3*Sin[(e + f*x)/2] - (c -
 d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2*(c + 8*d)*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(3*c - 2*d)*d^2*(e +
f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*d^3*Cos[e + f*x]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.41, size = 157, normalized size = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2\left(c^3-3cd^2+2d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2c^3+6c^2d-6cd^2+2d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2c^3-6c^2d+6cd^2-2d^3\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d^2\left(-\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3c-2d)\arctan\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}{a^2f}$ |
| default | $\frac{-\frac{2\left(c^3-3cd^2+2d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2c^3+6c^2d-6cd^2+2d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2c^3-6c^2d+6cd^2-2d^3\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d^2\left(-\frac{d}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3c-2d)\arctan\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}{a^2f}$ |
| risch | $\frac{3d^2xc}{a^2}-\frac{2d^3x}{a^2}-\frac{d^3e^{i(fx+e)}}{2a^2f}-\frac{d^3e^{-i(fx+e)}}{2a^2f}-\frac{2i\left(-9ic^2d\,e^{2i(fx+e)}+18icd^2e^{2i(fx+e)}-9id^3e^{2i(fx+e)}+ic^3+6ic^2d-\right.}{3fa^2(e}$ |
| norman | $\frac{\frac{d^2(3c-2d)x}{a}+\frac{\left(-2c^3+6cd^2-4d^3\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(-2c^3-6c^2d+18cd^2-16d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{d^2(3c-2d)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{-4c^3}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-(c^3-3*c*d^2+2*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*c^3+6*c^2*d-6*
c*d^2+2*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*c^3-6*c^2*d+6*c*d^2-2*d^3)/(ta
n(1/2*f*x+1/2*e)+1)^3+d^2*(-d/(1+tan(1/2*f*x+1/2*e)^2)+(3*c-2*d)*arctan(tan
(1/2*f*x+1/2*e))))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(120) = 240$.
time = 0.50, size = 641, normalized size = 5.34

$$-\frac{2\left(2d^3\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-3cd^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3c^2d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(2*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 3*c*
d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
```

```
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c^3*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) +
 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(120) = 240$.

time = 0.38, size = 318, normalized size = 2.65

$$-\frac{3\,d^3\cos\left(fx+e\right)^3-c^3+3\,c^2d-3\,cd^2+d^3+6\,(3\,cd^2-2\,d^3)fx-(c^3+6\,c^2d-15\,cd^2+11\,d^3+3\,(3\,cd^2-2\,d^3)fx)\cos\left(fx+e\right)^2-(2\,c^3+3\,c^2d-12\,cd^2+13\,d^3-3\,(3\,cd^2-2\,d^3)fx)\cos\left(fx+e\right)-\left(3\,d^3\cos\left(fx+e\right)^2-c^3+3\,c^2d-3\,cd^2+d^3-6\,(3\,cd^2-2\,d^3)fx+(c^3+6\,c^2d-15\,cd^2+14\,d^3-3\,(3\,cd^2-2\,d^3)fx)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d^3*cos(f*x + e)^3 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 + 6*(3*c*d^2 - 2
*d^3)*f*x - (c^3 + 6*c^2*d - 15*c*d^2 + 11*d^3 + 3*(3*c*d^2 - 2*d^3)*f*x)*c
os(f*x + e)^2 - (2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3 - 3*(3*c*d^2 - 2*d^3)*
f*x)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 -
 6*(3*c*d^2 - 2*d^3)*f*x + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3 - 3*(3*c*d^2
- 2*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos
(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(109) = 218$.

time = 4.99, size = 3585, normalized size = 29.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c**3*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**3*tan(e/2 + f*
x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a
**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 10*c**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**
3*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c**3/(3*a**2*f*tan(e/2 + f*x/2)**5 +
```

9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*c**2*d*tan(e
/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4
 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f
*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 18*c**2*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(
e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2
)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*d/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*
c*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 27*c*d**2*f*x*tan(e/2 + f*x
/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a*
*2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) + 36*c*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*c*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*
tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f
*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 27*c*d**2*f*x*tan(e/2 +
f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a*
*2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) + 9*c*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*c*d**2*tan(e/2 + f
*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*
a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e
/2 + f*x/2) + 3*a**2*f) + 54*c*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 42
*c*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/
2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)*
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 54*c*d**2*tan(e/2 + f*x/2)/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) + 24*c*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d**3*f*x*tan(e/2 + f*x/2)**5/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f

```
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*d**3*f*x*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*d**3*f*x*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d**3*f*x/(3*a**2
*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 +
 f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*...
```

**Giac [A]**
time = 0.46, size = 209, normalized size = 1.74

$$\frac{\frac{3\,(3\,cd^2-2\,d^3)(fx+e)}{a^2}-\frac{6\,d^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{2\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-27\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c^3+3\,c^2d-12\,cd^2+7\,d^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(3*c*d^2 - 2*d^3)*(f*x + e)/a^2 - 6*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1
)*a^2) - 2*(3*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
 6*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*c^3*tan(1/2*f*x + 1/2*e) + 9*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*c*d^2*tan(1/2*f*x + 1/2*e) + 15*d^3*tan(1/2*f*x + 1/2*e
) + 2*c^3 + 3*c^2*d - 12*c*d^2 + 7*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))
/f
```

**Mupad [B]**
time = 8.39, size = 298, normalized size = 2.48

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(3\,c-2\,d)}{6\,c\,d^2-4\,d^3}\right)\,(3\,c-2\,d)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,c^3+6\,c^2\,d-18\,c\,d^2+12\,d^3)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{10\,c^3}{3}+2\,c^2\,d-14\,c\,d^2+\frac{44\,d^3}{3}\right)-8\,c\,d^2+2\,c^2\,d+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,c^3-6\,c\,d^2+4\,d^3)+\frac{4\,c^3}{3}+\frac{20\,d^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c^3+6\,c^2\,d-18\,c\,d^2+16\,d^3)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(3*c - 2*d))/(6*c*d^2 - 4*d^3))*(3*c
- 2*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(6*c^2*d - 18*c*d^2 + 2*c^3 + 12*d^
3) + tan(e/2 + (f*x)/2)^2*(2*c^2*d - 14*c*d^2 + (10*c^3)/3 + (44*d^3)/3) -
8*c*d^2 + 2*c^2*d + tan(e/2 + (f*x)/2)^4*(2*c^3 - 6*c*d^2 + 4*d^3) + (4*c^3
)/3 + (20*d^3)/3 + tan(e/2 + (f*x)/2)*(6*c^2*d - 18*c*d^2 + 2*c^3 + 16*d^3)
)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e
/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)
))
```

# 3.464 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=85

$$\frac{d^2x}{a^2}-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}$$

```
[Out] d^2*x/a^2-1/3*(c-d)*(c+4*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*cos(f
*x+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2839, 2814, 2727}

$$-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/a^2 - ((c - d)*(c + 4*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x]))
 - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2839

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m*((c + d*Sin[e + f*x])/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(c^2+3cd-d^2)-3ad^2\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}$$
$$= \frac{d^2x}{a^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2} + \frac{((c-d)(c+4d))\int \frac{1}{a+a\sin(e+f}}{3a}$$
$$= \frac{d^2x}{a^2} - \frac{(c-d)(c+4d)\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.

time = 0.18, size = 172, normalized size = 2.02

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c^2+4cd-5d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3d^2(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{3a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c^2 + 4*c*d - 5*d^2)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*d^2*(e + f*x)*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)
```

**Maple** [A]

time = 0.40, size = 108, normalized size = 1.27

| method | result |
|---|---|
| derivativedivides | $\frac{2d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2\left(c^2-d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2c^2+4cd-2d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2c^2-4cd+2d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{a^2f}$ |
| default | $\frac{2d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2\left(c^2-d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2c^2+4cd-2d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2c^2-4cd+2d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{a^2f}$ |
| risch | $\frac{d^2x}{a^2}-\frac{2\left(-c^2-4cd+5d^2+3ic^2e^{i(fx+e)}+6icd\,e^{i(fx+e)}-9id^2e^{i(fx+e)}+6cd\,e^{2i(fx+e)}-6d^2e^{2i(fx+e)}\right)}{3f\,a^2\left(e^{i(fx+e)}+i\right)^3}$ |
| norman | $\frac{\frac{d^2x}{a}+\frac{d^2x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{\left(-2c^2+2d^2\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(-2c^2-4cd+6d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(-2c^2-4cd+6d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(d^2*arctan(tan(1/2*f*x+1/2*e))-(c^2-d^2)/(tan(1/2*f*x+1/2*e)+1)-1/
2*(-2*c^2+4*c*d-2*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*c^2-4*c*d+2*d^2)/(ta
n(1/2*f*x+1/2*e)+1)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(86) = 172$.

time = 0.50, size = 390, normalized size = 4.59

$$\frac{2\left(d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{2cd\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(86) = 172$.

time = 0.36, size = 205, normalized size = 2.41

$$-\frac{6d^2fx-(3d^2fx+c^2+4cd-5d^2)\cos(fx+e)^2-c^2+2cd-d^2+(3d^2fx-2c^2-2cd+4d^2)\cos(fx+e)+(6d^2fx+c^2-2cd+d^2+(3d^2fx-c^2-4cd+5d^2)\cos(fx+e))\sin(fx+e)}{3\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2a^2f-(a^2f\cos(fx+e)+2a^2f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^2*f*x - (3*d^2*f*x + c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 - c^2 +
2*c*d - d^2 + (3*d^2*f*x - 2*c^2 - 2*c*d + 4*d^2)*cos(f*x + e) + (6*d^2*f*x
 + c^2 - 2*c*d + d^2 + (3*d^2*f*x - c^2 - 4*c*d + 5*d^2)*cos(f*x + e))*sin(
f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(76) = 152$.

time = 2.45, size = 915, normalized size = 10.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*t
```

```
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c**2/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) - 12*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c*d/(3*a**2*f*tan
(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
 + 3*a**2*f) + 3*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9
*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9
*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2
)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 + 6*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*d**2*tan(e/2 +
 f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*d**2/(3*a**2*f*tan(e/2 + f*x/2)**3 +
9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f,
 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))
```

**Giac [A]**

time = 0.50, size = 132, normalized size = 1.55

$$\frac{\frac{3\,(fx+e)d^2}{a^2}-\frac{2\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c^2+2\,cd-4\,d^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*d^2/a^2 - 2*(3*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*d^2*tan(1/2*
f*x + 1/2*e)^2 + 3*c^2*tan(1/2*f*x + 1/2*e) + 6*c*d*tan(1/2*f*x + 1/2*e) -
9*d^2*tan(1/2*f*x + 1/2*e) + 2*c^2 + 2*c*d - 4*d^2)/(a^2*(tan(1/2*f*x + 1/2
*e) + 1)^3))/f
```

**Mupad [B]**

time = 7.43, size = 93, normalized size = 1.09

$$\frac{d^2\,x}{a^2}-\frac{\frac{4\,c\,d}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2-2\,d^2\right)+\frac{4\,c^2}{3}-\frac{8\,d^2}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+4\,c\,d-6\,d^2\right)}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d^2*x)/a^2 - ((4*c*d)/3 + tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + (4*c^2)/3
 - (8*d^2)/3 + tan(e/2 + (f*x)/2)*(4*c*d + 2*c^2 - 6*d^2))/(a^2*f*(tan(e/2
+ (f*x)/2) + 1)^3)
```

# 3.465 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=65

$$-\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

```
[Out] -1/3*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(c+2*d)*cos(f*x+e)/f/(a^2+a^
2*sin(f*x+e))
```

**Rubi** [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2829, 2727}

$$-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((c + 2*d)*Cos[e +
 f*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(c+2d)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

**Mathematica [A]**
time = 0.04, size = 43, normalized size = 0.66

$$-\frac{\cos(e+fx)(2c+d+(c+2d)\sin(e+fx))}{3a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(2*c + d + (c + 2*d)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
f*x])^2)
```

**Maple [A]**
time = 0.26, size = 70, normalized size = 1.08

| method | result | size |
|---|---|---|
| risch | $-\frac{2\left(-c+3ic\,{\rm e}^{i(fx+e)}+3id\,{\rm e}^{i(fx+e)}+3d\,{\rm e}^{2i(fx+e)}-2d\right)}{3f\,a^2\left({\rm e}^{i(fx+e)}+i\right)^3}$ | 68 |
| derivativedivides | $\frac{-\frac{-2c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2c-2d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2f}$ | 70 |
| default | $\frac{-\frac{-2c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2c-2d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2f}$ | 70 |
| norman | $\frac{-\frac{2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{(-2c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{-4c-2d}{3af}+\frac{2(-c-d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{2(-5c-d)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$ | 144 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-1/2*(-2*c+2*d)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*c-2*d)/(tan(1/2*f*
x+1/2*e)+1)^3-c/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(65) = 130.
time = 0.30, size = 232, normalized size = 3.57

$$-\frac{2\left(\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
```

```
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + d*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

**Fricas** [A]
time = 0.32, size = 125, normalized size = 1.92

$$\frac{(c+2\,d)\cos\left(fx+e\right)^2+(2\,c+d)\cos\left(fx+e\right)+((c+2\,d)\cos\left(fx+e\right)-c+d)\sin\left(fx+e\right)+c-d}{3\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((c + 2*d)*cos(f*x + e)^2 + (2*c + d)*cos(f*x + e) + ((c + 2*d)*cos(f*x
 + e) - c + d)*sin(f*x + e) + c - d)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(56) = 112$.
time = 1.41, size = 372, normalized size = 5.72

$$\begin{cases} -\frac{6c\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{6c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{4c}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{6d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2d}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for}\: f \neq 0 \\ \frac{x(c+d\sin(e))}{(a\sin(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*
a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*d/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f),
Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a)**2, True))
```

**Giac** [A]
time = 0.44, size = 68, normalized size = 1.05

$$-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c+d\right)}{3\,a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) + 3*d*tan(1/2*f*x + 1/2*e) + 2*c + d)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)

**Mupad** [B]
time = 7.21, size = 97, normalized size = 1.49

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,c}{2}+\frac{d}{2}-\frac{c\,\cos(e+f\,x)}{2}+\frac{d\,\cos(e+f\,x)}{2}+\frac{3\,c\,\sin(e+f\,x)}{2}+\frac{3\,d\,\sin(e+f\,x)}{2}\right)}{3\,a^2\,f\,\left(\frac{3\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}-\frac{\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)

[Out] -(2*cos(e/2 + (f*x)/2)*((5*c)/2 + d/2 - (c*cos(e + f*x))/2 + (d*cos(e + f*x))/2 + (3*c*sin(e + f*x))/2 + (3*d*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2))

# 3.466 $\int \frac{1}{(a+a\sin(e+fx))^2} dx$

**Optimal**. Leaf size=55

$$-\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))}$$

```
[Out] -1/3*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*cos(f*x+e)/f/(a^2+a^2*sin(f*x+e))
```

**Rubi** [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$-\frac{\cos(e+fx)}{3f\,(a^2\sin(e+fx)+a^2)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -1/3*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*f*(a^2 + a^2
*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^2}\,dx &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))}\end{aligned}$$

**Mathematica [A]**
time = 0.07, size = 54, normalized size = 0.98

$$-\frac{-3+4\cos(e+fx)+\cos(2(e+fx))-4\sin(e+fx)+\sin(2(e+fx))}{6a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -1/6*(-3 + 4*Cos[e + f*x] + Cos[2*(e + f*x)] - 4*Sin[e + f*x] + Sin[2*(e +
f*x)])/(a^2*f*(1 + Sin[e + f*x])^2)
```

**Maple [A]**
time = 0.20, size = 53, normalized size = 0.96

| method | result | size |
|---|---|---|
| risch | $-\frac{2i\left(i+3\,\mathrm{e}^{i(fx+e)}\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}}$ | 38 |
| derivativedivides | $\frac{-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^{2}f}$ | 53 |
| default | $\frac{-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^{2}f}$ | 53 |
| norman | $\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{4}{3af}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-2/3/(tan(1/2*f*x+1/2*e)+1)^3+1/(tan(1/2*f*x+1/2*e)+1)^2-1/(tan(1/
2*f*x+1/2*e)+1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(55) = 110.
time = 0.28, size = 127, normalized size = 2.31

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{3\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f)
```

**Fricas [A]**
time = 0.34, size = 103, normalized size = 1.87

$$\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+2\,\cos\left(fx+e\right)+1}{3\,\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + (cos(f*x + e) - 1)*sin(f*x + e) + 2*cos(f*x + e) + 1)
/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e)
 + 2*a^2*f)*sin(f*x + e))
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(46) = 92.
time = 0.73, size = 221, normalized size = 4.02

$$\begin{cases} -\frac{6\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{4}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f} & \text{for}\: f\neq 0\\ \frac{x}{\left(a\sin\left(e\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*tan(e/2 + f
*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (
x/(a*sin(e) + a)**2, True))
```

**Giac [A]**
time = 0.44, size = 50, normalized size = 0.91

$$-\frac{2\left(3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\right)}{3\,a^{2}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*f*x + 1/2*e)^2 + 3*tan(1/2*f*x + 1/2*e) + 2)/(a^2*f*(tan(1/
2*f*x + 1/2*e) + 1)^3)
```

**Mupad [B]**
time = 6.98, size = 76, normalized size = 1.38

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-3\right)}{3}}{a^{2}\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) - (2*cos(e/2 + (f*x)/2)*(cos(e/
2 + (f*x)/2)^2 - 3))/3)/(a^2*f*(cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^3)
```

# 3.467 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=131

$$\frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f}-\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))}-\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

```
[Out] -1/3*(c-4*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*cos(f*x+e)/(c-d)/f
/(a+a*sin(f*x+e))^2+2*d^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/
a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2845, 3057, 12, 2739, 632, 210}

$$\frac{2d^2\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(c-4d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^2*Sqr
t[c^2 - d^2]*f) - ((c - 4*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e +
f*x])) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(c-3d)-ad\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx}}{3a^2(c-d)}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+}$$

$$= \frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f} - \frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx)}$$

## Mathematica [A]

time = 0.24, size = 204, normalized size = 1.56

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c-4d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{6d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\sqrt{c^2-d^2}}\right)}{3a^2(c-d)^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 4*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*ArcTan[(d + c*Tan[(e + f*x)/2]
)/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]
))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2)
```

## Maple [A]

time = 0.48, size = 132, normalized size = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2d^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(c-2d)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{4}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2f}$ |

| | |
|---|---|
| default | $\frac{\frac{2d^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(c-2d)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{4}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2 f}$ |
| risch | $\frac{2d\,\mathrm{e}^{2i(fx+e)}+\frac{2c}{3}-\frac{8d}{3}-2ic\,\mathrm{e}^{i(fx+e)}+6id\,\mathrm{e}^{i(fx+e)}}{\left(\mathrm{e}^{i(fx+e)}+i\right)^3(c-d)^2 f a^2}-\frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)}{\sqrt{-c^2+d^2}\,(c-d)^2 f a^2}+\frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\ldots\right.}{\sqrt{-c^2\ldots}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))-(c-2*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)-2/3/(c-d)/(tan(1/
2*f*x+1/2*e)+1)^3+1/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2)
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(127) = 254$.
time = 0.39, size = 1022, normalized size = 7.80

$$\left[\frac{2c^3-2c^2d-2cd^2+2d^3+2(c^3-4c^2d-cd^2+4d^3)\cos(fx+e)^2-3\left(d^2\cos(fx+e)^2-d^2\cos(fx+e)-2d^2-(d^2\cos(fx+e)+2d^2)\sin(fx+e)\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2(2c^3-5c^2d-2cd^2+5d^3)\cos(fx+e)-2(c^3-c^2d-cd^2+d^3-(c^3-4c^2d-cd^2+4d^3)\cos(fx+e))\sin(fx+e)}{6\left((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)^2-(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)-2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f-((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)+2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f)\sin(fx+e)\right)}, \frac{c^3-c^2d-cd^2+d^3+(c^3-4c^2d-cd^2+4d^3)\cos(fx+e)^2-3\left(d^2\cos(fx+e)^2-d^2\cos(fx+e)-2d^2-(d^2\cos(fx+e)+2d^2)\sin(fx+e)\right)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+(2c^3-5c^2d-2cd^2+5d^3)\cos(fx+e)-(c^3-c^2d-cd^2+d^3-(c^3-4c^2d-cd^2+4d^3)\cos(fx+e))\sin(fx+e)}{3\left((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)^2-(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)-2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f-((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)+2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f)\sin(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^3 - 2*c^2*d - 2*c*d^2 + 2*d^3 + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3)
*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f*x + e) - 2*d^2 - (d^2*c
os(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(
f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e
) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2)) + 2*(2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - 2*
(c^3 - c^2*d - c*d^2 + d^3 - (c^3 - 4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
```

```
e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(
a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*
c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*(c^3 - c^2*d - c*d^2 + d^3 + (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f
*x + e) - 2*d^2 - (d^2*cos(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^3 - 5*c
^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*
d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^
2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3
- a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
 e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e))]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.46, size = 195, normalized size = 1.49

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^2}{\left(a^2c^2-2a^2cd+a^2d^2\right)\sqrt{c^2-d^2}}-\frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2c-5d}{\left(a^2c^2-2a^2cd+a^2d^2\right)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1
/2*e) + d)/sqrt(c^2 - d^2)))*d^2/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sqrt(c^2
- d^2)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 6*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*ta
n(1/2*f*x + 1/2*e) - 9*d*tan(1/2*f*x + 1/2*e) + 2*c - 5*d)/((a^2*c^2 - 2*a^
2*c*d + a^2*d^2)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad** [B]
time = 8.02, size = 250, normalized size = 1.91

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}}{2\,d^2}\right)}{a^2\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}-\frac{\frac{2\,\left(2\,c-5\,d\right)}{3\,{\left(c-d\right)}^2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c-3\,d\right)}{{\left(c-d\right)}^2}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(c-2\,d\right)}{{\left(c-d\right)}^2}}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)

[Out] (2*d^2*atan(((d^2*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(a^2*c^2 + a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*d^2)))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*c - 5*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(c - 3*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(c - 2*d))/(c - d)^2)/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))

# 3.468 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=221

$$\frac{2d^2(3c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f}-\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e}{3a^2(c-d)^2f(1+\sin(e+fx}$$

```
[Out] -1/3*d*(c^2-6*c*d-10*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))-1
/3*(c-6*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*cos
(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))+2*d^2*(3*c+2*d)*arctan(
(d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/
2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2845, 3057, 2833, 12, 2739, 632, 210}

$$\frac{2d^2(3c+2d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(
c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(c^2 - 6*c*d - 10*d^2)*Cos[e + f*x
])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((c - 6*d)*Cos[e + f*
x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - Cos[e + f
*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - $$

$$= -\frac{(c-6d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - $$

$$= -\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - $$

$$= -\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - $$

$$= -\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - $$

$$= -\frac{d(c^2-6cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - $$

$$= \frac{2d^2(3c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f} - $$

**Mathematica [A]**
time = 0.87, size = 267, normalized size = 1.21

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c-7d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{6d^2(3c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)\sqrt{c^2-d^2}}+\frac{3d^3\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)(c+d\sin(e+fx))}\right)}{3a^2(c-d)^3f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 7*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
 + d)*Sqrt[c^2 - d^2]) + (3*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1 + Sin[e +
 f*x])^2)
```

**Maple [A]**
time = 0.65, size = 205, normalized size = 0.93

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{2(c-3d)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{4}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{2d^2\left(\frac{\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{(c}}{a^2f}$ |
| default | $\frac{-\frac{2(c-3d)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{4}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{2}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{2d^2\left(\frac{\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{(c}}{a^2f}$ |
| risch | $\frac{-16id^3e^{i(fx+e)}+4c\,d^2+6c\,d^2e^{4i(fx+e)}-34d^2e^{2i(fx+e)}c-\frac{2c^2d}{3}+6ic^2d\,e^{3i(fx+e)}-6ic^2d\,e^{i(fx+e)}+\frac{4ic^3e^{i(fx+e)}}{3}-\frac{46d\,c^2e}{}}{\left(e^{i(fx+e)}+i\right)^3(c+d)\left(d\,e^{2i(fx+e)}-\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-(c-3*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)-2/3/(c-d)^2/(tan(1/2*f*x+1
/2*e)+1)^3+1/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2+d^2/(c-d)^3*((d^2/(c+d)/c*tan
(1/2*f*x+1/2*e)+d/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+
(3*c+2*d)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(219) = 438$.
time = 0.44, size = 2342, normalized size = 10.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
[Out] [1/6*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - 2*(c^4*d
- 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^3 + 2*(c^5 - 5*c^
4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^5)*cos(f*x + e)^2 - 3*(6*c^2*d^2
+ 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^
3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e) + (6
*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2
 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c
^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x +
e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d
^3 + 14*c*d^4 + 13*d^5)*cos(f*x + e) - 2*(c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d
^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(
f*x + e)^2 - (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a
^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c
^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*
c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2
+ 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*co
s(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2
*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5
*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f
*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*
a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*
c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2
*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e)), 1/3*(c^5 - c^4*d - 2*c^3*d^2
+ 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*
d^5)*cos(f*x + e)^3 + (c^5 - 5*c^4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^
5)*cos(f*x + e)^2 + 3*(6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos
(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5
*c*d^3 + 2*d^4)*cos(f*x + e) + (6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2
*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d^3 + 14*c*d^4 + 13*d^5)*co
s(f*x + e) - (c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d -
6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^2 - (c^5 - 4*c^4*d
- 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/
((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2
*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c
^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x
 + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*
d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^
2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a
^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c
```

```
^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*
d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*
f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.46, size = 320, normalized size = 1.45

$$\frac{2\left(\frac{3\,(3\,cd^2+2\,d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4)\sqrt{c^2-d^2}}+\frac{3\,(d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+cd^3)}{(a^2c^5-2\,a^2c^4d+2\,a^2c^2d^3-a^2cd^4)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c-8\,d}{(a^2c^3-3\,a^2c^2d+3\,a^2cd^2-a^2d^3)\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan(
(c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*
a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)) + 3*(d^4*tan(1/2*f*x + 1/2*e) + c*d^3
)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*f*x + 1/2
*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 9*d*
tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) - 15*d*tan(1/2*f*x + 1/2*
e) + 2*c - 8*d)/((a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f
*x + 1/2*e) + 1)^3))/f
```

**Mupad** [**B**]
time = 10.49, size = 625, normalized size = 2.83

$$\frac{\frac{2\,(-2\,c^3+6\,c^2\,d+8\,c\,d^2+3\,d^3)}{3\,(c+d)\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(-5\,c^3+11\,c^2\,d+30\,c\,d^2+9\,d^3)}{3\,c\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(-3\,c^4+8\,c^3\,d+27\,c^2\,d^2+25\,c\,d^3+3\,d^4)}{3\,c\,(c+d)\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(-c^4+2\,c^3\,d+9\,c^2\,d^2+7\,c\,d^3+3\,d^4)}{c\,(c+d)\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(-c^4+2\,c^3\,d+3\,c^2\,d^2+d^4)}{c\,(c+d)\,(c-d)\,(c^2-2\,c\,d+d^2)}}{f\,\left(a^2\,c+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(3\,a^2\,c+2\,a^2\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(3\,a^2\,c+2\,a^2\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(4\,a^2\,c+6\,a^2\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(4\,a^2\,c+6\,a^2\,d)+a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right)}-\frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,(3\,c+2\,d)\,\left(-2\,a^2\,c^4\,d+4\,a^2\,c^3\,d^2-4\,a^2\,c\,d^4+2\,a^2\,d^5\right)}{a^2\,(c+d)^{3/2}\,(c-d)^{7/2}}-\frac{2\,c\,d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(3\,c+2\,d)\,\left(a^2\,c^4-2\,a^2\,c^3\,d+2\,a^2\,c\,d^3-a^2\,d^4\right)}{a^2\,(c+d)^{3/2}\,(c-d)^{7/2}}}{4\,d^3+6\,c\,d^2}\right)\,(3\,c+2\,d)}{a^2\,f\,(c+d)^{3/2}\,(c-d)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*(8*c*d^2 + 6*c^2*d - 2*c^3 + 3*d^3))/(3*(c + d)*(c - d)*(c^2 - 2*c*d +
d^2)) + (2*tan(e/2 + (f*x)/2)^2*(30*c*d^2 + 11*c^2*d - 5*c^3 + 9*d^3))/(3*c
*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(25*c*d^3 + 8*c^3*d -
```

```
 3*c^4 + 3*d^4 + 27*c^2*d^2))/(3*c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (
2*tan(e/2 + (f*x)/2)^3*(7*c*d^3 + 2*c^3*d - c^4 + 3*d^4 + 9*c^2*d^2))/(c*(c
 + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(2*c^3*d - c^4
 + d^4 + 3*c^2*d^2))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)))/(f*(a^2*c + t
an(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 2*a
^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^3*(4*
a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5)) - (2*d^2*atan(((d^2*(3*c +
2*d)*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2))/(a^2*(c + d)^
(3/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)*(3*c + 2*d)*(a^2*c^4 - a
^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2)))/(6*
c*d^2 + 4*d^3))*(3*c + 2*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7/2))
```

# 3.469 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=294

$$\frac{d^2(12c^2+16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f}-\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}-\frac{(c}{3a^2(c-d)^2f(1+}$$

[Out] -1/6*d*(2*c^2-16*c*d-21*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(c-8*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e))+d^2*(12*c^2+16*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/f/(c^2-d^2)^(1/2)

**Rubi** [A]
time = 0.42, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2845, 3057, 2833, 12, 2739, 632, 210}

$$\frac{d^2(12c^2+16cd+7d^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^2}-\frac{d(2c^3-16c^2d-59cd^2-32d^3)\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}-\frac{(c-8d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))^2}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]

[Out] (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (d*(2*c^2 - 16*c*d - 21*d^2)*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((c - 8*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Cos[e + f*x])/(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \left(\cdots\right.}{\cdots}$$

$$= -\frac{(c-8d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\cdots}{3(\cdots}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c-d)^2f\cdots}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c-d)^2f\cdots}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c-d)^2f\cdots}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c-d)^2f\cdots}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c-d)^2f\cdots}$$

$$= \frac{d^2(12c^2+16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(2c^2\cdots}{6a^2(c-\cdots}$$

**Mathematica** [A]
time = 0.78, size = 338, normalized size = 1.15

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-2(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(c-10d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{6d^2(12c^2+16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)^2\sqrt{c^2-d^2}}+\frac{3(c-d)d^3\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)(c+d\sin(e+fx))^2}+\frac{3d^3(7c+4d)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)^2(c+d\sin(e+fx))}\right)}{6a^2(c-d)^4f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(c - d)*Sin[(e + f*x)/2] - 2*(c -
 d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 10*d)*Sin[(e + f*x)/2]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(12*c^2 + 16*c*d + 7*d^2)*A
rcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)/((c + d)^2*Sqrt[c^2 - d^2]) + (3*(c - d)*d^3*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x])^2) + (3
*d^3*(7*c + 4*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
+ d)^2*(c + d*Sin[e + f*x]))))/(6*a^2*(c - d)^4*f*(1 + Sin[e + f*x])^2)
```

**Maple** [A]
time = 0.86, size = 378, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-(c-4*d)/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)-2/3/(c-d)^3/(tan(1/2*f*x+1
/2*e)+1)^3+1/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2+d^2/(c-d)^4*((1/2*d^2*(9*c^2+
4*c*d-2*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(8*c^4+4*c^3*d+15
*c^2*d^2+8*c*d^3-2*d^4)/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(2
3*c^2+12*c*d-2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)+1/2*d*(8*c^2+4*c*d
-d^2)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+
1/2*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

**<u>Maxima</u> [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**<u>Fricas</u> [B]** Leaf count of result is larger than twice the leaf count of optimal. 1754 vs. $2(292) = 584$.
time = 0.50, size = 3597, normalized size = 12.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*c^7 - 4*c^6*d - 12*c^5*d^2 + 12*c^4*d^3 + 12*c^3*d^4 - 12*c^2*d^5
 - 4*c*d^6 + 4*d^7 - 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 +
59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - 2*(4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 -
 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e)^3 + 2*(2*c^7 -
 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c^3*d^4 + 39*c^2*d^5 + 73*c*d^6 +
27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^
5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^4 - (24*c^3*d^3 +
 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e)^3 - (12*c^4*d^2 + 64*c^3*d^3 +
 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3
 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e) + (24*c^4*d^2 + 80*c^3*d^3 +
 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x
```

+ e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)^2 + (1
2*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e))*sin(f
*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-
c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 4*(2*c
^7 - 5*c^6*d - 36*c^5*d^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6
 + 20*d^7)*cos(f*x + e) - 2*(2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^
3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4
 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 -
102*c^4*d^3 - 45*c^3*d^4 + 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 -
 2*(c^7 - 4*c^6*d - 33*c^5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*
c*d^6 + 19*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 -
 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^
9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d
^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^
2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^
8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 -
8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2
 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*
d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d
^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c
^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*
a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2
*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 +
4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*
c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos
(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 +
 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x + e)), -1/6*(2*c^7 - 2*c^6*d - 6*c^5*
d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 - (2*c^5*d^2 - 16
*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - (4
*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 +
43*d^7)*cos(f*x + e)^3 + (2*c^7 - 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c
^3*d^4 + 39*c^2*d^5 + 73*c*d^6 + 27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 8
0*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^
6)*cos(f*x + e)^4 - (24*c^3*d^3 + 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x +
e)^3 - (12*c^4*d^2 + 64*c^3*d^3 + 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x
+ e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x
+ e) + (24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2
*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c
*d^5 + 7*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c
*d^5 + 7*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*
x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(2*c^7 - 5*c^6*d - 36*c^5*d
^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6 + 20*d^7)*cos(f*x + e)
 - (2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d
^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 +

```
32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 - 102*c^4*d^3 - 45*c^3*d^4
+ 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 - 2*(c^7 - 4*c^6*d - 33*c^
5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*c*d^6 + 19*d^7)*cos(f*x +
 e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^
5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x +
 e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a
^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*c
os(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 1
8*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^
2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*
d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c...
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(292) = 584.
time = 0.57, size = 614, normalized size = 2.09

$$\frac{\frac{3\,(12\,c^2d^2+16\,cd^3+7\,d^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^6-2\,a^2c^5d-a^2c^4d^2+4\,a^2c^3d^3-a^2c^2d^4-2\,a^2cd^5+a^2d^6)\sqrt{c^2-d^2}}+\frac{3\left(9\,c^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+4\,c^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,cd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,c^4d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,c^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,c^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,cd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d^7\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+23\,c^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,c^2d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,cd^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,c^4d^3+4\,c^3d^4-c^2d^5\right)}{(a^2c^8-2\,a^2c^7d-a^2c^6d^2+4\,a^2c^5d^3-a^2c^4d^4-2\,a^2c^3d^5+a^2c^2d^6)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)^2}-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-21\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c-11\,d\right)}{(a^2c^4-4\,a^2c^3d+6\,a^2c^2d^2-4\,a^2cd^3+a^2d^4)\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*
a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d
^6)*sqrt(c^2 - d^2)) + 3*(9*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*c^2*d^5*tan(
1/2*f*x + 1/2*e)^3 - 2*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*c^4*d^3*tan(1/2*f*x
 + 1/2*e)^2 + 4*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 15*c^2*d^5*tan(1/2*f*x + 1
/2*e)^2 + 8*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*d^7*tan(1/2*f*x + 1/2*e)^2 + 2
3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 12*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*c*d^6*
tan(1/2*f*x + 1/2*e) + 8*c^4*d^3 + 4*c^3*d^4 - c^2*d^5)/((a^2*c^8 - 2*a^2*c
^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3 - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*
d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - 2*(3*c*
tan(1/2*f*x + 1/2*e)^2 - 12*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/
2*e) - 21*d*tan(1/2*f*x + 1/2*e) + 2*c - 11*d)/((a^2*c^4 - 4*a^2*c^3*d + 6*
a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

**Mupad [B]**
time = 10.81, size = 1199, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((12*c*d^4 + 14*c^4*d - 4*c^5 - 3*d^5 + 46*c^2*d^3 + 40*c^3*d^2)/(3*(c + d)
*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^5*(4*c*
d^5 + 4*c^5*d - 2*c^6 - 2*d^6 + 23*c^2*d^4 + 40*c^3*d^3 + 38*c^4*d^2))/(c^2
*(c^5 - 3*c^4*d - 3*c*d^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (2*tan(e/2 + (f
*x)/2)^3*(33*c*d^5 + 16*c^5*d - 6*c^6 - 9*d^6 + 177*c^2*d^4 + 212*c^3*d^3 +
 102*c^4*d^2))/(3*c^2*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e
/2 + (f*x)/2)*(33*c*d^4 + 20*c^4*d - 6*c^5 - 6*d^5 + 160*c^2*d^3 + 114*c^3*
d^2))/(3*c*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/
2)^2*(6*c*d^6 + 16*c^6*d - 14*c^7 - 6*d^7 + 232*c^2*d^5 + 583*c^3*d^4 + 532
*c^4*d^3 + 226*c^5*d^2))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^
3 - d^3)) + (tan(e/2 + (f*x)/2)^4*(48*c*d^6 + 14*c^6*d - 16*c^7 - 18*d^7 +
303*c^2*d^5 + 522*c^3*d^4 + 502*c^4*d^3 + 220*c^5*d^2))/(3*c^2*(c + d)*(c^2
 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^6*(4*c*d^5 +
 4*c^5*d - 2*c^6 - 2*d^6 + 9*c^2*d^4 + 8*c^3*d^3 + 14*c^4*d^2))/(c*(c - d)*
(2*c*d + c^2 + d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/2 + (f*x)/2
)*(3*a^2*c^2 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 1
2*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + ta
n(e/2 + (f*x)/2)^3*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/
2)^4*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(3*a^2*c^
2 + 4*a^2*c*d) + a^2*c^2 + a^2*c^2*tan(e/2 + (f*x)/2)^7)) - (d^2*atan(((d^2
*(16*c*d + 12*c^2 + 7*d^2)*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c
^2*d^5 - 8*a^2*c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2))/(2*a^2*(c + d)^(5/
2)*(c - d)^(9/2)) + (c*d^2*tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2)*(2*
a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 +
 a^2*c^4*d^2))/(a^2*(c + d)^(5/2)*(c - d)^(9/2)))/(16*c*d^3 + 7*d^4 + 12*c^
2*d^2))*(16*c*d + 12*c^2 + 7*d^2))/(a^2*f*(c + d)^(5/2)*(c - d)^(9/2))
```

# 3.470 $\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=354

$$\frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\,x}{2a^3}+\frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)\cos(e+fx)}{15a^3f}+\frac{d^2}{}$$

```
[Out] 1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*x/a^3+2/15*d*(2*c^5+18*c^4*d+107*
c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*cos(f*x+e)/a^3/f+1/30*d^2*(4*c^4+36*
c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*cos(f*x+e)*sin(f*x+e)/a^3/f+1/15*d*(2*
c^3+18*c^2*d+111*c*d^2-136*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a^3/f-1/15*(c
-d)*(2*c^2+18*c*d+115*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a^3+a^3*sin(f*x
+e))-1/15*(c-d)*(2*c+13*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/a/f/(a+a*sin(f*x+e
))^2-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^5/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.53, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2844, 3056, 2832, 2813}

$$-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d(2c^3+18c^2d+111cd^2-136d^3)\cos(e+fx)(c+d\sin(e+fx))^2}{15a^3f}+\frac{d^3x(40c^3-90c^2d+78cd^2-23d^3)}{2a^3}+\frac{d^2(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)\sin(e+fx)\cos(e+fx)}{30a^3f}+\frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)\cos(e+fx)}{15a^3f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^5}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+13d)\cos(e+fx)(c+d\sin(e+fx))^4}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*x)/(2*a^3) + (2*d*(2*c^5 + 18*
c^4*d + 107*c^3*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d^5)*Cos[e + f*x])/(15*
a^3*f) + (d^2*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 + 345*d^4)*Cos[e
+ f*x]*Sin[e + f*x])/(30*a^3*f) + (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^
3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a^3*f) - ((c - d)*(2*c^2 + 18*c
*d + 115*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(15*f*(a^3 + a^3*Sin[e +
 f*x])) - ((c - d)*(2*c + 13*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^5)/(
5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^5}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^4\left(-a\left(2c^2+8cd-5d^2\right)+a(3\cdots}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+13d)\cos(e+fx)(c+d\sin(e+fx))^4}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+\cdots}{5f(a+a\sin(e\cdots}$$

$$= -\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+\cdots}{\cdots}$$

$$= \frac{d(2c^3+18c^2d+111cd^2-136d^3)\cos(e+fx)(c+d\sin(e+fx))^2}{15a^3f} - \frac{(c-d)\,(2\cdots}{\cdots}$$

$$= \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\,x}{2a^3} + \frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+4\cdots}{15a^3f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.74, size = 560, normalized size = 1.58

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(c-d)^6\sin\left(\frac{1}{2}(e+fx)\right)-24(c-d)^6\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+32(c-d)^5(c+14d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-16(c-d)^5(c+14d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+16(c-d)^4(2c^2+26cd+197d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-60d^3(-40c^3+90c^2d-78cd^2+23d^3)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+10d^6\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-45d^4(20c^2-24cd+9d^2)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(\cos(e+fx)-i\sin(e+fx))-45d^4(20c^2-24cd+9d^2)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(\cos(e+fx)+i\sin(e+fx))-45i(2c-d)d^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(\cos(2(e+fx))-i\sin(2(e+fx)))+45i(2c-d)d^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(\cos(2(e+fx))+i\sin(2(e+fx)))\right)}{120a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(c - d)^6*Sin[(e + f*x)/2] - 24*
(c - d)^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 32*(c - d)^5*(c + 14*d)*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 16*(c - d)^5*(c +
 14*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 16*(c - d)^4*(2*c^2 + 26*c
*d + 197*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 60
*d^3*(-40*c^3 + 90*c^2*d - 78*c*d^2 + 23*d^3)*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5 + 10*d^6*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5 - 45*d^4*(20*c^2 - 24*c*d + 9*d^2)*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*(Cos[e + f*x] - I*Sin[e + f*x]) - 45*d^4*(20*c^2 - 24*c*d + 9*d
^2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[e + f*x] + I*Sin[e + f*x])
 - (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[2*(e +
 f*x)] - I*Sin[2*(e + f*x)]) + (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])))/(120*a^3*f*(1 +
Sin[e + f*x])^3)
```

**Maple** [A]
time = 0.63, size = 468, normalized size = 1.32 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-(c^6-20*c^3*d^3+45*c^2*d^4-36*c*d^5+10*d^6)/(tan(1/2*f*x+1/2*e)+1
)-1/2*(-4*c^6+12*c^5*d-40*c^3*d^3+60*c^2*d^4-36*c*d^5+8*d^6)/(tan(1/2*f*x+1
/2*e)+1)^2-1/3*(8*c^6-36*c^5*d+60*c^4*d^2-40*c^3*d^3+12*c*d^5-4*d^6)/(tan(1
/2*f*x+1/2*e)+1)^3-1/4*(-8*c^6+48*c^5*d-120*c^4*d^2+160*c^3*d^3-120*c^2*d^4
+48*c*d^5-8*d^6)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*c^6-24*c^5*d+60*c^4*d^2-80
*c^3*d^3+60*c^2*d^4-24*c*d^5+4*d^6)/(tan(1/2*f*x+1/2*e)+1)^5+d^3*(((3*c*d^2
-3/2*d^3)*tan(1/2*f*x+1/2*e)^5+(-15*c^2*d+18*c*d^2-6*d^3)*tan(1/2*f*x+1/2*e
)^4+(-30*c^2*d+36*c*d^2-14*d^3)*tan(1/2*f*x+1/2*e)^2+(-3*c*d^2+3/2*d^3)*tan
(1/2*f*x+1/2*e)-15*c^2*d+18*c*d^2-20/3*d^3)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*
(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*arctan(tan(1/2*f*x+1/2*e))))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. 2(354) = 708.
time = 0.55, size = 2169, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] -1/15*(d^6*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*c*d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 90*c^2*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 40*c^3*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^6*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)

```
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 60*c^4*d^2*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
+ 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 36*c^5
*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(354) = 708$.
time = 0.44, size = 841, normalized size = 2.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*d^6*cos(f*x + e)^6 + 6*c^6 - 36*c^5*d + 90*c^4*d^2 - 120*c^3*d^3 +
 90*c^2*d^4 - 36*c*d^5 + 6*d^6 + 15*(6*c*d^5 - d^6)*cos(f*x + e)^5 - 10*(45
*c^2*d^4 - 36*c*d^5 + 14*d^6)*cos(f*x + e)^4 - (4*c^6 + 36*c^5*d + 210*c^4*
d^2 - 1280*c^3*d^3 + 3510*c^2*d^4 - 2694*c*d^5 + 839*d^6 - 15*(40*c^3*d^3 -
 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^3 - 60*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (8*c^6 + 72*c^5*d - 30*c^4*d^2 - 760*c^3
*d^3 + 2520*c^2*d^4 - 2148*c*d^5 + 668*d^6 + 45*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 + 6*(3*c^6 + 12*c^5*d + 45*c^4*d^2 -
 360*c^3*d^3 + 945*c^2*d^4 - 768*c*d^5 + 233*d^6 - 5*(40*c^3*d^3 - 90*c^2*d
^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e) + (10*d^6*cos(f*x + e)^5 - 6*c^6
+ 36*c^5*d - 90*c^4*d^2 + 120*c^3*d^3 - 90*c^2*d^4 + 36*c*d^5 - 6*d^6 - 5*(
18*c*d^5 - 5*d^6)*cos(f*x + e)^4 - 5*(90*c^2*d^4 - 54*c*d^5 + 23*d^6)*cos(f
*x + e)^3 - 60*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (4*c^6 +
 36*c^5*d + 210*c^4*d^2 - 1280*c^3*d^3 + 3060*c^2*d^4 - 2424*c*d^5 + 724*d^
6 + 15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 +
6*(2*c^6 + 18*c^5*d + 30*c^4*d^2 - 340*c^3*d^3 + 930*c^2*d^4 - 762*c*d^5 +
232*d^6 - 5*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 28065 vs. $2(340) = 680$.
time = 51.50, size = 28065, normalized size = 79.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))**6/(a+a*sin(f*x+e))**3,x)

[Out] Piecewise((-60*c**6*tan(e/2 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 120*c**6*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 340*c**6*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 440*c**6*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 688*c**6*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 600*c**6*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 624*c**6*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 360*c**6*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*

```
f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e
/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 244*c**6*tan(e/2 + f*x/2
)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 39
0*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f
*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/
2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/
2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a
**3*f) - 80*c**6*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**
3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(
e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f
*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**
4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a
**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 28*c**6/(30*a**3*f*tan(e/2 + f*x/2)**
11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750
*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f
*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/
2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2
)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 360*c**5*d*tan(e/2 + f*x/
2)**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 3
90*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*
f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e
/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x
/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*
a**3*f) - 360*c**5*d*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 +
150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*ta...
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(354) = 708$.

time = 0.52, size = 776, normalized size = 2.19

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*(f*x + e)/a^3 + 10*(
18*c*d^5*tan(1/2*f*x + 1/2*e)^5 - 9*d^6*tan(1/2*f*x + 1/2*e)^5 - 90*c^2*d^4
*tan(1/2*f*x + 1/2*e)^4 + 108*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 36*d^6*tan(1/2
*f*x + 1/2*e)^4 - 180*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 216*c*d^5*tan(1/2*f*
x + 1/2*e)^2 - 84*d^6*tan(1/2*f*x + 1/2*e)^2 - 18*c*d^5*tan(1/2*f*x + 1/2*e
) + 9*d^6*tan(1/2*f*x + 1/2*e) - 90*c^2*d^4 + 108*c*d^5 - 40*d^6)/((tan(1/2
*f*x + 1/2*e)^2 + 1)^3*a^3) - 4*(15*c^6*tan(1/2*f*x + 1/2*e)^4 - 300*c^3*d^
3*tan(1/2*f*x + 1/2*e)^4 + 675*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 540*c*d^5*t
an(1/2*f*x + 1/2*e)^4 + 150*d^6*tan(1/2*f*x + 1/2*e)^4 + 30*c^6*tan(1/2*f*x
```

```
 + 1/2*e)^3 + 90*c^5*d*tan(1/2*f*x + 1/2*e)^3 - 1500*c^3*d^3*tan(1/2*f*x +
1/2*e)^3 + 3150*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2430*c*d^5*tan(1/2*f*x + 1
/2*e)^3 + 660*d^6*tan(1/2*f*x + 1/2*e)^3 + 40*c^6*tan(1/2*f*x + 1/2*e)^2 +
90*c^5*d*tan(1/2*f*x + 1/2*e)^2 + 300*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2900
*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5400*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 399
0*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 1060*d^6*tan(1/2*f*x + 1/2*e)^2 + 20*c^6*t
an(1/2*f*x + 1/2*e) + 90*c^5*d*tan(1/2*f*x + 1/2*e) + 150*c^4*d^2*tan(1/2*f
*x + 1/2*e) - 1900*c^3*d^3*tan(1/2*f*x + 1/2*e) + 3600*c^2*d^4*tan(1/2*f*x
+ 1/2*e) - 2670*c*d^5*tan(1/2*f*x + 1/2*e) + 710*d^6*tan(1/2*f*x + 1/2*e) +
 7*c^6 + 18*c^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 855*c^2*d^4 - 642*c*d^5 + 17
2*d^6)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 9.71, size = 898, normalized size = 2.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^6/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*atan((d^3*tan(e/2 + (f*x)/2)*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/
(78*c*d^5 - 23*d^6 - 90*c^2*d^4 + 40*c^3*d^3))*(78*c*d^2 - 90*c^2*d + 40*c^
3 - 23*d^3))/(a^3*f) - (tan(e/2 + (f*x)/2)^9*(12*c^5*d - 390*c*d^5 + 4*c^6
+ 115*d^6 + 450*c^2*d^4 - 200*c^3*d^3) - (608*c*d^5)/5 + (12*c^5*d)/5 + tan
(e/2 + (f*x)/2)^10*(2*c^6 - 78*c*d^5 + 23*d^6 + 90*c^2*d^4 - 40*c^3*d^3) +
tan(e/2 + (f*x)/2)*(12*c^5*d - 530*c*d^5 + (8*c^6)/3 + (475*d^6)/3 + 630*c^
2*d^4 - (760*c^3*d^3)/3 + 20*c^4*d^2) + (14*c^6)/15 + (544*d^6)/15 + 144*c^
2*d^4 - (176*c^3*d^3)/3 + 4*c^4*d^2 + tan(e/2 + (f*x)/2)^8*(12*c^5*d - 988*
c*d^5 + (34*c^6)/3 + (874*d^6)/3 + 1140*c^2*d^4 - (1520*c^3*d^3)/3 + 40*c^4
*d^2) + tan(e/2 + (f*x)/2)^3*(48*c^5*d - 2052*c*d^5 + 12*c^6 + (1846*d^6)/3
 + 2460*c^2*d^4 - 960*c^3*d^3 + 60*c^4*d^2) + tan(e/2 + (f*x)/2)^7*(48*c^5*
d - 1820*c*d^5 + (44*c^6)/3 + (1610*d^6)/3 + 2100*c^2*d^4 - (2560*c^3*d^3)/
3 + 20*c^4*d^2) + tan(e/2 + (f*x)/2)^5*(72*c^5*d - 2952*c*d^5 + 20*c^6 + (2
668*d^6)/3 + 3480*c^2*d^4 - 1360*c^3*d^3 + 60*c^4*d^2) + tan(e/2 + (f*x)/2)
^2*((96*c^5*d)/5 - (5954*c*d^5)/5 + (122*c^6)/15 + (5347*d^6)/15 + 1422*c^2
*d^4 - (1688*c^3*d^3)/3 + 52*c^4*d^2) + tan(e/2 + (f*x)/2)^4*((216*c^5*d)/5
 - (14004*c*d^5)/5 + (104*c^6)/5 + (12622*d^6)/15 + 3372*c^2*d^4 - 1376*c^3
*d^3 + 132*c^4*d^2) + tan(e/2 + (f*x)/2)^6*((192*c^5*d)/5 - (13208*c*d^5)/5
 + (344*c^6)/15 + (11684*d^6)/15 + 3144*c^2*d^4 - (4016*c^3*d^3)/3 + 124*c^
4*d^2))/(f*(13*a^3*tan(e/2 + (f*x)/2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*
a^3*tan(e/2 + (f*x)/2)^4 + 46*a^3*tan(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (
f*x)/2)^6 + 38*a^3*tan(e/2 + (f*x)/2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*
a^3*tan(e/2 + (f*x)/2)^9 + 5*a^3*tan(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x
)/2)^11 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

# 3.471 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=278

$$\frac{d^3(20c^2-30cd+13d^2)\,x}{2a^3}+\frac{2d(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\cos(e+fx)}{15a^3f}+\frac{d^2(4c^3+30c^2d+146cd^2$$

```
[Out] 1/2*d^3*(20*c^2-30*c*d+13*d^2)*x/a^3+2/15*d*(2*c^4+15*c^3*d+72*c^2*d^2-180*
c*d^3+76*d^4)*cos(f*x+e)/a^3/f+1/30*d^2*(4*c^3+30*c^2*d+146*c*d^2-195*d^3)*
cos(f*x+e)*sin(f*x+e)/a^3/f-1/15*(c-d)*(2*c^2+15*c*d+76*d^2)*cos(f*x+e)*(c+
d*sin(f*x+e))^2/f/(a^3+a^3*sin(f*x+e))-1/15*(c-d)*(2*c+11*d)*cos(f*x+e)*(c+
d*sin(f*x+e))^3/a/f/(a+a*sin(f*x+e))^2-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e)
)^4/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.41, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2844, 3056, 2813}

$$-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x(20c^2-30cd+13d^2)}{2a^3}+\frac{d^2(4c^3+30c^2d+146cd^2-195d^3)\sin(e+fx)\cos(e+fx)}{30a^3f}+\frac{2d(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\cos(e+fx)}{15a^3f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+11d)\cos(e+fx)(c+d\sin(e+fx))^3}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*x)/(2*a^3) + (2*d*(2*c^4 + 15*c^3*d + 72*c^
2*d^2 - 180*c*d^3 + 76*d^4)*Cos[e + f*x])/(15*a^3*f) + (d^2*(4*c^3 + 30*c^2
*d + 146*c*d^2 - 195*d^3)*Cos[e + f*x]*Sin[e + f*x])/(30*a^3*f) - ((c - d)*
(2*c^2 + 15*c*d + 76*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*f*(a^3 +
 a^3*Sin[e + f*x])) - ((c - d)*(2*c + 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x
])^3)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^4)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^3(-a(2c-d)(c+4d)+a(2c-}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+11d)\cos(e+fx)(c+d\sin(e+fx))^3}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+}{5f(a+a\sin(e}$$

$$= -\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+}{1}$$

$$= \frac{d^3\left(20c^2-30cd+13d^2\right)x}{2a^3} + \frac{2d\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)\cos(e-}{15a^3f}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 992 vs. 2(278) = 556.
time = 6.86, size = 992, normalized size = 3.57

---

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1200*c^4*d*Cos[(e + f*x)/2] + 4800*
c^3*d^2*Cos[(e + f*x)/2] - 21600*c^2*d^3*Cos[(e + f*x)/2] + 22500*c*d^4*Cos
[(e + f*x)/2] - 7560*d^5*Cos[(e + f*x)/2] + 12000*c^2*d^3*(e + f*x)*Cos[(e
+ f*x)/2] - 18000*c*d^4*(e + f*x)*Cos[(e + f*x)/2] + 7800*d^5*(e + f*x)*Cos
```

```
[(e + f*x)/2] - 160*c^5*Cos[(3*(e + f*x))/2] - 1200*c^4*d*Cos[(3*(e + f*x))
/2] - 3200*c^3*d^2*Cos[(3*(e + f*x))/2] + 18400*c^2*d^3*Cos[(3*(e + f*x))/2
] - 24300*c*d^4*Cos[(3*(e + f*x))/2] + 9230*d^5*Cos[(3*(e + f*x))/2] - 6000
*c^2*d^3*(e + f*x)*Cos[(3*(e + f*x))/2] + 9000*c*d^4*(e + f*x)*Cos[(3*(e +
f*x))/2] - 3900*d^5*(e + f*x)*Cos[(3*(e + f*x))/2] + 1500*c*d^4*Cos[(5*(e +
 f*x))/2] - 750*d^5*Cos[(5*(e + f*x))/2] - 1200*c^2*d^3*(e + f*x)*Cos[(5*(e
 + f*x))/2] + 1800*c*d^4*(e + f*x)*Cos[(5*(e + f*x))/2] - 780*d^5*(e + f*x)
*Cos[(5*(e + f*x))/2] + 300*c*d^4*Cos[(7*(e + f*x))/2] - 105*d^5*Cos[(7*(e
+ f*x))/2] - 15*d^5*Cos[(9*(e + f*x))/2] + 320*c^5*Sin[(e + f*x)/2] + 1200*
c^4*d*Sin[(e + f*x)/2] + 6400*c^3*d^2*Sin[(e + f*x)/2] - 29600*c^2*d^3*Sin[
(e + f*x)/2] + 35100*c*d^4*Sin[(e + f*x)/2] - 12760*d^5*Sin[(e + f*x)/2] +
12000*c^2*d^3*(e + f*x)*Sin[(e + f*x)/2] - 18000*c*d^4*(e + f*x)*Sin[(e + f
*x)/2] + 7800*d^5*(e + f*x)*Sin[(e + f*x)/2] + 2400*c^3*d^2*Sin[(3*(e + f*x
))/2] - 7200*c^2*d^3*Sin[(3*(e + f*x))/2] + 4500*c*d^4*Sin[(3*(e + f*x))/2]
 - 930*d^5*Sin[(3*(e + f*x))/2] + 6000*c^2*d^3*(e + f*x)*Sin[(3*(e + f*x))/
2] - 9000*c*d^4*(e + f*x)*Sin[(3*(e + f*x))/2] + 3900*d^5*(e + f*x)*Sin[(3*
(e + f*x))/2] - 32*c^5*Sin[(5*(e + f*x))/2] - 240*c^4*d*Sin[(5*(e + f*x))/2
] - 1120*c^3*d^2*Sin[(5*(e + f*x))/2] + 5120*c^2*d^3*Sin[(5*(e + f*x))/2] -
 7260*c*d^4*Sin[(5*(e + f*x))/2] + 2782*d^5*Sin[(5*(e + f*x))/2] - 1200*c^2
*d^3*(e + f*x)*Sin[(5*(e + f*x))/2] + 1800*c*d^4*(e + f*x)*Sin[(5*(e + f*x)
)/2] - 780*d^5*(e + f*x)*Sin[(5*(e + f*x))/2] + 300*c*d^4*Sin[(7*(e + f*x))
/2] - 105*d^5*Sin[(7*(e + f*x))/2] + 15*d^5*Sin[(9*(e + f*x))/2]))/(480*f*(
a + a*Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.58, size = 360, normalized size = 1.29

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(c^5-10c^2d^3+15c\,d^4-6d^5\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4c^5+10c^4d-20c^2d^3+20c\,d^4-6d^5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8c^5-30c^4d+40c^3d^2-20c^2d^3+2d^5\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8c^5+40c^4d-80c^3d^2+}{2\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |
| default | $-\frac{2\left(c^5-10c^2d^3+15c\,d^4-6d^5\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4c^5+10c^4d-20c^2d^3+20c\,d^4-6d^5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8c^5-30c^4d+40c^3d^2-20c^2d^3+2d^5\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8c^5+40c^4d-80c^3d^2+}{2\left(\tan\left(\frac{fx}{2}+\right.\right.}$ |
| risch | $\frac{10d^3x\,c^2}{a^3}-\frac{15d^4xc}{a^3}+\frac{13d^5x}{2a^3}+\frac{id^5\mathrm{e}^{2i(fx+e)}}{8f\,a^3}-\frac{5d^4\mathrm{e}^{i(fx+e)}c}{2f\,a^3}+\frac{3d^5\mathrm{e}^{i(fx+e)}}{2f\,a^3}-\frac{5d^4\mathrm{e}^{-i(fx+e)}c}{2f\,a^3}+\frac{3d^5\mathrm{e}^{-i(fx+e)}}{2f\,a^3}$ |
| norman | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-(c^5-10*c^2*d^3+15*c*d^4-6*d^5)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*c^
5+10*c^4*d-20*c^2*d^3+20*c*d^4-6*d^5)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*c^5-3
0*c^4*d+40*c^3*d^2-20*c^2*d^3+2*d^5)/(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*c^5+4
```

```
0*c^4*d-80*c^3*d^2+80*c^2*d^3-40*c*d^4+8*d^5)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*
(4*c^5-20*c^4*d+40*c^3*d^2-40*c^2*d^3+20*c*d^4-4*d^5)/(tan(1/2*f*x+1/2*e)+1
)^5+d^3*((1/2*d^2*tan(1/2*f*x+1/2*e)^3+(-5*c*d+3*d^2)*tan(1/2*f*x+1/2*e)^2-
1/2*d^2*tan(1/2*f*x+1/2*e)-5*c*d+3*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(20*
c^2-30*c*d+13*d^2)*arctan(tan(1/2*f*x+1/2*e))))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(278) = 556$.
time = 0.53, size = 1636, normalized size = 5.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22
75*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a
^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*c*d^4*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a^3) + 20*c^2*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/a^3) - 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
```

```
+ e)^5/(cos(f*x + e) + 1)^5) - 40*c^3*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^4*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(278) = 556$.
time = 0.37, size = 669, normalized size = 2.41

$$\frac{15\,d^5\cos(fx+e)^5 + 6\,c^5 - 30\,c^4d + 60\,c^3d^2 - 60\,c^2d^3 + 30\,cd^4 - 6\,d^5 - 30\,(5\,cd^4 - 2\,d^5)\cos(fx+e)^4 - (4\,c^5 + 30\,c^4d + 140\,c^3d^2 - 640\,c^2d^3 + 1170\,cd^4 - 449\,d^5 - 15\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx)\cos(fx+e)^3 - 60\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx + (8\,c^5 + 60\,c^4d - 20\,c^3d^2 - 380\,c^2d^3 + 840\,cd^4 - 358\,d^5 + 45\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx)\cos(fx+e)^2 + 6\,(3\,c^5 + 10\,c^4d + 30\,c^3d^2 - 180\,c^2d^3 + 315\,cd^4 - 128\,d^5 - 5\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx)\cos(fx+e) - (15\,d^5\cos(fx+e)^4 + 6\,c^5 - 30\,c^4d + 60\,c^3d^2 - 60\,c^2d^3 + 30\,cd^4 - 6\,d^5 + 15\,(10\,cd^4 - 3\,d^5)\cos(fx+e)^3 + 60\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx - (4\,c^5 + 30\,c^4d + 140\,c^3d^2 - 640\,c^2d^3 + 1020\,cd^4 - 404\,d^5 + 15\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx)\cos(fx+e)^2 - 6\,(2\,c^5 + 15\,c^4d + 20\,c^3d^2 - 170\,c^2d^3 + 310\,cd^4 - 127\,d^5 - 5\,(20\,c^2d^3 - 30\,cd^4 + 13\,d^5)fx)\cos(fx+e))\sin(fx+e)}{30\,(a^3f\cos(fx+e)^3 + 3\,a^3f\cos(fx+e)^2 - 2\,a^3f\cos(fx+e) - 4\,a^3f + (a^3f\cos(fx+e)^2 - 2\,a^3f\cos(fx+e) - 4\,a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*d^5*cos(f*x + e)^5 + 6*c^5 - 30*c^4*d + 60*c^3*d^2 - 60*c^2*d^3 +
30*c*d^4 - 6*d^5 - 30*(5*c*d^4 - 2*d^5)*cos(f*x + e)^4 - (4*c^5 + 30*c^4*d
+ 140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 449*d^5 - 15*(20*c^2*d^3 - 30*c*
d^4 + 13*d^5)*f*x)*cos(f*x + e)^3 - 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x
 + (8*c^5 + 60*c^4*d - 20*c^3*d^2 - 380*c^2*d^3 + 840*c*d^4 - 358*d^5 + 45*
(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x + e)^2 + 6*(3*c^5 + 10*c^4*d
+ 30*c^3*d^2 - 180*c^2*d^3 + 315*c*d^4 - 128*d^5 - 5*(20*c^2*d^3 - 30*c*d^4
 + 13*d^5)*f*x)*cos(f*x + e) - (15*d^5*cos(f*x + e)^4 + 6*c^5 - 30*c^4*d +
60*c^3*d^2 - 60*c^2*d^3 + 30*c*d^4 - 6*d^5 + 15*(10*c*d^4 - 3*d^5)*cos(f*x
+ e)^3 + 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x - (4*c^5 + 30*c^4*d + 140*
c^3*d^2 - 640*c^2*d^3 + 1020*c*d^4 - 404*d^5 + 15*(20*c^2*d^3 - 30*c*d^4 +
13*d^5)*f*x)*cos(f*x + e)^2 - 6*(2*c^5 + 15*c^4*d + 20*c^3*d^2 - 170*c^2*d^
3 + 310*c*d^4 - 127*d^5 - 5*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x +
 e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f
*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*
a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 15553 vs. $2(264) = 528$.
time = 31.07, size = 15553, normalized size = 55.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-60*c**5*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +
 f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 120*c**5*tan(e/2 + f*x/2)**7
/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3
*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/
2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2
)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 280*c**5*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a
**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan
(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*
x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 320*c**5*tan(e/2 + f*x/2)**5/(3
0*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*
tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 +
 f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**
3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
f) - 408*c**5*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 280*c**5*tan(e/2 + f*x/2)**3/(30*a
**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan
(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*
x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f)
- 216*c**5*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*
tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 +
 f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**
4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a
**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 80*c**5*tan(e/2 + f*x/2)/(30*a**3*f*t
an(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 +
f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5
 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a*
*3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 28*c*
*5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a*
*3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(
e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x
/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*
a**3*f) - 300*c**4*d*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 1
50*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f
*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2
+ f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)*
*2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 300*c**4*d*tan(e/2 + f*x/2)
```

```
**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a
**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan
(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*
x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30
*a**3*f) - 900*c**4*d*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 +
150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*
f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2
 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)
**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 660*c**4*d*tan(e/2 + f*x/2
)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f
*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 3
0*a**3*f) - 900*c**4*d*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 +
 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3
*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/
2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2
)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 420*c**4*d*tan(e/2 + f*x/
2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360
*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*t
an(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 +
f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15...
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(278) = 556$.
time = 0.45, size = 564, normalized size = 2.03

$$\frac{\frac{15\,(20\,c^2d^3-30\,cd^4+13\,d^5)(fx+e)}{a^3}+\frac{30\left(d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-10\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-10\,cd^4+6\,d^5\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}-\frac{4\left(15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-150\,c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+225\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+75\,c^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-750\,c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+1050\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-405\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^5 [illegible]}{[illegible]}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*(f*x + e)/a^3 + 30*(d^5*tan(1/2*f
*x + 1/2*e)^3 - 10*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*d^5*tan(1/2*f*x + 1/2*e
)^2 - d^5*tan(1/2*f*x + 1/2*e) - 10*c*d^4 + 6*d^5)/((tan(1/2*f*x + 1/2*e)^2
 + 1)^2*a^3) - 4*(15*c^5*tan(1/2*f*x + 1/2*e)^4 - 150*c^2*d^3*tan(1/2*f*x +
 1/2*e)^4 + 225*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 90*d^5*tan(1/2*f*x + 1/2*e)^
4 + 30*c^5*tan(1/2*f*x + 1/2*e)^3 + 75*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 750*c
^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 1050*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 405*d^5
*tan(1/2*f*x + 1/2*e)^3 + 40*c^5*tan(1/2*f*x + 1/2*e)^2 + 75*c^4*d*tan(1/2*
f*x + 1/2*e)^2 + 200*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 1450*c^2*d^3*tan(1/2*
f*x + 1/2*e)^2 + 1800*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 665*d^5*tan(1/2*f*x +
1/2*e)^2 + 20*c^5*tan(1/2*f*x + 1/2*e) + 75*c^4*d*tan(1/2*f*x + 1/2*e) + 10
0*c^3*d^2*tan(1/2*f*x + 1/2*e) - 950*c^2*d^3*tan(1/2*f*x + 1/2*e) + 1200*c*
d^4*tan(1/2*f*x + 1/2*e) - 445*d^5*tan(1/2*f*x + 1/2*e) + 7*c^5 + 15*c^4*d
```

```
+ 20*c^3*d^2 - 220*c^2*d^3 + 285*c*d^4 - 107*d^5)/(a^3*(tan(1/2*f*x + 1/2*e
) + 1)^5))/f
```

**Mupad [B]**
time = 9.54, size = 652, normalized size = 2.35

$$\frac{d^3\,\mathrm{atan}\left(\frac{d^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(20\,c^2-30\,c\,d+13\,d^2\right)}{20\,c^2\,d^3-30\,c\,d^4+13\,d^5}\right)\,\left(20\,c^2-30\,c\,d+13\,d^2\right)}{a^3\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{28\,c^5}{3}+10\,c^4\,d+\frac{80\,c^3\,d^2}{3}-\frac{700\,c^2\,d^3}{3}+350\,c\,d^4-\frac{455\,d^5}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{36\,c^5}{5}+14\,c^4\,d+32\,c^3\,d^2-252\,c^2\,d^3+426\,c\,d^4-\frac{891\,d^5}{5}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(\frac{32\,c^5}{3}+30\,c^4\,d+\frac{40\,c^3\,d^2}{3}-\frac{980\,c^2\,d^3}{3}+550\,c\,d^4-\frac{715\,d^5}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{28\,c^5}{3}+30\,c^4\,d+\frac{80\,c^3\,d^2}{3}-\frac{1060\,c^2\,d^3}{3}+610\,c\,d^4-\frac{761\,d^5}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{68\,c^5}{5}+22\,c^4\,d+56\,c^3\,d^2-436\,c^2\,d^3+698\,c\,d^4-\frac{1443\,d^5}{5}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(4\,c^5+10\,c^4\,d-100\,c^2\,d^3+150\,c\,d^4-65\,d^5\right)+48\,c\,d^4+2\,c^4\,d+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(2\,c^5-20\,c^2\,d^3+30\,c\,d^4-13\,d^5\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,c^5}{3}+10\,c^4\,d+\frac{40\,c^3\,d^2}{3}-\frac{380\,c^2\,d^3}{3}+210\,c\,d^4-\frac{265\,d^5}{3}\right)+\frac{14\,c^5}{15}-\frac{304\,d^5}{15}-\frac{88\,c^2\,d^3}{3}+\frac{8\,c^3\,d^2}{3}}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+12\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+20\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+26\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+26\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+20\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+12\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*atan((d^3*tan(e/2 + (f*x)/2)*(20*c^2 - 30*c*d + 13*d^2))/(13*d^5 - 30*
c*d^4 + 20*c^2*d^3))*(20*c^2 - 30*c*d + 13*d^2))/(a^3*f) - (tan(e/2 + (f*x)
/2)^6*(350*c*d^4 + 10*c^4*d + (28*c^5)/3 - (455*d^5)/3 - (700*c^2*d^3)/3 +
(80*c^3*d^2)/3) + tan(e/2 + (f*x)/2)^2*(426*c*d^4 + 14*c^4*d + (36*c^5)/5 -
 (891*d^5)/5 - 252*c^2*d^3 + 32*c^3*d^2) + tan(e/2 + (f*x)/2)^5*(550*c*d^4
+ 30*c^4*d + (32*c^5)/3 - (715*d^5)/3 - (980*c^2*d^3)/3 + (40*c^3*d^2)/3) +
 tan(e/2 + (f*x)/2)^3*(610*c*d^4 + 30*c^4*d + (28*c^5)/3 - (761*d^5)/3 - (1
060*c^2*d^3)/3 + (80*c^3*d^2)/3) + tan(e/2 + (f*x)/2)^4*(698*c*d^4 + 22*c^4
*d + (68*c^5)/5 - (1443*d^5)/5 - 436*c^2*d^3 + 56*c^3*d^2) + tan(e/2 + (f*x
)/2)^7*(150*c*d^4 + 10*c^4*d + 4*c^5 - 65*d^5 - 100*c^2*d^3) + 48*c*d^4 + 2
*c^4*d + tan(e/2 + (f*x)/2)^8*(30*c*d^4 + 2*c^5 - 13*d^5 - 20*c^2*d^3) + ta
n(e/2 + (f*x)/2)*(210*c*d^4 + 10*c^4*d + (8*c^5)/3 - (265*d^5)/3 - (380*c^2
*d^3)/3 + (40*c^3*d^2)/3) + (14*c^5)/15 - (304*d^5)/15 - (88*c^2*d^3)/3 + (
8*c^3*d^2)/3)/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

# 3.472 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=195

$$\frac{(4c-3d)d^3x}{a^3}+\frac{d^2(2c^2+10cd-27d^2)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\,(2c^2+12cd+45d^2)\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}-\frac{(c-d)(2c+9}{15}$$

```
[Out] (4*c-3*d)*d^3*x/a^3+1/15*d^2*(2*c^2+10*c*d-27*d^2)*cos(f*x+e)/a^3/f-1/15*(c
-d)^2*(2*c^2+12*c*d+45*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/15*(c-d)*(2
*c+9*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(c-d)*cos(
f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.42, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2844, 3056, 3047, 3102, 2814, 2727}

$$\frac{d^2(2c^2+10cd-27d^2)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\,(2c^2+12cd+45d^2)\cos(e+fx)}{15f\,(a^3\sin(e+fx)+a^3)}+\frac{d^3x(4c-3d)}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((4*c - 3*d)*d^3*x)/a^3 + (d^2*(2*c^2 + 10*c*d - 27*d^2)*Cos[e + f*x])/(15*
a^3*f) - ((c - d)^2*(2*c^2 + 12*c*d + 45*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^
3*Sin[e + f*x])) - ((c - d)*(2*c + 9*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2
)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^3)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
```

```
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^2(-a(2c^2+6cd-3d^2)+a(c-}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d}{5f(a+a\sin(e+}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d}{5f(a+a\sin(e+}$$

$$= \frac{d^2(2c^2+10cd-27d^2)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+}{15af(a+a\sin(e+fx))^2}$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2(2c^2+10cd-27d^2)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+}{15af(a+a\sin}$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2(2c^2+10cd-27d^2)\cos(e+fx)}{15a^3f} - \frac{(c-d)^2(2c^2+12cd+45}{15f(a^3+a^3\sin(e}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 683 vs. 2(195) = 390.
time = 0.94, size = 683, normalized size = 3.50

---

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*d*(16*c^3 + 48*c^2*d - 15*d^3*(-
5 + 4*e + 4*f*x) + 16*c*d^2*(-9 + 5*e + 5*f*x))*Cos[(e + f*x)/2] - 5*(8*c^4
 + 48*c^3*d + 96*c^2*d^2 - 9*d^4*(-27 + 10*e + 10*f*x) + 8*c*d^3*(-46 + 15*
e + 15*f*x))*Cos[(3*(e + f*x))/2] + 75*d^4*Cos[(5*(e + f*x))/2] - 120*c*d^3
*e*Cos[(5*(e + f*x))/2] + 90*d^4*e*Cos[(5*(e + f*x))/2] - 120*c*d^3*f*x*Cos
[(5*(e + f*x))/2] + 90*d^4*f*x*Cos[(5*(e + f*x))/2] + 15*d^4*Cos[(7*(e + f*
x))/2] + 80*c^4*Sin[(e + f*x)/2] + 240*c^3*d*Sin[(e + f*x)/2] + 960*c^2*d^2
*Sin[(e + f*x)/2] - 2960*c*d^3*Sin[(e + f*x)/2] + 1755*d^4*Sin[(e + f*x)/2]
 + 1200*c*d^3*e*Sin[(e + f*x)/2] - 900*d^4*e*Sin[(e + f*x)/2] + 1200*c*d^3*
f*x*Sin[(e + f*x)/2] - 900*d^4*f*x*Sin[(e + f*x)/2] + 360*c^2*d^2*Sin[(3*(e
 + f*x))/2] - 720*c*d^3*Sin[(3*(e + f*x))/2] + 225*d^4*Sin[(3*(e + f*x))/2]
 + 600*c*d^3*e*Sin[(3*(e + f*x))/2] - 450*d^4*e*Sin[(3*(e + f*x))/2] + 600*
c*d^3*f*x*Sin[(3*(e + f*x))/2] - 450*d^4*f*x*Sin[(3*(e + f*x))/2] - 8*c^4*S
in[(5*(e + f*x))/2] - 48*c^3*d*Sin[(5*(e + f*x))/2] - 168*c^2*d^2*Sin[(5*(e
 + f*x))/2] + 512*c*d^3*Sin[(5*(e + f*x))/2] - 363*d^4*Sin[(5*(e + f*x))/2]
 - 120*c*d^3*e*Sin[(5*(e + f*x))/2] + 90*d^4*e*Sin[(5*(e + f*x))/2] - 120*c
```

```
*d^3*f*x*Sin[(5*(e + f*x))/2] + 90*d^4*f*x*Sin[(5*(e + f*x))/2] + 15*d^4*Si
n[(7*(e + f*x))/2]))/(120*a^3*f*(1 + Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.56, size = 248, normalized size = 1.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-(c^4-4*c*d^3+3*d^4)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*c^4+8*c^3*d-8*
c*d^3+4*d^4)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-8*c^4+32*c^3*d-48*c^2*d^2+32*c*
d^3-8*d^4)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*c^4-16*c^3*d+24*c^2*d^2-16*c*d^3
+4*d^4)/(tan(1/2*f*x+1/2*e)+1)^5-8/3*c*(c^3-3*c^2*d+3*c*d^2-d^3)/(tan(1/2*f
*x+1/2*e)+1)^3+d^3*(-d/(1+tan(1/2*f*x+1/2*e)^2)+(4*c-3*d)*arctan(tan(1/2*f*
x+1/2*e))))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 1197 vs. $2(196) = 392$.
time = 0.53, size = 1197, normalized size = 6.14

$$-\frac{2\left(3\,d^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{189\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{200\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{160\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{75\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+24}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{11a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{11a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{5a^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)-4\,cd^3\left(\frac{\frac{95\sin(fx+e)}{\cos(fx+e)+1}+\frac{145\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{75\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+22}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^4\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{12\,c^2d^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{12\,c^3d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) - 4*c*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^2*d^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*
```

```
sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^3*d*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(196) = 392$.
time = 0.40, size = 508, normalized size = 2.61

$$-\frac{15\,d^4\cos(fx+e)^4-3\,c^4+12\,c^3d-18\,c^2d^2+12\,cd^3-3\,d^4+(2\,c^4+12\,c^3d+42\,c^2d^2-128\,cd^3+117\,d^4-15\,(4\,cd^3-3\,d^4)fx)\cos(fx+e)^3+60\,(4\,cd^3-3\,d^4)fx-(4\,c^4+24\,c^3d-6\,c^2d^2-76\,cd^3+84\,d^4+45\,(4\,cd^3-3\,d^4)fx)\cos(fx+e)^2-3\,(3\,c^4+8\,c^3d+18\,c^2d^2-72\,cd^3+63\,d^4-10\,(4\,cd^3-3\,d^4)fx)\cos(fx+e)+(15\,d^4\cos(fx+e)^3+3\,c^4-12\,c^3d+18\,c^2d^2-12\,cd^3+3\,d^4+60\,(4\,cd^3-3\,d^4)fx-(2\,c^4+12\,c^3d+42\,c^2d^2-128\,cd^3+102\,d^4+15\,(4\,cd^3-3\,d^4)fx)\cos(fx+e)^2-6\,(c^4+6\,c^3d+6\,c^2d^2-34\,cd^3+31\,d^4-5\,(4\,cd^3-3\,d^4)fx)\cos(fx+e))\sin(fx+e)}{15\,(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f+(a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*d^4*cos(f*x + e)^4 - 3*c^4 + 12*c^3*d - 18*c^2*d^2 + 12*c*d^3 - 3
*d^4 + (2*c^4 + 12*c^3*d + 42*c^2*d^2 - 128*c*d^3 + 117*d^4 - 15*(4*c*d^3 -
 3*d^4)*f*x)*cos(f*x + e)^3 + 60*(4*c*d^3 - 3*d^4)*f*x - (4*c^4 + 24*c^3*d
- 6*c^2*d^2 - 76*c*d^3 + 84*d^4 + 45*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2
- 3*(3*c^4 + 8*c^3*d + 18*c^2*d^2 - 72*c*d^3 + 63*d^4 - 10*(4*c*d^3 - 3*d^4
)*f*x)*cos(f*x + e) + (15*d^4*cos(f*x + e)^3 + 3*c^4 - 12*c^3*d + 18*c^2*d^
2 - 12*c*d^3 + 3*d^4 + 60*(4*c*d^3 - 3*d^4)*f*x - (2*c^4 + 12*c^3*d + 42*c^
2*d^2 - 128*c*d^3 + 102*d^4 + 15*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2 - 6*
(c^4 + 6*c^3*d + 6*c^2*d^2 - 34*c*d^3 + 31*d^4 - 5*(4*c*d^3 - 3*d^4)*f*x)*c
os(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 7373 vs. $2(177) = 354$.
time = 19.03, size = 7373, normalized size = 37.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**4*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75
*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*t
an(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**4*tan(e/2 + f*x
/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
```

an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 110*c**4*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*c**4*ta
n(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 94*c**4*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 4
0*c**4*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*
*4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**4/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**3*d*tan(
e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 120*c**3*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
240*c**3*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*c**3*d*tan(e/2 + f*x/2)**2
/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*
f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 120*c**3*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 24*c**3*d/(15*a**3
*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)
**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 240*c**2*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75
*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*t
an(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**2*d**2*tan(e/2
 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6

```
 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a*
*3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 264*c**2*d**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
120*c**2*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 24*c**2*d**2/(15*a**3*f*tan(e/
2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)
**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 60*
c*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16...
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(196) = 392$.
time = 0.46, size = 395, normalized size = 2.03

$$-\frac{\frac{30\,d^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\,(4\,cd^3-3\,d^4)(fx+e)}{a^3}+\frac{2\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+60\,c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-300\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+210\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+60\,c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+120\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-580\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+360\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+60\,c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+60\,c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-380\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+240\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,c^4+12\,c^3d+12\,c^2d^2-88\,cd^3+57\,d^4\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(30*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(4*c*d^3 - 3*d^4)*(f*
x + e)/a^3 + 2*(15*c^4*tan(1/2*f*x + 1/2*e)^4 - 60*c*d^3*tan(1/2*f*x + 1/2*
e)^4 + 45*d^4*tan(1/2*f*x + 1/2*e)^4 + 30*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*c
^3*d*tan(1/2*f*x + 1/2*e)^3 - 300*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*d^4*ta
n(1/2*f*x + 1/2*e)^3 + 40*c^4*tan(1/2*f*x + 1/2*e)^2 + 60*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + 120*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 580*c*d^3*tan(1/2*f*x +
1/2*e)^2 + 360*d^4*tan(1/2*f*x + 1/2*e)^2 + 20*c^4*tan(1/2*f*x + 1/2*e) + 6
0*c^3*d*tan(1/2*f*x + 1/2*e) + 60*c^2*d^2*tan(1/2*f*x + 1/2*e) - 380*c*d^3*
tan(1/2*f*x + 1/2*e) + 240*d^4*tan(1/2*f*x + 1/2*e) + 7*c^4 + 12*c^3*d + 12
*c^2*d^2 - 88*c*d^3 + 57*d^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad** [B]
time = 9.03, size = 440, normalized size = 2.26

$$\frac{2\,d^3\,\mathrm{atan}\left(\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,c-3\,d\right)}{8\,c\,d^3-6\,d^4}\right)\,\left(4\,c-3\,d\right)}{a^3\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{22\,c^4}{3}+8\,c^3\,d+16\,c^2\,d^2-\frac{256\,c\,d^3}{3}+64\,d^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{20\,c^4}{3}+16\,c^3\,d+8\,c^2\,d^2-\frac{272\,c\,d^3}{3}+80\,d^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{94\,c^4}{15}+\frac{48\,c^3\,d}{5}+\frac{88\,c^2\,d^2}{5}-\frac{1336\,c\,d^3}{15}+\frac{378\,d^4}{5}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(4\,c^4+8\,c^3\,d-40\,c\,d^3+30\,d^4\right)-\frac{176\,c\,d^3}{15}+\frac{8\,c^3\,d}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(2\,c^4-8\,c\,d^3+6\,d^4\right)+\frac{14\,c^4}{15}+\frac{48\,d^4}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,c^4}{3}+8\,c^3\,d+8\,c^2\,d^2-\frac{152\,c\,d^3}{3}+42\,d^4\right)+\frac{8\,c^2\,d^2}{5}}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+11\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+15\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+15\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+11\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*d^3*atan((2*d^3*tan(e/2 + (f*x)/2)*(4*c - 3*d))/(8*c*d^3 - 6*d^4))*(4*c
- 3*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^4*(8*c^3*d - (256*c*d^3)/3 + (22*c^4)
/3 + 64*d^4 + 16*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(16*c^3*d - (272*c*d^3)/3
+ (20*c^4)/3 + 80*d^4 + 8*c^2*d^2) + tan(e/2 + (f*x)/2)^2*((48*c^3*d)/5 - (
1336*c*d^3)/15 + (94*c^4)/15 + (378*d^4)/5 + (88*c^2*d^2)/5) + tan(e/2 + (f
*x)/2)^5*(8*c^3*d - 40*c*d^3 + 4*c^4 + 30*d^4) - (176*c*d^3)/15 + (8*c^3*d)
/5 + tan(e/2 + (f*x)/2)^6*(2*c^4 - 8*c*d^3 + 6*d^4) + (14*c^4)/15 + (48*d^4
)/5 + tan(e/2 + (f*x)/2)*(8*c^3*d - (152*c*d^3)/3 + (8*c^4)/3 + 42*d^4 + 8*
c^2*d^2) + (8*c^2*d^2)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2
+ (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 +
5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2
 + (f*x)/2)))
```

# 3.473 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=142

$$\frac{d^3x}{a^3}-\frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3}$$

```
[Out] d^3*x/a^3-1/15*(c-d)^2*(2*c+7*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(c-
d)*(2*c^2+11*c*d+29*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/5*(c-d)*cos(f*
x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2844, 3047, 3098, 2814, 2727}

$$-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*x)/a^3 - ((c - d)^2*(2*c + 7*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f
*x])^2) - ((c - d)*(2*c^2 + 11*c*d + 29*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3
*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a
*Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(2c^2+5cd-2d^2\right)-5ad^2 \cdots}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-ac\left(2c^2+5cd-2d^2\right)+\left(-5acd^2-ad\left(2c^2+5 \cdots\right.\right.}{(a+a\sin(e \cdots}}{5a^2}$$

$$= -\frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \cdots$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx)) \cdots}{5f(a+a\sin(e+fx))^3}$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+f \cdots}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.78, size = 1366, normalized size = 9.62

---

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/2160*(Cos[e + f*x]*(9450*Sqrt[2]*(c + d)^3*ArcSin[Sqrt[1 - Sin[e + f*x]]
/Sqrt[2]] - 4725*(c + d)^3*Sqrt[1 + Cos[2*e + 2*f*x]] + 142*(c + d)^3*Hyper
geometric2F1[3/2, 9/2, 11/2, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)
 + 60*(c + d)^3*HypergeometricPFQ[{3/2, 2, 9/2}, {1, 11/2}, (1 - Sin[e + f*
x])/2]*(1 - Sin[e + f*x])^(9/2) + 8*(c + d)^3*HypergeometricPFQ[{3/2, 2, 2,
 9/2}, {1, 1, 11/2}, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2) + 9450*
Sqrt[2]*d*(c + d)^2*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*(-1 + Sin[e + f*
x]) + 5670*Sqrt[2]*d^2*(c + d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*(-1 +
 Sin[e + f*x])^2 + 1350*Sqrt[2]*d^3*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*
(-1 + Sin[e + f*x])^3 - 1575*Sqrt[2]*(c + d)^3*(1 - Sin[e + f*x])^(3/2)*Sqr
t[1 + Sin[e + f*x]] - 630*Sqrt[2]*(c + d)^3*(1 - Sin[e + f*x])^(5/2)*Sqrt[1
 + Sin[e + f*x]] + 4725*Sqrt[2]*(c + d)^2*Sqrt[Cos[e + f*x]^2]*(d - d*Sin[e
 + f*x]) - 282*(c + d)^2*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - Sin[e + f*x
])/2]*(1 - Sin[e + f*x])^(9/2)*(d - d*Sin[e + f*x]) - 156*(c + d)^2*Hyperge
ometricPFQ[{3/2, 2, 9/2}, {1, 11/2}, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x
])^(9/2)*(d - d*Sin[e + f*x]) - 24*(c + d)^2*HypergeometricPFQ[{3/2, 2, 2,
9/2}, {1, 1, 11/2}, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)*(d - d*S
in[e + f*x]) + 1575*Sqrt[2]*(c + d)^2*(1 - Sin[e + f*x])^(3/2)*Sqrt[1 + Sin
[e + f*x]]*(d - d*Sin[e + f*x]) + 630*Sqrt[2]*(c + d)^2*(1 - Sin[e + f*x])^
(5/2)*Sqrt[1 + Sin[e + f*x]]*(d - d*Sin[e + f*x]) - 2835*Sqrt[2]*(c + d)*Sq
rt[Cos[e + f*x]^2]*(d - d*Sin[e + f*x])^2 + 186*(c + d)*Hypergeometric2F1[3
/2, 9/2, 11/2, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)*(d - d*Sin[e
+ f*x])^2 + 132*(c + d)*HypergeometricPFQ[{3/2, 2, 9/2}, {1, 11/2}, (1 - Si
n[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)*(d - d*Sin[e + f*x])^2 + 24*(c + d)
*HypergeometricPFQ[{3/2, 2, 2, 9/2}, {1, 1, 11/2}, (1 - Sin[e + f*x])/2]*(1
 - Sin[e + f*x])^(9/2)*(d - d*Sin[e + f*x])^2 - 945*Sqrt[2]*(c + d)*(1 - Si
n[e + f*x])^(3/2)*Sqrt[1 + Sin[e + f*x]]*(d - d*Sin[e + f*x])^2 - 378*Sqrt[
2]*(c + d)*(1 - Sin[e + f*x])^(5/2)*Sqrt[1 + Sin[e + f*x]]*(d - d*Sin[e + f
*x])^2 + 675*Sqrt[2]*Sqrt[Cos[e + f*x]^2]*(d - d*Sin[e + f*x])^3 - 46*Hyper
geometric2F1[3/2, 9/2, 11/2, (1 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)
*(d - d*Sin[e + f*x])^3 - 36*HypergeometricPFQ[{3/2, 2, 9/2}, {1, 11/2}, (1
 - Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(9/2)*(d - d*Sin[e + f*x])^3 - 8*Hyp
ergeometricPFQ[{3/2, 2, 2, 9/2}, {1, 1, 11/2}, (1 - Sin[e + f*x])/2]*(1 - S
in[e + f*x])^(9/2)*(d - d*Sin[e + f*x])^3 + 225*Sqrt[2]*(1 - Sin[e + f*x])^
(3/2)*Sqrt[1 + Sin[e + f*x]]*(d - d*Sin[e + f*x])^3 + 90*Sqrt[2]*(1 - Sin[e
 + f*x])^(5/2)*Sqrt[1 + Sin[e + f*x]]*(d - d*Sin[e + f*x])^3))/(Sqrt[2]*a^3
*f*(1 - Sin[e + f*x])^(7/2)*Sqrt[1 + Sin[e + f*x]])
```

**Maple** [A]
time = 0.47, size = 194, normalized size = 1.37

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{2\left(c^3-d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4c^3+6c^2d-2d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8c^3+24c^2d-24c\,d^2+8d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4c^3-12c^2d+12c\,d^2-4d^3\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(8c^3-18c^2d+12c\,d^2-2d^3\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+}{f\,a^3}$ |
| default | $\frac{-\frac{2\left(c^3-d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4c^3+6c^2d-2d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8c^3+24c^2d-24c\,d^2+8d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4c^3-12c^2d+12c\,d^2-4d^3\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(8c^3-18c^2d+12c\,d^2-2d^3\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+}{f\,a^3}$ |
| risch | $\frac{d^3x}{a^3}-\frac{2\left(-45d\,c^2\mathrm{e}^{2i(fx+e)}+21c\,d^2-45ic^2d\,\mathrm{e}^{i(fx+e)}-60ic\,d^2\mathrm{e}^{i(fx+e)}+45c\,d^2\mathrm{e}^{4i(fx+e)}+90ic\,d^2\mathrm{e}^{3i(fx+e)}+115id^3\mathrm{e}^{i(fx}\right.}{}$ |
| norman | $\frac{d^3x}{a}+\frac{d^3x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{\left(-2c^3+2d^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(-4c^3-6c^2d+10d^3\right)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(-12c^3-24c^2d-12c\,d^2+48\right.}{af}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-(c^3-d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*c^3+6*c^2*d-2*d^3)/(tan(
1/2*f*x+1/2*e)+1)^2-1/4*(-8*c^3+24*c^2*d-24*c*d^2+8*d^3)/(tan(1/2*f*x+1/2*e
)+1)^4-1/5*(4*c^3-12*c^2*d+12*c*d^2-4*d^3)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(8*
c^3-18*c^2*d+12*c*d^2-2*d^3)/(tan(1/2*f*x+1/2*e)+1)^3+d^3*arctan(tan(1/2*f*
x+1/2*e)))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(143) = 286$.
time = 0.52, size = 852, normalized size = 6.00

$$\frac{2\left(d^3\left(\frac{\frac{95\sin(fx+e)}{\cos(fx+e)+1}+\frac{145\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{75\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+22}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)-\frac{c^3\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{6cd^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{9c^2d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) -
 c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) - 6*c*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
```

```
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) - 9*c^2*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) +
 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(143) = 286$.
time = 0.39, size = 364, normalized size = 2.56

$$-\frac{60\,d^3fx - (15\,d^3fx - 2\,c^3 - 9\,c^2d - 21\,cd^2 + 32\,d^3)\cos\left(fx+e\right)^3 - 3\,c^3 + 9\,c^2d - 9\,cd^2 + 3\,d^3 - (45\,d^3fx + 4\,c^3 + 18\,c^2d - 3\,cd^2 - 19\,d^3)\cos\left(fx+e\right)^2 + 3\,(10\,d^3fx - 3\,c^3 - 6\,c^2d - 9\,cd^2 + 18\,d^3)\cos\left(fx+e\right) + \left(60\,d^3fx + 3\,c^3 - 9\,c^2d + 9\,cd^2 - 3\,d^3 - (15\,d^3fx + 2\,c^3 + 9\,c^2d + 21\,cd^2 - 32\,d^3)\cos\left(fx+e\right)^2 + 3\,(10\,d^3fx - 2\,c^3 - 9\,c^2d - 6\,cd^2 + 17\,d^3)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(fx+e\right)^2 - 2\,a^3f\cos\left(fx+e\right) - 4\,a^3f + \left(a^3f\cos\left(fx+e\right)^2 - 2\,a^3f\cos\left(fx+e\right) - 4\,a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(60*d^3*f*x - (15*d^3*f*x - 2*c^3 - 9*c^2*d - 21*c*d^2 + 32*d^3)*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - (45*d^3*f*x + 4*c^3 + 18*c
^2*d - 3*c*d^2 - 19*d^3)*cos(f*x + e)^2 + 3*(10*d^3*f*x - 3*c^3 - 6*c^2*d -
 9*c*d^2 + 18*d^3)*cos(f*x + e) + (60*d^3*f*x + 3*c^3 - 9*c^2*d + 9*c*d^2 -
 3*d^3 - (15*d^3*f*x + 2*c^3 + 9*c^2*d + 21*c*d^2 - 32*d^3)*cos(f*x + e)^2
+ 3*(10*d^3*f*x - 2*c^3 - 9*c^2*d - 6*c*d^2 + 17*d^3)*cos(f*x + e))*sin(f*x
 + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e
) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f
*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(126) = 252$.
time = 10.00, size = 2640, normalized size = 18.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**3*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c**3*tan(e/2 + f*x/2)**2/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 40*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**3/(15*
a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 90*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2
)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*
c**2*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e
/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*c**2*d*tan(e/2 + f*x/2
)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 18*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c*d**2*tan(e/
2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 12*c*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 +
f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*d**3*f*x*tan(e/2 + f*x/2)**
5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) + 75*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 150*d**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*d**3*f*x*t
an(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*d**3*f*x*tan(e/2 + f*x/2)/(15*
a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) + 15*d**3*f*x/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*t
an(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 30*d**3*tan(e/2 + f*x
/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150
*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) + 150*d**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e
/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2
)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) + 290*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
```

```
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 190*d**3*tan(e/2
 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 44*d**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (
x*(c + d*sin(e))**3/(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.46, size = 280, normalized size = 1.97

$$\frac{\frac{15\,(fx+e)d^3}{a^3} - \frac{2\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 45\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 75\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 40\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 45\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 60\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 145\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 20\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 45\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 30\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 95\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 7\,c^3 + 9\,c^2d + 6\,cd^2 - 22\,d^3\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*d^3/a^3 - 2*(15*c^3*tan(1/2*f*x + 1/2*e)^4 - 15*d^3*tan(
1/2*f*x + 1/2*e)^4 + 30*c^3*tan(1/2*f*x + 1/2*e)^3 + 45*c^2*d*tan(1/2*f*x +
 1/2*e)^3 - 75*d^3*tan(1/2*f*x + 1/2*e)^3 + 40*c^3*tan(1/2*f*x + 1/2*e)^2 +
 45*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*d^
3*tan(1/2*f*x + 1/2*e)^2 + 20*c^3*tan(1/2*f*x + 1/2*e) + 45*c^2*d*tan(1/2*f
*x + 1/2*e) + 30*c*d^2*tan(1/2*f*x + 1/2*e) - 95*d^3*tan(1/2*f*x + 1/2*e) +
 7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad** [B]
time = 9.90, size = 240, normalized size = 1.69

$$\frac{d^3\,x}{a^3} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,c^3-2\,d^3\right) + {\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{16\,c^3}{3}+6\,c^2\,d+8\,c\,d^2-\frac{58\,d^3}{3}\right) + \frac{4\,c\,d^2}{5} + \frac{6\,c^2\,d}{5} + {\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(4\,c^3+6\,c^2\,d-10\,d^3\right) + \frac{14\,c^3}{15} - \frac{44\,d^3}{15} + \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,c^3}{3}+6\,c^2\,d+4\,c\,d^2-\frac{38\,d^3}{3}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+10\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*x)/a^3 - (tan(e/2 + (f*x)/2)^4*(2*c^3 - 2*d^3) + tan(e/2 + (f*x)/2)^2*
(8*c*d^2 + 6*c^2*d + (16*c^3)/3 - (58*d^3)/3) + (4*c*d^2)/5 + (6*c^2*d)/5 +
 tan(e/2 + (f*x)/2)^3*(6*c^2*d + 4*c^3 - 10*d^3) + (14*c^3)/15 - (44*d^3)/1
5 + tan(e/2 + (f*x)/2)*(4*c*d^2 + 6*c^2*d + (8*c^3)/3 - (38*d^3)/3))/(f*(10
*a^3*tan(e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (
f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

# 3.474 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=125

$$-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3}$$

```
[Out] -1/15*(c-d)*(2*c+5*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*c^2+6*c*d+7
*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e
))/f/(a+a*sin(f*x+e))^3
```

**Rubi** [A]
time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2839, 2829, 2727}

$$-\frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f\,(a^3\sin(e+fx)+a^3)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*((c - d)*(2*c + 5*d)*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^2) - ((2
*c^2 + 6*c*d + 7*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((c -
 d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2839

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m*((c + d*Sin[e + f*x])/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
```

```
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2c^2+4cd-d^2)-ad(c+4d)\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} + \frac{(2c^2 \cdots}{\cdots}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))} - \frac{(c-d)\cos(e \cdots}{5f(a \cdots}$$

**Mathematica [A]**
time = 0.09, size = 84, normalized size = 0.67

$$-\frac{\cos(e+fx)\,(7c^2+6cd+2d^2+6(c^2+3cd+d^2)\sin(e+fx)+(2c^2+6cd+7d^2)\sin^2(e+fx))}{15a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*c^2 + 6*c*d + 2*d^2 + 6*(c^2 + 3*c*d + d^2)*Sin[e +
f*x] + (2*c^2 + 6*c*d + 7*d^2)*Sin[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3
)
```

**Maple [A]**
time = 0.40, size = 139, normalized size = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2c^2}{\tan(\frac{fx}{2}+\frac{e}{2})+1}-\frac{2(4c^2-8cd+4d^2)}{5(\tan(\frac{fx}{2}+\frac{e}{2})+1)^5}+\frac{4c(c-d)}{(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{-8c^2+16cd-8d^2}{2(\tan(\frac{fx}{2}+\frac{e}{2})+1)^4}-\frac{2(8c^2-12cd+4d^2)}{3(\tan(\frac{fx}{2}+\frac{e}{2})+1)^3}}{f\,a^3}$ |
| default | $\frac{-\frac{2c^2}{\tan(\frac{fx}{2}+\frac{e}{2})+1}-\frac{2(4c^2-8cd+4d^2)}{5(\tan(\frac{fx}{2}+\frac{e}{2})+1)^5}+\frac{4c(c-d)}{(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{-8c^2+16cd-8d^2}{2(\tan(\frac{fx}{2}+\frac{e}{2})+1)^4}-\frac{2(8c^2-12cd+4d^2)}{3(\tan(\frac{fx}{2}+\frac{e}{2})+1)^3}}{f\,a^3}$ |
| risch | $-\frac{2(-20id^2e^{i(fx+e)}+6cd+30icd\,e^{3i(fx+e)}+7d^2-20c^2e^{2i(fx+e)}-40d^2e^{2i(fx+e)}-30icd\,e^{i(fx+e)}-30cd\,e^{2i(fx+e)}+15d^2e^{4 \cdots}}{15f\,a^3(e^{i(fx+e)}+i)^5}$ |
| norman | $\frac{-\frac{2c^2(\tan^8(\frac{fx}{2}+\frac{e}{2}))}{af}+\frac{-14c^2-12cd-4d^2}{15af}+\frac{4(-c^2-cd)(\tan^7(\frac{fx}{2}+\frac{e}{2}))}{af}+\frac{4(-9c^2-7cd-4d^2)(\tan^2(\frac{fx}{2}+\frac{e}{2}))}{5af}+\frac{(-8c^2-12cd-4d^2 \cdots}{3af}}{(1 \cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-c^2/(tan(1/2*f*x+1/2*e)+1)-1/5*(4*c^2-8*c*d+4*d^2)/(tan(1/2*f*x+1
/2*e)+1)^5+2*c*(c-d)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-8*c^2+16*c*d-8*d^2)/(ta
n(1/2*f*x+1/2*e)+1)^4-1/3*(8*c^2-12*c*d+4*d^2)/(tan(1/2*f*x+1/2*e)+1)^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(126) = 252$.
time = 0.29, size = 601, normalized size = 4.81

$$-\frac{2\left(\frac{c^2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2d^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{6cd\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(126) = 252$.
time = 0.37, size = 254, normalized size = 2.03

$$-\frac{(2c^2+6cd+7d^2)\cos(fx+e)^3-(4c^2+12cd-d^2)\cos(fx+e)^2-3c^2+6cd-3d^2-3(3c^2+4cd+3d^2)\cos(fx+e)-((2c^2+6cd+7d^2)\cos(fx+e)^2-3c^2+6cd-3d^2+6(c^2+3cd+d^2)\cos(fx+e))\sin(fx+e)}{15(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^3 - (4*c^2 + 12*c*d - d^2)*cos(
f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 - 3*(3*c^2 + 4*c*d + 3*d^2)*cos(f*x + e)
 - ((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 + 6*(c^2
 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(114) = 228$.

time = 5.64, size = 1365, normalized size = 10.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**2*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c**2*tan(e/2 + f*x/2)**2/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 40*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**2/(15*
a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d
*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f
*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2
+ 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 12*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 +
 f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*d**2*tan(e/2 + f*x/2)**2/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) - 20*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 4*d**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a)**3,
True))
```

**Giac** [A]

time = 0.48, size = 181, normalized size = 1.45

$$-\frac{2\left(15\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+30\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+30\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+10\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,c^2+6\,cd+2\,d^2\right)}{15\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*c
*d*tan(1/2*f*x + 1/2*e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 + 30*c*d*tan(1/2*
f*x + 1/2*e)^2 + 20*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e
) + 30*c*d*tan(1/2*f*x + 1/2*e) + 10*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 + 6*c
*d + 2*d^2)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

**Mupad [B]**
time = 7.49, size = 218, normalized size = 1.74

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,c\,d-4\,c^2\,\cos\left(e+f\,x\right)+d^2\,\cos\left(e+f\,x\right)+\frac{25\,c^2\,\sin(e+f\,x)}{2}+\frac{5\,d^2\,\sin(e+f\,x)}{2}+\frac{53\,c^2}{4}+\frac{13\,d^2}{4}-\frac{9\,c^2\,\cos(2\,e+2\,f\,x)}{4}-\frac{9\,d^2\,\cos(2\,e+2\,f\,x)}{4}-\frac{5\,c^2\,\sin(2\,e+2\,f\,x)}{4}+\frac{5\,d^2\,\sin(2\,e+2\,f\,x)}{4}+3\,c\,d\,\cos\left(e+f\,x\right)+15\,c\,d\,\sin\left(e+f\,x\right)-3\,c\,d\,\cos\left(2\,e+2\,f\,x\right)\right)}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*(6*c*d - 4*c^2*cos(e + f*x) + d^2*cos(e + f*x) + (25*
c^2*sin(e + f*x))/2 + (5*d^2*sin(e + f*x))/2 + (53*c^2)/4 + (13*d^2)/4 - (9
*c^2*cos(2*e + 2*f*x))/4 - (9*d^2*cos(2*e + 2*f*x))/4 - (5*c^2*sin(2*e + 2*
f*x))/4 + (5*d^2*sin(2*e + 2*f*x))/4 + 3*c*d*cos(e + f*x) + 15*c*d*sin(e +
f*x) - 3*c*d*cos(2*e + 2*f*x)))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e
)/2 - pi/4 + (5*f*x)/2))/4))
```

# 3.475 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=102

$$-\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2c+3d)\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}$$

```
[Out] -1/5*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*c+3*d)*cos(f*x+e)/a/f/(a
+a*sin(f*x+e))^2-1/15*(2*c+3*d)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

**Rubi** [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {2829, 2729, 2727}

$$-\frac{(2c+3d)\cos(e+fx)}{15f\,(a^3\sin(e+fx)+a^3)}-\frac{(2c+3d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) - ((2*c + 3*d)*Cos[e
 + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c + 3*d)*Cos[e + f*x])/(15*f
*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(2c+3d)\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}$$
$$= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(2c+3d)\int \frac{1}{a+a\sin(e+f}}{15a^2}$$
$$= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2c+3d)\cos(e+f}{15f\,(a^3+a^3\sin(e+}$$

**Mathematica [A]**
time = 0.06, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\,(7c+3d+(6c+9d)\sin(e+fx)+(2c+3d)\sin^2(e+fx))}{15a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*c + 3*d + (6*c + 9*d)*Sin[e + f*x] + (2*c + 3*d)*Sin
[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.35, size = 114, normalized size = 1.12

| method | result |
|---|---|
| risch | $-\frac{2i\left(20ic\,e^{2i(fx+e)}+15id\,e^{2i(fx+e)}+15d\,e^{3i(fx+e)}-2ic-3id-10c\,e^{i(fx+e)}-15d\,e^{i(fx+e)}\right)}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$ |
| derivativedivides | $\frac{-\frac{2(4c-4d)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2(8c-6d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8c+8d}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-4c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^3}$ |
| default | $\frac{-\frac{2(4c-4d)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2(8c-6d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8c+8d}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-4c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^3}$ |
| norman | $\frac{-\frac{2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{(-4c-2d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{-14c-6d}{15af}+\frac{(-8c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3af}+\frac{(-22c-6d)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{(-20c-12d}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-1/5*(4*c-4*d)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(8*c-6*d)/(tan(1/2*f*x
+1/2*e)+1)^3-1/4*(-8*c+8*d)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-4*c+2*d)/(tan(1/
2*f*x+1/2*e)+1)^2-c/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(102) = 204$.
time = 0.29, size = 421, normalized size = 4.13

$$-\frac{2\left(\frac{c\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

**Fricas** [A]
time = 0.34, size = 202, normalized size = 1.98

$$-\frac{(2c+3d)\cos(fx+e)^3-2(2c+3d)\cos(fx+e)^2-3(3c+2d)\cos(fx+e)-\left((2c+3d)\cos(fx+e)^2+3(2c+3d)\cos(fx+e)-3c+3d\right)\sin(fx+e)-3c+3d}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c + 3*d)*cos(f*x + e)^3 - 2*(2*c + 3*d)*cos(f*x + e)^2 - 3*(3*c +
 2*d)*cos(f*x + e) - ((2*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x +
e) - 3*c + 3*d)*sin(f*x + e) - 3*c + 3*d)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(87) = 174$.
time = 3.12, size = 1015, normalized size = 9.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
```

```
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 40*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*d*tan(e/2 + f*x/2)**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 30*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*d/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.44, size = 130, normalized size = 1.27

$$-\frac{2\left(15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+20\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,c+3\,d\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 30*c*tan(1/2*f*x + 1/2*e)^3 + 15*d*tan
(1/2*f*x + 1/2*e)^3 + 40*c*tan(1/2*f*x + 1/2*e)^2 + 15*d*tan(1/2*f*x + 1/2*
e)^2 + 20*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e) + 7*c + 3*d)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

**Mupad** [B]
time = 7.25, size = 150, normalized size = 1.47

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{53\,c}{4}+3\,d-4\,c\,\cos\left(e+f\,x\right)+\frac{3\,d\,\cos(e+f\,x)}{2}+\frac{25\,c\,\sin(e+f\,x)}{2}+\frac{15\,d\,\sin(e+f\,x)}{2}-\frac{9\,c\,\cos(2\,e+2\,f\,x)}{4}-\frac{3\,d\,\cos(2\,e+2\,f\,x)}{2}-\frac{5\,c\,\sin(2\,e+2\,f\,x)}{4}\right)}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*c)/4 + 3*d - 4*c*cos(e + f*x) + (3*d*cos(e + f*x
))/2 + (25*c*sin(e + f*x))/2 + (15*d*sin(e + f*x))/2 - (9*c*cos(2*e + 2*f*x
))/4 - (3*d*cos(2*e + 2*f*x))/2 - (5*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*
2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f
*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

# 3.476 $\int \frac{1}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=83

$$-\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{2\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -1/5*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-2/15*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2
-2/15*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

**Rubi** [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$-\frac{2\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{2\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] -1/5*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^3) - (2*Cos[e + f*x])/(15*a*f*(a
+ a*Sin[e + f*x])^2) - (2*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^3}\,dx &= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{2\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}\\ &= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{2\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}\\ &= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{2\cos(e+fx)}{15f\left(a^3+a^3\sin(e+\right.}\end{aligned}$$

**Mathematica [A]**
time = 0.09, size = 76, normalized size = 0.92

$$\frac{10-15\cos(e+fx)-6\cos(2(e+fx))+\cos(3(e+fx))+15\sin(e+fx)-6\sin(2(e+fx))-\sin(3(e+fx))}{30a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] (10 - 15*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + Cos[3*(e + f*x)] + 15*Sin[e +
f*x] - 6*Sin[2*(e + f*x)] - Sin[3*(e + f*x)])/(30*a^3*f*(1 + Sin[e + f*x])^
3)
```

**Maple [A]**
time = 0.25, size = 85, normalized size = 1.02

| method | result | size |
|---|---|---|
| risch | $\frac{-\frac{4}{15}+\frac{8\,{\mathrm e}^{2 i (f x+e)}}{3}+\frac{4 i\,{\mathrm e}^{i (f x+e)}}{3}}{f\,a^{3} \left({\mathrm e}^{i (f x+e)}+i\right)^{5}}$ | 48 |
| derivativedivides | $\frac{\frac{4}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}+\frac{4}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{8}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{16}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}}{f\,a^{3}}$ | 85 |
| default | $\frac{\frac{4}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}+\frac{4}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{8}{5 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{16}{3 \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1}}{f\,a^{3}}$ | 85 |
| norman | $\frac{-\frac{2 \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a f}-\frac{14}{15 a f}-\frac{4 \left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a f}-\frac{8 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{3 a f}-\frac{16 \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 a f}}{a^{2} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}$ | 101 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(2/(tan(1/2*f*x+1/2*e)+1)^4+2/(tan(1/2*f*x+1/2*e)+1)^2-4/5/(tan(1/2
*f*x+1/2*e)+1)^5-8/3/(tan(1/2*f*x+1/2*e)+1)^3-1/(tan(1/2*f*x+1/2*e)+1))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(83) = 166.
time = 0.28, size = 221, normalized size = 2.66

$$-\frac{2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{15\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
```

```
*x + e) + 1)^4 + 7)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)*f)
```

**Fricas** [A]
time = 0.35, size = 159, normalized size = 1.92

$$-\frac{2 \cos\left(fx+e\right)^{3}-4 \cos\left(fx+e\right)^{2}-\left(2 \cos\left(fx+e\right)^{2}+6 \cos\left(fx+e\right)-3\right) \sin\left(fx+e\right)-9 \cos\left(fx+e\right)-3}{15 \left(a^{3} f \cos\left(fx+e\right)^{3}+3 a^{3} f \cos\left(fx+e\right)^{2}-2 a^{3} f \cos\left(fx+e\right)-4 a^{3} f+\left(a^{3} f \cos\left(fx+e\right)^{2}-2 a^{3} f \cos\left(fx+e\right)-4 a^{3} f\right) \sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(f*x + e)^3 - 4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 + 6*cos(f*x
+ e) - 3)*sin(f*x + e) - 9*cos(f*x + e) - 3)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(73) = 146.
time = 1.51, size = 558, normalized size = 6.72

$$\begin{cases} -\frac{30 \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^{3} f \tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^{3} f} - \frac{60 \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^{3} f \tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^{3} f} - \frac{80 \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^{3} f \tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^{3} f} - \frac{40 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^{3} f \tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^{3} f} - \frac{14}{15a^{3} f \tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3} f \tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3} f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^{3} f} & \text{for}\: f \neq 0 \\ \frac{x}{\left(a \sin\left(e\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*tan(e/2 + f*x/
2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 80*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
40*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0
)), (x/(a*sin(e) + a)**3, True))
```

**Giac** [A]
time = 0.48, size = 78, normalized size = 0.94

$$-\frac{2 \left(15 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}+30 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}+40 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}+20 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+7\right)}{15 a^{3} f \left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*f*x + 1/2*e)^4 + 30*tan(1/2*f*x + 1/2*e)^3 + 40*tan(1/2*f
*x + 1/2*e)^2 + 20*tan(1/2*f*x + 1/2*e) + 7)/(a^3*f*(tan(1/2*f*x + 1/2*e) +
 1)^5)
```

**Mupad [B]**
time = 6.96, size = 133, normalized size = 1.60

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+20\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+40\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+30\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+15\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right)}{15\,a^3\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*(7*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4 +
30*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 + 20*cos(e/2 + (f*x)/2)^3*sin(e/
2 + (f*x)/2) + 40*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*a^3*f*(co
s(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^5)
```

# 3.477 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=186

$$-\frac{2d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f}-\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3}-\frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}-\frac{(2c^2-}{15(c-}$$

```
[Out] -1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*c-7*d)*cos(f*x+e)/a/(c-d
)^2/f/(a+a*sin(f*x+e))^2-1/15*(2*c^2-9*c*d+22*d^2)*cos(f*x+e)/(c-d)^3/f/(a^
3+a^3*sin(f*x+e))-2*d^3*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^
3/(c-d)^3/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.35, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2845, 3057, 12, 2739, 632, 210}

$$-\frac{2d^3\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{(2c^2-9cd+22d^2)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c-7d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^3*Sq
rt[c^2 - d^2]*f) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3) - ((2*
c - 7*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2) - ((2*c^2
- 9*c*d + 22*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2c-5d)-2ad\sin(e+fx}{(a+a\sin(e+fx))^2(c+d\sin(e+}}{5a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+}{15a(c-d)^2f(a+a\sin(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+}{15a(c-d)^2f(a+a\sin(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+}{15a(c-d)^2f(a+a\sin(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+}{15a(c-d)^2f(a+a\sin(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+}{15a(c-d)^2f(a+a\sin(}$$

$$= -\frac{2d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f} - \frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+f}$$

**Mathematica** [A]
time = 0.49, size = 301, normalized size = 1.62

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(2c-7d)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(c-d)(-2c+7d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2(2c^2-9cd+22d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-\frac{30d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{\sqrt{c^2-d^2}}\right)}{15a^3(c-d)^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(2*c - 7*d)*(c - d)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-2*c + 7*d)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 9*c*d + 22*d^2)*Sin[(e
 + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*ArcTan[(d + c*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
/Sqrt[c^2 - d^2]))/(15*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

**Maple** [A]
time = 0.58, size = 200, normalized size = 1.08

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{-4c+6d}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8c-10d)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(c^2-3cd+3d^2\right)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{4}{(c-d)\left(\tan\left(\frac{fx}{2}\right.\right.}}{f\,a^3}$ |
| default | $\frac{-\frac{-4c+6d}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8c-10d)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(c^2-3cd+3d^2\right)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{4}{(c-d)\left(\tan\left(\frac{fx}{2}\right.\right.}}{f\,a^3}$ |
| risch | $-\frac{2\left(-95id^2e^{i(fx+e)}-9cd+22d^2-15icd\,e^{3i(fx+e)}+45icd\,e^{i(fx+e)}+75cd\,e^{2i(fx+e)}+75id^2e^{3i(fx+e)}-10ic^2e^{i(fx+e)}+15d^2e\right.}{15\left(e^{i(fx+e)}+i\right)^5(c-d)^3f\,a^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-1/2*(-4*c+6*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*c-10*d)/(c
-d)^2/(tan(1/2*f*x+1/2*e)+1)^3-(c^2-3*c*d+3*d^2)/(c-d)^3/(tan(1/2*f*x+1/2*e
)+1)-4/5/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5+2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4-d^
3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(182) = 364$.
time = 0.42, size = 1789, normalized size = 9.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^4 - 12*c^3*d + 12*c*d^3 - 6*d^4 - 2*(2*c^4 - 9*c^3*d + 20*c^2*d^
2 + 9*c*d^3 - 22*d^4)*cos(f*x + e)^3 + 2*(4*c^4 - 18*c^3*d + 25*c^2*d^2 + 1
```

```
8*c*d^3 - 29*d^4)*cos(f*x + e)^2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x +
 e)^2 - 2*d^3*cos(f*x + e) - 4*d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x +
e) - 4*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos
(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 6*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x +
e) - 2*(3*c^4 - 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 +
 9*c*d^3 - 22*d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d
^3 - 17*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^
3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^
5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*
cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 -
 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c
^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d
 + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
- 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e)), 1/15*(3*c^4 - 6*c^3*d +
6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*d^4)*cos(f*x
 + e)^3 + (4*c^4 - 18*c^3*d + 25*c^2*d^2 + 18*c*d^3 - 29*d^4)*cos(f*x + e)^
2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*
d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*d^3)*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3
*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x + e) - (3*c^4
- 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*
d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*
d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
 a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3
*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^
3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

**Giac [A]**
time = 0.45, size = 364, normalized size = 1.96

$$-\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^3}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\sqrt{c^2-d^2}}+\frac{15c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-105cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+135d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-135cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+185d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+20c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-75cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+115d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7c^2-24cd+32d^2}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x
+ 1/2*e) + d)/sqrt(c^2 - d^2)))*d^3/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
 a^3*d^3)*sqrt(c^2 - d^2)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 45*c*d*tan(1/
2*f*x + 1/2*e)^4 + 45*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2
*e)^3 - 105*c*d*tan(1/2*f*x + 1/2*e)^3 + 135*d^2*tan(1/2*f*x + 1/2*e)^3 + 4
0*c^2*tan(1/2*f*x + 1/2*e)^2 - 135*c*d*tan(1/2*f*x + 1/2*e)^2 + 185*d^2*tan
(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 75*c*d*tan(1/2*f*x + 1/
2*e) + 115*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 24*c*d + 32*d^2)/((a^3*c^3 -
3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 10.07, size = 466, normalized size = 2.51

$$\frac{2d^3\,\mathrm{atan}\left(\frac{\frac{d^3\left(-2a^3c^3d+6a^3c^2d^2-6a^3cd^3+2a^3d^4\right)}{a^3\sqrt{c+d}\,(c-d)^{7/2}}-\frac{2cd^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)}{a^3\sqrt{c+d}\,(c-d)^{7/2}}}{2d^3}\right)}{a^3f\sqrt{c+d}\,(c-d)^{7/2}}-\frac{\frac{2(7c^2-24cd+32d^2)}{15(c-d)(c^2-2cd+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c^2-15cd+23d^2)}{3(c-d)(c^2-2cd+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(c^2-3cd+3d^2)}{(c-d)(c^2-2cd+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(2c^2-7cd+9d^2)}{(c-d)(c^2-2cd+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(8c^2-27cd+37d^2)}{3(c-d)(c^2-2cd+d^2)}}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+10a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+10a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^3*atan(((d^3*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^3*c^2*d^2))/
(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^3*tan(e/2 + (f*x)/2)*(a^3*c^3 -
a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)))/(2
*d^3)))/(a^3*f*(c + d)^(1/2)*(c - d)^(7/2)) - ((2*(7*c^2 - 24*c*d + 32*d^2)
)/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*c^2 - 15*c*d
+ 23*d^2))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(c^2 -
 3*c*d + 3*d^2))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2
*c^2 - 7*c*d + 9*d^2))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(8*c^2 - 27*c*d + 37*d^2))/(3*(c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*
tan(e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/
2)^4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

# 3.478 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=298

$$-\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f}-\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}-\frac{\text{co}}{5(c-d)f(a+a\sin(e}$$

```
[Out] -1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+
d*sin(f*x+e))-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/
15*(2*c-9*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/1
5*(2*c^2-12*c*d+45*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))/(c+d*sin(
f*x+e))-2*d^3*(4*c+3*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^
3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.49, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2845, 3057, 2833, 12, 2739, 632, 210}

$$-\frac{2d^3(4c+3d)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{(2c^2-12cd+45d^2)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))}-\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))}-\frac{(2c-9d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^3*(4*c + 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*
(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72
*d^3)*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) - Cos
[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])) - ((2*c
 - 9*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) - ((2*c^2 - 12*c*d + 45*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 +
 a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \overline{(a}}{}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{}{15a(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{}{15a(}$$

$$= -\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{}{5(c-d)f}$$

$$= -\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{}{5(c-d)f}$$

$$= -\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{}{5(c-d)f}$$

$$= -\frac{d(2c^3-12c^2d+43cd^2+72d^3)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{}{5(c-d)f}$$

$$= -\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f} - \frac{d(2c^3-12c^2d+}{15a^3(c-d)^4(c}$$

**Mathematica [A]**
time = 1.64, size = 361, normalized size = 1.21

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(c-6d)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2(c-6d)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2(2c^2-14cd+57d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-\frac{30d^3(4c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{(c+d)\sqrt{c^2-d^2}}-\frac{15d^4\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{(c+d)(c+d\sin(e+fx))}\right)}{15a^3(c-d)^4f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 6*d)*(c - d)*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - 6*d)*(c - d)*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 14*c*d + 57*d^2)*Sin[(e +
 f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*(4*c + 3*d)*ArcT
an[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5)/((c + d)*Sqrt[c^2 - d^2]) - (15*d^4*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)*(c + d*Sin[e + f*x]))))/(15*a^3*(c - d)^
4*f*(1 + Sin[e + f*x])^3)
```

**Maple [A]**
time = 0.74, size = 273, normalized size = 0.92

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2(8c-12d)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8d-4c}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(c^2-4cd+6d^2\right)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{4}{(c-d)^2\left(\tan\left(\right.\right.}}{f\,a^3}$ |
| default | $\frac{-\frac{2(8c-12d)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{8d-4c}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(c^2-4cd+6d^2\right)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{4}{(c-d)^2\left(\tan\left(\right.\right.}}{f\,a^3}$ |
| risch | $-\frac{2\left(-2c^3d+12c^2d^2-43d^3c+60ic^2d^2\mathrm{e}^{5i(fx+e)}+345icd^3\mathrm{e}^{5i(fx+e)}-480d^4\mathrm{e}^{4i(fx+e)}+567d^4\mathrm{e}^{2i(fx+e)}+20c^4\mathrm{e}^{2i(fx+e)}+299\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-1/3*(8*c-12*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(8*d-4*c)/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^2-(c^2-4*c*d+6*d^2)/(c-d)^4/(tan(1/2*f*x+1/2*e)
+1)-4/5/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5+2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^4
-d^3/(c-d)^4*((d^2/(c+d)/c*tan(1/2*f*x+1/2*e)+d/(c+d))/(c*tan(1/2*f*x+1/2*e
)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(4*c+3*d)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1601 vs. 2(297) = 594.
time = 0.48, size = 3292, normalized size = 11.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/30*(6*c^6 - 12*c^5*d - 6*c^4*d^2 + 24*c^3*d^3 - 6*c^2*d^4 - 12*c*d^5 + 6*d^6 - 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^4 - 2*(2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + 2*(4*c^6 - 19*c^5*d + 22*c^4*d^2 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 + 15*(16*c^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^3 + 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 + 28*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 - 71*c*d^5 - 62*d^6)*cos(f*x + e) - 2*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2 + 106*c^3*d^3 + 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 - 9*c^5*d + 13*c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 - (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^4 - (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + (4*c^6 - 19*c^5*d + 22*c^4*d^2 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 - 15*(16*c^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^3

```
+ 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos(f
*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 + 28
*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^4
+ 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*sin
(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*co
s(f*x + e))) + 3*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 -
 71*c*d^5 - 62*d^6)*cos(f*x + e) - (3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^
3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c
^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2
+ 106*c^3*d^3 + 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 -
 9*c^5*d + 13*c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*
x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4
*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^
4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 -
11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 -
(3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4
 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e
)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*
d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 -
2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6
 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5
*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f
*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*
a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^
3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2
*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c...
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.47, size = 517, normalized size = 1.73

$$-\frac{2\left(\frac{15\left(4cd^3+3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^5-3a^3c^4d+2a^3c^3d^2+2a^3c^2d^3-3a^3cd^4+a^3d^5\right)\sqrt{c^2-d^2}}+\frac{15\left(d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+cd^4\right)}{\left(a^3c^6-3a^3c^5d+2a^3c^4d^2+2a^3c^3d^3-3a^3c^2d^4+a^3cd^5\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}+\frac{15c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-60cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+90d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-150cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+300d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-190cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+420d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+20c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-110cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+270d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7c^2-34cd+72d^2}{\left(a^3c^4-4a^3c^3d+6a^3c^2d^2-4a^3cd^3+a^3d^4\right)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d +
 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) +
15*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*a^3*c^4*d
^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 60*c*d*t
an(1/2*f*x + 1/2*e)^4 + 90*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 150*c*d*tan(1/2*f*x + 1/2*e)^3 + 300*d^2*tan(1/2*f*x + 1/2*e)^
3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 190*c*d*tan(1/2*f*x + 1/2*e)^2 + 420*d^
2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 110*c*d*tan(1/2*f*
x + 1/2*e) + 270*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 34*c*d + 72*d^2)/((a^3*
c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1
/2*e) + 1)^5))/f
```

**<u>Mupad</u> [B]**
time = 10.30, size = 987, normalized size = 3.31

$$-\frac{\frac{2\,\left(7\,c^4-27\,c^3\,d+38\,c^2\,d^2+72\,c\,d^3+15\,d^4\right)}{15\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(5\,c^4-18\,c^3\,d+19\,c^2\,d^2+84\,c\,d^3+15\,d^4\right)}{3\,c\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(20\,c^5-76\,c^4\,d+106\,c^3\,d^2+346\,c^2\,d^3+219\,c\,d^4+15\,d^5\right)}{15\,c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(c^5-3\,c^4\,d+2\,c^3\,d^2+6\,c^2\,d^3+d^5\right)}{c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(2\,c^5-6\,c^4\,d+4\,c^3\,d^2+24\,c^2\,d^3+13\,c\,d^4+5\,d^5\right)}{c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(11\,c^5-27\,c^4\,d+4\,c^3\,d^2+162\,c^2\,d^3+135\,c\,d^4+30\,d^5\right)}{3\,c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(47\,c^5-137\,c^4\,d+88\,c^3\,d^2+812\,c^2\,d^3+690\,c\,d^4+75\,d^5\right)}{15\,c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}}{f\,\left(a^3\,c+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,a^3\,c+2\,a^3\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(5\,a^3\,c+2\,a^3\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(11\,a^3\,c+10\,a^3\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(11\,a^3\,c+10\,a^3\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(15\,a^3\,c+20\,a^3\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(15\,a^3\,c+20\,a^3\,d\right)+a^3\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\right)}-\frac{2\,d^3\,\mathrm{atan}\left(\frac{\frac{d^3\,\left(4\,c+3\,d\right)\,\left(2\,a^3\,c^5\,d-6\,a^3\,c^4\,d^2+4\,a^3\,c^3\,d^3+4\,a^3\,c^2\,d^4-6\,a^3\,c\,d^5+2\,a^3\,d^6\right)}{a^3\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{9/2}}+\frac{2\,c\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,c+3\,d\right)\,\left(a^3\,c^5-3\,a^3\,c^4\,d+2\,a^3\,c^3\,d^2+2\,a^3\,c^2\,d^3-3\,a^3\,c\,d^4+a^3\,d^5\right)}{a^3\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{9/2}}}{6\,d^4+8\,c\,d^3}\right)\,\left(4\,c+3\,d\right)}{a^3\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] - ((2*(72*c*d^3 - 27*c^3*d + 7*c^4 + 15*d^4 + 38*c^2*d^2))/(15*(c + d)*(c -
 d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(84*c*d^3 -
18*c^3*d + 5*c^4 + 15*d^4 + 19*c^2*d^2))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d +
c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)*(219*c*d^4 - 76*c^4*d + 20*c^5 + 15*d^5
 + 346*c^2*d^3 + 106*c^3*d^2))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c
^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^6*(c^5 - 3*c^4*d + d^5 + 6*c^2*d^3 + 2*c
^3*d^2))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 +
 (f*x)/2)^5*(13*c*d^4 - 6*c^4*d + 2*c^5 + 5*d^5 + 24*c^2*d^3 + 4*c^3*d^2))/
(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)
^4*(135*c*d^4 - 27*c^4*d + 11*c^5 + 30*d^5 + 162*c^2*d^3 + 4*c^3*d^2))/(3*c
*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^2
*(690*c*d^4 - 137*c^4*d + 47*c^5 + 75*d^5 + 812*c^2*d^3 + 88*c^3*d^2))/(15*
c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(a^3*c + tan(e/2 + (
f*x)/2)*(5*a^3*c + 2*a^3*d) + tan(e/2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + ta
n(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^3*d) + tan(e/2 + (f*x)/2)^5*(11*a^3*c +
 10*a^3*d) + tan(e/2 + (f*x)/2)^3*(15*a^3*c + 20*a^3*d) + tan(e/2 + (f*x)/2
)^4*(15*a^3*c + 20*a^3*d) + a^3*c*tan(e/2 + (f*x)/2)^7)) - (2*d^3*atan(((d^
3*(4*c + 3*d)*(2*a^3*d^6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^
3*c^3*d^3 - 6*a^3*c^4*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^3*ta
n(e/2 + (f*x)/2)*(4*c + 3*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4 - 3*a^3*c^4*d
 + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)))/(8*c*
d^3 + 6*d^4))*(4*c + 3*d))/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

# 3.479 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=378

$$-\frac{d^3(20c^2+30cd+13d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f}-\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}-\frac{}{5(c-d)f}$$

```
[Out] -1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(
c+d*sin(f*x+e))^2-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)
)^2-1/15*(2*c-11*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+
e))^2-1/15*(2*c^2-15*c*d+76*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))/
(c+d*sin(f*x+e))^2-1/30*d*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4)*co
s(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^3*(20*c^2+30*c*d+13*d^2)*
arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2
-d^2)^(1/2)
```

**Rubi** [A]
time = 0.64, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2845, 3057, 2833, 12, 2739, 632, 210}

$$-\frac{d^3(20c^2+30cd+13d^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{(2c^2-15cd+76d^2)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^2}-\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^2}-\frac{d(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}-\frac{(2c-11d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}-\frac{\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(a^3*(c - d)^5*(c + d)^2*Sqrt[c^2 - d^2]*f)) - (d*(4*c^3 - 30*c^2*
d + 146*c*d^2 + 195*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*S
in[e + f*x])^2) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*S
in[e + f*x])^2) - ((2*c - 11*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[
e + f*x])^2*(c + d*Sin[e + f*x])^2) - ((2*c^2 - 15*c*d + 76*d^2)*Cos[e + f*
x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(
4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{}{(a\cdots}}{}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{}{15a(\cdots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{}{15a(\cdots}$$

$$= -\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)\cdots}$$

$$= -\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)\cdots}$$

$$= -\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)\cdots}$$

$$= -\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)\cdots}$$

$$= -\frac{d(4c^3-30c^2d+146cd^2+195d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)\cdots}$$

$$= -\frac{d^3(20c^2+30cd+13d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(4c^3\cdots}{30a^3\cdots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 914 vs. 2(378) = 756.
time = 4.06, size = 914, normalized size = 2.42

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-480*d^3*(20*c^2 + 30*c*d + 13*d^2
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^5)/Sqrt[c^2 - d^2] + (10*d*(-40*c^5 + 340*c^4*d + 1934*c^3*d^
```

2 + 3040*c^2*d^3 + 1994*c*d^4 + 481*d^5)*Cos[(e + f*x)/2] - 2*(80*c^6 - 424*c^5*d + 1200*c^4*d^2 + 9698*c^3*d^3 + 17640*c^2*d^4 + 12371*c*d^5 + 2905*d^6)*Cos[(3*(e + f*x))/2] - 1260*c^3*d^3*Cos[(5*(e + f*x))/2] - 2640*c^2*d^4*Cos[(5*(e + f*x))/2] - 2250*c*d^5*Cos[(5*(e + f*x))/2] - 870*d^6*Cos[(5*(e + f*x))/2] + 32*c^5*d*Cos[(7*(e + f*x))/2] - 200*c^4*d^2*Cos[(7*(e + f*x))/2] + 836*c^3*d^3*Cos[(7*(e + f*x))/2] + 4480*c^2*d^4*Cos[(7*(e + f*x))/2] + 5747*c*d^5*Cos[(7*(e + f*x))/2] + 2200*d^6*Cos[(7*(e + f*x))/2] - 135*c*d^5*Cos[(9*(e + f*x))/2] - 90*d^6*Cos[(9*(e + f*x))/2] + 320*c^6*Sin[(e + f*x)/2] - 1520*c^5*d*Sin[(e + f*x)/2] + 4568*c^4*d^2*Sin[(e + f*x)/2] + 27340*c^3*d^3*Sin[(e + f*x)/2] + 40904*c^2*d^4*Sin[(e + f*x)/2] + 26020*c*d^5*Sin[(e + f*x)/2] + 6318*d^6*Sin[(e + f*x)/2] + 800*c^4*d^2*Sin[(3*(e + f*x))/2] + 7500*c^3*d^3*Sin[(3*(e + f*x))/2] + 13280*c^2*d^4*Sin[(3*(e + f*x))/2] + 9690*c*d^5*Sin[(3*(e + f*x))/2] + 2750*d^6*Sin[(3*(e + f*x))/2] - 32*c^6*Sin[(5*(e + f*x))/2] + 80*c^5*d*Sin[(5*(e + f*x))/2] - 32*c^4*d^2*Sin[(5*(e + f*x))/2] - 6820*c^3*d^3*Sin[(5*(e + f*x))/2] - 18080*c^2*d^4*Sin[(5*(e + f*x))/2] - 15670*c*d^5*Sin[(5*(e + f*x))/2] - 4266*d^6*Sin[(5*(e + f*x))/2] - 60*c^2*d^4*Sin[(7*(e + f*x))/2] + 135*c*d^5*Sin[(7*(e + f*x))/2] + 60*d^6*Sin[(7*(e + f*x))/2] + 8*c^4*d^2*Sin[(9*(e + f*x))/2] - 60*c^3*d^3*Sin[(9*(e + f*x))/2] + 284*c^2*d^4*Sin[(9*(e + f*x))/2] + 915*c*d^5*Sin[(9*(e + f*x))/2] + 518*d^6*Sin[(9*(e + f*x))/2])/(c + d*Sin[e + f*x])^2))/(480*a^3*(c - d)^5*(c + d)^2*f*(1 + Sin[e + f*x])^3)

**Maple [A]**
time = 1.08, size = 446, normalized size = 1.18

| method | result |
|---|---|
| derivativedivides | $-\frac{-4c+10d}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8c-14d)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(c^2-5cd+10d^2\right)}{(c-d)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{}{(c-d)^3(\tan}$ |
| default | $-\frac{-4c+10d}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8c-14d)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(c^2-5cd+10d^2\right)}{(c-d)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{8}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{}{(c-d)^3(\tan}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-1/2*(-4*c+10*d)/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*c-14*d)/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3-(c^2-5*c*d+10*d^2)/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)-4/5/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5+2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1
```

```
)^4-d^3/(c-d)^5*((1/2*d^2*(11*c^2+6*c*d-2*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^3+1/2*d*(10*c^4+6*c^3*d+19*c^2*d^2+12*c*d^3-2*d^4)/c^2/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(29*c^2+18*c*d-2*d^2)/(c^2+2*c*d+d^2)/c*t
an(1/2*f*x+1/2*e)+1/2*d*(10*c^2+6*c*d-d^2)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+
1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*d+
d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2603 vs. 2(377) = 754.
time = 0.53, size = 5295, normalized size = 14.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*c^8 - 24*c^7*d - 24*c^6*d^2 + 72*c^5*d^3 - 72*c^3*d^5 + 24*c^2*d
^6 + 24*c*d^7 - 12*d^8 + 2*(4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*
d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - 2*(8*c^7*d - 52*c
^6*d^2 + 216*c^5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^
7 - 413*d^8)*cos(f*x + e)^4 - 2*(4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3
 + 2676*c^4*d^4 + 2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(
f*x + e)^3 + 4*(4*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 +
 345*c^3*d^5 - 526*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c
^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*
c*d^6 + 13*d^7)*cos(f*x + e)^5 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39
*d^7)*cos(f*x + e)^4 - (20*c^4*d^3 + 110*c^3*d^4 + 193*c^2*d^5 + 142*c*d^6
+ 39*d^7)*cos(f*x + e)^3 - (60*c^4*d^3 + 290*c^3*d^4 + 479*c^2*d^5 + 340*c*
d^6 + 91*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56
*c*d^6 + 13*d^7)*cos(f*x + e) + (80*c^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 2
24*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*cos(f*x + e)^4 - 2*(20
*c^3*d^4 + 50*c^2*d^5 + 43*c*d^6 + 13*d^7)*cos(f*x + e)^3 - (20*c^4*d^3 + 1
```

50*c^3*d^4 + 293*c^2*d^5 + 228*c*d^6 + 65*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d
^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*cos(f*x + e))*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 12*(3*c^8 -
11*c^7*d + 9*c^6*d^2 + 213*c^5*d^3 + 475*c^4*d^4 + 237*c^3*d^5 - 359*c^2*d^
6 - 439*c*d^7 - 128*d^8)*cos(f*x + e) - 2*(6*c^8 - 12*c^7*d - 12*c^6*d^2 +
36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*d^7 - 6*d^8 + (4*c^6*d^2 - 30*c
^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos
(f*x + e)^4 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3
*d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e)^3 - 2*(2*c^8 - 7*c^
7*d + 14*c^6*d^2 + 291*c^5*d^3 + 726*c^4*d^4 + 384*c^3*d^5 - 557*c^2*d^6 -
668*c*d^7 - 185*d^8)*cos(f*x + e)^2 - 6*(2*c^8 - 9*c^7*d + 11*c^6*d^2 + 207
*c^5*d^3 + 475*c^4*d^4 + 243*c^3*d^5 - 361*c^2*d^6 - 441*c*d^7 - 127*d^8)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*
cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7
*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6
*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^
3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 3
4*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*
d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9
*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 3
8*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^1
1)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*
d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5
*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a
^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*
a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d
^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d
^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^1
1)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3
*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 -
3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*
a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^
5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a
^3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3
*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6
+ 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*
f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 +
10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*
c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e)), -1/30*(6
*c^8 - 12*c^7*d - 12*c^6*d^2 + 36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*
d^7 - 6*d^8 + (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2
*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - (8*c^7*d - 52*c^6*d^2 + 216*c^

```
5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^7 - 413*d^8)*co
s(f*x + e)^4 - (4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3 + 2676*c^4*d^4 +
 2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(f*x + e)^3 + 2*(4
*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 + 345*c^3*d^5 - 52
6*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c^4*d^3 + 280*c^3*
d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^...
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. 2(377) = 754.
time = 0.61, size = 794, normalized size = 2.10

$$-\frac{\frac{15\,(20\,c^2d^3+30\,cd^4+13\,d^5)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^7-3\,a^3c^6d+a^3c^5d^2+5\,a^3c^4d^3-5\,a^3c^3d^4-a^3c^2d^5+3\,a^3cd^6-a^3d^7)\sqrt{c^2-d^2}}+\frac{15\left(11\,c^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6\,c^2d^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2\,cd^7\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+10\,c^4d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6\,c^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+19\,c^2d^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12\,cd^7\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2\,d^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+29\,c^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+18\,c^2d^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\,cd^7\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+10\,c^4d^4+6\,c^3d^5-c^2d^6\right)}{(a^3c^9-3\,a^3c^8d+a^3c^7d^2+5\,a^3c^6d^3-5\,a^3c^5d^4-a^3c^4d^5+3\,a^3c^3d^6-a^3c^2d^7)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}+\frac{2\left(15\,c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-75\,cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+150\,d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30\,c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-195\,cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+525\,d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40\,c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-245\,cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+745\,d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+20\,c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-145\,cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+485\,d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7\,c^2-44\,cd+127\,d^2\right)}{(a^3c^5-5\,a^3c^4d+10\,a^3c^3d^2-10\,a^3c^2d^3+5\,a^3cd^4-a^3d^5)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7
- 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 +
 3*a^3*c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) + 15*(11*c^3*d^5*tan(1/2*f*x + 1/2
*e)^3 + 6*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^7*tan(1/2*f*x + 1/2*e)^3 +
 10*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 19*
c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 2*d^8*ta
n(1/2*f*x + 1/2*e)^2 + 29*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*c^2*d^6*tan(1/2
*f*x + 1/2*e) - 2*c*d^7*tan(1/2*f*x + 1/2*e) + 10*c^4*d^4 + 6*c^3*d^5 - c^2
*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4
 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2
*d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(15*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*c*d
*tan(1/2*f*x + 1/2*e)^4 + 150*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f
*x + 1/2*e)^3 - 195*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*d^2*tan(1/2*f*x + 1/2*
e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*c*d*tan(1/2*f*x + 1/2*e)^2 + 745
*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 145*c*d*tan(1/2
*f*x + 1/2*e) + 485*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 44*c*d + 127*d^2)/((
a^3*c^5 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3
*d^5)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

**Mupad [B]**
time = 11.65, size = 1660, normalized size = 4.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)
[Out] (d^3*atan(((d^3*(30*c*d + 20*c^2 + 13*d^2)*(2*a^3*d^8 - 6*a^3*c*d^7 - 2*a^3
*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3*c^4*d^4 - 2*a^3*c^5*d^3 +
6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11/2)) - (c*d^3*tan(e/2 + (f*
x)/2)*(30*c*d + 20*c^2 + 13*d^2)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6 - 3*a^3*c
^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2))/(a^3*(c
+ d)^(5/2)*(c - d)^(11/2)))/(30*c*d^4 + 13*d^5 + 20*c^2*d^3))*(30*c*d + 20*
c^2 + 13*d^2))/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2)) - ((90*c*d^5 - 60*c^5*d
 + 14*c^6 - 15*d^6 + 404*c^2*d^4 + 420*c^3*d^3 + 92*c^4*d^2)/(15*(c + d)^2*
(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^
7*(2*c*d^7 - 10*c^7*d + 4*c^8 - 2*d^8 + 49*c^2*d^6 + 141*c^3*d^5 + 200*c^4*
d^4 + 122*c^5*d^3 - 2*c^6*d^2))/(c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c
^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^6*(114*c*d^7 - 54*
c^7*d + 28*c^8 - 30*d^8 + 759*c^2*d^6 + 1707*c^3*d^5 + 1960*c^4*d^4 + 870*c
^5*d^3 - 62*c^6*d^2))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4
*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(270*c*d^7 - 62*c^7*d +
32*c^8 - 60*d^8 + 1857*c^2*d^6 + 3763*c^3*d^5 + 3560*c^4*d^4 + 1294*c^5*d^3
 - 70*c^6*d^2))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3
 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^8*(6*c*d^6 - 6*c^6*d + 2*c^7 - 2
*d^7 + 11*c^2*d^5 + 20*c^3*d^4 + 30*c^4*d^3 + 2*c^5*d^2))/(c*(c - d)*(2*c*d
 + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*
x)/2)^2*(30*c*d^7 - 290*c^7*d + 108*c^8 - 30*d^8 + 2501*c^2*d^6 + 8725*c^3*
d^5 + 10616*c^4*d^4 + 4810*c^5*d^3 - 10*c^6*d^2))/(15*c^2*(c + d)^2*(c - d)
*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^3*(570*
c*d^7 - 314*c^7*d + 140*c^8 - 150*d^8 + 7945*c^2*d^6 + 19441*c^3*d^5 + 1860
0*c^4*d^4 + 6898*c^5*d^3 - 210*c^6*d^2))/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4
*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(1650*c*d^6 -
614*c^6*d + 204*c^7 - 300*d^7 + 10235*c^2*d^5 + 14330*c^3*d^4 + 7254*c^4*d^
3 + 316*c^5*d^2))/(15*c^2*(c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 +
6*c^2*d^2)) + (tan(e/2 + (f*x)/2)*(195*c*d^6 - 154*c^6*d + 40*c^7 - 30*d^7
+ 1901*c^2*d^5 + 3400*c^3*d^4 + 2018*c^4*d^3 + 190*c^5*d^2))/(15*c*(c + d)^
2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(tan(e/2 + (f*x)
/2)*(5*a^3*c^2 + 4*a^3*c*d) + tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3*d^2
+ 20*a^3*c*d) + tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*c*d)
+ tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (
f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (f*x)/2)^4*(26
*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^5*(26*a^3*c^2 + 40
*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d) + a^3
*c^2 + a^3*c^2*tan(e/2 + (f*x)/2)^9))
```

# 3.480 $\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx$

**Optimal**. Leaf size=75

$$-\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))}$$

```
[Out] -1/7*(A-B)*cos(x)/(1+sin(x))^4-1/35*(3*A+4*B)*cos(x)/(1+sin(x))^3-2/105*(3*
A+4*B)*cos(x)/(1+sin(x))^2-2/105*(3*A+4*B)*cos(x)/(1+sin(x))
```

**Rubi** [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2829, 2729, 2727}

$$-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)}-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)^2}-\frac{(3A+4B)\cos(x)}{35(\sin(x)+1)^3}-\frac{(A-B)\cos(x)}{7(\sin(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```

```
[Out] -1/7*((A - B)*Cos[x])/(1 + Sin[x])^4 - ((3*A + 4*B)*Cos[x])/(35*(1 + Sin[x]
)^3) - (2*(3*A + 4*B)*Cos[x])/(105*(1 + Sin[x])^2) - (2*(3*A + 4*B)*Cos[x])
/(105*(1 + Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx &= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}+\frac{1}{7}(3A+4B)\int \frac{1}{(1+\sin(x))^3}\,dx \\
&= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}+\frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\sin(x))^2}\,dx \\
&= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}+\frac{1}{105}(2(3A+4B))\int \\
&= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))}
\end{aligned}
$$

**Mathematica** [A]
time = 0.04, size = 55, normalized size = 0.73

$$
-\frac{\cos(x)\left(36A+13B+13(3A+4B)\sin(x)+8(3A+4B)\sin^2(x)+(6A+8B)\sin^3(x)\right)}{105(1+\sin(x))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```

```
[Out] -1/105*(Cos[x]*(36*A + 13*B + 13*(3*A + 4*B)*Sin[x] + 8*(3*A + 4*B)*Sin[x]^
2 + (6*A + 8*B)*Sin[x]^3))/(1 + Sin[x])^4
```

**Maple** [A]
time = 0.16, size = 115, normalized size = 1.53

| method | result |
|---|---|
| risch | $\frac{\frac{8B\,\mathrm{e}^{4ix}}{3}+\frac{8iB\,\mathrm{e}^{3ix}}{3}-\frac{12A\,\mathrm{e}^{2ix}}{5}-\frac{4iA\,\mathrm{e}^{ix}}{5}+4iA\,\mathrm{e}^{3ix}-\frac{16B\,\mathrm{e}^{2ix}}{5}-\frac{16iB\,\mathrm{e}^{ix}}{15}+\frac{4A}{35}+\frac{16B}{105}}{\left(\mathrm{e}^{ix}+i\right)^7}$ |
| default | $-\frac{2(18A-10B)}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{-24A+24B}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^6}-\frac{2A}{\tan\left(\frac{x}{2}\right)+1}-\frac{-6A+2B}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{-32A+24B}{2\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{2(36A-32B)}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{2(8A-8B)}{7\left(\tan\left(\frac{x}{2}\right)+1\right)^7}$ |
| norman | $\frac{-2A\left(\tan^8\left(\frac{x}{2}\right)\right)+\left(-\frac{318A}{35}-\frac{362B}{105}\right)\left(\tan^2\left(\frac{x}{2}\right)\right)+\left(-\frac{102A}{5}-\frac{98B}{15}\right)\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(-\frac{74A}{5}-\frac{106B}{15}\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-\frac{14A}{5}-\frac{26B}{15}\right)\tan\left(\frac{x}{2}\right)+(-}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1+sin(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(18*A-10*B)/(tan(1/2*x)+1)^3-1/3*(-24*A+24*B)/(tan(1/2*x)+1)^6-2*A/(ta
n(1/2*x)+1)-(-6*A+2*B)/(tan(1/2*x)+1)^2-1/2*(-32*A+24*B)/(tan(1/2*x)+1)^4-2
/5*(36*A-32*B)/(tan(1/2*x)+1)^5-2/7*(8*A-8*B)/(tan(1/2*x)+1)^7
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(67) = 134$.
time = 0.29, size = 309, normalized size = 4.12

$$-\frac{2B\left(\frac{91\sin(x)}{\cos(x)+1}+\frac{168\sin(x)^2}{(\cos(x)+1)^2}+\frac{280\sin(x)^3}{(\cos(x)+1)^3}+\frac{175\sin(x)^4}{(\cos(x)+1)^4}+\frac{105\sin(x)^5}{(\cos(x)+1)^5}+13\right)}{105\left(\frac{7\sin(x)}{\cos(x)+1}+\frac{21\sin(x)^2}{(\cos(x)+1)^2}+\frac{35\sin(x)^3}{(\cos(x)+1)^3}+\frac{35\sin(x)^4}{(\cos(x)+1)^4}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{7\sin(x)^6}{(\cos(x)+1)^6}+\frac{\sin(x)^7}{(\cos(x)+1)^7}+1\right)}-\frac{2A\left(\frac{49\sin(x)}{\cos(x)+1}+\frac{147\sin(x)^2}{(\cos(x)+1)^2}+\frac{210\sin(x)^3}{(\cos(x)+1)^3}+\frac{210\sin(x)^4}{(\cos(x)+1)^4}+\frac{105\sin(x)^5}{(\cos(x)+1)^5}+\frac{35\sin(x)^6}{(\cos(x)+1)^6}+12\right)}{35\left(\frac{7\sin(x)}{\cos(x)+1}+\frac{21\sin(x)^2}{(\cos(x)+1)^2}+\frac{35\sin(x)^3}{(\cos(x)+1)^3}+\frac{35\sin(x)^4}{(\cos(x)+1)^4}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{7\sin(x)^6}{(\cos(x)+1)^6}+\frac{\sin(x)^7}{(\cos(x)+1)^7}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) + 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 + 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 + 13)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 +
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1) - 2/35*A*(49*sin(x
)/(cos(x) + 1) + 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
+ 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 + 35*sin(x)^6/(
cos(x) + 1)^6 + 12)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
) + 1)^5 + 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(67) = 134$.
time = 0.41, size = 150, normalized size = 2.00

$$\frac{2(3A+4B)\cos(x)^4+8(3A+4B)\cos(x)^3-9(3A+4B)\cos(x)^2-15(4A+3B)\cos(x)+\left(2(3A+4B)\cos(x)^3-6(3A+4B)\cos(x)^2-15(3A+4B)\cos(x)+15A-15B\right)\sin(x)-15A+15B}{105\left(\cos(x)^4-3\cos(x)^3-8\cos(x)^2-\left(\cos(x)^3+4\cos(x)^2-4\cos(x)-8\right)\sin(x)+4\cos(x)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(x)^4 + 8*(3*A + 4*B)*cos(x)^3 - 9*(3*A + 4*B)*cos(
x)^2 - 15*(4*A + 3*B)*cos(x) + (2*(3*A + 4*B)*cos(x)^3 - 6*(3*A + 4*B)*cos(
x)^2 - 15*(3*A + 4*B)*cos(x) + 15*A - 15*B)*sin(x) - 15*A + 15*B)/(cos(x)^4
 - 3*cos(x)^3 - 8*cos(x)^2 - (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
+ 4*cos(x) + 8)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(75) = 150$.
time = 3.43, size = 889, normalized size = 11.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))**4,x)
```

```
[Out] -210*A*tan(x/2)**6/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 +
3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105
```

```
) - 630*A*tan(x/2)**5/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5
 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) +
105) - 1260*A*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2
)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2
) + 105) - 1260*A*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan
(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan
(x/2) + 105) - 882*A*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*
tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*
tan(x/2) + 105) - 294*A*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*
tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*
tan(x/2) + 105) - 72*A/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**
5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) +
 105) - 210*B*tan(x/2)**5/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2
)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2
) + 105) - 350*B*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(
x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(
x/2) + 105) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*t
an(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*t
an(x/2) + 105) - 336*B*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 220
5*tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 73
5*tan(x/2) + 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 220
5*tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 73
5*tan(x/2) + 105) - 26*B/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)
**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2)
 + 105)
```

**Giac** [A]
time = 0.44, size = 112, normalized size = 1.49

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^{6}+315\,A\tan\left(\frac{1}{2}\,x\right)^{5}+105\,B\tan\left(\frac{1}{2}\,x\right)^{5}+630\,A\tan\left(\frac{1}{2}\,x\right)^{4}+175\,B\tan\left(\frac{1}{2}\,x\right)^{4}+630\,A\tan\left(\frac{1}{2}\,x\right)^{3}+280\,B\tan\left(\frac{1}{2}\,x\right)^{3}+441\,A\tan\left(\frac{1}{2}\,x\right)^{2}+168\,B\tan\left(\frac{1}{2}\,x\right)^{2}+147\,A\tan\left(\frac{1}{2}\,x\right)+91\,B\tan\left(\frac{1}{2}\,x\right)+36\,A+13\,B\right)}{105\,\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 + 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 + 175*B*tan(1/2*x)^4 + 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 + 168*B*tan(1/2*x)^2 + 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A + 13*B)/(tan(1/2*x) + 1)^7
```

**Mupad** [B]
time = 7.07, size = 94, normalized size = 1.25

$$-\frac{2\,A\,{\tan\left(\frac{x}{2}\right)}^6+\left(6\,A+2\,B\right)\,{\tan\left(\frac{x}{2}\right)}^5+\left(12\,A+\frac{10\,B}{3}\right)\,{\tan\left(\frac{x}{2}\right)}^4+\left(12\,A+\frac{16\,B}{3}\right)\,{\tan\left(\frac{x}{2}\right)}^3+\left(\frac{42\,A}{5}+\frac{16\,B}{5}\right)\,{\tan\left(\frac{x}{2}\right)}^2+\left(\frac{14\,A}{5}+\frac{26\,B}{15}\right)\,\tan\left(\frac{x}{2}\right)+\frac{24\,A}{35}+\frac{26\,B}{105}}{{\left(\tan\left(\frac{x}{2}\right)+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(x))/(sin(x) + 1)^4,x)
```

```
[Out] -((24*A)/35 + (26*B)/105 + 2*A*tan(x/2)^6 + tan(x/2)*((14*A)/5 + (26*B)/15)
 + tan(x/2)^5*(6*A + 2*B) + tan(x/2)^4*(12*A + (10*B)/3) + tan(x/2)^3*(12*A
 + (16*B)/3) + tan(x/2)^2*((42*A)/5 + (16*B)/5))/(tan(x/2) + 1)^7
```

# 3.481 $\int \frac{A+B\sin(x)}{(1-\sin(x))^4} dx$

**Optimal**. Leaf size=81

$$\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}$$

```
[Out] 1/7*(A+B)*cos(x)/(1-sin(x))^4+1/35*(3*A-4*B)*cos(x)/(1-sin(x))^3+2/105*(3*A
-4*B)*cos(x)/(1-sin(x))^2+2/105*(3*A-4*B)*cos(x)/(1-sin(x))
```

**Rubi** [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2829, 2729, 2727}

$$\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] ((A + B)*Cos[x])/(7*(1 - Sin[x])^4) + ((3*A - 4*B)*Cos[x])/(35*(1 - Sin[x])
^3) + (2*(3*A - 4*B)*Cos[x])/(105*(1 - Sin[x])^2) + (2*(3*A - 4*B)*Cos[x])/
(105*(1 - Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx &= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4}+\frac{1}{7}(3A-4B)\int \frac{1}{(1-\sin(x))^3}\,dx \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{1}{35}(2(3A-4B))\int \frac{1}{(1-\sin(x))^2}\,dx \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{1}{105}(2(3A-4B))\int \frac{}{1-} \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}
\end{aligned}
$$

**Mathematica [A]**
time = 0.05, size = 54, normalized size = 0.67

$$
\frac{\cos(x)\left(36A-13B+(-39A+52B)\sin(x)+8(3A-4B)\sin^2(x)+(-6A+8B)\sin^3(x)\right)}{105(-1+\sin(x))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] (Cos[x]*(36*A - 13*B + (-39*A + 52*B)*Sin[x] + 8*(3*A - 4*B)*Sin[x]^2 + (-6
*A + 8*B)*Sin[x]^3))/(105*(-1 + Sin[x])^4)
```

**Maple [A]**
time = 0.17, size = 115, normalized size = 1.42

| method | result |
|---|---|
| risch | $\frac{\frac{8B\,e^{4ix}}{3}-\frac{8iB\,e^{3ix}}{3}+\frac{12A\,e^{2ix}}{5}-\frac{4iA\,e^{ix}}{5}+4iA\,e^{3ix}-\frac{16B\,e^{2ix}}{5}+\frac{16iB\,e^{ix}}{15}-\frac{4A}{35}+\frac{16B}{105}}{\left(e^{ix}-i\right)^7}$ |
| default | $-\frac{2(18A+10B)}{3\left(\tan\left(\frac{x}{2}\right)-1\right)^3}-\frac{32A+24B}{2\left(\tan\left(\frac{x}{2}\right)-1\right)^4}-\frac{24A+24B}{3\left(\tan\left(\frac{x}{2}\right)-1\right)^6}-\frac{6A+2B}{\left(\tan\left(\frac{x}{2}\right)-1\right)^2}-\frac{2(8A+8B)}{7\left(\tan\left(\frac{x}{2}\right)-1\right)^7}-\frac{2A}{\tan\left(\frac{x}{2}\right)-1}-\frac{2(36A+32B)}{5\left(\tan\left(\frac{x}{2}\right)-1\right)^5}$ |
| norman | $\frac{-2A\left(\tan^8\left(\frac{x}{2}\right)\right)+\left(-\frac{318A}{35}+\frac{362B}{105}\right)\left(\tan^2\left(\frac{x}{2}\right)\right)+\left(-\frac{102A}{5}+\frac{98B}{15}\right)\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(\frac{14A}{5}-\frac{26B}{15}\right)\tan\left(\frac{x}{2}\right)+\left(\frac{74A}{5}-\frac{106B}{15}\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-14A+\right.}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\left(\tan\left(\frac{x}{2}\right)-1\right)^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1-sin(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(18*A+10*B)/(tan(1/2*x)-1)^3-1/2*(32*A+24*B)/(tan(1/2*x)-1)^4-1/3*(24*
A+24*B)/(tan(1/2*x)-1)^6-(6*A+2*B)/(tan(1/2*x)-1)^2-2/7*(8*A+8*B)/(tan(1/2*
x)-1)^7-2*A/(tan(1/2*x)-1)-2/5*(36*A+32*B)/(tan(1/2*x)-1)^5
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(65) = 130$.
time = 0.29, size = 309, normalized size = 3.81

$$-\frac{2B\left(\frac{91\sin(x)}{\cos(x)+1} - \frac{168\sin(x)^2}{(\cos(x)+1)^2} + \frac{280\sin(x)^3}{(\cos(x)+1)^3} - \frac{175\sin(x)^4}{(\cos(x)+1)^4} + \frac{105\sin(x)^5}{(\cos(x)+1)^5} - 13\right)}{105\left(\frac{7\sin(x)}{\cos(x)+1} - \frac{21\sin(x)^2}{(\cos(x)+1)^2} + \frac{35\sin(x)^3}{(\cos(x)+1)^3} - \frac{35\sin(x)^4}{(\cos(x)+1)^4} + \frac{21\sin(x)^5}{(\cos(x)+1)^5} - \frac{7\sin(x)^6}{(\cos(x)+1)^6} + \frac{\sin(x)^7}{(\cos(x)+1)^7} - 1\right)} + \frac{2A\left(\frac{49\sin(x)}{\cos(x)+1} - \frac{147\sin(x)^2}{(\cos(x)+1)^2} + \frac{210\sin(x)^3}{(\cos(x)+1)^3} - \frac{210\sin(x)^4}{(\cos(x)+1)^4} + \frac{105\sin(x)^5}{(\cos(x)+1)^5} - \frac{35\sin(x)^6}{(\cos(x)+1)^6} - 12\right)}{35\left(\frac{7\sin(x)}{\cos(x)+1} - \frac{21\sin(x)^2}{(\cos(x)+1)^2} + \frac{35\sin(x)^3}{(\cos(x)+1)^3} - \frac{35\sin(x)^4}{(\cos(x)+1)^4} + \frac{21\sin(x)^5}{(\cos(x)+1)^5} - \frac{7\sin(x)^6}{(\cos(x)+1)^6} + \frac{\sin(x)^7}{(\cos(x)+1)^7} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) - 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 - 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 - 13)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 -
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1) + 2/35*A*(49*sin(x
)/(cos(x) + 1) - 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
- 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 - 35*sin(x)^6/(
cos(x) + 1)^6 - 12)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
) + 1)^5 - 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.
time = 0.33, size = 150, normalized size = 1.85

$$-\frac{2(3A-4B)\cos(x)^4 + 8(3A-4B)\cos(x)^3 - 9(3A-4B)\cos(x)^2 - 15(4A-3B)\cos(x) - (2(3A-4B)\cos(x)^3 - 6(3A-4B)\cos(x)^2 - 15(3A-4B)\cos(x) + 15A + 15B)\sin(x) - 15A - 15B}{105\left(\cos(x)^4 - 3\cos(x)^3 - 8\cos(x)^2 + (\cos(x)^3 + 4\cos(x)^2 - 4\cos(x) - 8)\sin(x) + 4\cos(x) + 8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(3*A - 4*B)*cos(x)^4 + 8*(3*A - 4*B)*cos(x)^3 - 9*(3*A - 4*B)*cos
(x)^2 - 15*(4*A - 3*B)*cos(x) - (2*(3*A - 4*B)*cos(x)^3 - 6*(3*A - 4*B)*cos
(x)^2 - 15*(3*A - 4*B)*cos(x) + 15*A + 15*B)*sin(x) - 15*A - 15*B)/(cos(x)^
4 - 3*cos(x)^3 - 8*cos(x)^2 + (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
 + 4*cos(x) + 8)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(75) = 150$.
time = 3.36, size = 887, normalized size = 10.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))**4,x)
```

```
[Out] -210*A*tan(x/2)**6/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 -
3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105
```

```
) + 630*A*tan(x/2)**5/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5
 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) -
105) - 1260*A*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2
)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2
) - 105) + 1260*A*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan
(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan
(x/2) - 105) - 882*A*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*
tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*
tan(x/2) - 105) + 294*A*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*
tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*
tan(x/2) - 105) - 72*A/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**
5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) -
 105) - 210*B*tan(x/2)**5/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2
)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2
) - 105) + 350*B*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(
x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(
x/2) - 105) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*t
an(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*t
an(x/2) - 105) + 336*B*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 220
5*tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 73
5*tan(x/2) - 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 220
5*tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 73
5*tan(x/2) - 105) + 26*B/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)
**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2)
 - 105)
```

**Giac** [A]
time = 0.51, size = 112, normalized size = 1.38

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^{6}-315\,A\tan\left(\frac{1}{2}\,x\right)^{5}+105\,B\tan\left(\frac{1}{2}\,x\right)^{5}+630\,A\tan\left(\frac{1}{2}\,x\right)^{4}-175\,B\tan\left(\frac{1}{2}\,x\right)^{4}-630\,A\tan\left(\frac{1}{2}\,x\right)^{3}+280\,B\tan\left(\frac{1}{2}\,x\right)^{3}+441\,A\tan\left(\frac{1}{2}\,x\right)^{2}-168\,B\tan\left(\frac{1}{2}\,x\right)^{2}-147\,A\tan\left(\frac{1}{2}\,x\right)+91\,B\tan\left(\frac{1}{2}\,x\right)+36\,A-13\,B\right)}{105\,\left(\tan\left(\frac{1}{2}\,x\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 - 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 - 175*B*tan(1/2*x)^4 - 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 - 168*B*tan(1/2*x)^2 - 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A - 13*B)/(tan(1/2*x) - 1)^7
```

**Mupad** [B]
time = 7.06, size = 97, normalized size = 1.20

$$-\frac{2\,A\,{\mathrm{tan}\left(\frac{x}{2}\right)}^6+\left(2\,B-6\,A\right)\,{\mathrm{tan}\left(\frac{x}{2}\right)}^5+\left(12\,A-\frac{10\,B}{3}\right)\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4+\left(\frac{16\,B}{3}-12\,A\right)\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3+\left(\frac{42\,A}{5}-\frac{16\,B}{5}\right)\,{\mathrm{tan}\left(\frac{x}{2}\right)}^2+\left(\frac{26\,B}{15}-\frac{14\,A}{5}\right)\,\mathrm{tan}\left(\frac{x}{2}\right)+\frac{24\,A}{35}-\frac{26\,B}{105}}{{\left(\mathrm{tan}\left(\frac{x}{2}\right)-1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(x))/(sin(x) - 1)^4,x)
```

```
[Out] -((24*A)/35 - (26*B)/105 + 2*A*tan(x/2)^6 - tan(x/2)*((14*A)/5 - (26*B)/15)
 - tan(x/2)^5*(6*A - 2*B) + tan(x/2)^4*(12*A - (10*B)/3) - tan(x/2)^3*(12*A
 - (16*B)/3) + tan(x/2)^2*((42*A)/5 - (16*B)/5))/(tan(x/2) - 1)^7
```

## 3.482 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=290

$$-\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5c+7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-$$

```
[Out] -2/35*a*(5*c+7*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/7*a*cos(f*x+e)*(c+d
*sin(f*x+e))^(5/2)/f-2/105*a*(15*c^2+56*c*d+25*d^2)*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/f-2/105*a*(15*c^3+161*c^2*d+145*c*d^2+63*d^3)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/105*a*(c^2-d^2)*(15*c^2+56*c*d+25*d^2)*(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+
e))^(1/2)
```

**Rubi** [A]
time = 0.31, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(c^2-d^2)(15c^2+56cd+25d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105df\sqrt{c+d\sin(e+fx)}}+\frac{2a(15c^3+161c^2d+145cd^2+63d^3)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}-\frac{2a(5c+7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*(15*c^2 + 56*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(10
5*f) - (2*a*(5*c + 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*f) - (
2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*a*(15*c^3 + 161*c^2
*d + 145*c*d^2 + 63*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c
^2 - d^2)*(15*c^2 + 56*c*d + 25*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}+\frac{2}{7}\int (c+d\sin(e+$$

$$= -\frac{2a(5c+7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-\frac{2a\cos(e$$

$$= -\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2a(15c^2+56cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.54, size = 3531, normalized size = 12.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*((c^3*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
```

]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(7*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (23*c^2
*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
 - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
(c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sq
rt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[
e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^
2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*S
in[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]]))/(15*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (29*c*d^2*S
ec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c +
 d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]
^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*C
sc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[
1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2
] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
 c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]
)) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e
]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt
[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]]))/(21*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^3*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*(-1/210*((180*c^2 + 308*c*d + 115*d^2)*Cos[e]*Cos[f*x
])/f + (d^2*Cos[3*e]*Cos[3*f*x])/(14*f) - (d*(15*c + 7*d)*Cos[2*f*x]*Sin[2*
e])/(35*f) + ((180*c^2 + 308*c*d + 115*d^2)*Sin[e]*Sin[f*x])/(210*f) - (d*(
15*c + 7*d)*Cos[2*e]*Sin[2*f*x])/(35*f) - (d^2*Sin[3*e]*Sin[3*f*x])/(14*f)

```
+ (2*(15*c^3 + 161*c^2*d + 145*c*d^2 + 63*d^3)*Tan[e])/(105*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (18*c^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*S
qrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c +
 d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]
^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan
[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[
1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]
) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt
[1 + Tan[e]^2]])/(7*f*(Cos[e/2 + (f*x)/2] + Sin...
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(332) = 664$.

time = 4.82, size = 1315, normalized size = 4.53

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1315 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*a*(60*c*d^4*sin(f*x+e)^4+90*c^2*d^3*sin(f*x+e)^3+98*c*d^4*sin(f*x+e)^
3+45*c^3*d^2*sin(f*x+e)^2+77*c^2*d^3*sin(f*x+e)^2-35*c*d^4*sin(f*x+e)^2-90*
c^2*d^3*sin(f*x+e)-98*c*d^4*sin(f*x+e)+98*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+145*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+1
5*d^5*sin(f*x+e)^5+21*d^5*sin(f*x+e)^4+10*d^5*sin(f*x+e)^3-21*d^5*sin(f*x+e
)^2-25*d^5*sin(f*x+e)-77*c^2*d^3-25*c*d^4-45*c^3*d^2+63*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-88*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*d^5-15*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^5-32*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c
+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3-176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4-161*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1
/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d+120
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
```

```
f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*c^4*d+176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^
(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))*c^3*d^2-130*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(
f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2)/d^2/cos(f*x+e)/(c+d*si
n(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 592, normalized size = 2.04

$$-\frac{\sqrt{2}\,(30\,ac^4+7\,ac^3d-115\,ac^2d^2-231\,acd^3-75\,ad^4)\sqrt{i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)-3i\,d\sin(fx+e)-2i\,c}{3\,d}\right)+\sqrt{2}\,(30\,ac^4+7\,ac^3d-115\,ac^2d^2-231\,acd^3-75\,ad^4)\sqrt{-i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)+3i\,d\sin(fx+e)+2i\,c}{3\,d}\right)+3\sqrt{2}\,(15i\,ac^3d+161i\,ac^2d^2+145i\,acd^3+63i\,ad^4)\sqrt{i\,d}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)-3i\,d\sin(fx+e)-2i\,c}{3\,d}\right)\right)+3\sqrt{2}\,(-15i\,ac^3d-161i\,ac^2d^2-145i\,acd^3-63i\,ad^4)\sqrt{-i\,d}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)+3i\,d\sin(fx+e)+2i\,c}{3\,d}\right)\right)-6\,(15\,ad^4\cos(fx+e)^3-3\,(15\,acd^3+7\,ad^4)\cos(fx+e)\sin(fx+e)-(45\,ac^2d^2+77\,acd^3+40\,ad^4)\cos(fx+e))\sqrt{d\sin(fx+e)+c}}{315\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(sqrt(2)*(30*a*c^4 + 7*a*c^3*d - 115*a*c^2*d^2 - 231*a*c*d^3 - 75*a*
d^4)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3
 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) +
 sqrt(2)*(30*a*c^4 + 7*a*c^3*d - 115*a*c^2*d^2 - 231*a*c*d^3 - 75*a*d^4)*sq
rt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*
I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sq
rt(2)*(15*I*a*c^3*d + 161*I*a*c^2*d^2 + 145*I*a*c*d^3 + 63*I*a*d^4)*sqrt(I*
d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^
3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2
)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*
(-15*I*a*c^3*d - 161*I*a*c^2*d^2 - 145*I*a*c*d^3 - 63*I*a*d^4)*sqrt(-I*d)*w
eierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/
d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(15*a*d^4*
cos(f*x + e)^3 - 3*(15*a*c*d^3 + 7*a*d^4)*cos(f*x + e)*sin(f*x + e) - (45*a
*c^2*d^2 + 77*a*c*d^3 + 40*a*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(
d^2*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^2\sqrt{c+d\sin(e+fx)}\,dx+\int c^2\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int d^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx+\int d^2\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(c + d*sin(e + f*x)), x) + Integral(c**2*sqrt(c + d*si
n(e + f*x))*sin(e + f*x), x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e
 + f*x)**2, x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x)
 + Integral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(2*c*
d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\,\sin\left(e + f\,x\right))\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.483 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=231

$$-\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}+\frac{2a(3c^2+20cd+9d^2$$

```
[Out] -2/5*a*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/15*a*(3*c+5*d)*cos(f*x+e)*(c+d
*sin(f*x+e))^(1/2)/f-2/15*a*(3*c^2+20*c*d+9*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/(c+d))^(
1/2)+2/15*a*(3*c+5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2
*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1
/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a(3c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2a\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}-\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*(3*c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*a*Cos[
e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*a*(3*c^2 + 20*c*d + 9*d^2)*
EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*
d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(3*c + 5*d)*(c^2 - d^2)*Elli
pticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f} + \frac{2}{5}\int \sqrt{c+d\sin(}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2a\cos(e}{}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2a\cos(e}{}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2a\cos(e}{}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2a\cos(e}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.34, size = 2625, normalized size = 11.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*((c^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (4*c*d*
Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c
```

+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e
]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x -
ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*
Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[
1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^
2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2]
+ c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^2*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*(6*c + 5*d)*Cos[e]*Cos[f*x])/(15*f) - (d*Cos[2*f
*x]*Sin[2*e])/(5*f) + (2*(6*c + 5*d)*Sin[e]*Sin[f*x])/(15*f) - (d*Cos[2*e]*
Sin[2*f*x])/(5*f) + (2*(3*c^2 + 20*c*d + 9*d^2)*Tan[e])/(15*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*
Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]
*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]
]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e
]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 +
 Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) +
 d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]])/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan
[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec
[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[
1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt
[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c
*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + A

```
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/
2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*d*AppellF1[1/
2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))
)), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/
(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Se
c[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt...
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(277) = 554$.
time = 5.10, size = 1034, normalized size = 4.48

| method | result |
|---|---|
| default | $\frac{2a\left(18\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\,\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\,\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\text{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3d+14c^2\sqrt{\vphantom{x}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a*(18*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*c^3*d+14*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-18*c*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-14*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-3*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*
x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))*c^4-20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^3*d-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2+20*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+9*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4+3*d
^4*sin(f*x+e)^4+9*c*d^3*sin(f*x+e)^3+5*d^4*sin(f*x+e)^3+6*c^2*d^2*sin(f*x+e
)^2+5*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2-9*c*d^3*sin(f*x+e)-5*d^4*sin(f*
x+e)-6*c^2*d^2-5*d^3*c)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 521, normalized size = 2.26

$$-\frac{\sqrt{2}\left(6ac^3-5ac^2d-18acd^2-15ad^3\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}\left(6ac^3-5ac^2d-18acd^2-15ad^3\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)+3\sqrt{2}\left(3iac^2d+20iacd^2+9iad^3\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3\sqrt{2}\left(-3iac^2d-20iacd^2-9iad^3\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)+6\left(3ad^3\cos(fx+e)\sin(fx+e)+\left(6acd^2+5ad^3\right)\cos(fx+e)\right)\sqrt{d\sin(fx+e)+c}}{45\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(sqrt(2)*(6*a*c^3 - 5*a*c^2*d - 18*a*c*d^2 - 15*a*d^3)*sqrt(I*d)*weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(6*a*c^3 -
 5*a*c^2*d - 18*a*c*d^2 - 15*a*d^3)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(3*I*a*c^2*d + 20*I*a*c*d^2 + 9
*I*a*d^3)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*
I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)
/d)) + 3*sqrt(2)*(-3*I*a*c^2*d - 20*I*a*c*d^2 - 9*I*a*d^3)*sqrt(-I*d)*weier
strassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(3*a*d^3*cos(f
*x + e)*sin(f*x + e) + (6*a*c*d^2 + 5*a*d^3)*cos(f*x + e))*sqrt(d*sin(f*x +
 e) + c))/(d^2*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{c+d\sin(e+fx)}\,dx+\int c\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int d\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int d\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(c*sqrt(c + d*sin(e +
f*x))*sin(e + f*x), x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x),
x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x))
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x)) (c + d \sin(e + f x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.484 $\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=179

$$-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c^2-d^2)}{}$$

```
[Out] -2/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f-2/3*a*(c+3*d)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/3*a*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+
1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2)
)*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*a*(c + 3*d)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sqrt
[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c + d*
Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

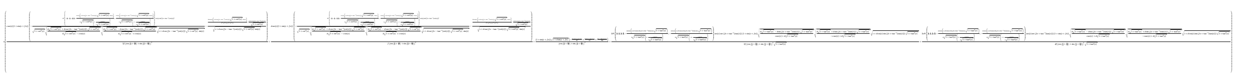
$$\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{1}{2}a(3c+d)+\frac{1}{2}\cdots}{\sqrt{c+d\cdots}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{(a(c+3d))\int\sqrt{c+\cdots}}{3d\cdots}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{\left(a(c+3d)\sqrt{c+d\,\mathrm{si}\cdots}\right.}{\cdots}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2a(c+3d)E\left(\frac{1}{2}(e-\cdots\right.}{3df\sqrt{\cdots}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.21, size = 1736, normalized size = 9.70



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]
])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]
*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan
```

```
[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e]
)/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Co
t[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + C
ot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d
*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Cs
c[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) -
((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(
d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 +
Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])
)/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*Cos[e]*Cos[f*x])/(3*f) + (2*Sin[e]*Sin[f*x])/(3*
f) + (2*(c + 3*d)*Tan[e])/(3*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
])^2 + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + Ar
cTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d
*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*
Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan
[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqr
t[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e]
 + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*f*(Cos[e/2 + (f*
x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2,
 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e
]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x +
 ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/
(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(229) = 458$.

time = 4.68, size = 657, normalized size = 3.67

| method | result |
|---|---|
| default | $\frac{2a\left(4\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\,\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\,\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\;\text{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^2d-4\sqrt{\frac{c+c}{}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*(4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^2*d-4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(
c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*
x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3-3*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*d^3+d^3*sin(f*x+e)^3+c*d^2*sin(f*x+e)^2-d^3*sin(f*x+e)-c*d^2)/d^2/c
os(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 453, normalized size = 2.53

$$-\frac{6\,\sqrt{d\sin\left(fx+e\right)+c}\,ad^{2}\cos\left(fx+e\right)+\sqrt{2}\,\left(2\,ac^{2}-3\,acd-3\,ad^{2}\right)\sqrt{i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(8i\,c^{3}-9i\,cd^{2}\right)}{27\,d^{3}},\frac{3\,d\cos\left(fx+e\right)-3i\,d\sin\left(fx+e\right)-2i\,c}{3\,d}\right)+\sqrt{2}\,\left(2\,ac^{2}-3\,acd-3\,ad^{2}\right)\sqrt{-i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(-8i\,c^{3}+9i\,cd^{2}\right)}{27\,d^{3}},\frac{3\,d\cos\left(fx+e\right)+3i\,d\sin\left(fx+e\right)+2i\,c}{3\,d}\right)+3\,\sqrt{2}\,\left(i\,acd+3i\,ad^{2}\right)\sqrt{i\,d}\,\mathrm{weierstrassZeta}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(8i\,c^{3}-9i\,cd^{2}\right)}{27\,d^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(8i\,c^{3}-9i\,cd^{2}\right)}{27\,d^{3}},\frac{3\,d\cos\left(fx+e\right)-3i\,d\sin\left(fx+e\right)-2i\,c}{3\,d}\right)\right)+3\,\sqrt{2}\,\left(-i\,acd-3i\,ad^{2}\right)\sqrt{-i\,d}\,\mathrm{weierstrassZeta}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(-8i\,c^{3}+9i\,cd^{2}\right)}{27\,d^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,\left(4\,c^{2}-3\,d^{2}\right)}{3\,d^{2}},-\frac{8\,\left(-8i\,c^{3}+9i\,cd^{2}\right)}{27\,d^{3}},\frac{3\,d\cos\left(fx+e\right)+3i\,d\sin\left(fx+e\right)+2i\,c}{3\,d}\right)\right)}{9\,d^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*sqrt(d*sin(f*x + e) + c)*a*d^2*cos(f*x + e) + sqrt(2)*(2*a*c^2 - 3*
a*c*d - 3*a*d^2)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) -
 2*I*c)/d) + sqrt(2)*(2*a*c^2 - 3*a*c*d - 3*a*d^2)*sqrt(-I*d)*weierstrassPI
nverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d
*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(I*a*c*d + 3*I*a
*d^2)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d))
 + 3*sqrt(2)*(-I*a*c*d - 3*I*a*d^2)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4
```

```
*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e)
+ 3*I*d*sin(f*x + e) + 2*I*c)/d)))/(d^2*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
[Out] a*(Integral(sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(c + d
*sin(e + f*x)), x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.485 $\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=138

$$\frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/d/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2*a*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)
^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2
)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2
)
```

**Rubi** [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2a\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*a*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c - d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{a\int\sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-ac+ad)\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-ac+ad)\sqrt{\frac{c-}{}}\right.}{}$$

$$= \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2a(c-d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right.}{df\sqrt{c+d\,\mathrm{si}}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 6.19, size = 880, normalized size = 6.38

$$a\left(\frac{\sec(e)(1+\sin(e+fx))\left(-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}\,\sin(e)\right)}{d\sqrt{1+\cot^2(e)}\left(1-\frac{c\csc(e)}{d\sqrt{1+\cot^2(e)}}\right)},-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}\,\sin(e)\right)}{d\sqrt{1+\cot^2(e)}\left(-1-\frac{c\csc(e)}{d\sqrt{1+\cot^2(e)}}\right)}\right)\cot(e)\sin\left(fx-\tan^{-1}(\cot(e))\right)}{\sqrt{1+\cot^2(e)}\sqrt{\frac{d\sqrt{1+\cot^2(e)}+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}}{d\sqrt{1+\cot^2(e)}-c\csc(e)}}\sqrt{\frac{d\sqrt{1+\cot^2(e)}-d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}}{d\sqrt{1+\cot^2(e)}+c\csc(e)}}\sqrt{c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}\,\sin(e)}}-\frac{\frac{2d\sin(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}\,\sin(e)\right)}{d^2\cos^2(e)+d^2\sin^2(e)}-\frac{\cot(e)\sin\left(fx-\tan^{-1}(\cot(e))\right)}{\sqrt{1+\cot^2(e)}}}{\sqrt{c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{1+\cot^2(e)}\,\sin(e)}}\right)}{f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2}+\frac{2(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,\tan(e)}{df\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2}+\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};-\frac{\sec(e)\left(c+d\cos(e)\sin\left(fx+\tan^{-1}(\tan(e))\right)\sqrt{1+\tan^2(e)}\right)}{d\sqrt{1+\tan^2(e)}\left(1-\frac{c\sec(e)}{d\sqrt{1+\tan^2(e)}}\right)},-\frac{\sec(e)\left(c+d\cos(e)\sin\left(fx+\tan^{-1}(\tan(e))\right)\sqrt{1+\tan^2(e)}\right)}{d\sqrt{1+\tan^2(e)}\left(-1-\frac{c\sec(e)}{d\sqrt{1+\tan^2(e)}}\right)}\right)\sec(e)\sec\left(fx+\tan^{-1}(\tan(e))\right)(1+\sin(e+fx))\sqrt{\frac{d\sqrt{1+\tan^2(e)}-d\sin\left(fx+\tan^{-1}(\tan(e))\right)\sqrt{1+\tan^2(e)}}{c\sec(e)+d\sqrt{1+\tan^2(e)}}}\sqrt{\frac{d\sqrt{1+\tan^2(e)}+d\sin\left(fx+\tan^{-1}(\tan(e))\right)\sqrt{1+\tan^2(e)}}{-c\sec(e)+d\sqrt{1+\tan^2(e)}}}\sqrt{c+d\cos(e)\sin\left(fx+\tan^{-1}(\tan(e))\right)\sqrt{1+\tan^2(e)}}}{df\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2\sqrt{1+\tan^2(e)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])
/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]]))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]*Tan[e])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*
Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (
c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcT
an[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*
Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])
*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2
])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2]
)]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

**Maple** [A]
time = 5.34, size = 203, normalized size = 1.47

| method | result |
|---|---|
| default | $-\frac{2a(c-d)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\left(\text{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c+\text{Ellipt}\right.}{d^2\cos(fx+e)\sqrt{c+d\sin(fx+e)}\,f}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*d-2*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d
)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 391, normalized size = 2.83

$$\frac{-3i\sqrt{2}\,a\sqrt{i\,d}\,d\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)-3i\,d\sin(fx+e)-2i\,c}{3\,d}\right)\right)+3i\sqrt{2}\,a\sqrt{-i\,d}\,d\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)+3i\,d\sin(fx+e)+2i\,c}{3\,d}\right)\right)-\sqrt{2}\,(2\,ac-3\,ad)\sqrt{i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(8i\,c^3-9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)-3i\,d\sin(fx+e)-2i\,c}{3\,d}\right)-\sqrt{2}\,(2\,ac-3\,ad)\sqrt{-i\,d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,c^2-3\,d^2)}{3\,d^2},-\frac{8\,(-8i\,c^3+9i\,cd^2)}{27\,d^3},\frac{3\,d\cos(fx+e)+3i\,d\sin(fx+e)+2i\,c}{3\,d}\right)}{3\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*sqrt(2)*a*sqrt(I*d)*d*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
 e) - 2*I*c)/d)) + 3*I*sqrt(2)*a*sqrt(-I*d)*d*weierstrassZeta(-4/3*(4*c^2 -
 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d)) - sqrt(2)*(2*a*c - 3*a*d)*sqrt(I*d)*weiers
trassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/
3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) - sqrt(2)*(2*a*c - 3*a
*d)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^
3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d))
/(d^2*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{\sin{\left(e + f x \right)}}{\sqrt{c + d \sin{\left(e + f x \right)}}}\, dx + \int \frac{1}{\sqrt{c + d \sin{\left(e + f x \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c +
 d*sin(e + f*x)), x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [B]**
time = 7.62, size = 176, normalized size = 1.28

$$\frac{a\left(2\,c\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)-2\,\left(c+d\right)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{c+d\,\sin\left(e+f\,x\right)}{c+d}}}{d\,f\,\cos\left(e+f\,x\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}-\frac{2\,a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|\frac{2\,d}{c+d}\right)\,\sqrt{\frac{c+d\,\sin\left(e+f\,x\right)}{c+d}}}{f\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] (a*(2*c*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(c + d)
) - 2*(c + d)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(
c + d)))*(cos(e + f*x)^2)^(1/2)*((c + d*sin(e + f*x))/(c + d))^(1/2))/(d*f*
cos(e + f*x)*(c + d*sin(e + f*x))^(1/2)) - (2*a*ellipticF(pi/4 - e/2 - (f*x
)/2, (2*d)/(c + d))*((c + d*sin(e + f*x))/(c + d))^(1/2))/(f*(c + d*sin(e +
 f*x))^(1/2))
```

# 3.486 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=169

$$-\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}}-\frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin}}$$

```
[Out] -2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(1/2)+2*a*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c+d)/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(
f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}-\frac{2a\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d*(c + d)*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}a(c-d)+\frac{1}{2}a(c-d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d} - \frac{a\int \sqrt{c+d}}{d(}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin}{}}}{d(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{d(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.29, size = 938, normalized size = 5.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*Csc[e]*Sec[e])/(d*(c +
 d)*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x
]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (Sec[e]*(1 + Sin[e + f*
x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(
d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Co
t[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*
Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqr
t[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTa
n[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c
+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d
```

```
^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[
c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c + d)*f*(Co
s[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2,
 -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*
Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c
+ d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e
]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Ta
n[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt
[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e
]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqr
t[1 + Tan[e]^2]])/(d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*
Sqrt[1 + Tan[e]^2]))
```

**Maple** [**A**]
time = 3.91, size = 246, normalized size = 1.46

| method | result |
|---|---|
| default | $\frac{2\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\,\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\,\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\text{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^2-\sqrt{\frac{c+d\sin(fx}{c-d}}\right.}{d^2(c+d)\cos(fx+e)\sqrt{c+}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c^2-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*d^2+d^2*sin(f*x+e)^2-d^2)/d^2*a/(c+d)/cos(f*x+e)/(c+d*
sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 543, normalized size = 3.21

$$-\frac{6\sqrt{d\sin(fx+e)+c}\,ad^2\cos(fx+e)-\left(\sqrt{2}\,(2acd+3ad^2)\sin(fx+e)+\sqrt{2}\,(2ac^2+3acd)\right)\sqrt{id}\,\text{weierstrassPInverse}([illegible])-\left(\sqrt{2}\,(2acd+3ad^2)\sin(fx+e)+\sqrt{2}\,(2ac^2+3acd)\right)\sqrt{-id}\,\text{weierstrassPInverse}([illegible])+3\left(-i\sqrt{2}\,ad^2\sin(fx+e)-i\sqrt{2}\,acd\right)\sqrt{id}\,\text{weierstrassZeta}([illegible],\text{weierstrassPInverse}([illegible]))+3\left(i\sqrt{2}\,ad^2\sin(fx+e)+i\sqrt{2}\,acd\right)\sqrt{-id}\,\text{weierstrassZeta}([illegible],\text{weierstrassPInverse}([illegible]))}{3\left((cd^3+d^4)f\sin(fx+e)+(c^2d^2+cd^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(d*sin(f*x + e) + c)*a*d^2*cos(f*x + e) - (sqrt(2)*(2*a*c*d + 3
*a*d^2)*sin(f*x + e) + sqrt(2)*(2*a*c^2 + 3*a*c*d))*sqrt(I*d)*weierstrassPI
nverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*
cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) - (sqrt(2)*(2*a*c*d + 3*a*d^2
)*sin(f*x + e) + sqrt(2)*(2*a*c^2 + 3*a*c*d))*sqrt(-I*d)*weierstrassPInvers
e(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(
f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(-I*sqrt(2)*a*d^2*sin(f*x + e
) - I*sqrt(2)*a*c*d)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
e) - 2*I*c)/d)) + 3*(I*sqrt(2)*a*d^2*sin(f*x + e) + I*sqrt(2)*a*c*d)*sqrt(-
I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)
/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c
*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)))/((c*d^3
 + d^4)*f*sin(f*x + e) + (c^2*d^2 + c*d^3)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\,\sin\left(e + f\,x\right)}{\left(c + d\,\sin\left(e + f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.487 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=237

$$-\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}}-\frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}}-\frac{2a(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{3(c-d)d(c+d)^2f\sqrt{\frac{c+}{}}}$$

```
[Out] -2/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-2/3*a*(c-3*d)*cos(f*x+e)/(
c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+2/3*a*(c-3*d)*(sin(1/2*e+1/4*Pi+1/2*f
*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),
2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)/d/(c+d)^2/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-2/3*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a(c-3d)\cos(e+fx)}{3f(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{2a(c-3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c-d)(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (2*a*(c - 3*
d)*Cos[e + f*x])/(3*(c - d)*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*(c
 - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x
]])/(3*(c - d)*d*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(3*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}a(c-d)-\frac{1}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3(c^2-d^2)}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} +$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} -$$

<u>**Mathematica**</u> [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.56, size = 1870, normalized size = 7.89

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(c - 3*d)*Csc[e]*Sec[e
])/(3*(c - d)*d*(c + d)^2*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(3*d*(c +
 d)*f*(c + d*Sin[e + f*x])^2) - (2*Csc[e]*(3*c*Cos[e] - d*Cos[e] - c*Sin[f*
x] + 3*d*Sin[f*x]))/(3*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c*Sec[e]*(1 + Sin[e + f*x])*(-((Appe
llF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + C
ot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]
*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*S
```

qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(3*(c - d)*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c - d)*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) - (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*(c - d)*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/((c - d)*d*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(283) = 566$.
time = 16.19, size = 884, normalized size = 3.73

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\, a\left(\frac{\left(-c+d\right)\left(\frac{2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{3\left(c^{2}-d^{2}\right)d\left(\sin\left(fx+e\right)+\frac{c}{d}\right)^{2}}+\frac{}{3(}\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*((-c+d)/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 942, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(2*a*c^2*d^2 + 3*a*c*d^3 - 3*a*d^4)*cos(f*x + e)^2 - 2*sqrt(2
)*(2*a*c^3*d + 3*a*c^2*d^2 - 3*a*c*d^3)*sin(f*x + e) - sqrt(2)*(2*a*c^4 + 3
*a*c^3*d - a*c^2*d^2 + 3*a*c*d^3 - 3*a*d^4))*sqrt(I*d)*weierstrassPInverse(
-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x
 + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(2*a*c^2*d^2 + 3*a*c*d^3
- 3*a*d^4)*cos(f*x + e)^2 - 2*sqrt(2)*(2*a*c^3*d + 3*a*c^2*d^2 - 3*a*c*d^3)
*sin(f*x + e) - sqrt(2)*(2*a*c^4 + 3*a*c^3*d - a*c^2*d^2 + 3*a*c*d^3 - 3*a*
d^4))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*
c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d
) - 3*(sqrt(2)*(-I*a*c*d^3 + 3*I*a*d^4)*cos(f*x + e)^2 + 2*sqrt(2)*(I*a*c^2
*d^2 - 3*I*a*c*d^3)*sin(f*x + e) + sqrt(2)*(I*a*c^3*d - 3*I*a*c^2*d^2 + I*a
*c*d^3 - 3*I*a*d^4))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
e) - 2*I*c)/d)) - 3*(sqrt(2)*(I*a*c*d^3 - 3*I*a*d^4)*cos(f*x + e)^2 + 2*sqr
t(2)*(-I*a*c^2*d^2 + 3*I*a*c*d^3)*sin(f*x + e) + sqrt(2)*(-I*a*c^3*d + 3*I*
a*c^2*d^2 - I*a*c*d^3 + 3*I*a*d^4))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4
*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e)
+ 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*((a*c*d^3 - 3*a*d^4)*cos(f*x + e)*sin
(f*x + e) + (2*a*c^2*d^2 - 3*a*c*d^3 - a*d^4)*cos(f*x + e))*sqrt(d*sin(f*x
+ e) + c))/((c^3*d^4 + c^2*d^5 - c*d^6 - d^7)*f*cos(f*x + e)^2 - 2*(c^4*d^3
 + c^3*d^4 - c^2*d^5 - c*d^6)*f*sin(f*x + e) - (c^5*d^2 + c^4*d^3 - c*d^6 -
 d^7)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + a\sin(e + fx)}{(c + d\sin(e + fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.488 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=318

$$-\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}}-\frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}-\frac{2a(3c^2-20cd+9d^2)\cos(}{15(c-d)^2(c+d)^3f\sqrt{c+d\,\mathrm{si}}}$$

```
[Out] -2/5*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(5/2)-2/15*a*(3*c-5*d)*cos(f*x+e
)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-2/15*a*(3*c^2-20*c*d+9*d^2)*cos(f*
x+e)/(c-d)^2/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+2/15*a*(3*c^2-20*c*d+9*d^2)*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)
^2/d/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/15*a*(3*c-5*d)*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(c-d)/d/
(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.34, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2a(3c^2-20cd+9d^2)\cos(e+fx)}{15f(c-d)^2(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{2a(3c^2-20cd+9d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)^2(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(3c-5d)\cos(e+fx)}{15f(c-d)(c+d)^2(c+d\sin(e+fx))^{3/2}}-\frac{2a\cos(e+fx)}{5f(c+d)(c+d\sin(e+fx))^{5/2}}+\frac{2a(3c-5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (2*a*(3*c -
5*d)*Cos[e + f*x])/(15*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2
*a*(3*c^2 - 20*c*d + 9*d^2)*Cos[e + f*x])/(15*(c - d)^2*(c + d)^3*f*Sqrt[c
+ d*Sin[e + f*x]]) - (2*a*(3*c^2 - 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)^2*d*(c + d)^3*f*
Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*(3*c - 5*d)*EllipticF[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*(c - d)*d*
(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}a(c-d)-\frac{3}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}dx}{5(c^2-d^2)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.79, size = 2815, normalized size = 8.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(3*c^2 - 20*c*d + 9*d^
2)*Csc[e]*Sec[e])/(15*(c - d)^2*d*(c + d)^3*f) + (2*Csc[e]*(c*Cos[e] + d*Si
n[f*x]))/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^3) - (2*Csc[e]*(5*c*Cos[e] - 3
*d*Cos[e] - 3*c*Sin[f*x] + 5*d*Sin[f*x]))/(15*(c - d)*(c + d)^2*f*(c + d*Si
n[e + f*x])^2) - (2*Csc[e]*(15*c^2*Cos[e] - 12*c*d*Cos[e] + 5*d^2*Cos[e] -
3*c^2*Sin[f*x] + 20*c*d*Sin[f*x] - 9*d^2*Sin[f*x]))/(15*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c
^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
```

t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
 - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
(c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sq
rt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[
e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^
2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*S
in[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
])^2) + (4*c*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2
, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d
*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c
 + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[
e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[C
ot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTa
n[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*S
qrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt
[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + C
ot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 +
 Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTa
n[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2]*Sin[e]]))/(3*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e
/2 + (f*x)/2])^2) - (3*d^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/
2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))),
-((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*S
qrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*
x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos
[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e]
)]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin
[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot
[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x
)/2] + Sin[e/2 + (f*x)/2])^2) - (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + T
an[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]
*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 -
 (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1
+ Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sq
rt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e
]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqr

```
t[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[
e]^2]])/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^
2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Se
c[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[
e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
 Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] +
 d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 +
Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]]
)/((c - d)^2*d*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt
[1 + Tan[e]^2]) + (2*d*AppellF1[1/2, 1/2, 1/2, ...
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(360) = 720$.

time = 23.27, size = 1046, normalized size = 3.29

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1046 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*(1/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2
)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^
2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2
-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(-c+d)/d*(2/5/
(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16
/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d
)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-s
in(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/
(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
```

```
+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e
)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 1489, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
[Out] 1/45*((3*sqrt(2)*(6*a*c^4*d^2 + 5*a*c^3*d^3 - 18*a*c^2*d^4 + 15*a*c*d^5)*co
s(f*x + e)^2 + (sqrt(2)*(6*a*c^3*d^3 + 5*a*c^2*d^4 - 18*a*c*d^5 + 15*a*d^6)
*cos(f*x + e)^2 - sqrt(2)*(18*a*c^5*d + 15*a*c^4*d^2 - 48*a*c^3*d^3 + 50*a*
c^2*d^4 - 18*a*c*d^5 + 15*a*d^6))*sin(f*x + e) - sqrt(2)*(6*a*c^6 + 5*a*c^5
*d + 30*a*c^3*d^3 - 54*a*c^2*d^4 + 45*a*c*d^5))*sqrt(I*d)*weierstrassPInver
se(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(
f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (3*sqrt(2)*(6*a*c^4*d^2 + 5*a*c
^3*d^3 - 18*a*c^2*d^4 + 15*a*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(6*a*c^3*d^3
+ 5*a*c^2*d^4 - 18*a*c*d^5 + 15*a*d^6)*cos(f*x + e)^2 - sqrt(2)*(18*a*c^5*d
 + 15*a*c^4*d^2 - 48*a*c^3*d^3 + 50*a*c^2*d^4 - 18*a*c*d^5 + 15*a*d^6))*sin
(f*x + e) - sqrt(2)*(6*a*c^6 + 5*a*c^5*d + 30*a*c^3*d^3 - 54*a*c^2*d^4 + 45
*a*c*d^5))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I
*c)/d) - 3*(3*sqrt(2)*(-3*I*a*c^3*d^3 + 20*I*a*c^2*d^4 - 9*I*a*c*d^5)*cos(f
*x + e)^2 + (sqrt(2)*(-3*I*a*c^2*d^4 + 20*I*a*c*d^5 - 9*I*a*d^6)*cos(f*x +
e)^2 + sqrt(2)*(9*I*a*c^4*d^2 - 60*I*a*c^3*d^3 + 30*I*a*c^2*d^4 - 20*I*a*c*
d^5 + 9*I*a*d^6))*sin(f*x + e) + sqrt(2)*(3*I*a*c^5*d - 20*I*a*c^4*d^2 + 18
*I*a*c^3*d^3 - 60*I*a*c^2*d^4 + 27*I*a*c*d^5))*sqrt(I*d)*weierstrassZeta(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInvers
e(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f
*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(3*sqrt(2)*(3*I*a*c^3*d^3 - 2
0*I*a*c^2*d^4 + 9*I*a*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(3*I*a*c^2*d^4 - 20*
I*a*c*d^5 + 9*I*a*d^6)*cos(f*x + e)^2 + sqrt(2)*(-9*I*a*c^4*d^2 + 60*I*a*c^
3*d^3 - 30*I*a*c^2*d^4 + 20*I*a*c*d^5 - 9*I*a*d^6))*sin(f*x + e) + sqrt(2)*
(-3*I*a*c^5*d + 20*I*a*c^4*d^2 - 18*I*a*c^3*d^3 + 60*I*a*c^2*d^4 - 27*I*a*c
```

```
*d^5))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3
 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*
I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)
/d)) - 6*((3*a*c^2*d^4 - 20*a*c*d^5 + 9*a*d^6)*cos(f*x + e)^3 - (9*a*c^3*d^
3 - 45*a*c^2*d^4 + 15*a*c*d^5 + 5*a*d^6)*cos(f*x + e)*sin(f*x + e) - (9*a*c
^4*d^2 - 25*a*c^3*d^3 + 3*a*c^2*d^4 - 15*a*c*d^5 + 12*a*d^6)*cos(f*x + e))*
sqrt(d*sin(f*x + e) + c))/(3*(c^6*d^4 + c^5*d^5 - 2*c^4*d^6 - 2*c^3*d^7 + c
^2*d^8 + c*d^9)*f*cos(f*x + e)^2 - (c^8*d^2 + c^7*d^3 + c^6*d^4 + c^5*d^5 -
 5*c^4*d^6 - 5*c^3*d^7 + 3*c^2*d^8 + 3*c*d^9)*f + ((c^5*d^5 + c^4*d^6 - 2*c
^3*d^7 - 2*c^2*d^8 + c*d^9 + d^10)*f*cos(f*x + e)^2 - (3*c^7*d^3 + 3*c^6*d^
4 - 5*c^5*d^5 - 5*c^4*d^6 + c^3*d^7 + c^2*d^8 + c*d^9 + d^10)*f)*sin(f*x +
e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.489 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=378

$$\frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2(5c(c-9d)-56d^2)\cos(e+fx)(c+$$

```
[Out] 4/315*a^2*(5*c*(c-9*d)-56*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/63*a
^2*(c-9*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f-2/9*a^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(7/2)/d/f+4/315*a^2*(5*c^3-45*c^2*d-141*c*d^2-75*d^3)*cos(f*x+e)*
(c+d*sin(f*x+e))^(1/2)/d/f+4/315*a^2*(5*c^4-45*c^3*d-381*c^2*d^2-435*c*d^3-
168*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(
1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/315*a^2*(c^2-d^2)*(5*c^3-45*c^2
*d-141*c*d^2-75*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1
/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*
sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.44, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2842, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2(c^2-d^2)(5c^3-45c^2d-141cd^2-75d^3)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(5c^4-45c^3d-381c^2d^2-435cd^3-168d^4)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(5c(c-9d)-56d^2)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315df}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df}+\frac{4a^2(c-9d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*(5*c^3 - 45*c^2*d - 141*c*d^2 - 75*d^3)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(315*d*f) + (4*a^2*(5*c*(c - 9*d) - 56*d^2)*Cos[e + f*x]*(c + d*
Sin[e + f*x])^(3/2))/(315*d*f) + (4*a^2*(c - 9*d)*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(5/2))/(63*d*f) - (2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/
(9*d*f) - (4*a^2*(5*c^4 - 45*c^3*d - 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c^2 - d^2)*(5*c^3 - 45*c^
2*d - 141*c*d^2 - 75*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
```

```
, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df} + \frac{2\int (8a^2d - a^2(c$$

$$= \frac{4a^2(c-9d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df} - \frac{2a^2\cos(e$$

$$= \frac{4a^2(5c(c-9d)-56d^2)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315df}$$

$$= \frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(}}{315df}$$

$$= \frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(}}{315df}$$

$$= \frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(}}{315df}$$

$$= \frac{4a^2(5c^3-45c^2d-141cd^2-75d^3)\cos(e+fx)\sqrt{c+d\sin(}}{315df}$$

**Mathematica** [A]
time = 1.41, size = 322, normalized size = 0.85

$$\frac{a^2(1+\sin(e+fx))^2\left(16\left(-d^2(235c^3+405c^2d+309cd^2+75d^3)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+(5c^4-45c^3d-381c^2d^2-435cd^3-168d^4)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-d(c+d\sin(e+fx))\left(2(20c^3+1080c^2d+1671cd^2+690d^3)\cos(e+fx)+2d(-5d(19c+18d)\cos(3(e+fx))+(150c^2+540cd+259d^2-35d^2\cos(2(e+fx)))\sin(2(e+fx)))\right)\right)}{1260d^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(16*(-(d^2*(235*c^3 + 405*c^2*d + 309*c*d^2 + 75*
d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (5*c^4 - 45*c^3*d -
 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/
4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqr
t[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(20*c^3 + 1080*
c^2*d + 1671*c*d^2 + 690*d^3)*Cos[e + f*x] + 2*d*(-5*d*(19*c + 18*d)*Cos[3*
```

```
(e + f*x)] + (150*c^2 + 540*c*d + 259*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*
(e + f*x)]))))/(1260*d^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c +
 d*Sin[e + f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. $2(416) = 832$.

time = 4.96, size = 1614, normalized size = 4.27

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1614 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*a^2*(486*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*d^6-10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-
1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6-336*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6-224*c^2*d^4*sin(f*x
+e)^2+210*c*d^5*sin(f*x+e)^2+80*c^3*d^3*sin(f*x+e)+540*c^2*d^4*sin(f*x+e)+5
06*c*d^5*sin(f*x+e)-130*c*d^5*sin(f*x+e)^5-170*c^2*d^4*sin(f*x+e)^4-360*c*d
^5*sin(f*x+e)^4-80*c^3*d^3*sin(f*x+e)^3-540*c^2*d^4*sin(f*x+e)^3-376*c*d^5*
sin(f*x+e)^3-5*c^4*d^2*sin(f*x+e)^2-270*c^3*d^3*sin(f*x+e)^2-35*d^6*sin(f*x
+e)^6-90*d^6*sin(f*x+e)^5-77*d^6*sin(f*x+e)^4-60*d^6*sin(f*x+e)^3+112*d^6*s
in(f*x+e)^2+150*d^6*sin(f*x+e)+150*c*d^5+5*c^4*d^2+270*c^3*d^3+394*c^2*d^4+
772*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^4*d^2+780*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c
+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3-426*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4-870*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^5+1
0*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin
(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*c^5*d-570*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*c^4*d^2-1012*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(si
n(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+84*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4+100
```

```
2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin
(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*c*d^5+90*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^
(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))*c^5*d)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 737, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
[Out] 2/945*(sqrt(2)*(10*a^2*c^5 - 90*a^2*c^4*d - 57*a^2*c^3*d^2 + 345*a^2*c^2*d^
3 + 591*a^2*c*d^4 + 225*a^2*d^5)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*
d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(10*a^2*c^5 - 90*a^2*c^4*d - 57*a^2*c^
3*d^2 + 345*a^2*c^2*d^3 + 591*a^2*c*d^4 + 225*a^2*d^5)*sqrt(-I*d)*weierstra
ssPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*
(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*sqrt(2)*(-5*I*a^2*c^
4*d + 45*I*a^2*c^3*d^2 + 381*I*a^2*c^2*d^3 + 435*I*a^2*c*d^4 + 168*I*a^2*d^
5)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I
*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) -
3*sqrt(2)*(5*I*a^2*c^4*d - 45*I*a^2*c^3*d^2 - 381*I*a^2*c^2*d^3 - 435*I*a^2
*c*d^4 - 168*I*a^2*d^5)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2
)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(
f*x + e) + 2*I*c)/d)) + 3*(5*(19*a^2*c*d^4 + 18*a^2*d^5)*cos(f*x + e)^3 - (
5*a^2*c^3*d^2 + 270*a^2*c^2*d^3 + 489*a^2*c*d^4 + 240*a^2*d^5)*cos(f*x + e)
 + (35*a^2*d^5*cos(f*x + e)^3 - 3*(25*a^2*c^2*d^3 + 90*a^2*c*d^4 + 49*a^2*d
^5)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^3*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c^{2}\sqrt{c+d\sin(e+fx)}\,dx+\int 2c^{2}\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int c^{2}\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)\,dx+\int d^{2}\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)\,dx+\int 2d^{2}\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)\,dx+\int d^{2}\sqrt{c+d\sin(e+fx)}\sin^{4}(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int 4cd\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(c + d*sin(e + f*x)), x) + Integral(2*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x), x) + Integral(c**2*sqrt(c + d*sin(e + f*x))*
sin(e + f*x)**2, x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**
2, x) + Integral(2*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + Inte
gral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4, x) + Integral(2*c*d*sqr
t(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(4*c*d*sqrt(c + d*sin(e +
f*x))*sin(e + f*x)**2, x) + Integral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e +
 f*x)**3, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin\left(e+f\,x\right))^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2), x)
```

## 3.490 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=298

$$\frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df}-\frac{2a^2\,\mathrm{c}}{}$$

```
[Out] 4/35*a^2*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f-2/7*a^2*cos(f*x+e)*(
c+d*sin(f*x+e))^(5/2)/d/f+4/35*a^2*(c^2-7*c*d-10*d^2)*cos(f*x+e)*(c+d*sin(f
*x+e))^(1/2)/d/f+4/35*a^2*(c+3*d)*(c^2-10*c*d-7*d^2)*(sin(1/2*e+1/4*Pi+1/2*
f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c
+d))^(1/2)-4/35*a^2*(c^2-7*c*d-10*d^2)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))
^(1/2)
```

**Rubi** [A]

time = 0.32, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2842, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2(c^2-7cd-10d^2)(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{35d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)(c^2-10cd-7d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{35d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df}+\frac{4a^2(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*a^2*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(35*d*
f) + (4*a^2*(c - 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d*f) - (
2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*f) - (4*a^2*(c + 3*d)*(
c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(35*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c
^2 - 7*c*d - 10*d^2)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(35*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int(6a^2d-a^2(c\ldots}{\ldots}$$

$$= \frac{4a^2(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2a^2\cos(e\ldots}{\ldots}$$

$$= \frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4\ldots}{\ldots}$$

$$= \frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4\ldots}{\ldots}$$

$$= \frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4\ldots}{\ldots}$$

$$= \frac{4a^2(c^2-7cd-10d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4\ldots}{\ldots}$$

**Mathematica** [A]
time = 1.40, size = 262, normalized size = 0.88

$$\frac{a^2\left(8(c^4-6c^3d-44c^2d^2-58cd^3-21d^4)\,E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-8(c^4-7c^3d-11c^2d^2+7cd^3+10d^4)\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d\cos(e+fx)\left(-4c^3-112c^2d-106cd^2-28d^3+2d^2(13c+14d)\cos(2(e+fx))-d(36c^2+168cd+95d^2)\sin(e+fx)+5d^3\sin(3(e+fx))\right)\right)}{70d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(8*(c^4 - 6*c^3*d - 44*c^2*d^2 - 58*c*d^3 - 21*d^4)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 8*(c^4 -
 7*c^3*d - 11*c^2*d^2 + 7*c*d^3 + 10*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(-4*c^3
- 112*c^2*d - 106*c*d^2 - 28*d^3 + 2*d^2*(13*c + 14*d)*Cos[2*(e + f*x)] - d
*(36*c^2 + 168*c*d + 95*d^2)*Sin[e + f*x] + 5*d^3*Sin[3*(e + f*x)])))/(70*d
^2*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(340) = 680$.
time = 4.87, size = 1316, normalized size = 4.42

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1316 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*a^2*(-13*c*d^4*sin(f*x+e)^4-9*c^2*d^3*sin(f*x+e)^3-42*c*d^4*sin(f*x+e
)^3-c^3*d^2*sin(f*x+e)^2-28*c^2*d^3*sin(f*x+e)^2-7*c*d^4*sin(f*x+e)^2+9*c^2
*d^3*sin(f*x+e)+42*c*d^4*sin(f*x+e)+28*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(si
n(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3-74*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4-5*d^5
*sin(f*x+e)^5-14*d^5*sin(f*x+e)^4-15*d^5*sin(f*x+e)^3+14*d^5*sin(f*x+e)^2+2
0*d^5*sin(f*x+e)+28*c^2*d^3+20*c*d^4+c^3*d^2-42*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5+62*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-2*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^5-64*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^2*d^3+68*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+14*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d+2*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d)
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d
-68*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^3*d^2+76*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+
d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)
/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 639, normalized size = 2.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(2*sqrt(2)*(a^2*c^4 - 7*a^2*c^3*d + 2*a^2*c^2*d^2 + 21*a^2*c*d^3 + 15
*a^2*d^4)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*
I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)
/d) + 2*sqrt(2)*(a^2*c^4 - 7*a^2*c^3*d + 2*a^2*c^2*d^2 + 21*a^2*c*d^3 + 15*
a^2*d^4)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8
*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c
)/d) - 3*sqrt(2)*(-I*a^2*c^3*d + 7*I*a^2*c^2*d^2 + 37*I*a^2*c*d^3 + 21*I*a^
2*d^4)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c
^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)
) - 3*sqrt(2)*(I*a^2*c^3*d - 7*I*a^2*c^2*d^2 - 37*I*a^2*c*d^3 - 21*I*a^2*d^
4)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9
*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^
3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d))
 + 3*(5*a^2*d^4*cos(f*x + e)^3 - 2*(4*a^2*c*d^3 + 7*a^2*d^4)*cos(f*x + e)*s
in(f*x + e) - (a^2*c^2*d^2 + 28*a^2*c*d^3 + 25*a^2*d^4)*cos(f*x + e))*sqrt(
d*sin(f*x + e) + c))/(d^3*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c\sqrt{c+d\sin(e+fx)}\,dx+\int 2c\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int c\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)\,dx+\int d\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int 2d\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)\,dx+\int d\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x), x) + Integral(c*sqrt(c + d*sin(e + f*x))*sin(e + f
*x)**2, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integra
l(2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(d*sqrt(c + d*
sin(e + f*x))*sin(e + f*x)**3, x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x))^2 (c + d \sin(e + f x))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2), x)
```

## 3.491 $\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=239

$$\frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df}-\frac{4a^2(c^2-5cd-12d^2)\,E}{15a}$$

```
[Out] -2/5*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/15*a^2*(c-5*d)*cos(f*x+e)*
(c+d*sin(f*x+e))^(1/2)/d/f+4/15*a^2*(c^2-5*c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-4/15*a^2*(c-5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.23, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2842, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^2(c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c^2-5cd-12d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df}+\frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*(c - 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*a^2*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) - (4*a^2*(c^2 - 5*c*d - 12*
d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 5*d)*(c^2 - d^
2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int (4a^2d-a^2(c\cdot$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+$$

**Mathematica** [A]
time = 0.94, size = 244, normalized size = 1.02

$$-\frac{a^2(1+\sin(e+fx))^2\left(-4(c^3-4c^2d-17cd^2-12d^3)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+4(c^3-5c^2d-cd^2+5d^3)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-d\cos(e+fx)\left(-2c^2-20cd-3d^2+3d^2\cos(2(e+fx))-4d(2c+5d)\sin(e+fx)\right)\right)}{15d^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/15*(a^2*(1 + Sin[e + f*x])^2*(-4*(c^3 - 4*c^2*d - 17*c*d^2 - 12*d^3)*Ell
ipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)] + 4*(c^3 - 5*c^2*d - c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(-2*c^2 -
 20*c*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] - 4*d*(2*c + 5*d)*Sin[e + f*x])))/
(d^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(285) = 570.
time = 5.17, size = 1035, normalized size = 4.33

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{2a^2\left(2\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\,\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\,\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3d-34c^2\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^3*d-34*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-2*c*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+34*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-2*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^3*d+26*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-
1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-24*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-
3*d^4*sin(f*x+e)^4-4*c*d^3*sin(f*x+e)^3-10*d^4*sin(f*x+e)^3-c^2*d^2*sin(f*x
+e)^2-10*c*d^3*sin(f*x+e)^2+3*d^4*sin(f*x+e)^2+4*c*d^3*sin(f*x+e)+10*d^4*si
n(f*x+e)+c^2*d^2+10*d^3*c)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 554, normalized size = 2.32

$$\frac{2\left(\sqrt{2}\left(2a^2c^3-10a^2c^2d+9a^2cd^2+15a^2d^3\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}\left(2a^2c^3-10a^2c^2d+9a^2cd^2+15a^2d^3\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)-3\sqrt{2}\left(-ia^2c^2d+5ia^2cd^2+12ia^2d^3\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)-3\sqrt{2}\left(ia^2c^2d-5ia^2cd^2-12ia^2d^3\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)-3\left(3a^2d^3\cos(fx+e)\sin(fx+e)+(a^2cd^2+10a^2d^3)\cos(fx+e)\right)\sqrt{d\sin(fx+e)+c}\right)}{45d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(sqrt(2)*(2*a^2*c^3 - 10*a^2*c^2*d + 9*a^2*c*d^2 + 15*a^2*d^3)*sqrt(I*
d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2
)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(2*
a^2*c^3 - 10*a^2*c^2*d + 9*a^2*c*d^2 + 15*a^2*d^3)*sqrt(-I*d)*weierstrassPI
nverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d
*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*sqrt(2)*(-I*a^2*c^2*d +
5*I*a^2*c*d^2 + 12*I*a^2*d^3)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2
)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3
*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*s
in(f*x + e) - 2*I*c)/d)) - 3*sqrt(2)*(I*a^2*c^2*d - 5*I*a^2*c*d^2 - 12*I*a^
2*d^3)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3
 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*
I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)
/d)) - 3*(3*a^2*d^3*cos(f*x + e)*sin(f*x + e) + (a^2*c*d^2 + 10*a^2*d^3)*co
s(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^3*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(
c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x))
, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.492 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=189

$$-\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df}-\frac{4a^2(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(c-2\ldots}{}$$

```
[Out] -2/3*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f+4/3*a^2*(c-3*d)*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(
f*x+e))/(c+d))^(1/2)-4/3*a^2*(c-2*d)*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2842, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^2(c-2d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c-3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*a^2*(c - 3*d)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 2*d)*(c - d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{2a^2d-a^2(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{(2a^2(c-3d))\int \sqrt{c+d\sin(e+fx)}\ d}{3d^2}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2a^2(c-3d)\sqrt{c+d\sin(e+fx)}\right)\int}{3d^2\sqrt{\frac{c+d\sin(e}{c+d}}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4a^2(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{}}{3d^2f\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

**Mathematica [A]**
time = 0.73, size = 193, normalized size = 1.02

$$-\frac{2a^2(1+\sin(e+fx))^2\left(d\cos(e+fx)(c+d\sin(e+fx))-2(c^2-2cd-3d^2)\,E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+2(c^2-3cd+2d^2)\,F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3d^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*(c^2
 - 2*c*d - 3*d^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)] + 2*(c^2 - 3*c*d + 2*d^2)*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(239) = 478.
time = 5.33, size = 758, normalized size = 4.01

| method | result |
|---|---|
| default | $-\frac{2a^2\left(2\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\ \mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^2d-12\sqrt{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^2*d-12*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2*c+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-2*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3+6*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^2*d+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*c*d^2-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-d^3*sin(f*x+e)^3-c*d^2*sin(f*x+e)^2+d^3
*sin(f*x+e)+c*d^2)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 475, normalized size = 2.51

$$-\frac{2\left(3\sqrt{d\sin\left(fx+e\right)+c}\,a^2d^2\cos\left(fx+e\right)-2\sqrt{2}\left(a^2c^2-3a^2cd+3a^2d^2\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)-2\sqrt{2}\left(a^2c^2-3a^2cd+3a^2d^2\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)+3\sqrt{2}\left(-ia^2cd+3ia^2d^2\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)\right)+3\sqrt{2}\left(ia^2cd-3ia^2d^2\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)\right)\right)}{9d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*sqrt(d*sin(f*x + e) + c)*a^2*d^2*cos(f*x + e) - 2*sqrt(2)*(a^2*c^2
- 3*a^2*c*d + 3*a^2*d^2)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*
x + e) - 2*I*c)/d) - 2*sqrt(2)*(a^2*c^2 - 3*a^2*c*d + 3*a^2*d^2)*sqrt(-I*d)
*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)
/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(-
I*a^2*c*d + 3*I*a^2*d^2)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)
```

```
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*
x + e) - 2*I*c)/d)) + 3*sqrt(2)*(I*a^2*c*d - 3*I*a^2*d^2)*sqrt(-I*d)*weiers
trassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weier
strassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)))/(d^3*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx+\int \frac{\sin^2(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx+\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e
 + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x
)), x))
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.493 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=189

$$\frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\mid\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d^2(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^2(c-d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\right.\right.}{d^2f\sqrt{c-}}$$

```
[Out] 2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-4*a^2*c*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)+4*a^2*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2841, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^2(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{d^2f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^
2*c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(d^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^2*(c - d)*Ellipti
cF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(
d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-ad-ac\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a^2(c-d))\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} + \frac{(}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^2c\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+d}+\frac{d}{}}}{d^2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{4a^2cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{d^2(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

**Mathematica** [A]
time = 0.61, size = 175, normalized size = 0.93

$$-\frac{2a^2(1+\sin(e+fx))^2\left(2c(c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-(c-d)\left(d\cos(e+fx)+2(c+d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\right)}{d^2(c+d)f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(2*c*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - (c - d)*(d*Cos[e + f*x]
 + 2*(c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Si
n[e + f*x])/(c + d)])))/(d^2*(c + d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [A]
time = 4.61, size = 463, normalized size = 2.45

| method | result |
|---|---|
| default | $-\frac{2\left(2\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3-2\sqrt{\frac{c+}{}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^2-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+c*d^2*sin(f*x+e)^2-d^3*sin(f*x+e)^2-c*d^2+d^3)/d^3*a^2/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 594, normalized size = 3.14

$$\frac{2\left(3\left(a^2cd^2-a^2d^3\right)\sqrt{d\sin(fx+e)+c}\,\cos(fx+e)-\left(\sqrt{2}\left(2a^2c^2d-3a^2d^3\right)\sin(fx+e)+\sqrt{2}\left(2a^2c^3-3a^2cd^2\right)\right)\sqrt{i d}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)-\left(\sqrt{2}\left(2a^2c^2d-3a^2d^3\right)\sin(fx+e)+\sqrt{2}\left(2a^2c^3-3a^2cd^2\right)\right)\sqrt{-i d}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)-3\left(i\sqrt{2}a^2cd^2\sin(fx+e)+i\sqrt{2}a^2c^2d\right)\sqrt{i d}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)-3\left(-i\sqrt{2}a^2cd^2\sin(fx+e)-i\sqrt{2}a^2c^2d\right)\sqrt{-i d}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)\right)}{3\left((cd^4+d^5)f\sin(fx+e)+(c^2d^3+cd^4)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/3*(3*(a^2*c*d^2 - a^2*d^3)*sqrt(d*sin(f*x + e) + c)*cos(f*x + e) - (sqrt(2)*(2*a^2*c^2*d - 3*a^2*d^3)*sin(f*x + e) + sqrt(2)*(2*a^2*c^3 - 3*a^2*c*d^2))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) - (sqrt(2)*(2*a^2*c^2*d - 3*a^2*d^3)*sin(f*x + e) + sqrt(2)*(2*a^2*c^3 - 3*a^2*c*d^2))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(I*sqrt(2)*a^2*c*d^2*sin(f*x + e) + I*sqrt(2)*a^2*c^2*d)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(-I*sqrt(2)*a^2*c*d^2*sin(f*x + e) - I*sqrt(2)*a^2*c^2*d)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*

```
cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)))/((c*d^4 + d^5)*f*sin(f*x +
e) + (c^2*d^3 + c*d^4)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin\left(e + f\,x\right))^2}{(c + d\,\sin\left(e + f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.494 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}} \, dx$

**Optimal**. Leaf size=247

$$\frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}}-\frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c}}{3d^2(c+d)^2f\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

```
[Out] 2/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-4/3*a^2*(c+3*d)*c
os(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+4/3*a^2*(c+3*d)*(sin(1/2*e+1/4
*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+
1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/((c+
d*sin(f*x+e))/(c+d))^(1/2)-4/3*a^2*(c+2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/(c+d)/f/(c+d*sin(f*x+e))
^(1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2841, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^2(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^2(c+3d)\cos(e+fx)}{3df(c+d)^2\sqrt{c+d\sin(e+fx)}}+\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (
4*a^2*(c + 3*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) -
(4*a^2*(c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^2*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^
2*(c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)])/(3*d^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-3ad-a(c+2d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(4a)\int}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(2a^2(c}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2(c\right.}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{4a^2(c}{}$$

**Mathematica** [A]
time = 1.21, size = 207, normalized size = 0.84

$$-\frac{2a^2(1+\sin(e+fx))^2\left(-2(c+d)^2(c+3d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}+2(c+d)^2(c+2d)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}+d\cos(e+fx)\left(c^2+6cd+d^2+2d(c+3d)\sin(e+fx)\right)\right)}{3d^2(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(-2*(c + d)^2*(c + 3*d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e + f*x])/(c + d))^(3/2) + 2*(c + d)^
2*(c + 2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e +
 f*x])/(c + d))^(3/2) + d*Cos[e + f*x]*(c^2 + 6*c*d + d^2 + 2*d*(c + 3*d)*S
in[e + f*x])))/(3*d^2*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(
c + d*Sin[e + f*x])^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. 2(293) = 586.
time = 4.80, size = 1221, normalized size = 4.94

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1221 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^2*((2*c*d^3+6*d^4)*sin(f*x+e)*cos(f*x+e)^2-2*(-d/(c-d)*sin(f*x+e)-d/
(c-d))^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2)*d*(EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3+3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*c^2*d-EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1
/2))*c*d^2-3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*d^3-EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2
))*c^2*d+EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*d^3)*sin(f*x+e)+(c^2*d^2+6*c*d^3+d^4)*cos(f*x+e)^2+2*(d/(c-d)*sin(f*x+e)+
1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d
/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3*d-2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-2*(d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^4-6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(
1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2
)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d
^2+6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3)/(c+d)^2/(c+d*sin(f*x+e))^(3/2)/d^3/c
os(f*x+e)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1013, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 2/9*(2*(sqrt(2)*(a^2*c^2*d^2 + 3*a^2*c*d^3 + 3*a^2*d^4)*cos(f*x + e)^2 - 2*sqrt(2)*(a^2*c^3*d + 3*a^2*c^2*d^2 + 3*a^2*c*d^3)*sin(f*x + e) - sqrt(2)*(a^2*c^4 + 3*a^2*c^3*d + 4*a^2*c^2*d^2 + 3*a^2*c*d^3 + 3*a^2*d^4))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + 2*(sqrt(2)*(a^2*c^2*d^2 + 3*a^2*c*d^3 + 3*a^2*d^4)*cos(f*x + e)^2 - 2*sqrt(2)*(a^2*c^3*d + 3*a^2*c^2*d^2 + 3*a^2*c*d^3)*sin(f*x + e) - sqrt(2)*(a^2*c^4 + 3*a^2*c^3*d + 4*a^2*c^2*d^2 + 3*a^2*c*d^3 + 3*a^2*d^4))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(I*a^2*c*d^3 + 3*I*a^2*d^4)*cos(f*x + e)^2 + 2*sqrt(2)*(-I*a^2*c^2*d^2 - 3*I*a^2*c*d^3)*sin(f*x + e) + sqrt(2)*(-I*a^2*c^3*d - 3*I*a^2*c^2*d^2 - I*a^2*c*d^3 - 3*I*a^2*d^4))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*a^2*c*d^3 - 3*I*a^2*d^4)*cos(f*x + e)^2 + 2*sqrt(2)*(I*a^2*c^2*d^2 + 3*I*a^2*c*d^3)*sin(f*x + e) + sqrt(2)*(I*a^2*c^3*d + 3*I*a^2*c^2*d^2 + I*a^2*c*d^3 + 3*I*a^2*d^4))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 3*(2*(a^2*c*d^3 + 3*a^2*d^4)*cos(f*x + e)*sin(f*x + e) + (a^2*c^2*d^2 + 6*a^2*c*d^3 + a^2*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f)

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)

[Out] Timed out

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^2}{(c+d\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2), x)
```

## 3.495 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=320

$$\frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}}-\frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}}-\frac{4a^2(c^2+5cd-12d^2)\cos(e+fx)}{15(c-d)d(c+d)^3f\sqrt{c+d\sin(e+f}$$

```
[Out] 2/5*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)-4/15*a^2*(c+5*d)*
cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-4/15*a^2*(c^2+5*c*d-12*d^2)*c
os(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+4/15*a^2*(c^2+5*c*d-12*d
^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/
(c-d)/d^2/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*a^2*(c+5*d)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2
/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.37, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2841, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^2(c^2+5cd-12d^2)\cos(e+fx)}{15df(c-d)(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c^2+5cd-12d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c-d)(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(c+5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+5d)\cos(e+fx)}{15df(c+d)^2(c+d\sin(e+fx))^{3/2}}+\frac{2a^2(c-d)\cos(e+fx)}{5df(c+d)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (
4*a^2*(c + 5*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2))
 - (4*a^2*(c^2 + 5*c*d - 12*d^2)*Cos[e + f*x])/(15*(c - d)*d*(c + d)^3*f*Sq
rt[c + d*Sin[e + f*x]]) - (4*a^2*(c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2
 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)*d^2*(c + d)
^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c + 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*(
c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-5ad-a(c+4d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} + \frac{(4a)}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4}{15(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4}{15(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4}{15(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4}{15(c}$$

**Mathematica [A]**
time = 1.38, size = 283, normalized size = 0.88

$$\frac{2a^2(1+\sin(e+fx))^2\left(-2\left((11c-5d)d^2F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-(c^2+5cd-12d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)(c+d\sin(e+fx))^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d\cos(e+fx)\left(3(c-d)^2(c+d)^2-2(c-d)(c+d)(c+5d)(c+d\sin(e+fx))-2(c^2+5cd-12d^2)(c+d\sin(e+fx))^2\right)\right)}{15(c-d)d^2(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(1 + Sin[e + f*x])^2*(-2*((11*c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*
f*x)/4, (2*d)/(c + d)] - (c^2 + 5*c*d - 12*d^2)*((c + d)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[
e + f*x]*(3*(c - d)^2*(c + d)^2 - 2*(c - d)*(c + d)*(c + 5*d)*(c + d*Sin[e
+ f*x]) - 2*(c^2 + 5*c*d - 12*d^2)*(c + d*Sin[e + f*x])^2)))/(15*(c - d)*d^
```

```
2*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c + d*Sin[e + f*x])^
(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(362) = 724$.
time = 25.84, size = 1436, normalized size = 4.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1436 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^2*(2*(-c+d)/d^2*(2/3/(c^2-d^2)/d*
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)
^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c
^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8
/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d
^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/
(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)
*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^2-2*c*d+d^2)/d^2
*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d
)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+
e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*
d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9
*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos
(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 1631, normalized size = 5.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/45*((3*sqrt(2)*(2*a^2*c^4*d^2 + 10*a^2*c^3*d^3 + 9*a^2*c^2*d^4 - 15*a^2*c
*d^5)*cos(f*x + e)^2 + (sqrt(2)*(2*a^2*c^3*d^3 + 10*a^2*c^2*d^4 + 9*a^2*c*d
^5 - 15*a^2*d^6)*cos(f*x + e)^2 - sqrt(2)*(6*a^2*c^5*d + 30*a^2*c^4*d^2 + 2
9*a^2*c^3*d^3 - 35*a^2*c^2*d^4 + 9*a^2*c*d^5 - 15*a^2*d^6))*sin(f*x + e) -
sqrt(2)*(2*a^2*c^6 + 10*a^2*c^5*d + 15*a^2*c^4*d^2 + 15*a^2*c^3*d^3 + 27*a^
2*c^2*d^4 - 45*a^2*c*d^5))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(
f*x + e) - 2*I*c)/d) + (3*sqrt(2)*(2*a^2*c^4*d^2 + 10*a^2*c^3*d^3 + 9*a^2*c
^2*d^4 - 15*a^2*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(2*a^2*c^3*d^3 + 10*a^2*c^
2*d^4 + 9*a^2*c*d^5 - 15*a^2*d^6)*cos(f*x + e)^2 - sqrt(2)*(6*a^2*c^5*d + 3
0*a^2*c^4*d^2 + 29*a^2*c^3*d^3 - 35*a^2*c^2*d^4 + 9*a^2*c*d^5 - 15*a^2*d^6)
)*sin(f*x + e) - sqrt(2)*(2*a^2*c^6 + 10*a^2*c^5*d + 15*a^2*c^4*d^2 + 15*a^
2*c^3*d^3 + 27*a^2*c^2*d^4 - 45*a^2*c*d^5))*sqrt(-I*d)*weierstrassPInverse(
-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*
x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(3*sqrt(2)*(I*a^2*c^3*d^3 + 5*I
*a^2*c^2*d^4 - 12*I*a^2*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(I*a^2*c^2*d^4 + 5
*I*a^2*c*d^5 - 12*I*a^2*d^6)*cos(f*x + e)^2 + sqrt(2)*(-3*I*a^2*c^4*d^2 - 1
5*I*a^2*c^3*d^3 + 35*I*a^2*c^2*d^4 - 5*I*a^2*c*d^5 + 12*I*a^2*d^6))*sin(f*x
 + e) + sqrt(2)*(-I*a^2*c^5*d - 5*I*a^2*c^4*d^2 + 9*I*a^2*c^3*d^3 - 15*I*a^
2*c^2*d^4 + 36*I*a^2*c*d^5))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*
d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*si
n(f*x + e) - 2*I*c)/d)) + 3*(3*sqrt(2)*(-I*a^2*c^3*d^3 - 5*I*a^2*c^2*d^4 +
12*I*a^2*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(-I*a^2*c^2*d^4 - 5*I*a^2*c*d^5 +
 12*I*a^2*d^6)*cos(f*x + e)^2 + sqrt(2)*(3*I*a^2*c^4*d^2 + 15*I*a^2*c^3*d^3
 - 35*I*a^2*c^2*d^4 + 5*I*a^2*c*d^5 - 12*I*a^2*d^6))*sin(f*x + e) + sqrt(2)
*(I*a^2*c^5*d + 5*I*a^2*c^4*d^2 - 9*I*a^2*c^3*d^3 + 15*I*a^2*c^2*d^4 - 36*I
*a^2*c*d^5))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8
```

```
*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/2
7*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) +
2*I*c)/d)) - 3*(2*(a^2*c^2*d^4 + 5*a^2*c*d^5 - 12*a^2*d^6)*cos(f*x + e)^3 -
 2*(3*a^2*c^3*d^3 + 15*a^2*c^2*d^4 - 25*a^2*c*d^5 - 5*a^2*d^6)*cos(f*x + e)
*sin(f*x + e) - (a^2*c^4*d^2 + 20*a^2*c^3*d^3 - 18*a^2*c^2*d^4 - 27*a^2*d^6
)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(3*(c^5*d^5 + 2*c^4*d^6 - 2*c^2*d
^8 - c*d^9)*f*cos(f*x + e)^2 - (c^7*d^3 + 2*c^6*d^4 + 3*c^5*d^5 + 4*c^4*d^6
 - c^3*d^7 - 6*c^2*d^8 - 3*c*d^9)*f + ((c^4*d^6 + 2*c^3*d^7 - 2*c*d^9 - d^1
0)*f*cos(f*x + e)^2 - (3*c^6*d^4 + 6*c^5*d^5 + c^4*d^6 - 4*c^3*d^7 - 3*c^2*
d^8 - 2*c*d^9 - d^10)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^2}{(c+d\,\sin\left(e+f\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.496 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=467

$$-\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{693d^2f}-\frac{4a^3(4c^3-33c^2d+182cd^2}{}$$

```
[Out] -4/693*a^3*(4*c^3-33*c^2*d+182*c*d^2+231*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^(
3/2)/d^2/f-4/693*a^3*(4*c^2-33*c*d+189*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/
2)/d^2/f+8/99*a^3*(c-6*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d^2/f-2/11*cos(
f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(7/2)/d/f-4/693*a^3*(4*c^4-33*
c^3*d+177*c^2*d^2+561*c*d^3+315*d^4)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/
f-4/693*a^3*(c+3*d)*(4*c^4-45*c^3*d+309*c^2*d^2+525*c*d^3+231*d^4)*(sin(1/2
*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1
/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+4/693*a^3*(c^2-d^2)*(4*c^4-33*c^3*d+177*c^2*d^2+56
1*c*d^3+315*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(
f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.64, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2842, 3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2-33cd+189d^2)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{4a^3(4c^3-33c^2d+182cd^2+231d^3)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{693d^2f}-\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{693d^2f}-\frac{4a^3(c^2-d^2)(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{693d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(c+3d)(4c^4-45c^3d+309c^2d^2+525cd^3+231d^4)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{693d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-6d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f}-\frac{2\cos(e+fx)(a^3\sin(e+fx)+a^3)(c+d\sin(e+fx))^{7/2}}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-4*a^3*(4*c^4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*Cos[e + f*x]
*Sqrt[c + d*Sin[e + f*x]])/(693*d^2*f) - (4*a^3*(4*c^3 - 33*c^2*d + 182*c*d
^2 + 231*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(693*d^2*f) - (4*a^3
*(4*c^2 - 33*c*d + 189*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d
^2*f) + (8*a^3*(c - 6*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f
) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(1
1*d*f) + (4*a^3*(c + 3*d)*(4*c^4 - 45*c^3*d + 309*c^2*d^2 + 525*c*d^3 + 231
*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(693*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^
4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*EllipticF[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(693*d^3*f*Sqrt[c +
 d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
```

```
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{7/}}{11df}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{7/}}{11df}$$

$$= \frac{8a^3(c-6d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f} - \frac{2\cos(e+}{}$$

$$= -\frac{4a^3(4c^2-33cd+189d^2)\cos(e+fx)(c+d\sin(e+fx)}{693d^2f}$$

$$= -\frac{4a^3(4c^3-33c^2d+182cd^2+231d^3)\cos(e+fx)(c+d\,\mathrm{si}}{693d^2f}$$

$$= -\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+f}{693d^2f}$$

$$= -\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+f}{693d^2f}$$

$$= -\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+f}{693d^2f}$$

$$= -\frac{4a^3(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4)\cos(e+f}{693d^2f}$$

**Mathematica** [A]
time = 1.30, size = 377, normalized size = 0.81

$$\frac{a^3(1+\sin(e+fx))^3\left(-32\left(d^2(c^4+858c^3d+1668c^2d^2+1254cd^3+315d^4)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(4c^5-33c^4d+174c^3d^2+1452c^2d^3+1806cd^4+693d^5)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d(c+d\sin(e+fx))\left(2(32c^4-264c^3d-8994c^2d^2-13926cd^3-5859d^4)\cos(e+fx)+d^2(452c^2+2508cd+1701d^2)\cos(3(e+fx))-63d^4\cos(5(e+fx))-4d(6c^3+990c^2d+2401cd^2+1155d^3)\sin(2(e+fx))+14d^3(23c+33d)\sin(4(e+fx))\right)\right)}{5544d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-32*(d^2*(c^4 + 858*c^3*d + 1668*c^2*d^2 + 1254*
c*d^3 + 315*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^5 -
 33*c^4*d + 174*c^3*d^2 + 1452*c^2*d^3 + 1806*c*d^4 + 693*d^5)*((c + d)*Ell
ipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e
```

```
 + f*x])*(2*(32*c^4 - 264*c^3*d - 8994*c^2*d^2 - 13926*c*d^3 - 5859*d^4)*Co
s[e + f*x] + d^2*(452*c^2 + 2508*c*d + 1701*d^2)*Cos[3*(e + f*x)] - 63*d^4*
Cos[5*(e + f*x)] - 4*d*(6*c^3 + 990*c^2*d + 2401*c*d^2 + 1155*d^3)*Sin[2*(e
 + f*x)] + 14*d^3*(23*c + 33*d)*Sin[4*(e + f*x)])))/(5544*d^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(501) = 1002$.
time = 5.45, size = 1926, normalized size = 4.12

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1926 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693*a^3*(224*c*d^6*sin(f*x+e)^6+274*c^2*d^5*sin(f*x+e)^5+858*c*d^6*sin(f*
x+e)^5+116*c^3*d^4*sin(f*x+e)^4+1122*c^2*d^5*sin(f*x+e)^4+1274*c*d^6*sin(f*
x+e)^4-c^4*d^3*sin(f*x+e)^3+528*c^3*d^4*sin(f*x+e)^3+1942*c^2*d^5*sin(f*x+e
)^3+1188*c*d^6*sin(f*x+e)^3-4*c^5*d^2*sin(f*x+e)^2+33*c^4*d^3*sin(f*x+e)^2+
980*c^3*d^4*sin(f*x+e)^2+462*c^2*d^5*sin(f*x+e)^2-868*c*d^6*sin(f*x+e)^2+c^
4*d^3*sin(f*x+e)-528*c^3*d^4*sin(f*x+e)-2216*c^2*d^5*sin(f*x+e)-2046*c*d^6*
sin(f*x+e)-33*c^4*d^3+1518*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*
d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^5+3612*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^6+4*c^5*d^2-1584*
c^2*d^5+63*d^7*sin(f*x+e)^7+231*d^7*sin(f*x+e)^6+315*d^7*sin(f*x+e)^5+231*d
^7*sin(f*x+e)^4+252*d^7*sin(f*x+e)^3-462*d^7*sin(f*x+e)^2-630*d^7*sin(f*x+e
)-2016*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*d^7-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*c^7+1386*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f
*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^7-1096*c^3*d^4-630*c*d^6+8*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*c^6*d-72*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^5*d^2+2128*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^3+4176*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^4-120*((c
```

```
+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*c^2*d^5-4104*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(
1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))*c*d^6+66*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6*d-340*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5*d^2-2970*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^4*d^3-3264*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*c^3*d^4)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 843, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2079*(sqrt(2)*(8*a^3*c^6 - 66*a^3*c^5*d + 345*a^3*c^4*d^2 + 330*a^3*c^3*
d^3 - 1392*a^3*c^2*d^4 - 2376*a^3*c*d^5 - 945*a^3*d^6)*sqrt(I*d)*weierstras
sPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3
*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(8*a^3*c^6 - 66*
a^3*c^5*d + 345*a^3*c^4*d^2 + 330*a^3*c^3*d^3 - 1392*a^3*c^2*d^4 - 2376*a^3
*c*d^5 - 945*a^3*d^6)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
 + e) + 2*I*c)/d) + 3*sqrt(2)*(4*I*a^3*c^5*d - 33*I*a^3*c^4*d^2 + 174*I*a^3
*c^3*d^3 + 1452*I*a^3*c^2*d^4 + 1806*I*a^3*c*d^5 + 693*I*a^3*d^6)*sqrt(I*d)
*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
 weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/
d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*(-
4*I*a^3*c^5*d + 33*I*a^3*c^4*d^2 - 174*I*a^3*c^3*d^3 - 1452*I*a^3*c^2*d^4 -
```

```
1806*I*a^3*c*d^5 - 693*I*a^3*d^6)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 -
 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 3*(63*a^3*d^6*cos(f*x + e)^5 - (113*a^3*
c^2*d^4 + 627*a^3*c*d^5 + 504*a^3*d^6)*cos(f*x + e)^3 - (4*a^3*c^4*d^2 - 33
*a^3*c^3*d^3 - 1209*a^3*c^2*d^4 - 2211*a^3*c*d^5 - 1071*a^3*d^6)*cos(f*x +
e) - (7*(23*a^3*c*d^5 + 33*a^3*d^6)*cos(f*x + e)^3 - 3*(a^3*c^3*d^3 + 165*a
^3*c^2*d^4 + 427*a^3*c*d^5 + 231*a^3*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c^2\sqrt{c+d\sin(e+fx)}\,dx+\int 3c^2\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int 3c^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx+\int c^2\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)\,dx+\int d^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx+\int 3d^2\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)\,dx+\int 3d^2\sqrt{c+d\sin(e+fx)}\sin^4(e+fx)\,dx+\int d^2\sqrt{c+d\sin(e+fx)}\sin^5(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int 6cd\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)\,dx+\int 6cd\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)\,dx+\int 2cd\sqrt{c+d\sin(e+fx)}\sin^4(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(c**2*sqrt(c + d*sin(e + f*x)), x) + Integral(3*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*c**2*sqrt(c + d*sin(e + f*x)
)*sin(e + f*x)**2, x) + Integral(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)
**3, x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Inte
gral(3*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + Integral(3*d**2*
sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4, x) + Integral(d**2*sqrt(c + d*sin
(e + f*x))*sin(e + f*x)**5, x) + Integral(2*c*d*sqrt(c + d*sin(e + f*x))*si
n(e + f*x), x) + Integral(6*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x
) + Integral(6*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + Integral(
2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin(e+f\,x))^3\,(c+d\,\sin(e+f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2), x)
```

## 3.497 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=390

$$-\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\frac{4a^3(4c^2-27cd+119d^2)\cos(e+fx}{315d^2f}$$

```
[Out] -4/315*a^3*(4*c^2-27*c*d+119*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f+8
/63*a^3*(c-5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f-2/9*cos(f*x+e)*(a^3
+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2)/d/f-4/315*a^3*(4*c^3-27*c^2*d+114*c
*d^2+165*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-4/315*a^3*(4*c^4-27*c
^3*d+111*c^2*d^2+579*c*d^3+357*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin
(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d)
)^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/315*
a^3*(c^2-d^2)*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))
^(1/2)
```

**Rubi** [A]
time = 0.51, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2842, 3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2-27cd+119d^2)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\frac{4a^3(c^2-d^2)\left(4c^3-27c^2d+114cd^2+165d^3\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^4-27c^3d+111c^2d^2+579cd^3+357d^4\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-5d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f}-\frac{2\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^{5/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^3*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(315*d^2*f) - (4*a^3*(4*c^2 - 27*c*d + 119*d^2)*Cos[e + f*x]*(
c + d*Sin[e + f*x])^(3/2))/(315*d^2*f) + (8*a^3*(c - 5*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^(5/2))/(63*d^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x
])*(c + d*Sin[e + f*x])^(5/2))/(9*d*f) + (4*a^3*(4*c^4 - 27*c^3*d + 111*c^2
*d^2 + 579*c*d^3 + 357*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sq
rt[c + d*Sin[e + f*x]])/(315*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c^2 - d^2)*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^3*f*
Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
```

```
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{5/2}}{9df}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{5/2}}{9df}$$

$$= \frac{8a^3(c-5d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f} - \frac{2\cos(e+$$

$$= -\frac{4a^3(4c^2-27cd+119d^2)\cos(e+fx)(c+d\sin(e+fx))^{3}}{315d^2f}$$

$$= -\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{315d^2f}$$

$$= -\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{315d^2f}$$

$$= -\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{315d^2f}$$

$$= -\frac{4a^3(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{315d^2f}$$

**Mathematica** [A]

time = 1.50, size = 318, normalized size = 0.82

$$\frac{a^3(1+\sin(e+fx))^3\left(-16\left(d^2(c^3+387c^2d+471cd^2+165d^3)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(4c^4-27c^3d+111c^2d^2+579cd^3+357d^4)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d(c+d\sin(e+fx))\left((32c^3-216c^2d-3828cd^2-2910d^3)\cos(e+fx)+2d(5d(10c+27d)\cos(3(e+fx))-(6c^2+432cd+511d^2-35d^2\cos(2(e+fx)))\sin(2(e+fx)))\right)\right)}{1260d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-16*(d^2*(c^3 + 387*c^2*d + 471*c*d^2 + 165*d^3)
*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^4 - 27*c^3*d + 111*
c^2*d^2 + 579*c*d^3 + 357*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*((32*c^3 - 216*c^2*d -
3828*c*d^2 - 2910*d^3)*Cos[e + f*x] + 2*d*(5*d*(10*c + 27*d)*Cos[3*(e + f*x
)] - (6*c^2 + 432*c*d + 511*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]
```

```
))))/(1260*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e +
 f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(428) = 856$.
time = 4.93, size = 1613, normalized size = 4.14

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1613 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*a^3*(-1044*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(
1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))*d^6-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-
1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6+714*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6+413*c^2*d^4*sin(f*x
+e)^2-21*c*d^5*sin(f*x+e)^2+c^3*d^3*sin(f*x+e)-243*c^2*d^4*sin(f*x+e)-704*c
*d^5*sin(f*x+e)+85*c*d^5*sin(f*x+e)^5+53*c^2*d^4*sin(f*x+e)^4+351*c*d^5*sin
(f*x+e)^4-c^3*d^3*sin(f*x+e)^3+243*c^2*d^4*sin(f*x+e)^3+619*c*d^5*sin(f*x+e
)^3-4*c^4*d^2*sin(f*x+e)^2+27*c^3*d^3*sin(f*x+e)^2+35*d^6*sin(f*x+e)^6+135*
d^6*sin(f*x+e)^5+203*d^6*sin(f*x+e)^4+195*d^6*sin(f*x+e)^3-238*d^6*sin(f*x+
e)^2-330*d^6*sin(f*x+e)-330*c*d^5+4*c^4*d^2-27*c^3*d^3-466*c^2*d^4-214*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e
))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))*c^4*d^2-1212*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c^3*d^3-492*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*
x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4+1158*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*
EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^5+8*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^5*d-60*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*c^4*d^2+1048*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+1104*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4-1056*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e
```

```
))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))*c*d^5+54*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^5*d)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 735, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/945*(sqrt(2)*(8*a^3*c^5 - 54*a^3*c^4*d + 219*a^3*c^3*d^2 - 3*a^3*c^2*d^3
 - 699*a^3*c*d^4 - 495*a^3*d^5)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 -
 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d
*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(8*a^3*c^5 - 54*a^3*c^4*d + 219*a^3*c^3
*d^2 - 3*a^3*c^2*d^3 - 699*a^3*c*d^4 - 495*a^3*d^5)*sqrt(-I*d)*weierstrassP
Inverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*
d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(4*I*a^3*c^4*d
- 27*I*a^3*c^3*d^2 + 111*I*a^3*c^2*d^3 + 579*I*a^3*c*d^4 + 357*I*a^3*d^5)*s
qrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d
^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I
*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sq
rt(2)*(-4*I*a^3*c^4*d + 27*I*a^3*c^3*d^2 - 111*I*a^3*c^2*d^3 - 579*I*a^3*c*
d^4 - 357*I*a^3*d^5)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
 + e) + 2*I*c)/d)) - 3*(5*(10*a^3*c*d^4 + 27*a^3*d^5)*cos(f*x + e)^3 + (4*a
^3*c^3*d^2 - 27*a^3*c^2*d^3 - 516*a^3*c*d^4 - 465*a^3*d^5)*cos(f*x + e) + (
35*a^3*d^5*cos(f*x + e)^3 - 3*(a^3*c^2*d^3 + 72*a^3*c*d^4 + 91*a^3*d^5)*cos
(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int c\sqrt{c+d\sin{\left(e+fx\right)}}\,dx+\int 3c\sqrt{c+d\sin{\left(e+fx\right)}}\sin{\left(e+fx\right)}\,dx+\int 3c\sqrt{c+d\sin{\left(e+fx\right)}}\sin^{2}{\left(e+fx\right)}\,dx+\int c\sqrt{c+d\sin{\left(e+fx\right)}}\sin^{3}{\left(e+fx\right)}\,dx+\int d\sqrt{c+d\sin{\left(e+fx\right)}}\sin{\left(e+fx\right)}\,dx+\int 3d\sqrt{c+d\sin{\left(e+fx\right)}}\sin^{2}{\left(e+fx\right)}\,dx+\int 3d\sqrt{c+d\sin{\left(e+fx\right)}}\sin^{3}{\left(e+fx\right)}\,dx+\int d\sqrt{c+d\sin{\left(e+fx\right)}}\sin^{4}{\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(3*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x), x) + Integral(3*c*sqrt(c + d*sin(e + f*x))*sin(e +
 f*x)**2, x) + Integral(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + In
tegral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*d*sqrt(c +
d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(3*d*sqrt(c + d*sin(e + f*x))
*sin(e + f*x)**3, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4,
 x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\,\sin(e + f\,x))^3\,(c + d\,\sin(e + f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.498 $\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=318

$$-\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(c-4d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2}{}$$

```
[Out] 8/35*a^3*(c-4*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f-2/7*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/d/f-4/105*a^3*(4*c^2-21*c*d+65*d^2
)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-4/105*a^3*(4*c^3-21*c^2*d+62*c*d^
2+147*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))
^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/105*a^3*(c^2-d^2)*(4*c^2-21*c
*d+65*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e)
)/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.39, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2842, 3047, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}-\frac{4a^3(c^2-d^2)(4c^2-21cd+65d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(4c^3-21c^2d+62cd^2+147d^3)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-4d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2\cos(e+fx)(a^3\sin(e+fx)+a^3)(c+d\sin(e+fx))^{3/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*(4*c^2 - 21*c*d + 65*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(1
05*d^2*f) + (8*a^3*(c - 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d
^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
)/(7*d*f) + (4*a^3*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^2 - 21*c*d + 65*d^2)*E
llipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}$$

$$= \frac{8a^3(c-4d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f} - \frac{2\cos(e+f}{}$$

$$= -\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= -\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= -\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= -\frac{4a^3(4c^2-21cd+65d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

**Mathematica [A]**

time = 1.81, size = 266, normalized size = 0.84

$$-\frac{a^3 \left(16(4c^4-17c^3d+41c^2d^2+209cd^3+147d^4) E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right) \sqrt{\frac{c+d\sin(e+fx)}{c+d}} - 16(4c^4-21c^3d+61c^2d^2+21cd^3-65d^4) F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right) \sqrt{\frac{c+d\sin(e+fx)}{c+d}} - 2d\cos(e+fx)\left(16c^3-84c^2d-556cd^2-126d^3+18d^2(2c+7d)\cos(2(e+fx))+d(4c^2-336cd-565d^2)\sin(e+fx)+15d^3\sin(3(e+fx))\right)\right)}{420d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/420*(a^3*(16*(4*c^4 - 17*c^3*d + 41*c^2*d^2 + 209*c*d^3 + 147*d^4)*Ellip
ticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)] - 16*(4*c^4 - 21*c^3*d + 61*c^2*d^2 + 21*c*d^3 - 65*d^4)*EllipticF[(-2*e
 + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*d*C
os[e + f*x]*(16*c^3 - 84*c^2*d - 556*c*d^2 - 126*d^3 + 18*d^2*(2*c + 7*d)*C
os[2*(e + f*x)] + d*(4*c^2 - 336*c*d - 565*d^2)*Sin[e + f*x] + 15*d^3*Sin[3
*(e + f*x)])))/(d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(360) = 720$.
time = 5.97, size = 1316, normalized size = 4.14

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1316 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*a^3*(18*c*d^4*sin(f*x+e)^4-c^2*d^3*sin(f*x+e)^3+84*c*d^4*sin(f*x+e)^3
-4*c^3*d^2*sin(f*x+e)^2+21*c^2*d^3*sin(f*x+e)^2+112*c*d^4*sin(f*x+e)^2+c^2*
d^3*sin(f*x+e)-84*c*d^4*sin(f*x+e)-336*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(si
n(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+124*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+15*d
^5*sin(f*x+e)^5+63*d^5*sin(f*x+e)^4+115*d^5*sin(f*x+e)^3-63*d^5*sin(f*x+e)^
2-130*d^5*sin(f*x+e)-21*c^2*d^3-130*c*d^4+4*c^3*d^2+294*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-424*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e
))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))*d^5-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^5+416*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(
c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+48*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+42*((c+d*sin(f*x+e))
```

```
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d+8*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x
+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))*c^4*d-48*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^3*d^2-116*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2)/d^4/cos(f*x+e)/(c+d*sin(f*
x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 638, normalized size = 2.01

$$-\frac{2\left(\sqrt{2}\left(8a^3c^4-42a^3c^3d+121a^3c^2d^2-84a^3cd^3-195a^3d^4\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}\left(8a^3c^4-42a^3c^3d+121a^3c^2d^2-84a^3cd^3-195a^3d^4\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)+3\sqrt{2}\left(4ia^3c^3d-21ia^3c^2d^2+62ia^3cd^3+147ia^3d^4\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3\sqrt{2}\left(-4ia^3c^3d+21ia^3c^2d^2-62ia^3cd^3-147ia^3d^4\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)-3\left(15a^3d^4\cos(fx+e)^3-3(a^3cd^3+21a^3d^4)\cos(fx+e)\sin(fx+e)+(4a^3c^2d^2-21a^3cd^3-145a^3d^4)\cos(fx+e)\right)\sqrt{d\sin(fx+e)+c}\right)}{315d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(8*a^3*c^4 - 42*a^3*c^3*d + 121*a^3*c^2*d^2 - 84*a^3*c*d^3
- 195*a^3*d^4)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/2
7*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2
*I*c)/d) + sqrt(2)*(8*a^3*c^4 - 42*a^3*c^3*d + 121*a^3*c^2*d^2 - 84*a^3*c*d
^3 - 195*a^3*d^4)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2,
-8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e
) + 2*I*c)/d) + 3*sqrt(2)*(4*I*a^3*c^3*d - 21*I*a^3*c^2*d^2 + 62*I*a^3*c*d^
3 + 147*I*a^3*d^4)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/2
7*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2,
-8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e)
 - 2*I*c)/d)) + 3*sqrt(2)*(-4*I*a^3*c^3*d + 21*I*a^3*c^2*d^2 - 62*I*a^3*c*d
^3 - 147*I*a^3*d^4)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
+ e) + 2*I*c)/d)) - 3*(15*a^3*d^4*cos(f*x + e)^3 - 3*(a^3*c*d^3 + 21*a^3*d^
4)*cos(f*x + e)*sin(f*x + e) + (4*a^3*c^2*d^2 - 21*a^3*c*d^3 - 145*a^3*d^4)
*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sqrt{c+d\sin(e+fx)}\,\sin(e+fx)\,dx+\int 3\sqrt{c+d\sin(e+fx)}\,\sin^2(e+fx)\,dx+\int \sqrt{c+d\sin(e+fx)}\,\sin^3(e+fx)\,dx+\int \sqrt{c+d\sin(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*sqr
t(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x
))*sin(e + f*x)**3, x) + Integral(sqrt(c + d*sin(e + f*x)), x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin(e+f\,x))^3\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.499 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=258

$$\frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f}-\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df}+\frac{4a^3(}{}$$

```
[Out] 8/15*a^3*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/5*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/d/f-4/15*a^3*(4*c^2-15*c*d+27*d^2)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/15*a^3*(c-d)*(4*c^2-11*c*d+15*d^2)*(sin
(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2
*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/
d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.31, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2842, 3047, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(c-d)\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2-15cd+27d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{15d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f}-\frac{2\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)\sqrt{c+d\sin(e+fx)}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (8*a^3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) + (4*a
^3*(4*c^2 - 15*c*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c - d)*(4*c^2 - 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))(a}{\sqrt{c+}}}{}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{a^3(c+3d)+(-2a^3(c}{}}{}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+f\right.}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+f\right.}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+f\right.}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+f\right.}{5df}$$

**Mathematica [A]**
time = 1.10, size = 246, normalized size = 0.95

$$-\frac{a^3(1+\sin(e+fx))^3\left(4(4c^3-11c^2d+12cd^2+27d^3)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-4(4c^3-15c^2d+26cd^2-15d^3)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-d\cos(e+fx)\left(8c^2-30cd-3d^2+3d^2\cos(2(e+fx))+2(c-15d)d\sin(e+fx)\right)\right)}{15d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/15*(a^3*(1 + Sin[e + f*x])^3*(4*(4*c^3 - 11*c^2*d + 12*c*d^2 + 27*d^3)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
```

```
 + d)] - 4*(4*c^3 - 15*c^2*d + 26*c*d^2 - 15*d^3)*EllipticF[(-2*e + Pi - 2*
f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*
(8*c^2 - 30*c*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] + 2*(c - 15*d)*d*Sin[e + f
*x])))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*
x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(304) = 608$.

time = 5.10, size = 1035, normalized size = 4.01

| method | result |
|---|---|
| default | $\frac{2a^3\left(8\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3d-36c^2\sqrt{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a^3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^3*d-36*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2+112*c*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-84*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d)
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-8
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*c^4+30*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*c^3*d-46*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-30*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+54*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4
+3*d^4*sin(f*x+e)^4-c*d^3*sin(f*x+e)^3+15*d^4*sin(f*x+e)^3-4*c^2*d^2*sin(f*
x+e)^2+15*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)-15*d^4*sin
(f*x+e)+4*c^2*d^2-15*d^3*c)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 556, normalized size = 2.16

$$-\frac{2\left(\sqrt{2}\left(8a^3c^3-30a^3c^2d+51a^3cd^2-45a^3d^3\right)\sqrt{i d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)+\sqrt{2}\left(8a^3c^3-30a^3c^2d+51a^3cd^2-45a^3d^3\right)\sqrt{-i d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)+3\sqrt{2}\left(4ia^3c^2d-15ia^3cd^2+27ia^3d^3\right)\sqrt{i d}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)\right)+3\sqrt{2}\left(-4ia^3c^2d+15ia^3cd^2-27ia^3d^3\right)\sqrt{-i d}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)\right)+3\left(3a^3d^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(4a^3cd^2-15a^3d^3\right)\cos\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}\right)}{45d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(sqrt(2)*(8*a^3*c^3 - 30*a^3*c^2*d + 51*a^3*c*d^2 - 45*a^3*d^3)*sqrt(
I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d
^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(
8*a^3*c^3 - 30*a^3*c^2*d + 51*a^3*c*d^2 - 45*a^3*d^3)*sqrt(-I*d)*weierstras
sPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(
3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(4*I*a^3*c^2*
d - 15*I*a^3*c*d^2 + 27*I*a^3*d^3)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 -
3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^
2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*
I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*(-4*I*a^3*c^2*d + 15*I*a^3*c*d^2
- 27*I*a^3*d^3)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*
(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e)
 + 2*I*c)/d)) + 3*(3*a^3*d^3*cos(f*x + e)*sin(f*x + e) - (4*a^3*c*d^2 - 15*
a^3*d^3)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{3\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int\frac{3\sin^2\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int\frac{\sin^3\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int\frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(3*sin
(e + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e + f*x)**3/sqrt(c
 + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x)), x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin(e + f\,x))^3}{\sqrt{c + d\,\sin(e + f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2), x)
```

## 3.500 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=270

$$\frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2(c+d)f}-\frac{4a^3(4c^2-5cd-3\cdots}{3\cdots}$$

```
[Out] 2*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-4/
3*a^3*(2*c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)/f+4/3*a^3*(4*c^2-
5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e
))^(1/2)/d^3/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/3*a^3*(4*c-5*d)*(c-d)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(
1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.32, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2841, 3047, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2-5cd-3d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^3(4c-5d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f(c+d)}+\frac{2(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*Sqrt[c + d*S
in[e + f*x]]) - (4*a^3*(2*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*(c + d)*f) - (4*a^3*(4*c^2 - 5*c*d - 3*d^2)*EllipticE[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + (4*a^3*(4*c - 5*d)*(c - d)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-2d)-a(2c-d}{\sqrt{c+d\sin(e+fx}}}{d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{a^2(c-2d)+\left(a^2(c-2d)-a^2(2c-d)\right)s}{\sqrt{c+d\sin}}}{d(c+d}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\,s}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\,s}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\,s}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\,s}}{3d^2(c+d)f}$$

**Mathematica** [A]
time = 0.99, size = 234, normalized size = 0.87

$$-\frac{2a^3(1+\sin(e+fx))^3\left(-2(4c^3-c^2d-8cd^2-3d^3)\,E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+2(4c^3-5c^2d-4cd^2+5d^3)\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d\cos(e+fx)\left(4c^2-5cd+3d^2+d(c+d)\sin(e+fx)\right)\right)}{3d^3(c+d)f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(4*c^3 - c^2*d - 8*c*d^2 - 3*d^3)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
```

```
+ 2*(4*c^3 - 5*c^2*d - 4*c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(4*c^2 - 5
*c*d + 3*d^2 + d*(c + d)*Sin[e + f*x])))/(3*d^3*(c + d)*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(318) = 636$.
time = 5.33, size = 1031, normalized size = 3.82

| method | result |
|---|---|
| default | $-\frac{2\left(8\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3d-16c^2\sqrt{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))*c^3*d-16*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-8*c*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+16*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-8*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^3*d+14*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-6*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-c*d^3*
sin(f*x+e)^3-d^4*sin(f*x+e)^3-4*c^2*d^2*sin(f*x+e)^2+5*c*d^3*sin(f*x+e)^2-3
*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)+d^4*sin(f*x+e)+4*c^2*d^2-5*d^3*c+3*d^4)*
a^3/d^4/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 803, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/9*((sqrt(2)*(8*a^3*c^3*d - 10*a^3*c^2*d^2 - 9*a^3*c*d^3 + 15*a^3*d^4)*sin(f*x + e) + sqrt(2)*(8*a^3*c^4 - 10*a^3*c^3*d - 9*a^3*c^2*d^2 + 15*a^3*c*d^3))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*a^3*c^3*d - 10*a^3*c^2*d^2 - 9*a^3*c*d^3 + 15*a^3*d^4)*sin(f*x + e) + sqrt(2)*(8*a^3*c^4 - 10*a^3*c^3*d - 9*a^3*c^2*d^2 + 15*a^3*c*d^3))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-4*I*a^3*c^2*d^2 + 5*I*a^3*c*d^3 + 3*I*a^3*d^4)*sin(f*x + e) + sqrt(2)*(-4*I*a^3*c^3*d + 5*I*a^3*c^2*d^2 + 3*I*a^3*c*d^3))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(4*I*a^3*c^2*d^2 - 5*I*a^3*c*d^3 - 3*I*a^3*d^4)*sin(f*x + e) + sqrt(2)*(4*I*a^3*c^3*d - 5*I*a^3*c^2*d^2 - 3*I*a^3*c*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 3*((a^3*c*d^3 + a^3*d^4)*cos(f*x + e)*sin(f*x + e) + (4*a^3*c^2*d^2 - 5*a^3*c*d^3 + 3*a^3*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f)

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)

[Out] Timed out

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.501 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=280

$$\frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}}+\frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(4c^2+5cd-3d^2)\,E\left(\frac{1}{2}\left(e-\right.\right.}{3d^3(c+d)^2f}$$

```
[Out] 2/3*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)+
8/3*a^3*(c-d)*(c+2*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)-4/3*a
^3*(4*c^2+5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi
+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d
*sin(f*x+e))^(1/2)/d^3/(c+d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/3*a^3*(c-
d)*(4*c+5*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+
e))/(c+d))^(1/2)/d^3/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.38, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2841, 3047, 3100, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a^3(4c^2+5cd-3d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^3f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^3(c-d)(4c+5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{c+d\sin(e+fx)}}+\frac{2(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{3df(c+d)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*f*(c + d*Sin
[e + f*x])^(3/2)) + (8*a^3*(c - d)*(c + 2*d)*Cos[e + f*x])/(3*d^2*(c + d)^2
*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^3*(4*c^2 + 5*c*d - 3*d^2)*EllipticE[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)^2*
f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c - d)*(4*c + 5*d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^3*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-4d)-a(2c+d}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{a^2(c-4d)+(a^2(c-4d)-a^2(2c+d))\,s}{(c+d\sin(e-}}{3d(c+}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}}$$

## Mathematica [A]

time = 1.05, size = 232, normalized size = 0.83

$$-\frac{2a^3(1+\sin(e+fx))^3\left(2(c+d)\left(d^2(c+5d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+\left(4c^2+5cd-3d^2\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}+d(-c+d)\cos(e+fx)\left(4c^2+9cd+d^2+d(5c+9d)\sin(e+fx)\right)\right)}{3d^3(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(2*(c + d)*(d^2*(c + 5*d)*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)] + (4*c^2 + 5*c*d - 3*d^2)*((c + d)*EllipticE[(-
```

```
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))^(3/2) + d*(-c + d)*Cos[e + f*
x]*(4*c^2 + 9*c*d + d^2 + d*(5*c + 9*d)*Sin[e + f*x])))/(3*d^3*(c + d)^2*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(326) = 652$.
time = 20.67, size = 1257, normalized size = 4.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1257 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(-2/d^4/(sin(f*x+e)*cos(f*x+e)^
2*d+cos(f*x+e)^2*c)^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c-d)*si
n(f*x+e)-d/(c-d))^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(EllipticE((d/(
c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-EllipticE((d/(c-d
)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2+2*EllipticF((d/(c-d)
*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-6*EllipticF((d/(c-d)*
sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d+4*EllipticF((d/(c-d)*s
in(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2)+1/d^3*(-c^3+3*c^2*d-3*
c*d^2+d^3)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+3/d^3*(c^2-2*c*d+d^2)*(2*d*cos(f*x+e)^2/(c^2-d^2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 1173, normalized size = 4.19

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -2/9*((sqrt(2)*(8*a^3*c^3*d^2 + 10*a^3*c^2*d^3 - 9*a^3*c*d^4 - 15*a^3*d^5)*cos(f*x + e)^2 - 2*sqrt(2)*(8*a^3*c^4*d + 10*a^3*c^3*d^2 - 9*a^3*c^2*d^3 - 15*a^3*c*d^4)*sin(f*x + e) - sqrt(2)*(8*a^3*c^5 + 10*a^3*c^4*d - a^3*c^3*d^2 - 5*a^3*c^2*d^3 - 9*a^3*c*d^4 - 15*a^3*d^5))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*a^3*c^3*d^2 + 10*a^3*c^2*d^3 - 9*a^3*c*d^4 - 15*a^3*d^5)*cos(f*x + e)^2 - 2*sqrt(2)*(8*a^3*c^4*d + 10*a^3*c^3*d^2 - 9*a^3*c^2*d^3 - 15*a^3*c*d^4)*sin(f*x + e) - sqrt(2)*(8*a^3*c^5 + 10*a^3*c^4*d - a^3*c^3*d^2 - 5*a^3*c^2*d^3 - 9*a^3*c*d^4 - 15*a^3*d^5))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-4*I*a^3*c^2*d^3 - 5*I*a^3*c*d^4 + 3*I*a^3*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*a^3*c^3*d^2 + 5*I*a^3*c^2*d^3 - 3*I*a^3*c*d^4)*sin(f*x + e) + sqrt(2)*(4*I*a^3*c^4*d + 5*I*a^3*c^3*d^2 + I*a^3*c^2*d^3 + 5*I*a^3*c*d^4 - 3*I*a^3*d^5))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(4*I*a^3*c^2*d^3 + 5*I*a^3*c*d^4 - 3*I*a^3*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*a^3*c^3*d^2 - 5*I*a^3*c^2*d^3 + 3*I*a^3*c*d^4)*sin(f*x + e) + sqrt(2)*(-4*I*a^3*c^4*d - 5*I*a^3*c^3*d^2 - I*a^3*c^2*d^3 - 5*I*a^3*c*d^4 + 3*I*a^3*d^5))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 3*((5*a^3*c^2*d^3 + 4*a^3*c*d^4 - 9*a^3*d^5)*cos(f*x + e)*sin(f*x + e) + (4*a^3*c^3*d^2 + 5*a^3*c^2*d^3 - 8*a^3*c*d^4 - a^3*d^5)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f)

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^3}{(c+d\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.502 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=336

$$\frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}}+\frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}}-\frac{4a^3(4c^2+15cd+27d^2)\cos}{15d^2(c+d)^3f\sqrt{c+d\sin(}}$$

```
[Out] 2/5*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)+
8/15*a^3*(c-d)*(c+3*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-4/15
*a^3*(4*c^2+15*c*d+27*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+
4/15*a^3*(4*c^2+15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/1
5*a^3*(4*c^2+11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.48, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2841, 3047, 3100, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2+15cd+27d^2)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(4c^2+11cd+15d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(4c^2+15cd+27d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2f(c+d)^2(c+d\sin(e+fx))^{3/2}}+\frac{2(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{5df(c+d)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(5*d*(c + d)*f*(c + d*Sin
[e + f*x])^(5/2)) + (8*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(15*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^2 + 15*c*d + 27*d^2)*Cos[e +
f*x])/(15*d^2*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^2 + 15*c*
d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e +
 f*x]])/(15*d^3*(c + d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4
*c^2 + 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(15*d^3*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-6d)-a(2c+3}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{a^2(c-6d)+\left(a^2(c-6d)-a^2(2c+3d)\right)}{(c+d\sin(e}}{5d(c+}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx}{15d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx}{15d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx}{15d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx}{15d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx}{15d^2(c+d)^2f(c+d\sin(e+fx))}$$

**Mathematica [A]**
time = 1.41, size = 298, normalized size = 0.89

$$-\frac{2a^3(1+\sin(e+fx))^3\left(-2\left((c-15d)d^2F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+(4c^2+15cd+27d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)(c+d\sin(e+fx))^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d\cos(e+fx)\left(4c^4+15c^3d+55c^2d^2+15cd^3+3d^4+d(9c^3+45c^2d+115cd^2+15d^3)\sin(e+fx)+2d^2(4c^2+15cd+27d^2)\sin^2(e+fx)\right)\right)}{15d^3(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*((c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Co
s[e + f*x]*(4*c^4 + 15*c^3*d + 55*c^2*d^2 + 15*c*d^3 + 3*d^4 + d*(9*c^3 + 4
5*c^2*d + 115*c*d^2 + 15*d^3)*Sin[e + f*x] + 2*d^2*(4*c^2 + 15*c*d + 27*d^2
)*Sin[e + f*x]^2)))/(15*d^3*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^6*(c + d*Sin[e + f*x])^(5/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(378) = 756$.
time = 27.17, size = 1589, normalized size = 4.73

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1589 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(2/d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+1/d^3*(-c^3+3*c^2*d-3*c*d^2+d^3)*(2/5/(c^2-d^
2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(
c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/1
5*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+
e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^
2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-
c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3/d^3*(c^2-2*c*d+
d^2)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c
/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
```

```
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))))+3*(-c+d)/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 1622, normalized size = 4.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/45*((3*sqrt(2)*(8*a^3*c^4*d^2 + 30*a^3*c^3*d^3 + 51*a^3*c^2*d^4 + 45*a^3*
c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(8*a^3*c^3*d^3 + 30*a^3*c^2*d^4 + 51*a^3*c
*d^5 + 45*a^3*d^6)*cos(f*x + e)^2 - sqrt(2)*(24*a^3*c^5*d + 90*a^3*c^4*d^2
+ 161*a^3*c^3*d^3 + 165*a^3*c^2*d^4 + 51*a^3*c*d^5 + 45*a^3*d^6))*sin(f*x +
 e) - sqrt(2)*(8*a^3*c^6 + 30*a^3*c^5*d + 75*a^3*c^4*d^2 + 135*a^3*c^3*d^3
+ 153*a^3*c^2*d^4 + 135*a^3*c*d^5))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c
^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3
*I*d*sin(f*x + e) - 2*I*c)/d) + (3*sqrt(2)*(8*a^3*c^4*d^2 + 30*a^3*c^3*d^3
+ 51*a^3*c^2*d^4 + 45*a^3*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(8*a^3*c^3*d^3 +
 30*a^3*c^2*d^4 + 51*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e)^2 - sqrt(2)*(24*a
^3*c^5*d + 90*a^3*c^4*d^2 + 161*a^3*c^3*d^3 + 165*a^3*c^2*d^4 + 51*a^3*c*d^
5 + 45*a^3*d^6))*sin(f*x + e) - sqrt(2)*(8*a^3*c^6 + 30*a^3*c^5*d + 75*a^3*
```

```
c^4*d^2 + 135*a^3*c^3*d^3 + 153*a^3*c^2*d^4 + 135*a^3*c*d^5))*sqrt(-I*d)*we
ierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^
3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(3*sqrt(2)*(-
4*I*a^3*c^3*d^3 - 15*I*a^3*c^2*d^4 - 27*I*a^3*c*d^5)*cos(f*x + e)^2 + (sqrt
(2)*(-4*I*a^3*c^2*d^4 - 15*I*a^3*c*d^5 - 27*I*a^3*d^6)*cos(f*x + e)^2 + sqr
t(2)*(12*I*a^3*c^4*d^2 + 45*I*a^3*c^3*d^3 + 85*I*a^3*c^2*d^4 + 15*I*a^3*c*d
^5 + 27*I*a^3*d^6))*sin(f*x + e) + sqrt(2)*(4*I*a^3*c^5*d + 15*I*a^3*c^4*d^
2 + 39*I*a^3*c^3*d^3 + 45*I*a^3*c^2*d^4 + 81*I*a^3*c*d^5))*sqrt(I*d)*weiers
trassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weiers
trassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/
3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(3*sqrt(2)*(4*I*a
^3*c^3*d^3 + 15*I*a^3*c^2*d^4 + 27*I*a^3*c*d^5)*cos(f*x + e)^2 + (sqrt(2)*(
4*I*a^3*c^2*d^4 + 15*I*a^3*c*d^5 + 27*I*a^3*d^6)*cos(f*x + e)^2 + sqrt(2)*(
-12*I*a^3*c^4*d^2 - 45*I*a^3*c^3*d^3 - 85*I*a^3*c^2*d^4 - 15*I*a^3*c*d^5 -
27*I*a^3*d^6))*sin(f*x + e) + sqrt(2)*(-4*I*a^3*c^5*d - 15*I*a^3*c^4*d^2 -
39*I*a^3*c^3*d^3 - 45*I*a^3*c^2*d^4 - 81*I*a^3*c*d^5))*sqrt(-I*d)*weierstra
ssZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstr
assPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 3*(2*(4*a^3*c^2*d^4
+ 15*a^3*c*d^5 + 27*a^3*d^6)*cos(f*x + e)^3 - (9*a^3*c^3*d^3 + 45*a^3*c^2*d
^4 + 115*a^3*c*d^5 + 15*a^3*d^6)*cos(f*x + e)*sin(f*x + e) - (4*a^3*c^4*d^2
 + 15*a^3*c^3*d^3 + 63*a^3*c^2*d^4 + 45*a^3*c*d^5 + 57*a^3*d^6)*cos(f*x + e
))*sqrt(d*sin(f*x + e) + c))/(3*(c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 + c*d^9)*f
*cos(f*x + e)^2 - (c^6*d^4 + 3*c^5*d^5 + 6*c^4*d^6 + 10*c^3*d^7 + 9*c^2*d^8
 + 3*c*d^9)*f + ((c^3*d^7 + 3*c^2*d^8 + 3*c*d^9 + d^10)*f*cos(f*x + e)^2 -
(3*c^5*d^5 + 9*c^4*d^6 + 10*c^3*d^7 + 6*c^2*d^8 + 3*c*d^9 + d^10)*f)*sin(f*
x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sin(e + fx))^3}{(c + d\sin(e + fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.503 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=419

$$\frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}}+\frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}}-\frac{4a^3(4c^2+21cd+65d^2)\,\mathrm{c}}{105d^2(c+d)^3f(c+d\sin(}$$

```
[Out] 2/7*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)+
8/35*a^3*(c-d)*(c+4*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)-4/10
5*a^3*(4*c^2+21*c*d+65*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^(3/2)
-4/105*a^3*(4*c^3+21*c^2*d+62*c*d^2-147*d^3)*cos(f*x+e)/(c-d)/d^2/(c+d)^4/f
/(c+d*sin(f*x+e))^(1/2)+4/105*a^3*(4*c^3+21*c^2*d+62*c*d^2-147*d^3)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)/d^3/(
c+d)^4/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/105*a^3*(4*c^2+21*c*d+65*d^2)*(si
n(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/
2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)
/d^3/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.61, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2841, 3047, 3100, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a^3(4c^2+21cd+65d^2)\cos(e+fx)}{105d^2f(c+d)^3(c+d\sin(e+fx))^{3/2}}+\frac{4a^3(4c^2+21cd+65d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(4c^3+21c^2d+62cd^2-147d^3)\cos(e+fx)}{105d^2f(c-d)(c+d)^4\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(4c^3+21c^2d+62cd^2-147d^3)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f(c-d)(c+d)^4\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2f(c+d)^2(c+d\sin(e+fx))^{5/2}}+\frac{2(c-d)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{7df(c+d)(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(7*d*(c + d)*f*(c + d*Sin
[e + f*x])^(7/2)) + (8*a^3*(c - d)*(c + 4*d)*Cos[e + f*x])/(35*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(5/2)) - (4*a^3*(4*c^2 + 21*c*d + 65*d^2)*Cos[e +
f*x])/(105*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^3 + 21
*c^2*d + 62*c*d^2 - 147*d^3)*Cos[e + f*x])/(105*(c - d)*d^2*(c + d)^4*f*Sqr
t[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^3)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*(c
 - d)*d^3*(c + d)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4*c^2 +
 21*c*d + 65*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(105*d^3*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-8d)-a(2c+5}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{a^2(c-8d)+\left(a^2(c-8d)-a^2(2c+5d)\right)}{(c+d\sin(e}}{7d(c+}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx}{35d^2(c+d)^2f(c+d\sin(e+fx))}$$

**Mathematica [A]**
time = 2.43, size = 351, normalized size = 0.84

$$-\frac{2a^3(1+\sin(e+fx))^3\left(-2\left(d^2\left(c^2-126cd+65d^2\right)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+\left(4c^3+21c^2d+62cd^2-147d^3\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)(c+d\sin(e+fx))^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d\cos(e+fx)\left(15(c-d)^3(c+d)^3-9(c-d)^2(c+d)^2(3c+7d)(c+d\sin(e+fx))+2(c-d)(c+d)\left(4c^2+21cd+65d^2\right)(c+d\sin(e+fx))^2+2\left(4c^3+21c^2d+62cd^2-147d^3\right)(c+d\sin(e+fx))^3\right)\right)}{105(c-d)d^3(c+d)^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(d^2*(c^2 - 126*c*d + 65*d^2)*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^
3)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(
-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(c + d*Sin[e + f*x])^3*Sqrt[(c + d*S
in[e + f*x])/(c + d)] + d*Cos[e + f*x]*(15*(c - d)^3*(c + d)^3 - 9*(c - d)^
```

```
2*(c + d)^2*(3*c + 7*d)*(c + d*Sin[e + f*x]) + 2*(c - d)*(c + d)*(4*c^2 + 2
1*c*d + 65*d^2)*(c + d*Sin[e + f*x])^2 + 2*(4*c^3 + 21*c^2*d + 62*c*d^2 - 1
47*d^3)*(c + d*Sin[e + f*x])^3)))/(105*(c - d)*d^3*(c + d)^4*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(7/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(457) = 914$.
time = 38.30, size = 2079, normalized size = 4.96

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2079 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(3*(c^2-2*c*d+d^2)/d^3*(2/5/(c^
2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15
*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2
+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(
f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^
2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(-c+d)/d^3*
(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2
+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2
*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))))+1/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(-
c^3+3*c^2*d-3*c*d^2+d^3)/d^3*(2/7/(c^2-d^2)/d^3*(-(-d*sin(f*x+e)-c)*cos(f*x
```

```
+e)^2)^(1/2)/(sin(f*x+e)+c/d)^4+24/35/(c^2-d^2)^2/d^2*c*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+2/105*(71*c^2+25*d^2)/d/(c^2-d^2)^3
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+32/105*d*cos(f*
x+e)^2/(c^2-d^2)^4*c*(11*c^2+13*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)+2*(105*c^4+254*c^2*d^2+25*d^4)/(105*c^8-420*c^6*d^2+630*c^4*d^4-420*c^2*d
^6+105*d^8)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+32/105*c*
d*(11*c^2+13*d^2)/(c^2-d^2)^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 2483, normalized size = 5.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*((sqrt(2)*(8*a^3*c^4*d^4 + 42*a^3*c^3*d^5 + 121*a^3*c^2*d^6 + 84*a^3*
c*d^7 - 195*a^3*d^8)*cos(f*x + e)^4 - 2*sqrt(2)*(24*a^3*c^6*d^2 + 126*a^3*c
^5*d^3 + 371*a^3*c^4*d^4 + 294*a^3*c^3*d^5 - 464*a^3*c^2*d^6 + 84*a^3*c*d^7
 - 195*a^3*d^8)*cos(f*x + e)^2 - 4*(sqrt(2)*(8*a^3*c^5*d^3 + 42*a^3*c^4*d^4
 + 121*a^3*c^3*d^5 + 84*a^3*c^2*d^6 - 195*a^3*c*d^7)*cos(f*x + e)^2 - sqrt(
2)*(8*a^3*c^7*d + 42*a^3*c^6*d^2 + 129*a^3*c^5*d^3 + 126*a^3*c^4*d^4 - 74*a
^3*c^3*d^5 + 84*a^3*c^2*d^6 - 195*a^3*c*d^7))*sin(f*x + e) + sqrt(2)*(8*a^3
*c^8 + 42*a^3*c^7*d + 169*a^3*c^6*d^2 + 336*a^3*c^5*d^3 + 539*a^3*c^4*d^4 +
 546*a^3*c^3*d^5 - 1049*a^3*c^2*d^6 + 84*a^3*c*d^7 - 195*a^3*d^8))*sqrt(I*d
)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)
/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*
a^3*c^4*d^4 + 42*a^3*c^3*d^5 + 121*a^3*c^2*d^6 + 84*a^3*c*d^7 - 195*a^3*d^8
)*cos(f*x + e)^4 - 2*sqrt(2)*(24*a^3*c^6*d^2 + 126*a^3*c^5*d^3 + 371*a^3*c^
4*d^4 + 294*a^3*c^3*d^5 - 464*a^3*c^2*d^6 + 84*a^3*c*d^7 - 195*a^3*d^8)*cos
```

(f*x + e)^2 - 4*(sqrt(2)*(8*a^3*c^5*d^3 + 42*a^3*c^4*d^4 + 121*a^3*c^3*d^5 + 84*a^3*c^2*d^6 - 195*a^3*c*d^7)*cos(f*x + e)^2 - sqrt(2)*(8*a^3*c^7*d + 42*a^3*c^6*d^2 + 129*a^3*c^5*d^3 + 126*a^3*c^4*d^4 - 74*a^3*c^3*d^5 + 84*a^3*c^2*d^6 - 195*a^3*c*d^7))*sin(f*x + e) + sqrt(2)*(8*a^3*c^8 + 42*a^3*c^7*d + 169*a^3*c^6*d^2 + 336*a^3*c^5*d^3 + 539*a^3*c^4*d^4 + 546*a^3*c^3*d^5 - 1049*a^3*c^2*d^6 + 84*a^3*c*d^7 - 195*a^3*d^8))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-4*I*a^3*c^3*d^5 - 21*I*a^3*c^2*d^6 - 62*I*a^3*c*d^7 + 147*I*a^3*d^8)*cos(f*x + e)^4 + 2*sqrt(2)*(12*I*a^3*c^5*d^3 + 63*I*a^3*c^4*d^4 + 190*I*a^3*c^3*d^5 - 420*I*a^3*c^2*d^6 + 62*I*a^3*c*d^7 - 147*I*a^3*d^8)*cos(f*x + e)^2 + 4*(sqrt(2)*(4*I*a^3*c^4*d^4 + 21*I*a^3*c^3*d^5 + 62*I*a^3*c^2*d^6 - 147*I*a^3*c*d^7)*cos(f*x + e)^2 + sqrt(2)*(-4*I*a^3*c^6*d^2 - 21*I*a^3*c^5*d^3 - 66*I*a^3*c^4*d^4 + 126*I*a^3*c^3*d^5 - 62*I*a^3*c^2*d^6 + 147*I*a^3*c*d^7))*sin(f*x + e) + sqrt(2)*(-4*I*a^3*c^7*d - 21*I*a^3*c^6*d^2 - 86*I*a^3*c^5*d^3 + 21*I*a^3*c^4*d^4 - 376*I*a^3*c^3*d^5 + 861*I*a^3*c^2*d^6 - 62*I*a^3*c*d^7 + 147*I*a^3*d^8))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(4*I*a^3*c^3*d^5 + 21*I*a^3*c^2*d^6 + 62*I*a^3*c*d^7 - 147*I*a^3*d^8)*cos(f*x + e)^4 + 2*sqrt(2)*(-12*I*a^3*c^5*d^3 - 63*I*a^3*c^4*d^4 - 190*I*a^3*c^3*d^5 + 420*I*a^3*c^2*d^6 - 62*I*a^3*c*d^7 + 147*I*a^3*d^8)*cos(f*x + e)^2 + 4*(sqrt(2)*(-4*I*a^3*c^4*d^4 - 21*I*a^3*c^3*d^5 - 62*I*a^3*c^2*d^6 + 147*I*a^3*c*d^7)*cos(f*x + e)^2 + sqrt(2)*(4*I*a^3*c^6*d^2 + 21*I*a^3*c^5*d^3 + 66*I*a^3*c^4*d^4 - 126*I*a^3*c^3*d^5 + 62*I*a^3*c^2*d^6 - 147*I*a^3*c*d^7))*sin(f*x + e) + sqrt(2)*(4*I*a^3*c^7*d + 21*I*a^3*c^6*d^2 + 86*I*a^3*c^5*d^3 - 21*I*a^3*c^4*d^4 + 376*I*a^3*c^3*d^5 - 861*I*a^3*c^2*d^6 + 62*I*a^3*c*d^7 - 147*I*a^3*d^8))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 3*(2*(16*a^3*c^4*d^4 + 84*a^3*c^3*d^5 + 247*a^3*c^2*d^6 - 462*a^3*c*d^7 - 65*a^3*d^8)*cos(f*x + e)^3 - (4*a^3*c^6*d^2 + 21*a^3*c^5*d^3 + 287*a^3*c^4*d^4 - 42*a^3*c^3*d^5 + 382*a^3*c^2*d^6 - 987*a^3*c*d^7 - 145*a^3*d^8)*cos(f*x + e) + (2*(4*a^3*c^3*d^5 + 21*a^3*c^2*d^6 + 62*a^3*c*d^7 - 147*a^3*d^8)*cos(f*x + e)^3 - (13*a^3*c^5*d^3 + 147*a^3*c^4*d^4 + 678*a^3*c^3*d^5 - 798*a^3*c^2*d^6 - 163*a^3*c*d^7 - 357*a^3*d^8)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c^5*d^8 + 3*c^4*d^9 + 2*c^3*d^10 - 2*c^2*d^11 - 3*c*d^12 - d^13)*f*cos(f*x + e)^4 - 2*(3*c^7*d^6 + 9*c^6*d^7 + 7*c^5*d^8 - 3*c^4*d^9 - 7*c^3*d^10 - 5*c^2*d^11 - 3*c*d^12 - d^13)*f*cos(f*x + e)^2 + (c^9*d^4 + 3*c^8*d^5 + 8*c^7*d^6 + 16*c^6*d^7 + 10*c^5*d^8 - 10*c^4*d^9 - 16*c^3*d^10 - 8*c^2*d^11 - 3*c*d^12 - d^13)*f - 4*((c^6*d^7 + 3*c^5*d^8 + 2*c^4*d^9 - 2*c^3*d^10 - 3*c^2*d^11 - c*d^12)*f*cos(f*x + e)^2 - (c^8*d^5 + 3*c^7*d^6 + 3*c^6*d^7 + c^5*d^8 - c^4*d^9 - 3*c^3*d^10 - 3*c^2*d^11 - c*d^12)*f)*sin(f*x + e))

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin(e + f\,x))^3}{(c + d\,\sin(e + f\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2), x)
```

# 3.504 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=246

$$\frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))}-\frac{(3c^2-20cd+9d^2)}{}$$

```
[Out] -(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))+1/3*(3*c-5*d)*d
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f+1/3*(3*c^2-20*c*d+9*d^2)*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*sin(f
*x+e))/(c+d))^(1/2)-1/3*(3*c-5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.25, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2846, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{(3c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)}+\frac{d(3c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*c - 5*d)*d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f*x])) - ((3*c^2 -
 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c - 5*d)*(c^2
 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(3*a*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{d\int\left(-\frac{1}{2}a(5c-3d)+\frac{1}{2}a(3c-\right.}{}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{f(a+a\sin(e+fx)}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{f(a+a\sin(e+fx)}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{f(a+a\sin(e+fx)}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{f(a+a\sin(e+fx)}$$

**Mathematica** [A]
time = 0.95, size = 298, normalized size = 1.21

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-3(c-d)^2(c+d\sin(e+fx))-2d^2\cos(e+fx)(c+d\sin(e+fx))+\frac{6(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}-d(15c^2-12cd+5d^2)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(3c^2-20cd+9d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3af(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-3*(c - d)^2*(c + d*Sin[e + f*x])
 - 2*d^2*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (6*(c - d)^2*Sin[(e + f*x)/2]*
(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - d*(15*c^2 - 1
2*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)] + (3*c^2 - 20*c*d + 9*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*a*f*(1 + Sin[e + f*x])*Sq
rt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(294) = 588$.
time = 6.17, size = 1372, normalized size = 5.58

| method | result | size |
|---|---|---|

| default | Expression too large to display | 1372 |
|---|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(sin(f*x+e)*cos(f*x+e)^2*d+cos(f*x+e)^2*c)^(1/2)*(3*(d/(c-d)*sin(f*x+e)
+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-
d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))
^(1/2))*c^4-20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+6*(d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^2*d^2+20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c
+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e
)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-9*(d/(c-d)*sin(f*x+e)+1/(c-d)
*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))
^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*
d^4+12*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(
1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c
-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1
/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)
*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^
2-12*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2
)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*E
llipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^4-2*d^
4*sin(f*x+e)*cos(f*x+e)^2-3*cos(f*x+e)^2*c^2*d^2+4*cos(f*x+e)^2*c*d^3-3*cos
(f*x+e)^2*d^4+3*c^3*d*sin(f*x+e)-9*c^2*d^2*sin(f*x+e)+9*c*d^3*sin(f*x+e)-3*
d^4*sin(f*x+e)-3*c^3*d+9*c^2*d^2-9*d^3*c+3*d^4)/d/(-(c+d*sin(f*x+e))*(sin(f
*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 843, normalized size = 3.43

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/18*((sqrt(2)*(6*c^3 + 5*c^2*d - 18*c*d^2 + 15*d^3)*cos(f*x + e) + sqrt(2)
*(6*c^3 + 5*c^2*d - 18*c*d^2 + 15*d^3)*sin(f*x + e) + sqrt(2)*(6*c^3 + 5*c^
2*d - 18*c*d^2 + 15*d^3))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2
)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f
*x + e) - 2*I*c)/d) + (sqrt(2)*(6*c^3 + 5*c^2*d - 18*c*d^2 + 15*d^3)*cos(f*
x + e) + sqrt(2)*(6*c^3 + 5*c^2*d - 18*c*d^2 + 15*d^3)*sin(f*x + e) + sqrt(
2)*(6*c^3 + 5*c^2*d - 18*c*d^2 + 15*d^3))*sqrt(-I*d)*weierstrassPInverse(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x
+ e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-3*I*c^2*d + 20*I*c*d^2
 - 9*I*d^3)*cos(f*x + e) + sqrt(2)*(-3*I*c^2*d + 20*I*c*d^2 - 9*I*d^3)*sin(
f*x + e) + sqrt(2)*(-3*I*c^2*d + 20*I*c*d^2 - 9*I*d^3))*sqrt(I*d)*weierstra
ssZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstra
ssPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(
3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(3*I*c^2*d
- 20*I*c*d^2 + 9*I*d^3)*cos(f*x + e) + sqrt(2)*(3*I*c^2*d - 20*I*c*d^2 + 9*
I*d^3)*sin(f*x + e) + sqrt(2)*(3*I*c^2*d - 20*I*c*d^2 + 9*I*d^3))*sqrt(-I*d
)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^
3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^
2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(2*d^3*
cos(f*x + e)^2 + 3*c^2*d - 6*c*d^2 + 3*d^3 + (3*c^2*d - 6*c*d^2 + 5*d^3)*co
s(f*x + e) + (2*d^3*cos(f*x + e) - 3*c^2*d + 6*c*d^2 - 3*d^3)*sin(f*x + e))
*sqrt(d*sin(f*x + e) + c))/(a*d*f*cos(f*x + e) + a*d*f*sin(f*x + e) + a*d*f
)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2\sqrt{c+d\sin{(e+fx)}}}{\sin{(e+fx)}+1}\,dx + \int \frac{d^2\sqrt{c+d\sin{(e+fx)}}\sin^2{(e+fx)}}{\sin{(e+fx)}+1}\,dx + \int \frac{2cd\sqrt{c+d\sin{(e+fx)}}\sin{(e+fx)}}{\sin{(e+fx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x) + Integral(d
**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2/(sin(e + f*x) + 1), x) + Integ
ral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,\sin(e + f\,x))^{5/2}}{a + a\,\sin(e + f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)), x)
```

# 3.505 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=186

$$-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}-\frac{(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c^2-d^2)}{}$$

```
[Out] -(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))+(c-3*d)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1
/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2846, 2831, 2742, 2740, 2734, 2732}

$$\frac{(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])))
 - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c^2 - d^2)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*
f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{d\int \frac{-\frac{1}{2}a(3c-d)+\frac{1}{2}a(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)\int \sqrt{c+d\sin(e+fx)}}{2a}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\left((c-3d)\sqrt{c+d\sin(e+fx)}\right)}{2a\sqrt{\frac{c+d\sin}{c}}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)}{af\sqrt{\frac{c+d\sin(e}{c+d}}}$$

**Mathematica [A]**
time = 1.07, size = 223, normalized size = 1.20

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-\left((c^2-2cd-3d^2)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)+(c-d)\left(c+d\sin(e+fx)+(c+d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\right)\right)}{af(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2]*(c + d*S
in[e + f*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-((c^2 - 2*c*d - 3*d^
2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)]) + (c - d)*(c + d*Sin[e + f*x] + (c + d)*EllipticF[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))))/(a*f*(1 + S
in[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(242) = 484$.
time = 5.68, size = 925, normalized size = 4.97

| method | result |
|---|---|
| default | $\frac{\sqrt{\sin(fx+e)(\cos^2(fx+e))d+(\cos^2(fx+e))c}\left(\sqrt{\frac{d\sin(fx+e)}{c-d}+\frac{c}{c-d}}\sqrt{-\frac{d\sin(fx+e)}{c+d}+\frac{d}{c+d}}\sqrt{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)*cos(f*x+e)^2*d+cos(f*x+e)^2*c)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^3-3*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/
2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*
EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3
*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(
-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)
^(1/2),((c-d)/(c+d))^(1/2))*d^3+2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/
(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipti
cF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-2*(d/(c-
d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d
)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),
((c-d)/(c+d))^(1/2))*d^3-cos(f*x+e)^2*c*d^2+cos(f*x+e)^2*d^3+c^2*d*sin(f*x+
e)-2*c*d^2*sin(f*x+e)+d^3*sin(f*x+e)-c^2*d+2*c*d^2-d^3)/d/(-(c+d*sin(f*x+e)
)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 695, normalized size = 3.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(2)*(2*c^2 + 3*c*d - 3*d^2)*cos(f*x + e) + sqrt(2)*(2*c^2 + 3*c*d
 - 3*d^2)*sin(f*x + e) + sqrt(2)*(2*c^2 + 3*c*d - 3*d^2))*sqrt(I*d)*weierst
```

```
rassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(2*c^2 + 3*c
*d - 3*d^2)*cos(f*x + e) + sqrt(2)*(2*c^2 + 3*c*d - 3*d^2)*sin(f*x + e) + s
qrt(2)*(2*c^2 + 3*c*d - 3*d^2))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2
 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-I*c*d + 3*I*d^2)*cos(f*x + e) +
sqrt(2)*(-I*c*d + 3*I*d^2)*sin(f*x + e) + sqrt(2)*(-I*c*d + 3*I*d^2))*sqrt(
I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/
d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d
^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(
2)*(I*c*d - 3*I*d^2)*cos(f*x + e) + sqrt(2)*(I*c*d - 3*I*d^2)*sin(f*x + e)
+ sqrt(2)*(I*c*d - 3*I*d^2))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2
)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 -
3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d
*sin(f*x + e) + 2*I*c)/d)) - 6*(c*d - d^2 + (c*d - d^2)*cos(f*x + e) - (c*d
 - d^2)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/(a*d*f*cos(f*x + e) + a*d*f
*sin(f*x + e) + a*d*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sin(e+fx)}}{\sin(e+fx)+1}\,dx+\int \frac{d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)), x)
```

# 3.506 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=170

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c+d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right.}{af\sqrt{c+d}}$$

```
[Out] -cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))+(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*sin(f*x+e))/(c+
d))^(1/2)-(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f
*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2848, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x]))) - (Elli
pticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(a*f*Sqr
t[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2848

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(
a*f*(a + b*Sin[e + f*x]))), x] + Dist[d*(n/(a*b)), Int[(c + d*Sin[e + f*x])
^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2
*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} + \frac{d\int \frac{a-a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2a} + \frac{(c+d)\int -}{\sqrt{}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\sqrt{c+d\sin(e+fx)}\int \sqrt{\frac{c}{c+d}+\frac{d\sin(}{c}}}{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

**Mathematica [A]**
time = 0.76, size = 201, normalized size = 1.18

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(c+d\sin(e+fx)-(c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(c+d)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\right)}{af(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]])
```

**Maple [A]**
time = 5.81, size = 382, normalized size = 2.25

| method | result |
|---|---|
| default | $\frac{\sqrt{\sin(fx+e)\left(\cos^2(fx+e)\right)d+\left(\cos^2(fx+e)\right)c}\left(\sqrt{\frac{d\sin(fx+e)}{c-d}+\frac{c}{c-d}}\sqrt{-\frac{d\sin(fx+e)}{c+d}+\frac{d}{c+d}}\sqrt{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)*cos(f*x+e)^2*d+cos(f*x+e)^2*c)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^2-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1
/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2-cos(f*x+e)^2*d^2+c*d*sin(f*x+e)-d^2*si
n(f*x+e)-c*d+d^2)/d/(-(c+d*sin(f*x+e))*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)
/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 576, normalized size = 3.39

$$\frac{\left(\sqrt{2}\,(2c+3d)\cos(fx+e)+\sqrt{2}\,(2c+3d)\sin(fx+e)+\sqrt{2}\,(2c+3d)\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\left(\sqrt{2}\,(2c+3d)\cos(fx+e)+\sqrt{2}\,(2c+3d)\sin(fx+e)+\sqrt{2}\,(2c+3d)\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)-3\left(-i\sqrt{2}\,d\cos(fx+e)-i\sqrt{2}\,d\sin(fx+e)-i\sqrt{2}\,d\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)-3\left(i\sqrt{2}\,d\cos(fx+e)+i\sqrt{2}\,d\sin(fx+e)+i\sqrt{2}\,d\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)-6\left(d\cos(fx+e)-d\sin(fx+e)+d\right)\sqrt{d\sin(fx+e)+c}}{6\left(adf\cos(fx+e)+adf\sin(fx+e)+adf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(2)*(2*c + 3*d)*cos(f*x + e) + sqrt(2)*(2*c + 3*d)*sin(f*x + e) +
 sqrt(2)*(2*c + 3*d))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
 e) - 2*I*c)/d) + (sqrt(2)*(2*c + 3*d)*cos(f*x + e) + sqrt(2)*(2*c + 3*d)*s
in(f*x + e) + sqrt(2)*(2*c + 3*d))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c
^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(-I*sqrt(2)*d*cos(f*x + e) - I*sqrt(2)*d
*sin(f*x + e) - I*sqrt(2)*d)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*
d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*si
n(f*x + e) - 2*I*c)/d)) - 3*(I*sqrt(2)*d*cos(f*x + e) + I*sqrt(2)*d*sin(f*x
 + e) + I*sqrt(2)*d)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
 + e) + 2*I*c)/d)) - 6*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(d*sin(f*x
 + e) + c))/(a*d*f*cos(f*x + e) + a*d*f*sin(f*x + e) + a*d*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sin{(e+fx)}}}{\sin{(e+fx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x)/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin{(e+f\,x)}}}{a+a\,\sin{(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)), x)
```

# 3.507 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=181

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{}}{af\sqrt{c+d\sin(}}$$

```
[Out] -cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))+(sin(1/2*e+1/4*
Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1
/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+
1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d
*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2847, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(c-d)(a\sin(e+fx)+a)}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((c - d)*f*(a + a*Sin[e + f*x])))
 - (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(a*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (EllipticF[(e - Pi/2 + f
*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*S
in[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} + \frac{d\int \frac{-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)}{\sqrt{c+d\sin(e+fx}}}{a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} + \frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}}}{2a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} - \frac{\sqrt{c+d\sin(e+fx)}\int}{2a(c-d)\sqrt{}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{a(c-d)f\sqrt{\frac{c+}{}}}$$

## Mathematica [A]

time = 0.78, size = 210, normalized size = 1.16

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(c+d\sin(e+fx)-(c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(c-d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\right)}{a(c-d)f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c - d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*(c - d)*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin
[e + f*x]])
```

## Maple [A]

time = 11.99, size = 443, normalized size = 2.45

| method | result |
|---|---|
| default | $\frac{\sqrt{-(-d\sin(fx+e)-c)(\cos^2(fx+e))}\left(-\frac{-\left(\sin^2(fx+e)\right)d-c\sin(fx+e)+d\sin(fx+e)+c}{(c-d)\sqrt{(-d\sin(fx+e)-c)(\sin(fx+e)-1)(1+s}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2
)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-
d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 603, normalized size = 3.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(2)*(2*c - 3*d)*cos(f*x + e) + sqrt(2)*(2*c - 3*d)*sin(f*x + e) +
 sqrt(2)*(2*c - 3*d))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
 e) - 2*I*c)/d) + (sqrt(2)*(2*c - 3*d)*cos(f*x + e) + sqrt(2)*(2*c - 3*d)*s
in(f*x + e) + sqrt(2)*(2*c - 3*d))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c
^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(I*sqrt(2)*d*cos(f*x + e) + I*sqrt(2)*d*
sin(f*x + e) + I*sqrt(2)*d)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/
d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin
(f*x + e) - 2*I*c)/d)) + 3*(-I*sqrt(2)*d*cos(f*x + e) - I*sqrt(2)*d*sin(f*x
```

```
 + e) - I*sqrt(2)*d)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
 + e) + 2*I*c)/d)) - 6*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(d*sin(f*x
 + e) + c))/((a*c*d - a*d^2)*f*cos(f*x + e) + (a*c*d - a*d^2)*f*sin(f*x + e
) + (a*c*d - a*d^2)*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin{(e+fx)}}\sin{(e+fx)}+\sqrt{c+d\sin{(e+fx)}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x)
)), x)/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\,\sin{(e+f\,x)})\,\sqrt{c+d\,\sin{(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.508 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=244

$$-\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)E\left(\frac{1}{2}\left(e\right.\right.}{a(c-d}$$

```
[Out] -d*(c+3*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-cos(f*x+e)/(
c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2)+(c+3*d)*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/(c+d)/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)-(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*
Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((
c+d*sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2847, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{d(c+3d)\cos(e+fx)}{af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af(c-d)^2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((d*(c + 3*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]
])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[
e + f*x]])/(a*(c - d)^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(a*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}} + \frac{d\int}{\quad}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\quad}{(c-d)f(a+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\quad}{(c-d)f(a+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\quad}{(c-d)f(a+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\quad}{(c-d)f(a+}$$

**Mathematica** [A]
time = 1.40, size = 264, normalized size = 1.08

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\frac{2\left((c+d)^2\cos\left(\frac{1}{2}(e+fx)\right)+d(2(c+d)+(c+3d)\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+(c+3d)(c+d\sin(e+fx))+(c^2+4cd+3d^2)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-(c^2-d^2)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{a(c-d)^2(c+d)f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*((c + d)^2*Cos[(e + f*x)/2] +
 d*(2*(c + d) + (c + 3*d)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]) + (c + 3*d)*(c + d*Sin[e + f*x]) + (c^2 + 4*c*d + 3*d
^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)]))/(a*(c - d)^2*(c + d)*f*(1 + Sin[e + f*x
])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(298) = 596$.
time = 6.78, size = 926, normalized size = 3.80

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{\sqrt{\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)d+\left(\cos^{2}\left(fx+e\right)\right)c}\,\left(4\sqrt{\frac{d\sin(fx+e)}{c-d}+\frac{c}{c-d}}\,\sqrt{-\frac{d\sin(fx+e)}{c+d}+\frac{d}{c+d}}\,\sqrt{\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(sin(f*x+e)*cos(f*x+e)^2*d+cos(f*x+e)^2*c)^(1/2)*(4*(d/(c-d)*sin(f*x+e)+1/
(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(
c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1
/2))*c^2*d-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c
+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e
)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^3-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^
(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/
2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3-
3*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*
(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c
)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d
/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipt
icE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3*(d/(c
-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-
d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)
,((c-d)/(c+d))^(1/2))*d^3+cos(f*x+e)^2*c*d^2+3*cos(f*x+e)^2*d^3-c^2*d*sin(f
*x+e)+d^3*sin(f*x+e)+c^2*d-d^3)/d/(c^2-d^2)/(-(c+d*sin(f*x+e))*(sin(f*x+e)-
1)*(1+sin(f*x+e)))^(1/2)/(c-d)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 1205, normalized size = 4.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(2)*(2*c^2*d - 3*c*d^2 - 3*d^3)*cos(f*x + e)^2 - sqrt(2)*(2*c^3 -
 3*c^2*d - 3*c*d^2)*cos(f*x + e) - (sqrt(2)*(2*c^2*d - 3*c*d^2 - 3*d^3)*cos
(f*x + e) + sqrt(2)*(2*c^3 - c^2*d - 6*c*d^2 - 3*d^3))*sin(f*x + e) - sqrt(
2)*(2*c^3 - c^2*d - 6*c*d^2 - 3*d^3))*sqrt(I*d)*weierstrassPInverse(-4/3*(4
*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) -
 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(2*c^2*d - 3*c*d^2 - 3*d^3)*cos(
f*x + e)^2 - sqrt(2)*(2*c^3 - 3*c^2*d - 3*c*d^2)*cos(f*x + e) - (sqrt(2)*(2
*c^2*d - 3*c*d^2 - 3*d^3)*cos(f*x + e) + sqrt(2)*(2*c^3 - c^2*d - 6*c*d^2 -
 3*d^3))*sin(f*x + e) - sqrt(2)*(2*c^3 - c^2*d - 6*c*d^2 - 3*d^3))*sqrt(-I*
d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^
2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)
*(I*c*d^2 + 3*I*d^3)*cos(f*x + e)^2 + sqrt(2)*(-I*c^2*d - 3*I*c*d^2)*cos(f*
x + e) + (sqrt(2)*(-I*c*d^2 - 3*I*d^3)*cos(f*x + e) + sqrt(2)*(-I*c^2*d - 4
*I*c*d^2 - 3*I*d^3))*sin(f*x + e) + sqrt(2)*(-I*c^2*d - 4*I*c*d^2 - 3*I*d^3
))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I
*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) +
3*(sqrt(2)*(-I*c*d^2 - 3*I*d^3)*cos(f*x + e)^2 + sqrt(2)*(I*c^2*d + 3*I*c*d
^2)*cos(f*x + e) + (sqrt(2)*(I*c*d^2 + 3*I*d^3)*cos(f*x + e) + sqrt(2)*(I*c
^2*d + 4*I*c*d^2 + 3*I*d^3))*sin(f*x + e) + sqrt(2)*(I*c^2*d + 4*I*c*d^2 +
3*I*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*
c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I
*c)/d)) + 6*(c^2*d - d^3 + (c*d^2 + 3*d^3)*cos(f*x + e)^2 + (c^2*d + c*d^2
+ 2*d^3)*cos(f*x + e) - (c^2*d - d^3 - (c*d^2 + 3*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(d*sin(f*x + e) + c))/((a*c^3*d^2 - a*c^2*d^3 - a*c*d^4 + a*d^5
)*f*cos(f*x + e)^2 - (a*c^4*d - a*c^3*d^2 - a*c^2*d^3 + a*c*d^4)*f*cos(f*x
+ e) - (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f - ((a*c^3*d^2 - a*c^2*d^3 - a*c*d^
4 + a*d^5)*f*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f)*sin(f*x + e)
)
```

**Sympy** [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c\sqrt{c+d\sin(e+fx)}+d\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e +
f*x)) + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*sqrt(c + d*sin(e + f
*x))*sin(e + f*x)), x)/a
```

**Giac** [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.509 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=333

$$-\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}-\frac{d(3c^2+}{3a(c-d)^3(}$$

```
[Out] -1/3*d*(3*c+5*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/3*d*(3*c^2+20*c*d+9*
d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+1/3*(3*c^2+20*c*
d+9*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(
1/2)/a/(c-d)^3/(c+d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(3*c+5*d)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a
/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.33, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2847, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{d(3c^2+20cd+9d^2)\cos(e+fx)}{3af(c-d)^3(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{(3c^2+20cd+9d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^3(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{d(3c+5d)\cos(e+fx)}{3af(c-d)^2(c+d)(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^{3/2}}+\frac{(3c+5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/3*(d*(3*c + 5*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x
])^(3/2)) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x
])^(3/2)) - (d*(3*c^2 + 20*c*d + 9*d^2)*Cos[e + f*x])/(3*a*(c - d)^3*(c + d
)^2*f*Sqrt[c + d*Sin[e + f*x]]) - ((3*c^2 + 20*c*d + 9*d^2)*EllipticE[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a*(c - d)^3*(c +
 d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c + 5*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a*(c -
d)^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}} + \frac{d\int}{}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{(c-d)f(a}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{(c-d)f(a}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{(c-d)f(a}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{(c-d)f(a}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{(c-d)f(a}$$

**<u>Mathematica</u> [A]**
time = 2.95, size = 367, normalized size = 1.10

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(3c^2+20cd+9d^2)(c+d\sin(e+fx))+d(15c^2+12cd+5d^2)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(3c^2+20cd+9d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(c+d)^2}+2(c+d\sin(e+fx))\left(\frac{3\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}-\frac{3c^2+13cd+6d^2+\frac{d^2\cos(e+fx)\left(8c^2+3cd-d^2+d(7c+3d)\sin(e+fx)\right)}{(c+d\sin(e+fx))^2}}{(c+d)^2}\right)\right)}{3a(c-d)^3f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(((3*c^2 + 20*c*d + 9*d^2)*(c + d*
Sin[e + f*x]) + d*(15*c^2 + 12*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (3*c^2 + 20*c*d + 9*d
^2)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(c + d)^2 + 2*(c + d*Sin[e + f*x])*((3*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]) - (3*c^2 + 13*c*d + 6*d^2 + (d^2*Cos[e + f*x]*(8*c^2 +
3*c*d - d^2 + d*(7*c + 3*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2)/(c + d)^
2)))/(3*a*(c - d)^3*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(377) = 754$.
time = 24.00, size = 1291, normalized size = 3.88

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1291 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-d/(c-d)*(2/3/(c^2-d^2)/d*(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^
2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c
^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d
/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)^2
*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(
sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))))-d/(c-d)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 0.22, size = 2243, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*((sqrt(2)*(6*c^3*d^2 - 5*c^2*d^3 - 18*c*d^4 - 15*d^5)*cos(f*x + e)^3 +
 sqrt(2)*(12*c^4*d - 4*c^3*d^2 - 41*c^2*d^3 - 48*c*d^4 - 15*d^5)*cos(f*x +
e)^2 - sqrt(2)*(6*c^5 - 5*c^4*d - 12*c^3*d^2 - 20*c^2*d^3 - 18*c*d^4 - 15*d
^5)*cos(f*x + e) + (sqrt(2)*(6*c^3*d^2 - 5*c^2*d^3 - 18*c*d^4 - 15*d^5)*cos
(f*x + e)^2 - 2*sqrt(2)*(6*c^4*d - 5*c^3*d^2 - 18*c^2*d^3 - 15*c*d^4)*cos(f
*x + e) - sqrt(2)*(6*c^5 + 7*c^4*d - 22*c^3*d^2 - 56*c^2*d^3 - 48*c*d^4 - 1
5*d^5))*sin(f*x + e) - sqrt(2)*(6*c^5 + 7*c^4*d - 22*c^3*d^2 - 56*c^2*d^3 -
 48*c*d^4 - 15*d^5))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
e) - 2*I*c)/d) + (sqrt(2)*(6*c^3*d^2 - 5*c^2*d^3 - 18*c*d^4 - 15*d^5)*cos(f
*x + e)^3 + sqrt(2)*(12*c^4*d - 4*c^3*d^2 - 41*c^2*d^3 - 48*c*d^4 - 15*d^5)
*cos(f*x + e)^2 - sqrt(2)*(6*c^5 - 5*c^4*d - 12*c^3*d^2 - 20*c^2*d^3 - 18*c
*d^4 - 15*d^5)*cos(f*x + e) + (sqrt(2)*(6*c^3*d^2 - 5*c^2*d^3 - 18*c*d^4 -
15*d^5)*cos(f*x + e)^2 - 2*sqrt(2)*(6*c^4*d - 5*c^3*d^2 - 18*c^2*d^3 - 15*c
*d^4)*cos(f*x + e) - sqrt(2)*(6*c^5 + 7*c^4*d - 22*c^3*d^2 - 56*c^2*d^3 - 4
8*c*d^4 - 15*d^5))*sin(f*x + e) - sqrt(2)*(6*c^5 + 7*c^4*d - 22*c^3*d^2 - 5
6*c^2*d^3 - 48*c*d^4 - 15*d^5))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(3*I*c^2*d^3 + 20*I*c*d^4 + 9*I*d^
5)*cos(f*x + e)^3 + sqrt(2)*(6*I*c^3*d^2 + 43*I*c^2*d^3 + 38*I*c*d^4 + 9*I*
d^5)*cos(f*x + e)^2 + sqrt(2)*(-3*I*c^4*d - 20*I*c^3*d^2 - 12*I*c^2*d^3 - 2
0*I*c*d^4 - 9*I*d^5)*cos(f*x + e) + (sqrt(2)*(3*I*c^2*d^3 + 20*I*c*d^4 + 9*
I*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(-3*I*c^3*d^2 - 20*I*c^2*d^3 - 9*I*c*d^4)
*cos(f*x + e) + sqrt(2)*(-3*I*c^4*d - 26*I*c^3*d^2 - 52*I*c^2*d^3 - 38*I*c*
d^4 - 9*I*d^5))*sin(f*x + e) + sqrt(2)*(-3*I*c^4*d - 26*I*c^3*d^2 - 52*I*c^
2*d^3 - 38*I*c*d^4 - 9*I*d^5))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 -
3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*
sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-3*I*c^2*d^3 - 20*I*c*d^4 - 9*I*d^5
)*cos(f*x + e)^3 + sqrt(2)*(-6*I*c^3*d^2 - 43*I*c^2*d^3 - 38*I*c*d^4 - 9*I*
d^5)*cos(f*x + e)^2 + sqrt(2)*(3*I*c^4*d + 20*I*c^3*d^2 + 12*I*c^2*d^3 + 20
*I*c*d^4 + 9*I*d^5)*cos(f*x + e) + (sqrt(2)*(-3*I*c^2*d^3 - 20*I*c*d^4 - 9*
I*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(3*I*c^3*d^2 + 20*I*c^2*d^3 + 9*I*c*d^4)*
cos(f*x + e) + sqrt(2)*(3*I*c^4*d + 26*I*c^3*d^2 + 52*I*c^2*d^3 + 38*I*c*d^
4 + 9*I*d^5))*sin(f*x + e) + sqrt(2)*(3*I*c^4*d + 26*I*c^3*d^2 + 52*I*c^2*d
```

```
^3 + 38*I*c*d^4 + 9*I*d^5))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3
*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*
sin(f*x + e) + 2*I*c)/d)) + 6*(3*c^4*d - 6*c^2*d^3 + 3*d^5 - (3*c^2*d^3 + 2
0*c*d^4 + 9*d^5)*cos(f*x + e)^3 + (6*c^3*d^2 + 25*c^2*d^3 + 6*c*d^4 - 5*d^5
)*cos(f*x + e)^2 + (3*c^4*d + 6*c^3*d^2 + 22*c^2*d^3 + 26*c*d^4 + 7*d^5)*co
s(f*x + e) - (3*c^4*d - 6*c^2*d^3 + 3*d^5 - (3*c^2*d^3 + 20*c*d^4 + 9*d^5)*
cos(f*x + e)^2 - 2*(3*c^3*d^2 + 14*c^2*d^3 + 13*c*d^4 + 2*d^5)*cos(f*x + e)
)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((a*c^5*d^3 - a*c^4*d^4 - 2*a*c^3
*d^5 + 2*a*c^2*d^6 + a*c*d^7 - a*d^8)*f*cos(f*x + e)^3 + (2*a*c^6*d^2 - a*c
^5*d^3 - 5*a*c^4*d^4 + 2*a*c^3*d^5 + 4*a*c^2*d^6 - a*c*d^7 - a*d^8)*f*cos(f
*x + e)^2 - (a*c^7*d - a*c^6*d^2 - a*c^5*d^3 + a*c^4*d^4 - a*c^3*d^5 + a*c^
2*d^6 + a*c*d^7 - a*d^8)*f*cos(f*x + e) - (a*c^7*d + a*c^6*d^2 - 3*a*c^5*d^
3 - 3*a*c^4*d^4 + 3*a*c^3*d^5 + 3*a*c^2*d^6 - a*c*d^7 - a*d^8)*f + ((a*c^5*
d^3 - a*c^4*d^4 - 2*a*c^3*d^5 + 2*a*c^2*d^6 + a*c*d^7 - a*d^8)*f*cos(f*x +
e)^2 - 2*(a*c^6*d^2 - a*c^5*d^3 - 2*a*c^4*d^4 + 2*a*c^3*d^5 + a*c^2*d^6 - a
*c*d^7)*f*cos(f*x + e) - (a*c^7*d + a*c^6*d^2 - 3*a*c^5*d^3 - 3*a*c^4*d^4 +
 3*a*c^3*d^5 + 3*a*c^2*d^6 - a*c*d^7 - a*d^8)*f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c^2\sqrt{c+d\sin(e+fx)}+2cd\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)+d^2\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+d^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c**2*sqrt(c + d*si
n(e + f*x)) + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*d*sqrt(c
 + d*sin(e + f*x))*sin(e + f*x) + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x
)**3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad [F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

# 3.510 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=256

$$-\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2}-\frac{(c^2+5cd-1}{}$$

```
[Out] -1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^2-1/3*(c-d)
*(c+5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/f/(1+sin(f*x+e))+1/3*(c^2+5*
c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e)
)^(1/2)/a^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c+5*d)*(c^2-d^2)*(sin(1/2
*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1
/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/
f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.36, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2844, 3056, 2831, 2742, 2740, 2734, 2732}

$$\frac{(c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((c - d)*(c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a^2*f*(1 +
Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(3*f*(a
+ a*Sin[e + f*x])^2) - ((c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2 + f*x)/2
, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x
])/(c + d)]) + ((c + 5*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x
]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\,\left(-\frac{1}{2}a(2c^2+\cdots\right.}{a+a\sin(e+\cdots}}{3a^2\cdots}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+\cdots}{3f(a+a\sin(e-\cdots}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+\cdots}{3f(a+a\sin(e-\cdots}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+\cdots}{3f(a+a\sin(e-\cdots}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+\cdots}{3f(a+a\sin(e-\cdots}$$

**Mathematica [A]**
time = 1.74, size = 310, normalized size = 1.21

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\left(\left(c^2+5cd-6d^2\right)(c+d\sin(e+fx))\right)+\frac{(c-d)\left(7d\cos\left(\frac{1}{2}(e+fx)\right)-(c+6d)\cos\left(\frac{3}{2}(e+fx)\right)+(3c+11d)\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+d^2(-11c+5d)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+\left(c^2+5cd-12d^2\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3a^2f(1+\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c^2 + 5*c*d - 6*d^2)*(c + d*Si
n[e + f*x])) + ((c - d)*(7*d*Cos[(e + f*x)/2] - (c + 6*d)*Cos[(3*(e + f*x))
/2] + (3*c + 11*d)*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3 + d^2*(-11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 + 5*c*d - 12*d^2
)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(
3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. 2(302) = 604.
time = 21.12, size = 1372, normalized size = 5.36

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1372 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))+6*c*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4*d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+
(c^3-3*c^2*d+3*c*d^2-d^3)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)
^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3
*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3
*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*d
*(c^2-2*c*d+d^2)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d
*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e
))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1185, normalized size = 4.63

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/18*((sqrt(2)*(2*c^3 + 10*c^2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e)^2 - sqrt(
2)*(2*c^3 + 10*c^2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e) - (sqrt(2)*(2*c^3 + 1
0*c^2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e) + 2*sqrt(2)*(2*c^3 + 10*c^2*d + 9*
c*d^2 - 15*d^3))*sin(f*x + e) - 2*sqrt(2)*(2*c^3 + 10*c^2*d + 9*c*d^2 - 15*
d^3))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)
+ (sqrt(2)*(2*c^3 + 10*c^2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e)^2 - sqrt(2)*(
2*c^3 + 10*c^2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e) - (sqrt(2)*(2*c^3 + 10*c^
2*d + 9*c*d^2 - 15*d^3)*cos(f*x + e) + 2*sqrt(2)*(2*c^3 + 10*c^2*d + 9*c*d^
2 - 15*d^3))*sin(f*x + e) - 2*sqrt(2)*(2*c^3 + 10*c^2*d + 9*c*d^2 - 15*d^3)
)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3
+ 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) +
3*(sqrt(2)*(I*c^2*d + 5*I*c*d^2 - 12*I*d^3)*cos(f*x + e)^2 + sqrt(2)*(-I*c^
2*d - 5*I*c*d^2 + 12*I*d^3)*cos(f*x + e) + (sqrt(2)*(-I*c^2*d - 5*I*c*d^2 +
 12*I*d^3)*cos(f*x + e) + 2*sqrt(2)*(-I*c^2*d - 5*I*c*d^2 + 12*I*d^3))*sin(
f*x + e) + 2*sqrt(2)*(-I*c^2*d - 5*I*c*d^2 + 12*I*d^3))*sqrt(I*d)*weierstra
ssZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstra
ssPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(
3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*c^2*d -
 5*I*c*d^2 + 12*I*d^3)*cos(f*x + e)^2 + sqrt(2)*(I*c^2*d + 5*I*c*d^2 - 12*I
*d^3)*cos(f*x + e) + (sqrt(2)*(I*c^2*d + 5*I*c*d^2 - 12*I*d^3)*cos(f*x + e)
 + 2*sqrt(2)*(I*c^2*d + 5*I*c*d^2 - 12*I*d^3))*sin(f*x + e) + 2*sqrt(2)*(I*
c^2*d + 5*I*c*d^2 - 12*I*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
*d*sin(f*x + e) + 2*I*c)/d)) + 6*(c^2*d - 2*c*d^2 + d^3 + (c^2*d + 5*c*d^2
- 6*d^3)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e) - (c^2*d
 - 2*c*d^2 + d^3 - (c^2*d + 5*c*d^2 - 6*d^3)*cos(f*x + e))*sin(f*x + e))*sq
rt(d*sin(f*x + e) + c))/(a^2*d*f*cos(f*x + e)^2 - a^2*d*f*cos(f*x + e) - 2*
a^2*d*f - (a^2*d*f*cos(f*x + e) + 2*a^2*d*f)*sin(f*x + e))
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{5/2}}{(a+a\,\sin\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2, x)
```

# 3.511 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2} \, dx$

**Optimal**. Leaf size=237

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{(c+3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\right.\right.}{3a^2f\sqrt{\cdots}}$$

```
[Out] -1/3*(c+3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/f/(1+sin(f*x+e))-1/3*(c-
d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^2+1/3*(c+3*d)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c+d)*(c+2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)
^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2
)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/f/(c+d*sin(f*x+e))^(1
/2)
```

**Rubi** [A]
time = 0.36, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2844, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}+\frac{(c+d)(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a^2*f*(1 + Sin[e +
f*x])) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e
+ f*x])^2) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c
 + d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)
*(c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
 f*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(2c^2+5cd-d^2)-\frac{1}{2}ad(c+5d)\sin(e+f}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx}}}{3a^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e}}{3f(a+a\sin(e+fx))^2}$$

**Mathematica** [**A**]
time = 1.71, size = 283, normalized size = 1.19

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-((c+3d)(c+d\sin(e+fx)))+\frac{\left(4d\cos\left(\frac{1}{2}(e+fx)\right)-(c+3d)\cos\left(\frac{3}{2}(e+fx)\right)+(3c+5d)\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-2d^2F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(c+3d)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3a^2f(1+\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c + 3*d)*(c + d*Sin[e + f*x]))
 + ((4*d*Cos[(e + f*x)/2] - (c + 3*d)*Cos[(3*(e + f*x))/2] + (3*c + 5*d)*Si
n[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)] + (c + 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]))/(3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e
+ f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(283) = 566$.
time = 18.48, size = 1049, normalized size = 4.43

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1049 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+(c^2-2*c*d+d^2)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x
+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))))+2*d*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/
(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1
-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)
/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 969, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*(sqrt(2)*(c^2 + 3*c*d + 3*d^2)*cos(f*x + e)^2 - sqrt(2)*(c^2 + 3*c*
d + 3*d^2)*cos(f*x + e) - (sqrt(2)*(c^2 + 3*c*d + 3*d^2)*cos(f*x + e) + 2*s
qrt(2)*(c^2 + 3*c*d + 3*d^2))*sin(f*x + e) - 2*sqrt(2)*(c^2 + 3*c*d + 3*d^2
))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + 2
*(sqrt(2)*(c^2 + 3*c*d + 3*d^2)*cos(f*x + e)^2 - sqrt(2)*(c^2 + 3*c*d + 3*d
^2)*cos(f*x + e) - (sqrt(2)*(c^2 + 3*c*d + 3*d^2)*cos(f*x + e) + 2*sqrt(2)*
(c^2 + 3*c*d + 3*d^2))*sin(f*x + e) - 2*sqrt(2)*(c^2 + 3*c*d + 3*d^2))*sqrt
(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*
c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqr
t(2)*(I*c*d + 3*I*d^2)*cos(f*x + e)^2 + sqrt(2)*(-I*c*d - 3*I*d^2)*cos(f*x
+ e) + (sqrt(2)*(-I*c*d - 3*I*d^2)*cos(f*x + e) + 2*sqrt(2)*(-I*c*d - 3*I*d
^2))*sin(f*x + e) + 2*sqrt(2)*(-I*c*d - 3*I*d^2))*sqrt(I*d)*weierstrassZeta
(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInv
erse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*co
s(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*c*d - 3*I*d^2
)*cos(f*x + e)^2 + sqrt(2)*(I*c*d + 3*I*d^2)*cos(f*x + e) + (sqrt(2)*(I*c*d
 + 3*I*d^2)*cos(f*x + e) + 2*sqrt(2)*(I*c*d + 3*I*d^2))*sin(f*x + e) + 2*sq
rt(2)*(I*c*d + 3*I*d^2))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin
(f*x + e) + 2*I*c)/d)) + 6*((c*d + 3*d^2)*cos(f*x + e)^2 + c*d - d^2 + 2*(c
*d + d^2)*cos(f*x + e) - (c*d - d^2 - (c*d + 3*d^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(d*sin(f*x + e) + c))/(a^2*d*f*cos(f*x + e)^2 - a^2*d*f*cos(f*x +
e) - 2*a^2*d*f - (a^2*d*f*cos(f*x + e) + 2*a^2*d*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sin{(e+fx)}}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx+\int \frac{d\sqrt{c+d\sin{(e+fx)}}\sin{(e+fx)}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1)
, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x)**2 +
2*sin(e + f*x) + 1), x))/a**2
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2, x)
```

# 3.512 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=233

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(}{3a^2(c-d)f\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

```
[Out] -1/3*c*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)/f/(1+sin(f*x+e))-1/3*cos
(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^2+1/3*c*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-1/3*(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2
*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1
/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.27, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2843, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)(\sin(e+fx)+1)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(c*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a^2*(c - d)*f*(1 + Sin[e +
f*x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e + f*x])^
2) - (c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x
]])/(3*a^2*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*Ellipti
cF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(
3*a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{\frac{1}{2}a(2c+d)+\frac{1}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} -$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} -$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} -$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} -$$

**Mathematica [A]**
time = 1.69, size = 256, normalized size = 1.10

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-c(c+d\sin(e+fx))+\frac{\left(d\cos\left(\frac{1}{2}(e+fx)\right)-c\cos\left(\frac{3}{2}(e+fx)\right)+(3c-d)\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+c(c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-(c^2-d^2)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3a^2(c-d)f(1+\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-(c*(c + d*Sin[e + f*x])) + ((d*C
os[(e + f*x)/2] - c*Cos[(3*(e + f*x))/2] + (3*c - d)*Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + c*(c + d)*Ellip
ticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)*f*(1 + Sin[e + f*x])^2*Sqrt[c + d
*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(279) = 558.
time = 19.84, size = 906, normalized size = 3.89

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\,\left((c-d)\left(-\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{3(c-d)(1+\sin(fx+e))^{2}}-\cdots\right.\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*((c-d)*(-1/3/(c-d)*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x
+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+si
n(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))))+d*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-
d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c
/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-si
n(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 913, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/18*((sqrt(2)*(2*c^2 - 3*d^2)*cos(f*x + e)^2 - sqrt(2)*(2*c^2 - 3*d^2)*cos
(f*x + e) - (sqrt(2)*(2*c^2 - 3*d^2)*cos(f*x + e) + 2*sqrt(2)*(2*c^2 - 3*d^
2))*sin(f*x + e) - 2*sqrt(2)*(2*c^2 - 3*d^2))*sqrt(I*d)*weierstrassPInverse
(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*
x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(2*c^2 - 3*d^2)*cos(f*x
+ e)^2 - sqrt(2)*(2*c^2 - 3*d^2)*cos(f*x + e) - (sqrt(2)*(2*c^2 - 3*d^2)*co
s(f*x + e) + 2*sqrt(2)*(2*c^2 - 3*d^2))*sin(f*x + e) - 2*sqrt(2)*(2*c^2 - 3
*d^2))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I
*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/
d) - 3*(-I*sqrt(2)*c*d*cos(f*x + e)^2 + I*sqrt(2)*c*d*cos(f*x + e) + 2*I*sq
rt(2)*c*d + (I*sqrt(2)*c*d*cos(f*x + e) + 2*I*sqrt(2)*c*d)*sin(f*x + e))*sq
rt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^
2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*
c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(I*
sqrt(2)*c*d*cos(f*x + e)^2 - I*sqrt(2)*c*d*cos(f*x + e) - 2*I*sqrt(2)*c*d +
 (-I*sqrt(2)*c*d*cos(f*x + e) - 2*I*sqrt(2)*c*d)*sin(f*x + e))*sqrt(-I*d)*w
eierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/
d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(c*d*cos(f
*x + e)^2 + c*d - d^2 + (2*c*d - d^2)*cos(f*x + e) + (c*d*cos(f*x + e) - c*
d + d^2)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((a^2*c*d - a^2*d^2)*f*cos
(f*x + e)^2 - (a^2*c*d - a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c*d - a^2*d^2)*f
- ((a^2*c*d - a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c*d - a^2*d^2)*f)*sin(f*x +
e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sin{(e+fx)}}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x
)/a**2
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2, x)
```

# 3.513 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=257

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2}-\frac{(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+\cdots}\right.}{3a^2(c-d)^2f\sqrt{\frac{c+\cdots}{\cdots}}}$$

```
[Out] -1/3*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)^2/f/(1+sin(f*x+e))
-1/3*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^2+1/3*(c-3*
d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a
^2/(c-d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c-2*d)*(sin(1/2*e+1/4*Pi+1
/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f
*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/(c-d)/f/(c+
d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2845, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2(\sin(e+fx)+1)}+\frac{(c-2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -1/3*((c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a^2*(c - d)^2*f*(1
+ Sin[e + f*x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(c - d)*f*(a
+ a*Sin[e + f*x])^2) - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c +
d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/
(c + d)]) + ((c - 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c
 + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(2c-5d)-\frac{1}{2}ac}{(a+a\sin(e+fx))\sqrt{c+}}}{3a^2(c-}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a}$$

**Mathematica [A]**
time = 1.69, size = 290, normalized size = 1.13

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-((c-3d)(c+d\sin(e+fx)))-\frac{\left(2d\cos\left(\frac{1}{2}(e+fx)\right)+(c-3d)\cos\left(\frac{3}{2}(e+fx)\right)+(-3c+7d)\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-2d^2F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(c-3d)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3a^2(c-d)^2f(1+\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c - 3*d)*(c + d*Sin[e + f*x]))
 - ((2*d*Cos[(e + f*x)/2] + (c - 3*d)*Cos[(3*(e + f*x))/2] + (-3*c + 7*d)*S
in[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)] + (c - 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (
2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2*Sqrt[c
 + d*Sin[e + f*x]])
```

**Maple [A]**
time = 14.21, size = 507, normalized size = 1.97

| method | result |
|---|---|
| default | $\frac{\sqrt{-(-d\sin(fx+e)-c)(\cos^2(fx+e))}\left(-\frac{\sqrt{-(-d\sin(fx+e)-c)(\cos^2(fx+e))}}{3(c-d)(1+\sin(fx+e))^2}-\frac{}{3(c-d)^2}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*s
in(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e
)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 1076, normalized size = 4.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/18*(2*(sqrt(2)*(c^2 - 3*c*d + 3*d^2)*cos(f*x + e)^2 - sqrt(2)*(c^2 - 3*c*
d + 3*d^2)*cos(f*x + e) - (sqrt(2)*(c^2 - 3*c*d + 3*d^2)*cos(f*x + e) + 2*s
```

```
qrt(2)*(c^2 - 3*c*d + 3*d^2))*sin(f*x + e) - 2*sqrt(2)*(c^2 - 3*c*d + 3*d^2
))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + 2
*(sqrt(2)*(c^2 - 3*c*d + 3*d^2)*cos(f*x + e)^2 - sqrt(2)*(c^2 - 3*c*d + 3*d
^2)*cos(f*x + e) - (sqrt(2)*(c^2 - 3*c*d + 3*d^2)*cos(f*x + e) + 2*sqrt(2)*
(c^2 - 3*c*d + 3*d^2))*sin(f*x + e) - 2*sqrt(2)*(c^2 - 3*c*d + 3*d^2))*sqrt
(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*
c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqr
t(2)*(I*c*d - 3*I*d^2)*cos(f*x + e)^2 + sqrt(2)*(-I*c*d + 3*I*d^2)*cos(f*x
+ e) + (sqrt(2)*(-I*c*d + 3*I*d^2)*cos(f*x + e) + 2*sqrt(2)*(-I*c*d + 3*I*d
^2))*sin(f*x + e) + 2*sqrt(2)*(-I*c*d + 3*I*d^2))*sqrt(I*d)*weierstrassZeta
(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInv
erse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*co
s(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*c*d + 3*I*d^2
)*cos(f*x + e)^2 + sqrt(2)*(I*c*d - 3*I*d^2)*cos(f*x + e) + (sqrt(2)*(I*c*d
 - 3*I*d^2)*cos(f*x + e) + 2*sqrt(2)*(I*c*d - 3*I*d^2))*sin(f*x + e) + 2*sq
rt(2)*(I*c*d - 3*I*d^2))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin
(f*x + e) + 2*I*c)/d)) + 6*((c*d - 3*d^2)*cos(f*x + e)^2 + c*d - d^2 + 2*(c
*d - 2*d^2)*cos(f*x + e) - (c*d - d^2 - (c*d - 3*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(d*sin(f*x + e) + c))/((a^2*c^2*d - 2*a^2*c*d^2 + a^2*d^3)*f*cos
(f*x + e)^2 - (a^2*c^2*d - 2*a^2*c*d^2 + a^2*d^3)*f*cos(f*x + e) - 2*(a^2*c
^2*d - 2*a^2*c*d^2 + a^2*d^3)*f - ((a^2*c^2*d - 2*a^2*c*d^2 + a^2*d^3)*f*co
s(f*x + e) + 2*(a^2*c^2*d - 2*a^2*c*d^2 + a^2*d^3)*f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2\sqrt{c+d\sin(e+fx)}\sin(e+fx)+\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x))), x)/a**2
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\sin(e + f x))^2 \sqrt{c + d\sin(e + f x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.514 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=326

$$-\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}}-\frac{(c-5d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}}-\frac{}{3(c-d)f(c}$$

```
[Out] -1/3*d*(c^2-5*c*d-12*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^(
1/2)-1/3*(c-5*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^(
1/2)-1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2)+1/3*(
c^2-5*c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*
f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(
f*x+e))^(1/2)/a^2/(c-d)^3/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c-5*d
)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(c
os(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^
(1/2)/a^2/(c-d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.43, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{(c^2-5cd-12d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-5d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)\sqrt{c+d\sin(e+fx)}}+\frac{(c-5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/3*(d*(c^2 - 5*c*d - 12*d^2)*Cos[e + f*x])/(a^2*(c - d)^3*(c + d)*f*Sqrt[
c + d*Sin[e + f*x]]) - ((c - 5*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin
[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((c^2 - 5*c*d - 12*d^2)*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)
^3*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c - 5*d)*EllipticF[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*
(c - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

<u>Rubi steps</u>

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}} - \int$$

$$= -\frac{(c-5d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}} - \frac{}{3(}$$

$$= -\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{3a^2(c-d)^2}$$

$$= -\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{3a^2(c-d)^2}$$

$$= -\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{3a^2(c-d)^2}$$

$$= -\frac{d(c^2-5cd-12d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{3a^2(c-d)^2}$$

**<u>Mathematica</u> [A]**
time = 3.28, size = 405, normalized size = 1.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left((c+d\sin(e+fx))\left(-\frac{2(c^2-5cd-9d^2)}{c+d}+\frac{2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{-c+d}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{2(c-6d)\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{6d^3\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}\right)+\frac{(c^2-5cd-12d^2)(c+d\sin(e+fx))-d^2(11c+5d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(c^2-5cd-12d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{c+d}\right)}{3a^2(c-d)^3f(1+\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((c + d*Sin[e + f*x])*((-2*(c^2 -
5*c*d - 9*d^2))/(c + d) + (2*(c - d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (-c + d)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2
*(c - 6*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (6*d^3
*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))) + ((c^2 - 5*c*d - 12*d^2)*(c
 + d*Sin[e + f*x]) - d^2*(11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 - 5*c*d - 12*d^2)*((c
+ d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(c + d))
)/(3*a^2*(c - d)^3*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1298 vs. $2(368) = 736$.

time = 23.01, size = 1299, normalized size = 3.98

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1299 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(1/(c-d)*(-1/3/(c-d)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f
*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+
sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))))-d/(c-d)^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x
+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2
*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f
*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+d^2/(c-d)^2*(2*d*cos(f*x+e)
^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+
e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*s
```

```
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 2093, normalized size = 6.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/18*((sqrt(2)*(2*c^3*d - 10*c^2*d^2 + 9*c*d^3 + 15*d^4)*cos(f*x + e)^3 + s
qrt(2)*(2*c^4 - 6*c^3*d - 11*c^2*d^2 + 33*c*d^3 + 30*d^4)*cos(f*x + e)^2 -
sqrt(2)*(2*c^4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4)*cos(f*x + e) + (sqr
t(2)*(2*c^3*d - 10*c^2*d^2 + 9*c*d^3 + 15*d^4)*cos(f*x + e)^2 - sqrt(2)*(2*
c^4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4)*cos(f*x + e) - 2*sqrt(2)*(2*c^
4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4))*sin(f*x + e) - 2*sqrt(2)*(2*c^4
 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4))*sqrt(I*d)*weierstrassPInverse(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x +
 e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(2*c^3*d - 10*c^2*d^2 + 9*c
*d^3 + 15*d^4)*cos(f*x + e)^3 + sqrt(2)*(2*c^4 - 6*c^3*d - 11*c^2*d^2 + 33*
c*d^3 + 30*d^4)*cos(f*x + e)^2 - sqrt(2)*(2*c^4 - 8*c^3*d - c^2*d^2 + 24*c*
d^3 + 15*d^4)*cos(f*x + e) + (sqrt(2)*(2*c^3*d - 10*c^2*d^2 + 9*c*d^3 + 15*
d^4)*cos(f*x + e)^2 - sqrt(2)*(2*c^4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^
4)*cos(f*x + e) - 2*sqrt(2)*(2*c^4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4)
)*sin(f*x + e) - 2*sqrt(2)*(2*c^4 - 8*c^3*d - c^2*d^2 + 24*c*d^3 + 15*d^4))
*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 +
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3
*(sqrt(2)*(I*c^2*d^2 - 5*I*c*d^3 - 12*I*d^4)*cos(f*x + e)^3 + sqrt(2)*(I*c^
3*d - 3*I*c^2*d^2 - 22*I*c*d^3 - 24*I*d^4)*cos(f*x + e)^2 + sqrt(2)*(-I*c^3
*d + 4*I*c^2*d^2 + 17*I*c*d^3 + 12*I*d^4)*cos(f*x + e) + (sqrt(2)*(I*c^2*d^
2 - 5*I*c*d^3 - 12*I*d^4)*cos(f*x + e)^2 + sqrt(2)*(-I*c^3*d + 4*I*c^2*d^2
```

```
+ 17*I*c*d^3 + 12*I*d^4)*cos(f*x + e) + 2*sqrt(2)*(-I*c^3*d + 4*I*c^2*d^2 +
 17*I*c*d^3 + 12*I*d^4))*sin(f*x + e) + 2*sqrt(2)*(-I*c^3*d + 4*I*c^2*d^2 +
 17*I*c*d^3 + 12*I*d^4))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*
x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*c^2*d^2 + 5*I*c*d^3 + 12*I*d^4)*cos(f*
x + e)^3 + sqrt(2)*(-I*c^3*d + 3*I*c^2*d^2 + 22*I*c*d^3 + 24*I*d^4)*cos(f*x
 + e)^2 + sqrt(2)*(I*c^3*d - 4*I*c^2*d^2 - 17*I*c*d^3 - 12*I*d^4)*cos(f*x +
 e) + (sqrt(2)*(-I*c^2*d^2 + 5*I*c*d^3 + 12*I*d^4)*cos(f*x + e)^2 + sqrt(2)
*(I*c^3*d - 4*I*c^2*d^2 - 17*I*c*d^3 - 12*I*d^4)*cos(f*x + e) + 2*sqrt(2)*(
I*c^3*d - 4*I*c^2*d^2 - 17*I*c*d^3 - 12*I*d^4))*sin(f*x + e) + 2*sqrt(2)*(I
*c^3*d - 4*I*c^2*d^2 - 17*I*c*d^3 - 12*I*d^4))*sqrt(-I*d)*weierstrassZeta(-
4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInve
rse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*co
s(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(c^3*d - c^2*d^2 - c*d^3 +
 d^4 - (c^2*d^2 - 5*c*d^3 - 12*d^4)*cos(f*x + e)^3 + (c^3*d - 4*c^2*d^2 - 6
*c*d^3 - 7*d^4)*cos(f*x + e)^2 + 2*(c^3*d - 2*c^2*d^2 - 6*c*d^3 - 9*d^4)*co
s(f*x + e) - (c^3*d - c^2*d^2 - c*d^3 + d^4 - (c^2*d^2 - 5*c*d^3 - 12*d^4)*
cos(f*x + e)^2 - (c^3*d - 3*c^2*d^2 - 11*c*d^3 - 19*d^4)*cos(f*x + e))*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c))/((a^2*c^4*d^2 - 2*a^2*c^3*d^3 + 2*a^2*c
*d^5 - a^2*d^6)*f*cos(f*x + e)^3 + (a^2*c^5*d - 4*a^2*c^3*d^3 + 2*a^2*c^2*d
^4 + 3*a^2*c*d^5 - 2*a^2*d^6)*f*cos(f*x + e)^2 - (a^2*c^5*d - a^2*c^4*d^2 -
 2*a^2*c^3*d^3 + 2*a^2*c^2*d^4 + a^2*c*d^5 - a^2*d^6)*f*cos(f*x + e) - 2*(a
^2*c^5*d - a^2*c^4*d^2 - 2*a^2*c^3*d^3 + 2*a^2*c^2*d^4 + a^2*c*d^5 - a^2*d^
6)*f + ((a^2*c^4*d^2 - 2*a^2*c^3*d^3 + 2*a^2*c*d^5 - a^2*d^6)*f*cos(f*x + e
)^2 - (a^2*c^5*d - a^2*c^4*d^2 - 2*a^2*c^3*d^3 + 2*a^2*c^2*d^4 + a^2*c*d^5
- a^2*d^6)*f*cos(f*x + e) - 2*(a^2*c^5*d - a^2*c^4*d^2 - 2*a^2*c^3*d^3 + 2*
a^2*c^2*d^4 + a^2*c*d^5 - a^2*d^6)*f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2c\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c\sqrt{c+d\sin(e+fx)}+d\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+2d\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e + f*x)) + d*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**3 + 2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*s
qrt(c + d*sin(e + f*x))*sin(e + f*x)), x)/a**2
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\,\sin(e+f\,x))^2\,(c+d\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.515 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=405

$$-\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}}-\frac{(c-7d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^{3/2}}-\frac{}{3(c-d}$$

```
[Out] -1/3*d*(c^2-7*c*d-10*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^(
3/2)-1/3*(c-7*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^(
3/2)-1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2)-1/3*d
*(c+3*d)*(c^2-10*c*d-7*d^2)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e
))^(1/2)+1/3*(c+3*d)*(c^2-10*c*d-7*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)^4/(c+d)^2/f/((c+d*sin(f*x+e))
/(c+d))^(1/2)-1/3*(c^2-7*c*d-10*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d
))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e
))^(1/2)
```

**Rubi** [A]
time = 0.56, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{3a^2f(c-d)^4(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(c^2-7cd-10d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\left(c^2-10cd-7d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^4(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-7d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/3*(d*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x])/(a^2*(c - d)^3*(c + d)*f*(c +
d*Sin[e + f*x])^(3/2)) - ((c - 7*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + S
in[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(3*(c - d)*f*(a + a
*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - (d*(c + 3*d)*(c^2 - 10*c*d -
 7*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]
) - ((c + 3*d)*(c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[(c + d*
Sin[e + f*x])/(c + d)]) + ((c^2 - 7*c*d - 10*d^2)*EllipticF[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)^3*(c
 + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{(c-7d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{3a^2(c-d}$$

$$= -\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{3a^2(c-d}$$

$$= -\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{3a^2(c-d}$$

$$= -\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{3a^2(c-d}$$

$$= -\frac{d(c^2-7cd-10d^2)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{3a^2(c-d}$$

**Mathematica** [A]
time = 6.54, size = 674, normalized size = 1.66

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}\left(-\frac{2(c^3-7c^2d-27cd^2-15d^3)}{3(c-d)^4(c+d)^2}+\frac{2\sin\left(\frac{1}{2}(e+fx)\right)}{3(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{1}{3(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{2\left(c\sin\left(\frac{1}{2}(e+fx)\right)-9d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{3(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{2d^3\cos(e+fx)}{3(c-d)^3(c+d)(c+d\sin(e+fx))^2}+\frac{4(5cd^3\cos(e+fx)+3d^4\cos(e+fx))}{3(c-d)^4(c+d)^2(c+d\sin(e+fx))}\right)}{f(a+a\sin(e+fx))^2}+\frac{d\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\frac{2(26c^2d+28cd^2+10d^3)F\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}+\frac{2(c^3-7c^2d-37cd^2-21d^3)\cos^2(e+fx)\sqrt{c+d\sin(e+fx)}}{d(1-\sin^2(e+fx))}-\frac{(-c^3+7c^2d+37cd^2+21d^3)\left(\frac{2(c+d)E\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}-\frac{2cF\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}\right)}{d}\right)}{6(c-d)^4(c+d)^2f(a+a\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]]*((-2*(c^3
 - 7*c^2*d - 27*c*d^2 - 15*d^3))/(3*(c - d)^4*(c + d)^2) + (2*Sin[(e + f*x)
/2])/(3*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 1/(3*(c - d)^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(c*Sin[(e + f*x)/2] - 9*d*Si
n[(e + f*x)/2]))/(3*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (2*d
^3*Cos[e + f*x])/(3*(c - d)^3*(c + d)*(c + d*Sin[e + f*x])^2) + (4*(5*c*d^3
*Cos[e + f*x] + 3*d^4*Cos[e + f*x]))/(3*(c - d)^4*(c + d)^2*(c + d*Sin[e +
f*x]))))/(f*(a + a*Sin[e + f*x])^2) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4*((-2*(26*c^2*d + 28*c*d^2 + 10*d^3)*EllipticF[(-e + Pi/2 - f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] +
 (2*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f
*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-c^3 + 7*c^2*d + 37*c*d^2 + 21*d^3)*((2*
(c + d)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*
x]]))/d))/(6*(c - d)^4*(c + d)^2*f*(a + a*Sin[e + f*x])^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1757 vs. $2(443) = 886$.
time = 33.35, size = 1758, normalized size = 4.34

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1758 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(1/(c-d)^2*(-1/3/(c-d)*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin
(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(
1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
```

```
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))))+d^2/(c-d)^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-
sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-s
in(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(c-d)^3*d*(-(-sin(f*x
+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1
)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
))+2*d^2/(c-d)^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(
f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 3328, normalized size = 8.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)

[Out] 1/18*(2*(sqrt(2)*(c^4*d^2 - 7*c^3*d^3 + 2*c^2*d^4 + 21*c*d^5 + 15*d^6)*cos(
f*x + e)^4 - sqrt(2)*(2*c^5*d - 13*c^4*d^2 - 3*c^3*d^3 + 44*c^2*d^4 + 51*c*
d^5 + 15*d^6)*cos(f*x + e)^3 - sqrt(2)*(c^6 - 3*c^5*d - 23*c^4*d^2 + 8*c^3*
d^3 + 105*c^2*d^4 + 123*c*d^5 + 45*d^6)*cos(f*x + e)^2 + sqrt(2)*(c^6 - 5*c
^5*d - 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6)*cos(f*x +
e) - (sqrt(2)*(c^4*d^2 - 7*c^3*d^3 + 2*c^2*d^4 + 21*c*d^5 + 15*d^6)*cos(f*x
 + e)^3 + 2*sqrt(2)*(c^5*d - 6*c^4*d^2 - 5*c^3*d^3 + 23*c^2*d^4 + 36*c*d^5
+ 15*d^6)*cos(f*x + e)^2 - sqrt(2)*(c^6 - 5*c^5*d - 11*c^4*d^2 + 18*c^3*d^3
 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6)*cos(f*x + e) - 2*sqrt(2)*(c^6 - 5*c^5*d
- 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6))*sin(f*x + e) +
 2*sqrt(2)*(c^6 - 5*c^5*d - 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d^5
 + 15*d^6))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*
c)/d) + 2*(sqrt(2)*(c^4*d^2 - 7*c^3*d^3 + 2*c^2*d^4 + 21*c*d^5 + 15*d^6)*co
s(f*x + e)^4 - sqrt(2)*(2*c^5*d - 13*c^4*d^2 - 3*c^3*d^3 + 44*c^2*d^4 + 51*
c*d^5 + 15*d^6)*cos(f*x + e)^3 - sqrt(2)*(c^6 - 3*c^5*d - 23*c^4*d^2 + 8*c^
3*d^3 + 105*c^2*d^4 + 123*c*d^5 + 45*d^6)*cos(f*x + e)^2 + sqrt(2)*(c^6 - 5
*c^5*d - 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6)*cos(f*x
+ e) - (sqrt(2)*(c^4*d^2 - 7*c^3*d^3 + 2*c^2*d^4 + 21*c*d^5 + 15*d^6)*cos(f
*x + e)^3 + 2*sqrt(2)*(c^5*d - 6*c^4*d^2 - 5*c^3*d^3 + 23*c^2*d^4 + 36*c*d^
5 + 15*d^6)*cos(f*x + e)^2 - sqrt(2)*(c^6 - 5*c^5*d - 11*c^4*d^2 + 18*c^3*d
^3 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6)*cos(f*x + e) - 2*sqrt(2)*(c^6 - 5*c^5*
d - 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d^5 + 15*d^6))*sin(f*x + e)
 + 2*sqrt(2)*(c^6 - 5*c^5*d - 11*c^4*d^2 + 18*c^3*d^3 + 59*c^2*d^4 + 51*c*d
^5 + 15*d^6))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/2
7*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) +
2*I*c)/d) + 3*(sqrt(2)*(I*c^3*d^3 - 7*I*c^2*d^4 - 37*I*c*d^5 - 21*I*d^6)*co
s(f*x + e)^4 + sqrt(2)*(-2*I*c^4*d^2 + 13*I*c^3*d^3 + 81*I*c^2*d^4 + 79*I*c
*d^5 + 21*I*d^6)*cos(f*x + e)^3 + sqrt(2)*(-I*c^5*d + 3*I*c^4*d^2 + 62*I*c^
3*d^3 + 190*I*c^2*d^4 + 195*I*c*d^5 + 63*I*d^6)*cos(f*x + e)^2 + sqrt(2)*(I
*c^5*d - 5*I*c^4*d^2 - 50*I*c^3*d^3 - 102*I*c^2*d^4 - 79*I*c*d^5 - 21*I*d^6
)*cos(f*x + e) + (sqrt(2)*(-I*c^3*d^3 + 7*I*c^2*d^4 + 37*I*c*d^5 + 21*I*d^6
)*cos(f*x + e)^3 + 2*sqrt(2)*(-I*c^4*d^2 + 6*I*c^3*d^3 + 44*I*c^2*d^4 + 58*
I*c*d^5 + 21*I*d^6)*cos(f*x + e)^2 + sqrt(2)*(I*c^5*d - 5*I*c^4*d^2 - 50*I*
c^3*d^3 - 102*I*c^2*d^4 - 79*I*c*d^5 - 21*I*d^6)*cos(f*x + e) + 2*sqrt(2)*(
I*c^5*d - 5*I*c^4*d^2 - 50*I*c^3*d^3 - 102*I*c^2*d^4 - 79*I*c*d^5 - 21*I*d^
6))*sin(f*x + e) + 2*sqrt(2)*(I*c^5*d - 5*I*c^4*d^2 - 50*I*c^3*d^3 - 102*I*
c^2*d^4 - 79*I*c*d^5 - 21*I*d^6))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3
*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2
 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I
*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-I*c^3*d^3 + 7*I*c^2*d^4 + 37*I*
c*d^5 + 21*I*d^6)*cos(f*x + e)^4 + sqrt(2)*(2*I*c^4*d^2 - 13*I*c^3*d^3 - 81
```

```
*I*c^2*d^4 - 79*I*c*d^5 - 21*I*d^6)*cos(f*x + e)^3 + sqrt(2)*(I*c^5*d - 3*I
*c^4*d^2 - 62*I*c^3*d^3 - 190*I*c^2*d^4 - 195*I*c*d^5 - 63*I*d^6)*cos(f*x +
 e)^2 + sqrt(2)*(-I*c^5*d + 5*I*c^4*d^2 + 50*I*c^3*d^3 + 102*I*c^2*d^4 + 79
*I*c*d^5 + 21*I*d^6)*cos(f*x + e) + (sqrt(2)*(I*c^3*d^3 - 7*I*c^2*d^4 - 37*
I*c*d^5 - 21*I*d^6)*cos(f*x + e)^3 + 2*sqrt(2)*(I*c^4*d^2 - 6*I*c^3*d^3 - 4
4*I*c^2*d^4 - 58*I*c*d^5 - 21*I*d^6)*cos(f*x + e)^2 + sqrt(2)*(-I*c^5*d + 5
*I*c^4*d^2 + 50*I*c^3*d^3 + 102*I*c^2*d^4 + 79*I*c*d^5 + 21*I*d^6)*cos(f*x
+ e) + 2*sqrt(2)*(-I*c^5*d + 5*I*c^4*d^2 + 50*I*c^3*d^3 + 102*I*c^2*d^4 + 7
9*I*c*d^5 + 21*I*d^6))*sin(f*x + e) + 2*sqrt(2)*(-I*c^5*d + 5*I*c^4*d^2 + 5
0*I*c^3*d^3 + 102*I*c^2*d^4 + 79*I*c*d^5 + 21*I*d^6))*sqrt(-I*d)*weierstras
sZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstra
ssPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*
(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(c^5*d - c^4*d^2 -
2*c^3*d^3 + 2*c^2*d^4 + c*d^5 - d^6 - (c^3*d^3 - 7*c^2*d^4 - 37*c*d^5 - 21*
d^6)*cos(f*x + e)^4 - 2*(c^4*d^2 - 6*c^3*d^3 - 31*c^2*d^4 - 44*c*d^5 - 16*d
^6)*cos(f*x + e)^3 + (c^5*d - 5*c^4*d^2 - 15*c^3*d^3 - 41*c^2*d^4 - 50*c*d^
5 - 18*d^6)*cos(f*x + e)^2 + 2*(c^5*d - 2*c^4*d^2 - 15*c^3*d^3 - 47*c^2*d^4
 - 50*c*d^5 - 15*d^6)*cos(f*x + e) - (c^5*d - c^4*d^2 - 2*c^3*d^3 + 2*c^2*d
^4 + c*d^5 - d^6 + (c^3*d^3 - 7*c^2*d^4 - 37*c*d^5 - 21*d^6)*cos(f*x + e)^3
 - (2*c^4*d^2 - 13*c^3*d^3 - 55*c^2*d^4 - 51*c*d^5 - 11*d^6)*cos(f*x + e)^2
 - (c^5*d - 3*c^4*d^2 - 28*c^3*d^3 - 96*c^2*d^4 - 101*c*d^5 - 29*d^6)*cos(f
*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((a^2*c^6*d^3 - 2*a^2*c^5*
d^4 - a^2*c^4*d^5 + 4*a^2*c^3*d^6 - a^2*c^2*d^7 - 2*a^2*c*d^8 + a^2*d^9)*f*
cos(f*x + e)^4 - (2*a^2*c^7*d^2 - 3*a^2*c^6*d^3 - 4*a^2*c^5*d^4 + 7*a^2*c^4
*d^5 + 2*a^2*c^3*d^6 - 5*a^2*c^2*d^7 + a^2*d^9)...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2c^2\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c^2\sqrt{c+d\sin(e+fx)}+2cd\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+4cd\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)+d^2\sqrt{c+d\sin(e+fx)}\sin^4(e+fx)+2d^2\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+d^2\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x) + c**2*sqrt(c + d*sin(e + f*x)) + 2*c*d*sqrt(
c + d*sin(e + f*x))*sin(e + f*x)**3 + 4*c*d*sqrt(c + d*sin(e + f*x))*sin(e
+ f*x)**2 + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + d**2*sqrt(c + d*s
in(e + f*x))*sin(e + f*x)**4 + 2*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)
**3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a**2
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

# 3.516 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=322

$$-\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}-\frac{(4c^2+15cd+27d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3+a^3\sin(e+fx)\right)}-$$

```
[Out] -1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^3-2/15*(c-d
)*(c+3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^2-1/30*(4*
c^2+15*c*d+27*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a^3+a^3*sin(f*x+e))
+1/30*(4*c^2+15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*(c+d*sin(f*x+e))^(1/2)/a^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(c+d)*(4*
c^2+11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2
*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*si
n(f*x+e))/(c+d))^(1/2)/a^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.57, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2844, 3056, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2+15cd+27d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3\sin(e+fx)+a^3\right)}+\frac{(c+d)\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f\sqrt{c+d\sin(e+fx)}}-\frac{(4c^2+15cd+27d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f(a\sin(e+fx)+a)^3}-\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*(c - d)*(c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a
*Sin[e + f*x])^2) - ((4*c^2 + 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(30*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(5*f*(a + a*Sin[e + f*x])^3) - ((4*c^2 + 15*c*d + 27*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
30*a^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c^2 + 11*c*d + 1
5*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)])/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\,\left(-\frac{1}{2}a(4c^2\right.}{(a+a\sin(e}}{5a}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c}{5f(a+a\sin(}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2+15cd+27d^2)\,\mathrm{co}}{30f\,(a^3}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2+15cd+27d^2)\,\mathrm{co}}{30f\,(a^3}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2+15cd+27d^2)\,\mathrm{co}}{30f\,(a^3}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2+15cd+27d^2)\,\mathrm{co}}{30f\,(a^3}$$

**Mathematica [A]**
time = 3.88, size = 385, normalized size = 1.20

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left((4c^2+15cd+27d^2)(c+d\sin(e+fx))\right)-\frac{\left(-2d(35c+57d)\cos\left(\frac{1}{2}(e+fx)\right)+(20c^2+74cd+90d^2)\cos\left(\frac{3}{2}(e+fx)\right)+2\left(-3(6c^2+11cd+29d^2)+2(2c^2+7cd-9d^2)\cos(e+fx)+(4c^2+15cd+27d^2)\cos(2(e+fx))\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+(c-15d)d^2F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(4c^2+15cd+27d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{30a^3f(1+\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 + 15*c*d + 27*d^2)*(c +
d*Sin[e + f*x])) - ((-2*d*(35*c + 57*d)*Cos[(e + f*x)/2] + (20*c^2 + 74*c*d
 + 90*d^2)*Cos[(3*(e + f*x))/2] + 2*(-3*(6*c^2 + 11*c*d + 29*d^2) + 2*(2*c^
2 + 7*c*d - 9*d^2)*Cos[e + f*x] + (4*c^2 + 15*c*d + 27*d^2)*Cos[2*(e + f*x)
])*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(2*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5) + (c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*f*(1
+ Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. $2(364) = 728$.
time = 28.40, size = 1615, normalized size = 5.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1615 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+3*d*(c^2-2*c*d+d^2)*(-1/3/(c-d)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+
e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin
(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))))+3*d^2*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+
e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*
c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*
x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elli
pticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*s
```

```
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^3-3*c^2*d+3*c*d^2-d^3)*(-
1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-
3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*
(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)
/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/
(60*c^3-180*c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^3, x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1558, normalized size = 4.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/180*((sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e)^3 + 3*s
qrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8
*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 + 30*c^
2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 30*c^2*d + 51*
c*d^2 + 45*d^3)*cos(f*x + e) - 4*sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*
d^3))*sin(f*x + e) - 4*sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3))*sqrt
(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*
d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)
*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(8*c^3 +
 30*c^2*d + 51*c*d^2 + 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 30*c^2*d
 + 51*c*d^2 + 45*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2
+ 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3)
*cos(f*x + e) - 4*sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3))*sin(f*x +
 e) - 4*sqrt(2)*(8*c^3 + 30*c^2*d + 51*c*d^2 + 45*d^3))*sqrt(-I*d)*weierstr
```

```
assPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-4*I*c^2*
d - 15*I*c*d^2 - 27*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(-4*I*c^2*d - 15*I*c*
d^2 - 27*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^2*d + 15*I*c*d^2 + 27*I*d
^3)*cos(f*x + e) + (sqrt(2)*(-4*I*c^2*d - 15*I*c*d^2 - 27*I*d^3)*cos(f*x +
e)^2 + 2*sqrt(2)*(4*I*c^2*d + 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e) + 4*sqrt(
2)*(4*I*c^2*d + 15*I*c*d^2 + 27*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(4*I*c^2*d
 + 15*I*c*d^2 + 27*I*d^3))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(
f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(4*I*c^2*d + 15*I*c*d^2 + 27*I*d^3)*cos(
f*x + e)^3 + 3*sqrt(2)*(4*I*c^2*d + 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e)^2 +
 2*sqrt(2)*(-4*I*c^2*d - 15*I*c*d^2 - 27*I*d^3)*cos(f*x + e) + (sqrt(2)*(4*
I*c^2*d + 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^2*d - 1
5*I*c*d^2 - 27*I*d^3)*cos(f*x + e) + 4*sqrt(2)*(-4*I*c^2*d - 15*I*c*d^2 - 2
7*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(-4*I*c^2*d - 15*I*c*d^2 - 27*I*d^3))*sq
rt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*
d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9
*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*
((4*c^2*d + 15*c*d^2 + 27*d^3)*cos(f*x + e)^3 - 6*c^2*d + 12*c*d^2 - 6*d^3
- (8*c^2*d + 29*c*d^2 + 9*d^3)*cos(f*x + e)^2 - 2*(9*c^2*d + 16*c*d^2 + 21*
d^3)*cos(f*x + e) + (6*c^2*d - 12*c*d^2 + 6*d^3 - (4*c^2*d + 15*c*d^2 + 27*
d^3)*cos(f*x + e)^2 - 4*(3*c^2*d + 11*c*d^2 + 9*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(d*sin(f*x + e) + c))/(a^3*d*f*cos(f*x + e)^3 + 3*a^3*d*f*cos(f*x
 + e)^2 - 2*a^3*d*f*cos(f*x + e) - 4*a^3*d*f + (a^3*d*f*cos(f*x + e)^2 - 2*
a^3*d*f*cos(f*x + e) - 4*a^3*d*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3, x)
```

# 3.517 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=323

$$-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}-\frac{(4c^2+5cd-3d^2)\cos}{30(c-d)f}$$

```
[Out] -1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^3-2/15*(c+2
*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^2-1/30*(4*c^2+5*
c*d-3*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a^3+a^3*sin(f*x+e))+1
/30*(4*c^2+5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+
d*sin(f*x+e))^(1/2)/a^3/(c-d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(c+d)*(
4*c+5*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/a^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.58, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2844, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2+5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)\left(a^3\sin(e+fx)+a^3\right)}-\frac{(4c^2+5cd-3d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c+d)(4c+5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f\sqrt{c+d\sin(e+fx)}}-\frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x]
)^3) - (2*(c + 2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a*S
in[e + f*x])^2) - ((4*c^2 + 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(30*(c - d)*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 + 5*c*d - 3*d^2)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a
^3*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c + 5*d)*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a\left(4c^2+7cd-d^2\right)-\frac{1}{2}ad(3c+7d)\sin(e+\ldots}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+f\ldots}}}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{15af(a+a\sin(e+fx))\ldots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{15af(a+a\sin(e+fx))\ldots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{15af(a+a\sin(e+fx))\ldots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{15af(a+a\sin(e+fx))\ldots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{15af(a+a\sin(e+fx))\ldots}$$

**Mathematica** [A]
time = 4.11, size = 441, normalized size = 1.37

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\left(\frac{2\left(6(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+3(-c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(c+2d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2(c+2d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\frac{\left(4c^2+5cd-3d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{c-d}\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}-\frac{\left(4c^2+5cd-3d^2\right)(c+d\sin(e+fx))-d^2(c+5d)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+\left(-4c^2-5cd+3d^2\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{c-d}\right)}{30a^3f(1+\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((2*(6*(c - d)*Sin[(e + f*x)/2] +
3*(-c + d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c + 2*d)*Sin[(e + f*x
)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c + 2*d)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3 + ((4*c^2 + 5*c*d - 3*d^2)*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d))*(c + d*Sin[e + f*x]))/(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5 - ((4*c^2 + 5*c*d - 3*d^2)*(c + d*Sin[e + f*x
```

```
]) - d^2*(c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)] + (-4*c^2 - 5*c*d + 3*d^2)*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(c - d)))/(30*a^3*f*(1 +
Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(365) = 730$.
time = 28.63, size = 1462, normalized size = 4.53

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1462 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(2*d*(c-d)*(-1/3/(c-d)*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin
(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(
1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))))+d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+
c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2
*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e
))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^2-2*c*d+d^2)*(-1/5/(c-d)*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^
2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x
+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c
^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))
```

```
^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 1634, normalized size = 5.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
[Out] 1/180*((sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3)*cos(f*x + e)^3 + 3*sq
rt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c
^3 + 10*c^2*d - 9*c*d^2 - 15*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 + 10*c^2*d
 - 9*c*d^2 - 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2
 - 15*d^3)*cos(f*x + e) - 4*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3))*
sin(f*x + e) - 4*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3))*sqrt(I*d)*w
eierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^
3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*c^3
 + 10*c^2*d - 9*c*d^2 - 15*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(8*c^3 + 10*c^2*
d - 9*c*d^2 - 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^
2 - 15*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3)*c
os(f*x + e)^2 - 2*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3)*cos(f*x + e
) - 4*sqrt(2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3))*sin(f*x + e) - 4*sqrt(
2)*(8*c^3 + 10*c^2*d - 9*c*d^2 - 15*d^3))*sqrt(-I*d)*weierstrassPInverse(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x
+ e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(4*I*c^2*d + 5*I*c*d^2 -
 3*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(4*I*c^2*d + 5*I*c*d^2 - 3*I*d^3)*cos(
f*x + e)^2 + 2*sqrt(2)*(-4*I*c^2*d - 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e) + (s
qrt(2)*(4*I*c^2*d + 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*c
^2*d - 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e) + 4*sqrt(2)*(-4*I*c^2*d - 5*I*c*d^
2 + 3*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(-4*I*c^2*d - 5*I*c*d^2 + 3*I*d^3))*
sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*
d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*
I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(
sqrt(2)*(-4*I*c^2*d - 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(-4*I
```

```
*c^2*d - 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^2*d + 5*I*c
*d^2 - 3*I*d^3)*cos(f*x + e) + (sqrt(2)*(-4*I*c^2*d - 5*I*c*d^2 + 3*I*d^3)*
cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^2*d + 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e) +
 4*sqrt(2)*(4*I*c^2*d + 5*I*c*d^2 - 3*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(4*I
*c^2*d + 5*I*c*d^2 - 3*I*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
*d*sin(f*x + e) + 2*I*c)/d)) - 6*((4*c^2*d + 5*c*d^2 - 3*d^3)*cos(f*x + e)^
3 - 6*c^2*d + 12*c*d^2 - 6*d^3 - (8*c^2*d + 9*c*d^2 - 11*d^3)*cos(f*x + e)^
2 - 2*(9*c^2*d + c*d^2 - 4*d^3)*cos(f*x + e) + (6*c^2*d - 12*c*d^2 + 6*d^3
- (4*c^2*d + 5*c*d^2 - 3*d^3)*cos(f*x + e)^2 - 2*(6*c^2*d + 7*c*d^2 - 7*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((a^3*c*d - a^3*d^2
)*f*cos(f*x + e)^3 + 3*(a^3*c*d - a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c*d -
a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c*d - a^3*d^2)*f + ((a^3*c*d - a^3*d^2)*f*
cos(f*x + e)^2 - 2*(a^3*c*d - a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c*d - a^3*d^
2)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\sin(e+f\,x))^{3/2}}{(a+a\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3, x)
```

# 3.518 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=334

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}-\frac{(4c^2-5cd-3d^2)\cos(e+fx}{30(c-d)^2f\,(a^3+a^3}$$

```
[Out] -1/5*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^3-1/15*(2*c-d)*co
s(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/(a+a*sin(f*x+e))^2-1/30*(4*c^2-5*
c*d-3*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)^2/f/(a^3+a^3*sin(f*x+e))
+1/30*(4*c^2-5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(
c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(4*
c-5*d)*(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+
e))/(c+d))^(1/2)/a^3/(c-d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.51, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2843, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2-5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^2\,(a^3\sin(e+fx)+a^3)}-\frac{(4c^2-5cd-3d^2)\,\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(4c-5d)(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(c-d)(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])^3) - (
(2*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)*f*(a + a*Sin
[e + f*x])^2) - ((4*c^2 - 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(30*(c - d)^2*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 5*c*d - 3*d^2)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a
^3*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c - 5*d)*(c + d)*E
llipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/(30*a^3*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}+\frac{\int \frac{\frac{1}{2}a(4c+d)+\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3}-\frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2}$$

**Mathematica** [A]
time = 3.89, size = 449, normalized size = 1.34

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left(\left(4c^2-5cd-3d^2\right)(c+d\sin(e+fx))\right)+\frac{2\left(6(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c-d)(2c-d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(c-d)(2c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\left(4c^2-5cd-3d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+(c-5d)d^2F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+\left(4c^2-5cd-3d^2\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{30a^3(c-d)^2f(1+\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 - 5*c*d - 3*d^2)*(c + d*
Sin[e + f*x])) + (2*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c - d)^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*(2*c - d)*Sin[(e + f*x)/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(2*c - d)*(Cos[(e + f*x)/2] + Sin[
```

```
(e + f*x)/2])^3 + (4*c^2 - 5*c*d - 3*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4)*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5 + (c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 - 5*c*d - 3*d^2)*((c + d)*Elli
pticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*
x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*(c - d)^
2*f*(1 + Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(376) = 752$.

time = 24.84, size = 1056, normalized size = 3.16

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1056 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(d*(-1/3/(c-d)*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+
d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*
x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))+(c-d)*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-
d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e))
)^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)
^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*si
n(f*x+e))^(1/2)/f
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 1703, normalized size = 5.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/180*((sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3)*cos(f*x + e)^3 + 3*sq
rt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c
^3 - 10*c^2*d - 9*c*d^2 + 15*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 - 10*c^2*d
 - 9*c*d^2 + 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2
 + 15*d^3)*cos(f*x + e) - 4*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3))*
sin(f*x + e) - 4*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3))*sqrt(I*d)*w
eierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^
3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*c^3
 - 10*c^2*d - 9*c*d^2 + 15*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(8*c^3 - 10*c^2*
d - 9*c*d^2 + 15*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^
2 + 15*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3)*c
os(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3)*cos(f*x + e
) - 4*sqrt(2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3))*sin(f*x + e) - 4*sqrt(
2)*(8*c^3 - 10*c^2*d - 9*c*d^2 + 15*d^3))*sqrt(-I*d)*weierstrassPInverse(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x
+ e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(-4*I*c^2*d + 5*I*c*d^2
+ 3*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(-4*I*c^2*d + 5*I*c*d^2 + 3*I*d^3)*co
s(f*x + e)^2 + 2*sqrt(2)*(4*I*c^2*d - 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e) + (
sqrt(2)*(-4*I*c^2*d + 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*
c^2*d - 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e) + 4*sqrt(2)*(4*I*c^2*d - 5*I*c*d^
2 - 3*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(4*I*c^2*d - 5*I*c*d^2 - 3*I*d^3))*s
qrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d
^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I
*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(s
qrt(2)*(4*I*c^2*d - 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(4*I*c^
2*d - 5*I*c*d^2 - 3*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^2*d + 5*I*c*d
^2 + 3*I*d^3)*cos(f*x + e) + (sqrt(2)*(4*I*c^2*d - 5*I*c*d^2 - 3*I*d^3)*cos
(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^2*d + 5*I*c*d^2 + 3*I*d^3)*cos(f*x + e) + 4
*sqrt(2)*(-4*I*c^2*d + 5*I*c*d^2 + 3*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(-4*I
*c^2*d + 5*I*c*d^2 + 3*I*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
```

```
*d*sin(f*x + e) + 2*I*c)/d)) - 6*((4*c^2*d - 5*c*d^2 - 3*d^3)*cos(f*x + e)^
3 - 6*c^2*d + 12*c*d^2 - 6*d^3 - (8*c^2*d - 11*c*d^2 - d^3)*cos(f*x + e)^2
- 2*(9*c^2*d - 14*c*d^2 + d^3)*cos(f*x + e) + (6*c^2*d - 12*c*d^2 + 6*d^3 -
 (4*c^2*d - 5*c*d^2 - 3*d^3)*cos(f*x + e)^2 - 4*(3*c^2*d - 4*c*d^2 - d^3)*c
os(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/((a^3*c^2*d - 2*a^3*c*
d^2 + a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*f*c
os(f*x + e)^2 - 2*(a^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*f*cos(f*x + e) - 4*(a
^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*f + ((a^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*
f*cos(f*x + e)^2 - 2*(a^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*f*cos(f*x + e) - 4
*(a^3*c^2*d - 2*a^3*c*d^2 + a^3*d^3)*f)*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sin(e+fx)}}{\sin^3(e+fx)+3\sin^2(e+fx)+3\sin(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*
sin(e + f*x) + 1), x)/a**3
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin(e+f\,x)}}{\left(a+a\,\sin(e+f\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3, x)
```

# 3.519 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=344

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3}-\frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)^2f(a+a\sin(e+fx))^2}-\frac{(4c^2-15cd+27d^2)\cos(e+}{30(c-d)^3f\,(a^3+}$$

```
[Out] -1/5*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^3-2/15*(c-3
*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/30*(
4*c^2-15*c*d+27*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)^3/f/(a^3+a^3*s
in(f*x+e))+1/30*(4*c^2-15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+
d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1
/2)-1/30*(4*c^2-11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^3/(c-d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.50, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2845, 3057, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2-15cd+27d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^3\,(a^3\sin(e+fx)+a^3)}+\frac{(4c^2-11cd+15d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{c+d\sin(e+fx)}}-\frac{(4c^2-15cd+27d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(c-d)(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -1/5*(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((c - d)*f*(a + a*Sin[e + f*x]
)^3) - (2*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)^2*
f*(a + a*Sin[e + f*x])^2) - ((4*c^2 - 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 15
*c*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[
e + f*x]])/(30*a^3*(c - d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^
2 - 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(4c-9d)-\frac{3}{2}a}{(a+a\sin(e+fx))^2\sqrt{c+}}}{5a^2(c-}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)}{15a(c-d)^2f(a+}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)}{15a(c-d)^2f(a+}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)}{15a(c-d)^2f(a+}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)}{15a(c-d)^2f(a+}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)}{15a(c-d)^2f(a+}$$

**Mathematica [A]**
time = 4.19, size = 445, normalized size = 1.29

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left((4c^2-15cd+27d^2)(c+d\sin(e+fx))\right)+\frac{2\left(6(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(c-3d)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2(c-3d)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(4c^2-15cd+27d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+d^2(c+15d)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+(4c^2-15cd+27d^2)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{30a^3(c-d)^3f(1+\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 - 15*c*d + 27*d^2)*(c +
d*Sin[e + f*x])) + (2*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c - d)^2*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 3*d)*(c - d)*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - 3*d)*(c - d)*(Cos[(e + f*x)/2] +
```

Sin[(e + f*x)/2])^3 + (4*c^2 - 15*c*d + 27*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + d^2*(c + 15*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 - 15*c*d + 27*d^2)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])

**Maple** [A]
time = 17.37, size = 593, normalized size = 1.72

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{5(c-d)(1+\sin(fx+e))^{3}}-\frac{2(c-3d)}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1789, normalized size = 5.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/180*((sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e)^3 + 3*s
qrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8
*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 - 30*c^
2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 30*c^2*d + 51*
c*d^2 - 45*d^3)*cos(f*x + e) - 4*sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*
d^3))*sin(f*x + e) - 4*sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3))*sqrt
(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*
d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)
*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(8*c^3 -
 30*c^2*d + 51*c*d^2 - 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 30*c^2*d
 + 51*c*d^2 - 45*d^3)*cos(f*x + e) + (sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2
- 45*d^3)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3)
*cos(f*x + e) - 4*sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3))*sin(f*x +
 e) - 4*sqrt(2)*(8*c^3 - 30*c^2*d + 51*c*d^2 - 45*d^3))*sqrt(-I*d)*weierstr
assPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(4*I*c^2*d
 - 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e)^3 + 3*sqrt(2)*(4*I*c^2*d - 15*I*c*d^
2 + 27*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^
3)*cos(f*x + e) + (sqrt(2)*(4*I*c^2*d - 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e)
^2 + 2*sqrt(2)*(-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^3)*cos(f*x + e) + 4*sqrt(2
)*(-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(-4*I*c^2*
d + 15*I*c*d^2 - 27*I*d^3))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/
d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin
(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^3)*co
s(f*x + e)^3 + 3*sqrt(2)*(-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^3)*cos(f*x + e)^
2 + 2*sqrt(2)*(4*I*c^2*d - 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e) + (sqrt(2)*(
-4*I*c^2*d + 15*I*c*d^2 - 27*I*d^3)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^2*d -
 15*I*c*d^2 + 27*I*d^3)*cos(f*x + e) + 4*sqrt(2)*(4*I*c^2*d - 15*I*c*d^2 +
27*I*d^3))*sin(f*x + e) + 4*sqrt(2)*(4*I*c^2*d - 15*I*c*d^2 + 27*I*d^3))*sq
rt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*
d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9
*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*
((4*c^2*d - 15*c*d^2 + 27*d^3)*cos(f*x + e)^3 - 6*c^2*d + 12*c*d^2 - 6*d^3
- (8*c^2*d - 31*c*d^2 + 39*d^3)*cos(f*x + e)^2 - 2*(9*c^2*d - 29*c*d^2 + 36
*d^3)*cos(f*x + e) + (6*c^2*d - 12*c*d^2 + 6*d^3 - (4*c^2*d - 15*c*d^2 + 27
```

```
*d^3)*cos(f*x + e)^2 - 2*(6*c^2*d - 23*c*d^2 + 33*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(d*sin(f*x + e) + c))/((a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3
 - a^3*d^4)*f*cos(f*x + e)^3 + 3*(a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 -
 a^3*d^4)*f*cos(f*x + e)^2 - 2*(a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 - a
^3*d^4)*f*cos(f*x + e) - 4*(a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 - a^3*d
^4)*f + ((a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 - a^3*d^4)*f*cos(f*x + e)
^2 - 2*(a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 - a^3*d^4)*f*cos(f*x + e) -
 4*(a^3*c^3*d - 3*a^3*c^2*d^2 + 3*a^3*c*d^3 - a^3*d^4)*f)*sin(f*x + e))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+3\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+3\sqrt{c+d\sin(e+fx)}\sin(e+fx)+\sqrt{c+d\sin(e+fx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**2 + 3*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c +
 d*sin(e + f*x))), x)/a**3
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.520 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=423

$$-\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}-\frac{}{15a(c-}$$

```
[Out] -1/30*d*(4*c^3-21*c^2*d+62*c*d^2+147*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c
+d*sin(f*x+e))^(1/2)-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x
+e))^(1/2)-2/15*(c-4*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(
f*x+e))^(1/2)-1/30*(4*c^2-21*c*d+65*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(
f*x+e))/(c+d*sin(f*x+e))^(1/2)+1/30*(4*c^3-21*c^2*d+62*c*d^2+147*d^3)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)
^4/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(4*c^2-21*c*d+65*d^2)*(sin(1
/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e
+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^
3/(c-d)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.66, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2-21cd+65d^2)\cos(e+fx)}{30f(c-d)^3(a^3\sin(e+fx)+a^3)\sqrt{c+d\sin(e+fx)}}+\frac{(4c^2-21cd+65d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^3\sqrt{c+d\sin(e+fx)}}-\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3f(c-d)^4(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{(4c^3-21c^2d+62cd^2+147d^3)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^4(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(c-4d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/30*(d*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*Cos[e + f*x])/(a^3*(c - d)
^4*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(5*(c - d)*f*(a + a*S
in[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]) - (2*(c - 4*d)*Cos[e + f*x])/(15*a
*(c - d)^2*f*(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((4*c^2 - 2
1*c*d + 65*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*Sqrt
[c + d*Sin[e + f*x]]) - ((4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c - d
)^4*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2 - 21*c*d + 65*d
^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(30*a^3*(c - d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{\int}{}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{}{15}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{}{15}$$

$$= -\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{5(c-d)}$$

$$= -\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{5(c-d)}$$

$$= -\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{5(c-d)}$$

$$= -\frac{d(4c^3-21c^2d+62cd^2+147d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{}{5(c-d)}$$

**Mathematica [A]**
time = 6.39, size = 745, normalized size = 1.76

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}\left(-\frac{4c^3-21c^2d+62cd^2+117d^3}{15(c-d)^4(c+d)}+\frac{2\sin\left(\frac{1}{2}(e+fx)\right)}{5(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}-\frac{1}{5(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}+\frac{-2c+11d}{15(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{2\left(2c\sin\left(\frac{1}{2}(e+fx)\right)-11d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{15(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{4c^2\sin\left(\frac{1}{2}(e+fx)\right)-25cd\sin\left(\frac{1}{2}(e+fx)\right)+87d^2\sin\left(\frac{1}{2}(e+fx)\right)}{15(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{2d^4\cos(e+fx)}{(c-d)^4(c+d)(c+d\sin(e+fx))}\right)}{f(a+a\sin(e+fx))^3}+\frac{d\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\frac{2(-c^2d-126cd^2-65d^3)F\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}+\frac{2(4c^3-21c^2d+62cd^2+147d^3)\cos^2(e+fx)\sqrt{c+d\sin(e+fx)}}{d(1-\sin^2(e+fx))}-\frac{(-4c^3+21c^2d-62cd^2-147d^3)\left(\frac{2(c+d)E\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}-\frac{2cF\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{\sqrt{c+d\sin(e+fx)}}\right)}{d}\right)}{60(c-d)^4(c+d)f(a+a\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-1/15*(4
*c^3 - 21*c^2*d + 62*c*d^2 + 117*d^3)/((c - d)^4*(c + d)) + (2*Sin[(e + f*x
)/2])/(5*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - 1/(5*(c - d)^
2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (-2*c + 11*d)/(15*(c - d)^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(2*c*Sin[(e + f*x)/2] - 11*d*Si
n[(e + f*x)/2]))/(15*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (
4*c^2*Sin[(e + f*x)/2] - 25*c*d*Sin[(e + f*x)/2] + 87*d^2*Sin[(e + f*x)/2])
/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d^4*Cos[e + f*x]
)/((c - d)^4*(c + d)*(c + d*Sin[e + f*x]))))/(f*(a + a*Sin[e + f*x])^3) + (
d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-2*(-(c^2*d) - 126*c*d^2 - 65*d
^3)*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/Sqrt[c + d*Sin[e + f*x]] + (2*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147
*d^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - (
(-4*c^3 + 21*c^2*d - 62*c*d^2 - 147*d^3)*((2*(c + d)*EllipticE[(-e + Pi/2 -
 f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[
e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d)^4*(c + d
)*f*(a + a*Sin[e + f*x])^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1850 vs. $2(461) = 922$.
time = 32.72, size = 1851, normalized size = 4.38

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1851 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-d/(c-d)^2*(-1/3/(c-d)*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*si
n(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*
(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/
```

```
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))))+1/(c-d)*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*
x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+
sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-60*d^3)*(c
/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-si
n(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*d+27
*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+d^2/(c-d
)^3*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c
)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))-d^3/(c-d)^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.32, size = 3124, normalized size = 7.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/180*((sqrt(2)*(8*c^4*d - 42*c^3*d^2 + 121*c^2*d^3 - 84*c*d^4 - 195*d^5)*c
os(f*x + e)^4 - sqrt(2)*(8*c^5 - 26*c^4*d + 37*c^3*d^2 + 158*c^2*d^3 - 363*
c*d^4 - 390*d^5)*cos(f*x + e)^3 - sqrt(2)*(24*c^5 - 86*c^4*d + 153*c^3*d^2
+ 353*c^2*d^3 - 1005*c*d^4 - 975*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^5 - 3
4*c^4*d + 79*c^3*d^2 + 37*c^2*d^3 - 279*c*d^4 - 195*d^5)*cos(f*x + e) - (sq
rt(2)*(8*c^4*d - 42*c^3*d^2 + 121*c^2*d^3 - 84*c*d^4 - 195*d^5)*cos(f*x + e
)^3 + sqrt(2)*(8*c^5 - 18*c^4*d - 5*c^3*d^2 + 279*c^2*d^3 - 447*c*d^4 - 585
*d^5)*cos(f*x + e)^2 - 2*sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^3*d^2 + 37*c^2*d^
3 - 279*c*d^4 - 195*d^5)*cos(f*x + e) - 4*sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^
3*d^2 + 37*c^2*d^3 - 279*c*d^4 - 195*d^5))*sin(f*x + e) + 4*sqrt(2)*(8*c^5
- 34*c^4*d + 79*c^3*d^2 + 37*c^2*d^3 - 279*c*d^4 - 195*d^5))*sqrt(I*d)*weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(8*c^4*d
- 42*c^3*d^2 + 121*c^2*d^3 - 84*c*d^4 - 195*d^5)*cos(f*x + e)^4 - sqrt(2)*(
8*c^5 - 26*c^4*d + 37*c^3*d^2 + 158*c^2*d^3 - 363*c*d^4 - 390*d^5)*cos(f*x
+ e)^3 - sqrt(2)*(24*c^5 - 86*c^4*d + 153*c^3*d^2 + 353*c^2*d^3 - 1005*c*d^
4 - 975*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^3*d^2 + 37
*c^2*d^3 - 279*c*d^4 - 195*d^5)*cos(f*x + e) - (sqrt(2)*(8*c^4*d - 42*c^3*d
^2 + 121*c^2*d^3 - 84*c*d^4 - 195*d^5)*cos(f*x + e)^3 + sqrt(2)*(8*c^5 - 18
*c^4*d - 5*c^3*d^2 + 279*c^2*d^3 - 447*c*d^4 - 585*d^5)*cos(f*x + e)^2 - 2*
sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^3*d^2 + 37*c^2*d^3 - 279*c*d^4 - 195*d^5)*
cos(f*x + e) - 4*sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^3*d^2 + 37*c^2*d^3 - 279*
c*d^4 - 195*d^5))*sin(f*x + e) + 4*sqrt(2)*(8*c^5 - 34*c^4*d + 79*c^3*d^2 +
 37*c^2*d^3 - 279*c*d^4 - 195*d^5))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(4*I*c^3*d^2 - 21*I*c^2*d^3 +
62*I*c*d^4 + 147*I*d^5)*cos(f*x + e)^4 + sqrt(2)*(-4*I*c^4*d + 13*I*c^3*d^2
 - 20*I*c^2*d^3 - 271*I*c*d^4 - 294*I*d^5)*cos(f*x + e)^3 + sqrt(2)*(-12*I*
c^4*d + 43*I*c^3*d^2 - 81*I*c^2*d^3 - 751*I*c*d^4 - 735*I*d^5)*cos(f*x + e)
^2 + 2*sqrt(2)*(4*I*c^4*d - 17*I*c^3*d^2 + 41*I*c^2*d^3 + 209*I*c*d^4 + 147
*I*d^5)*cos(f*x + e) + (sqrt(2)*(-4*I*c^3*d^2 + 21*I*c^2*d^3 - 62*I*c*d^4 -
 147*I*d^5)*cos(f*x + e)^3 + sqrt(2)*(-4*I*c^4*d + 9*I*c^3*d^2 + I*c^2*d^3
- 333*I*c*d^4 - 441*I*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^4*d - 17*I*c^3
*d^2 + 41*I*c^2*d^3 + 209*I*c*d^4 + 147*I*d^5)*cos(f*x + e) + 4*sqrt(2)*(4*
I*c^4*d - 17*I*c^3*d^2 + 41*I*c^2*d^3 + 209*I*c*d^4 + 147*I*d^5))*sin(f*x +
 e) + 4*sqrt(2)*(4*I*c^4*d - 17*I*c^3*d^2 + 41*I*c^2*d^3 + 209*I*c*d^4 + 14
7*I*d^5))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
```

```
3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*
I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)
/d)) + 3*(sqrt(2)*(-4*I*c^3*d^2 + 21*I*c^2*d^3 - 62*I*c*d^4 - 147*I*d^5)*co
s(f*x + e)^4 + sqrt(2)*(4*I*c^4*d - 13*I*c^3*d^2 + 20*I*c^2*d^3 + 271*I*c*d
^4 + 294*I*d^5)*cos(f*x + e)^3 + sqrt(2)*(12*I*c^4*d - 43*I*c^3*d^2 + 81*I*
c^2*d^3 + 751*I*c*d^4 + 735*I*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^4*d +
 17*I*c^3*d^2 - 41*I*c^2*d^3 - 209*I*c*d^4 - 147*I*d^5)*cos(f*x + e) + (sqr
t(2)*(4*I*c^3*d^2 - 21*I*c^2*d^3 + 62*I*c*d^4 + 147*I*d^5)*cos(f*x + e)^3 +
 sqrt(2)*(4*I*c^4*d - 9*I*c^3*d^2 - I*c^2*d^3 + 333*I*c*d^4 + 441*I*d^5)*co
s(f*x + e)^2 + 2*sqrt(2)*(-4*I*c^4*d + 17*I*c^3*d^2 - 41*I*c^2*d^3 - 209*I*
c*d^4 - 147*I*d^5)*cos(f*x + e) + 4*sqrt(2)*(-4*I*c^4*d + 17*I*c^3*d^2 - 41
*I*c^2*d^3 - 209*I*c*d^4 - 147*I*d^5))*sin(f*x + e) + 4*sqrt(2)*(-4*I*c^4*d
 + 17*I*c^3*d^2 - 41*I*c^2*d^3 - 209*I*c*d^4 - 147*I*d^5))*sqrt(-I*d)*weier
strassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(6*c^4*d - 12*
c^3*d^2 + 12*c*d^4 - 6*d^5 - (4*c^3*d^2 - 21*c^2*d^3 + 62*c*d^4 + 147*d^5)*
cos(f*x + e)^4 - (4*c^4*d - 9*c^3*d^2 - 2*c^2*d^3 + 207*c*d^4 + 376*d^5)*co
s(f*x + e)^3 + (8*c^4*d - 33*c^3*d^2 + 31*c^2*d^3 + 165*c*d^4 + 213*d^5)*co
s(f*x + e)^2 + 2*(9*c^4*d - 29*c^3*d^2 + 25*c^2*d^3 + 161*c*d^4 + 218*d^5)*
cos(f*x + e) - (6*c^4*d - 12*c^3*d^2 + 12*c*d^4 - 6*d^5 + (4*c^3*d^2 - 21*c
^2*d^3 + 62*c*d^4 + 147*d^5)*cos(f*x + e)^3 - (4*c^4*d - 13*c^3*d^2 + 19*c^
2*d^3 + 145*c*d^4 + 229*d^5)*cos(f*x + e)^2 - 2*(6*c^4*d - 23*c^3*d^2 + 25*
c^2*d^3 + 155*c*d^4 + 221*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c))/((a^3*c^5*d^2 - 3*a^3*c^4*d^3 + 2*a^3*c^3*d^4 + 2*a^3*c^2*d^5 - 3
*a^3*c*d^6 + a^3*d^7)*f*cos(f*x + e)^4 - (a^3*c^6*d - a^3*c^5*d^2 - 4*a^3*c
^4*d^3 + 6*a^3*c^3*d^4 + a^3*c^2*d^5 - 5*a^3*c*d^6 + 2*a^3*d^7)*f*cos(f*x +
 e)^3 - (3*a^3*c^6*d - 4*a^3*c^5*d^2 - 9*a^3*c^4*d^3 + 16*a^3*c^3*d^4 + a^3
*c^2*d^5 - 12*a^3*c*d^6 + 5*a^3*d^7)*f*cos(f*x + e)^2 + 2*(a^3*c^6*d - 2*a^
3*c^5*d^2 - a^3*c^4*d^3 + 4*a^3*c^3*d^4 - a^3*c...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{c\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+3c\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+3c\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c\sqrt{c+d\sin(e+fx)}+d\sqrt{c+d\sin(e+fx)}\sin^4(e+fx)+3d\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+3d\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}}{a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x)**2 + 3*c*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c*
sqrt(c + d*sin(e + f*x)) + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4 + 3*d
*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*d*sqrt(c + d*sin(e + f*x))*si
n(e + f*x)**2 + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)), x)/a**3
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)),x)
[Out] \text{Hanged}
```

## 3.521 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=518

$$-\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}-\frac{}{15a(c}$$

```
[Out] -1/30*d*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(
c+d*sin(f*x+e))^(3/2)-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*
x+e))^(3/2)-2/15*(c-5*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin
(f*x+e))^(3/2)-1/30*(4*c^2-27*c*d+119*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*si
n(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/30*d*(4*c^4-27*c^3*d+111*c^2*d^2+579*c*d
^3+357*d^4)*cos(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+1/30*(4
*c^4-27*c^3*d+111*c^2*d^2+579*c*d^3+357*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^5/(c+d)^2/f/((c+d*sin(f*
x+e))/(c+d))^(1/2)-1/30*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^3/(c-d)^4/
(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.83, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4c^2-27cd+119d^2)\cos(e+fx)}{30f(c-d)^3(a^3\sin(e+fx)+a^3)(c+d\sin(e+fx))^{3/2}}-\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{(4c^3-27c^2d+114cd^2+165d^3)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^4(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{d(4c^4-27c^3d+111c^2d^2+579cd^3+357d^4)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{(4c^4-27c^3d+111c^2d^2+579cd^3+357d^4)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^5(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(c-5d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/30*(d*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x])/(a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(5*(c - d)*f*(a +
a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)) - (2*(c - 5*d)*Cos[e + f*x])/
(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - ((4*
c^2 - 27*c*d + 119*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*
x])*(c + d*Sin[e + f*x])^(3/2)) - (d*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*
c*d^3 + 357*d^4)*Cos[e + f*x])/(30*a^3*(c - d)^5*(c + d)^2*f*Sqrt[c + d*Sin
[e + f*x]]) - ((4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*Ellip
ticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(
c - d)^5*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^3 - 27*c^2
*d + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
```

```
[(c + d*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^4*(c + d)*f*Sqrt[c + d*Sin[
e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-}$$

$$= -\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-}$$

$$= -\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-}$$

$$= -\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-}$$

$$= -\frac{d(4c^3-27c^2d+114cd^2+165d^3)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-}$$

**<u>Mathematica</u> [A]**
time = 6.73, size = 828, normalized size = 1.60

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}\left(-\frac{4c^4-27c^3d+111c^2d^2+449cd^3+267d^4}{15(c-d)^5(c+d)^2}+\frac{2\sin\left(\frac{1}{2}(e+fx)\right)}{5(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}-\frac{1}{5(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}-\frac{2(c-8d)}{15(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{4\left(c\sin\left(\frac{1}{2}(e+fx)\right)-8d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{15(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{4c^2\sin\left(\frac{1}{2}(e+fx)\right)-35cd\sin\left(\frac{1}{2}(e+fx)\right)+177d^2\sin\left(\frac{1}{2}(e+fx)\right)}{15(c-d)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{2d^4\cos(e+fx)}{3(c-d)^4(c+d)(c+d\sin(e+fx))^2}-\frac{2(13cd^4\cos(e+fx)+9d^5\cos(e+fx))}{3(c-d)^5(c+d)^2(c+d\sin(e+fx))}\right)}{f(a+a\sin(e+fx))^3}+$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-1/15*(4
*c^4 - 27*c^3*d + 111*c^2*d^2 + 449*c*d^3 + 267*d^4)/((c - d)^5*(c + d)^2)
+ (2*Sin[(e + f*x)/2])/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
) - 1/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - (2*(c - 8*d))
```

```
/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (4*(c*Sin[(e + f*
x)/2] - 8*d*Sin[(e + f*x)/2]))/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3) + (4*c^2*Sin[(e + f*x)/2] - 35*c*d*Sin[(e + f*x)/2] + 177*d^2*Si
n[(e + f*x)/2])/(15*(c - d)^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d
^4*Cos[e + f*x])/(3*(c - d)^4*(c + d)*(c + d*Sin[e + f*x])^2) - (2*(13*c*d^
4*Cos[e + f*x] + 9*d^5*Cos[e + f*x]))/(3*(c - d)^5*(c + d)^2*(c + d*Sin[e +
 f*x]))))/(f*(a + a*Sin[e + f*x])^3) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*((-2*(-(c^3*d) - 387*c^2*d^2 - 471*c*d^3 - 165*d^4)*EllipticF[(-e +
Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c +
d*Sin[e + f*x]] + (2*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)
*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-4*c
^4 + 27*c^3*d - 111*c^2*d^2 - 579*c*d^3 - 357*d^4)*((2*(c + d)*EllipticE[(-
e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[
c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sq
rt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d
)^5*(c + d)^2*f*(a + a*Sin[e + f*x])^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2310 vs. $2(552) = 1104$.
time = 45.30, size = 2311, normalized size = 4.46

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2311 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-2/(c-d)^3*d*(-1/3/(c-d)*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*
sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1
)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)^2*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*si
n(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)
*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-60*d^3
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*
```

```
d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-d^3/
(c-d)^3*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e
)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))))+3/(c-d)^4*d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f
*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/
(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-1-sin(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin
(f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*E
llipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-3*d^3/(c-d)^4*(2*d*cos(f*
x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(
f*x+e))*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-1-sin(f*x+e))
*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2
)/f
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 4857, normalized size = 9.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/180*((sqrt(2)*(8*c^5*d^2 - 54*c^4*d^3 + 219*c^3*d^4 - 3*c^2*d^5 - 699*c*d
^6 - 495*d^7)*cos(f*x + e)^5 + sqrt(2)*(16*c^6*d - 84*c^5*d^2 + 276*c^4*d^3
 + 651*c^3*d^4 - 1407*c^2*d^5 - 3087*c*d^6 - 1485*d^7)*cos(f*x + e)^4 - sqr
t(2)*(8*c^7 - 22*c^6*d + 27*c^5*d^2 + 711*c^4*d^3 - 54*c^3*d^4 - 3300*c^2*d
^5 - 4077*c*d^6 - 1485*d^7)*cos(f*x + e)^3 - sqrt(2)*(24*c^7 - 82*c^6*d + 1
73*c^5*d^2 + 1803*c^4*d^3 - 594*c^3*d^4 - 8496*c^2*d^5 - 9843*c*d^6 - 3465*
d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^7 - 38*c^6*d + 119*c^5*d^2 + 381*c^4*d
^3 - 486*c^3*d^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7)*cos(f*x + e) + (sqr
t(2)*(8*c^5*d^2 - 54*c^4*d^3 + 219*c^3*d^4 - 3*c^2*d^5 - 699*c*d^6 - 495*d^
7)*cos(f*x + e)^4 - 2*sqrt(2)*(8*c^6*d - 46*c^5*d^2 + 165*c^4*d^3 + 216*c^3
*d^4 - 702*c^2*d^5 - 1194*c*d^6 - 495*d^7)*cos(f*x + e)^3 - sqrt(2)*(8*c^7
- 6*c^6*d - 65*c^5*d^2 + 1041*c^4*d^3 + 378*c^3*d^4 - 4704*c^2*d^5 - 6465*c
*d^6 - 2475*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^7 - 38*c^6*d + 119*c^5*d^2
 + 381*c^4*d^3 - 486*c^3*d^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7)*cos(f*x
 + e) + 4*sqrt(2)*(8*c^7 - 38*c^6*d + 119*c^5*d^2 + 381*c^4*d^3 - 486*c^3*d
^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7))*sin(f*x + e) + 4*sqrt(2)*(8*c^7
- 38*c^6*d + 119*c^5*d^2 + 381*c^4*d^3 - 486*c^3*d^4 - 1896*c^2*d^5 - 1689*
c*d^6 - 495*d^7))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e)
- 2*I*c)/d) + (sqrt(2)*(8*c^5*d^2 - 54*c^4*d^3 + 219*c^3*d^4 - 3*c^2*d^5 -
699*c*d^6 - 495*d^7)*cos(f*x + e)^5 + sqrt(2)*(16*c^6*d - 84*c^5*d^2 + 276*
c^4*d^3 + 651*c^3*d^4 - 1407*c^2*d^5 - 3087*c*d^6 - 1485*d^7)*cos(f*x + e)^
4 - sqrt(2)*(8*c^7 - 22*c^6*d + 27*c^5*d^2 + 711*c^4*d^3 - 54*c^3*d^4 - 330
0*c^2*d^5 - 4077*c*d^6 - 1485*d^7)*cos(f*x + e)^3 - sqrt(2)*(24*c^7 - 82*c^
6*d + 173*c^5*d^2 + 1803*c^4*d^3 - 594*c^3*d^4 - 8496*c^2*d^5 - 9843*c*d^6
- 3465*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^7 - 38*c^6*d + 119*c^5*d^2 + 38
1*c^4*d^3 - 486*c^3*d^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7)*cos(f*x + e)
 + (sqrt(2)*(8*c^5*d^2 - 54*c^4*d^3 + 219*c^3*d^4 - 3*c^2*d^5 - 699*c*d^6 -
 495*d^7)*cos(f*x + e)^4 - 2*sqrt(2)*(8*c^6*d - 46*c^5*d^2 + 165*c^4*d^3 +
216*c^3*d^4 - 702*c^2*d^5 - 1194*c*d^6 - 495*d^7)*cos(f*x + e)^3 - sqrt(2)*
(8*c^7 - 6*c^6*d - 65*c^5*d^2 + 1041*c^4*d^3 + 378*c^3*d^4 - 4704*c^2*d^5 -
 6465*c*d^6 - 2475*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(8*c^7 - 38*c^6*d + 119*
c^5*d^2 + 381*c^4*d^3 - 486*c^3*d^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7)*
cos(f*x + e) + 4*sqrt(2)*(8*c^7 - 38*c^6*d + 119*c^5*d^2 + 381*c^4*d^3 - 48
6*c^3*d^4 - 1896*c^2*d^5 - 1689*c*d^6 - 495*d^7))*sin(f*x + e) + 4*sqrt(2)*
(8*c^7 - 38*c^6*d + 119*c^5*d^2 + 381*c^4*d^3 - 486*c^3*d^4 - 1896*c^2*d^5
- 1689*c*d^6 - 495*d^7))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2
)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(
f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(4*I*c^4*d^3 - 27*I*c^3*d^4 + 111*I*c^2*d
^5 + 579*I*c*d^6 + 357*I*d^7)*cos(f*x + e)^5 + sqrt(2)*(8*I*c^5*d^2 - 42*I*
```

```
c^4*d^3 + 141*I*c^3*d^4 + 1491*I*c^2*d^5 + 2451*I*c*d^6 + 1071*I*d^7)*cos(f
*x + e)^4 + sqrt(2)*(-4*I*c^6*d + 11*I*c^5*d^2 - 15*I*c^4*d^3 - 942*I*c^3*d
^4 - 3006*I*c^2*d^5 - 3165*I*c*d^6 - 1071*I*d^7)*cos(f*x + e)^3 + sqrt(2)*(
-12*I*c^6*d + 41*I*c^5*d^2 - 91*I*c^4*d^3 - 2658*I*c^3*d^4 - 7638*I*c^2*d^5
 - 7623*I*c*d^6 - 2499*I*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(4*I*c^6*d - 19*I*
c^5*d^2 + 61*I*c^4*d^3 + 774*I*c^3*d^4 + 1626*I*c^2*d^5 + 1293*I*c*d^6 + 35
7*I*d^7)*cos(f*x + e) + (sqrt(2)*(4*I*c^4*d^3 - 27*I*c^3*d^4 + 111*I*c^2*d^
5 + 579*I*c*d^6 + 357*I*d^7)*cos(f*x + e)^4 + 2*sqrt(2)*(-4*I*c^5*d^2 + 23*
I*c^4*d^3 - 84*I*c^3*d^4 - 690*I*c^2*d^5 - 936*I*c*d^6 - 357*I*d^7)*cos(f*x
 + e)^3 + sqrt(2)*(-4*I*c^6*d + 3*I*c^5*d^2 + 31*I*c^4*d^3 - 1110*I*c^3*d^4
 - 4386*I*c^2*d^5 - 5037*I*c*d^6 - 1785*I*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(
4*I*c^6*d - 19*I*c^5*d^2 + 61*I*c^4*d^3 + 774*I*c^3*d^4 + 1626*I*c^2*d^5 +
1293*I*c*d^6 + 357*I*d^7)*cos(f*x + e) + 4*sqrt(2)*(4*I*c^6*d - 19*I*c^5*d^
2 + 61*I*c^4*d^3 + 774*I*c^3*d^4 + 1626*I*c^2*d^5 + 1293*I*c*d^6 + 357*I*d^
7))*sin(f*x + e) + 4*sqrt(2)*(4*I*c^6*d - 19*I*c^5*d^2 + 61*I*c^4*d^3 + 774
*I*c^3*d^4 + 1626*I*c^2*d^5 + 1293*I*c*d^6 + 357*I*d^7))*sqrt(I*d)*weierstr
assZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstr
assPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*
(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-4*I*c^4*
d^3 + 27*I*c^3*d^4 - 111*I*c^2*d^5 - 579*I*c*d^6 - 357*I*d^7)*cos(f*x + e)^
5 + sqrt(2)*(-8*I*c^5*d^2 + 42*I*c^4*d^3 - 141*I*c^3*d^4 - 1491*I*c^2*d^5 -
 2451*I*c*d^6 - 1071*I*d^7)*cos(f*x + e)^4 + sqrt(2)*(4*I*c^6*d - 11*I*c^5*
d^2 + 15*I*c^4*d^3 + 942*I*c^3*d^4 + 3006*I*c^2*d^5 + 3165*I*c*d^6 + 1071*I
*d^7)*cos(f*x + e)^3 + sqrt(2)*(12*I*c^6*d - 41*I*c^5*d^2 + 91*I*c^4*d^3 +
2658*I*c^3*d^4 + 7638*I*c^2*d^5 + 7623*I*c*d^6 + 2499*I*d^7)*cos(f*x + e)^2
 + 2*sqrt(2)*(-4*I*c^6*d + 19*I*c^5*d^2 - 61*I*c^4*d^3 - 774*I*c^3*d^4 - 16
26*I*c^2*d^5 - 1293*I*c*d^6 - 357*I*d^7)*cos(f*...
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^{2}\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)+3c^{2}\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)+3c^{2}\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c^{2}\sqrt{c+d\sin(e+fx)}+2cd\sqrt{c+d\sin(e+fx)}\sin^{4}(e+fx)+6cd\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)+6cd\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)+2cd\sqrt{c+d\sin(e+fx)}\sin(e+fx)+d^{2}\sqrt{c+d\sin(e+fx)}\sin^{5}(e+fx)+3d^{2}\sqrt{c+d\sin(e+fx)}\sin^{4}(e+fx)+3d^{2}\sqrt{c+d\sin(e+fx)}\sin^{3}(e+fx)+d^{2}\sqrt{c+d\sin(e+fx)}\sin^{2}(e+fx)}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x)**2 + 3*c**2*sqrt(c + d*sin(e + f*x))*sin(e +
f*x) + c**2*sqrt(c + d*sin(e + f*x)) + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e
 + f*x)**4 + 6*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 6*c*d*sqrt(c
+ d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e +
f*x) + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**5 + 3*d**2*sqrt(c + d*si
n(e + f*x))*sin(e + f*x)**4 + 3*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)*
*3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a**3
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

# 3.522 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=161

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a+a\sin(e+fx)}}-\frac{8(5c-d)d(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{35f}-\frac{12d^2(c+$$

```
[Out] -12/35*d^2*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-4/35*a*(c+d)*(15*c^2
+10*c*d+7*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/7*a*cos(f*x+e)*(c+d*si
n(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-8/35*(5*c-d)*d*(c+d)*cos(f*x+e)*(a+a*s
in(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2849, 2840, 2830, 2725}

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a\sin(e+fx)+a}}-\frac{12d^2(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}-\frac{8d(5c-d)(c+d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-4*a*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(35*f*Sqrt[a + a*Sin[
e + f*x]]) - (8*(5*c - d)*d*(c + d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(35*f) - (12*d^2*(c + d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f)
- (2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
```

```
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}+\frac{1}{7}(6(c+d))\int \sqrt{a\ldots}$$

$$= -\frac{12d^2(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af}-\frac{2a\cos(e\ldots}{7f\ldots}$$

$$= -\frac{8(5c-d)d(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{35f}-\frac{12d\ldots}{\ldots}$$

$$= -\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a+a\sin(e+fx)}}-\frac{8(5c-d)d(\ldots}{\ldots}$$

**Mathematica [A]**
time = 0.34, size = 146, normalized size = 0.91

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(140c^3+280c^2d+266cd^2+76d^3-6d^2(7c+2d)\cos(2(e+fx))+d\left(140c^2+112cd+47d^2\right)\sin(e+fx)-5d^3\sin(3(e+fx))\right)}{70f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/70*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(14
0*c^3 + 280*c^2*d + 266*c*d^2 + 76*d^3 - 6*d^2*(7*c + 2*d)*Cos[2*(e + f*x)]
 + d*(140*c^2 + 112*c*d + 47*d^2)*Sin[e + f*x] - 5*d^3*Sin[3*(e + f*x)]))/(
f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 3.20, size = 141, normalized size = 0.88

| method | result |
|---|---|
| default | $\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(5d^3(\sin^3(fx+e))+21c\,d^2(\sin^2(fx+e))+6d^3(\sin^2(fx+e))+35c^2d\sin(fx+e)+28c\,d^2\sin(fx+e)+8}{35\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(5*d^3*sin(f*x+e)^3+21*c*d^2*sin(f*x+e
)^2+6*d^3*sin(f*x+e)^2+35*c^2*d*sin(f*x+e)+28*c*d^2*sin(f*x+e)+8*d^3*sin(f*
x+e)+35*c^3+70*c^2*d+56*c*d^2+16*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^3, x)
```

**Fricas [A]**
time = 0.35, size = 251, normalized size = 1.56

$$\frac{2\left(5\,d^3\cos\left(fx+e\right)^4+3\left(7\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^3-35\,c^3-35\,c^2d-49\,cd^2-9\,d^3-\left(35\,c^2d+7\,cd^2+12\,d^3\right)\cos\left(fx+e\right)^2-\left(35\,c^3+70\,c^2d+77\,cd^2+22\,d^3\right)\cos\left(fx+e\right)+\left(5\,d^3\cos\left(fx+e\right)^3+35\,c^3+35\,c^2d+49\,cd^2+9\,d^3-\left(21\,cd^2+d^3\right)\cos\left(fx+e\right)^2-\left(35\,c^2d+28\,cd^2+13\,d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{35\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/35*(5*d^3*cos(f*x + e)^4 + 3*(7*c*d^2 + 2*d^3)*cos(f*x + e)^3 - 35*c^3 -
35*c^2*d - 49*c*d^2 - 9*d^3 - (35*c^2*d + 7*c*d^2 + 12*d^3)*cos(f*x + e)^2
- (35*c^3 + 70*c^2*d + 77*c*d^2 + 22*d^3)*cos(f*x + e) + (5*d^3*cos(f*x + e
)^3 + 35*c^3 + 35*c^2*d + 49*c*d^2 + 9*d^3 - (21*c*d^2 + d^3)*cos(f*x + e)^
2 - (35*c^2*d + 28*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**3, x)
```

**Giac [A]**
time = 0.60, size = 266, normalized size = 1.65

$$\frac{\sqrt{2}\left(5\,d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+35\left(8\,c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+12\,c^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+12\,cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+35\left(4\,c^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+7\left(6\,cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{a}}{140\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*(5*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7/2*
f*x + 7/2*e) + 35*(8*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*c^2*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1
/2*e) + 35*(4*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*c*d^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-
3/4*pi + 3/2*f*x + 3/2*e) + 7*(6*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*
sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3, x)
```

## 3.523 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=112

$$-\frac{2a(15c^2+10cd+7d^2)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{4(5c-d)d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}$$

```
[Out] -2/5*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*c^2+10*c*d+7*d^2)
*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-4/15*(5*c-d)*d*cos(f*x+e)*(a+a*sin(f*x
+e))^(1/2)/f
```

**Rubi** [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2840, 2830, 2725}

$$-\frac{2a(15c^2+10cd+7d^2)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{4d(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]
]) - (4*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d^2*
Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int \sqrt{a+a\sin(e}{}$$
$$= -\frac{4(5c-d)d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2d^2\cos(e+}{}$$
$$= -\frac{2a(15c^2+10cd+7d^2)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{4(5c-d)d\cos(e+}{}$$

**Mathematica [A]**
time = 0.20, size = 111, normalized size = 0.99

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(30c^2+40cd+19d^2-3d^2\cos(2(e+fx))+4d(5c+2d)\sin(e+fx))}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30
*c^2 + 40*c*d + 19*d^2 - 3*d^2*Cos[2*(e + f*x)] + 4*d*(5*c + 2*d)*Sin[e + f
*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.86, size = 92, normalized size = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(3d^2\left(\sin^2(fx+e)\right)+10cd\sin(fx+e)+4d^2\sin(fx+e)+15c^2+20cd+8d^2\right)}{15\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$ | 92 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*d^2*sin(f*x+e)^2+10*c*d*sin(f*x+e)+
4*d^2*sin(f*x+e)+15*c^2+20*c*d+8*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2, x)
```

**Fricas [A]**
time = 0.32, size = 166, normalized size = 1.48

$$\frac{2\left(3\,d^2\cos\left(fx+e\right)^3-\left(10\,cd+d^2\right)\cos\left(fx+e\right)^2-15\,c^2-10\,cd-7\,d^2-\left(15\,c^2+20\,cd+11\,d^2\right)\cos\left(fx+e\right)-\left(3\,d^2\cos\left(fx+e\right)^2-15\,c^2-10\,cd-7\,d^2+2\left(5\,cd+2\,d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*d^2*cos(f*x + e)^3 - (10*c*d + d^2)*cos(f*x + e)^2 - 15*c^2 - 10*c*
d - 7*d^2 - (15*c^2 + 20*c*d + 11*d^2)*cos(f*x + e) - (3*d^2*cos(f*x + e)^2
 - 15*c^2 - 10*c*d - 7*d^2 + 2*(5*c*d + 2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**2, x)
```

**Giac [A]**
time = 0.54, size = 167, normalized size = 1.49

$$\frac{\sqrt{2}\left(3\,d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+30\left(2\,c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\left(4\,cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\right)\sqrt{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*c*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(4*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*
sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2, x)
```

# 3.524 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=62

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

```
[Out] -2/3*a*(3*c+d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*d*cos(f*x+e)*(a+a*si
n(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2830, 2725}

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2d\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*c + d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))\,dx &= -\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3c+d)\int \sqrt{a+\\ &= -\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+\end{aligned}$$

**Mathematica** [A]
time = 0.09, size = 82, normalized size = 1.32

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(3c+2d+d\sin(e+fx))}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 2*d + d*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 2.59, size = 58, normalized size = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(d\sin(fx+e)+3c+2d)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ | 58 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(d*sin(f*x+e)+3*c+2*d)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c), x)
```

**Fricas** [A]
time = 0.33, size = 92, normalized size = 1.48

$$-\frac{2\,\left(d\cos\left(fx+e\right)^{2}+\left(3\,c+2\,d\right)\cos\left(fx+e\right)+\left(d\cos\left(fx+e\right)-3\,c-d\right)\sin\left(fx+e\right)+3\,c+d\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\,\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] -2/3*(d*cos(f*x + e)^2 + (3*c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) - 3*c -
 d)*sin(f*x + e) + 3*c + d)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x)), x)
```

**Giac [A]**
time = 0.52, size = 90, normalized size = 1.45

$$\frac{\sqrt{2}\left(d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\left(2c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sqrt{a}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x +
3/2*e) + 3*(2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + d*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x)
```

# 3.525 $\int \sqrt{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=26

$$-\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,dx = -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.02, size = 65, normalized size = 2.50

$$\frac{2\left(-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 1.80, size = 43, normalized size = 1.65

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)}{\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$ | 43 |
| risch | $-\frac{i\sqrt{2}\,\sqrt{-a\left(-2-2\sin\left(fx+e\right)\right)}\,\left(\mathrm{e}^{i(fx+e)}-i\right)\left(\mathrm{e}^{i(fx+e)}+i\right)}{\left(\mathrm{e}^{2i(fx+e)}+2i\mathrm{e}^{i(fx+e)}-1\right)f}$ | 74 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(f*x+e))*a*(sin(f*x+e)-1)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(26) = 52$.
time = 0.33, size = 55, normalized size = 2.12

$$-\frac{2\sqrt{a\sin\left(fx+e\right)+a}\,\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x +
e) + f*sin(f*x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(e+fx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*sin(e + f*x) + a), x)
```

**Giac** [**A**]
time = 0.46, size = 38, normalized size = 1.46

$$\frac{2\sqrt{2}\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)/f
```

**Mupad** [**B**]
time = 7.42, size = 33, normalized size = 1.27

$$-\frac{2\cos\left(e+f\,x\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}}{f\,\left(\sin\left(e+f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2),x)
[Out] -(2*cos(e + f*x)*(a*(sin(e + f*x) + 1))^(1/2))/(f*(sin(e + f*x) + 1))
```

# 3.526 $\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=61

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}\,\sqrt{c+d}\,f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*a
^(1/2)/f/d^(1/2)/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2852, 214}

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}\,f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}\,f}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.84, size = 657, normalized size = 10.77

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\#1^4\&,\frac{(1+i)d\sqrt{e^{-ie}}fx-(2-2i)d\sqrt{e^{-ie}}\log\left(e^{\frac{ifx}{2}}-\#1\right)-i\sqrt{d}\sqrt{c+d}fx\#1+2\sqrt{d}\sqrt{c+d}\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1+\frac{(1-i)cfx\#1^2}{\sqrt{e^{-ie}}}+\frac{(2+2i)c\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1^2}{\sqrt{e^{-ie}}}-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c+d}e^{ie}\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1^3}{d-ice^{ie}\#1^2}\&\right]-i\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\#1^4\&,\frac{(1-i)d\sqrt{e^{-ie}}fx+(2+2i)d\sqrt{e^{-ie}}\log\left(e^{\frac{ifx}{2}}-\#1\right)+\sqrt{d}\sqrt{c+d}fx\#1+2i\sqrt{d}\sqrt{c+d}\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1-\frac{(1+i)cfx\#1^2}{\sqrt{e^{-ie}}}+\frac{(2-2i)c\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1^2}{\sqrt{e^{-ie}}}-i\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3+2\sqrt{d}\sqrt{c+d}e^{ie}\log\left(e^{\frac{ifx}{2}}-\#1\right)\#1^3}{d-ice^{ie}\#1^2}\&\right]\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\sqrt{a(1+\sin(e+fx))}}{\sqrt{d}\sqrt{c+d}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((1/8 + I/8)*(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((
1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f
*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/
2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log
[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f
*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(
d - I*c*E^(I*e)*#1^2) & ] - I*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I
)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)
]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt
[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
+ ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*S
qrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*
x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ])*(Cos[e/2] + I*Sin[e/2])*Sqrt[a*(
1 + Sin[e + f*x])])/(Sqrt[d]*Sqrt[c + d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]))
```

**Maple** [A]
time = 2.80, size = 80, normalized size = 1.31

| method | result | size |
|---|---|---|
| default | $-\frac{2a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)}{\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ | 80 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c), x)
```

**Fricas [A]**
time = 0.43, size = 484, normalized size = 7.93

$$\left[\frac{\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2\,acd-ad^2-(6\,acd+7\,ad^2)\cos(fx+e)^2+4\left(c^2d+4\,cd^2+3\,d^3-(cd^2+d^3)\cos(fx+e)^2+(c^2d+3\,cd^2+2\,d^3)\cos(fx+e)-(c^2d+4\,cd^2+3\,d^3+(cd^2+d^3)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{\frac{a}{cd+d^2}}-(ac^2+8\,acd+9\,ad^2)\cos(fx+e)+\left(ad^2\cos(fx+e)^2-ac^2-2\,acd-ad^2+2\,(3\,acd+4\,ad^2)\cos(fx+e)\right)\sin(fx+e)}{d^2\cos(fx+e)^3+(2\,cd+d^2)\cos(fx+e)^2-c^2-2\,cd-d^2-(c^2+d^2)\cos(fx+e)+\left(d^2\cos(fx+e)^2-2\,cd\cos(fx+e)-c^2-2\,cd-d^2\right)\sin(fx+e)}\right)}{2\,f}, -\frac{\sqrt{-\frac{a}{cd+d^2}}\arctan\left(\frac{\sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)-c-2\,d)\sqrt{-\frac{a}{cd+d^2}}}{2\,a\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^
2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^
2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d +
 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
 (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*
c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e)))/f, -sqrt(-a/(c*d + d^2
))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(
c*d + d^2))/(a*cos(f*x + e)))/f]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x)), x)
```

**Giac [A]**
time = 0.47, size = 65, normalized size = 1.07

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-cd-d^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2)
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(sqrt(-c*d - d^2)*f)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x)), x)
```

# 3.527 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=105

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}(c+d)^{3/2}f}-\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(
1/2)/(c+d)^(3/2)/f/d^(1/2)-a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f
*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2851, 2852, 214}

$$-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*S
in[e + f*x]])])/(Sqrt[d]*(c + d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx = -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2(c+d)}$$

$$= -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{a\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x \ldots\right.}{(c+ \ldots}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}(c+d)^{3/2}f} - \frac{a\cos(e+f \ldots}{(c+d)f\sqrt{a+a\sin(e+fx)} \ldots}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.18, size = 871, normalized size = 8.30

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((Cos[e/2] + I*Sin[e/2])*((-1 + I)
*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1
 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*
x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2
)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[
E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*
x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d
 - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]
)/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e
]))*Sqrt[Cos[e] - I*Sin[e]]) + ((Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] -
 (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4
 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^(
(I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*L
```

```
og[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2
*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d
]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*
#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + S
in[e]))/(4*f)))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[
e] - I*Sin[e]]) - ((2 - 2*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d
)*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
```

**Maple [A]**
time = 4.82, size = 155, normalized size = 1.48

| method | result |
|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)\sin(fx+e)ad+\operatorname{arctanh}\left(\frac{\sqrt{-}\cdots\right.\right.}{(c+d)(c+d\sin(fx+e))\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{a+\cdots}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*d+arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a*c+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/(c+d)/(c
+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^2, x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(94) = 188.
time = 0.43, size = 828, normalized size = 7.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((d*cos(f*x + e)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x
 + e) - c - d)*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*
```

```
c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d
^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e)
- (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(
f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4
*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x +
 e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*sqrt(a*s
in(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x
+ e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*
f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*sin(f*x + e)), -1/2*((d*cos(f*x + e
)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x + e) - c - d)*sqrt(
-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)
*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^
2 + 2*c*d + d^2)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.56, size = 139, normalized size = 1.32

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{-cd-d^2}\,(c+d)}+\frac{2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)(c+d)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e)/sqrt(-c*d - d^2))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(sqrt(-c*d - d^2)*
(c + d)) + 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^2, x)
```

# 3.528 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=154

$$-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{d}\,(c+d)^{5/2}f}-\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}-\frac{}{4(c+}$$

```
[Out] -3/4*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))
*a^(1/2)/(c+d)^(5/2)/f/d^(1/2)-1/2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^2/
(a+a*sin(f*x+e))^(1/2)-3/4*a*cos(f*x+e)/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin
(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2851, 2852, 214}

$$-\frac{3a\cos(e+fx)}{4f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{d}\,f(c+d)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(4*Sqrt[d]*(c + d)^(5/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*a*Cos[e + f*x])/(4*
(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

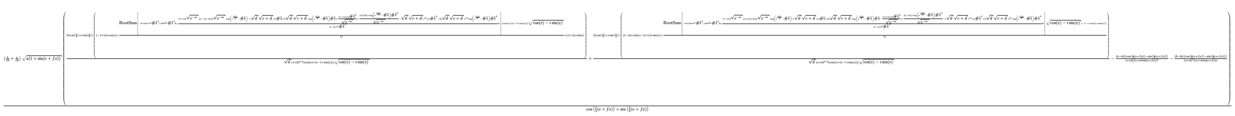
$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx = -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{3\int \frac{\sqrt{a+a\sin(e+f}}{(c+d\sin(e+fx))^2}}{4(c+d)}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{3a\,c}{4(c+d)^2f\sqrt{a+a\sin}}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{3a\,c}{4(c+d)^2f\sqrt{a+a\sin}}$$

$$= -\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{d}\,(c+d)^{5/2}f} - \frac{a\cos(e}{2(c+d)f\sqrt{a+a\sin(e+}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.23, size = 920, normalized size = 5.97



Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*((3*(Cos[e/2] + I*Sin[e/2])*((-1
+ I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & ,
 ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2
)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^(
(I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*
Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e
)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3
)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin
[e]])/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + S
in[e]))*Sqrt[Cos[e] - I*Sin[e]]) + (3*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Co
```

```
s[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e
)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*L
og[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c
+ d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + (
(2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt
[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x)
- #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[
e] + Sin[e]))/(4*f)))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*f*(c + d*Sin[e + f*x])^2) - ((6 - 6*I)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])
```

**Maple** [A]
time = 4.88, size = 254, normalized size = 1.65

| method | result |
|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)(\sin^2(fx+e))\,a\,d^2+6\,\mathrm{arctanh}\left(\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a*d^2+6*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*c*d+3*(-a*(sin(f*x+e)-1))^(1/2)*
(a*(c+d)*d)^(1/2)*sin(f*x+e)*d+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a*c^2+5*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*d)/(c+d)^2/(c+d*sin(f*x+e))^2/(a*(c+d
)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^3, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(138) = 276$.
time = 0.52, size = 1304, normalized size = 8.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/16*(3*(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*d*cos(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x + e) - 5*c + d)*sin(f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e)), -1/8*(3*(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*d*cos(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x + e) - 5*c + d)*sin(f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))]

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)

[Out] Timed out

**Giac** [**A**]

time = 0.51, size = 222, normalized size = 1.44

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c^2+2\,cd+d^2\right)\sqrt{-cd-d^2}}+\frac{2\left(6\,d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-5\,c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-5\,d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)^2\left(c^2+2\,cd+d^2\right)}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(3*sqrt(2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/sqrt(-c*d - d^2))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2 + 2*c*d +
d^2)*sqrt(-c*d - d^2)) + 2*(6*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 5*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e) - 5*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2*(c
^2 + 2*c*d + d^2)))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^3, x)
```

# 3.529 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=231

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}}+\frac{8a(c-17d)(5c-d)(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f}$$

```
[Out] 4/105*(c-17*d)*d*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f+4/315*a^2*(c-17*
d)*(c+d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/2)+2/63*a
^2*(c-17*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)-2/9*a^
2*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*sin(f*x+e))^(1/2)+8/315*a*(c-17*d)
*(5*c-d)*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2842, 21, 2849, 2840, 2830, 2725}

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-17d)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a\sin(e+fx)+a}}+\frac{4d(c-17d)(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f}+\frac{8a(c-17d)(5c-d)(c+d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (4*a^2*(c - 17*d)*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(315*d*f*
Sqrt[a + a*Sin[e + f*x]]) + (8*a*(c - 17*d)*(5*c - d)*(c + d)*Cos[e + f*x]*
Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*(c - 17*d)*d*(c + d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(c - 17*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^3)/(63*d*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\left(-\frac{1}{2}a^2(c-17d)- \cdots\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-17d))\int \sqrt{\cdots}}{\cdots}$$

$$= \frac{2a^2(c-17d)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e \cdots}{9df \cdots}$$

$$= \frac{4(c-17d)d(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2 \cdots}{\cdots}$$

$$= \frac{8a(c-17d)(5c-d)(c+d)\cos(e+fx)\sqrt{a+a\sin(e+f \cdots}}{315f}$$

$$= \frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} + \frac{8 \cdots}{\cdots}$$

**Mathematica [A]**
time = 1.08, size = 203, normalized size = 0.88

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(4200c^3+9828c^2d+8892cd^2+2689d^3-4d\left(189c^2+351cd+137d^2\right)\cos(2(e+fx))+35d^3\cos(4(e+fx))+840c^3\sin(e+fx)+4536c^2d\sin(e+fx)+4554cd^2\sin(e+fx)+1598d^3\sin(e+fx)-270cd^2\sin(3(e+fx))-170d^3\sin(3(e+fx))\right)}{1260f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(4200*c^3 + 9828*c^2*d + 8892*c*d^2 + 2689*d^3 - 4*d*(189*c^2 + 351*c*d +
137*d^2)*Cos[2*(e + f*x)] + 35*d^3*Cos[4*(e + f*x)] + 840*c^3*Sin[e + f*x]
+ 4536*c^2*d*Sin[e + f*x] + 4554*c*d^2*Sin[e + f*x] + 1598*d^3*Sin[e + f*x]
 - 270*c*d^2*Sin[3*(e + f*x)] - 170*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.57, size = 195, normalized size = 0.84

| method | result |
|---|---|
| default | $\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(35d^3\left(\sin^4(fx+e)\right)+135c\,d^2\left(\sin^3(fx+e)\right)+85d^3\left(\sin^3(fx+e)\right)+189c^2d\left(\sin^2(fx+e)\right)+351c\,d^2\left(\sin \cdots\right.\right.}{315\cos(fx+e)\sqrt{\cdots}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(35*d^3*sin(f*x+e)^4+135*c*d^2*sin(
f*x+e)^3+85*d^3*sin(f*x+e)^3+189*c^2*d*sin(f*x+e)^2+351*c*d^2*sin(f*x+e)^2+
102*d^3*sin(f*x+e)^2+105*c^3*sin(f*x+e)+567*c^2*d*sin(f*x+e)+468*c*d^2*sin(
f*x+e)+136*d^3*sin(f*x+e)+525*c^3+1134*c^2*d+936*c*d^2+272*d^3)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^3, x)
```

**Fricas [A]**
time = 0.36, size = 352, normalized size = 1.52

$$-\frac{2\left(35\,ad^{3}\cos\left(fx+e\right)^{5}-5\left(27\,acd^{2}+10\,ad^{3}\right)\cos\left(fx+e\right)^{4}+420\,ac^{3}+756\,ac^{2}d+684\,acd^{2}+188\,ad^{3}-\left(189\,ac^{2}d+351\,acd^{2}+172\,ad^{3}\right)\cos\left(fx+e\right)^{3}+\left(105\,ac^{3}+378\,ac^{2}d+387\,acd^{2}+134\,ad^{3}\right)\cos\left(fx+e\right)^{2}+\left(525\,ac^{3}+1323\,ac^{2}d+1287\,acd^{2}+409\,ad^{3}\right)\cos\left(fx+e\right)-\left(35\,ad^{3}\cos\left(fx+e\right)^{4}+420\,ac^{3}+756\,ac^{2}d+684\,acd^{2}+188\,ad^{3}+5\left(27\,acd^{2}+17\,ad^{3}\right)\cos\left(fx+e\right)^{3}-3\left(63\,ac^{2}d+72\,acd^{2}+29\,ad^{3}\right)\cos\left(fx+e\right)^{2}-\left(105\,ac^{3}+567\,ac^{2}d+603\,acd^{2}+221\,ad^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{315\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*d^3*cos(f*x + e)^5 - 5*(27*a*c*d^2 + 10*a*d^3)*cos(f*x + e)^4
+ 420*a*c^3 + 756*a*c^2*d + 684*a*c*d^2 + 188*a*d^3 - (189*a*c^2*d + 351*a*
c*d^2 + 172*a*d^3)*cos(f*x + e)^3 + (105*a*c^3 + 378*a*c^2*d + 387*a*c*d^2
+ 134*a*d^3)*cos(f*x + e)^2 + (525*a*c^3 + 1323*a*c^2*d + 1287*a*c*d^2 + 40
9*a*d^3)*cos(f*x + e) - (35*a*d^3*cos(f*x + e)^4 + 420*a*c^3 + 756*a*c^2*d
+ 684*a*c*d^2 + 188*a*d^3 + 5*(27*a*c*d^2 + 17*a*d^3)*cos(f*x + e)^3 - 3*(6
3*a*c^2*d + 72*a*c*d^2 + 29*a*d^3)*cos(f*x + e)^2 - (105*a*c^3 + 567*a*c^2*
d + 603*a*c*d^2 + 221*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**3, x)
```

**Giac [A]**
time = 0.64, size = 374, normalized size = 1.62

$$\frac{\sqrt{2}\left(35\,ad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)+1890\left(4\,ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,ac^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7\,acd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,ad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+210\left(4\,ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+18\,ac^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15\,acd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,ad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+378\left(2\,ac^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,acd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+ad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+135\left(2\,acd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+ad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\right)\sqrt{a}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi +
9/2*f*x + 9/2*e) + 1890*(4*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*a*
c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*a*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 2*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 210*(4*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*a
*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*a*c*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 5*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi
 + 3/2*f*x + 3/2*e) + 378*(2*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
3*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 135*(2*a*c*d^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-
7/4*pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\sin(e + f\,x))^{3/2}\,(c + d\sin(e + f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3, x)
```

# 3.530 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=157

$$-\frac{8a^2(35c^2+42cd+19d^2)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{2a(35c^2+42cd+19d^2)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{4(7c-d)}{}$$

```
[Out] -4/35*(7*c-d)*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*d^2*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*c^2+42*c*d+19*d^2)*cos(f*x+e)/f/(a+a*s
in(f*x+e))^(1/2)-2/105*a*(35*c^2+42*c*d+19*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/f
```

**Rubi** [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2840, 2830, 2726, 2725}

$$-\frac{8a^2(35c^2+42cd+19d^2)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a(35c^2+42cd+19d^2)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{4d(7c-d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-8*a^2*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e +
f*x]]) - (2*a*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(105*f) - (4*(7*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*
f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}+\frac{2\int(a+a\sin(e$$

$$= -\frac{4(7c-d)d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}-\frac{2d^2\cos$$

$$= -\frac{2a(35c^2+42cd+19d^2)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}$$

$$= -\frac{8a^2(35c^2+42cd+19d^2)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{2a(35c^2+42cd$$

**Mathematica [A]**
time = 0.57, size = 136, normalized size = 0.87

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(700c^2+1092cd+494d^2-6d(14c+13d)\cos(2(e+fx))+(140c^2+504cd+253d^2)\sin(e+fx)-15d^2\sin(3(e+fx))\right)}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(700*c^2 + 1092*c*d + 494*d^2 - 6*d*(14*c + 13*d)*Cos[2*(e + f*x)] + (140*c
^2 + 504*c*d + 253*d^2)*Sin[e + f*x] - 15*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.94, size = 130, normalized size = 0.83

| method | result |
|---|---|

| default | $\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(15d^2(\sin^3(fx+e))+42cd(\sin^2(fx+e))+39d^2(\sin^2(fx+e))+35c^2\sin(fx+e)+126cd\sin(fx+e)+52d}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*d^2*sin(f*x+e)^3+42*c*d*sin(f*x
+e)^2+39*d^2*sin(f*x+e)^2+35*c^2*sin(f*x+e)+126*c*d*sin(f*x+e)+52*d^2*sin(f
*x+e)+175*c^2+252*c*d+104*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^2, x)
```

**Fricas [A]**
time = 0.38, size = 240, normalized size = 1.53

$$\frac{2\left(15\,ad^2\cos\left(fx+e\right)^4+3\left(14\,acd+13\,ad^2\right)\cos\left(fx+e\right)^3-140\,ac^2-168\,acd-76\,ad^2-\left(35\,ac^2+84\,acd+43\,ad^2\right)\cos\left(fx+e\right)^2-\left(175\,ac^2+294\,acd+143\,ad^2\right)\cos\left(fx+e\right)+\left(15\,ad^2\cos\left(fx+e\right)^3+140\,ac^2+168\,acd+76\,ad^2-6\left(7\,acd+4\,ad^2\right)\cos\left(fx+e\right)^2-\left(35\,ac^2+126\,acd+67\,ad^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{105\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*d^2*cos(f*x + e)^4 + 3*(14*a*c*d + 13*a*d^2)*cos(f*x + e)^3 - 1
40*a*c^2 - 168*a*c*d - 76*a*d^2 - (35*a*c^2 + 84*a*c*d + 43*a*d^2)*cos(f*x
+ e)^2 - (175*a*c^2 + 294*a*c*d + 143*a*d^2)*cos(f*x + e) + (15*a*d^2*cos(f
*x + e)^3 + 140*a*c^2 + 168*a*c*d + 76*a*d^2 - 6*(7*a*c*d + 4*a*d^2)*cos(f*
x + e)^2 - (35*a*c^2 + 126*a*c*d + 67*a*d^2)*cos(f*x + e))*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**2, x)
```

**Giac [A]**
time = 0.55, size = 250, normalized size = 1.59

$$\frac{\sqrt{2}\left(15\,ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+105\left(12\,ac^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+16\,acd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+35\left(4\,ac^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+12\,acd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)+21\left(4\,acd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\right)\sqrt{a}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 105*(12*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*a*
c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*a*c^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 12*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*a*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(4
*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a*d^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin\left(e+f\,x\right))^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2, x)
```

# 3.531 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=101

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2d\cos(e+fx)(a+a\sin(e+fx)}{5f}$$

```
[Out] -2/5*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*c+3*d)*cos(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*c+3*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*c + 3*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*c + 3*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5c+3d)\int (a$$
$$= -\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2d\cos(e$$
$$= -\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a}}{15f}$$

**Mathematica** [A]
time = 0.27, size = 101, normalized size = 1.00

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(50c+39d-3d\cos(2(e+fx))+2(5c+9d)\sin(e+fx))}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
50*c + 39*d - 3*d*Cos[2*(e + f*x)] + 2*(5*c + 9*d)*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple** [A]
time = 2.94, size = 77, normalized size = 0.76

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)(5c+9d)-3(\cos^2(fx+e))d+25c+21d)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ | 77 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)*(5*c+9*d)-3*cos(f*x+e)^2
*d+25*c+21*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c), x)
```

**Fricas [A]**
time = 0.33, size = 145, normalized size = 1.44

$$\frac{2\left(3\,ad\cos\left(fx+e\right)^{3}-\left(5\,ac+6\,ad\right)\cos\left(fx+e\right)^{2}-20\,ac-12\,ad-\left(25\,ac+21\,ad\right)\cos\left(fx+e\right)-\left(3\,ad\cos\left(fx+e\right)^{2}-20\,ac-12\,ad+\left(5\,ac+9\,ad\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*d*cos(f*x + e)^3 - (5*a*c + 6*a*d)*cos(f*x + e)^2 - 20*a*c - 12*a
*d - (25*a*c + 21*a*d)*cos(f*x + e) - (3*a*d*cos(f*x + e)^2 - 20*a*c - 12*a
*d + (5*a*c + 9*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(
f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x)), x)
```

**Giac [A]**
time = 0.57, size = 147, normalized size = 1.46

$$\frac{\sqrt{2}\left(3\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+30\left(3\,ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+5\left(2\,ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\right)\sqrt{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(3*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*a*c*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x)
```

# 3.532 $\int (a+a\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=59

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

```
[Out] -8/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

**Rubi** [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}\,dx &= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(4a)\int\sqrt{a+a\sin(e+fx)}\,dx\\ &= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}\end{aligned}$$

**Mathematica** [A]
time = 0.10, size = 89, normalized size = 1.51

$$-\frac{(a(1+\sin(e+fx)))^{3/2}\left(9\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)-9\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)\right)}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[e + f*x]))^(3/2)*(9*Cos[(e + f*x)/2] + Cos[(3*(e + f*x))/
2] - 9*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)
```

**Maple** [A]
time = 2.12, size = 53, normalized size = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ | 53 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)+5)/cos(f*x+e)/(a+a*sin(f*
x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [A]
time = 0.46, size = 83, normalized size = 1.41

$$-\frac{2\left(a\cos\left(fx+e\right)^2+5\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)-4\,a\right)\sin\left(fx+e\right)+4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 + 5*a*cos(f*x + e) + (a*cos(f*x + e) - 4*a)*sin(f*x
+ e) + 4*a)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin{(e+fx)}+a)^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(3/2), x)
```

**Giac [A]**
time = 0.51, size = 71, normalized size = 1.20

$$\frac{\sqrt{2}\left(9\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\right)\sqrt{a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(9*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x + 3/
2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2), x)
```

# 3.533 $\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=98

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f}-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*a^(3/2)*(c-d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x
+e))^(1/2))/d^(3/2)/f/(c+d)^(1/2)-2*a^2*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/
2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2842, 21, 2852, 214}

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*(c - d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^2*Cos[e + f*x])/(d*f
*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
```

```
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx=-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{2\int\frac{-\frac{1}{2}a^2(c-d)-\frac{1}{2}a^2(c-d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx}{d}$$

$$=-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}-\frac{(a(c-d))\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{d}$$

$$=-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{(2a^2(c-d))\,\text{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{df}$$

$$=\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f}-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(98) = 196.
time = 1.64, size = 233, normalized size = 2.38

$$\frac{\left(-2\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)+(c-d)\left(\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)+2\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}{d^{3/2}\sqrt{c+d}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] + (c - d)*(Log[-(Sec[(e + f*x)/4]
^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[
```

```
(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1
+ 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]) + 2*Sqrt[d]*Sqrt[c + d]*Sin[(e
 + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))/(d^(3/2)*Sqrt[c + d]*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [A]
time = 4.38, size = 137, normalized size = 1.40

| method | result |
|---|---|
| default | $-\frac{2a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ac+a\operatorname{arctanh}\left(\frac{\sqrt{-a(\mathrm{s}}{\sqrt{}}\right.\right.}{d\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{a+a\sin(fx-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a*c+a*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*d+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c+d)*d)^
(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(86) = 172.
time = 0.44, size = 685, normalized size = 6.99

$$\left[-\frac{(ac-ad+(ac-ad)\cos(fx+e)+(ac-ad)\sin(fx+e))\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{[illegible]}{[illegible]}\right)+4\,(a\cos(fx+e)-a\sin(fx+e)+a)\sqrt{a\sin(fx+e)+a}}{2\,(df\cos(fx+e)+df\sin(fx+e)+df)},\ \frac{(ac-ad+(ac-ad)\cos(fx+e)+(ac-ad)\sin(fx+e))\sqrt{-\frac{a}{cd+d^2}}\arctan\left(\frac{\sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)-c-2d)\sqrt{-\frac{a}{cd+d^2}}}{2a\cos(fx+e)}\right)-2\,(a\cos(fx+e)-a\sin(fx+e)+a)\sqrt{a\sin(fx+e)+a}}{df\cos(fx+e)+df\sin(fx+e)+df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
```

```
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(a*cos(f*x + e) - a*sin(f
*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*cos(f*x + e) - a*si
n(f*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.50, size = 130, normalized size = 1.33

$$\frac{\sqrt{2}\left(\frac{2\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d}+\frac{\sqrt{2}\,\left(ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\sqrt{-cd-d^2}\,d}\right)\sqrt{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] sqrt(2)*(2*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/d + sqrt(2)*(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/s
qrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d))*sqrt(a)/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x)), x)
```

# 3.534 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=119

$$-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f}+\frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}$$

```
[Out] -a^(3/2)*(c+3*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))/d^(3/2)/(c+d)^(3/2)/f+a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(
f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2841, 21, 2852, 214}

$$\frac{a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^(3/2)*(c + 3*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sin[e + f*x]])]])/(d^(3/2)*(c + d)^(3/2)*f)) + (a^2*(c - d)*Cos[e +
 f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
```

```
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{a\int \frac{-\frac{1}{2}a(c+3d)-\frac{1}{2}a(c+3}{\sqrt{a+a\sin(e+fx)}}}{d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{(a(c+3d))\int \frac{\sqrt{a+a\,\mathrm{si}}}{c+d\,\mathrm{si}}}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{(a^2(c+3d))\,\mathrm{Subst}\left(\int \frac{}{a}\right.}{}$$

$$= -\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a^2(c}{d(c+d)f\sqrt{a+a}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(119) = 238.
time = 1.65, size = 268, normalized size = 2.25

$$-\frac{(a(1+\sin(e+fx)))^{3/2}\left(-2(c-d)\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)+2(c-d)\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)+(c+3d)\left(\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)(c+d\sin(e+fx))\right)}{2d^{3/2}(c+d)^{3/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - d)*Sqrt[d]*Sqrt[c + d]*Cos[(e +
 f*x)/2] + 2*(c - d)*Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2] + (c + 3*d)*(Log[
-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d
```

```
]*Sqrt[c + d]*Sin[(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d
]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)])*(c + d*Sin[e
 + f*x])))/(d^(3/2)*(c + d)^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
*(c + d*Sin[e + f*x]))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(103) = 206$.
time = 4.53, size = 233, normalized size = 1.96

| method | result |
|---|---|
| default | $\frac{a(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+d^2a}}\right)ad(c+3d)-\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{d(c+d)(c+d\sin}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(c+3*d)-arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a
*d^2)^(1/2))*a*c*d+(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c-(a-a*sin(f*x+
e))^(1/2)*(a*(c+d)*d)^(1/2)*d)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^2, x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(108) = 216$.
time = 0.45, size = 1012, normalized size = 8.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*c^2 + 4*a*c*d + 3*a*d^2 - (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c
^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)
```

```
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*
cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) +
(c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e)), 1/2*((a*c^2 + 4*a*c*d + 3*a*d^2 -
 (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2
 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-
a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*
d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*c - a*d + (a*c - a*d)*cos(
f*x + e) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 + d^
3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d
^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x
+ e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.50, size = 204, normalized size = 1.71

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\left(ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{(cd+d^2)\sqrt{-cd-d^2}}-\frac{2\,\left(ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)(cd+d^2)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*
a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d + d^2)*sqrt(-c*d - d^2)) - 2*(a*c*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - a*d*sgn(c
```

```
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^{3/2}}{(c + d\,\sin(e + f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^2, x)
```

# 3.535 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=179

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f}+\frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}$$

```
[Out] -1/4*a^(3/2)*(c+7*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*si
n(f*x+e))^(1/2))/d^(3/2)/(c+d)^(5/2)/f+1/2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(
c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(c+7*d)*cos(f*x+e)/d/(c+d)
^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2841, 21, 2851, 2852, 214}

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a^2(c+7d)\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/4*(a^(3/2)*(c + 7*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]
*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(5/2)*f) + (a^2*(c - d)*Cos[e
 + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) -
(a^2*(c + 7*d)*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a\int \frac{-\frac{1}{2}a(c+7d)-\frac{1}{2}a(c+}{\sqrt{a+a\sin(e+fx}}}{2d(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{(a(c+7d))\int \frac{\sqrt{a+a\,s}}{(c+d\,si}}{4d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a^2(c+7}{4d(c+d)^2f\sqrt{a+a\sin}}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a^2(c+7}{4d(c+d)^2f\sqrt{a+a\sin}}$$

$$= -\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{a^2(c}{2d(c+d)f\sqrt{a+a\,s}}$$

**Mathematica [A]**
time = 2.72, size = 313, normalized size = 1.75

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-\frac{2(c+7d)\left(\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{5/2}}-\frac{4\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\left(-c^2+7cd+2d^2+d(c+7d)\sin(e+fx)\right)}{(c+d)^2(c+d\sin(e+fx))^2}+\frac{4\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\left(-c^2+7cd+2d^2+d(c+7d)\sin(e+fx)\right)}{(c+d)^2(c+d\sin(e+fx))^2}\right)}{16d^{3/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-2*(c + 7*d)*(Log[-(Sec[(e + f*x)/4]^2*(c +
 d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*
x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan
[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt[d]*Cos[(e +
f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c +
d*Sin[e + f*x])^2) + (4*Sqrt[d]*Sin[(e + f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*
(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e + f*x])^2)))/(16*d^(3/2)*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(155) = 310$.
time = 5.72, size = 429, normalized size = 2.40

| method | result |
|---|---|
| default | $\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)(\sin^2(fx+e))a^2cd^2-7\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)(\sin^2(fx\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a
^2*c*d^2-7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e
)^2*a^2*d^3-2*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*
x+e)*a^2*c^2*d-14*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*si
n(f*x+e)*a^2*c*d^2+(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+7*(-a*(s
in(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^2*c^3-7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^2*c^2*d+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-8*(
-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-9*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d
)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^3, x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(163) = 326$.
time = 0.52, size = 1612, normalized size = 9.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos
(f*x + e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 +
7*a*c^2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d
^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^
2 - 7*a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 +
 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*
f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*
x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*
c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x +
 e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e)), 1/8
*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x +
 e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 + 7*a*c^
2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7
*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)*cos(f
*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e)
+ a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*
(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^2 - 7*
a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d + 7*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 + 2*c*d
^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*f*cos(
```

```
f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)
 - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*c*d^4
+ d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x + e) -
(c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.59, size = 325, normalized size = 1.82

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\left(ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\left(c^2d+2\,cd^2+d^3\right)\sqrt{-cd-d^2}}+\frac{2\left(2\,acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+14\,ad^2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+ac^2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,ad^2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c^2d+2\,cd^2+d^3\right)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)^2}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*
a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^2*d + 2*c*d^2 + d^3)*sqrt(-c*d - d^2))
+ 2*(2*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^3 + 14*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^3 + a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 8*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e) - 9*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e))/((c^2*d + 2*c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - c - d)^2))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^3, x)
```

# 3.536 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=328

$$-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)}{3465df}$$

```
[Out] -4/1155*a*(c+d)*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-
4/3465*a^3*(c+d)*(15*c^2+10*c*d+7*d^2)*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)/d^
2/f/(a+a*sin(f*x+e))^(1/2)-2/693*a^3*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/99*a^3*(3*c-23*d)*cos(f*x+e)*
(c+d*sin(f*x+e))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)-8/3465*a^2*(5*c-d)*(c+d)*(3
*c^2-38*c*d+355*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

**Rubi** [A]
time = 0.44, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2842, 3060, 2849, 2840, 2830, 2725}

$$-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right)\cos(e+fx)}{3465d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-23d)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465df}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^4}{11df}-\frac{4a(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{1155f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e
+ f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(3
*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f)
- (4*a*(c + d)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(1155*f) - (2*a^3*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*(3*c - 23*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (
2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f
)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^4}{11df}$$

$$= \frac{2a^3(3c-23d)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e$$

$$= -\frac{2a^3(3c^2-38cd+355d^2)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4a(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)(a+a\sin(e+}{1155f}$$

$$= -\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a}}{3465df}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right)\cos(}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica [A]**
time = 3.99, size = 246, normalized size = 0.75

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(164472c^3+411840c^2d+373098cd^2+114640d^3-8(693c^3+5940c^2d+8382cd^2+3250d^3)\cos(2(e+fx))+70d^2(33c+32d)\cos(4(e+fx))+51744c^3\sin(e+fx)+199980c^2d\sin(e+fx)+205656cd^2\sin(e+fx)+69890d^3\sin(e+fx)-5940c^2d\sin(3(e+fx))-17160cd^2\sin(3(e+fx))-8675d^3\sin(3(e+fx))+315d^3\sin(5(e+fx)))}{27720f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/27720*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(164472*c^3 + 411840*c^2*d + 373098*c*d^2 + 114640*d^3 - 8*(693*c^3 + 5
940*c^2*d + 8382*c*d^2 + 3250*d^3)*Cos[2*(e + f*x)] + 70*d^2*(33*c + 32*d)*
Cos[4*(e + f*x)] + 51744*c^3*Sin[e + f*x] + 199980*c^2*d*Sin[e + f*x] + 205
656*c*d^2*Sin[e + f*x] + 69890*d^3*Sin[e + f*x] - 5940*c^2*d*Sin[3*(e + f*x
)] - 17160*c*d^2*Sin[3*(e + f*x)] - 8675*d^3*Sin[3*(e + f*x)] + 315*d^3*Sin
[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 2.58, size = 249, normalized size = 0.76

| method | result |
|---|---|
| default | $\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(315d^3(\sin^5(fx+e))+1155c\,d^2(\sin^4(fx+e))+1120d^3(\sin^4(fx+e))+1485c^2d(\sin^3(fx+e))+4290c\,d}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*d^3*sin(f*x+e)^5+1155*c*d^2*s
in(f*x+e)^4+1120*d^3*sin(f*x+e)^4+1485*c^2*d*sin(f*x+e)^3+4290*c*d^2*sin(f*
x+e)^3+1775*d^3*sin(f*x+e)^3+693*c^3*sin(f*x+e)^2+5940*c^2*d*sin(f*x+e)^2+7
227*c*d^2*sin(f*x+e)^2+2130*d^3*sin(f*x+e)^2+3234*c^3*sin(f*x+e)+11385*c^2*
d*sin(f*x+e)+9636*c*d^2*sin(f*x+e)+2840*d^3*sin(f*x+e)+9933*c^3+22770*c^2*d
+19272*c*d^2+5680*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^3, x)
```

**Fricas [A]**
time = 0.35, size = 508, normalized size = 1.55

$$-\frac{2\,\left(315\,a^2d^3\cos\left(fx+e\right)^6+35\,\left(33\,a^2cd^2+32\,a^2d^3\right)\cos\left(fx+e\right)^5+7392\,a^2c^3+15840\,a^2c^2d+13728\,a^2cd^2+4000\,a^2d^3-5\,\left(297\,a^2c^2d+627\,a^2cd^2+320\,a^2d^3\right)\cos\left(fx+e\right)^4-\left(693\,a^2c^3+5940\,a^2c^2d+9537\,a^2cd^2+4370\,a^2d^3\right)\cos\left(fx+e\right)^3+\left(2541\,a^2c^3+8415\,a^2c^2d+8679\,a^2cd^2+2965\,a^2d^3\right)\cos\left(fx+e\right)^2+2\,\left(5313\,a^2c^3+14355\,a^2c^2d+13827\,a^2cd^2+4465\,a^2d^3\right)\cos\left(fx+e\right)+\left(315\,a^2d^3\cos\left(fx+e\right)^5-7392\,a^2c^3-15840\,a^2c^2d-13728\,a^2cd^2-4000\,a^2d^3-35\,\left(33\,a^2cd^2+23\,a^2d^3\right)\cos\left(fx+e\right)^4-5\,\left(297\,a^2c^2d+858\,a^2cd^2+481\,a^2d^3\right)\cos\left(fx+e\right)^3+3\,\left(231\,a^2c^3+1485\,a^2c^2d+1749\,a^2cd^2+655\,a^2d^3\right)\cos\left(fx+e\right)^2+2\,\left(1617\,a^2c^3+6435\,a^2c^2d+6963\,a^2cd^2+2465\,a^2d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{3465\,\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3465*(315*a^2*d^3*cos(f*x + e)^6 + 35*(33*a^2*c*d^2 + 32*a^2*d^3)*cos(f*
x + e)^5 + 7392*a^2*c^3 + 15840*a^2*c^2*d + 13728*a^2*c*d^2 + 4000*a^2*d^3
- 5*(297*a^2*c^2*d + 627*a^2*c*d^2 + 320*a^2*d^3)*cos(f*x + e)^4 - (693*a^2
*c^3 + 5940*a^2*c^2*d + 9537*a^2*c*d^2 + 4370*a^2*d^3)*cos(f*x + e)^3 + (25
41*a^2*c^3 + 8415*a^2*c^2*d + 8679*a^2*c*d^2 + 2965*a^2*d^3)*cos(f*x + e)^2
 + 2*(5313*a^2*c^3 + 14355*a^2*c^2*d + 13827*a^2*c*d^2 + 4465*a^2*d^3)*cos(
f*x + e) + (315*a^2*d^3*cos(f*x + e)^5 - 7392*a^2*c^3 - 15840*a^2*c^2*d - 1
3728*a^2*c*d^2 - 4000*a^2*d^3 - 35*(33*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)
^4 - 5*(297*a^2*c^2*d + 858*a^2*c*d^2 + 481*a^2*d^3)*cos(f*x + e)^3 + 3*(23
1*a^2*c^3 + 1485*a^2*c^2*d + 1749*a^2*c*d^2 + 655*a^2*d^3)*cos(f*x + e)^2 +
 2*(1617*a^2*c^3 + 6435*a^2*c^2*d + 6963*a^2*c*d^2 + 2465*a^2*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(c + d*sin(e + f*x))**3, x)
```

**Giac [A]**
time = 0.69, size = 510, normalized size = 1.55

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/4*
pi + 11/2*f*x + 11/2*e) + 6930*(40*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 90*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 78*a^2*c*d^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 2310*(20*a^2*c^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 66*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 60*a^
2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*a^2*d^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 693*(8*a^2*c^3*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 60*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 72*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 25*a^2*d^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 495*(12*a^2*
c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*a^2*c*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 13*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/
4*pi + 7/2*f*x + 7/2*e) + 385*(6*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 5*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-9/4*pi + 9/2*f*x
+ 9/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3, x)
```

# 3.537 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=202

$$-\frac{64a^3(21c^2+30cd+13d^2)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(21c^2+30cd+13d^2)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f}-\frac{2a(2\cdots}{}$$

```
[Out] -2/105*a*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-4/63*(9
*c-d)*d*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*d^2*cos(f*x+e)*(a+a*sin(f*x
+e))^(7/2)/a/f-64/315*a^3*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+
e))^(1/2)-16/315*a^2*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/
2)/f
```

**Rubi** [A]

time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2840, 2830, 2726, 2725}

$$-\frac{64a^3(21c^2+30cd+13d^2)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(21c^2+30cd+13d^2)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{2a(21c^2+30cd+13d^2)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f}-\frac{4d(9c-d)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{63f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-64*a^3*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x])/(315*f*Sqrt[a + a*Sin[e +
 f*x]]) - (16*a^2*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e
+ f*x]])/(315*f) - (2*a*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(3/2))/(105*f) - (4*(9*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(63*f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af} + \frac{2\int(a+a\sin(e\ldots}{\ldots}$$
$$= -\frac{4(9c-d)d\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} - \frac{2d^2\cos(e\ldots}{\ldots}$$
$$= -\frac{2a(21c^2+30cd+13d^2)\cos(e+fx)(a+a\sin(e+fx))^{3/\ldots}}{105f}$$
$$= -\frac{16a^2(21c^2+30cd+13d^2)\cos(e+fx)\sqrt{a+a\sin(e+fx\ldots}}{315f}$$
$$= -\frac{64a^3(21c^2+30cd+13d^2)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(21c^2+30\ldots}{\ldots}$$

**Mathematica [A]**
time = 2.11, size = 180, normalized size = 0.89

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(7476c^2+12480cd+5653d^2-4(63c^2+360cd+254d^2)\cos(2(e+fx))+35d^2\cos(4(e+fx))+2352c^2\sin(e+fx)+6060cd\sin(e+fx)+3116d^2\sin(e+fx)-180cd\sin(3(e+fx))-260d^2\sin(3(e+fx))\right)}{1260f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(7476*c^2 + 12480*c*d + 5653*d^2 - 4*(63*c^2 + 360*c*d + 254*d^2)*Cos[2*
(e + f*x)] + 35*d^2*Cos[4*(e + f*x)] + 2352*c^2*Sin[e + f*x] + 6060*c*d*Sin
[e + f*x] + 3116*d^2*Sin[e + f*x] - 180*c*d*Sin[3*(e + f*x)] - 260*d^2*Sin[
3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [A]**
time = 3.09, size = 168, normalized size = 0.83

| method | result |
|---|---|
| default | $\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(35d^2(\sin^4(fx+e))+90cd(\sin^3(fx+e))+130d^2(\sin^3(fx+e))+63c^2(\sin^2(fx+e))+360cd(\sin^2(fx-}{315\cos(fx+e)\sqrt{a+a\sin(fx}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(35*d^2*sin(f*x+e)^4+90*c*d*sin(f*x
+e)^3+130*d^2*sin(f*x+e)^3+63*c^2*sin(f*x+e)^2+360*c*d*sin(f*x+e)^2+219*d^2
*sin(f*x+e)^2+294*c^2*sin(f*x+e)+690*c*d*sin(f*x+e)+292*d^2*sin(f*x+e)+903*
c^2+1380*c*d+584*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^2, x)
```

**Fricas [A]**
time = 0.34, size = 352, normalized size = 1.74

$$-\frac{2\left(35\,a^{2}d^{2}\cos\left(fx+e\right)^{5}-5\left(18\,a^{2}cd+19\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{4}+672\,a^{2}c^{2}+960\,a^{2}cd+416\,a^{2}d^{2}-\left(63\,a^{2}c^{2}+360\,a^{2}cd+289\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{3}+\left(231\,a^{2}c^{2}+510\,a^{2}cd+263\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{2}+2\left(483\,a^{2}c^{2}+870\,a^{2}cd+419\,a^{2}d^{2}\right)\cos\left(fx+e\right)-\left(35\,a^{2}d^{2}\cos\left(fx+e\right)^{4}+672\,a^{2}c^{2}+960\,a^{2}cd+416\,a^{2}d^{2}+10\left(9\,a^{2}cd+13\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{3}-3\left(21\,a^{2}c^{2}+90\,a^{2}cd+53\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{2}-2\left(147\,a^{2}c^{2}+390\,a^{2}cd+211\,a^{2}d^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{315\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*d^2*cos(f*x + e)^5 - 5*(18*a^2*c*d + 19*a^2*d^2)*cos(f*x + e
)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 - (63*a^2*c^2 + 360*a^2*c*d +
 289*a^2*d^2)*cos(f*x + e)^3 + (231*a^2*c^2 + 510*a^2*c*d + 263*a^2*d^2)*co
s(f*x + e)^2 + 2*(483*a^2*c^2 + 870*a^2*c*d + 419*a^2*d^2)*cos(f*x + e) - (
35*a^2*d^2*cos(f*x + e)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 + 10*(9
*a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^3 - 3*(21*a^2*c^2 + 90*a^2*c*d + 53*a^2
*d^2)*cos(f*x + e)^2 - 2*(147*a^2*c^2 + 390*a^2*c*d + 211*a^2*d^2)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(c + d*sin(e + f*x))**2, x)
```

**Giac [A]**
time = 0.63, size = 348, normalized size = 1.72

$$\frac{\sqrt{2}\left(35\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{9}{4}\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)+630\left(20\,a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+30\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+13\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+420\left(5\,a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+11\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)+252\left(a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+45\left(4\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\right)\sqrt{a}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
+ 9/2*f*x + 9/2*e) + 630*(20*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
30*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 13*a^2*d^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 420*(5*a^2*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 5*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x
 + 3/2*e) + 252*(a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*a^2*c*d*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(4*a^2*c*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 5*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*p
i + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2, x)
```

# 3.538 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=138

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a+}{35f}$$

```
[Out] -2/35*a*(7*c+5*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*d*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*c+5*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-16/105*a^2*(7*c+5*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(7*c + 5*d)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*c + 5*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*c +
 5*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*d*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}+\frac{1}{7}(7c+5d)\int (a+$$
$$= -\frac{2a(7c+5d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}-\frac{2d\cos(e$$
$$= -\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{2a(7c+$$
$$= -\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{}}{105f}$$

**Mathematica [A]**
time = 1.01, size = 119, normalized size = 0.86

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(1246c+1040d-6(7c+20d)\cos(2(e+fx))+(392c+505d)\sin(e+fx)-15d\sin(3(e+fx)))}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1246*c + 1040*d - 6*(7*c + 20*d)*Cos[2*(e + f*x)] + (392*c + 505*d)*Sin[
e + f*x] - 15*d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
)
```

**Maple [A]**
time = 3.66, size = 99, normalized size = 0.72

| method | result |
|---|---|
| default | $\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-15\sin(fx+e)\left(\cos^2(fx+e)\right)d+(98c+130d)\sin(fx+e)+(-21c-60d)\left(\cos^2(fx+e)\right)+322c+290d\right)}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*sin(f*x+e)*cos(f*x+e)^2*d+(98*
c+130*d)*sin(f*x+e)+(-21*c-60*d)*cos(f*x+e)^2+322*c+290*d)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c), x)
```

**Fricas** [A]
time = 0.37, size = 217, normalized size = 1.57

$$\frac{2\left(15\,a^{2}d\cos\left(fx+e\right)^{4}+3\left(7\,a^{2}c+20\,a^{2}d\right)\cos\left(fx+e\right)^{3}-224\,a^{2}c-160\,a^{2}d-\left(77\,a^{2}c+85\,a^{2}d\right)\cos\left(fx+e\right)^{2}-2\left(161\,a^{2}c+145\,a^{2}d\right)\cos\left(fx+e\right)+\left(15\,a^{2}d\cos\left(fx+e\right)^{3}+224\,a^{2}c+160\,a^{2}d-3\left(7\,a^{2}c+15\,a^{2}d\right)\cos\left(fx+e\right)^{2}-2\left(49\,a^{2}c+65\,a^{2}d\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{105\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^2*d*cos(f*x + e)^4 + 3*(7*a^2*c + 20*a^2*d)*cos(f*x + e)^3 - 22
4*a^2*c - 160*a^2*d - (77*a^2*c + 85*a^2*d)*cos(f*x + e)^2 - 2*(161*a^2*c +
 145*a^2*d)*cos(f*x + e) + (15*a^2*d*cos(f*x + e)^3 + 224*a^2*c + 160*a^2*d
 - 3*(7*a^2*c + 15*a^2*d)*cos(f*x + e)^2 - 2*(49*a^2*c + 65*a^2*d)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(c + d*sin(e + f*x)), x)
```

**Giac** [A]
time = 0.52, size = 213, normalized size = 1.54

$$\frac{\sqrt{2}\left(15\,a^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+525\left(4\,a^{2}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,a^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+35\left(10\,a^{2}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+11\,a^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)+21\left(2\,a^{2}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,a^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\right)\sqrt{a}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 525*(4*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a^2*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*
(10*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*a^2*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*a^2*c*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 5*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x)
```

# 3.539 $\int (a+a\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=89

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

```
[Out] -2/5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-64/15*a^3*cos(f*x+e)/f/(a+a*sin(
f*x+e))^(1/2)-16/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(3/2))/(5*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}\,dx &= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(8a)\int (a+a\sin(e+fx))^{3/2}\,dx\\ &= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3}}{5f}\\ &= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2a\cos}{}\end{aligned}$$

**Mathematica** [A]
time = 0.23, size = 117, normalized size = 1.31

$$-\frac{(a(1+\sin(e+fx)))^{5/2}\left(150\cos\left(\frac{1}{2}(e+fx)\right)+25\cos\left(\frac{3}{2}(e+fx)\right)-3\cos\left(\frac{5}{2}(e+fx)\right)-150\sin\left(\frac{1}{2}(e+fx)\right)+25\sin\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{5}{2}(e+fx)\right)\right)}{30f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/30*((a*(1 + Sin[e + f*x]))^(5/2)*(150*Cos[(e + f*x)/2] + 25*Cos[(3*(e +
f*x))/2] - 3*Cos[(5*(e + f*x))/2] - 150*Sin[(e + f*x)/2] + 25*Sin[(3*(e + f
*x))/2] + 3*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5)
```

**Maple** [A]
time = 2.71, size = 65, normalized size = 0.73

| method | result | size |
|---|---|---|
| default | $\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(3\left(\sin^2(fx+e)\right)+14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ | 65 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(3*sin(f*x+e)^2+14*sin(f*x+e)+43)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [A]
time = 0.33, size = 124, normalized size = 1.39

$$\frac{2\left(3a^2\cos(fx+e)^3-11a^2\cos(fx+e)^2-46a^2\cos(fx+e)-32a^2-\left(3a^2\cos(fx+e)^2+14a^2\cos(fx+e)-32a^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{15\left(f\cos(fx+e)+f\sin(fx+e)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

[Out] 2/15*(3*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 46*a^2*cos(f*x + e) - 32*a^2 - (3*a^2*cos(f*x + e)^2 + 14*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e) )*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2),x)

[Out] Integral((a*sin(e + f*x) + a)**(5/2), x)

**Giac** [A]
time = 0.48, size = 108, normalized size = 1.21

$$\frac{\sqrt{2}\left(150\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+25\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] 1/30*sqrt(2)*(150*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2 *f*x + 1/2*e) + 25*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/ 2*f*x + 3/2*e) + 3*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/ 2*f*x + 5/2*e))*sqrt(a)/f

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin(e+f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^(5/2),x)

[Out] int((a + a*sin(e + f*x))^(5/2), x)

# 3.540 $\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=142

$$-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}\,f}+\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}$$

```
[Out] -2*a^(5/2)*(c-d)^2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(
f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^3*(3*c-7*d)*cos(f*x+e)/d^2/f/(a+
a*sin(f*x+e))^(1/2)-2/3*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f
```

**Rubi** [A]
time = 0.27, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2842, 3060, 2852, 214}

$$-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}+\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a^(5/2)*(c - d)^2*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*S
qrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^3*(3*c - 7*d)*Cos
[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\,(\frac{1}{2}a^2(c+3d)-}{c+d\sin(e+fx)}}{3d}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{(a^2(c-}{}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} - \frac{(2a^3(c-}{}$$

$$= -\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}\,f} + \frac{2a^3(3c-7d)\cos(}{3d^2f\sqrt{a+a\sin(}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 330 vs. $2(142) = 284$.
time = 2.51, size = 330, normalized size = 2.32

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(6(2c-5d)\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-2d^{3/2}\cos\left(\frac{3}{2}(e+fx)\right)-\frac{3(c-d)^2\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}+\frac{3(c-d)^2\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}+6\sqrt{d}(-2c+5d)\sin\left(\frac{1}{2}(e+fx)\right)-2d^{3/2}\sin\left(\frac{3}{2}(e+fx)\right)\right)}{6d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(6*(2*c - 5*d)*Sqrt[d]*Cos[(e + f*x)/2] - 2*d
^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)^2*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x
)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/Sqrt[c + d] + (3*(c - d)^2
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 +
Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[
c + d] + 6*Sqrt[d]*(-2*c + 5*d)*Sin[(e + f*x)/2] - 2*d^(3/2)*Sin[(3*(e + f*
x))/2]))/(6*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

**Maple** [**A**]
time = 4.45, size = 229, normalized size = 1.61

| method | result |
|---|---|
| default | $-\frac{2a(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a\left(c+d\right)d}}\right)a^{2}c^{2}-6\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\right.}}{}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2-6*arctanh((-a*(sin(f*x+e)-1))^(1/2)*
d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^2*d^2-(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*(-a*(sin
(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c+9*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*a*d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

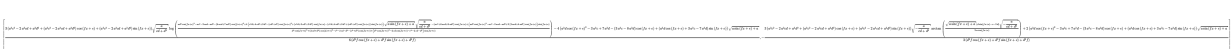
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(126) = 252$.
time = 0.46, size = 906, normalized size = 6.38



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*co
s(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x + e))*sqrt(a/(c*d + d^
2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^
2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)
^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c
*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*
d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(a^2*d*cos(f*x + e)^2 - 3*a^2*c + 7*a^2
*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*x + e) + 3*a^2*c - 7*a
^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f), -1/3*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2
*a^2*c*d + a^2*d^2)*cos(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x
+ e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
 e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(a^2*d*cos(f*x +
e)^2 - 3*a^2*c + 7*a^2*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*
x + e) + 3*a^2*c - 7*a^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*
cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.57, size = 239, normalized size = 1.68

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\left(a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\sqrt{-cd-d^2}\,d^2}+\frac{2\left(2\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{d^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(a)*(3*sqrt(2)*(a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) - 2*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a^2*d^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(
-c*d - d^2))/(sqrt(-c*d - d^2)*d^2) + 2*(2*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 3*a^2*c*d*sgn(cos(-1/4*pi +
```

```
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*a^2*d^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^3)/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^{5/2}}{c + d \sin(e + f x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x)), x)
```

# 3.541 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=166

$$\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f}-\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}}+\frac{a^2(c-d)\,c}{d(c}$$

```
[Out] a^(5/2)*(c-d)*(3*c+5*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a
*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^3*(3*c+d)*cos(f*x+e)/d^2/(c+d)/
f/(a+a*sin(f*x+e))^(1/2)+a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d
)/f/(c+d*sin(f*x+e))
```

**Rubi** [A]

time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2841, 3060, 2852, 214}

$$\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^3(3c+d)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^(5/2)*(c - d)*(3*c + 5*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d)^(3/2)*f) - (a^3*(3*c + d
)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\,(\frac{1}{2}a(c-5d}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx}}{d(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx}}{d(c+d)f(c+d\sin(e+fx))}$$

$$= \frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} - \frac{a^3(3c+c}{d^2(c+d)f\sqrt{}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(166) = 332.
time = 2.83, size = 350, normalized size = 2.11

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(-8\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\frac{(3c^2+2cd-5d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+\frac{(-3c^2-2cd+5d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+8\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)-\frac{4(c-d)^2\sqrt{d}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}\right)}{4d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-8*Sqrt[d]*Cos[(e + f*x)/2] + ((3*c^2 + 2*c*
d - 5*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^
2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(
e + f*x)/2]))]))/(c + d)^(3/2) + ((-3*c^2 - 2*c*d + 5*d^2)*(e + f*x - 2*Log
[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c +
d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*Sqrt
[d]*Sin[(e + f*x)/2] - (4*(c - d)^2*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(148) = 296$.
time = 5.43, size = 393, normalized size = 2.37

| method | result |
|---|---|
| default | $-\frac{a^{2}(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\sin(fx+e)d\left(3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+d^{2}a}}\right)a\,c^{2}+2\operatorname{arctanh}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(3*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2+2*arctanh((a-a*sin(f*x+e))
^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d-5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*a*d^2-2*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c-2*(a-a*
sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d)-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(
a*c*d+a*d^2)^(1/2))*a*c^3-2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a*c^2*d+5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*
d^2+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2+(a-a*sin(f*x+e))^(1/2)*(
a*(c+d)*d)^(1/2)*d^2)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+
e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^2, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(155) = 310$.

time = 0.45, size = 1368, normalized size = 8.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c
*d^2)*cos(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (
3*a^2*c^2*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(
c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
+ 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(
f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*
d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f
*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*s
in(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x +
 e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^
2 + 2*(a^2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x +
e) - (3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 -
(c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 +
d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*
a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2*a^2*c*d^
2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c*d^2)*cos
(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2
*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^
2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/
(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 + 2*(a^
2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x + e) - (3*a
^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2
+ c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*co
s(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

[Out] Timed out

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(155) = 310.
time = 0.56, size = 314, normalized size = 1.89

$$\frac{\sqrt{2}\left(\frac{4\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d^2}+\frac{\sqrt{2}\left(3\,a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-5\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{(cd^2+d^3)\sqrt{-cd-d^2}}-\frac{2\left(a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{(cd^2+d^3)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}\right)\sqrt{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/d^2 + sqrt(2)*(3*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2
*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*a^2*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*
d - d^2))/((c*d^2 + d^3)*sqrt(-c*d - d^2)) - 2*(a^2*c^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*a^2*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + a^2*d^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c*d^2 + d^3)*(2*d*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))*sqrt(a)/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^2, x)
```

# 3.542 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=194

$$-\frac{a^{5/2}(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2}$$

```
[Out] -1/4*a^(5/2)*(3*c^2+10*c*d+19*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)
^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+3/4*a^3*(c-d)*(c+3*d)*
cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+1/2*a^2*(c
-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

**Rubi** [A]
time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2841, 3059, 2852, 214}

$$-\frac{a^{5/2}(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/4*(a^(5/2)*(3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*
x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d)^(5/2)*f) + (a
^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c + d*Sin
[e + f*x])^2) + (3*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(4*d^2*(c + d)^2*f*S
qrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\,\left(\frac{1}{2}a(c-9\right.}{(c+d\sin(e+f}}{2d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\,c}{4d^2(c+d)^2f\sqrt{a+a\sin(e+f}}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\,c}{4d^2(c+d)^2f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{a^{5/2}(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a^2(c-}{2}$$

**Mathematica [A]**
time = 3.20, size = 379, normalized size = 1.95

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(-\frac{(3c^2+10cd+19d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{5/2}}+\frac{(3c^2+10cd+19d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{5/2}}-\frac{8(c-d)^2\sqrt{d}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))^2}-\frac{4\sqrt{d}\left(-5c^2-6cd+11d^2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}\right)}{16d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-(((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Lo
g[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c
+ d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(
5/2)) + ((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)
/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (8*(c - d)^2*Sqrt[d]*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]
*(-5*c^2 - 6*c*d + 11*d^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^
2*(c + d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(170) = 340$.
time = 5.64, size = 567, normalized size = 2.92

| method | result |
|---|---|
| default | $-\frac{a\left(2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+d^2a}}\right)a^2cd(3c^2+10cd+19d^2)-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+d^2a}}\right)a^2d^2\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*(2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*c^2+10*c*d+19*d^2)-arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*d^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2+3*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^4+10*arctanh((a-a*sin(f*x+e))^(1/2)*
d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+22*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d
+a*d^2)^(1/2))*a^2*c^2*d^2+10*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*c*d^3+19*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a^2*d^4+5*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d+6*(a-a*sin(f*x+e)
)^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-11*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*d^3-3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-13*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a
*c*d^2+13*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1
))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^3, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(178) = 356$.
time = 0.56, size = 2048, normalized size = 10.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*
d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6*a^2*c
^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (3*a^2*
c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x +
e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4
 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*a^2*c^
3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*
d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d +
7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*
x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^
3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x
 + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin
(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d
 - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e
) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2
*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x + e)^
2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e) - (3*a
^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^2*c*d^
2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2
*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 +
 d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*
f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^
2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f
*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*
x + e)), 1/8*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 1
9*a^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6
*a^2*c^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (
3*a^2*c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(
f*x + e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a
^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*
a^2*c^3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
```

```
2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^3 + 3*a^2*c^2*d -
15*a^2*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x
 + e)^2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)
- (3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^
2*c*d^2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c
*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f
+ ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*
d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*
sin(f*x + e))]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(178) = 356.
time = 0.56, size = 448, normalized size = 2.31

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\left(3a^2c^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+10a^2cd\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+19a^2d^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(c^2d^2+2cd^3+d^4)\sqrt{-cd-d^2}}-\frac{2\left(10a^2c^2d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+12a^2cd^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-22a^2d^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2c^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-13a^2c^2d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+3a^2cd^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+13a^2d^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{(c^2d^2+2cd^3+d^4)\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(3*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 10*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*a^2*d^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/s
qrt(-c*d - d^2))/((c^2*d^2 + 2*c*d^3 + d^4)*sqrt(-c*d - d^2)) - 2*(10*a^2*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 +
 12*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^3 - 22*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^3 - 3*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e) - 13*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e) + 3*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 13*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2*d^2 + 2*c*d^3 + d^4)*(2*d*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\sin(e + fx))^{5/2}}{(c + d\sin(e + fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^3, x)
```

# 3.543 $\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=178

$$-\frac{\sqrt{2}\,(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4d(21c^2-12cd+7d^2)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2(9c-d)d^2\cos(e+\cdots}{\cdots}$$

```
[Out] -(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(
1/2)/f/a^(1/2)-4/15*d*(21*c^2-12*c*d+7*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(
1/2)-2/5*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/15*(9*c
-d)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

**Rubi** [A]

time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2857, 3047, 3102, 2830, 2728, 212}

$$-\frac{4d(21c^2-12cd+7d^2)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d^2(9c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*d*(21*c^2 - 12*c*d + 7*d^2)*Cos[e + f*x])/(
15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*(9*c - d)*d^2*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(15*a*f) - (2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqr
t[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{(c+d\sin(e+fx))(-a(5c^2-cd+4d^2)-a(9c-d)d\sin}{\sqrt{a+a\sin(e+fx)}}}{5a}$$

$$= -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-ac(5c^2-cd+4d^2)+(-ac(9c-d)d-ad(5c^2-cd+4}{\sqrt{a+a\sin(e}}}{5a}$$

$$= -\frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2d\cos(e+fx)(c+d\sin(e+}{5f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4d(21c^2-12cd+7d^2)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+}}{15af}$$

$$= -\frac{4d(21c^2-12cd+7d^2)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+}}{15af}$$

$$= -\frac{\sqrt{2}(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4d(21c^2-12cd+7d^2)\mathrm{co}}{15f\sqrt{a+a\sin(e+}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.42, size = 155, normalized size = 0.87

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-60-60i)(-1)^{3/4}(c-d)^3\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-90c^2+30cd-29d^2+3d^2\cos(2(e+fx))-2(15c-d)d\sin(e+fx)\right)\right)}{30f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/30*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-60 - 60*I)*(-1)^(3/4)*(c -
d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 2*d*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])*(-90*c^2 + 30*c*d - 29*d^2 + 3*d^2*Cos[2*(e +
 f*x)] - 2*(15*c - d)*d*Sin[e + f*x])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [A]
time = 3.92, size = 285, normalized size = 1.60

| method | result |
|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(15a^{\frac{5}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)c^3-45a^{\frac{5}{2}}\sqrt{2}\,\mathrm{arcta}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-45*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+45*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-15*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3+6*(a-a*sin(f*x+e))^(5/2
)*d^3-30*(a-a*sin(f*x+e))^(3/2)*a*c*d^2-10*(a-a*sin(f*x+e))^(3/2)*a*d^3+90*
c^2*d*a^2*(a-a*sin(f*x+e))^(1/2)+30*d^3*a^2*(a-a*sin(f*x+e))^(1/2))/a^3/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(166) = 332.
time = 0.38, size = 409, normalized size = 2.30

$$-\frac{\frac{15\sqrt{2}\left(ac^3-3ac^2d+3acd^2-ad^3+\left(ac^3-3ac^2d+3acd^2-ad^3\right)\cos(fx+e)+\left(ac^3-3ac^2d+3acd^2-ad^3\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}-4\left(3d^3\cos(fx+e)^3-45c^2d+30cd^2-17d^3-\left(15cd^2-4d^3\right)\cos(fx+e)^2-\left(45c^2d-15cd^2+16d^3\right)\cos(fx+e)-\left(3d^3\cos(fx+e)^2-45c^2d+30cd^2-17d^3+\left(15cd^2-d^3\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{30\left(af\cos(fx+e)+af\sin(fx+e)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3 + (a*c^3 - 3*a*c^2
*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e) + (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d
^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a
) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2))/sqrt(a) - 4*(3*d^3*cos(f*x + e)^3 - 45*c^2*d + 30*c*d^2
 - 17*d^3 - (15*c*d^2 - 4*d^3)*cos(f*x + e)^2 - (45*c^2*d - 15*c*d^2 + 16*d
^3)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - 45*c^2*d + 30*c*d^2 - 17*d^3 + (
15*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*
cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sin{(e + fx)})^3}{\sqrt{a (\sin{(e + fx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac [A]**
time = 0.54, size = 294, normalized size = 1.65

$$\frac{\frac{15 \sqrt{2} \left(\sqrt{a} c^3 - 3 \sqrt{a} c^2 d + 3 \sqrt{a} c d^2 - \sqrt{a} d^3\right) \log\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)}{a \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{15 \sqrt{2} \left(\sqrt{a} c^3 - 3 \sqrt{a} c^2 d + 3 \sqrt{a} c d^2 - \sqrt{a} d^3\right) \log\left(-\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)}{a \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} + \frac{4 \sqrt{2} \left(12 a^{\frac{9}{2}} d^3 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^5 - 30 a^{\frac{9}{2}} c d^2 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^3 - 10 a^{\frac{9}{2}} d^3 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^3 + 45 a^{\frac{9}{2}} c^2 d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 15 a^{\frac{9}{2}} d^3 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{a^5 \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}}{30 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(15*sqrt(2)*(sqrt(a)*c^3 - 3*sqrt(a)*c^2*d + 3*sqrt(a)*c*d^2 - sqrt(a)
*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - 15*sqrt(2)*(sqrt(a)*c^3 - 3*sqrt(a)*c^2*d + 3*sqrt(a)*c*d^2 -
sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))) + 4*sqrt(2)*(12*a^(9/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^5 - 30*a^(9/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*a^(9/2)*d^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 45*a^(9/2)*c^2*d*sin(-1/4*pi + 1/2*f*x +
1/2*e) + 15*a^(9/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^5*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d \sin{(e + f x)})^3}{\sqrt{a + a \sin{(e + f x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.544 $\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{\sqrt{2}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

```
[Out] -(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(
1/2)/f/a^(1/2)-4/3*(3*c-d)*d*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*d^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

**Rubi** [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2840, 2830, 2728, 212}

$$-\frac{4d(3c-d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*(3*c - d)*d*Cos[e + f*x])/(3*f*Sqrt[a + a*S
in[e + f*x]]) - (2*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + \frac{2\int \frac{\frac{1}{2}a(3c^2+d^2)+a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a}$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + (c-d)^2\int$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} - \frac{(2(c-d)^2}{}$$

$$= -\frac{\sqrt{2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.27, size = 125, normalized size = 1.02

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-3-3i)(-1)^{3/4}(c-d)^2\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(6c-d+d\sin(e+fx))\right)}{3f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-3 - 3*I)*(-1)^(3/4)*(c - d)^2*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])*(6*c - d + d*Sin[e + f*x])))/(3*f*Sqrt[a*(1 + Sin[e
+ f*x])])
```

**Maple** [A]
time = 3.48, size = 185, normalized size = 1.50

| method | result |
|---|---|
| default | $\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{2}+6a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctan}\right.}{3a^{2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+6*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+2*(a-a*sin(f*x+e))^(3/2)*d^2-12*a*c*d*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) + a), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(112) = 224$.
time = 0.35, size = 306, normalized size = 2.49

$$\frac{\frac{3\sqrt{2}\left(ac^2-2acd+ad^2+(ac^2-2acd+ad^2)\cos(fx+e)+(ac^2-2acd+ad^2)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}-4\left(d^2\cos\left(fx+e\right)^2+6\,cd-2\,d^2+\left(6\,cd-d^2\right)\cos\left(fx+e\right)+\left(d^2\cos\left(fx+e\right)-6\,cd+2\,d^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{6\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/6*(3*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(d^2*cos(f*x + e)^2 + 6*c*d - 2*d^2 + (6*c*d - d^2)*cos(f*x + e) + (d^2*cos(f*x + e) - 6*c*d + 2*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin(e + fx))^2}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac [A]**
time = 0.58, size = 199, normalized size = 1.62

$$\frac{\frac{3\sqrt{2}\left(\sqrt{a}\,c^2 - 2\sqrt{a}\,cd + \sqrt{a}\,d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(\sqrt{a}\,c^2 - 2\sqrt{a}\,cd + \sqrt{a}\,d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{8\sqrt{2}\left(a^{\frac{5}{2}}d^2\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3a^{\frac{5}{2}}cd\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(2)*(sqrt(a)*c^2 - 2*sqrt(a)*c*d + sqrt(a)*d^2)*log(sin(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)
*(sqrt(a)*c^2 - 2*sqrt(a)*c*d + sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 8*sqrt(2)*(a^(5/2)*d^2
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*a^(5/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,\sin(e + f\,x))^2}{\sqrt{a + a\,\sin(e + f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.545 $\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

[Out] -(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)-2*d*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)

**Rubi** [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2728, 212}

$$-\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*d*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2(c-d))\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.15, size = 106, normalized size = 1.34

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}(c-d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+d\left(-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(c - d)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [A]
time = 4.68, size = 128, normalized size = 1.62

| method | result |
|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(\sqrt{a}\,\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c-\sqrt{a}\,\sqrt{2}\,\mathrm{arct}\right.}{a\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d+2*(a-a*sin(f*x+e))^(1/2)*d)/a/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(72) = 144$.
time = 0.35, size = 232, normalized size = 2.94

$$-\frac{\frac{\sqrt{2}\left(ac-ad+(ac-ad)\cos(fx+e)+(ac-ad)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}+4\left(d\cos(fx+e)-d\sin(fx+e)+d\right)\sqrt{a\sin(fx+e)+a}}{2\left(af\cos(fx+e)+af\sin(fx+e)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x +
 e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x
 + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2))/sqrt(a) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e
) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+d\sin(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(72) = 144$.
time = 0.61, size = 149, normalized size = 1.89

$$\frac{\frac{4\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\sqrt{a}\,c-\sqrt{a}\,d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\sqrt{a}\,c-\sqrt{a}\,d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(sqrt(a)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))) + sqrt(2)*(sqrt(a)*c - sqrt(a)*d)*log(sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(sqrt(a
)*c - sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

**Mupad [B]**
time = 8.96, size = 151, normalized size = 1.91

$$-\frac{c\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}-\frac{d\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{a+a\,\sin\left(e+f\,x\right)}{2\,a}}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] - (c*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))
/(f*(a + a*sin(e + f*x))^(1/2)) - (d*(4*ellipticE(asin((2^(1/2)*(1 - sin(e
+ f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))
/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*cos(
e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

# 3.546 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^(1/2)/a
^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2728, 212}

$$-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*f))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.04, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [A]
time = 2.22, size = 75, normalized size = 1.60

| method | result | size |
|---|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$ | 75 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas [A]**
time = 0.38, size = 180, normalized size = 3.83

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{2\sqrt{a}f}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{-\frac{1}{a}}}{\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqr
t(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3
*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2))/(sqrt(a)*f), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(f*
x + e) + a)*sqrt(-1/a)/cos(f*x + e))/f]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(e+fx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(e + f*x) + a), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(40) = 80.
time = 0.53, size = 117, normalized size = 2.49

$$\frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(-1/4*pi + 1/2*f*x + 1/2*e) + sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*l
og(abs(1/sin(-1/4*pi + 1/2*f*x + 1/2*e) + sin(-1/4*pi + 1/2*f*x + 1/2*e) -
2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [B]**
time = 7.72, size = 49, normalized size = 1.04

$$-\frac{\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(1/2),x)
[Out] -(ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))/(f
*(a + a*sin(e + f*x))^(1/2))
```

# 3.547 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=123

$$-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{c+d}\,f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2859, 2728, 212, 2852, 214}

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)]])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x]
)/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2859

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\text{Subs}}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{d}\tanh^{-1}}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.30, size = 215, normalized size = 1.75

$$\frac{\left((2+2i)(-1)^{3/4}\sqrt{c+d}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt{d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)\sqrt{c+d}\,f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (((2 + 2*I)*(-1)^(3/4)*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan
[(e + f*x)/4])] + Sqrt[d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)/4]^2*(Sqrt[
c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x
])])
```

**Maple [A]**
time = 5.18, size = 131, normalized size = 1.07

| method | result |
|---|---|
| default | $-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-2d\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^{\frac{3}{2}}+\sqrt{2}\,\mathrm{arctanh}\left(\sqrt{\right.\right.}{(c-d)\sqrt{a(c+d)d}\,a^{\frac{3}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-2*d*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)+2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*a*(a*(c+d)*d)^(1/2))/(c-d)/(a*(c+d)*d)^(1/2)/a^(3/2
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(104) = 208.
time = 0.43, size = 727, normalized size = 5.91

$$\left[-\frac{\sqrt{\frac{d}{ac+ad}}\log\left(\frac{d^2\cos(fx+e)^3-(6cd+7d^2)\cos(fx+e)^2-c^2-2cd-d^2-4\left((cd+d^2)\cos(fx+e)^2-c^2-4cd-3d^2-(c^2+3cd+2d^2)\cos(fx+e)+(c^2+4cd+3d^2+(cd+d^2)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{\frac{d}{ac+ad}}-(c^2+8cd+9d^2)\cos(fx+e)+\left(d^2\cos(fx+e)^2-c^2-2cd-d^2+2(3cd+4d^2)\cos(fx+e)\right)\sin(fx+e)}{d^2\cos(fx+e)^3+(2cd+d^2)\cos(fx+e)^2-c^2-2cd-d^2-(c^2+d^2)\cos(fx+e)+\left(d^2\cos(fx+e)^2-2cd\cos(fx+e)-c^2-2cd-d^2\right)\sin(fx+e)}\right)+\frac{\sqrt{2}\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{2(c-d)f},\ -\frac{2\sqrt{-\frac{d}{ac+ad}}\arctan\left(\frac{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)-c-2d)\sqrt{-\frac{d}{ac+ad}}}{2d\cos(fx+e)}\right)-\frac{\sqrt{2}\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{2(c-d)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
```

```
x + e))) + sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(
a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(a))/((c - d)*f), 1/2*(2*sqrt(-d/(a*c + a*d))*arct
an(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a
*d))/(d*cos(f*x + e))) - sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*
sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x +
 e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((c - d)*f)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))), x)
```

**Giac** [A]
time = 0.51, size = 206, normalized size = 1.67

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\sqrt{-cd-d^2}\,\left(c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}+\frac{\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{2\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*d*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sq
rt(-c*d - d^2))/(sqrt(-c*d - d^2)*(c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))) + log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - d*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))) - log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/(sqrt(a)*f)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)
```

## 3.548 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=175

$$-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f}+\frac{\sqrt{d}\,(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2(c+d)^{3/2} f}+\frac{}{(c^2-d}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-
d)^2/f/a^(1/2)+(3*c+d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*
sin(f*x+e))^(1/2))*d^(1/2)/(c-d)^2/(c+d)^(3/2)/f/a^(1/2)+d*cos(f*x+e)/(c^2-
d^2)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2858, 3064, 2728, 212, 2852, 214}

$$\frac{d\cos(e+fx)}{f\,(c^2-d^2)\,\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)^2}+\frac{\sqrt{d}\,(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^2*f)) + (Sqrt[d]*(3*c + d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Co
s[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^2*(c
+ d)^(3/2)*f) + (d*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx = \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{2\text{Sub}}{}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f} + \frac{\sqrt{d}\,(3c+d)}{}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 2.48, size = 324, normalized size = 1.85

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\frac{\sqrt{d}(3c+d)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}-\frac{\sqrt{d}(3c+d)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+\frac{4(c-d)d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}\right)}{4(c-d)^2f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (Sqrt[d]*(3*c + d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*
Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)^(3/2) - (Sqrt[d]*(3
*c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sq
rt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)
^(3/2) + (4*(c - d)*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c +
d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(150) = 300$.
time = 5.64, size = 453, normalized size = 2.59

| method | result |
|---|---|
| default | $\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(\sin(fx+e)\left(3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+d^2a}}\right)a^{\frac{7}{2}}c\,d^2+a^{\frac{7}{2}}\operatorname{arctanh}\left(\right.\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)*(sin(f*x+e)*(3*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d^2+a^(7/2)*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^3-(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d-(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^2)+3*ar
ctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c^2*d+arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d^2+(a-a*sin(f*x+e))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d-(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(5/2)*d^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^3*c^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+
d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maple [F]**

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(157) = 314$.
time = 0.59, size = 1566, normalized size = 8.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/4*((3*a*c^2 + 4*a*c*d + a*d^2 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a
*c^2 + a*c*d)*cos(f*x + e) + (3*a*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 -
(6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x
 + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 +
 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(
f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x +
e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
- (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2
 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*
d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d
 + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2
 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^
2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(c*d -
 d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e)^2 - (a*c
^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*a*c^2*d^2 +
 a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e) + (a*c^
4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e)), -1/2*((3*a*c^2 + 4*a*c*d + a*d^2
 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a*c^2 + a*c*d)*cos(f*x + e) + (3*a
*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + sqrt(2)*(a*c^2 + 2*a*c*d + a*
d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^
2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos
(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x +
e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(co
```

```
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a)
 + 2*(c*d - d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
e)^2 - (a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*
a*c^2*d^2 + a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
 e) + (a*c^4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(157) = 314.
time = 0.65, size = 380, normalized size = 2.17

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,(3\,cd+d^2)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\left(c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-c^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{-cd-d^2}}+\frac{\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}\right)}{2\,\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*(3*c*d + d^2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x +
 1/2*e)/sqrt(-c*d - d^2))/((c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - c^2*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + log(si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
- 2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))) - log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/((c^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))/(sqrt(a)*f)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2), x)
```

# 3.549 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=247

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3 f}+\frac{\sqrt{d}\,(15c^2+10cd+7d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{a}(c-d)^3(c+d)^{5/2}f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-
d)^3/f/a^(1/2)+1/4*(15*c^2+10*c*d+7*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)
/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)+
1/2*d*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*
d*(7*c+d)*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.51, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2858, 3063, 3064, 2728, 212, 2852, 214}

$$\frac{d(7c+d)\cos(e+fx)}{4f(c^2-d^2)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{d\cos(e+fx)}{2f(c^2-d^2)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{\sqrt{d}\,(15c^2+10cd+7d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{a}f(c-d)^3(c+d)^{5/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^3*f)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*ArcTanh[(Sq
rt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqr
t[a]*(c - d)^3*(c + d)^(5/2)*f) + (d*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (d*(7*c + d)*Cos[e + f*x])/(4*(
c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx = \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\int \cdots}{\cdots}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\cdots}{4(\cdots}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\cdots}{4(\cdots}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\cdots}{4(\cdots}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3f} + \frac{\sqrt{d}\left(15c^2+\cdots\right.}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 3.41, size = 414, normalized size = 1.68

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{(32+32i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)}{(c-d)^3}+\frac{\sqrt{d}\left(15c^2+10cd+7d^2\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c-d)^3(c+d)^{5/2}}+\frac{\sqrt{d}\left(15c^2+10cd+7d^2\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(-c+d)^3(c+d)^{5/2}}+\frac{8d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)(c+d)(c+d\sin(e+fx))^2}+\frac{4d(7c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))}\right)}{16f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])])/(c - d)^3 + (Sqrt[d]*(15*c^2
+ 10*c*d + 7*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x
)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
))/((c - d)^3*(c + d)^(5/2)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[
d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/((-c + d)^3*(c + d)^(5/2
)) + (8*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c - d)*(c + d)*(c + d*Si
n[e + f*x])^2) + (4*d*(7*c + d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c
- d)^2*(c + d)^2*(c + d*Sin[e + f*x]))))/(16*f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(214) = 428$.
time = 7.18, size = 1065, normalized size = 4.31

| method | result | size |
|---|---|---|

| default | Expression too large to display | 1065 |
|---|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4+7*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)^2*d^5+15*arc
tanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*c^4*d+10*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*c^3*d^2+7*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*c^2*d^3+(-a*(sin(
f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*d^4-4*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*c^2
*d^2-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)^2*a^5*c*d^3-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^5*c^3*d-16*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)*a^5*c^2*d^2-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^5*c*d^3+10*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)^2*c*d^4+30*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c^3*d^2+20
*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)
*c^2*d^3+14*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)
*sin(f*x+e)*c*d^4+9*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c^3
*d-(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c^2*d^2-9*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c*d^3-7*(-a*(sin(f*x+e)-1))^(3/2)
*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^
(1/2)*a^(7/2)*c*d^3-4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*c^4+15*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)^2*c^2*d^3-8*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*c^3*d-4*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
a^5*c^2*d^2-4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*d^4)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(
f*x+e))/a^(11/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c-d)^3/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^3), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(224) = 448$.
time = 0.79, size = 2991, normalized size = 12.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [1/16*((15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a
*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^
2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^
2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e) + (15*a*c^4 + 40*a*c^3*d + 42*a*
c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(
f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f
*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
 - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
 + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c +
a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*
c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)
^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
 + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f
*x + e))) + 8*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2
*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d
^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d +
2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(9*c^3*d -
 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e)^2
+ (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d - 15*c^2*
d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4 + 2*a*
c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2 - 5*a*
c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - (
a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*c*d^6 -
 a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a
*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^3 - 2*a
*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^6*d - a
```

```
*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*x + e)
- (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c^2*d^5
- a*c*d^6 - a*d^7)*f)*sin(f*x + e)), -1/8*((15*a*c^4 + 40*a*c^3*d + 42*a*c^
2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*
x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^
2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x +
 e) + (15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*
c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d
^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2
*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(
d*cos(f*x + e))) - 4*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*
c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*
d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c
^3*d + 2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e
)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*
(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(9*
c^3*d - 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x
+ e)^2 + (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d -
15*c^2*d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4
 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2
 - 5*a*c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e
)^2 - (a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*
c*d^6 - a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^
3 + 3*a*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^
3 - 2*a*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^
6*d - a*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*
x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c
^2*d^5 - a*c*d^6 - a*d^7)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(224) = 448$.

time = 0.67, size = 585, normalized size = 2.37

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,(15\,c^{2}d+10\,cd^{2}+7\,d^{3})\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^{2}}}\right)}{\left(c^{5}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-c^{4}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,c^{3}d^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,c^{2}d^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+cd^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-d^{5}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{-cd-d^{2}}}+\frac{4\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,c^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,cd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-d^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,c^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,cd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-d^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\left(14\,cd^{2}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,d^{3}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-9\,c^{2}d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,cd^{2}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d^{3}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,c^{2}d^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+d^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c-d\right)^{2}}\right)}{8\,\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*(15*c^2*d + 10*c*d^2 + 7*d^3)*arctan(sqrt(2)*d*sin(-1/
4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^5*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c^3*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 2*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - d^5*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*sqrt(-c*d - d^2)) + 4*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - d^3*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))) - 4*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) + 2*(14*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2*d^3*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 9*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
 8*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e
))/((c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c^2*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))/(sqrt(a)*f)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3), x)
```

# 3.550 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=192

$$-\frac{(c-d)^2(c+11d)\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(e+fx)}{\sqrt{2}\ \sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\ a^{3/2}f}+\frac{d(3c^2-24cd+13d^2)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}}+\frac{(3c-7d)d^2\ \text{co}}{}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
^2*(c+11*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(3/2)/f*2^(1/2)+1/3*d*(3*c^2-24*c*d+13*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e
))^(1/2)+1/6*(3*c-7*d)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

**Rubi** [A]
time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2844, 3047, 3102, 2830, 2728, 212}

$$-\frac{(c+11d)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(e+fx)}{\sqrt{2}\ \sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\ a^{3/2}f}+\frac{d^2(3c-7d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3c^2-24cd+13d^2)\cos(e+fx)}{3af\sqrt{a\sin(e+fx)+a}}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c - d)^2*(c + 11*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + (d*(3*c^2 - 24*c*d + 13*d^2)*Cos[
e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*c - 7*d)*d^2*Cos[e + f*x]*
Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e +
f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a(c^2+7cd-4d^2)+\frac{1}{2}\cdots\right.}{\sqrt{a+a\sin(e+\cdots}}}{2a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}ac(c^2+7cd-4d^2)+\left(\frac{1}{2}ac(3c-7d)d-\frac{1}{2}\cdots\right.}{\sqrt{a\cdots}}}{\cdots}$$

$$= \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin\cdots}{2f(a+a\sin(e+fx)\cdots}$$

$$= \frac{d(3c^2-24cd+13d^2)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+\cdots}}{6a^2f}$$

$$= \frac{d(3c^2-24cd+13d^2)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+\cdots}}{6a^2f}$$

$$= -\frac{(c-d)^2(c+11d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{d(3c^2-24cd+1\cdots}{3af\sqrt{a+a\cdots}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.38, size = 328, normalized size = 1.71

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(6(c-d)^3\sin\left(\tfrac{1}{2}(e+fx)\right)-3(c-d)^3\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+(3+3i)(-1)^{3/4}(c-d)^2(c+11d)\tanh^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\tfrac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-18(2c-d)d^2\cos\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-2d^3\cos\left(\tfrac{3}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+18(2c-d)d^2\sin\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-2d^3\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2\sin\left(\tfrac{3}{2}(e+fx)\right)\right)}{6f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^3*Sin[(e + f*x)/2] - 3*(c
 - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(-1)^(3/4)*(c - d
)^2*(c + 11*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 18*(2*c - d)*d^2*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 + 18*(2*c - d)*d^2*Sin[(e + f*x)/2]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2*Sin[(3*(e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(169) = 338$.
time = 3.32, size = 490, normalized size = 2.55

| method | result |
|---|---|
| default | $\frac{\left(\sin(fx+e)\left(8(a-a\sin(fx+e))^{\frac{3}{2}}d^3\sqrt{a}-72a^{\frac{3}{2}}cd^2\sqrt{a-a\sin\left(fx+e\right)}+24a^{\frac{3}{2}}d^3\sqrt{a-a\sin\left(fx+e\right)}-3\sqrt{2}\ \text{arc}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/a^(7/2)*(sin(f*x+e)*(8*(a-a*sin(f*x+e))^(3/2)*d^3*a^(1/2)-72*a^(3/2)*c
*d^2*(a-a*sin(f*x+e))^(1/2)+24*a^(3/2)*d^3*(a-a*sin(f*x+e))^(1/2)-3*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-27*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+63*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-33*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3)+8*(a-a*sin(f*x+e))^(3
/2)*d^3*a^(1/2)-6*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3+18*(a-a*sin(f*x+e))^(1
/2)*a^(3/2)*c^2*d-90*a^(3/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)+30*a^(3/2)*d^3*(a
-a*sin(f*x+e))^(1/2)-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c^3-27*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^2*d+63*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c*d^2-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*d^3)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(178) = 356$.
time = 0.38, size = 520, normalized size = 2.71

$$-\frac{3\sqrt{2}\left(2c^3+18c^2d-42cd^2+22d^3-\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx+e\right)^2+\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx+e\right)+\left(2c^3+18c^2d-42cd^2+22d^3+\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos(fx+e)-\sin(fx+e)+1\right)+3a\cos(fx+e)-\left(a\cos(fx+e)-2a\right)\sin(fx+e)+2a}{\cos(fx+e)^2-\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(4d^3\cos\left(fx+e\right)^3-3c^3+9c^2d-9cd^2+3d^3-4\left(9cd^2-4d^3\right)\cos\left(fx+e\right)^2-3\left(c^3-3c^2d+15cd^2-5d^3\right)\cos\left(fx+e\right)-\left(4d^3\cos\left(fx+e\right)^2-3c^3+9c^2d-9cd^2+3d^3+12\left(3cd^2-d^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{24\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 - (c^3 + 9*c^2*d - 2
1*c*d^2 + 11*d^3)*cos(f*x + e)^2 + (c^3 + 9*c^2*d - 21*c*d^2 + 11*d^3)*cos(
```

```
f*x + e) + (2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 + (c^3 + 9*c^2*d - 21*c*d^
2 + 11*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1)
+ 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*d^3*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - 4*(9*c*d^2 - 4*d^3)*cos(f*
x + e)^2 - 3*(c^3 - 3*c^2*d + 15*c*d^2 - 5*d^3)*cos(f*x + e) - (4*d^3*cos(f
*x + e)^2 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 + 12*(3*c*d^2 - d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin(e + fx))^3}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**3/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(178) = 356.
time = 0.57, size = 376, normalized size = 1.96

$$\frac{\frac{3\sqrt{2}\left(\sqrt{a}\,c^3+9\sqrt{a}\,c^2d-21\sqrt{a}\,cd^2+11\sqrt{a}\,d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(\sqrt{a}\,c^3+9\sqrt{a}\,c^2d-21\sqrt{a}\,cd^2+11\sqrt{a}\,d^3\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{6\sqrt{2}\left(\sqrt{a}\,c^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\sqrt{a}\,c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+3\sqrt{a}\,cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a}\,d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{16\sqrt{2}\left(2a^{\frac{9}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-9a^{\frac{9}{2}}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+3a^{\frac{9}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*(sqrt(a)*c^3 + 9*sqrt(a)*c^2*d - 21*sqrt(a)*c*d^2 + 11*sqrt
(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) - 3*sqrt(2)*(sqrt(a)*c^3 + 9*sqrt(a)*c^2*d - 21*sqrt(a)*c*d
^2 + 11*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)*(sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) - 3*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*sqrt(a)*c*d^
2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))) - 16*sqrt(2)*(2*a^(9/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 9*
a^(9/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*a^(9/2)*d^3*sin(-1/4*pi +
1/2*f*x + 1/2*e))/(a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,\sin(e + f\,x))^3}{(a + a\,\sin(e + f\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.551 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*(
c+7*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/
2)/f*2^(1/2)+1/2*(c-5*d)*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2839, 2830, 2728, 212}

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-5d)\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c - d)*(c + 7*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + ((c - 5*d)*d*Cos[e + f*x])/(2*a*f*Sq
rt[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*f*
(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2839

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m*((c + d*Sin[e + f*x])/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a\left(c^2+5cd-2d^2\right)+\frac{1}{2}a(c-5d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} + \frac{((c-d)}{}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((c-d)}{}$$

$$= -\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{(c-5d)d\cos(e+f}{2af\sqrt{a+a\sin(e+}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.23, size = 239, normalized size = 1.73

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}\left(c^2+6cd-7d^2\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-4d^2\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+4d^2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)}{2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c^2 + 6*c
*d - 7*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*d^2*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 + 4*d^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(119) = 238$.
time = 3.51, size = 316, normalized size = 2.29

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a\,c^2+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d
^2+8*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a*d^2+2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c^2-4*(a-a*sin(f*x+e))^(
1/2)*a^(1/2)*c*d+10*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)*(-a*(sin(f*x+e)-1))
^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(126) = 252$.
time = 0.35, size = 404, normalized size = 2.93

$$-\frac{\sqrt{2}\left(\left(c^2+6cd-7d^2\right)\cos(fx+e)^2-2c^2-12cd+14d^2-\left(c^2+6cd-7d^2\right)\cos(fx+e)-\left(2c^2+12cd-14d^2+\left(c^2+6cd-7d^2\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(4d^2\cos(fx+e)^2+c^2-2cd+d^2+\left(c^2-2cd+5d^2\right)\cos(fx+e)+\left(4d^2\cos(fx+e)-c^2+2cd-d^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{8\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2a^2f-\left(a^2f\cos(fx+e)+2a^2f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((c^2 + 6*c*d - 7*d^2)*cos(f*x + e)^2 - 2*c^2 - 12*c*d + 14*d
^2 - (c^2 + 6*c*d - 7*d^2)*cos(f*x + e) - (2*c^2 + 12*c*d - 14*d^2 + (c^2 +
 6*c*d - 7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2
```

```
+ 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*
x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(4*d^2*
cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 5*d^2)*cos(f*x + e) + (
4*d^2*cos(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
+ e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^2}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(126) = 252.
time = 0.53, size = 277, normalized size = 2.01

$$\frac{\frac{16\sqrt{2}\,d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\sqrt{a}\,c^2+6\sqrt{a}\,cd-7\sqrt{a}\,d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\sqrt{a}\,c^2+6\sqrt{a}\,cd-7\sqrt{a}\,d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sqrt{2}\left(\sqrt{a}\,c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-2\sqrt{a}\,cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+\sqrt{a}\,d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*sqrt(2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(a^(3/2)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))) + sqrt(2)*(sqrt(a)*c^2 + 6*sqrt(a)*c*d - 7*sqrt(a)*d^
2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))) - sqrt(2)*(sqrt(a)*c^2 + 6*sqrt(a)*c*d - 7*sqrt(a)*d^2)*log(-sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) -
 2*sqrt(2)*(sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*sqrt(a)*c*d*sin(
-1/4*pi + 1/2*f*x + 1/2*e) + sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin(e+f\,x))^2}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.552 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c+3*d)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

**Rubi** [A]

time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2829, 2728, 212}

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c + 3*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f
*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(c+3d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(c+3d)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+f}}\right.}{2af}$$

$$= -\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} - \frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.15, size = 150, normalized size = 1.72

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(c+3d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)}{2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] + (-c +
d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c + 3*d)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(72) = 144.
time = 3.08, size = 176, normalized size = 2.02

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a(c+3d)+\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a\right.}{4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)*a*(c+3*d)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
```

```
/2))*a*c+3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)*(-a*
(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(76) = 152$.
time = 0.34, size = 315, normalized size = 3.62

$$\frac{\sqrt{2}\left((c+3d)\cos(fx+e)^2-(c+3d)\cos(fx+e)-((c+3d)\cos(fx+e)+2c+6d)\sin(fx+e)-2c-6d\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left((c-d)\cos(fx+e)-(c-d)\sin(fx+e)+c-d\right)\sqrt{a\sin(fx+e)+a}}{8\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2a^2f-(a^2f\cos(fx+e)+2a^2f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((c + 3*d)*cos(f*x + e)^2 - (c + 3*d)*cos(f*x + e) - ((c + 3*d
)*cos(f*x + e) + 2*c + 6*d)*sin(f*x + e) - 2*c - 6*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+d\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(76) = 152$.
time = 0.55, size = 192, normalized size = 2.21

$$\frac{\frac{\sqrt{2}\left(\sqrt{a}c+3\sqrt{a}d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\sqrt{a}c+3\sqrt{a}d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sqrt{2}\left(\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(sqrt(a)*c + 3*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(sqrt(a)*c + 3*sqrt
(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))) - 2*sqrt(2)*(sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - sqrt
(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.553 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sq
rt[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.13, size = 108, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)+(1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)(1+\sin(e+fx))\right)}{2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(1 + Sin[e + f*x])))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [A]
time = 2.40, size = 125, normalized size = 1.62

| method | result |
|---|---|
| default | $-\frac{\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-}\right.\right.}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-3/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(66) = 132$.
time = 0.35, size = 274, normalized size = 3.56

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos(fx+e)-\sin(fx+e)+1\right)+3a\cos(fx+e)-\left(a\cos(fx+e)-2a\right)\sin(fx+e)+2a}{\cos(fx+e)^2-\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{8\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)
*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
+ e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sin\left(e+fx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(-3/2), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(66) = 132$.
time = 0.54, size = 140, normalized size = 1.82

$$\frac{\sqrt{2}\left(\frac{\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{8\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))) - log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/
```

```
4*pi + 1/2*f*x + 1/2*e))) - 2*sin(-1/4*pi + 1/2*f*x + 1/2*e)/((sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/(sqrt(a)*
f)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a\,\sin(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(1/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.554 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=164

$$-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^2f}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}\,f}-\frac{c}{2(c-d)f(}$$

```
[Out] -1/2*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-5*d)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^2/f*2^(1/2)-2*
d^(3/2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

**Rubi** [A]

time = 0.30, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2845, 3064, 2728, 212, 2852, 214}

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] -1/2*((c - 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^2*f) - (2*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d
]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^2
*Sqrt[c + d]*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(c-4d)-\frac{1}{2}ad\,s}{\sqrt{a+a\sin(e+fx)}}}{2a^2(c-}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-5d)\int \frac{1}{\sqrt{a+a\sin}}}{4a(c-d)^2}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-5d)\text{Subst}\left(\int \frac{1}{2a-}\right.}{2}$$

$$= -\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^2 f} - \frac{2d^{3/2}\tan}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.29, size = 385, normalized size = 2.35

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(c-5d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-\frac{d^{3/2}\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\sqrt{c+d}}+\frac{d^{3/2}\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\sqrt{c+d}}\right)}{2(c-d)^2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c - 5*d)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - (d^(3/2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Se
c[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e +
 f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (d^(3/2)
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(135) = 270$.
time = 4.43, size = 338, normalized size = 2.06

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\left(8d^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+d^2a}}\right)a^{\frac{3}{2}}+\sqrt{a(c+d)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e}}{2\sqrt{a}}\right.\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+2*(a*(c+d)*d)^(1/2)*(a-a*sin(f
*x+e))^(1/2)*a^(1/2)*c-2*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d
)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

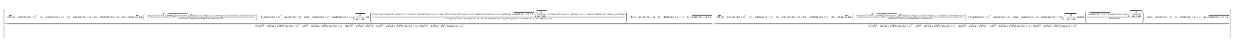
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(141) = 282$.
time = 0.55, size = 1371, normalized size = 8.36



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((c - 5*d)*cos(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5
*d)*cos(f*x + e) + 2*c - 10*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*c
os(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
+ 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) - 4*(a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) +
 2*a*d)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
+ 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2
 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
+ 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*
sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^
2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
+ d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*
d - d^2)*sin(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) +
c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a
^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(sqrt(2)*((c - 5*d)*c
os(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5*d)*cos(f*x + e) + 2*c - 10
*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos
(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (c
os(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*d*cos(f*x + e)^2
- a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(
-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c -
 d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d +
 a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)
 - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f
*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(141) = 282.
time = 0.56, size = 419, normalized size = 2.55

$$-\frac{\frac{8\sqrt{a}\,d^{2}\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^{2}}}\right)}{\left(a^{2}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2a^{2}cd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+a^{2}d^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^{2}}}-\frac{\left(\sqrt{a}\,c-5\sqrt{a}\,d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^{2}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^{2}cd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^{2}d^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\left(\sqrt{a}\,c-5\sqrt{a}\,d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^{2}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^{2}cd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^{2}d^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\sqrt{2}\,a^{2}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*(8*sqrt(a)*d^2*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c
*d - d^2))/((a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c*d*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*sqrt(-c*d - d^2)) - (sqrt(a)*c - 5*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)/(sqrt(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(
2)*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))) + (sqrt(a)*c - 5*sqrt(a)*d)*log(-sin(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 2*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*
e)/((sqrt(2)*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)))/f
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))), x)
```

# 3.555 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=243

$$-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f}-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f}-\frac{}{2(c}$$

```
[Out] -d^(3/2)*(5*c+3*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(
f*x+e))^(1/2))/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f-1/2*cos(f*x+e)/(c-d)/f/(a+a*si
n(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*(c-9*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-1/2*d*(c+3*d)*co
s(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.50, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2845, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3}-\frac{d(c+3d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -1/2*((c - 9*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^3*f) - (d^(3/2)*(5*c + 3*d)*ArcTanh[(Sqrt
[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)
*(c - d)^3*(c + d)^(3/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x]
)^(3/2)*(c + d*Sin[e + f*x])) - (d*(c + 3*d)*Cos[e + f*x])/(2*a*(c - d)^2*(
c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \cdots$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\cdots}{2c\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\cdots}{2c\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\cdots}{2c\cdots}$$

$$= -\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^3 f} - \frac{d^{3/2}(5c\cdots}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 3.19, size = 491, normalized size = 2.02

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{4\sin\left(\frac{1}{2}(e+fx)\right)}{(c-d)^2}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)^2}+\frac{(2+2i)(-1)^{3/4}(c-9d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^3}+\frac{d^{3/2}(5c+3d)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(-c+d)^3(c+d)^{3/2}}+\frac{d^{3/2}(5c+3d)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^3(c+d)^{3/2}}-\frac{4d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^2(c+d)(c+d\sin(e+fx))}\right)}{4f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((4*Sin[(e + f*x)/2])/(c - d)^2 - (2
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d)^2 + ((2 + 2*I)*(-1)^(3/4)*(
c - 9*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^3 + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]
*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((-c + d)^3*(c + d)^(3/2)) + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((c - d)^3*(c + d)^(3/2)) - (4*d^2*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c - d)^2*(c + d)*(c
+ d*Sin[e + f*x]))))/(4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. 2(210) = 420.
time = 6.06, size = 978, normalized size = 4.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 978 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*((a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2-9*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)^2*a*d^3+20*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(3/2)*sin(f*x+e)^2*c*d^3+12*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3-7*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)*a*c^2*d-17*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-9*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3+20*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^2*d
^2+32*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*
x+e)*c*d^3+12*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2
)*sin(f*x+e)*d^4+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(
f*x+e)*c^2*d+4*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+
e)*c*d^2-6*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d
^3+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*a*c^3-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2+20*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+12*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(1/2)*c^3+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(
1/2)*c*d^2-4*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^3)*(-a*(
sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/(c+d)/(c-d)^3/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

[Out] Timed out

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(220) = 440$.
time = 0.82, size = 2603, normalized size = 10.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2*d
+ 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2 -
(c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34*c*
d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d -
17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^
2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e)
 + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(10*a
*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (5*a
*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*
a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a
*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c
*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c
*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c
+ a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 -
2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x +
e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f
*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin
(f*x + e))) + 4*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(
f*x + e)^2 + (c^3 + c*d^2 - 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^
3 - (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e
)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f
*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^
2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2
+ 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^
2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*
c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e
)), 1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2
*d + 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2
```

```
 - (c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34
*c*d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d
 - 17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x +
e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
 e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(c
os(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(1
0*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (
5*a*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 +
 3*a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 +
3*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*s
in(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*si
n(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*(c^3 - c^2
*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e)^2 + (c^3 + c*d^2
- 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 2*c*d^2 - 3*d
^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a
^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3
*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f
*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^
4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos
(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*
d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2), x)
```

## 3.556 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=318

$$-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^4f}-\frac{d^{3/2}(35c^2+42cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{3/2}(c-d)^4(c+d)^{5/2}f}$$

```
[Out] -1/4*d^(3/2)*(35*c^2+42*c*d+19*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d
)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f-1/2*cos(f*x+e
)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4*(c-13*d)*arctanh(1/
2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1
/2)-1/2*d*(c+2*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(
f*x+e))^(1/2)-1/4*d*(2*c+d)*(c+7*d)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin
(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.77, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2845, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{d^{3/2}(35c^2+42cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^4}-\frac{d(2c+d)(c+7d)\cos(e+fx)}{4af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{d(c+2d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -1/2*((c - 13*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^4*f) - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^
2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(
a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(c + 2*d)*Cos[e + f*
x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^
2) - (d*(2*c + d)*(c + 7*d)*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a
 + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^4 f} - \frac{d^{3/2}(}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 4.49, size = 570, normalized size = 1.79

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(16(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-8(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(8+8i)(-1)^{3/4}(c-13d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-\frac{d^{3/2}(35c^2+42cd+19d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c+d)^{5/2}}+\frac{d^{3/2}(35c^2+42cd+19d^2)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c+d)^{5/2}}-\frac{8(c-d)^2d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c+d)(c+d\sin(e+fx))^2}-\frac{4(c-d)d^2(11c+5d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c+d)^2(c+d\sin(e+fx))}\right)}{16(c-d)^4f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*(c - d)*Sin[(e + f*x)/2] - 8*(c
- d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (8 + 8*I)*(-1)^(3/4)*(c - 13*d
)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2)/(c + d)^(5/2) + (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/(c + d)^(5/2) - (8*(c - d)^2*d^2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*Sin[e + f
*x])^2) - (4*(c - d)*d^2*(11*c + 5*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
```

```
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)^2*(c + d*Sin[e + f*x])))
)/(16*(c - d)^4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2221 vs. $2(279) = 558$.
time = 7.87, size = 2222, normalized size = 6.99

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2222 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(7/2)*(-a*(sin(f*x+e)-1))^(1/2)*(-2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*a^(3/2)*c^5+3*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2
)*d^5-19*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin
(f*x+e)^3*d^6-19*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(
5/2)*sin(f*x+e)^2*d^6-5*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)
*d^5-35*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*
d^2-42*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d
^3-19*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^
4+11*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3+51*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4+9*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)*a^2*c^4*d+47*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2+63*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a
^2*c^2*d^3+26*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+25*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4-2*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^2*a^2*c^4*d+21*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2+61*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin
(f*x+e)^2*a^2*c^2*d^3-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^2+13*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*
a^2*d^5-2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*
c^2*d^3+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*
c*d^4-4*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^4*
d-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4+13
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*a^2*c^2*d^3+11*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)
```

```
^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(
1/2)*a^(1/2)*c*d^4+11*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c
^2*d^3-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*a^2*c^5-119*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4-80*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^5+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*
(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^5-35*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2-112*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3-103*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^4-38*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c
*d^5+3*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^5-2
*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d-11*(-a*(sin(f*x+
e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2-(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3-5*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*a^(1/2)*sin(f*x+e)*d^5+13*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*c*d^4-35*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/
2)*sin(f*x+e)^3*c^2*d^4-42*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^5-70*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3+17*(-a*(sin(f*x+e)-1))^(1/2)*
(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4-17*(-a*(sin(f*x+e)-1))^(1/2)*(a*
(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d^2+25*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d^2+11*(-a*(sin(
f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3-2*(-a*(sin(f*
x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2+13*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)^3*a^2*d^5-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e
))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1968 vs. 2(292) = 584.
time = 1.33, size = 4237, normalized size = 13.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(2)*(2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4
- 26*d^5 + (c^3*d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c
^4*d - 21*c^3*d^2 - 61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 -
 7*c^4*d - 66*c^3*d^2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 +
(c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)
+ (2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*
d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^
2 - 36*c^2*d^3 - 38*c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^
3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)
*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin
(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*
x + e) - 2)) - (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 +
38*a*d^5 + (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^
3*d^2 + 119*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d
 + 182*a*c^3*d^2 + 292*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 +
 (35*a*c^4*d + 112*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f
*x + e) + (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*
d^5 - (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - 2*(35*a*c^3*d
^2 + 77*a*c^2*d^3 + 61*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 1
12*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e))*sin(f*x
 + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
x + e))) + 4*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (
2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x + e)^3 + (4*c^4*d + 15*c^
3*d^2 - 14*c^2*d^3 - 9*c*d^4 + 4*d^5)*cos(f*x + e)^2 + (2*c^5 + 2*c^4*d + 1
3*c^3*d^2 + 3*c^2*d^3 - 15*c*d^4 - 5*d^5)*cos(f*x + e) - (2*c^5 - 2*c^4*d -
 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^
4 - 7*d^5)*cos(f*x + e)^2 - (4*c^4*d + 17*c^3*d^2 - c^2*d^3 - 17*c*d^4 - 3*
d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^6*d^2 -
2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a
^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7
*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^3 -
(a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6
```

```
*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 +
 (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(
f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2
*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2
*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6
*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*
c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^
4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 +
6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e)), -1/8*(sqrt(2)*(2
*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 + (c^3*d^2
- 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c^4*d - 21*c^3*d^2 -
61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 - 7*c^4*d - 66*c^3*d^
2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*
c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e) + (2*c^5 - 18*c^4*d
- 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*d^2 - 11*c^2*d^3 - 2
5*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^2 - 36*c^2*d^3 - 38*
c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 -
 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e
)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (70*a
*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*d^5 + (35*a*c^2
*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^3*d^2 + 119*a*c^2*d^
3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d + 182*a*c^3*d^2 + 2
92*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 112*a
*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e) + (70*a*c^4*
d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

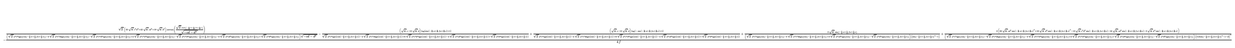
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(292) = 584$.
time = 0.81, size = 1036, normalized size = 3.26



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*(35*sqrt(a)*c^2*d^2 + 42*sqrt(a)*c*d^3 + 19*sqrt(a)*d^4)*arct
an(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^2
*c^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c^5*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*c^4*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 4*sqrt(2)*a^2*c^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2
)*a^2*c^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c*d^5*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^6*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)))*sqrt(-c*d - d^2)) - (sqrt(a)*c - 13*sqrt(a)*d)*log(sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - 4*sqrt(2)*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^
2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))) + (sqrt(a)*c - 13*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c^
3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*d^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sqrt(a)*
sin(-1/4*pi + 1/2*f*x + 1/2*e)/((sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 3*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqr
t(2)*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)) + 2*
(22*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 10*sqrt(a)*d^4*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 13*sqrt(a)*c^2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*
e) - 16*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(a)*d^4*sin(-1
/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a^2*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - sqrt(2)*a^2*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^
2*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^2*c^2*d^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*c*d^4*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) - sqrt(2)*a^2*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin
(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3), x)
```

# 3.557 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=194

$$-\frac{3(c-d)\left(c^2+6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(e+fx)}{\sqrt{2}\ \sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\ a^{5/2}f}-\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}+\frac{(c-9}{4a^2f\sqrt{}}$$

```
[Out] -1/16*(c-d)^2*(3*c+13*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*co
s(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-3/32*(c-d)*(c^2+6*c*d+
25*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5
/2)/f*2^(1/2)+1/4*(c-9*d)*d^2*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2844, 3047, 3098, 2830, 2728, 212}

$$-\frac{3\left(c^2+6cd+25d^2\right)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(e+fx)}{\sqrt{2}\ \sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\ a^{5/2}f}+\frac{d^2(c-9d)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(3c+13d)(c-d)^2\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c - d)*(c^2 + 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)^2*(3*c + 13*d
)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) + ((c - 9*d)*d^2*Cos[e
+ f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^2)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a(3c^2+9cd-4d^2)+\right.}{(a+a\sin(e+fx))^{3/}}}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}ac(3c^2+9cd-4d^2)+\left(\frac{1}{2}ac(c-9d)d-\frac{1}{2}\right.}{(a}}{}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(}{4f(}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(}{4f(}$$

$$= -\frac{3(c-d)\left(c^2+6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{(c-d)^2}{16af(}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.50, size = 400, normalized size = 2.06

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-11c^3\cos\left(\frac{1}{2}(e+fx)\right)+9c^2d\cos\left(\frac{1}{2}(e+fx)\right)+15cd^2\cos\left(\frac{1}{2}(e+fx)\right)-45d^3\cos\left(\frac{1}{2}(e+fx)\right)-3c^3\cos\left(\frac{3}{2}(e+fx)\right)-15c^2d\cos\left(\frac{3}{2}(e+fx)\right)+39cd^2\cos\left(\frac{3}{2}(e+fx)\right)-69d^3\cos\left(\frac{3}{2}(e+fx)\right)+16d^3\cos\left(\frac{5}{2}(e+fx)\right)+11c^3\sin\left(\frac{1}{2}(e+fx)\right)-9c^2d\sin\left(\frac{1}{2}(e+fx)\right)-15cd^2\sin\left(\frac{1}{2}(e+fx)\right)+45d^3\sin\left(\frac{1}{2}(e+fx)\right)+(6+6i)(-1)^{3/4}\left(c^3+5c^2d+19cd^2-25d^3\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-3c^3\sin\left(\frac{3}{2}(e+fx)\right)-15c^2d\sin\left(\frac{3}{2}(e+fx)\right)+39cd^2\sin\left(\frac{3}{2}(e+fx)\right)-69d^3\sin\left(\frac{3}{2}(e+fx)\right)-16d^3\sin\left(\frac{5}{2}(e+fx)\right)\right)}{32f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*c^3*Cos[(e + f*x)/2] + 9*c^2*d*
Cos[(e + f*x)/2] + 15*c*d^2*Cos[(e + f*x)/2] - 45*d^3*Cos[(e + f*x)/2] - 3*
c^3*Cos[(3*(e + f*x))/2] - 15*c^2*d*Cos[(3*(e + f*x))/2] + 39*c*d^2*Cos[(3*
(e + f*x))/2] - 69*d^3*Cos[(3*(e + f*x))/2] + 16*d^3*Cos[(5*(e + f*x))/2] +
 11*c^3*Sin[(e + f*x)/2] - 9*c^2*d*Sin[(e + f*x)/2] - 15*c*d^2*Sin[(e + f*x
)/2] + 45*d^3*Sin[(e + f*x)/2] + (6 + 6*I)*(-1)^(3/4)*(c^3 + 5*c^2*d + 19*c
*d^2 - 25*d^3)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 3*c^3*Sin[(3*(e + f*x))/2] - 15*c^2*d
*Sin[(3*(e + f*x))/2] + 39*c*d^2*Sin[(3*(e + f*x))/2] - 69*d^3*Sin[(3*(e +
f*x))/2] - 16*d^3*Sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(171) = 342$.
time = 4.74, size = 688, normalized size = 3.55

| method | result |
|---|---|
| default | $-\frac{\left(2\sin(fx+e)\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2c^3+15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(2*sin(f*x+e)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c^3+15*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^2*d+57*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c*d^2-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*d^3+64*a^(3/2)*d^3*(a-a*sin(f*x+e))^(1/2))+(-3*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-15*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-57*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3-64*a^(3/2)*d^3*(a-a*sin(f*x+e))^(1/2))*
cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c^3+30*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c^2*d+114*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
*d^2-150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^
3+20*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3+36*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c
^2*d-132*a^(3/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)+204*a^(3/2)*d^3*(a-a*sin(f*x+
e))^(1/2)-6*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-30*(a-a*sin(f*x+e))^(3/2)*a^
(1/2)*c^2*d+78*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2-42*(a-a*sin(f*x+e))^(3/
2)*d^3*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(180) = 360$.
time = 0.36, size = 638, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(3*sqrt(2)*((c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e)^3 - 4*c^
3 - 20*c^2*d - 76*c*d^2 + 100*d^3 + 3*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*c
os(f*x + e)^2 - 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e) - (4*c^3
 + 20*c^2*d + 76*c*d^2 - 100*d^3 - (c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(
f*x + e)^2 + 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e))*sin(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*(32*d^3*cos(f*x + e)^3 - 4*c^3 + 12*c^2*d - 12*
c*d^2 + 4*d^3 - (3*c^3 + 15*c^2*d - 39*c*d^2 + 53*d^3)*cos(f*x + e)^2 - (7*
c^3 + 3*c^2*d - 27*c*d^2 + 81*d^3)*cos(f*x + e) - (32*d^3*cos(f*x + e)^2 -
4*c^3 + 12*c^2*d - 12*c*d^2 + 4*d^3 + (3*c^3 + 15*c^2*d - 39*c*d^2 + 85*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f
*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(180) = 360.
time = 0.62, size = 420, normalized size = 2.16

$$\frac{\frac{128\sqrt{2}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(\sqrt{a}c^3+5\sqrt{a}c^2d+19\sqrt{a}cd^2-25\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(\sqrt{a}c^3+5\sqrt{a}c^2d+19\sqrt{a}cd^2-25\sqrt{a}d^3\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sqrt{2}\left(3\sqrt{a}c^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+15\sqrt{a}c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-39\sqrt{a}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+21\sqrt{a}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{a}c^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-9\sqrt{a}c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+33\sqrt{a}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-19\sqrt{a}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(128*sqrt(2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(a^(5/2)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(sqrt(a)*c^3 + 5*sqrt(a)*c^2*d + 19*sqr
t(a)*c*d^2 - 25*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(sqrt(a)*c^3 + 5*sqrt(a)*c^
2*d + 19*sqrt(a)*c*d^2 - 25*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)*(3*sqrt(a)*c^3
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 15*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x
```

```
+ 1/2*e)^3 - 39*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 21*sqrt(a)
*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) - 9*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 33*sqrt(a)*c*d
^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 19*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x +
1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.558 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=147

$$-\frac{(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}-\frac{(c-d)\cos(e+}{4f(a+a}$$

```
[Out] -3/16*(c-d)*(c+3*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*cos(f*x
+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^(5/2)-1/32*(3*c^2+10*c*d+19*d^2)*ar
ctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2839, 2829, 2728, 212}

$$-\frac{(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/16*((3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) - (3*(c - d)*(c + 3*d)*Cos[e
+ f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*
Sin[e + f*x]))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
```

```
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2839

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m*((c + d*Sin[e + f*x])/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c^2+7cd-2d^2)-\frac{1}{2}ad(c+7d)\sin(e+fx}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3}{}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3}{}$$

$$= -\frac{(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{3(c-d)(c+3d}{16af(a+a\sin}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.35, size = 252, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-4(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(3c^2+10cd-13d^2)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(c-d)(3c+13d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(1+i)(-1)^{3/4}(3c^2+10cd+19d^2)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c^2 + 10*c*d - 13*d^2)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(3*c +
13*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 +
 10*c*d + 19*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(
```

```
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(128) = 256$.
time = 4.72, size = 378, normalized size = 2.57

| method | result |
|---|---|
| default | $\frac{\left(-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2(3c^2+10cd+19d^2)+\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(9/2)*(-2*sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*2^(1/2)*a^2*(3*c^2+10*c*d+19*d^2)+arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*2^(1/2)*a^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2-6*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-20*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-38*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-20*(a-a*sin(f*x+e))^(1/2)
*a^(3/2)*c^2-24*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d+44*(a-a*sin(f*x+e))^(1/2
)*a^(3/2)*d^2+6*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+20*(a-a*sin(f*x+e))^(3/2
)*a^(1/2)*c*d-26*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2)*(-a*(sin(f*x+e)-1))^(1
/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(135) = 270$.
time = 0.35, size = 520, normalized size = 3.54

$$\frac{\sqrt{2}\left(\left(3c^2+10cd+19d^2\right)\cos(fx+e)^3+3\left(3c^2+10cd+19d^2\right)\cos(fx+e)^2-12c^2-40cd-76d^2-2\left(3c^2+10cd+19d^2\right)\cos(fx+e)+\left(\left(3c^2+10cd+19d^2\right)\cos(fx+e)^2-12c^2-40cd-76d^2-2\left(3c^2+10cd+19d^2\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}\left(\cos(fx+e)-\sin(fx+e)+1\right)+3a\cos(fx+e)-\left(a\cos(fx+e)-2a\right)\sin(fx+e)+2a}{\cos(fx+e)^2-\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(\left(3c^2+10cd-13d^2\right)\cos(fx+e)^2+4c^2-8cd+4d^2+\left(7c^2+2cd-9d^2\right)\cos(fx+e)-\left(4c^2-8cd+4d^2-\left(3c^2+10cd-13d^2\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 + 10*c*d
 + 19*d^2)*cos(f*x + e)^2 - 12*c^2 - 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d +
19*d^2)*cos(f*x + e) + ((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 -
 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) + 4*((3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e)^2 + 4*c^2 -
 8*c*d + 4*d^2 + (7*c^2 + 2*c*d - 9*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*
d^2 - (3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
 + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*s
in(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^2}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(135) = 270.
time = 0.53, size = 309, normalized size = 2.10

$$\frac{\frac{\sqrt{2}\left(3\sqrt{a}\,c^2+10\sqrt{a}\,cd+19\sqrt{a}\,d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3\sqrt{a}\,c^2+10\sqrt{a}\,cd+19\sqrt{a}\,d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sqrt{2}\left(3\sqrt{a}\,c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+10\sqrt{a}\,cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-13\sqrt{a}\,d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{a}\,c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-6\sqrt{a}\,cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+11\sqrt{a}\,d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*sqrt(a)*c^2 + 10*sqrt(a)*c*d + 19*sqrt(a)*d^2)*log(sin(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sq
rt(2)*(3*sqrt(a)*c^2 + 10*sqrt(a)*c*d + 19*sqrt(a)*d^2)*log(-sin(-1/4*pi +
1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)
*(3*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 10*sqrt(a)*c*d*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^3 - 13*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 -
 5*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 6*sqrt(a)*c*d*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 11*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1
/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))
/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.559 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c+5*d)*cos(f*x+e)/a/
f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*c+5*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

**Rubi** [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2829, 2729, 2728, 212}

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(3c+5d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/16*((3*c + 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[2]*a^(5/2)*f) - ((c - d)*Cos[e + f*x])/(4*f*(a + a*Sin[e
+ f*x])^(5/2)) - ((3*c + 5*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3c+5d)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3c+5d)\int \frac{}{\sqrt{a}}}{3}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3c+5d)\mathrm{Subst}\Big(}{}$$

$$= -\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.25, size = 227, normalized size = 1.80

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+4(-c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(3c+5d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(3c+5d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(1+i)(-1)^{3/4}(3c+5d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)*Sin[(e + f*x)/2] + 4*(-c
+ d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c + 5*d)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*c + 5*d)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c + 5*d)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(107) = 214$.
time = 3.87, size = 278, normalized size = 2.21

| method | result |
|---|---|
| default | $\frac{\left(-2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3c+5d)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(11/2)*(-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^3*(3*c+5*d)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^3*(3*c+5*d)*cos(f*x+e)^2-20*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c
-12*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*d+6*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c+10*
(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d-6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^3*c-10*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^3*d)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(113) = 226$.
time = 0.37, size = 420, normalized size = 3.33

$$\frac{\sqrt{2}\left((3c+5d)\cos(fx+e)^3+3(3c+5d)\cos(fx+e)^2-2(3c+5d)\cos(fx+e)+\left((3c+5d)\cos(fx+e)^2-2(3c+5d)\cos(fx+e)-12c-20d\right)\sin(fx+e)-12c-20d\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left((3c+5d)\cos(fx+e)^2+(7c+d)\cos(fx+e)+\left((3c+5d)\cos(fx+e)-4c+4d\right)\sin(fx+e)+4c-4d\right)\sqrt{a\sin(fx+e)+a}}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c + 5*d)*cos(f*x + e)^3 + 3*(3*c + 5*d)*cos(f*x + e)^2 -
2*(3*c + 5*d)*cos(f*x + e) + ((3*c + 5*d)*cos(f*x + e)^2 - 2*(3*c + 5*d)*co
s(f*x + e) - 12*c - 20*d)*sin(f*x + e) - 12*c - 20*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*c + 5*d)*cos(f*x + e)^2 + (7*c + d)*cos(f*x + e) + ((3*c + 5*d)*cos(
```

```
f*x + e) - 4*c + 4*d)*sin(f*x + e) + 4*c - 4*d)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d\sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(113) = 226.
time = 0.61, size = 246, normalized size = 1.95

$$\frac{\frac{\sqrt{2}\left(3\sqrt{a}\,c+5\sqrt{a}\,d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3\sqrt{a}\,c+5\sqrt{a}\,d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(3\sqrt{2}\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+5\sqrt{2}\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{2}\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\sqrt{2}\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*sqrt(a)*c + 5*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*sqrt(a)*c + 5
*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^3 + 5*sqrt(2)*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*sqrt(
a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*sqrt(a)*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + d\,\sin(e + f\,x)}{(a + a\,\sin(e + f\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.560 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-3/16*cos(f*x+e)/a/f/(a+a*sin(f*x+e
))^(3/2)-3/32*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/a^(5/2)/f*2^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{3\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16
*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - (3*Co
s[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(\ldots}}}{32a^2 \ldots}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{2a-x^2} \ldots\right.}{\ldots}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\ldots}{16af \ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.12, size = 196, normalized size = 1.83

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8\sin\left(\frac{1}{2}(e+fx)\right)-4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+6\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(3+3i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*Sin[(e + f*x)/2] - 4*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]) + 6*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.
time = 3.87, size = 195, normalized size = 1.82

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\left(6\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)-3\sqrt{2}\,\operatorname{arctanh}\left(\ldots\right.\right.}{32a \ldots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^(9/2)*(sin(f*x+e)*(6*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+14*(a-a*sin(f*x+e))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))
^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-5/2), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(94) = 188.
time = 0.36, size = 348, normalized size = 3.25

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)-4\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)-4\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos(fx+e)-\sin(fx+e)+1\right)+3a\cos(fx+e)-\left(a\cos(fx+e)-2a\right)\sin(fx+e)+2a}{\cos(fx+e)^{2}-\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(3\cos\left(fx+e\right)^{2}+\left(3\cos\left(fx+e\right)-4\right)\sin\left(fx+e\right)+7\cos\left(fx+e\right)+4\right)\sqrt{a\sin\left(fx+e\right)+a}}{64\left(a^{3}f\cos\left(fx+e\right)^{3}+3a^{3}f\cos\left(fx+e\right)^{2}-2a^{3}f\cos\left(fx+e\right)-4a^{3}f+\left(a^{3}f\cos\left(fx+e\right)^{2}-2a^{3}f\cos\left(fx+e\right)-4a^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*co
s(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)*sqrt(a)*log(-(a*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x
 + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*
(3*cos(f*x + e)^2 + (3*cos(f*x + e) - 4)*sin(f*x + e) + 7*cos(f*x + e) + 4)
*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(e+fx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(-5/2), x)
```

**Giac [A]**
time = 0.54, size = 161, normalized size = 1.50

$$\frac{\sqrt{2}\left(\frac{3\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-5\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{2}a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{64\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(3*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))) - 3*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 5*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)
^2*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/(sqrt(a)*f)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(1/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.561 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=218

$$-\frac{(3c^2-14cd+43d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^3f}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^3\sqrt{c+d}\,f}-$$

```
[Out] -1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c-11*d)*cos(f*x+e)/a
/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*c^2-14*c*d+43*d^2)*arctanh(1/2*co
s(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)+
2*d^(5/2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(
1/2))/a^(5/2)/(c-d)^3/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.50, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2845, 3057, 3064, 2728, 212, 2852, 214}

$$-\frac{(3c^2-14cd+43d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^3}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3c-11d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] -1/16*((3*c^2 - 14*c*d + 43*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*(c - d)^3*f) + (2*d^(5/2)*ArcTan
h[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(
a^(5/2)*(c - d)^3*Sqrt[c + d]*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e +
 f*x])^(5/2)) - ((3*c - 11*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e
+ f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-8d)-\frac{3}{2}ad\sin(}{(a+a\sin(e+fx))^{3/2}(c+d\sin}}{4a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e}{16a(c-d)^2f(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e}{16a(c-d)^2f(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e}{16a(c-d)^2f(a+a\sin}$$

$$= -\frac{(3c^2-14cd+43d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 2.26, size = 501, normalized size = 2.30

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{8\sin\left(\frac{1}{2}(e+fx)\right)}{c-d}-\frac{4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{c-d}+\frac{2(3c-11d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^2}+\frac{(-3c+11d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c-d)^2}+\frac{(1+i)(-1)^{3/4}(3c^2-14cd+43d^2)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^3}+\frac{8d^{5/2}\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^3\sqrt{c+d}}+\frac{8d^{5/2}\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(-c+d)^3\sqrt{c+d}}\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8*Sin[(e + f*x)/2])/(c - d) - (4*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d) + (2*(3*c - 11*d)*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^2 + ((-3*c + 11*d)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(c - d)^2 + ((1 + I)*(-1)^(3/4)*(3*
c^2 - 14*c*d + 43*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d)^3 + (8*d^(5/2)*(e + f*
x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqr
t[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4)/((c - d)^3*Sqrt[c + d]) + (8*d^(5/2)*(e + f*x - 2*Log[Sec
[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e +
 f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4)/((-c + d)^3*Sqrt[c + d])))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(185) = 370$.
time = 5.59, size = 732, normalized size = 3.36

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\left(-128d^3 \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+d^2a}}\right)a^{\frac{5}{2}}+6\sqrt{a(c+d)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx}{2\sqrt{a}}\right.\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(sin(f*x+e)*(-128*d^3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a^(5/2)+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*c^2-28*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(64*d^3*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)-3*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+14*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c*d-43*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*d^2)*cos(f*x+e)^2-128*d^3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^(5/2)+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+20*(a*(
c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-72*(a*(c+d)*d)^(1/2)*(a-a*
sin(f*x+e))^(1/2)*a^(3/2)*c*d+52*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a
^(3/2)*d^2-6*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+28*(a*(c+
d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-22*(a*(c+d)*d)^(1/2)*(a-a*si
n(f*x+e))^(3/2)*a^(1/2)*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e
))/(a*(c+d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(193) = 386.
time = 0.71, size = 2103, normalized size = 9.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 64*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^

```
3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos
(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a
^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d
 + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

**<u>Sympy</u>** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**<u>Giac</u>** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(193) = 386.
time = 0.67, size = 619, normalized size = 2.84

$$\frac{\frac{64\sqrt{a}d^3\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(a^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3a^3c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3a^3cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-a^3d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}+\frac{\left(3\sqrt{a}c^2-14\sqrt{a}cd+43\sqrt{a}d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}a^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}a^3c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}a^3cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^3d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\left(3\sqrt{a}c^2-14\sqrt{a}cd+43\sqrt{a}d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}a^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}a^3c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}a^3cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^3d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(3\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-11\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+13\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sqrt{2}a^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}a^3cd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}a^3d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*(64*sqrt(a)*d^3*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-
c*d - d^2))/((a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^2*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) +
(3*sqrt(a)*c^2 - 14*sqrt(a)*c*d + 43*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)/(sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqr
t(2)*a^3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))) - (3*sqrt(a)*c^2 - 14*sqrt(a)*c*d + 43*sqrt(a)*d^2)*log(-sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqr
t(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(3*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 - 11*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(a)*c*sin(-1/
4*pi + 1/2*f*x + 1/2*e) + 13*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sq
rt(2)*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3*c*d*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\,\sin(e + f\,x))^{5/2}\,\,(c + d\,\sin(e + f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),x)
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))), x)
```

# 3.562 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=313

$$-\frac{(3c^2-22cd+115d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^4f}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^4(c+d)^{3/2}f}$$

```
[Out] d^(5/2)*(7*c+5*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4*cos(f*x+e)/(c-d)/f/(a+a*sin
(f*x+e))^(5/2)/(c+d*sin(f*x+e))-3/16*(c-5*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*si
n(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/32*(3*c^2-22*c*d+115*d^2)*arctanh(1/2*co
s(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/f*2^(1/2)-
1/16*(c-7*d)*d*(3*c+5*d)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a
+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.75, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{(3c^2-22cd+115d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^4}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(c-7d)(3c+5d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{3(c-5d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -1/16*((3*c^2 - 22*c*d + 115*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*(c - d)^4*f) + (d^(5/2)*(7*c +
5*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
 f*x]])])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(
a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - (3*(c - 5*d)*Cos[e + f*x]
)/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) - ((c
- 7*d)*d*(3*c + 5*d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*S
in[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} -$$

$$= -\frac{(3c^2-22cd+115d^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right.}{16\sqrt{2}\,a^{5/2}(c-d)^4 f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 3.90, size = 570, normalized size = 1.82

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c - 19*d)*(c - d)*Sin[
(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-3*c + 19*d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 - 22*
c*d + 115*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4)/(c + d)^(3/2) - (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*Log[Sec[(e +
 f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)
/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/
(c + d)^(3/2) + (16*(c - d)*d^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c + d)*(c + d*Sin[e + f*x]))))/(16*(c
 - d)^4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1971 vs. $2(276) = 552$.
time = 7.97, size = 1972, normalized size = 6.30

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1972 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^4-13*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^2*a^2*c^3*d+167*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+323*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e
)*a^2*c*d^3+55*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2+301*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*
d^3-35*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d-19*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^3*a^2*c^2*d^2+93*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3+3*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-38*
(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4+38*(-a*(
sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d-224*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-224*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^
3-608*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*
```

```
x+e)^2*c*d^4+32*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x
+e)^2*d^4-448*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2
)*sin(f*x+e)*c^2*d^3-544*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(5/2)*sin(f*x+e)*c*d^4+148*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(3/2)*sin(f*x+e)*d^4-84*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*c^3*d-20*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2+
52*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+6*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-38*(-a*(sin(f*x+e)-1))^(3/2
)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4+230*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^2*a^2*d^4-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*si
n(f*x+e)*c^3*d+38*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f
*x+e)*c^2*d^2+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*c*d^3+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)
*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+115*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4-19*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c^3*d+93*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+115*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3-32*(-a*(sin(f*x+e)-1))^(
1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3-84*(-a*(sin(f*x+e)-1))^(1
/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^3+20*(-a*(sin(f*x+e)-1))^(1/2)
*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d-84*(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^2+115*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^4-2
24*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-1
60*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c*d^4+20*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4+32*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^4-160*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5-320*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5-160*arctanh((-a*(
sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5)*(-a*(sin(
f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/
(c+d)/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(289) = 578$.
time = 1.24, size = 3823, normalized size = 12.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^4
+ 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^4 - 13*c^3*d
 + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e)^3 - (9*c^4 - 42*c^3*d + 1
84*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74
*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (12*c^4 - 64*c^3*d + 296*c^2
*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*co
s(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 + 394*c*d^3 + 345*d^4)*cos(f*
x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) -
 (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e)
 + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 16*(28*a*c^2*d^2 + 48*a*c*d^3 + 2
0*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 - (7*a*c^2*d^2 + 19*a*c*d^3
+ 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*a*c*d^3 + 25*a*d^4)*cos(f*x
 + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e) + (28*a*c^2*d
^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^3 - (7*a*c^
2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3
 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*
x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d
^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x +
e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e)
+ (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))
*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2
*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x
 + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 -
4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4
- 15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d
 - 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^
3 - 4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^2 - (3*
c^4 - 12*c^3*d - 26*c^2*d^2 - 20*c*d^3 + 55*d^4)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2
*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5
*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)
*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d
```

```
^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6
- 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a
^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3
*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2
 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3
+ (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 +
3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^
3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^
6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 +
 a^3*d^6)*f)*sin(f*x + e)), 1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3
 + 115*d^4)*cos(f*x + e)^4 + 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 +
460*d^4 - (3*c^4 - 13*c^3*d + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e
)^3 - (9*c^4 - 42*c^3*d + 184*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2
 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (
12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d
^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 +
394*c*d^3 + 345*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 20
8*c*d^3 + 115*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)
^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e
) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos
(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(28
*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 -
 (7*a*c^2*d^2 + 19*a*c*d^3 + 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*
a*c*d^3 + 25*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)
*cos(f*x + e) + (28*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^
4)*cos(f*x + e)^3 - (7*a*c^2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 +
2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/
(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)
*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*
d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4 -
15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d -
 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) -...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(289) = 578.

time = 1.14, size = 936, normalized size = 2.99

$$\frac{\frac{64\sqrt{a}\,d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\sqrt{2}\,a^3c^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^3c^3d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\sqrt{2}\,a^3cd^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^3d^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}+\frac{32\sqrt{2}\left(7\sqrt{a}\,cd^3+5\sqrt{a}\,d^4\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}\,a^3c^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}\,a^3c^4d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\sqrt{2}\,a^3c^3d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\sqrt{2}\,a^3c^2d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}\,a^3cd^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^3d^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}+\cdots}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/32*(64*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)/((sqrt(2)*a^3*c^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 2*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - s
qrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^2 - c - d)) + 32*sqrt(2)*(7*sqrt(a)*c*d^3 + 5*sqrt(a)*d^4)*arc
tan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^
3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^4*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 2*sqrt(2)*a^3*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*s
qrt(2)*a^3*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^5*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (3*sqrt(a)*c^2 - 22*sq
rt(a)*c*d + 115*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(
2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c^3*d*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (3*sqrt(a)*c^2 - 22*
sqrt(a)*c*d + 115*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sq
rt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c^3*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(3*sqrt(a)*c*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 19*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 - 5*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 21*sqrt(a)*d*sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/((sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
- 3*sqrt(2)*a^3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2), x)
```

# 3.563 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=400

$$-\frac{3(c^2-10cd+73d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^5f}+\frac{3d^{5/2}(21c^2+30cd+13d^2)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+d}\,\sqrt{}}\right.}{4a^{5/2}(c-d)^5(c+d)^{5/2}f}$$

```
[Out] 3/4*d^(5/2)*(21*c^2+30*c*d+13*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)
^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*cos(f*x+e)
/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*c-19*d)*cos(f*x+
e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-3/32*(c^2-10*c*d+7
3*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/
2)/(c-d)^5/f*2^(1/2)-1/16*d*(3*c^2-20*c*d-31*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c
+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-3/16*d*(c+3*d)*(c^2-10*c*d-
7*d^2)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(
1/2)
```

**Rubi** [A]

time = 1.04, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2845, 3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{3(c^2-10cd+73d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^5}+\frac{3d^{5/2}(21c^2+30cd+13d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{5/2}f(c-d)^5(c+d)^{5/2}}-\frac{3d(c+3d)(c^2-10cd-7d^2)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{d(3c^2-20cd-31d^2)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{(3c-19d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}-\frac{\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (-3*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^5*f) + (3*d^(5/2)*(21*c^2
+ 30*c*d + 13*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - Cos[e + f*x
]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2) - ((3*c -
 19*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Si
n[e + f*x])^2) - (d*(3*c^2 - 20*c*d - 31*d^2)*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*d*(c + 3
*d)*(c^2 - 10*c*d - 7*d^2)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
```

```
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} -$$

$$= -\frac{3(c^2-10cd+73d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right.}{16\sqrt{2}\,a^{5/2}(c-d)^5 f}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 7.60, size = 683, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{8\sin\left(\frac{1}{2}(e+fx)\right)}{(c-d)^3}-\frac{4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)^3}+\frac{6(c-9d)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^4}-\frac{3(c-9d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c-d)^4}+\frac{(3+3i)(-1)^{3/4}\left(c^2-10cd+73d^2\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^5}+\frac{3d^{5/2}\left(21c^2+30cd+13d^2\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^5(c+d)^{5/2}}+\frac{3d^{5/2}\left(21c^2+30cd+13d^2\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(-c+d)^5(c+d)^{5/2}}+\frac{8d^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^3(c+d)(c+d\sin(e+fx))^2}+\frac{12d^3(5c+3d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}{(c-d)^4(c+d)^2(c+d\sin(e+fx))}\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8*Sin[(e + f*x)/2])/(c - d)^3 - (4
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d)^3 + (6*(c - 9*d)*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^4 - (3*(c - 9*d)*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(c - d)^4 + ((3 + 3*I)*(-1)^(3/4)*(c
^2 - 10*c*d + 73*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d)^5 + (3*d^(5/2)*(21*c^2
+ 30*c*d + 13*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*
x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c - d)^5*(c + d)^(5/2)) + (3*
d^(5/2)*(21*c^2 + 30*c*d + 13*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*S
in[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((-c + d)^5*(c
+ d)^(5/2)) + (8*d^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4)/((c - d)^3*(c + d)*(c + d*Sin[e + f*x])^2) + (12*d
^3*(5*c + 3*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4)/((c - d)^4*(c + d)^2*(c + d*Sin[e + f*x]))))/(16*f*(a*(1
 + Sin[e + f*x]))^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 3534 vs. $2(357) = 714$.
time = 11.20, size = 3535, normalized size = 8.84

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3535 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(9/2)*(-a*(sin(f*x+e)-1))^(1/2)*(40*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(3/2)*c^3*d^3-60*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a^(3/2)*c^2*d^4-136*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d
^5+2064*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(
f*x+e)^3*c*d^6+504*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*sin(f*x+e)^2*c^4*d^3-48*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*
a^(1/2)*c^5*d+2736*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*sin(f*x+e)^2*c^3*d^4+3696*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^5+1968*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^6-172*(-a*(sin(f*x+e)-1
```

))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^6+126*(-a*(sin(f*x+e)-1))
^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*d^6+144*(-a*(sin(f*x+e)-1))^(
3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^6+504*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*c^2*d^5+720*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*c*d^6+1008*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3
*c^3*d^4+2448*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2
)*sin(f*x+e)^3*c^2*d^5+312*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)^2*d^7+504*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*c^4*d^3+720*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*c^3*d^4+312*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*c^2*d^5-20*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1
/2)*a^(3/2)*c^6-56*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^6+
6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^6+72*(-a*(sin(f*x+e
)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^6+312*arctanh((-a*(sin(f*x+e)-1))^(
1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*d^7+624*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^7+24*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^4*a^2*c^3*d^3-162*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e
)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*c^2*d^4-408*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^
4*a^2*c*d^5-6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^5*d+42*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^4*d^
2-276*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^3-1140*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^4-12
54*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)^3*a^2*c*d^5+12*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^5*d-69*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^2*a^2*c^4*d^2-1032*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^3-2013*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f
*x+e)^2*a^2*c^2*d^4-1284*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^5+42*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a
^2*c^5*d-276*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d^2-1140*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^3
-1254*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^4-438*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^5-3*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^4*a^2*c^4*d^2-60*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1

```
/2)*c^4*d^2+48*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d^3-
66*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^4+48*(-a*(sin(
f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^5-3*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^6+1008*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^
3+2448*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f
*x+e)*c^3*d^4+2064*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*sin(f*x+e)*c^2*d^5+624*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^6-112*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(3/2)*sin(f*x+e)*d^6+96*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(3/2)*c^5*d+136*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^
4*d^2+232*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)...
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2910 vs. $2(373) = 746$.
time = 2.08, size = 6119, normalized size = 15.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/64*(3*sqrt(2)*(4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*d^3 + 1596*c^2*
d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d^4 + 136*c*d^5
+ 73*d^6)*cos(f*x + e)^5 + (2*c^5*d - 13*c^4*d^2 + 84*c^3*d^3 + 434*c^2*d^4
 + 554*c*d^5 + 219*d^6)*cos(f*x + e)^4 - (c^6 - 4*c^5*d + 25*c^4*d^2 + 328*
c^3*d^3 + 779*c^2*d^4 + 700*c*d^5 + 219*d^6)*cos(f*x + e)^3 - (3*c^6 - 14*c
^5*d + 89*c^4*d^2 + 892*c^3*d^3 + 1957*c^2*d^4 + 1682*c*d^5 + 511*d^6)*cos(
f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 28
2*c*d^5 + 73*d^6)*cos(f*x + e) + (4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*
d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d
^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^4 - 2*(c^5*d - 7*c^4*d^2 + 46*c^3*d^3
 + 190*c^2*d^4 + 209*c*d^5 + 73*d^6)*cos(f*x + e)^3 - (c^6 - 2*c^5*d + 11*c
^4*d^2 + 420*c^3*d^3 + 1159*c^2*d^4 + 1118*c*d^5 + 365*d^6)*cos(f*x + e)^2
```

+ 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 282*c*d^5 + 7
3*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 12*(84*a*c^4*d^2 +
 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (21*a*c^2*d^4 + 3
0*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^5 + (42*a*c^3*d^3 + 123*a*c^2*d^4 + 116*
a*c*d^5 + 39*a*d^6)*cos(f*x + e)^4 - (21*a*c^4*d^2 + 114*a*c^3*d^3 + 196*a*
c^2*d^4 + 142*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^3 - (63*a*c^4*d^2 + 300*a*c^
3*d^3 + 486*a*c^2*d^4 + 340*a*c*d^5 + 91*a*d^6)*cos(f*x + e)^2 + 2*(21*a*c^
4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e) +
 (84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (
21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^4 - 2*(21*a*c^3*d^3 + 51
*a*c^2*d^4 + 43*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^3 - (21*a*c^4*d^2 + 156*a*
c^3*d^3 + 298*a*c^2*d^4 + 228*a*c*d^5 + 65*a*d^6)*cos(f*x + e)^2 + 2*(21*a*
c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e)
)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2
 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^
2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(
f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)
*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^
2)*sin(f*x + e))) + 4*(4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4
 - 8*c*d^5 + 4*d^6 - 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^
6)*cos(f*x + e)^4 - (6*c^5*d - 41*c^4*d^2 - 152*c^3*d^3 - 78*c^2*d^4 + 170*
c*d^5 + 95*d^6)*cos(f*x + e)^3 + (3*c^6 - 16*c^5*d - 31*c^4*d^2 - 84*c^3*d^
3 - 23*c^2*d^4 + 100*c*d^5 + 51*d^6)*cos(f*x + e)^2 + (7*c^6 - 18*c^5*d - 7
9*c^4*d^2 - 196*c^3*d^3 - 15*c^2*d^4 + 214*c*d^5 + 87*d^6)*cos(f*x + e) - (
4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 + 3*
(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^3 - 2*(
3*c^5*d - 22*c^4*d^2 - 64*c^3*d^3 + 6*c^2*d^4 + 61*c*d^5 + 16*d^6)*cos(f*x
+ e)^2 - (3*c^6 - 10*c^5*d - 75*c^4*d^2 - 212*c^3*d^3 - 11*c^2*d^4 + 222*c*
d^5 + 83*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c
^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*
c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^
7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3
*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c
^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*
a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*
c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 -
36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^
2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4
- 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*co

```
s(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3
*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*
d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^
3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*
c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a
^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8
*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a
^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2
*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a
^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1453 vs. 2(373) = 746.
time = 1.05, size = 1453, normalized size = 3.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/32*(24*sqrt(2)*(21*sqrt(a)*c^2*d^3 + 30*sqrt(a)*c*d^4 + 13*sqrt(a)*d^5)*a
rctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*
a^3*c^7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^6*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*c^5*d^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 5*sqrt(2)*a^3*c^4*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*s
qrt(2)*a^3*c^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*c^2*d^
5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^6*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
*sqrt(-c*d - d^2)) + 3*(sqrt(a)*c^2 - 10*sqrt(a)*c*d + 73*sqrt(a)*d^2)*log(
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) - 5*sqrt(2)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
0*sqrt(2)*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*sqrt(2)*a^3*
c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*a^3*c*d^4*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))) - 3*(sqrt(a)*c^2 - 10*sqrt(a)*c*d + 73*sqrt(a)*d^2)*log(-sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
```

```
)) - 5*sqrt(2)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*sqrt(2)*a
^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*sqrt(2)*a^3*c^2*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*a^3*c*d^4*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - sqrt(2)*a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*
(12*sqrt(a)*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 - 84*sqrt(a)*c^2*d^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^7 - 444*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^7 - 252*sqrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^7 - 12*sqrt(a)*c
^4*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 52*sqrt(a)*c^3*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^5 + 500*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 1
196*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 568*sqrt(a)*d^5*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^5 + 3*sqrt(a)*c^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
 + 5*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 146*sqrt(a)*c^3*d^2*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 710*sqrt(a)*c^2*d^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^3 - 1057*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 399*sqr
t(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(a)*c^5*sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 9*sqrt(a)*c^4*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 86*sqrt(a)
*c^3*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 290*sqrt(a)*c^2*d^3*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 303*sqrt(a)*c*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 85*s
qrt(a)*d^5*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a^3*c^6*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3*c^5*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - sqrt(2)*a^3*c^4*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*sqrt(2)*
a^3*c^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*c^2*d^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3*c*d^5*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + sqrt(2)*a^3*d^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 - c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 3*d*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + c + d)^2))/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3),x)
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3), x)
```

# 3.564 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=203

$$-\frac{5\sqrt{a}\,(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{8\sqrt{d}\,f}-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+f}}{8f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -5/8*(c+d)^3*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*
sin(f*x+e))^(1/2))*a^(1/2)/f/d^(1/2)-5/12*a*(c+d)*cos(f*x+e)*(c+d*sin(f*x+e
))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/f
/(a+a*sin(f*x+e))^(1/2)-5/8*a*(c+d)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(
a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.26, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2849, 2854, 211}

$$-\frac{5\sqrt{a}\,(c+d)^3\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{8\sqrt{d}\,f}-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a\sin(e+fx)+a}}-\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*Sqrt[a]*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*Sqrt[d]*f) - (5*a*(c + d)^2*Cos[e
 + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*f*Sqrt[a + a*Sin[e + f*x]]) - (5*a*(c
+ d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \frac{1}{6}(5(c+d))\int \sqrt{\quad}$$

$$= -\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e}{3}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}} - \frac{5a(c+}{}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}} - \frac{5a(c+}{}$$

$$= -\frac{5\sqrt{a}\,(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(}}\right.}{8\sqrt{d}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 2.49, size = 391, normalized size = 1.93

$$-\frac{\sqrt{a(1+\sin(e+fx))}\left(\frac{15(c+d)^3\left(\log\left(\frac{e^{-ie}\left(2\sqrt[4]{-1}\,c-2(-1)^{3/4}de^{i(e+fx)}+2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)}{\sqrt{d}}\right)-\log\left(\frac{2e^{\frac{1}{2}i(e-2fx)}\left((-1)^{3/4}d+\sqrt[4]{-1}\,ce^{i(e+fx)}+i\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)f}{\sqrt{d}}\right)\right)\left(i\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{(\cos(e+fx)+i\sin(e+fx))(c+d\sin(e+fx))}}{\sqrt{d}}+2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))\left(33c^2+40cd+19d^2-4d^2\cos(2(e+fx))+2d(13c+5d)\sin(e+fx)\right)\right)}{48f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/48*(Sqrt[a*(1 + Sin[e + f*x])]*((15*(c + d)^3*(Log[(2*(-1)^(1/4)*c - 2*(
-1)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
+ E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*
((-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e +
 f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(I*Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x
])])/Sqrt[d] + 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])
*(33*c^2 + 40*c*d + 19*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(13*c + 5*d)*Sin[
```

```
e + f*x])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x
]])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + a\sin(fx + e)}\,(c + d\sin(fx + e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(183) = 366.
time = 0.74, size = 1311, normalized size = 6.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)
*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e))*sqrt(-a/d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*
c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
+ 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
+ e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*
(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqr
```

```
t(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*co
s(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
- 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*
d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1
)) + 8*(8*d^2*cos(f*x + e)^3 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 1
4*c*d - 13*d^2 - (33*c^2 + 40*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x +
 e)^2 - 33*c^2 - 14*c*d - 13*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) +
 f*sin(f*x + e) + f), 1/96*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^
2*d + 3*c*d^2 + d^3)*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8
*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(8*d^2*cos(f*x + e)^3
 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13*d^2 - (33*c^2 + 4
0*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13
*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.565 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=156

$$-\frac{3\sqrt{a}\,(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}\,f}-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -3/4*(c+d)^2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*
sin(f*x+e))^(1/2))*a^(1/2)/f/d^(1/2)-1/2*a*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2
)/f/(a+a*sin(f*x+e))^(1/2)-3/4*a*(c+d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/
(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2849, 2854, 211}

$$-\frac{3\sqrt{a}\,(c+d)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}\,f}-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[d]*f) - (3*a*(c + d)*Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
```

```
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{1}{4}(3(c+d))\int \sqrt{\ }$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+\ }{2f\sqrt{\ }}$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+\ }{2f\sqrt{\ }}$$

$$= -\frac{3\sqrt{a}\,(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e}}\right.}{4\sqrt{d}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.27, size = 365, normalized size = 2.34

$$\frac{\sqrt{a(1+\sin(e+fx))}\left(-\frac{3i(c+d)^2\left(\log\left(\frac{e^{-ie}\left(2\sqrt[4]{-1}\,c-2(-1)^{3/4}de^{i(e+fx)}+2\sqrt{d}\,\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)}{\sqrt{d}}\right)-\log\left(\frac{2e^{\frac{1}{2}i(e-2fx)}\left((-1)^{3/4}d+\sqrt[4]{-1}\,ce^{i(e+fx)}+i\sqrt{d}\,\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)f}{\sqrt{d}}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{(\cos(e+fx)+i\sin(e+fx))(c+d\sin(e+fx))}}{\sqrt{d}}-2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))(5c+3d+2d\sin(e+fx))\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(((-3*I)*(c + d)^2*(Log[(2*(-1)^(1/4)*c - 2*(-1
)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 +
E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((
-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f
*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*
Sin[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])
])/Sqrt[d] - 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])*(
5*c + 3*d + 2*d*Sin[e + f*x])))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(fx+e)}\,\left(c+d\sin(fx+e)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(139) = 278$.
time = 0.64, size = 1119, normalized size = 7.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) + (c^2 + 2*c
*d + d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 +
4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f
*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a
*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(
f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(
f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*
c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d
- 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(
5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*
c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d
^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2
+ 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
 e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d*cos(f*x + e)^2 + (5*c + 3*
d)*cos(f*x + e) + (2*d*cos(f*x + e) - 5*c - d)*sin(f*x + e) + 5*c + d)*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x
+ e) + f), 1/16*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) +
```

```
(c^2 + 2*c*d + d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^
2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d -
 a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))
) - 4*(2*d*cos(f*x + e)^2 + (5*c + 3*d)*cos(f*x + e) + (2*d*cos(f*x + e) -
5*c - d)*sin(f*x + e) + 5*c + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.566 $\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=105

$$-\frac{\sqrt{a}\,(c+d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -(c+d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*
x+e))^(1/2))*a^(1/2)/f/d^(1/2)-a*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2849, 2854, 211}

$$-\frac{\sqrt{a}\,(c+d)\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[d]*f)) - (a*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
```

```
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{1}{2}(c+d)\int\frac{\sqrt{a+a\cdots}}{\sqrt{c+d\cdots}}$$

$$= -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}-\frac{(a(c+d))\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a}\,(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+\cdots}}\right.}{\sqrt{d}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.90, size = 350, normalized size = 3.33

$$\frac{\sqrt{a(1+\sin(e+fx))}\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{f}-\frac{i(c+d)\left(\log\left(\frac{e^{-ie}\left(2\sqrt[4]{-1}\,c-2(-1)^{3/4}de^{i(e+fx)}+2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)}{\sqrt{d}}\right)-\log\left(\frac{2e^{\frac{1}{2}i(e-2fx)}\left((-1)^{3/4}d+\sqrt[4]{-1}\,ce^{i(e+fx)}+i\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)f}{\sqrt{d}}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{(\cos(e+fx)+i\sin(e+fx))(c+d\sin(e+fx))}}{\sqrt{d}\,f}\right)}{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*((-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/f - (I*(c + d)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I
*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e +
f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((-1)^(3/4)*d +
(-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/
2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sqrt[d]*f)
))/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(fx+e)}\,\sqrt{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(95) = 190.
time = 0.64, size = 991, normalized size = 9.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(((c + d)*cos(f*x + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(-a/d)*log(
(128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a
*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^
3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*
d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*
d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 +
33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x +
e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3
*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(
f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*
a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 -
6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^
3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1))
 - 8*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(
f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f), 1/4*(((c + d)*cos(f*x
 + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
 e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
```

```
e))) - 4*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) -
sin(f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.567 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=61

$$-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

```
[Out] -2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)
)^(1/2))*a^(1/2)/f/d^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2854, 211}

$$-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.79, size = 305, normalized size = 5.00

$$-\frac{i\left(\log\left(\frac{e^{-ie}\left(2\sqrt[4]{-1}\,c-2(-1)^{3/4}de^{i(e+fx)}+2\sqrt{d}\,\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)}{\sqrt{d}}\right)-\log\left(-\frac{(1+i)e^{\frac{1}{2}i(e-2fx)}\left(-\sqrt[4]{-1}\,d+(-1)^{3/4}ce^{i(e+fx)}-\sqrt{d}\,\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)f}{\sqrt{d}}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\sqrt{(\cos(e+fx)+i\sin(e+fx))(c+d\sin(e+fx))}}{\sqrt{d}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((-I)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqr
t[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))]
 - Log[((-1 - I)*E^((I/2)*(e - 2*f*x))*(-((-1)^(1/4)*d) + (-1)^(3/4)*c*E^(I
*(e + f*x)) - Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*
x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin
[e + f*x])]*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sq
rt[d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2706 vs. $2(49) = 98$.
time = 11.34, size = 2707, normalized size = 44.38

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2707 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*(a*(1+sin(
f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(sin(f*x+e)*cos(f*x+e)*(d^2/c^2)^(1/2
)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d^
2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*s
in(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d
)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c
^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d
```

^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*c*d-cos(f*x+e)*(d^2/c^2)^
(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^
2*d^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)
*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/
2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c
^2*d^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*c^2-cos(f*x+e)*(-(d^2
/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d^2+d^4*(d
^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e
)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/
2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*
c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d^2+d^4*(
d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*c*d+sin(f*x+e)*cos(f*x+e)*(-(d^2/
c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d^2+d^4*(d^
2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c
*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*c^2*d^2+d^4*(d
^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*d^2-sin(f*x+e)*arctan(((c+d*sin(f*
x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))
*c^2*d^3-sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4+(d^2/c^2)^(1/2)*arctan(((c+d*
sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^
(1/2))*c^4*d-(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*si
n(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2-(d^2/c^2)^(1/2)*ar
ctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2
)^(1/2)*c)^(1/2))*c^2*d^3+(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^
2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4+cos(f*x
+e)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-
(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-2*cos(f*x+e)^2*arctan(((c+d*sin(f*x+e))/(
(d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4+
sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/
2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2-cos(f*x+e)^2*(d^2/c^2)^(1/2)*arctan(
((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/
2)*c)^(1/2))*c^3*d^2+2*cos(f*x+e)^2*(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2
*d^3-cos(f*x+e)^2*(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)
*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4-sin(f*x+e)*(d^2
/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1
/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^4*d+sin(f*x+e)*(d^2/c^2)^(1/2)*arctan(((c
+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*
c)^(1/2))*c^3*d^2+sin(f*x+e)*(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2
/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-si
n(f*x+e)*(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*
x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4+cos(f*x+e)^2*arctan(((c+

```
d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c
)^(1/2))*d^5+sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x
+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^5-arctan(((c+d*sin(f*x+e))/((
d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2
+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/
c^2)^(1/2)*c)^(1/2))*c^2*d^3+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*si
n(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4-arctan(((c+d*sin(f*x
+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*
d^5)/d^2/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)/(-(d^2/c^2)^(1/2)*c)^(1/2)/(
c^2-2*c*d+d^2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

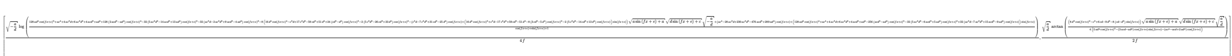
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(52) = 104.
time = 0.60, size = 807, normalized size = 13.23



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2
*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a
*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^
2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d
+ 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c
^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3
- 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c
*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^
3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d
^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*
d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3
- 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a
```

```
*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)
+ sin(f*x + e) + 1))/f, 1/2*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^
2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2
)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)))/f]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e+fx)+1)}}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.568 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=45

$$-\frac{2a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2850}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.14, size = 84, normalized size = 1.87

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}{(c+d)f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]])/((c +
 d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.
time = 0.87, size = 99, normalized size = 2.20

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,\left(\left(\cos^{2}\left(fx+e\right)\right)d+c\sin\left(fx+e\right)+d\sin\left(fx+e\right)-c-d\right)}{f\cos\left(fx+e\right)\left(\left(\cos^{2}\left(fx+e\right)\right)d^{2}+c^{2}-d^{2}\right)\left(c+d\right)}$ | 99 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(cos(f*x+e)^2*d+c*sin(f
*x+e)+d*sin(f*x+e)-c-d)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)/(c+d)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(44) = 88$.
time = 0.56, size = 193, normalized size = 4.29

$$-\frac{2\left(\sqrt{a}\,c-\frac{\sqrt{a}\,(c-2\,d)\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{a}\,(c-2\,d)\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{\sqrt{a}\,c\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)}{\left(c+d+\frac{(c+d)\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}\right)\left(c+\frac{2\,d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a)*c - sqrt(a)*(c - 2*d)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*(c - 2*d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(a)*c*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c + d + (c
 + d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
 e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(3/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(44) = 88$.
time = 0.36, size = 137, normalized size = 3.04

$$\frac{2\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}\,\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{(cd+d^{2})f\cos\left(fx+e\right)^{2}-(c^{2}+cd)f\cos\left(fx+e\right)-(c^{2}+2\,cd+d^{2})f-\left((cd+d^{2})f\cos\left(fx+e\right)+(c^{2}+2\,cd+d^{2})f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(f*x
 + e) + 1)/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^
2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*s
in(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(44) = 88.
time = 0.66, size = 157, normalized size = 3.49

$$\frac{4\sqrt{2}\left(c^2d^2-2\,cd^3+d^4\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)}{\left(c^3d^2-c^2d^3-cd^4+d^5\right)\sqrt{c\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^4+d\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^4+2\,c\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^2-6\,d\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^2+c+d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 4*sqrt(2)*(c^2*d^2 - 2*c*d^3 + d^4)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)/((c^3*d^2 - c^2*d^3 - c*d^4 + d^5)*sqrt
(c*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + d*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 +
2*c*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 6*d*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2
 + c + d)*f)
```

**Mupad [B]**
time = 9.00, size = 145, normalized size = 3.22

$$-\frac{4\left(2\,c\,\cos\left(e+f\,x\right)+d\,\sin\left(2\,e+2\,f\,x\right)\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}{f\,\left(c+d\right)\,\left(4\,c\,d+4\,c^2\,\sin\left(e+f\,x\right)+3\,d^2\,\sin\left(e+f\,x\right)+4\,c^2+2\,d^2-2\,d^2\,\cos\left(2\,e+2\,f\,x\right)-d^2\,\sin\left(3\,e+3\,f\,x\right)+8\,c\,d\,\sin\left(e+f\,x\right)-4\,c\,d\,\cos\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] -(4*(2*c*cos(e + f*x) + d*sin(2*e + 2*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*(c
 + d*sin(e + f*x))^(1/2))/(f*(c + d)*(4*c*d + 4*c^2*sin(e + f*x) + 3*d^2*si
n(e + f*x) + 4*c^2 + 2*d^2 - 2*d^2*cos(2*e + 2*f*x) - d^2*sin(3*e + 3*f*x)
+ 8*c*d*sin(e + f*x) - 4*c*d*cos(2*e + 2*f*x)))
```

# 3.569 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=95

$$-\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}-\frac{4a\cos(e+fx)}{3(c+d)^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-4/3
*a*cos(f*x+e)/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2851, 2850}

$$-\frac{4a\cos(e+fx)}{3f(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(3/2)) - (4*a*Cos[e + f*x])/(3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{a+a\sin(e+ \ldots}}{(c+d\sin(e+fx))^{3/ \ldots}}}{3(c+d)}$$
$$= -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{4c \ldots}{3(c+d)^2f\sqrt{a+a\,\mathrm{si} \ldots}}$$

**Mathematica** [A]
time = 0.20, size = 100, normalized size = 1.05

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(3c+d+2d\sin(e+fx))}{3(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 d + 2*d*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(c + d*Sin[e + f*x])^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(83) = 166.
time = 0.18, size = 222, normalized size = 2.34

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,\left(2\left(\cos^{4}\left(fx+e\right)\right)d^{3}+\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)c\,d^{2}+\sin\left(fx+e\right)\left(\cos^{2}\left(fx+ \ldots\right.\right.\right.}{3f\cos\left(fx+e\right)\left(\left( \ldots\right.\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(2*cos(f*x+e)^4*d^3+s
in(f*x+e)*cos(f*x+e)^2*c*d^2+sin(f*x+e)*cos(f*x+e)^2*d^3+4*cos(f*x+e)^2*c^2
*d+cos(f*x+e)^2*c*d^2-3*cos(f*x+e)^2*d^3+3*c^3*sin(f*x+e)+5*c^2*d*sin(f*x+e
)+c*d^2*sin(f*x+e)-d^3*sin(f*x+e)-3*c^3-5*c^2*d-c*d^2+d^3)/cos(f*x+e)/(cos(
f*x+e)^2*d^2+c^2-d^2)^2/(c+d)^2
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(89) = 178.
time = 0.61, size = 360, normalized size = 3.79

$$-\frac{2\left((3c^2+cd)\sqrt{a} - \frac{(3c^2-9cd-2d^2)\sqrt{a}\sin(fx+e)}{\cos(fx+e)+1} + \frac{2(3c^2-4cd+3d^2)\sqrt{a}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2(3c^2-4cd+3d^2)\sqrt{a}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{(3c^2-9cd-2d^2)\sqrt{a}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{(3c^2+cd)\sqrt{a}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^2}{3\left(c^2+2cd+d^2+\frac{2(c^2+2cd+d^2)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(c^2+2cd+d^2)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)\left(c+\frac{2d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((3*c^2 + c*d)*sqrt(a) - (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - (3*c^2 + c*d)*sqrt(a)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^2 + 2*c*d + d^2 + 2*(c^2 + 2*c*d
+ d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^2 + 2*c*d + d^2)*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(89) = 178$.
time = 0.36, size = 312, normalized size = 3.28

$$\frac{2\left(2\,d\cos\left(fx+e\right)^{2}+\left(3\,c+d\right)\cos\left(fx+e\right)+\left(2\,d\cos\left(fx+e\right)-3\,c+d\right)\sin\left(fx+e\right)+3\,c-d\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{3\left(\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{3}+\left(2\,c^{3}d+5\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{2}-\left(c^{4}+2\,c^{3}d+2\,c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)-\left(c^{4}+4\,c^{3}d+6\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f+\left(\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{2}-2\left(c^{3}d+2\,c^{2}d^{2}+cd^{3}\right)f\cos\left(fx+e\right)-\left(c^{4}+4\,c^{3}d+6\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(2*d*cos(f*x + e)^2 + (3*c + d)*cos(f*x + e) + (2*d*cos(f*x + e) - 3*c
+ d)*sin(f*x + e) + 3*c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*
c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)
*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2
 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*
x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(89) = 178$.
time = 0.86, size = 479, normalized size = 5.04

$$\frac{4\sqrt{2}\left(\left(\frac{3\left(c^{6}d^{4}-2\,c^{5}d^{5}-c^{4}d^{6}+4\,c^{3}d^{7}-c^{2}d^{8}-2\,cd^{9}+d^{10}\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}}{c^{7}d^{4}-c^{6}d^{5}-3\,c^{5}d^{6}+3\,c^{4}d^{7}+3\,c^{3}d^{8}-3\,c^{2}d^{9}-cd^{10}+d^{11}}+\frac{2\left(3\,c^{6}d^{4}-14\,c^{5}d^{5}+21\,c^{4}d^{6}-4\,c^{3}d^{7}-19\,c^{2}d^{8}+18\,cd^{9}-5\,d^{10}\right)}{c^{7}d^{4}-c^{6}d^{5}-3\,c^{5}d^{6}+3\,c^{4}d^{7}+3\,c^{3}d^{8}-3\,c^{2}d^{9}-cd^{10}+d^{11}}\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+\frac{3\left(c^{6}d^{4}-2\,c^{5}d^{5}-c^{4}d^{6}+4\,c^{3}d^{7}-c^{2}d^{8}-2\,cd^{9}+d^{10}\right)}{c^{7}d^{4}-c^{6}d^{5}-3\,c^{5}d^{6}+3\,c^{4}d^{7}+3\,c^{3}d^{8}-3\,c^{2}d^{9}-cd^{10}+d^{11}}\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)}{3\left(c\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}+d\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}+2\,c\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-6\,d\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+c+d\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 4/3*sqrt(2)*((3*(c^6*d^4 - 2*c^5*d^5 - c^4*d^6 + 4*c^3*d^7 - c^2*d^8 - 2*c*
d^9 + d^10)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2/(c^7*d^4 - c^6*d^5 - 3*c^5*d^6
 + 3*c^4*d^7 + 3*c^3*d^8 - 3*c^2*d^9 - c*d^10 + d^11) + 2*(3*c^6*d^4 - 14*c
^5*d^5 + 21*c^4*d^6 - 4*c^3*d^7 - 19*c^2*d^8 + 18*c*d^9 - 5*d^10)/(c^7*d^4
- c^6*d^5 - 3*c^5*d^6 + 3*c^4*d^7 + 3*c^3*d^8 - 3*c^2*d^9 - c*d^10 + d^11))
*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 3*(c^6*d^4 - 2*c^5*d^5 - c^4*d^6 + 4*c^
3*d^7 - c^2*d^8 - 2*c*d^9 + d^10)/(c^7*d^4 - c^6*d^5 - 3*c^5*d^6 + 3*c^4*d^
7 + 3*c^3*d^8 - 3*c^2*d^9 - c*d^10 + d^11))*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)/((c*tan(-1/8*pi + 1/4*f*x + 1/4
*e)^4 + d*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 2*c*tan(-1/8*pi + 1/4*f*x + 1/
4*e)^2 - 6*d*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + c + d)^(3/2)*f)
```

**Mupad [B]**
time = 13.90, size = 353, normalized size = 3.72

$$-\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{3\,d\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}+\frac{8\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,d\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}+\frac{8\,c\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{d^2\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}+\frac{c\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{d^2\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}\right)}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}-\frac{{\left(c+d\right)}^2\,1{}\mathrm{i}}{{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}-\frac{2\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(2\,c^2+2\,c\,d+d^2\right)}{d^2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(4\,c+d\right)}{d}+\frac{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(c+d\right)}^2\,\left(2\,c^2+2\,c\,d+d^2\right)\,2{}\mathrm{i}}{d^2\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}-\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(c+d\right)}^2\,\left(4\,c+d\right)\,1{}\mathrm{i}}{d\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2
)*8i)/(3*d*f*(c*1i + d*1i)^2) + (8*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^
(1/2))/(3*d*f*(c*1i + d*1i)^2) + (8*c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x
))^(1/2))/(d^2*f*(c*1i + d*1i)^2) + (c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*
x))^(1/2)*8i)/(d^2*f*(c*1i + d*1i)^2)))/(exp(e*5i + f*x*5i) - ((c + d)^2*1i
)/(c*1i + d*1i)^2 - (2*exp(e*3i + f*x*3i)*(2*c*d + 2*c^2 + d^2))/d^2 + (exp
(e*1i + f*x*1i)*(4*c + d))/d + (exp(e*2i + f*x*2i)*(c + d)^2*(2*c*d + 2*c^2
 + d^2)*2i)/(d^2*(c*1i + d*1i)^2) - (exp(e*4i + f*x*4i)*(c + d)^2*(4*c + d)
*1i)/(d*(c*1i + d*1i)^2))
```

# 3.570 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=142

$$-\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}-\frac{8a\cos(e+fx)}{15(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}$$

```
[Out] -2/5*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-8/1
5*a*cos(f*x+e)/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-16/1
5*a*cos(f*x+e)/(c+d)^3/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2851, 2850}

$$-\frac{16a\cos(e+fx)}{15f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a\cos(e+fx)}{15f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a\cos(e+fx)}{5f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(5/2)) - (8*a*Cos[e + f*x])/(15*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) - (16*a*Cos[e + f*x])/(15*(c + d)^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} + \frac{4\int \frac{\sqrt{a+a\sin(e+}}{(c+d\sin(e+fx))^{5/}}}{5(c+d)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{8}{15(c+d)^2f\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{8}{15(c+d)^2f\sqrt{a+a\,\mathrm{s}}}$$

**Mathematica** [A]
time = 0.28, size = 128, normalized size = 0.90

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(15c^2+10cd+3d^2+4d(5c+d)\sin(e+fx)+8d^2\sin^2(e+fx))}{15(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(15*c^
2 + 10*c*d + 3*d^2 + 4*d*(5*c + d)*Sin[e + f*x] + 8*d^2*Sin[e + f*x]^2))/(1
5*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5
/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(124) = 248.
time = 0.24, size = 430, normalized size = 3.03

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\,(1+\sin(fx+e))}\,\sqrt{c+d\sin(fx+e)}\,(-15c^5-7d^5-6c^2d^3-11c\,d^4-22c^3d^2-35c^4d+15c^5\sin(fx+e)-23(\cos^4($ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(-15*c^5-7*d^5-6*c^2
*d^3-11*c*d^4-22*c^3*d^2-35*c^4*d+15*c^5*sin(f*x+e)+4*sin(f*x+e)*cos(f*x+e)
^4*c*d^4+7*sin(f*x+e)*cos(f*x+e)^2*c^3*d^2+3*sin(f*x+e)*cos(f*x+e)^2*c^2*d^
3-15*sin(f*x+e)*cos(f*x+e)^2*c*d^4+22*cos(f*x+e)^2*d^5+7*sin(f*x+e)*d^5+8*c
os(f*x+e)^6*d^5-23*cos(f*x+e)^4*d^5+21*cos(f*x+e)^4*c^2*d^3-2*cos(f*x+e)^4*
c*d^4-11*sin(f*x+e)*cos(f*x+e)^2*d^5+25*cos(f*x+e)^2*c^4*d+19*cos(f*x+e)^2*
c^3*d^2-15*cos(f*x+e)^2*c^2*d^3+13*cos(f*x+e)^2*c*d^4+35*sin(f*x+e)*c^4*d+2
```

```
2*sin(f*x+e)*c^3*d^2+6*sin(f*x+e)*c^2*d^3+11*sin(f*x+e)*c*d^4+4*sin(f*x+e)*
cos(f*x+e)^4*d^5)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)^3/(c+d)^3
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(133) = 266$.
time = 0.61, size = 570, normalized size = 4.01

$$-\frac{2\left((15c^3+10c^2d+3cd^2)\sqrt{a}-\frac{(15c^3-60c^2d-25cd^2-6d^3)\sqrt{a}\sin(fx+e)}{\cos(fx+e)+1}+\frac{(45c^3-40c^2d+93cd^2+10d^3)\sqrt{a}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5(9c^3-22c^2d+13cd^2-12d^3)\sqrt{a}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5(9c^3-22c^2d+13cd^2-12d^3)\sqrt{a}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{(45c^3-40c^2d+93cd^2+10d^3)\sqrt{a}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{(15c^3-60c^2d-25cd^2-6d^3)\sqrt{a}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{(15c^3+10c^2d+3cd^2)\sqrt{a}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^3}{15\left(c^3+3c^2d+3cd^2+d^3+\frac{3(c^3+3c^2d+3cd^2+d^3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3(c^3+3c^2d+3cd^2+d^3)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{(c^3+3c^2d+3cd^2+d^3)\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)\left(c+\frac{2d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a) - (15*c^3 - 60*c^2*d - 25*c*d^
2 - 6*d^3)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + (45*c^3 - 40*c^2*d + 9
3*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(9*c^3 -
22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*(9*c^3 - 22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (45*c^3 - 40*c^2*d + 93*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + (15*c^3 - 60*c^2*d - 25*c*d^2 - 6*d^3)*sqrt(a)*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - (15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a)*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)
^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(133) = 266$.
time = 0.38, size = 572, normalized size = 4.03

$$\frac{2\left(8d^2\cos(fx+e)^3-4(5cd-d^2)\cos(fx+e)^2-15c^2+10cd-7d^2-(15c^2+10cd+11d^2)\cos(fx+e)-(8d^2\cos(fx+e)^2-15c^2+10cd-7d^2+4(5cd+d^2)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}}{15\left((c^3d^3+3c^2d^4+3cd^5+d^6)f\cos(fx+e)^4-3(c^4d^2+3c^3d^3+3c^2d^4+cd^5)f\cos(fx+e)^3-(3c^5d+12c^4d^2+20c^3d^3+18c^2d^4+9cd^5+2d^6)f\cos(fx+e)^2+(c^6+3c^5d+6c^4d^2+10c^3d^3+9c^2d^4+3cd^5)f\cos(fx+e)+(c^6+6c^5d+15c^4d^2+20c^3d^3+15c^2d^4+6cd^5+d^6)f-\left((c^3d^3+3c^2d^4+3cd^5+d^6)f\cos(fx+e)^3+(3c^4d^2+10c^3d^3+12c^2d^4+6cd^5+d^6)f\cos(fx+e)^2-(3c^5d+9c^4d^2+10c^3d^3+6c^2d^4+3cd^5+d^6)f\cos(fx+e)-(c^6+6c^5d+15c^4d^2+20c^3d^3+15c^2d^4+6cd^5+d^6)f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(8*d^2*cos(f*x + e)^3 - 4*(5*c*d - d^2)*cos(f*x + e)^2 - 15*c^2 + 10*c
*d - 7*d^2 - (15*c^2 + 10*c*d + 11*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^
2 - 15*c^2 + 10*c*d - 7*d^2 + 4*(5*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 + 3*
c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5)
*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*d
^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9*
```

```
c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^
3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)*
f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f*
cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^4
 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(133) = 266$.
time = 1.27, size = 1060, normalized size = 7.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] 4/15*sqrt(2)*(((5*(3*(c^10*d^6 - 2*c^9*d^7 - 3*c^8*d^8 + 8*c^7*d^9 + 2*c^6*
d^10 - 12*c^5*d^11 + 2*c^4*d^12 + 8*c^3*d^13 - 3*c^2*d^14 - 2*c*d^15 + d^16
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2/(c^11*d^6 - c^10*d^7 - 5*c^9*d^8 + 5*c^8
*d^9 + 10*c^7*d^10 - 10*c^6*d^11 - 10*c^5*d^12 + 10*c^4*d^13 + 5*c^3*d^14 -
 5*c^2*d^15 - c*d^16 + d^17) + 4*(3*c^10*d^6 - 14*c^9*d^7 + 15*c^8*d^8 + 24
*c^7*d^9 - 58*c^6*d^10 + 12*c^5*d^11 + 54*c^4*d^12 - 40*c^3*d^13 - 9*c^2*d^
14 + 18*c*d^15 - 5*d^16)/(c^11*d^6 - c^10*d^7 - 5*c^9*d^8 + 5*c^8*d^9 + 10*
c^7*d^10 - 10*c^6*d^11 - 10*c^5*d^12 + 10*c^4*d^13 + 5*c^3*d^14 - 5*c^2*d^1
5 - c*d^16 + d^17))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*(45*c^10*d^6 - 250
*c^9*d^7 + 601*c^8*d^8 - 664*c^7*d^9 - 166*c^6*d^10 + 1444*c^5*d^11 - 1510*
c^4*d^12 + 104*c^3*d^13 + 889*c^2*d^14 - 634*c*d^15 + 141*d^16)/(c^11*d^6 -
 c^10*d^7 - 5*c^9*d^8 + 5*c^8*d^9 + 10*c^7*d^10 - 10*c^6*d^11 - 10*c^5*d^12
 + 10*c^4*d^13 + 5*c^3*d^14 - 5*c^2*d^15 - c*d^16 + d^17))*tan(-1/8*pi + 1/
4*f*x + 1/4*e)^2 + 20*(3*c^10*d^6 - 14*c^9*d^7 + 15*c^8*d^8 + 24*c^7*d^9 -
58*c^6*d^10 + 12*c^5*d^11 + 54*c^4*d^12 - 40*c^3*d^13 - 9*c^2*d^14 + 18*c*d
^15 - 5*d^16)/(c^11*d^6 - c^10*d^7 - 5*c^9*d^8 + 5*c^8*d^9 + 10*c^7*d^10 -
10*c^6*d^11 - 10*c^5*d^12 + 10*c^4*d^13 + 5*c^3*d^14 - 5*c^2*d^15 - c*d^16
+ d^17))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 15*(c^10*d^6 - 2*c^9*d^7 - 3*c^
8*d^8 + 8*c^7*d^9 + 2*c^6*d^10 - 12*c^5*d^11 + 2*c^4*d^12 + 8*c^3*d^13 - 3*
```

```
c^2*d^14 - 2*c*d^15 + d^16)/(c^11*d^6 - c^10*d^7 - 5*c^9*d^8 + 5*c^8*d^9 +
10*c^7*d^10 - 10*c^6*d^11 - 10*c^5*d^12 + 10*c^4*d^13 + 5*c^3*d^14 - 5*c^2*
d^15 - c*d^16 + d^17))*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8
*pi + 1/4*f*x + 1/4*e)/((c*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + d*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^4 + 2*c*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 6*d*tan(-1/8
*pi + 1/4*f*x + 1/4*e)^2 + c + d)^(5/2)*f)
```

**Mupad** [B]
time = 16.35, size = 501, normalized size = 3.53

$$-\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{15\,d\,f\,{\left(c+d\right)}^3}-\frac{32\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d\,f\,{\left(c+d\right)}^3}+\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\left(240\,c^2+80\,d^2\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^3\,f\,{\left(c+d\right)}^3}-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(c^2\,240{}\mathrm{i}+d^2\,80{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^3\,f\,{\left(c+d\right)}^3}+\frac{32\,c\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,d^2\,f\,{\left(c+d\right)}^3}-\frac{c\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{3\,d^2\,f\,{\left(c+d\right)}^3}\right)}{{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+\frac{{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3}{{\left(c+d\right)}^3}-\frac{3\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(6\,c+d\right)}{d}+\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{d^3}+\frac{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(c\,6{}\mathrm{i}+d\,1{}\mathrm{i}\right)}{d}-\frac{3\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2\,{\left(c+d\right)}^3}+\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{d^3\,{\left(c+d\right)}^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2
)*32i)/(15*d*f*(c + d)^3) - (32*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/
2))/(15*d*f*(c + d)^3) + (exp(e*4i + f*x*4i)*(240*c^2 + 80*d^2)*(a + a*sin(
e + f*x))^(1/2))/(15*d^3*f*(c + d)^3) - (exp(e*3i + f*x*3i)*(c^2*240i + d^2
*80i)*(a + a*sin(e + f*x))^(1/2))/(15*d^3*f*(c + d)^3) + (32*c*exp(e*2i + f
*x*2i)*(a + a*sin(e + f*x))^(1/2))/(3*d^2*f*(c + d)^3) - (c*exp(e*5i + f*x*
5i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*d^2*f*(c + d)^3)))/(exp(e*7i + f*x*7
i) + (c*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2
))/d^2 - (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 +
 12*c^2*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3
*exp(e*2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3)
+ (exp(e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3)
)/(d^3*(c + d)^3))
```

# 3.571 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=285

$$\frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}+\frac{5a^2(c-15d)(c+d)^2\cos(e+fx)}{64df\sqrt{a+a\sin(e}$$

```
[Out] 5/64*a^(3/2)*(c-15*d)*(c+d)^3*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f+5/96*a^2*(c-15*d)*(c+d)*cos(f
*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/24*a^2*(c-15*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^(7/2)/d/f/(a+a*sin(f*x+e))^(1/2)+5/64*a^2*(c-15*d)*(c+
d)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.38, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2842, 21, 2849, 2854, 211}

$$\frac{5a^{3/2}(c-15d)(c+d)^3\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-15d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24df\sqrt{a\sin(e+fx)+a}}+\frac{5a^2(c-15d)(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96df\sqrt{a\sin(e+fx)+a}}+\frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*a^(3/2)*(c - 15*d)*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(3/2)*f) + (5*a^2*(c
 - 15*d)*(c + d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(64*d*f*Sqrt[a +
a*Sin[e + f*x]]) + (5*a^2*(c - 15*d)*(c + d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(96*d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - 15*d)*Cos[e + f*x]
*(c + d*Sin[e + f*x])^(5/2))/(24*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e
 + f*x]*(c + d*Sin[e + f*x])^(7/2))/(4*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{(-\frac{1}{2}a^2(c-15d)-\frac{1}{2}\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-15d))\int \cdots}{\cdots}$$

$$= \frac{a^2(c-15d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e\cdots}{4d\cdots}$$

$$= \frac{5a^2(c-15d)(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96df\sqrt{a+a\sin(e+fx)}} + \cdots$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \cdots$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \cdots$$

$$= \frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+\cdots}{\sqrt{a+a\sin(e+fx)}\sqrt{c+\cdots}}\right.\cdots}{64d^{3/2}f}$$

**Mathematica** [**A**]
time = 0.95, size = 318, normalized size = 1.12

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-\frac{5(c-15d)(c+d)^3\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{3/2}}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(15c^3+455c^2d+653cd^2+285d^3-4d^2(17c+15d)\cos(2(e+fx))+2d(59c^2+190cd+93d^2)\sin(e+fx)-12d^3\sin(3(e+fx))\right)}{3d}\right)}{128f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-5*(c - 15*d)*(c + d)^3*(2*ArcTan[(Sqrt[2]*
Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqr
t[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqr
t[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2
) - (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(15*c
^3 + 455*c^2*d + 653*c*d^2 + 285*d^3 - 4*d^2*(17*c + 15*d)*Cos[2*(e + f*x)]
 + 2*d*(59*c^2 + 190*c*d + 93*d^2)*Sin[e + f*x] - 12*d^3*Sin[3*(e + f*x)]))
/(3*d)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{3}{2}}(c+d\sin(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(262) = 524$.
time = 1.11, size = 1637, normalized size = 5.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/1536*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4 + (
a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*cos(f*x + e) + (
a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*sin(f*x + e))*sq
rt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^
2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a
*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^
2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2
- 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^
4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 -
51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*
d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 28
9*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a
*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5
*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^
2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f
*x + e) + 1)) - 8*(48*a*d^3*cos(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a
*c*d^2 - 147*a*d^3 + 8*(17*a*c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2
```

```
*d + 122*a*c*d^2 + 63*a*d^3)*cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721
*a*c*d^2 + 345*a*d^3)*cos(f*x + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 +
337*a*c^2*d + 341*a*c*d^2 + 147*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x +
e)^2 - 2*(59*a*c^2*d + 190*a*c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*
sin(f*x + e) + d*f), -1/768*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c
*d^3 - 15*a*d^4 + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*cos(f*x + e) + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d -
 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) - 4*(48*a*d^3*c
os(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a*c*d^2 - 147*a*d^3 + 8*(17*a*
c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2*d + 122*a*c*d^2 + 63*a*d^3)*
cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721*a*c*d^2 + 345*a*d^3)*cos(f*x
 + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 + 337*a*c^2*d + 341*a*c*d^2 + 1
47*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x + e)^2 - 2*(59*a*c^2*d + 190*a*
c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\,\sin\left(e + f\,x\right))^{3/2}\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.572 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=228

$$\frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f}+\frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c}}{8df\sqrt{a+a\sin(e+}}$$

```
[Out] 1/8*a^(3/2)*(c-11*d)*(c+d)^2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x
+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f+1/12*a^2*(c-11*d)*cos(f*x+e)*(
c+d*sin(f*x+e))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)-1/3*a^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(5/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/8*a^2*(c-11*d)*(c+d)*cos(f*x+e
)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2842, 21, 2849, 2854, 211}

$$\frac{a^{3/2}(c-11d)(c+d)^2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-11d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(3/2)*(c - 11*d)*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(3/2)*f) + (a^2*(c - 11
*d)*(c + d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a^2*(c - 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*d*f
*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/
(3*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-11d)-\frac{1}{2}\cdots\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-11d))\int \cdots}{\cdots}$$

$$= \frac{a^2(c-11d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e\cdots}{3d\cdots}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2\cdots}{\cdots}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2\cdots}{\cdots}$$

$$= \frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+f\cdots}{\sqrt{a+a\sin(e+fx)}\sqrt{c+\cdots}}\right.}{8d^{3/2}f}$$

**Mathematica** [A]
time = 0.65, size = 281, normalized size = 1.23

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(\frac{(c-11d)(c+d)^2\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{3/2}}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(3c^2+52cd+37d^2-4d^2\cos(2(e+fx))+2d(7c+11d)\sin(e+fx)\right)}{3d}\right)}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 11*d)*(c + d)^2*(-2*ArcTan[(Sqrt[2]*Sq
rt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2)
- (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(3*c^2
+ 52*c*d + 37*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(7*c + 11*d)*Sin[e + f*x])
)/(3*d)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{3}{2}}(c+d\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(208) = 416$.
time = 0.76, size = 1391, normalized size = 6.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/192*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3 - 9*a*c^2*d
 - 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 1
1*a*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4
*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*
x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*
c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f
*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f
*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c
^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c
^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
+ a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^
4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^
2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d + a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52
*a*c*d + 41*a*d^2)*cos(f*x + e) - (8*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*
```

```
c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x
 + e) + d*f), -1/96*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3
- 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21
*a*c*d^2 - 11*a*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)
^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d
- a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)
)) - 4*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d
+ a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52*a*c*d + 41*a*d^2)*cos(f*x + e) - (8
*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^{\frac{3}{2}}\,(c+d\sin (e+fx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin \left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.573 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=171

$$\frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}+\frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(}}{4df\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/4*a^(3/2)*(c-7*d)*(c+d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f-1/2*a^2*cos(f*x+e)*(c+d*sin(f*x+e
))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/4*a^2*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2842, 21, 2849, 2854, 211}

$$\frac{a^{3/2}(c-7d)(c+d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (a^(3/2)*(c - 7*d)*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(3/2)*f) + (a^2*(c - 7*d)*
Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + a*Sin[e + f*x]]) - (
a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*d*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-7d)-\frac{1}{2}a^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-7d))\int\sqrt{c \cdots}}{\cdots}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+ \cdots}{2df\sqrt{\cdots}}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+ \cdots}{2df\sqrt{\cdots}}$$

$$= \frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin \cdots}}\right.}{4d^{3/2}f}$$

**Mathematica [A]**
time = 0.42, size = 247, normalized size = 1.44

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(\frac{(c-7d)(c+d)\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{3/2}}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\,(c+7d+2d\sin(e+fx))}{d}\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 7*d)*(c + d)*(-2*ArcTan[(Sqrt[2]*Sqrt[
d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2) - (
2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(c + 7*d +
 2*d*Sin[e + f*x]))/d))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{3}{2}}\sqrt{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(154) = 308.
time = 0.68, size = 1177, normalized size = 6.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/32*((a*c^2 - 6*a*c*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x +
e) + (a*c^2 - 6*a*c*d - 7*a*d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*co
s(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2
*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)
*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x
+ e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4
+ 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f
*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*
cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^
4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^
4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (1
28*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d
^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5
*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*
cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*a*d*c
os(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(f*x +
 e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f), -1/16*((a*c^2 - 6*a*c
*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x + e) + (a*c^2 - 6*a*c*d
- 7*a*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 +
 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*c
os(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(2*
a*d*cos(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(
f*x + e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin(e + f x))^{3/2} \sqrt{c + d\sin(e + f x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.574 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=111

$$\frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}$$

```
[Out] a^(3/2)*(c-3*d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c
+d*sin(f*x+e))^(1/2))/d^(3/2)/f-a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(
a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2842, 21, 2854, 211}

$$\frac{a^{3/2}(c-3d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (a^(3/2)*(c - 3*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) - (a^2*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
```

```
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2(c-3d)-\frac{1}{2}a^2(c-3d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e}}}{d}$$

$$= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-3d))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,d}{2d}$$

$$= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} + \frac{(a^2(c-3d))\,\mathrm{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, -\right.}{\sqrt{}}$$

$$= \frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f} - \frac{a^2\cos(e}{df}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(111) = 222.

time = 0.41, size = 301, normalized size = 2.71

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-2(c-3d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-(c-3d)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+c\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)-3d\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)-2\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{c+d\sin(e+fx)}+2\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\sqrt{c+d\sin(e+fx)}\right)}{2d^{3/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - 3*d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*
e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - (c - 3*d)*ArcTanh[(Sqrt[2]*
```

```
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + c*Log[Sqrt[2
]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 3*d*Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 2*Sq
rt[d]*Cos[(e + f*x)/2]*Sqrt[c + d*Sin[e + f*x]] + 2*Sqrt[d]*Sin[(e + f*x)/2
]*Sqrt[c + d*Sin[e + f*x]]))/(2*d^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3)
```

**Maple [F]**
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^{\frac{3}{2}}}{\sqrt{c + d\sin(fx + e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(101) = 202.
time = 0.72, size = 1035, normalized size = 9.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin(f*x +
e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*
d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*
c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2
 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d +
 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^
2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 -
```

```
 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*
d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3
 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^
3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d
 + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 -
 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*
c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) +
 sin(f*x + e) + 1)) + 8*(a*cos(f*x + e) - a*sin(f*x + e) + a)*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
+ d*f), -1/4*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin
(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2
 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*si
n(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(a*cos(f*x + e) -
 a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*
f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.575 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}+\frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}}$$

```
[Out] -2*a^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/d^(3/2)/f+2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(a+a*sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 21, 2854, 211}

$$\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) + (2*a^2*(c - d)*Cos[e + f*x])/(d*
(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
```

```
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+d)}{\sqrt{a+a\sin(e+}}}{d(}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}}{d}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+dx^2}\,dx\right.}{}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f} + \frac{2}{d(c+d)f\sqrt{a+}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 377 vs. 2(117) = 234.
time = 5.95, size = 377, normalized size = 3.22

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(2c\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-2d^{3/2}\cos\left(\frac{1}{2}(e+fx)\right)-2c\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+2d^{3/2}\sin\left(\frac{1}{2}(e+fx)\right)+2(c+d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)\sqrt{c+d\sin(e+fx)}+(c+d)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)\sqrt{c+d\sin(e+fx)}-c\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\sqrt{c+d\sin(e+fx)}-d\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\sqrt{c+d\sin(e+fx)}\right)}{d^{3/2}(c+d)f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(2*c*Sqrt[d]*Cos[(e + f*x)/2] - 2*d^(3/2)*Cos
[(e + f*x)/2] - 2*c*Sqrt[d]*Sin[(e + f*x)/2] + 2*d^(3/2)*Sin[(e + f*x)/2] +
```

```
 2*(c + d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Si
n[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]] + (c + d)*ArcTanh[(Sqrt[2]*Sqrt[d]*Co
s[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]
 - c*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x
]]]*Sqrt[c + d*Sin[e + f*x]] - d*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)
/4] + Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]))/(d^(3/2)*(c + d)
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 6626 vs. $2(101) = 202$.
time = 9.23, size = 6627, normalized size = 56.64

| method | result | size |
|---|---|---|
| default | Expression too large to display | 6627 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(107) = 214$.
time = 0.68, size = 1351, normalized size = 11.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/4*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 +
 a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c
^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*
```

```
d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x +
 e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x +
 e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d
^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*
c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2
*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d
^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*
c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*c
os(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*
(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/
(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(a*c - a*d + (a*c - a*d)*cos(f*x + e
) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c
^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2
+ d^3)*f)*sin(f*x + e)), -1/2*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*c
os(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (
a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(
f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (
3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(
f*x + e))) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^2 + d^3)*f*c
os(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f
- ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin{(e+fx)}+1))^{\frac{3}{2}}}{(c+d\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

[Out] Timed out

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^{3/2}}{(c + d \sin(e + f x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.576 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=115

$$\frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3d(c+d)^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^
(1/2)-2/3*a^2*(c+5*d)*cos(f*x+e)/d/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2841, 21, 2850}

$$\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3df(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(3/2)) - (2*a^2*(c + 5*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+5c}{\sqrt{a+a\sin(e}}}{3c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{(a(c+5d))\int \frac{\sqrt{a+}}{(c+}}{3d(c-}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{2a^2(c}{3d(c+d)^2f\sqrt{a+a}}$$

**Mathematica [A]**
time = 0.38, size = 104, normalized size = 0.90

$$-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(5c+d+(c+5d)\sin(e+fx))}{3(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*c
 + d + (c + 5*d)*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(c + d*Sin[e + f*x])^(3/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(103) = 206$.
time = 0.50, size = 345, normalized size = 3.00

| method | result |
|---|---|
| default | $-\frac{2(a(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{c+d\sin(fx+e)}\,(\sin(fx+e)(\cos^4(fx+e))c\,d^2+5\sin(fx+e)(\cos^4(fx+e))d^3-2(\cos^4(fx+e))c^2c}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(sin(f*x+e)*cos(f*x+
e)^4*c*d^2+5*sin(f*x+e)*cos(f*x+e)^4*d^3-2*cos(f*x+e)^4*c^2*d-7*cos(f*x+e)^
4*c*d^2-9*cos(f*x+e)^4*d^3-sin(f*x+e)*cos(f*x+e)^2*c^3+sin(f*x+e)*cos(f*x+e
)^2*c^2*d-11*sin(f*x+e)*cos(f*x+e)^2*c*d^2-13*sin(f*x+e)*cos(f*x+e)^2*d^3-3
*cos(f*x+e)^2*c^3-5*cos(f*x+e)^2*c^2*d+15*cos(f*x+e)^2*c*d^2+17*cos(f*x+e)^
2*d^3-8*c^3*sin(f*x+e)-8*c^2*d*sin(f*x+e)+8*c*d^2*sin(f*x+e)+8*d^3*sin(f*x+
e)+8*c^3+8*c^2*d-8*c*d^2-8*d^3)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d^2)^2/(
c+d)^2
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(109) = 218$.
time = 0.58, size = 325, normalized size = 2.83

$$-\frac{2\left((5\,c^2+cd)a^{\frac{3}{2}}-\frac{(3\,c^2-19\,cd-2\,d^2)a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,(4\,c^2-7\,cd+9\,d^2)a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\,(4\,c^2-7\,cd+9\,d^2)a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{(3\,c^2-19\,cd-2\,d^2)a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{(5\,c^2+cd)a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{3\left(c^2+2\,cd+d^2+\frac{(c^2+2\,cd+d^2)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)\left(c+\frac{2\,d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((5*c^2 + c*d)*a^(3/2) - (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - (5*c^2 + c*d)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c^2 + 2*c*d + d^2 + (c^2 + 2*c*d +
d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(109) = 218$.
time = 0.35, size = 335, normalized size = 2.91

$$\frac{2\left((ac+5\,ad)\cos(fx+e)^2+4\,ac-4\,ad+(5\,ac+ad)\cos(fx+e)-(4\,ac-4\,ad-(ac+5\,ad)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\,\sqrt{d\sin(fx+e)+c}}{3\left((c^2d^2+2\,cd^3+d^4)f\cos(fx+e)^3+(2\,c^3d+5\,c^2d^2+4\,cd^3+d^4)f\cos(fx+e)^2-(c^4+2\,c^3d+2\,c^2d^2+2\,cd^3+d^4)f\cos(fx+e)-(c^4+4\,c^3d+6\,c^2d^2+4\,cd^3+d^4)f+\left((c^2d^2+2\,cd^3+d^4)f\cos(fx+e)^2-2\,(c^3d+2\,c^2d^2+cd^3)f\cos(fx+e)-(c^4+4\,c^3d+6\,c^2d^2+4\,cd^3+d^4)f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*((a*c + 5*a*d)*cos(f*x + e)^2 + 4*a*c - 4*a*d + (5*a*c + a*d)*cos(f*x +
 e) - (4*a*c - 4*a*d - (a*c + 5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*
x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 +
2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^
2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*
```

```
(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4
*c*d^3 + d^4)*f)*sin(f*x + e))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}}{(c+d\sin(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad [B]**
time = 14.40, size = 387, normalized size = 3.37

$$-\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{a\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,(c+5\,d)\,\sqrt{a+a\,\sin(e+f\,x)}\,4\mathrm{i}}{3\,d^2\,f\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}+\frac{a\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,(3\,c-d)\,\sqrt{a+a\,\sin(e+f\,x)}\,4\mathrm{i}}{d^2\,f\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}-\frac{a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(c\,3\mathrm{i}-d\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,4\mathrm{i}}{d^2\,f\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}-\frac{a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(c\,1\mathrm{i}+d\,5\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,4\mathrm{i}}{3\,d^2\,f\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}\right)}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-\frac{(c+d)^2\,1\mathrm{i}}{(c\,1\mathrm{i}+d\,1\mathrm{i})^2}-\frac{2\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,(2\,c^2+2\,c\,d+d^2)}{d^2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,(4\,c+d)}{d}+\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(c+d)^2\,(2\,c^2+2\,c\,d+d^2)\,2\mathrm{i}}{d^2\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}-\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(c+d)^2\,(4\,c+d)\,1\mathrm{i}}{d\,(c\,1\mathrm{i}+d\,1\mathrm{i})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((a*exp(e*1i + f*x*1i)*(c + 5*d)*(a + a*sin(e
+ f*x))^(1/2)*4i)/(3*d^2*f*(c*1i + d*1i)^2) + (a*exp(e*3i + f*x*3i)*(3*c -
d)*(a + a*sin(e + f*x))^(1/2)*4i)/(d^2*f*(c*1i + d*1i)^2) - (a*exp(e*2i + f
*x*2i)*(c*3i - d*1i)*(a + a*sin(e + f*x))^(1/2)*4i)/(d^2*f*(c*1i + d*1i)^2)
 - (a*exp(e*4i + f*x*4i)*(c*1i + d*5i)*(a + a*sin(e + f*x))^(1/2)*4i)/(3*d^
2*f*(c*1i + d*1i)^2)))/(exp(e*5i + f*x*5i) - ((c + d)^2*1i)/(c*1i + d*1i)^2
 - (2*exp(e*3i + f*x*3i)*(2*c*d + 2*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(
4*c + d))/d + (exp(e*2i + f*x*2i)*(c + d)^2*(2*c*d + 2*c^2 + d^2)*2i)/(d^2*
(c*1i + d*1i)^2) - (exp(e*4i + f*x*4i)*(c + d)^2*(4*c + d)*1i)/(d*(c*1i + d
*1i)^2))
```

## 3.577 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=172

$$\frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}-\frac{2a^2(c+9d)\cos(e+fx)}{15d(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3}}$$

```
[Out] 2/5*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^
(1/2)-2/15*a^2*(c+9*d)*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+a*s
in(f*x+e))^(1/2)-4/15*a^2*(c+9*d)*cos(f*x+e)/d/(c+d)^3/f/(a+a*sin(f*x+e))^(
1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 21, 2851, 2850}

$$-\frac{4a^2(c+9d)\cos(e+fx)}{15df(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^2(c+9d)\cos(e+fx)}{15df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+\frac{2a^2(c-d)\cos(e+fx)}{5df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(5/2)) - (2*a^2*(c + 9*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^2*(c + 9*d)*Cos[e
 + f*x])/(15*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+9 \ldots}{\sqrt{a+a\sin(\ldots}}}{5 \ldots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} + \frac{(a(c+9d))\int \frac{\sqrt{a+\ldots}}{(c+\ldots}}{5d(c\ldots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{2a^2 \ldots}{15d(c+d)^2f\sqrt{a+\ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{2a^2 \ldots}{15d(c+d)^2f\sqrt{a+\ldots}}$$

**Mathematica** [A]
time = 0.61, size = 140, normalized size = 0.81

$$-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(25c^2+13cd+12d^2-d(c+9d)\cos(2(e+fx))+\left(5c^2+46cd+9d^2\right)\sin(e+fx)\right)}{15(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(25*
c^2 + 13*c*d + 12*d^2 - d*(c + 9*d)*Cos[2*(e + f*x)] + (5*c^2 + 46*c*d + 9*
d^2)*Sin[e + f*x]))/(15*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
c + d*Sin[e + f*x])^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(154) = 308$.
time = 0.28, size = 625, normalized size = 3.63

| method | result |
|---|---|
| default | $-\frac{2(a(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{c+d\sin(fx+e)}\,(40c^5+24d^5-80c^2d^3+8c\,d^4-48c^3d^2+56c^4d-40c^5\sin(fx+e)+78(\cos^4(fx+e))d^5}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(40*c^5+24*d^5-80*c
^2*d^3+8*c*d^4-48*c^3*d^2+56*c^4*d-40*c^5*sin(f*x+e)-9*sin(f*x+e)*cos(f*x+e
)^4*c*d^4-90*sin(f*x+e)*cos(f*x+e)^2*c^3*d^2-146*sin(f*x+e)*cos(f*x+e)^2*c^
2*d^3+15*sin(f*x+e)*cos(f*x+e)^2*c*d^4-75*cos(f*x+e)^2*d^5-24*sin(f*x+e)*d^
5-27*cos(f*x+e)^6*d^5+78*cos(f*x+e)^4*d^5-105*cos(f*x+e)^4*c^2*d^3+23*cos(f
*x+e)^4*c*d^4+63*sin(f*x+e)*cos(f*x+e)^2*d^5-31*cos(f*x+e)^2*c^4*d+114*cos(
f*x+e)^2*c^3*d^2+186*cos(f*x+e)^2*c^2*d^3-19*cos(f*x+e)^2*c*d^4-56*sin(f*x+
e)*c^4*d+48*sin(f*x+e)*c^3*d^2+80*sin(f*x+e)*c^2*d^3-8*sin(f*x+e)*c*d^4-57*
sin(f*x+e)*cos(f*x+e)^4*d^5-15*cos(f*x+e)^2*c^5-13*cos(f*x+e)^4*c^4*d-5*sin
(f*x+e)*cos(f*x+e)^2*c^5+18*sin(f*x+e)*cos(f*x+e)^6*d^5-cos(f*x+e)^6*c^2*d^
3-63*cos(f*x+e)^4*c^3*d^2+2*sin(f*x+e)*cos(f*x+e)^6*c*d^4+9*sin(f*x+e)*cos(
f*x+e)^4*c^3*d^2+57*sin(f*x+e)*cos(f*x+e)^4*c^2*d^3+3*sin(f*x+e)*cos(f*x+e)
^2*c^4*d-12*cos(f*x+e)^6*c*d^4)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d^2)^3/(
c+d)^3
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(163) = 326$.
time = 0.60, size = 529, normalized size = 3.08

$$-\frac{2\left(\left(25\,c^3+12\,c^2d+3\,cd^2\right)a^{\frac{3}{2}}-\frac{\left(15\,c^3-130\,c^2d-39\,cd^2-6\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\left(65\,c^3-78\,c^2d+223\,cd^2+30\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5\left(11\,c^3-44\,c^2d+33\,cd^2-24\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5\left(11\,c^3-44\,c^2d+33\,cd^2-24\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{\left(65\,c^3-78\,c^2d+223\,cd^2+30\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{\left(15\,c^3-130\,c^2d-39\,cd^2-6\,d^3\right)a^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{\left(25\,c^3+12\,c^2d+3\,cd^2\right)a^{\frac{3}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^2}{15\left(c^3+3\,c^2d+3\,cd^2+d^3+\frac{2\left(c^3+3\,c^2d+3\,cd^2+d^3\right)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\left(c^3+3\,c^2d+3\,cd^2+d^3\right)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)\left(c+\frac{2\,d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/2) - (15*c^3 - 130*c^2*d - 39*c*d
^2 - 6*d^3)*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (65*c^3 - 78*c^2*d +
223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(11*c^3
 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
```

```
3 + 5*(11*c^3 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - (65*c^3 - 78*c^2*d + 223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 130*c^2*d - 39*c*d^2 - 6*d^3)*a^(3/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1)^2/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 2*(c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1)
+ c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(163) = 326$.
time = 0.38, size = 614, normalized size = 3.57

$$\frac{2\,\left(2\,\left(acd + 9\,ad^2\right)\cos\left(fx+e\right)^3 - 20\,ac^2 + 32\,acd - 12\,ad^2 - \left(5\,ac^2 + 44\,acd - 9\,ad^2\right)\cos\left(fx+e\right)^2 - \left(25\,ac^2 + 14\,acd + 21\,ad^2\right)\cos\left(fx+e\right) + \left(20\,ac^2 - 32\,acd + 12\,ad^2 - 2\,\left(acd + 9\,ad^2\right)\cos\left(fx+e\right)^2 - \left(5\,ac^2 + 46\,acd + 9\,ad^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{15\,\left(\left(c^3d^3 + 3\,c^2d^4 + 3\,cd^5 + d^6\right)f\cos\left(fx+e\right)^4 - 3\,\left(c^4d^2 + 3\,c^3d^3 + 3\,c^2d^4 + cd^5\right)f\cos\left(fx+e\right)^3 - \left(3\,c^5d + 12\,c^4d^2 + 20\,c^3d^3 + 18\,c^2d^4 + 9\,cd^5 + 2\,d^6\right)f\cos\left(fx+e\right)^2 + \left(c^6 + 3\,c^5d + 6\,c^4d^2 + 10\,c^3d^3 + 9\,c^2d^4 + 3\,cd^5\right)f\cos\left(fx+e\right) + \left(c^6 + 6\,c^5d + 15\,c^4d^2 + 20\,c^3d^3 + 15\,c^2d^4 + 6\,cd^5 + d^6\right)f - \left(\left(c^3d^3 + 3\,c^2d^4 + 3\,cd^5 + d^6\right)f\cos\left(fx+e\right)^3 + \left(3\,c^4d^2 + 10\,c^3d^3 + 12\,c^2d^4 + 6\,cd^5 + d^6\right)f\cos\left(fx+e\right)^2 - \left(3\,c^5d + 9\,c^4d^2 + 10\,c^3d^3 + 6\,c^2d^4 + 3\,cd^5 + d^6\right)f\cos\left(fx+e\right) - \left(c^6 + 6\,c^5d + 15\,c^4d^2 + 20\,c^3d^3 + 15\,c^2d^4 + 6\,cd^5 + d^6\right)f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(2*(a*c*d + 9*a*d^2)*cos(f*x + e)^3 - 20*a*c^2 + 32*a*c*d - 12*a*d^2 -
 (5*a*c^2 + 44*a*c*d - 9*a*d^2)*cos(f*x + e)^2 - (25*a*c^2 + 14*a*c*d + 21*
a*d^2)*cos(f*x + e) + (20*a*c^2 - 32*a*c*d + 12*a*d^2 - 2*(a*c*d + 9*a*d^2)
*cos(f*x + e)^2 - (5*a*c^2 + 46*a*c*d + 9*a*d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 +
 3*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d
^5)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*
c*d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 +
 9*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3
*d^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)
*f*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5
 + d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*
d^4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**B**]
time = 17.48, size = 541, normalized size = 3.15

$$\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(c+9\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^2\,f\,{\left(c+d\right)}^3}-\frac{8\,a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(9\,c^2-4\,c\,d+3\,d^2\right)}{3\,d^3\,f\,{\left(c+d\right)}^3}+\frac{8\,a\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c^2\,9{}\mathrm{i}-c\,d\,4{}\mathrm{i}+d^2\,3{}\mathrm{i}\right)}{3\,d^3\,f\,{\left(c+d\right)}^3}-\frac{8\,a\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(c\,1{}\mathrm{i}+d\,9{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^2\,f\,{\left(c+d\right)}^3}+\frac{8\,a\,c\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\left(c\,1{}\mathrm{i}+d\,9{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,d^3\,f\,{\left(c+d\right)}^3}-\frac{8\,a\,c\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\left(c+9\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,d^3\,f\,{\left(c+d\right)}^3}\right)}{{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+\frac{{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3}{{\left(c+d\right)}^3}-\frac{3\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(6\,c+d\right)}{d}+\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{d^3}+\frac{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(c\,6{}\mathrm{i}+d\,1{}\mathrm{i}\right)}{d}-\frac{3\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2\,{\left(c+d\right)}^3}+\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^3\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{d^3\,{\left(c+d\right)}^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((8*a*exp(e*6i + f*x*6i)*(c + 9*d)*(a + a*sin(e
 + f*x))^(1/2))/(15*d^2*f*(c + d)^3) - (8*a*exp(e*4i + f*x*4i)*(a + a*sin(e
 + f*x))^(1/2)*(9*c^2 - 4*c*d + 3*d^2))/(3*d^3*f*(c + d)^3) + (8*a*exp(e*3i
 + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*9i - c*d*4i + d^2*3i))/(3*d^3*f*
(c + d)^3) - (8*a*exp(e*1i + f*x*1i)*(c*1i + d*9i)*(a + a*sin(e + f*x))^(1/
2))/(15*d^2*f*(c + d)^3) + (8*a*c*exp(e*5i + f*x*5i)*(c*1i + d*9i)*(a + a*s
in(e + f*x))^(1/2))/(3*d^3*f*(c + d)^3) - (8*a*c*exp(e*2i + f*x*2i)*(c + 9*
d)*(a + a*sin(e + f*x))^(1/2))/(3*d^3*f*(c + d)^3)))/(exp(e*7i + f*x*7i) +
(c*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2))/d^
2 - (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 + 12*c
^2*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3*exp(
e*2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3) + (ex
p(e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(d^
3*(c + d)^3))
```

# 3.578 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=229

$$\frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}-\frac{2a^2(c+13d)\cos(e+fx)}{35d(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx)}$$

```
[Out] 2/7*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^
(1/2)-2/35*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)/(a+a*
sin(f*x+e))^(1/2)-8/105*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^3/f/(c+d*sin(f*x+e)
)^(3/2)/(a+a*sin(f*x+e))^(1/2)-16/105*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^4/f/(
a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 21, 2851, 2850}

$$-\frac{16a^2(c+13d)\cos(e+fx)}{105df(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^2(c+13d)\cos(e+fx)}{105df(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+13d)\cos(e+fx)}{35df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{5/2}}+\frac{2a^2(c-d)\cos(e+fx)}{7df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(7*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(7/2)) - (2*a^2*(c + 13*d)*Cos[e + f*x])/(35*d*(c + d)^2*f*S
qrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^2*(c + 13*d)*Cos
[e + f*x])/(105*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])
^(3/2)) - (16*a^2*(c + 13*d)*Cos[e + f*x])/(105*d*(c + d)^4*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+13d)\cdots}{\sqrt{a+a\sin(e\cdots}}}{7d(\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} + \frac{(a(c+13d))\int \frac{\sqrt{a+\cdots}}{(c+\cdots}}{7d(c+\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c\cdots}{35d(c+d)^2f\sqrt{a+a\cdots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c\cdots}{35d(c+d)^2f\sqrt{a+a\cdots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c\cdots}{35d(c+d)^2f\sqrt{a+a\cdots}}$$

**Mathematica [A]**
time = 1.02, size = 193, normalized size = 0.84

$$-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(175c^3+147c^2d+253cd^2+41d^3-2d(7c^2+92cd+13d^2)\cos(2(e+fx))+(35c^3+469c^2d+191cd^2+117d^3)\sin(e+fx)-2cd^2\sin(3(e+fx))-26d^3\sin(3(e+fx))\right)}{105(c+d)^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(175
*c^3 + 147*c^2*d + 253*c*d^2 + 41*d^3 - 2*d*(7*c^2 + 92*c*d + 13*d^2)*Cos[2
*(e + f*x)] + (35*c^3 + 469*c^2*d + 191*c*d^2 + 117*d^3)*Sin[e + f*x] - 2*c
*d^2*Sin[3*(e + f*x)] - 26*d^3*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(7/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(205) = 410$.
time = 2.23, size = 979, normalized size = 4.28

| method | result | size |
|---|---|---|
| default | Expression too large to display | 979 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(776*c^4*d^3*sin(f
*x+e)-136*c^3*d^4*sin(f*x+e)-424*c^2*d^5*sin(f*x+e)+120*c*d^6*sin(f*x+e)-77
6*c^4*d^3+280*c^7-296*c^5*d^2-152*d^7+424*c^2*d^5+152*d^7*sin(f*x+e)+136*c^
3*d^4-120*c*d^6+504*c^6*d-4*cos(f*x+e)^8*c^2*d^5-64*cos(f*x+e)^8*c*d^6-455*
sin(f*x+e)*cos(f*x+e)^6*d^7+4*cos(f*x+e)^6*c^4*d^3-149*cos(f*x+e)^6*c^3*d^4
-443*cos(f*x+e)^6*c^2*d^5+345*cos(f*x+e)^6*c*d^6+750*sin(f*x+e)*cos(f*x+e)^
4*d^7-112*cos(f*x+e)^4*c^6*d-670*cos(f*x+e)^4*c^5*d^2-1398*cos(f*x+e)^4*c^4
*d^3+56*cos(f*x+e)^4*c^3*d^4+1232*cos(f*x+e)^4*c^2*d^5-618*cos(f*x+e)^4*c*d
^6-35*sin(f*x+e)*cos(f*x+e)^2*c^7-551*sin(f*x+e)*cos(f*x+e)^2*d^7-259*cos(f
*x+e)^2*c^6*d+1035*cos(f*x+e)^2*c^5*d^2+2185*cos(f*x+e)^2*c^4*d^3-43*cos(f*
x+e)^2*c^3*d^4-1209*cos(f*x+e)^2*c^2*d^5+457*cos(f*x+e)^2*c*d^6-504*sin(f*x
+e)*c^6*d+296*sin(f*x+e)*c^5*d^2+104*sin(f*x+e)*cos(f*x+e)^8*d^7+7*sin(f*x+
e)*cos(f*x+e)^2*c^6*d-887*sin(f*x+e)*cos(f*x+e)^2*c^5*d^2-1797*sin(f*x+e)*c
os(f*x+e)^2*c^4*d^3-25*sin(f*x+e)*cos(f*x+e)^2*c^3*d^4+997*sin(f*x+e)*cos(f
*x+e)^2*c^2*d^5-397*sin(f*x+e)*cos(f*x+e)^2*c*d^6+8*sin(f*x+e)*cos(f*x+e)^8
*c*d^6+29*sin(f*x+e)*cos(f*x+e)^6*c^3*d^4+371*sin(f*x+e)*cos(f*x+e)^6*c^2*d
^5-113*sin(f*x+e)*cos(f*x+e)^6*c*d^6+106*sin(f*x+e)*cos(f*x+e)^4*c^5*d^2+75
4*sin(f*x+e)*cos(f*x+e)^4*c^4*d^3+72*sin(f*x+e)*cos(f*x+e)^4*c^3*d^4-944*si
n(f*x+e)*cos(f*x+e)^4*c^2*d^5+382*sin(f*x+e)*cos(f*x+e)^4*c*d^6-156*cos(f*x
+e)^8*d^7+635*cos(f*x+e)^6*d^7-954*cos(f*x+e)^4*d^7-105*cos(f*x+e)^2*c^7+62
7*cos(f*x+e)^2*d^7-280*sin(f*x+e)*c^7)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d
^2)^4/(c+d)^4
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(217) = 434$.
time = 0.64, size = 780, normalized size = 3.41

$$-\frac{2\left((175c^4+133c^3d+69c^2d^2+15cd^3)a^{\frac{3}{2}}-\frac{3(35c^4-385c^3d-189c^2d^2-67cd^3-10d^4)a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{18(35c^4-28c^3d+166c^2d^2+44cd^3+7d^4)a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14(35c^4-220c^3d+102c^2d^2-244cd^3-25d^4)a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{42(20c^4-61c^3d+117c^2d^2-55cd^3+35d^4)a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{42(20c^4-61c^3d+117c^2d^2-55cd^3+35d^4)a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{14(35c^4-220c^3d+102c^2d^2-244cd^3-25d^4)a^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{18(35c^4-28c^3d+166c^2d^2+44cd^3+7d^4)a^{\frac{3}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{3(35c^4-385c^3d-189c^2d^2-67cd^3-10d^4)a^{\frac{3}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{(175c^4+133c^3d+69c^2d^2+15cd^3)a^{\frac{3}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^3}{105\left(c^4+4c^3d+6c^2d^2+4cd^3+d^4+\frac{3(c^4+4c^3d+6c^2d^2+4cd^3+d^4)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3(c^4+4c^3d+6c^2d^2+4cd^3+d^4)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{(c^4+4c^3d+6c^2d^2+4cd^3+d^4)\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)\left(c+\frac{2d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/105*((175*c^4 + 133*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2) - 3*(35*c^4 -
 385*c^3*d - 189*c^2*d^2 - 67*c*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(35*c^4 - 220*c^3*d + 102*c^2*d^
2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 42*(2
0*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*d^4)*a^(3/2)*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 42*(20*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*
d^4)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(35*c^4 - 220*c^3*d +
 102*c^2*d^2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 385*c^3*d - 189*c^2*d^2 - 67*c
*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (175*c^4 + 133
*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^4 + 4*c^3*d + 6*c^2*d^2 +
 4*c*d^3 + d^4 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d
^4)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e
) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(217) = 434$.

time = 0.40, size = 957, normalized size = 4.18

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 2/105*(8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^4 - 140*a*c^3 + 308*a*c^2*d - 24
4*a*c*d^2 + 76*a*d^3 + 4*(7*a*c^2*d + 92*a*c*d^2 + 13*a*d^3)*cos(f*x + e)^3
 - (35*a*c^3 + 441*a*c^2*d - 167*a*c*d^2 + 195*a*d^3)*cos(f*x + e)^2 - (175
*a*c^3 + 161*a*c^2*d + 437*a*c*d^2 + 67*a*d^3)*cos(f*x + e) + (140*a*c^3 -
308*a*c^2*d + 244*a*c*d^2 - 76*a*d^3 + 8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^
3 - 4*(7*a*c^2*d + 90*a*c*d^2 - 13*a*d^3)*cos(f*x + e)^2 - (35*a*c^3 + 469*
a*c^2*d + 193*a*c*d^2 + 143*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4
*c*d^7 + d^8)*f*cos(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*
c^2*d^6 + 8*c*d^7 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*
c^4*d^4 + 16*c^3*d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c
```

```
^7*d + 11*c^6*d^2 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c
*d^7 + d^8)*f*cos(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 3
8*c^4*d^4 + 28*c^3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8
+ 8*c^7*d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6
+ 8*c*d^7 + d^8)*f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^4 - 4*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*c
os(f*x + e)^3 - 2*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^
2*d^6 + 6*c*d^7 + d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3
+ 8*c^4*d^4 + 7*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*
d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^
7 + d^8)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**B**]
time = 20.21, size = 807, normalized size = 3.52

$$\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{32\,a\,{\mathrm{e}}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\left(c+13\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{105\,d^2\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(9\,c^3-5\,c^2\,d+9\,c\,d^2-d^3\right)}{3\,d^4\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c^3\,9\mathrm{i}-c^2\,d\,5\mathrm{i}+c\,d^2\,9\mathrm{i}-d^3\,1\mathrm{i}\right)}{3\,d^4\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(5\,c^3+65\,c^2\,d+c\,d^2+13\,d^3\right)}{15\,d^4\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c^3\,5\mathrm{i}+c^2\,d\,65\mathrm{i}+c\,d^2\,1\mathrm{i}+d^3\,13\mathrm{i}\right)}{15\,d^4\,f\,{\left(c+d\right)}^4}+\frac{32\,a\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(c\,1\mathrm{i}+d\,13\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{105\,d^2\,f\,{\left(c+d\right)}^4}+\frac{32\,a\,c\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\left(c\,1\mathrm{i}+d\,13\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^3\,f\,{\left(c+d\right)}^4}+\frac{32\,a\,c\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(c+13\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^3\,f\,{\left(c+d\right)}^4}\right)}{{\mathrm{e}}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}+\frac{{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^4\,1\mathrm{i}}{{\left(c+d\right)}^4}-\frac{4\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\left(8\,c^3+6\,c^2\,d+6\,c\,d^2+d^3\right)}{d^3}-\frac{4\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\left(6\,c^2+2\,c\,d+d^2\right)}{d^2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(8\,c+d\right)}{d}+\frac{2\,{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{d^4}-\frac{{\mathrm{e}}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^4\,\left(8\,c^3+6\,c^2\,d+6\,c\,d^2+d^3\right)\,4\mathrm{i}}{d^3\,{\left(c+d\right)}^4}-\frac{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^4\,\left(6\,c^2+2\,c\,d+d^2\right)\,4\mathrm{i}}{d^2\,{\left(c+d\right)}^4}+\frac{{\mathrm{e}}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\left(8\,c+d\right)\,{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^4\,1\mathrm{i}}{d\,{\left(c+d\right)}^4}+\frac{{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^4\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)\,2\mathrm{i}}{d^4\,{\left(c+d\right)}^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((32*a*exp(e*8i + f*x*8i)*(c + 13*d)*(a + a*sin
(e + f*x))^(1/2))/(105*d^2*f*(c + d)^4) - (16*a*exp(e*4i + f*x*4i)*(a + a*s
in(e + f*x))^(1/2)*(9*c*d^2 - 5*c^2*d + 9*c^3 - d^3))/(3*d^4*f*(c + d)^4) -
 (16*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*9i - c^2*d*5i +
 c^3*9i - d^3*1i))/(3*d^4*f*(c + d)^4) - (16*a*exp(e*6i + f*x*6i)*(a + a*si
```

```
n(e + f*x))^(1/2)*(c*d^2 + 65*c^2*d + 5*c^3 + 13*d^3))/(15*d^4*f*(c + d)^4)
 - (16*a*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*1i + c^2*d*65
i + c^3*5i + d^3*13i))/(15*d^4*f*(c + d)^4) + (32*a*exp(e*1i + f*x*1i)*(c*1
i + d*13i)*(a + a*sin(e + f*x))^(1/2))/(105*d^2*f*(c + d)^4) + (32*a*c*exp(
e*7i + f*x*7i)*(c*1i + d*13i)*(a + a*sin(e + f*x))^(1/2))/(15*d^3*f*(c + d)
^4) + (32*a*c*exp(e*2i + f*x*2i)*(c + 13*d)*(a + a*sin(e + f*x))^(1/2))/(15
*d^3*f*(c + d)^4)))/(exp(e*9i + f*x*9i) + ((c*1i + d*1i)^4*1i)/(c + d)^4 -
(4*exp(e*3i + f*x*3i)*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3))/d^3 - (4*exp(e*7i
+ f*x*7i)*(2*c*d + 6*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(8*c + d))/d + (
2*exp(e*5i + f*x*5i)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/d^
4 - (exp(e*6i + f*x*6i)*(c*1i + d*1i)^4*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3)*4
i)/(d^3*(c + d)^4) - (exp(e*2i + f*x*2i)*(c*1i + d*1i)^4*(2*c*d + 6*c^2 + d
^2)*4i)/(d^2*(c + d)^4) + (exp(e*8i + f*x*8i)*(8*c + d)*(c*1i + d*1i)^4*1i)
/(d*(c + d)^4) + (exp(e*4i + f*x*4i)*(c*1i + d*1i)^4*(12*c*d^3 + 16*c^3*d +
 8*c^4 + 3*d^4 + 24*c^2*d^2)*2i)/(d^4*(c + d)^4))
```

# 3.579 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=377

$$-\frac{a^{5/2}(c+d)^3\left(3c^2-34cd+283d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{128d^{5/2}f}-\frac{a^3(c+d)^2\left(3c^2-3\right.}{}$$

```
[Out] -1/128*a^(5/2)*(c+d)^3*(3*c^2-34*c*d+283*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(
1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f-1/192*a^3*(c+
d)*(3*c^2-34*c*d+283*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(
f*x+e))^(1/2)-1/240*a^3*(3*c^2-34*c*d+283*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^
(5/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)+3/40*a^3*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^(7/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*cos(f*x+e)*(c+d*sin(f*x+e))
^(7/2)*(a+a*sin(f*x+e))^(1/2)/d/f-1/128*a^3*(c+d)^2*(3*c^2-34*c*d+283*d^2)*
cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.57, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2842, 3060, 2849, 2854, 211}

$$-\frac{a^{5/2}(c+d)^3\left(3c^2-34cd+283d^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{128d^{5/2}f}-\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{192d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)^2\left(3c^2-34cd+283d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{128d^2f\sqrt{a\sin(e+fx)+a}}+\frac{3a^3(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{40d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{7/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/128*(a^(5/2)*(c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*ArcTan[(Sqrt[a]*Sqrt[d
]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5
/2)*f) - (a^3*(c + d)^2*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/(128*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(c + d)*(3*c^2 -
 34*c*d + 283*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(192*d^2*f*Sqrt
[a + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*(c + d
*Sin[e + f*x])^(5/2))/(240*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*a^3*(c - 7*
d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(40*d^2*f*Sqrt[a + a*Sin[e + f*
x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2
))/(5*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx)}{5df}$$

$$= \frac{3a^3(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{40d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(}{}$$

$$= -\frac{a^3(3c^2-34cd+283d^2)\cos(e+fx)(c+d\sin(e+fx)}{240d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)\,(3c^2-34cd+283d^2)\cos(e+fx)(c+d\sin(}{192d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)^2\,(3c^2-34cd+283d^2)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{128d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)^2\,(3c^2-34cd+283d^2)\cos(e+fx)\sqrt{c+d\,\mathrm{si}}}{128d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^{5/2}(c+d)^3\,(3c^2-34cd+283d^2)\tan^{-1}\Big(\frac{}{\sqrt{a+a\sin}}}{128d^{5/2}f}$$

**Mathematica** [A]
time = 1.89, size = 395, normalized size = 1.05

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(c+d)^3(3c^2-34cd+283d^2)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{5/2}}+\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\,(45c^4-390c^3d-8396c^2d^2-12762cd^3-5521d^4+4d^2(93c^2+488cd+331d^2)\cos(2(e+fx))-48d^4\cos(4(e+fx))-30c^3d\sin(e+fx)-3322c^2d^2\sin(e+fx)-7774cd^3\sin(e+fx)-3874d^4\sin(e+fx)+252cd^3\sin(3(e+fx))+348d^4\sin(3(e+fx)))}{15d^2}\right)}{256f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(45*c^4 - 390*c^3*d - 8396*c^2*d^2 - 12762*c*d^3 - 5521*d^4 + 4*d^2
*(93*c^2 + 488*c*d + 331*d^2)*Cos[2*(e + f*x)] - 48*d^4*Cos[4*(e + f*x)] -
30*c^3*d*Sin[e + f*x] - 3322*c^2*d^2*Sin[e + f*x] - 7774*c*d^3*Sin[e + f*x]
 - 3874*d^4*Sin[e + f*x] + 252*c*d^3*Sin[3*(e + f*x)] + 348*d^4*Sin[3*(e +
f*x)]))/(15*d^2)))/(256*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{5}{2}} (c+d\sin(fx+e))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(351) = 702.
time = 1.33, size = 2145, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] [1/15360*(15*(3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3
+ 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2
 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5
 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a
^2*d^5)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*
a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x
 + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c
^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*
x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*
x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^
2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*
c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^
```

```
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(384*a^2*d^4*cos(f*x + e)^5 - 45*
a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4
- 1008*(a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c
*d^3 + 379*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2
565*a^2*c*d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d
- 8768*a^2*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^
2*d^4*cos(f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688
*a^2*c*d^3 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 -
 8*(93*a^2*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^
2*c^3*d + 1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(
f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/7680*(15*(3*a^2*c^5 - 25*a^2*c^4*
d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^
2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 +
283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 7
50*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*sin(f*x + e))*sqrt(a/d)*arcta
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*c
os(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*
d + 2*a*d^2)*cos(f*x + e))) - 4*(384*a^2*d^4*cos(f*x + e)^5 - 45*a^2*c^4 +
360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4 - 1008*(a^
2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c*d^3 + 379
*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2565*a^2*c*
d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d - 8768*a^2
*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^2*d^4*cos(
f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3
 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 - 8*(93*a^2
*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^2*c^3*d +
1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) +
 d^2*f*sin(f*x + e) + d^2*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.580 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=312

$$-\frac{a^{5/2}(c+d)^2\left(3c^2-26cd+163d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{5/2}f}-\frac{a^3(c+d)\left(3c^2-2\right.}{}$$

```
[Out] -1/64*a^(5/2)*(c+d)^2*(3*c^2-26*c*d+163*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f-1/96*a^3*(3*c^
2-26*c*d+163*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(f*x+e))^
(1/2)+1/24*a^3*(3*c-17*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f/(a+a*sin(
f*x+e))^(1/2)-1/4*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1
/2)/d/f-1/64*a^3*(c+d)*(3*c^2-26*c*d+163*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(
1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.47, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2842, 3060, 2849, 2854, 211}

$$-\frac{a^{5/2}(c+d)^2\left(3c^2-26cd+163d^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{5/2}f}-\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64d^2f\sqrt{a\sin(e+fx)+a}}+\frac{a^3(3c-17d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{5/2}}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/64*(a^(5/2)*(c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]
*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/
2)*f) - (a^3*(c + d)*(3*c^2 - 26*c*d + 163*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(64*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 26*c*d + 16
3*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(96*d^2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a^3*(3*c - 17*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(24*d
^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(5/2))/(4*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
```

```
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx)}{4df}$$

$$= \frac{a^3(3c-17d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos}{}$$

$$= -\frac{a^3(3c^2-26cd+163d^2)\cos(e+fx)(c+d\sin(e+fx)}{96d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)\,(3c^2-26cd+163d^2)\cos(e+fx)\sqrt{c+d\sin}}{64d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)\,(3c^2-26cd+163d^2)\cos(e+fx)\sqrt{c+d\sin}}{64d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^{5/2}(c+d)^2\,(3c^2-26cd+163d^2)\tan^{-1}\left(\frac{}{\sqrt{a+a\sin}}\right.}{64d^{5/2}f}$$

**Mathematica [A]**
time = 1.17, size = 327, normalized size = 1.05

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(c+d)^2(3c^2-26cd+163d^2)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{5/2}}+\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(9c^3-63c^2d-773cd^2-581d^3+4d^2(9c+23d)\cos(2(e+fx))-2d(3c^2+158cd+181d^2)\sin(e+fx)+12d^3\sin(3(e+fx))\right)}{3d^2}\right)}{128f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(9*c^3 - 63*c^2*d - 773*c*d^2 - 581*d^3 + 4*d^2*(9*c + 23*d)*Cos[2*
(e + f*x)] - 2*d*(3*c^2 + 158*c*d + 181*d^2)*Sin[e + f*x] + 12*d^3*Sin[3*(e
 + f*x)]))/(3*d^2)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{5}{2}}(c+d\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(289) = 578$.
time = 1.12, size = 1809, normalized size = 5.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/1536*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 16
3*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 1
63*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 30
0*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x
+ e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d
^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f
*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2
 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(
c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e
)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*
x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos
(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))
*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28
*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d
^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 2
56*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4
)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*
x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(48*a^2*d^3*co
s(f*x + e)^4 + 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(
```

```
9*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 +
 119*a^2*d^3)*cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 -
673*a^2*d^3)*cos(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2
*c^2*d + 493*a^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x
 + e)^2 - 2*(3*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
+ e) + d^2*f*sin(f*x + e) + d^2*f), 1/768*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 11
4*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 1
14*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 2
0*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*
sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
 d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e)
- (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(48*a^2*d^3*cos(f*x + e)^4 +
 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(9*a^2*c*d^2 +
23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 + 119*a^2*d^3)*
cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 - 673*a^2*d^3)*c
os(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2*c^2*d + 493*a
^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x + e)^2 - 2*(3
*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f)]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.581 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=241

$$-\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}-\frac{a^3\left(c^2-6cd+25d^2\right)\text{co}}{8d^2f\sqrt{a}}$$

[Out] -1/8*a^(5/2)*(c+d)*(c^2-6*c*d+25*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+1/12*a^3*(3*c-13*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/3*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/d/f-1/8*a^3*(c^2-6*c*d+25*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)

**Rubi** [A]
time = 0.37, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$,
Rules used = {2842, 3060, 2849, 2854, 211}

$$-\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}-\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a\sin(e+fx)+a}}+\frac{a^3(3c-13d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/8*(a^(5/2)*(c + d)*(c^2 - 6*c*d + 25*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e
+ f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) -
 (a^3*(c^2 - 6*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d^2*
f*Sqrt[a + a*Sin[e + f*x]]) + (a^3*(3*c - 13*d)*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^(3/2))/(12*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
```

```
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}{3df}$$

$$= \frac{a^3(3c-13d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(}{}$$

$$= -\frac{a^3(c^2-6cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{a^3(c^2-6cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+a\sin(e+f}}\right.}{8d^{5/2}f}$$

## Mathematica [A]

time = 0.64, size = 285, normalized size = 1.18

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(c+d)(c^2-6cd+25d^2)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{5/2}}+\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(3c^2-16cd-79d^2+4d^2\cos(2(e+fx))-2d(c+17d)\sin(e+fx)\right)}{3d^2}\right)}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)*(c^2 - 6*c*d + 25*d^2)*(2*ArcTan[(S
qrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTa
nh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] -
Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))
/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]*(3*c^2 - 16*c*d - 79*d^2 + 4*d^2*Cos[2*(e + f*x)] - 2*d*(c + 17*d)*Sin[e
+ f*x]))/(3*d^2)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

## Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{5}{2}}\sqrt{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(221) = 442$.
time = 0.82, size = 1509, normalized size = 6.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5
*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2
*d + 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos
(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*
a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*
cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x +
 e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 +
 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*
x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*c
os(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4
)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(8*a^2*d^
2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 - 2*(a^2*c*d + 13*a^
2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83*a^2*d^2)*cos(f*x + e)
- (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 + 2*(a^2*
c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/
96*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5*a^2
```

```
*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2*d +
 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3
*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f
*x + e))) + 4*(8*a^2*d^2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d
^2 - 2*(a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83
*a^2*d^2)*cos(f*x + e) - (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d
 - 49*a^2*d^2 + 2*(a^2*c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*si
n(f*x + e) + d^2*f)]
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x))^{5/2} \sqrt{c + d \sin(e + f x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2), x)
```

## 3.582 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=178

$$-\frac{a^{5/2}(3c^2-10cd+19d^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+}}{4d^2f\sqrt{a+a\sin(e+}}$$

```
[Out] -1/4*a^(5/2)*(3*c^2-10*c*d+19*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*s
in(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+3/4*a^3*(c-3*d)*cos(f*x+
e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/2*a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

**Rubi** [A]
time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2842, 3060, 2854, 211}

$$-\frac{a^{5/2}(3c^2-10cd+19d^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/4*(a^(5/2)*(3*c^2 - 10*c*d + 19*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (3*a
^3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + a*Sin
[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]])/(2*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
```

```
, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \sqrt{a+a\sin(e}{}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e}{2d}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e}{2d}$$

$$= -\frac{a^{5/2}(3c^2-10cd+19d^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}$$

## Mathematica [A]

time = 0.53, size = 256, normalized size = 1.44

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(3c^2-10cd+19d^2)\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\,\sqrt{d}\,\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{5/2}}+\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3c-11d-2d\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{d^2}\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^2 - 10*c*d + 19*d^2)*(2*ArcTan[(Sqrt[2
]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5
/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c - 11*d - 2*d*Sin[e + f*
x])*Sqrt[c + d*Sin[e + f*x]])/d^2))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5)
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{c + d\sin(fx+e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(161) = 322.
time = 0.70, size = 1269, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/32*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*
a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))
```

```
*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2
*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 +
9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17
*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d
^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25
*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3
 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 +
13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 +
 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d +
6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32
*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2
*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + si
n(f*x + e) + 1)) - 8*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c
 - 11*a^2*d)*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
 + e) + d^2*f*sin(f*x + e) + d^2*f), 1/16*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2
*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10
*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*
d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
e))) - 4*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c - 11*a^2*d)
*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*
f*sin(f*x + e) + d^2*f)]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

[Out] Timed out

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^{5/2}}{\sqrt{c + d\,\sin(e + f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.583 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=180

$$\frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+f}{d(c+d)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] a^(5/2)*(3*c-5*d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+2*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(
1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-a^3*(3*c-d)*cos(f*x+e)*(c+d*sin(f*x+e
))^(1/2)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.29, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 3060, 2854, 211}

$$\frac{a^{5/2}(3c-5d)\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}-\frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(5/2)*(3*c - 5*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (a^
3*(3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c + d)*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\,(\frac{1}{2}a(\ldots}{\sqrt{c+d\sin\ldots}}}{d(c+\ldots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\ldots}}{d^2(c+d)f\sqrt{a+a\sin(e\ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\ldots}}{d^2(c+d)f\sqrt{a+a\sin(e\ldots}}$$

$$= \frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d\ldots}{d(c\ldots}$$

## **Mathematica** [A]

time = 0.71, size = 263, normalized size = 1.46

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(-3c+5d)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)\right)}{d^{5/2}}-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3c^2-3cd+2d^2+d(c+d)\sin(e+fx)\right)}{d^2(c+d)\sqrt{c+d\sin(e+fx)}}\right)}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-3*c + 5*d)*(2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin
[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5/2) - (2*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c^2 - 3*c*d + 2*d^2 + d*(c + d)*Sin[e
+ f*x]))/(d^2*(c + d)*Sqrt[c + d*Sin[e + f*x]])))/(2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5)
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^{\frac{5}{2}}}{(c + d\sin(fx + e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(169) = 338$.
time = 0.72, size = 1723, normalized size = 9.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*((3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a
^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d
```

```
^2)*cos(f*x + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^
2*c^2*d - 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/d)*l
og((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
+ a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c
*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4
*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59
*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
 + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8
*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sq
rt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*c
os(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c
*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) +
1)) + 8*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d^2)*cos(f*x + e)
^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*a^2*c^2 - 4*a^2*
c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (
c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d
^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), 1/4*((3*a^
2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a^2*c*d^2 -
5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d^2)*cos(f*x
 + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2*d - 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8
*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x +
 e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*
d^2)*cos(f*x + e))) + 4*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d
^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*
a^2*c^2 - 4*a^2*c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*co
s(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)
*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x +
 e))]
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^{5/2}}{(c + d \sin(e + f x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.584 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=183

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}}+\frac{}{3d}$$

```
[Out] -2*a^(5/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/d^(5/2)/f+2/3*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/
d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)+2/3*a^3*(c-d)*(3*c+7*d)*cos(f*x+e)/d^2/(c+
d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.29, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 3059, 2854, 211}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df(c+d)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) + (2*a^3*(
c - d)*(3*c + 7*d)*Cos[e + f*x])/(3*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
```

```
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\,(\tfrac{1}{2}}{(c+d\sin(e}}{3d(c+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d}{3d^2(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d}{3d^2(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d)\cos}{3d(c+d}$$

## **Mathematica** [A]

time = 6.23, size = 261, normalized size = 1.43

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sqrt{c+d\sin(e+fx)}\right)}{d^{5/2}}+\frac{2(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3c^2+8cd+d^2+4d(c+2d)\sin(e+fx)\right)}{3d^2(c+d)^2(c+d\sin(e+fx))^{3/2}}\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*((2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2
*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]])/d^(5/2) + (2*(c - d)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(3*c^2 + 8*c*d + d^2 + 4*d*(c + 2*d)*Sin[e + f*x
]))/(3*d^2*(c + d)^2*(c + d*Sin[e + f*x])^(3/2))))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 16222 vs. $2(159) = 318$.
time = 0.40, size = 16223, normalized size = 88.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(168) = 336$.
time = 0.72, size = 2363, normalized size = 12.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/12*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 -
(a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2*a^2*c^3*d + 5*a^2
*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c^4 + 2*a^2*c^3*d +
```

2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a^2*c^4 + 4*a^2*c^3
*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a
^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a^2*c*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*
c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3
*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x
+ e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x
+ e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*
d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17
*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^
2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*
d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a
*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*
cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32
*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))
/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d
^2 + 7*a^2*d^3 + 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*
a^2*c^3 + 5*a^2*c^2*d - 7*a^2*c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 +
a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4
+ 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*
d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^
6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4
+ c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6
)*f)*sin(f*x + e)), -1/6*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*
c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2
*a^2*c^3*d + 5*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c
^4 + 2*a^2*c^3*d + 2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a
^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2
+ 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a
^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
e))) + 4*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 + 4*(a^2*c^2*d
+ a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 5*a^2*c^2*d - 7*a^2*
c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a
^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c^2*d^4 + 2*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e
)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c
^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d

```
^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^
4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^{5/2}}{(c + d \sin(e + f x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.585 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=189

$$\frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}-\frac{}{15c}$$

```
[Out] 2/15*a^3*(c-d)*(3*c+11*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+2/5*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+
d)/f/(c+d*sin(f*x+e))^(5/2)-2/15*a^3*(3*c^2+14*c*d+43*d^2)*cos(f*x+e)/d^2/(
c+d)^3/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2841, 3059, 2850}

$$-\frac{2a^3(3c^2+14cd+43d^2)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5df(c+d)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*d*(c + d)*f*(c + d
*Sin[e + f*x])^(5/2)) + (2*a^3*(c - d)*(3*c + 11*d)*Cos[e + f*x])/(15*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (2*a^3*(3
*c^2 + 14*c*d + 43*d^2)*Cos[e + f*x])/(15*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
```

```
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(\ldots\right.}{(c+d\sin(e\ldots}}{5d(c+\ldots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11\ldots}{15d^2(c+d)^2f\sqrt{a+a\sin(e+\ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11\ldots}{15d^2(c+d)^2f\sqrt{a+a\sin(e+\ldots}}$$

**Mathematica [A]**
time = 1.56, size = 152, normalized size = 0.80

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(89c^2+42cd+49d^2-\left(3c^2+14cd+43d^2\right)\cos(2(e+fx))+4\left(7c^2+46cd+7d^2\right)\sin(e+fx)\right)}{15(c+d)^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(89*c^2 + 42*c*d + 49*d^2 - (3*c^2 + 14*c*d + 43*d^2)*Cos[2*(e + f*x)] + 4
*(7*c^2 + 46*c*d + 7*d^2)*Sin[e + f*x]))/((c + d)^3*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(171) = 342$.
time = 7.18, size = 793, normalized size = 4.20

| method | result |
|---|---|
| default | $-\frac{2(a(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{c+d\sin(fx+e)}\,(43(\cos^8(fx+e))d^5-256c^3d^2+128c\,d^4+128d^5-256c^2d^3-128c^5\sin(fx+e)+12}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(-256*c^3*d^2+128*c
*d^4+128*d^5-256*c^2*d^3-128*c^5*sin(f*x+e)+128*c^5+544*cos(f*x+e)^2*c^2*d^
3-400*cos(f*x+e)^2*c*d^4-128*sin(f*x+e)*c^4*d+256*sin(f*x+e)*c^3*d^2+256*si
n(f*x+e)*c^2*d^3-128*sin(f*x+e)*c*d^4-9*cos(f*x+e)^6*c^4*d-27*cos(f*x+e)^6*
c^3*d^2-cos(f*x+e)^6*c^2*d^3-181*cos(f*x+e)^6*c*d^4-3*sin(f*x+e)*cos(f*x+e)
^4*c^5-355*sin(f*x+e)*cos(f*x+e)^4*d^5+7*cos(f*x+e)^4*c^4*d-194*cos(f*x+e)^
4*c^3*d^2-290*cos(f*x+e)^4*c^2*d^3+439*cos(f*x+e)^4*c*d^4+16*sin(f*x+e)*cos
(f*x+e)^2*c^5+368*sin(f*x+e)*cos(f*x+e)^2*d^5+3*cos(f*x+e)^8*c^2*d^3+14*cos
(f*x+e)^8*c*d^4+115*sin(f*x+e)*cos(f*x+e)^6*d^5+480*cos(f*x+e)^2*c^3*d^2+12
8*d*c^4-112*cos(f*x+e)^2*c^4*d+43*cos(f*x+e)^8*d^5-262*cos(f*x+e)^6*d^5-5*c
os(f*x+e)^4*c^5+523*cos(f*x+e)^4*d^5-80*cos(f*x+e)^2*c^5-432*cos(f*x+e)^2*d
^5-128*sin(f*x+e)*d^5+336*sin(f*x+e)*cos(f*x+e)^2*c*d^4+79*sin(f*x+e)*cos(f
*x+e)^6*c*d^4+sin(f*x+e)*cos(f*x+e)^4*c^4*d+50*sin(f*x+e)*cos(f*x+e)^4*c^3*
d^2+114*sin(f*x+e)*cos(f*x+e)^4*c^2*d^3-287*sin(f*x+e)*cos(f*x+e)^4*c*d^4+4
8*sin(f*x+e)*cos(f*x+e)^2*c^4*d-352*sin(f*x+e)*cos(f*x+e)^2*c^3*d^2-416*sin
(f*x+e)*cos(f*x+e)^2*c^2*d^3+9*sin(f*x+e)*cos(f*x+e)^6*c^3*d^2+37*sin(f*x+e
)*cos(f*x+e)^6*c^2*d^3)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^3/(c+d)^3
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(180) = 360$.
time = 0.67, size = 486, normalized size = 2.57

$$-\frac{2\left(\left(43\,c^3+14\,c^2d+3\,cd^2\right)a^{\frac{5}{2}}-\frac{\left(15\,c^3-256\,c^2d-53\,cd^2-6\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\left(113\,c^3-116\,c^2d+493\,cd^2+50\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)^2}{\left(\cos(fx+e)+1\right)^2}-\frac{5\left(17\,c^3-82\,c^2d+65\,cd^2-60\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)^3}{\left(\cos(fx+e)+1\right)^3}+\frac{5\left(17\,c^3-82\,c^2d+65\,cd^2-60\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)^4}{\left(\cos(fx+e)+1\right)^4}-\frac{\left(113\,c^3-116\,c^2d+493\,cd^2+50\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)^5}{\left(\cos(fx+e)+1\right)^5}+\frac{\left(15\,c^3-256\,c^2d-53\,cd^2-6\,d^3\right)a^{\frac{5}{2}}\sin(fx+e)^6}{\left(\cos(fx+e)+1\right)^6}-\frac{\left(43\,c^3+14\,c^2d+3\,cd^2\right)a^{\frac{5}{2}}\sin(fx+e)^7}{\left(\cos(fx+e)+1\right)^7}\right)\left(\frac{\sin(fx+e)^2}{\left(\cos(fx+e)+1\right)^2}+1\right)}{15\left(c^3+3\,c^2d+3\,cd^2+d^3+\frac{\left(c^3+3\,c^2d+3\,cd^2+d^3\right)\sin(fx+e)^2}{\left(\cos(fx+e)+1\right)^2}\right)\left(c+\frac{2\,d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{\left(\cos(fx+e)+1\right)^2}\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((43*c^3 + 14*c^2*d + 3*c*d^2)*a^(5/2) - (15*c^3 - 256*c^2*d - 53*c*d
^2 - 6*d^3)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (113*c^3 - 116*c^2*d
+ 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(17*c
^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*(17*c^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - (113*c^3 - 116*c^2*d + 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 256*c^2*d - 53*c*d^2 - 6*d^3)*a^
(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (43*c^3 + 14*c^2*d + 3*c*d^2)*a
```

```
^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 1)/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
+ 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(180) = 360$.
time = 0.39, size = 670, normalized size = 3.54

$$-\frac{2\left(32\,a^{2}c^{2}-64\,a^{2}cd+32\,a^{2}d^{2}-\left(3\,a^{2}c^{2}+14\,a^{2}cd+43\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{3}+\left(11\,a^{2}c^{2}+78\,a^{2}cd-29\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{2}+2\left(23\,a^{2}c^{2}+14\,a^{2}cd+23\,a^{2}d^{2}\right)\cos\left(fx+e\right)-\left(32\,a^{2}c^{2}-64\,a^{2}cd+32\,a^{2}d^{2}-\left(3\,a^{2}c^{2}+14\,a^{2}cd+43\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{2}-2\left(7\,a^{2}c^{2}+46\,a^{2}cd+7\,a^{2}d^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{15\left(\left(c^{3}d^{3}+3\,c^{2}d^{4}+3\,cd^{5}+d^{6}\right)f\cos\left(fx+e\right)^{4}-3\left(c^{4}d^{2}+3\,c^{3}d^{3}+3\,c^{2}d^{4}+cd^{5}\right)f\cos\left(fx+e\right)^{3}-\left(3\,c^{5}d+12\,c^{4}d^{2}+20\,c^{3}d^{3}+18\,c^{2}d^{4}+9\,cd^{5}+2\,d^{6}\right)f\cos\left(fx+e\right)^{2}+\left(c^{6}+3\,c^{5}d+6\,c^{4}d^{2}+10\,c^{3}d^{3}+9\,c^{2}d^{4}+3\,cd^{5}\right)f\cos\left(fx+e\right)+\left(c^{6}+6\,c^{5}d+15\,c^{4}d^{2}+20\,c^{3}d^{3}+15\,c^{2}d^{4}+6\,cd^{5}+d^{6}\right)f-\left(\left(c^{3}d^{3}+3\,c^{2}d^{4}+3\,cd^{5}+d^{6}\right)f\cos\left(fx+e\right)^{3}+\left(3\,c^{4}d^{2}+10\,c^{3}d^{3}+12\,c^{2}d^{4}+6\,cd^{5}+d^{6}\right)f\cos\left(fx+e\right)^{2}-\left(3\,c^{5}d+9\,c^{4}d^{2}+10\,c^{3}d^{3}+6\,c^{2}d^{4}+3\,cd^{5}+d^{6}\right)f\cos\left(fx+e\right)-\left(c^{6}+6\,c^{5}d+15\,c^{4}d^{2}+20\,c^{3}d^{3}+15\,c^{2}d^{4}+6\,cd^{5}+d^{6}\right)f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(32*a^2*c^2 - 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*
a^2*d^2)*cos(f*x + e)^3 + (11*a^2*c^2 + 78*a^2*c*d - 29*a^2*d^2)*cos(f*x +
e)^2 + 2*(23*a^2*c^2 + 14*a^2*c*d + 23*a^2*d^2)*cos(f*x + e) - (32*a^2*c^2
- 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*a^2*d^2)*cos(f*x +
 e)^2 - 2*(7*a^2*c^2 + 46*a^2*c*d + 7*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 + 3
*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5
)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*
d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9
*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d
^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)
*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f
*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^
4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad [B]**
time = 17.46, size = 590, normalized size = 3.12

$$-\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{8a^2e^{e4i+fx4i}\sqrt{a+a\sin(e+fx)}(15c^2-10cd+7d^2)}{3d^3f(c+d)^3}+\frac{4a^2e^{e2i+fx2i}\sqrt{a+a\sin(e+fx)}(5c^2+34cd-3d^2)}{3d^3f(c+d)^3}-\frac{8a^2e^{e3i+fx3i}\sqrt{a+a\sin(e+fx)}(c^2 15i-cd\,10i+d^2 7i)}{3d^3f(c+d)^3}-\frac{4a^2e^{e5i+fx5i}\sqrt{a+a\sin(e+fx)}(c^2 5i+cd\,34i-d^2 3i)}{3d^3f(c+d)^3}-\frac{4a^2e^{e6i+fx6i}\sqrt{a+a\sin(e+fx)}(3c^2+14cd+43d^2)}{15d^3f(c+d)^3}+\frac{4a^2e^{e1i+fx1i}\sqrt{a+a\sin(e+fx)}(c^2 3i+cd\,14i+d^2 43i)}{15d^3f(c+d)^3}\right)}{e^{e7i+fx7i}+\frac{(c1i+d1i)^3}{(c+d)^3}-\frac{3e^{e5i+fx5i}(4c^2+2cd+d^2)}{d^2}-\frac{e^{e1i+fx1i}(6c+d)}{d}+\frac{e^{e3i+fx3i}(8c^3+12c^2d+12cd^2+3d^3)}{d^3}+\frac{e^{e6i+fx6i}(c6i+d1i)}{d}-\frac{3e^{e2i+fx2i}(c1i+d1i)^3(4c^2+2cd+d^2)}{d^2(c+d)^3}+\frac{e^{e4i+fx4i}(c1i+d1i)^3(8c^3+12c^2d+12cd^2+3d^3)}{d^3(c+d)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((8*a^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x)
)^(1/2)*(15*c^2 - 10*c*d + 7*d^2))/(3*d^3*f*(c + d)^3) + (4*a^2*exp(e*2i +
f*x*2i)*(a + a*sin(e + f*x))^(1/2)*(34*c*d + 5*c^2 - 3*d^2))/(3*d^3*f*(c +
d)^3) - (8*a^2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*15i - c*d
*10i + d^2*7i))/(3*d^3*f*(c + d)^3) - (4*a^2*exp(e*5i + f*x*5i)*(a + a*sin(
e + f*x))^(1/2)*(c*d*34i + c^2*5i - d^2*3i))/(3*d^3*f*(c + d)^3) - (4*a^2*e
xp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*(14*c*d + 3*c^2 + 43*d^2))/(15
*d^3*f*(c + d)^3) + (4*a^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2)*(c
*d*14i + c^2*3i + d^2*43i))/(15*d^3*f*(c + d)^3)))/(exp(e*7i + f*x*7i) + (c
*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2))/d^2
- (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 + 12*c^2
*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3*exp(e*
2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3) + (exp(
e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(d^3*
(c + d)^3))
```

# 3.586 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=254

$$\frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}}+\frac{6a^3(c-d)(c+5d)\cos(e+fx)}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}-\frac{\ldots}{105d\ldots}$$

```
[Out] 6/35*a^3*(c-d)*(c+5*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)/(a+a
*sin(f*x+e))^(1/2)-2/105*a^3*(3*c^2+22*c*d+115*d^2)*cos(f*x+e)/d^2/(c+d)^3/
f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2/7*a^2*(c-d)*cos(f*x+e)*(a
+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)-4/105*a^3*(3*c^2+22*c
*d+115*d^2)*cos(f*x+e)/d^2/(c+d)^4/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)
)^(1/2)
```

**Rubi** [A]
time = 0.41, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 3059, 2851, 2850}

$$-\frac{4a^3(3c^2+22cd+115d^2)\cos(e+fx)}{105d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^3(3c^2+22cd+115d^2)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+\frac{6a^3(c-d)(c+5d)\cos(e+fx)}{35d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{5/2}}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{7df(c+d)(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(7*d*(c + d)*f*(c + d
*Sin[e + f*x])^(7/2)) + (6*a^3*(c - d)*(c + 5*d)*Cos[e + f*x])/(35*d^2*(c +
 d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (2*a^3*(3*c^
2 + 22*c*d + 115*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(3*c^2 + 22*c*d + 115*d^2)*Cos[
e + f*x])/(105*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\,(\tfrac{1}{2}}{(c+d\sin}}{7d(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5}{35d^2(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5}{35d^2(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5}{35d^2(c+d)^2f\sqrt{a+a\sin(e+}}$$

**Mathematica** [A]

time = 2.73, size = 216, normalized size = 0.85

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-623c^3-495c^2d-977cd^2-145d^3+(21c^3+157c^2d+827cd^2+115d^3)\cos(2(e+fx))-(196c^3+1865c^2d+694cd^2+465d^3)\sin(e+fx)+3c^2d\sin(3(e+fx))+22cd^2\sin(3(e+fx))+115d^3\sin(3(e+fx))\right)}{105(c+d)^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-623
*c^3 - 495*c^2*d - 977*c*d^2 - 145*d^3 + (21*c^3 + 157*c^2*d + 827*c*d^2 +
115*d^3)*Cos[2*(e + f*x)] - (196*c^3 + 1865*c^2*d + 694*c*d^2 + 465*d^3)*Si
n[e + f*x] + 3*c^2*d*Sin[3*(e + f*x)] + 22*c*d^2*Sin[3*(e + f*x)] + 115*d^3
*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
(c + d*Sin[e + f*x])^(7/2))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(230) = 460$.
time = 7.29, size = 1223, normalized size = 4.81

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1223 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(6*cos(f*x+e)^10*c
^2*d^5-2176*c^5*d^2+1664*c^3*d^4-384*c*d^6+368*cos(f*x+e)^2*sin(f*x+e)*c^6*
d-3344*cos(f*x+e)^2*sin(f*x+e)*c^5*d^2-5008*cos(f*x+e)^2*sin(f*x+e)*c^4*d^3
+4560*cos(f*x+e)^2*sin(f*x+e)*c^3*d^4-640*d^7+44*cos(f*x+e)^10*c*d^6+575*co
s(f*x+e)^8*sin(f*x+e)*d^7+48*cos(f*x+e)^8*c^4*d^3+257*cos(f*x+e)^8*c^3*d^4+
895*cos(f*x+e)^8*c^2*d^5-485*cos(f*x+e)^8*c*d^6-2350*cos(f*x+e)^6*sin(f*x+e
)*d^7-78*cos(f*x+e)^6*c^6*d-302*cos(f*x+e)^6*c^5*d^2-172*cos(f*x+e)^6*c^4*d
^3-2968*cos(f*x+e)^6*c^3*d^4-5370*cos(f*x+e)^6*c^2*d^5+1590*cos(f*x+e)^6*c*
d^6-21*cos(f*x+e)^4*sin(f*x+e)*c^7+3615*cos(f*x+e)^4*sin(f*x+e)*d^7+39*cos(
f*x+e)^4*c^6*d-1879*cos(f*x+e)^4*c^5*d^2-3397*cos(f*x+e)^4*c^4*d^3+6439*cos
(f*x+e)^4*c^3*d^4+10373*cos(f*x+e)^4*c^2*d^5-2285*cos(f*x+e)^4*c*d^6+112*co
s(f*x+e)^2*sin(f*x+e)*c^7-2480*cos(f*x+e)^2*sin(f*x+e)*d^7-944*cos(f*x+e)^2
*c^6*d+4432*cos(f*x+e)^2*c^5*d^2+6480*cos(f*x+e)^2*c^4*d^3-5392*cos(f*x+e)^
2*c^3*d^4-8336*cos(f*x+e)^2*c^2*d^5+1520*cos(f*x+e)^2*c*d^6-1152*sin(f*x+e)
*c^6*d+2176*sin(f*x+e)*c^5*d^2+2944*sin(f*x+e)*c^4*d^3-1664*sin(f*x+e)*c^3*
d^4-2432*sin(f*x+e)*c^2*d^5+384*sin(f*x+e)*c*d^6-560*cos(f*x+e)^2*c^7+2800*
cos(f*x+e)^2*d^7-896*sin(f*x+e)*c^7+640*sin(f*x+e)*d^7+230*cos(f*x+e)^10*d^
7-1555*cos(f*x+e)^8*d^7+3940*cos(f*x+e)^6*d^7-35*cos(f*x+e)^4*c^7-4775*cos(
f*x+e)^4*d^7+896*c^7+7120*cos(f*x+e)^2*sin(f*x+e)*c^2*d^5-1328*cos(f*x+e)^2
*sin(f*x+e)*c*d^6+cos(f*x+e)^4*sin(f*x+e)*c^6*d+479*cos(f*x+e)^4*sin(f*x+e)
*c^5*d^2+1261*cos(f*x+e)^4*sin(f*x+e)*c^4*d^3-4367*cos(f*x+e)^4*sin(f*x+e)*
```

```
c^3*d^4-7117*cos(f*x+e)^4*sin(f*x+e)*c^2*d^5+1669*cos(f*x+e)^4*sin(f*x+e)*c
*d^6+3*cos(f*x+e)^8*sin(f*x+e)*c^3*d^4+37*cos(f*x+e)^8*sin(f*x+e)*c^2*d^5+2
25*cos(f*x+e)^8*sin(f*x+e)*c*d^6+102*cos(f*x+e)^6*sin(f*x+e)*c^5*d^2+518*co
s(f*x+e)^6*sin(f*x+e)*c^4*d^3+1408*cos(f*x+e)^6*sin(f*x+e)*c^3*d^4+2392*cos
(f*x+e)^6*sin(f*x+e)*c^2*d^5-950*cos(f*x+e)^6*sin(f*x+e)*c*d^6-2944*c^4*d^3
+2432*c^2*d^5+1152*c^6*d)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^4/(c+d)^4
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(242) = 484$.
time = 0.74, size = 731, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/105*((301*c^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2) - 3*(35*c^4 -
 763*c^3*d - 297*c^2*d^2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^
(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(50*c^4 - 421*c^3*d + 201*c^
2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
26*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 + 25*d^4)*a^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 126*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 +
 25*d^4)*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(50*c^4 - 421*c^3
*d + 201*c^2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^(5/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 763*c^3*d - 297*c^2*d^
2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (301*c
^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e
) + 1)^9)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^4 + 4*c^3*d + 6*c
^2*d^2 + 4*c*d^3 + d^4 + 2*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^
4)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
 + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(242) = 484$.
time = 0.41, size = 1053, normalized size = 4.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -2/105*(224*a^2*c^3 - 608*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 - 2*(3*a^
2*c^2*d + 22*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^4 - (21*a^2*c^3 + 157*a^
2*c^2*d + 827*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^3 + (77*a^2*c^3 + 783*a
^2*c^2*d - 425*a^2*c*d^2 + 405*a^2*d^3)*cos(f*x + e)^2 + 2*(161*a^2*c^3 + 1
63*a^2*c^2*d + 451*a^2*c*d^2 + 65*a^2*d^3)*cos(f*x + e) - (224*a^2*c^3 - 60
8*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 + 2*(3*a^2*c^2*d + 22*a^2*c*d^2 +
 115*a^2*d^3)*cos(f*x + e)^3 - (21*a^2*c^3 + 151*a^2*c^2*d + 783*a^2*c*d^2
- 115*a^2*d^3)*cos(f*x + e)^2 - 2*(49*a^2*c^3 + 467*a^2*c^2*d + 179*a^2*c*d
^2 + 145*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*c^2*d^6 + 8*c*d^7
 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*c^4*d^4 + 16*c^3*
d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c^7*d + 11*c^6*d^2
 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c*d^7 + d^8)*f*cos
(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 38*c^4*d^4 + 28*c^
3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^
6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*
f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^4 - 4
*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e)^3 - 2
*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^2*d^6 + 6*c*d^7 +
 d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3 + 8*c^4*d^4 + 7*c
^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^6*d^2 +
56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*f)*sin(f
*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad [B]**
time = 20.61, size = 862, normalized size = 3.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((8*a^2*exp(e*8i + f*x*8i)*(a + a*sin(e + f*x))
^(1/2)*(22*c*d + 3*c^2 + 115*d^2))/(105*d^3*f*(c + d)^4) + (8*a^2*exp(e*1i
+ f*x*1i)*(a + a*sin(e + f*x))^(1/2)*(c*d*22i + c^2*3i + d^2*115i))/(105*d^
3*f*(c + d)^4) - (8*a^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^(1/2)*(36*c
*d^2 - 25*c^2*d + 30*c^3 - 5*d^3))/(3*d^4*f*(c + d)^4) - (8*a^2*exp(e*5i +
f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*36i - c^2*d*25i + c^3*30i - d^3*5
i))/(3*d^4*f*(c + d)^4) - (8*a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1
/2)*(244*c^2*d - 19*c*d^2 + 25*c^3 + 50*d^3))/(15*d^4*f*(c + d)^4) - (8*a^2
*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*d*244i - c*d^2*19i + c^
3*25i + d^3*50i))/(15*d^4*f*(c + d)^4) + (8*a^2*c*exp(e*2i + f*x*2i)*(a + a
*sin(e + f*x))^(1/2)*(22*c*d + 3*c^2 + 115*d^2))/(15*d^4*f*(c + d)^4) + (8*
a^2*c*exp(e*7i + f*x*7i)*(a + a*sin(e + f*x))^(1/2)*(c*d*22i + c^2*3i + d^2
*115i))/(15*d^4*f*(c + d)^4)))/(exp(e*9i + f*x*9i) + ((c*1i + d*1i)^4*1i)/(
c + d)^4 - (4*exp(e*3i + f*x*3i)*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3))/d^3 - (
4*exp(e*7i + f*x*7i)*(2*c*d + 6*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(8*c
+ d))/d + (2*exp(e*5i + f*x*5i)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c
^2*d^2))/d^4 - (exp(e*6i + f*x*6i)*(c*1i + d*1i)^4*(6*c*d^2 + 6*c^2*d + 8*c
^3 + d^3)*4i)/(d^3*(c + d)^4) - (exp(e*2i + f*x*2i)*(c*1i + d*1i)^4*(2*c*d
+ 6*c^2 + d^2)*4i)/(d^2*(c + d)^4) + (exp(e*8i + f*x*8i)*(8*c + d)*(c*1i +
d*1i)^4*1i)/(d*(c + d)^4) + (exp(e*4i + f*x*4i)*(c*1i + d*1i)^4*(12*c*d^3 +
 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2)*2i)/(d^4*(c + d)^4))
```

# 3.587 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx$

**Optimal**. Leaf size=317

$$\frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}}+\frac{2a^3(c-d)(3c+19d)\cos(e+fx)}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}-\frac{}{105d}$$

```
[Out] 2/63*a^3*(c-d)*(3*c+19*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(7/2)/(
a+a*sin(f*x+e))^(1/2)-2/105*a^3*(c^2+10*c*d+73*d^2)*cos(f*x+e)/d^2/(c+d)^3/
f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-8/315*a^3*(c^2+10*c*d+73*d^
2)*cos(f*x+e)/d^2/(c+d)^4/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2
/9*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(
9/2)-16/315*a^3*(c^2+10*c*d+73*d^2)*cos(f*x+e)/d^2/(c+d)^5/f/(a+a*sin(f*x+e
))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.51, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2841, 3059, 2851, 2850}

$$-\frac{16a^3(c^2+10cd+73d^2)\cos(e+fx)}{315d^2f(c+d)^5\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^3(c^2+10cd+73d^2)\cos(e+fx)}{315d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^3(c^2+10cd+73d^2)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{5/2}}+\frac{2a^3(c-d)(3c+19d)\cos(e+fx)}{63d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{7/2}}+\frac{2a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{9df(c+d)(c+d\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(9*d*(c + d)*f*(c + d
*Sin[e + f*x])^(9/2)) + (2*a^3*(c - d)*(3*c + 19*d)*Cos[e + f*x])/(63*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2)) - (2*a^3*(c
^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e +
 f*x])/(315*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(
3/2)) - (16*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(315*d^2*(c + d)^5*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\,(\frac{1}{2}a \ldots}{(c+d\sin(e \ldots}}{9d(c+ \ldots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19 \ldots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+ \ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19 \ldots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+ \ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19 \ldots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+ \ldots}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19 \ldots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+ \ldots}}$$

**Mathematica** [A]
time = 5.21, size = 304, normalized size = 0.96

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(1869c^4+2088c^3d+5776c^2d^2+1804cd^3+727d^4-(63c^4+648c^3d+4790c^2d^2+1424cd^3+803d^4)\cos(2(e+fx))+2d^2(c^2+10cd+73d^2)\cos(4(e+fx))+588c^4\sin(e+fx)+7326c^3d\sin(e+fx)+4370c^2d^2\sin(e+fx)+5498cd^3\sin(e+fx)+698d^4\sin(e+fx)-18c^3d\sin(3(e+fx))-182c^2d^2\sin(3(e+fx))-1334cd^3\sin(3(e+fx))-146d^4\sin(3(e+fx)))}{315(c+d)^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```

```
[Out] -1/315*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1869*c^4 + 2088*c^3*d + 5776*c^2*d^2 + 1804*c*d^3 + 727*d^4 - (63*c^4 +
648*c^3*d + 4790*c^2*d^2 + 1424*c*d^3 + 803*d^4)*Cos[2*(e + f*x)] + 2*d^2*(
c^2 + 10*c*d + 73*d^2)*Cos[4*(e + f*x)] + 588*c^4*Sin[e + f*x] + 7326*c^3*d
*Sin[e + f*x] + 4370*c^2*d^2*Sin[e + f*x] + 5498*c*d^3*Sin[e + f*x] + 698*d
^4*Sin[e + f*x] - 18*c^3*d*Sin[3*(e + f*x)] - 182*c^2*d^2*Sin[3*(e + f*x)]
- 1334*c*d^3*Sin[3*(e + f*x)] - 146*d^4*Sin[3*(e + f*x)]))/((c + d)^5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(9/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. 2(287) = 574.
time = 7.51, size = 1730, normalized size = 5.46

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1730 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(4224*d*c^8-12288*
d^3*c^6+13568*d^5*c^4+8*cos(f*x+e)^12*c^2*d^7+584*cos(f*x+e)^12*d^9-4599*co
s(f*x+e)^10*d^9+14245*cos(f*x+e)^8*d^9-22645*cos(f*x+e)^6*d^9-105*cos(f*x+e
)^4*c^9+19695*cos(f*x+e)^4*d^9-1680*cos(f*x+e)^2*c^9-8944*cos(f*x+e)^2*d^9-
2688*sin(f*x+e)*c^9-1664*sin(f*x+e)*d^9+1664*d^9-35*cos(f*x+e)^8*sin(f*x+e)
*c^5*d^4+5*cos(f*x+e)^8*sin(f*x+e)*c^4*d^5+1650*cos(f*x+e)^8*sin(f*x+e)*c^3
*d^6+5850*cos(f*x+e)^8*sin(f*x+e)*c^2*d^7-3295*cos(f*x+e)^8*sin(f*x+e)*c*d^
8+458*cos(f*x+e)^6*sin(f*x+e)*c^7*d^2+2730*cos(f*x+e)^6*sin(f*x+e)*c^6*d^3+
8774*cos(f*x+e)^6*sin(f*x+e)*c^5*d^4+17070*cos(f*x+e)^6*sin(f*x+e)*c^4*d^5-
6490*cos(f*x+e)^6*sin(f*x+e)*c^3*d^6-24522*cos(f*x+e)^6*sin(f*x+e)*c^2*d^7+
8010*cos(f*x+e)^6*sin(f*x+e)*c*d^8-15*cos(f*x+e)^4*sin(f*x+e)*c^8*d+1812*co
s(f*x+e)^4*sin(f*x+e)*c^7*d^2+5380*cos(f*x+e)^4*sin(f*x+e)*c^6*d^3-22482*co
s(f*x+e)^4*sin(f*x+e)*c^5*d^4-44418*cos(f*x+e)^4*sin(f*x+e)*c^4*d^5+13860*c
os(f*x+e)^4*sin(f*x+e)*c^3*d^6+38772*cos(f*x+e)^4*sin(f*x+e)*c^2*d^7+2688*c
^9+80*cos(f*x+e)^12*c*d^8+1460*cos(f*x+e)^10*sin(f*x+e)*d^9+37*cos(f*x+e)^1
0*c^4*d^5+360*cos(f*x+e)^10*c^3*d^6+2538*cos(f*x+e)^10*c^2*d^7-1360*cos(f*x
+e)^10*c*d^8-7535*cos(f*x+e)^8*sin(f*x+e)*d^9+310*cos(f*x+e)^8*c^6*d^3+1875
*cos(f*x+e)^8*c^5*d^4+6805*cos(f*x+e)^8*c^4*d^5-2930*cos(f*x+e)^8*c^3*d^6-1
6320*cos(f*x+e)^8*c^2*d^7+6095*cos(f*x+e)^8*c*d^8+15474*cos(f*x+e)^6*sin(f*
x+e)*d^9-279*cos(f*x+e)^6*c^8*d-1310*cos(f*x+e)^6*c^7*d^2-1482*cos(f*x+e)^6
*c^6*d^3-17010*cos(f*x+e)^6*c^5*d^4-35980*cos(f*x+e)^6*c^4*d^5+11406*cos(f*
x+e)^6*c^3*d^6+41570*cos(f*x+e)^6*c^2*d^7-11902*cos(f*x+e)^6*c*d^8-63*cos(f
*x+e)^4*sin(f*x+e)*c^9-15847*cos(f*x+e)^4*sin(f*x+e)*d^9+87*cos(f*x+e)^4*c^
8*d-7220*cos(f*x+e)^4*c^7*d^2-15204*cos(f*x+e)^4*c^6*d^3+31650*cos(f*x+e)^4
*c^5*d^4+63090*cos(f*x+e)^4*c^4*d^5-19908*cos(f*x+e)^4*c^3*d^6-51540*cos(f*
x+e)^4*c^2*d^7+11711*cos(f*x+e)^4*c*d^8+336*cos(f*x+e)^2*sin(f*x+e)*c^9+811
2*cos(f*x+e)^2*sin(f*x+e)*d^9-3312*cos(f*x+e)^2*c^8*d+16192*cos(f*x+e)^2*c^
7*d^2+28864*cos(f*x+e)^2*c^6*d^3-23904*cos(f*x+e)^2*c^5*d^4-47520*cos(f*x+e
)^2*c^4*d^5+15168*cos(f*x+e)^2*c^3*d^6+30912*cos(f*x+e)^2*c^2*d^7-5776*cos(
f*x+e)^2*c*d^8-4224*sin(f*x+e)*c^8*d+7168*sin(f*x+e)*c^7*d^2+12288*sin(f*x+
e)*c^6*d^3-7424*sin(f*x+e)*c^5*d^4-13568*sin(f*x+e)*c^4*d^5+4096*sin(f*x+e)
*c^3*d^6+7168*sin(f*x+e)*c^2*d^7-1152*sin(f*x+e)*c*d^8-7168*c^7*d^2+7424*c^
5*d^4-4096*c^3*d^6+1152*c*d^8+1200*cos(f*x+e)^2*sin(f*x+e)*c^8*d-12608*cos(
f*x+e)^2*sin(f*x+e)*c^7*d^2-22720*cos(f*x+e)^2*sin(f*x+e)*c^6*d^3+20192*cos
(f*x+e)^2*sin(f*x+e)*c^5*d^4+40736*cos(f*x+e)^2*sin(f*x+e)*c^4*d^5-13120*co
s(f*x+e)^2*sin(f*x+e)*c^3*d^6-27328*cos(f*x+e)^2*sin(f*x+e)*c^2*d^7+5200*co
s(f*x+e)^2*sin(f*x+e)*c*d^8-9255*cos(f*x+e)^4*sin(f*x+e)*c*d^8+4*cos(f*x+e)
^10*sin(f*x+e)*c^3*d^6+60*cos(f*x+e)^10*sin(f*x+e)*c^2*d^7+492*cos(f*x+e)^1
0*sin(f*x+e)*c*d^8-7168*c^2*d^7)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^5/
(c+d)^5
```

**Maxima** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(302) = 604$.

time = 0.84, size = 1018, normalized size = 3.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] -2/315*((903*c^5 + 720*c^4*d + 494*c^3*d^2 + 200*c^2*d^3 + 35*c*d^4)*a^(5/2
) - (315*c^5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^
5)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (4179*c^5 - 1710*c^4*d + 30878
*c^3*d^2 + 11540*c^2*d^3 + 3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 3*(805*c^5 - 9912*c^4*d + 2330*c^3*d^2 - 18504*c^2*d^3
- 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*(12
39*c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*
a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 42*(149*c^5 - 894*c^4*d + 140
2*c^3*d^2 - 2052*c^2*d^3 + 745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 42*(149*c^5 - 894*c^4*d + 1402*c^3*d^2 - 2052*c^2*d^3 +
745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6*(1239*
c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(805*c^5 - 9912*c^4*d + 2330*c
^3*d^2 - 18504*c^2*d^3 - 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - (4179*c^5 - 1710*c^4*d + 30878*c^3*d^2 + 11540*c^2*d^3 +
3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (315*c^
5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^5)*a^(5/2)*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - (903*c^5 + 720*c^4*d + 494*c^3*d^2
+ 200*c^2*d^3 + 35*c*d^4)*a^(5/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^5 + 5*c^4*d + 10*c^3*d^2 + 10
*c^2*d^3 + 5*c*d^4 + d^5 + 3*(c^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c
*d^4 + d^5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^5 + 5*c^4*d + 10*c^3
*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c
^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)^(11/2)*f)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1516 vs. $2(302) = 604$.
time = 0.43, size = 1516, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(672*a^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 4
16*a^2*d^4 + 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^5 - 4
*(9*a^2*c^3*d + 89*a^2*c^2*d^2 + 647*a^2*c*d^3 - 73*a^2*d^4)*cos(f*x + e)^4
 - (63*a^2*c^4 + 648*a^2*c^3*d + 4798*a^2*c^2*d^2 + 1504*a^2*c*d^3 + 1387*a
^2*d^4)*cos(f*x + e)^3 + (231*a^2*c^4 + 3060*a^2*c^3*d - 2158*a^2*c^2*d^2 +
 4580*a^2*c*d^3 - 673*a^2*d^4)*cos(f*x + e)^2 + 2*(483*a^2*c^4 + 684*a^2*c^
3*d + 2642*a^2*c^2*d^2 + 812*a^2*c*d^3 + 419*a^2*d^4)*cos(f*x + e) - (672*a
^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 416*a^2*d^4 +
 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^4 + 4*(9*a^2*c^3*
d + 91*a^2*c^2*d^2 + 667*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^3 - 3*(21*a^2
*c^4 + 204*a^2*c^3*d + 1478*a^2*c^2*d^2 - 388*a^2*c*d^3 + 365*a^2*d^4)*cos(
f*x + e)^2 - 2*(147*a^2*c^4 + 1836*a^2*c^3*d + 1138*a^2*c^2*d^2 + 1708*a^2*
c*d^3 + 211*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*s
qrt(d*sin(f*x + e) + c)/((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^8 + 5
*c*d^9 + d^10)*f*cos(f*x + e)^6 - 5*(c^6*d^4 + 5*c^5*d^5 + 10*c^4*d^6 + 10*
c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^5 - (10*c^7*d^3 + 55*c^6*d^4 +
128*c^5*d^5 + 165*c^4*d^6 + 130*c^3*d^7 + 65*c^2*d^8 + 20*c*d^9 + 3*d^10)*f
*cos(f*x + e)^4 + 10*(c^8*d^2 + 5*c^7*d^3 + 11*c^6*d^4 + 15*c^5*d^5 + 15*c^
4*d^6 + 11*c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^3 + (5*c^9*d + 35*c^
8*d^2 + 120*c^7*d^3 + 260*c^6*d^4 + 378*c^5*d^5 + 370*c^4*d^6 + 240*c^3*d^7
 + 100*c^2*d^8 + 25*c*d^9 + 3*d^10)*f*cos(f*x + e)^2 - (c^10 + 5*c^9*d + 20
*c^8*d^2 + 60*c^7*d^3 + 110*c^6*d^4 + 126*c^5*d^5 + 100*c^4*d^6 + 60*c^3*d^
7 + 25*c^2*d^8 + 5*c*d^9)*f*cos(f*x + e) - (c^10 + 10*c^9*d + 45*c^8*d^2 +
120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 210*c^4*d^6 + 120*c^3*d^7 + 45*c^
2*d^8 + 10*c*d^9 + d^10)*f - ((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^
8 + 5*c*d^9 + d^10)*f*cos(f*x + e)^5 + (5*c^6*d^4 + 26*c^5*d^5 + 55*c^4*d^6
 + 60*c^3*d^7 + 35*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*x + e)^4 - 2*(5*c^7*d
^3 + 25*c^6*d^4 + 51*c^5*d^5 + 55*c^4*d^6 + 35*c^3*d^7 + 15*c^2*d^8 + 5*c*d
^9 + d^10)*f*cos(f*x + e)^3 - 2*(5*c^8*d^2 + 30*c^7*d^3 + 80*c^6*d^4 + 126*
c^5*d^5 + 130*c^4*d^6 + 90*c^3*d^7 + 40*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*
x + e)^2 + (5*c^9*d + 25*c^8*d^2 + 60*c^7*d^3 + 100*c^6*d^4 + 126*c^5*d^5 +
 110*c^4*d^6 + 60*c^3*d^7 + 20*c^2*d^8 + 5*c*d^9 + d^10)*f*cos(f*x + e) + (
c^10 + 10*c^9*d + 45*c^8*d^2 + 120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 21
0*c^4*d^6 + 120*c^3*d^7 + 45*c^2*d^8 + 10*c*d^9 + d^10)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

**Mupad [B]**
time = 26.14, size = 1155, normalized size = 3.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(11/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((32*a^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x
))^(1/2)*(c*d*10i + c^2*1i + d^2*73i))/(315*d^3*f*(c + d)^5) - (32*a^2*exp(
e*10i + f*x*10i)*(a + a*sin(e + f*x))^(1/2)*(10*c*d + c^2 + 73*d^2))/(315*d
^3*f*(c + d)^5) - (32*a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*(25
*c^4 - 25*c^3*d - 15*c*d^3 + 6*d^4 + 57*c^2*d^2))/(5*d^5*f*(c + d)^5) + (32
*a^2*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c^4*25i - c^3*d*25i - c
*d^3*15i + d^4*6i + c^2*d^2*57i))/(5*d^5*f*(c + d)^5) - (16*a^2*exp(e*4i +
f*x*4i)*(a + a*sin(e + f*x))^(1/2)*(194*c*d^3 + 318*c^3*d + 25*c^4 - 5*d^4
- 20*c^2*d^2))/(15*d^5*f*(c + d)^5) + (16*a^2*exp(e*7i + f*x*7i)*(a + a*sin
(e + f*x))^(1/2)*(c*d^3*194i + c^3*d*318i + c^4*25i - d^4*5i - c^2*d^2*20i)
)/(15*d^5*f*(c + d)^5) + (16*a^2*exp(e*8i + f*x*8i)*(a + a*sin(e + f*x))^(1
/2)*(10*c*d^3 + 70*c^3*d + 7*c^4 + 73*d^4 + 512*c^2*d^2))/(35*d^5*f*(c + d)
^5) - (16*a^2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c*d^3*10i + c^
3*d*70i + c^4*7i + d^4*73i + c^2*d^2*512i))/(35*d^5*f*(c + d)^5) + (32*a^2*
c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^(1/2)*(10*c*d + c^2 + 73*d^2))/(3
5*d^4*f*(c + d)^5) - (32*a^2*c*exp(e*9i + f*x*9i)*(a + a*sin(e + f*x))^(1/2
)*(c*d*10i + c^2*1i + d^2*73i))/(35*d^4*f*(c + d)^5)))/(exp(e*11i + f*x*11i
) - (c*1i + d*1i)^5/(c + d)^5 + (10*exp(e*7i + f*x*7i)*(4*c*d^3 + 8*c^3*d +
 8*c^4 + d^4 + 12*c^2*d^2))/d^4 + (5*exp(e*3i + f*x*3i)*(8*c*d^2 + 8*c^2*d
+ 16*c^3 + d^3))/d^3 - (5*exp(e*9i + f*x*9i)*(2*c*d + 8*c^2 + d^2))/d^2 - (
2*exp(e*5i + f*x*5i)*(30*c*d^4 + 40*c^4*d + 16*c^5 + 5*d^5 + 60*c^2*d^3 + 8
0*c^3*d^2))/d^5 - (exp(e*1i + f*x*1i)*(10*c + d))/d - (5*exp(e*8i + f*x*8i)
*(c*1i + d*1i)^5*(8*c*d^2 + 8*c^2*d + 16*c^3 + d^3))/(d^3*(c + d)^5) + (5*e
xp(e*2i + f*x*2i)*(c*1i + d*1i)^5*(2*c*d + 8*c^2 + d^2))/(d^2*(c + d)^5) +
(2*exp(e*6i + f*x*6i)*(c*1i + d*1i)^5*(30*c*d^4 + 40*c^4*d + 16*c^5 + 5*d^5
 + 60*c^2*d^3 + 80*c^3*d^2))/(d^5*(c + d)^5) + (exp(e*10i + f*x*10i)*(10*c
+ d)*(c*1i + d*1i)^5)/(d*(c + d)^5) - (10*exp(e*4i + f*x*4i)*(c*1i + d*1i)^
5*(4*c*d^3 + 8*c^3*d + 8*c^4 + d^4 + 12*c^2*d^2))/(d^4*(c + d)^5))
```

# 3.588 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=249

$$-\frac{\sqrt{d}\,(15c^2-10cd+7d^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)^{5/2}\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

```
[Out] -(c-d)^(5/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)-1/4*(15*c^2-10*c*d+7*
d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+
e))^(1/2))*d^(1/2)/f/a^(1/2)-1/2*d*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a
*sin(f*x+e))^(1/2)-1/4*(7*c-d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.63, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2857, 3062, 3061, 2861, 214, 2854, 211}

$$-\frac{\sqrt{d}\,(15c^2-10cd+7d^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}\,f}-\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{d(7c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[d]*(15*c^2 - 10*c*d + 7*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (Sqr
t[2]*(c - d)^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - ((7*c - d)*d
*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (d
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\,(-a(4c^2-cd+3d^2)}{\sqrt{a+a\sin(e+fx}}}{4a}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+f}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+f}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+f}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{d}\,(15c^2-10cd+7d^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 15.62, size = 1893, normalized size = 7.60

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*((d*(-9*c +
 2*d)*Cos[(e + f*x)/2])/4 - (d^2*Cos[(3*(e + f*x))/2])/4 - (d*(-9*c + 2*d)*
Sin[(e + f*x)/2])/4 - (d^2*Sin[(3*(e + f*x))/2])/4))/(f*Sqrt[a*(1 + Sin[e +
 f*x])]) + ((Sqrt[2]*(c - d)^(5/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c -
 d)^(5/2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (I/8)*Sqrt[d]*(15*c^2 - 10*c
*d + 7*d^2)*Log[(2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3
/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2]))] + (I/8)*Sqrt[d]*(15*
c^2 - 10*c*d + 7*d^2)*Log[(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + C
os[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/
(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2]))])*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])*(c^3/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
```

```
 + d*Sin[e + f*x]]) - (9*c^2*d)/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sq
rt[c + d*Sin[e + f*x]]) + (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sqrt[c + d*Sin[e + f*x]]) - d^3/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]]) + (15*c^2*d*Sin[e + f*x])/(8*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (5*c*d^2*Sin[e + f*x])/(4*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*d^3*Sin[e
 + f*x])/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(5/2)*Sec[(e + f*x)/2]^2)/(Sqrt[
2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(5/2)*(((-c + d)*Sec[(e + f*x
)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt
[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - ((I/16)*d
^2*(15*c^2 - 10*c*d + 7*d^2)^2*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Se
c[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*
x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[
e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]
^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(15*c^2 - 10
*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[
d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*T
an[(e + f*x)/2]) + ((I/16)*d^2*(15*c^2 - 10*c*d + 7*d^2)^2*(-I + Tan[(e + f
*x)/2])*((2*(((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*
x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1
+ I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/Sqrt[2]))/(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2
])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[
e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1
+ I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + (I*c + d)*Tan[(e + f*x)/2])))
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^{\frac{5}{2}}}{\sqrt{a+a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(220) = 440.
time = 0.80, size = 3063, normalized size = 12.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(16*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(
f*x + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*s
qrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(c
os(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(
f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(
cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15
*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e)
+ (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos
(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^
2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*
(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x +
e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3
 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d
^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*
d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 +
 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^
4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5
*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d^2*cos(f*x + e
)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - 9*
c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), 1/16*(8*sqrt(2)*(a*c^2 - 2*
a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*c*d +
```

```
 a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) +
1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x
 + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15*a*c^2 - 10*a*c*d + 7*a*d^2 +
 (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*
d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - 4*(2*d^2*cos(f*x
+ e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) -
 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/32*(32*sqrt(2)*(a*c^2
 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*
c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e)))
 - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x
 + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d
^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*
c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3
 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(
f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 +
51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d +
31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d +
 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c
*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 +
289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2
 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d
^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(
f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*d^2*cos(f
*x + e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e
) - 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/16*(16*sqrt(2)*(a*
c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2
*a*c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e
))) - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(
f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1
/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*
x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x...
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.589 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=188

$$-\frac{(3c-d)\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\,\sqrt{a+a\sin(e}}\right.}{\sqrt{a}\,f}$$

```
[Out] -(c-d)^(3/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)-(3*c-d)*arctan(cos(f*
x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*d^(1/2)
/f/a^(1/2)-d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.40, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2857, 3061, 2861, 214, 2854, 211}

$$-\frac{\sqrt{d}\,(3c-d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((3*c - d)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f)) - (Sqrt[2]*(c - d)^(3/2)
*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (d*Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-a(2c^2-cd+d^2)-a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}}}{2a}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + (c-d)^2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+}}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2a(c-d)^2)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx,\right.}{}$$

$$= -\frac{(3c-d)\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\sqrt{2}\,(c-}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 15.45, size = 1639, normalized size = 8.72

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-(d*Cos[(e + f*x)/2]) + d*Sin[(e +
f*x)/2])*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Sqrt[
2]*(c - d)^(3/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c - d)^(3/2)*Log[c -
d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] +
(-c + d)*Tan[(e + f*x)/2]] + (I/2)*Sqrt[d]*(-3*c + d)*(Log[((2*I)*(I*c + d
+ (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)
/2]))] - Log[(-2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])
^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3
*c + d)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
c^2/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (c*d
)/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + d^2/
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (3*c*d
*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
f*x]]) - (d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 + d*Sin[e + f*x]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(3/2)*Sec[(e
+ f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(3/2)*(((-
```

```
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) + (I/2)*Sqrt[d]*(-3*c + d)*(((-1/2*I)*d^(3/2)*(-3*c + d)*(I + Tan[(
e + f*x)/2])*(((2*I)*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*C
os[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]
]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)/2])) - (
I*Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f
*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)
*(-3*c + d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*
Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e +
 f*x)/2]) + (d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])*((-2*(((I*c + d)*Se
c[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])
^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e +
 f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2
)*(-3*c + d)*(-I + Tan[(e + f*x)/2])) + (Sec[(e + f*x)/2]^2*(c + I*d + (1 +
 I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
+ (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])^
2)))/(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))))
```

**Maple** [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^{\frac{3}{2}}}{\sqrt{a+a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
[Out] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(166) = 332$.

time = 0.73, size = 2655, normalized size = 14.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*(4*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*
x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d
)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c -
 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (
3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*
c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 3
2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^
3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*
d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
+ e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c
*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e)
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a
) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (12
8*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d
^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2
+ 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(co
s(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f), -1/4*(2*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) +
 (a*c - a*d)*sin(f*x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x +
 e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*c
os(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f
*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (3*a*c - a*d + (3*a*c - a*
d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*c
os(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 -
(3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x
 + e))) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*
sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/8
*(8*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e
))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
```

```
+ e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) + (3*a*c - a*d + (3*a*c
 - a*d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*
cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2
*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 -
32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x
 + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*
d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*
c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59
*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^
2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289
*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 +
4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3
+ 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x
 + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e)
 - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(
a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/4*(4*sqrt(2)*(a*c - a*d + (a
*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(
-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)
/((c - d)*cos(f*x + e))) - (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (3*a
*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 +
6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*
x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 4*(d*cos(f*x
 + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin(e + fx))^{\frac{3}{2}}}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.590 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right.}{\sqrt{a}f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)-2*arctan(cos(f*x+e)*a
^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*d^(1/2)/f/a^(
1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2856, 2854, 211, 2861, 214}

$$-\frac{2\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sq
rt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])])/(Sqrt[a]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2856

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx + \frac{d\int \frac{\sqrt{a+a\sin(e+}}{\sqrt{c+d\sin(e+}}}{a}$$

$$= -\frac{(2a(c-d))\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+}}\right.}{f}$$

$$= -\frac{2\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\sqrt{2}\,\sqrt{c-d}\,\tan}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 14.11, size = 1251, normalized size = 8.87

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Sqrt[2]*Sqrt[c - d]*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*Sqrt[c - d]*Log[c
 - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + (-c + d)*Tan[(e + f*x)/2]] - I*Sqrt[d]*(Log[(2*(c - I*d + (1 - I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c
+ d)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2]))] - Log[(2*(c + I*d
 + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
 f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2]))]))*
Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]*((Sqrt[c - d]*Sec[(
e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*Sqrt[c - d]*(((-
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) - I*Sqrt[d]*((d^(3/2)*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Sec
[(e + f*x)/2]^2)/2 + ((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^
(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e +
f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)
*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*d + (1 - I)*Sqrt[2]*S
qrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c + d
)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2])^2)))/(2*(c - I*d + (1
- I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + ((-I)*c + d)*Tan[(e + f*x)/2])) - (d^(3/2)*(-I + Tan[(e + f*x)/2])*((2*(
((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 +
Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*
((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt
[2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1
 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]
] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])^2)))/(2*(
c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])))))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(114) = 228$.
time = 0.20, size = 3353, normalized size = 23.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)*2^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*c^2*d-2*cos(f*x+e)*2^(1
/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
```

sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos
(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*(2*c-2*d)^(1/2)*c*d^2+cos(f*x+e)*2^(1/2)*ln(-2*((2*c-2*d)^(1/2
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d
*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*(-(d
^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1
/2)*d^3+2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*ln(-2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+
e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e))
)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*d*sin(f*x+e)-2*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f
*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1
/2)*(2*c-2*d)^(1/2)*d^2*sin(f*x+e)+2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1
+cos(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*d^3*sin
(f*x+e)+2*c^3*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)
^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d*cos(f*x+e)-4*c^2*(d^2/c^2)^(1/2)*((c+d
*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x
+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*
d^2*cos(f*x+e)+2*c*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin
(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^3*cos(f*x+e)+2^(1/2)*ln(-2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f
*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+
e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*
c-2*d)^(1/2)*c^2*d-2*2^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*c*d^2+2^(1/2)*ln(-2*(
(2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin
(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*(2*c-2*d)^(1/2)*d^3-2*c^2*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(
1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^2*cos(f*x+e)+4*c*((c+d*sin(f*x+e))/((d^2
/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^3*cos(f*x+e)-2*(
(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin
(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/
2))*d^4*cos(f*x+e)-2*c*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c
*sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d
)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1

```
/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/
2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4
*c^2*d^2-4*d^4)*c)^(1/2))*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d
^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*sin
(f*x+e)-2*c*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin
(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f
*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^
2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*
d^4)*c)^(1/2))*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2
)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)-2*((c+d*sin(f*
x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*s
in(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d
)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c
^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c
^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(((d^2/c^2)^(1/2)*c^4+6*(
d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^...
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(120) = 240$.
time = 0.67, size = 2034, normalized size = 14.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(2)*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (
c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e)
- 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) + sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5
 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3
 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*
```

c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(
c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2
*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3
)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^
3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
+ e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
- 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f
*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x
 + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt((c - d)/a)*
log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c -
d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c -
 d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c
- 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) + sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(
c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*
sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e)
 - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))))/f, -1/4*(4*sqrt(2)*sqrt(-(c - d)
/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(-d/a)*log((128*d^4*cos(f*x + e)^
5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^
3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2
*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*
(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^
2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^
3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d
^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*c
os(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
 + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(
f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*
x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, -1/2*(2*sqrt(2)*sqrt(-(c - d
)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt
(-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x
+ e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^
2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e)))
)/f]

**Sympy** [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sin(e + fx)}}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + d\,\sin(e + f\,x)}}{\sqrt{a + a\,\sin(e + f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2), x)
```

## 3.591 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2861, 214}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+}{\sqrt{a+a\sin(e+fx)}\ \sqrt{}}\right.}{f}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}}\right.}{\sqrt{a}\sqrt{c-d}\,f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(79) = 158.
time = 2.70, size = 283, normalized size = 3.58

$$\frac{\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]])/(f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan
[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Co
s[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*S
in[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(64) = 128.
time = 10.04, size = 191, normalized size = 2.42

| method | result |
|---|---|
| default | $-\frac{(1-\cos(fx+e)+\sin(fx+e))\sqrt{c+d\sin(fx+e)}\,\ln\left(\frac{2\sqrt{2c-2d}\sqrt{2}\sqrt{\frac{c+d\sin(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)+2\cos(fx+e)c-2d}{1-\cos(fx+e)+\sin(fx+e)}\right.}{f\sqrt{a(1+\sin(fx+e))}\sin(fx+e)\sqrt{\frac{c+d\sin(fx+e)}{\cos(fx+e)+1}}\sqrt{2c-2d}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/f*(1-cos(f*x+e)+sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)*ln(2*((2*c-2*d)^(1/2)
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*
cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))/(a*(1+
sin(f*x+e)))^(1/2)/sin(f*x+e)*2^(1/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)/(2*c-2*d)^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas [A]**
time = 0.52, size = 499, normalized size = 6.32

$$\left[\frac{\sqrt{2}\log\left(\frac{(c^2-14cd+17d^2)\cos(fx+e)^3-(13c^2-22cd-3d^2)\cos(fx+e)^2-\frac{4\sqrt{2}\left((c^2-4cd+3d^2)\cos(fx+e)^2-4c^2+8cd-4d^2-(3c^2-4cd+d^2)\cos(fx+e)+(4c^2-8cd+4d^2+(c^2-4cd+3d^2)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}}{\sqrt{ac-ad}}-4c^2-8cd-4d^2-2(9c^2-14cd+9d^2)\cos(fx+e)+\left((c^2-14cd+17d^2)\cos(fx+e)^2-4c^2-8cd-4d^2+2(7c^2-18cd+7d^2)\cos(fx+e)\right)\sin(fx+e)}{\cos(fx+e)^3+3\cos(fx+e)^2+\left(\cos(fx+e)^2-2\cos(fx+e)-4\right)\sin(fx+e)-2\cos(fx+e)-4}\right)}{4\sqrt{ac-ad}\,f}, \frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\arctan\left(-\frac{\sqrt{2}\sqrt{a\sin(fx+e)+a}((c-3d)\sin(fx+e)-3c+d)\sqrt{d\sin(fx+e)+c}\sqrt{-\frac{1}{ac-ad}}}{4(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))}\right)}{2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*sqrt(2)*log(((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d
 - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*((c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2
- 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c
*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d
^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos
(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*
cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4))/(sqrt(a*c - a*d)*f),
1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c
 - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))/f]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.592 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=131

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^{3/2}f}+\frac{2d\cos(e+fx)}{(c^2-d^2)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)^(3/2)/f/a^(1/2)+2*d*cos(f*x+e)/(c^2-d
^2)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2858, 12, 2861, 214}

$$\frac{2d\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)^(3/2)*f)) + (2*d*
Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} + $$

$$= \frac{2d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} + $$

$$= \frac{2d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} - $$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}\right.}{\sqrt{a}\,(c-d)^{3/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 306 vs. 2(131) = 262.
time = 6.68, size = 306, normalized size = 2.34

$$\frac{\frac{2d\cos(e+fx)}{c+d}+\frac{\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}}{(c-d)f\sqrt{a(1+\sin(e+fx))}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((2*d*Cos[e + f*x])/(c + d) + (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
*Tan[(e + f*x)/2]])/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((
c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(
d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*
Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c +
d)*Tan[(e + f*x)/2]])))/((c - d)*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin
[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(112) = 224$.

time = 9.53, size = 874, normalized size = 6.67

| method | result |
|---|---|
| default | $-\frac{\ln\left(\frac{2\sqrt{2c-2d}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)+2\cos(fx+e)c-2d\cos(fx+e)+2c\sin(fx+e)-2d\sin(fx+e)-2c+2d}{1-\cos(fx+e)+\sin(fx+e)}\right)\sqrt{2}\,\sqrt{\frac{c+d}{\cos}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/f*(ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*sin(
f*x+e)+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d*sin
(f*x+e)+cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*c+cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*d+2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)
-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+
2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-2*d*(2*c-2
*d)^(1/2)*cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(c+d*sin(f*x+e))^(1/2)/(c+d)
/(2*c-2*d)^(1/2)/(c-d)
```

**Maple [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(118) = 236$.
time = 0.59, size = 1064, normalized size = 8.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/4*(8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c) - sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*
cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 +
(a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d^2)*co
s(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 + 4*sqrt(2)*((c^2 -
 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d
^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a
*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e)
 + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c
^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*
x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x +
 e) - 4))/sqrt(a*c - a*d))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a
*c*d^2)*f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d
- a*d^3)*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x +
e)), -1/2*(sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^
2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^
2)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e
) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))
 + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a*c*d^2)*
f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d - a*d^3)
*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x + e))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(3/2)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.593 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^{5/2}f}+\frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)^(5/2)/f/a^(1/2)+2/3*d*cos(f*x+e)/(c^2
-d^2)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2/3*d*(5*c+d)*cos(f*x
+e)/(c^2-d^2)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2858, 3063, 12, 2861, 214}

$$\frac{2d(5c+d)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2d\cos(e+fx)}{3f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f(c-d)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)^(5/2)*f)) + (2*d*
Cos[e + f*x])/(3*(c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]
)^(3/2)) + (2*d*(5*c + d)*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
```

```
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)}{3\,(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2d\cos(e+fx)}{3\,(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2d\cos(e+fx)}{3\,(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2d\cos(e+fx)}{3\,(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+}\right.}{\sqrt{a}\,(c-d)^{5/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 387 vs. 2(191) = 382.
time = 6.71, size = 387, normalized size = 2.03

$$\frac{\frac{2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6c^2+cd-d^2+d(5c+d)\sin(e+fx)\right)}{(c+d)^2(c+d\sin(e+fx))}+\frac{3\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}}{3(c-d)^2f\sqrt{a(1+\sin(e+fx))}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((2*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(6*c^2 + c*d - d^2 + d*(5*c + d)*Sin[e + f*x]))/((c + d)^2*(c + d*Si
n[e + f*x])) + (3*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sq
rt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*
x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[
(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e
 + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]
*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e +
 f*x)/2])))/(3*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*
x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2571 vs. 2(164) = 328.
time = 10.59, size = 2572, normalized size = 13.47

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2572 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/3/f*(3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1
-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)*cos(f*x+e)*c^2*d+6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d^2+3*2^(1/2)*sin(f*x+e)*cos(f*x+e)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3-3*2^(1/2)*cos(f
```

*x+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d-6*2^(1
/2)*cos(f*x+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1
/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c
-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d
^2-3*2^(1/2)*cos(f*x+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos
(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*d^3+3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*c^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+9*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+9*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+3*2^(1/2)*sin(f*x+e)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+
d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3+3*2^(1/2)*cos(f*x+e)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+3*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+3*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+9*ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+9*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)-10*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c
*d^2-2*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*d^3-12*(2*c-2*d)^(1/2)*cos(f*x
+e)*c^2*d-2*(2*c-2*d)^(1/2)*cos(f*x+e)*c*d^2+2*(2*c-2*d)^(1/2)*cos(f*x+e)*d

```
^3)*(c+d*sin(f*x+e))^(1/2)/(-cos(f*x+e)^2*d^2+2*sin(f*x+e)*c*d+c^2+d^2)/(a*
(1+sin(f*x+e)))^(1/2)/(c+d)^2/(2*c-2*d)^(1/2)/(c-d)^2
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(173) = 346$.
time = 0.63, size = 1915, normalized size = 10.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/12*(8*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 + (6*
c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d^2 +
 d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
 e) + c) + 3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 -
 (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
+ 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*
a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2
*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d
^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*(
(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c
*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/
sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*
x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 +
2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*
cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos
(f*x + e) - 4))/sqrt(a*c - a*d))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f
*x + e)^3 + (2*a*c^5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5
+ a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f
*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*
d^5 + a*d^6)*f + ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a
```

```
*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4
*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e)), -1/6*
(3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (a*c^2*d^
2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*a*c*d^3 +
a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2*a*c^2*d^2
 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*si
n(f*x + e) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f
*x + e))) + 4*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 +
 (6*c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d
^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^3 + (2*a*c^
5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f*cos(f*x
+ e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f*x + e) - (a*c^6 +
2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f + ((
a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a*c^5*d - 2*a*c^3*d^
3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3
- a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(5/2)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.594 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=251

$$-\frac{(5c-3d)d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c-d)^{3/2}(c+9d)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{a+a}}\right.}{2\sqrt{2}\,a^{3}}$$

```
[Out] -(5*c-3*d)*d^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)
/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f-1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3
/2)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)^(3/2)*(c+9*d)*arctanh(1/2*cos(f*x+e)
*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))
/a^(3/2)/f*2^(1/2)+1/2*(c-3*d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a
*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.60, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2844, 3062, 3061, 2861, 214, 2854, 211}

$$-\frac{d^{3/2}(5c-3d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c+9d)(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((5*c - 3*d)*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f)) - ((c - d)^(3/2)*(c +
9*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 3*d)*d*Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((c
- d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\,(-\frac{1}{2}a(c^2+6\ldots}{\sqrt{a+a\sin(e\ldots}}}{2a^2}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e\ldots}{2f(a+a\sin(e+fx))^{3/\ldots}}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e\ldots}{2f(a+a\sin(e+fx))^{3/\ldots}}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e\ldots}{2f(a+a\sin(e+fx))^{3/\ldots}}$$

$$= -\frac{(5c-3d)d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{(c-d)^{\ldots}}{\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 17.39, size = 1844, normalized size = 7.35

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-(d^2*Cos[(e + f*x)/2]) + d^2*Sin
[(e + f*x)/2] - (c - d)^2/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (c^2*
Sin[(e + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e
+ f*x]))^(3/2)) + ((((c - d)^(3/2)*(c + 9*d)*Log[1 + Tan[(e + f*x)/2]])/Sqr
t[2] + I*(5*c - 3*d)*d^(3/2)*Log[((-I)*((-I)*c + d + (1 - I)*Sqrt[2]*Sqrt[d
]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)*Tan[(e
 + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2]))] + I*d^(3/2)*(-
5*c + 3*d)*Log[(I*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x]
)^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5/2)*(-
5*c + 3*d)*(I + Tan[(e + f*x)/2]))] - ((c - d)^(3/2)*(c + 9*d)*Log[c - d +
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]])/Sqrt[2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c
```

```
^3/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*
c^2*d)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) -
 (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]) + (3*d^3)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*
x]]) + (5*c*d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt
[c + d*Sin[e + f*x]]) - (3*d^3*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*Sqrt[c + d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c
 - d)^(3/2)*(c + 9*d)*Sec[(e + f*x)/2]^2)/(2*Sqrt[2]*(1 + Tan[(e + f*x)/2])
) - ((c - d)^(3/2)*(c + 9*d)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d
]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] +
Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e +
 f*x]]))/(Sqrt[2]*(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt
[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])) - ((5*c - 3*d)*d^4*(-5*c
 + 3*d)*(-I + Tan[(e + f*x)/2])*(((-I)*(((c - I*d)*Sec[(e + f*x)/2]^2)/2 +
((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[
c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(-I + T
an[(e + f*x)/2])) + ((I/2)*Sec[(e + f*x)/2]^2*((-I)*c + d + (1 - I)*Sqrt[2]
*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)
*Tan[(e + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2])^2)))/((-I
)*c + d + (1 - I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (c - I*d)*Tan[(e + f*x)/2]) + (d^4*(-5*c + 3*d)^2*(I + Tan[
(e + f*x)/2])*((I*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])) - ((
I/2)*Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5
/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqr
t[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan
[(e + f*x)/2])))
```

**Maple [F]**
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^{\frac{5}{2}}}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(222) = 444$.
time = 0.86, size = 3582, normalized size = 14.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(2*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d
^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e) + (2*a*c^2 +
16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d)*c
os(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c - 2*
d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*
x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*
d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x +
e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d
- 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 +
24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5
*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25
*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 5
1*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(
f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*c
os(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin
(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c
^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e)
 - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*c
os(f*x + e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*(2*a*c^2 + 16*a*c*d -
18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d -
9*a*d^2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d -
```

```
 9*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e
) - sin(f*x + e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) -
 ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (10*a*c*d - 6
*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x +
 e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))
*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d
- d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt
(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (
c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c
*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 +
 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x +
 e) + 2*a^2*f)*sin(f*x + e)), 1/8*(4*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d
^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^
2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f
*x + e))) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a
*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c
*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32
*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9
*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3
)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d
^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x +
 e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*
d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a)
 + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128
*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^
3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 +
 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos
(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c*d + d^
2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 + 2*c*d
- d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^
2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2...
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{5/2}}{(a+a\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.595 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=194

$$-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}\,(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}}{\sqrt{2}\,\sqrt{a+a\sin(e+}\right.}{2\sqrt{2}\,a^{3/2}f}$$

```
[Out] -2*d^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/a^(3/2)/f-1/4*(c+5*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)
^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*(c-d)^(1/2)/a
^(3/2)/f*2^(1/2)-1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x
+e))^(3/2)
```

**Rubi** [A]

time = 0.37, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2844, 3061, 2861, 214, 2854, 211}

$$-\frac{2d^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}\,(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f) - (Sqrt[c - d]*(c + 5*d)*ArcTanh[(
Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(c^2+4cd-d^2)-2ad^2\sin(\cdots}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\cdots}}}{2a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{d^2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \cdots$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{\cdots}{\sqrt{a\cdots}}\right.}{\cdots}$$

$$= -\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{c-d}\,(c+5\cdots}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 17.42, size = 1625, normalized size = 8.38

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((-c + d)/(2*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])) + (c*Sin[(e + f*x)/2] - d*Sin[(e + f*x)/2])/(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e + f*
x]))^(3/2)) + ((Sqrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[1 + Tan[(e + f*x)/2]] - S
qrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (4*I)*Sqr
t[c - d]*d^(3/2)*(Log[(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2])/(2*
d^(5/2)*(I + Tan[(e + f*x)/2]))] - Log[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e +
f*x)/2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3*(c^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
 + f*x]]) + (c*d)/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
 f*x]]) - d^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]]) + (d^2*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c +
d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c^2 + 4*c*d - 5*d^2)*
Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c^2 + 4*c*
```

```
d - 5*d^2)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*S
qrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 +
Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d +
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]) - (4*I)*Sqrt[c - d]*d^(3/2)*((2*d^(5/2)*(I + Tan[(e
+ f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(2*d^(5/2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x
)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(4*d^(5/2)*(I + T
an[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]) - (
2*d^(5/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1
+ I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c +
d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2])
) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(4*d^
(5/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[
(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)
/2]))))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6674 vs. $2(159) = 318$.
time = 0.22, size = 6675, normalized size = 34.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(168) = 336$.
time = 0.85, size = 3037, normalized size = 15.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d - (a*c + 5*a
*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e))*sin(f*x +
e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*c
os(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*
d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d -
 (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x +
 e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*
c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d
 - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 +
 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(
5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 2
5*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 -
51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos
(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
 d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*
cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*si
n(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 2*((c - d)*cos(f*x + e) -
(c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
 + e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^
2 - 2*a*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c +
5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) -
sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c
 - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(a*d*cos(f*x +
e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*
sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
 d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c
^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*((c - d)*cos(f*x + e) - (c - d)*si
```

```
n(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2
*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*
a^2*f)*sin(f*x + e)), -1/4*(2*sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a
*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*
cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x +
 e))) - (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e)
+ 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^
3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*
d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*c
os(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2
+ 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e)
 + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2
 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) +
(c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^
4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 -
 d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32
*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*
x + e) + sin(f*x + e) + 1)) - 2*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e
) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e)), -1/2*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d
 - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e
))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - (a*
d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*si
n(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^
2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f
*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin(e + fx))^{\frac{3}{2}}}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin{(e+f\,x)})^{3/2}}{(a+a\,\sin{(e+f\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.596 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}\sqrt{c-d}f}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(c+d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+
e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)/(c-d)^(1/2)-1/2*cos(f*x
+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2843, 12, 2861, 214}

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{c-d}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*Sqrt[c - d]*f)
 - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{a(c+d)}{2\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+ \cdots}}{2a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(c+d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d \cdots}}{4a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(c+d)\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \cdots\right.}{2 \cdots}$$

$$= -\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}\sqrt{c-d}f} - \frac{\cos(e+ \cdots}{2f \cdots}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 372 vs. 2(126) = 252.

time = 5.28, size = 372, normalized size = 2.95

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{(c+d)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{4f(a(1+\sin(e+fx)))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
```

```
+ d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^
2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*S
in[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/
(4*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. $2(103) = 206$.
time = 11.17, size = 1373, normalized size = 10.90

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1373 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*((2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)*2^(1/2)*c+(2*c-
2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(
1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)*2^(1/2)*d+(2*c-2*d)^(1/2)*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c+(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*c
os(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x
+e)^2*2^(1/2)*d-2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x
+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*c-2*(2*
c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*d+(2*c-2*d)^(1/2)*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(
f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*cos(f*x+e)*2^(1/2)*c+(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)
*d-2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c-2*(2*c-2*d)^(1/2)*ln(2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f
*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e
```

```
)))*2^(1/2)*d-4*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*c+4*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d)*(c+
d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)/(c-d)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(109) = 218$.
time = 0.53, size = 944, normalized size = 7.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/16*(((c + d)*cos(f*x + e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x +
e) + 2*c + 2*d)*sin(f*x + e) - 2*c - 2*d)*sqrt(2*a*c - 2*a*d)*log(((a*c^2 -
 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*
c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3
*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) -
4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d
+ 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*
a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x +
e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e)
- 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
 (a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e)), -1/8*(((c + d)*cos(f*x +
 e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x + e) + 2*c + 2*d)*sin(f*x +
 e) - 2*c - 2*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(
a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e)
+ c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x +
 e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
```

```
(a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e))]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sin{(e + fx)}}}{(a(\sin{(e + fx)} + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] Timed out
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + d\,\sin{(e + f\,x)}}}{(a + a\,\sin{(e + f\,x)})^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.597 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=135

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^{3/2}f}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(c-3*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(3/2)/f*2^(1/2)-1/2*cos(f
*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2845, 12, 2861, 214}

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -1/2*((c - 3*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^(3/2
)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(c - d)*f*(a + a*Sin[e +
f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int -\frac{\cdots}{2\sqrt{a+a\sin(e-\cdots}}}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-3d)\int \frac{\cdots}{\sqrt{a+a\,\mathrm{s}\cdots}}}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-3d)\mathrm{Subst}\left(\int \frac{\cdots}{2a\cdots}\right.}{\cdots}$$

$$= -\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e\cdots}}\right.}{2\sqrt{2}\,a^{3/2}(c-d)^{3/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(135) = 270.

time = 5.77, size = 381, normalized size = 2.82

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{(c-3d)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{4(c-d)f(a(1+\sin(e+fx)))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
- 3*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos
[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(S
ec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2
]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c
*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 +
Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
)/(4*(c - d)*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(112) = 224$.
time = 9.46, size = 1268, normalized size = 9.39

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1268 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-
cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)*2^(1/2)*c+3*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*
c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*si
n(f*x+e)*cos(f*x+e)*2^(1/2)*d-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c+3*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*cos(f*x+e)^2*2^(1/2)*d+2*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*c
-6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*
x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*d-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)*c+
3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x
+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)*d+2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c-6*ln(2*((2
```

```
*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+co
s(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*
x+e)))*2^(1/2)*d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(3/2
)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/(2*c-2*d)^(1/2)/(c-d)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(118) = 236.
time = 0.58, size = 1056, normalized size = 7.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(((c - 3*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) + 2*c - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(2*a*c - 2*a*d)*log(((a
*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 -
(13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^
2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x +
 e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*
a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2
 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(
f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x
 + e) - 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f
*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(((c - 3
*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) + 2*c
 - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a
*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqr
t(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 -
 a*c*d)*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c
```

```
 - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*
c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\,\sin(e+f\,x))^{3/2}\,\sqrt{c+d\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.598 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=197

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^{5/2}f}-\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d}}$$

```
[Out] -1/4*(c-7*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(5/2)/f*2^(1/2)-1/2*cos(f
*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2)-1/2*d*(c+5*d)*c
os(f*x+e)/a/(c-d)^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.33, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2845, 3063, 12, 2861, 214}

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^{5/2}}-\frac{d(c+5d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*((c - 7*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^(5/2
)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[
e + f*x]]) - (d*(c + 5*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin}}\right.}{2\sqrt{2}\,a^{3/2}(c-d)^{5/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(197) = 394.
time = 6.19, size = 401, normalized size = 2.04

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(c^2+cd+4d^2+d(c+5d)\sin(e+fx)\right)}{(c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{(c-7d)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{4(c-d)^2f(a(1+\sin(e+fx)))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c^2 + c*d + 4*d^2 + d*(c + 5*d)*Sin[e + f*x]))/((c + d)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])) + ((c - 7*d)*(Log[1 + Tan[(e + f*x)/2]] - Log
[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x
]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/
2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])
^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]
])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(4*(c - d)^2*f*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. 2(168) = 336.
time = 11.91, size = 2246, normalized size = 11.40

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2246 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)*cos(f*x+e)*c^2-6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d-7*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2-cos(f*x+e)^2*((c+d*si
```

n(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+6*cos(f*x+e)^2*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+7*cos(f*x
+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+
e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+2*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)-14*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+
d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*cos(f*x+e)-6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)-7*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)+2*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d+10*
(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2+2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)-14*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x
+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)+2*(2*c-2*d)^(1/2)*cos(f*x+e)*c^2+2*(2*c-2*d)^(1/2
)*cos(f*x+e)*c*d+8*(2*c-2*d)^(1/2)*cos(f*x+e)*d^2)/(a*(1+sin(f*x+e)))^(3/2)
/(c+d*sin(f*x+e))^(1/2)/(c+d)/(2*c-2*d)^(1/2)/(c-d)^2

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(177) = 354$.
time = 0.71, size = 2014, normalized size = 10.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*c^3 + 10*c^2*d + 26*c
*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*cos(f*x + e)^2 - (c^3 -
 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 10*c^2*d - 26*c*d^2 -
14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c^3 - 5*c^2*d - 13*c*d
^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 1
4*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^
2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f*x + e)^2 - (3*c
 - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*
c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d +
4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*
c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) -
4)) - 8*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)
^2 + (c^3 + 3*c*d^2 - 4*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c
^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2
*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c
*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4
*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d -
 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c
^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)
- 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*
d^5)*f)*sin(f*x + e)), -1/8*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*
c^3 + 10*c^2*d + 26*c*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*co
s(f*x + e)^2 - (c^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 1
0*c^2*d - 26*c*d^2 - 14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c
^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c +
```

```
2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*
sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(c^3 - c^2*d - c*d^2
+ d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)^2 + (c^3 + 3*c*d^2 - 4*d^3)*
cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 4*c*d^2 - 5*d^3)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(
(a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2
*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x +
 e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 -
a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^
2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4
- a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*
c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*
a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**(3/2)), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.599 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=271

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{7/2}f}-\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin\ldots}$$

```
[Out] -1/2*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2)-1/4*(
c-11*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))
^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(7/2)/f*2^(1/2)-1/6*d*(3*c+7*d
)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
)-1/6*d*(3*c^2+38*c*d+19*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(a+a*sin(f*x+e
))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.57, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2845, 3063, 12, 2861, 214}

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{7/2}}-\frac{d(3c^2+38cd+19d^2)\cos(e+fx)}{6af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{d(3c+7d)\cos(e+fx)}{6af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/2*((c - 11*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^(7/
2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
 f*x])^(3/2)) - (d*(3*c + 7*d)*Cos[e + f*x])/(6*a*(c - d)^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(3*c^2 + 38*c*d + 19*d
^2)*Cos[e + f*x])/(6*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin}}\right.}{2\sqrt{2}a^{3/2}(c-d)^{7/2}f}$$

**Mathematica** [A]
time = 9.71, size = 478, normalized size = 1.76

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6c^4+12c^3d+81c^2d^2+70cd^3+11d^4-d^2\left(3c^2+38cd+19d^2\right)\cos(2(e+fx))+12d\left(c^3+8c^2d+9cd^2+2d^3\right)\sin(e+fx)\right)}{(c+d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}+\frac{3(c-11d)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{12(c-d)^3f(a(1+\sin(e+fx)))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-(((Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(6*c^4 + 12*c^3*d + 81*c^2*d^2 + 70*c*d^3 + 11*d^4 - d^2*(3*c^2 + 3
8*c*d + 19*d^2)*Cos[2*(e + f*x)] + 12*d*(c^3 + 8*c^2*d + 9*c*d^2 + 2*d^3)*S
in[e + f*x]))/((c + d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e
 + f*x]))) + (3*(c - 11*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[
c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Ta
n[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c
- d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d
+ d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
```

```
*Tan[(e + f*x)/2]))))/(12*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c +
 d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 5039 vs. $2(236) = 472$.
time = 11.35, size = 5040, normalized size = 18.60

| method | result | size |
|---|---|---|
| default | Expression too large to display | 5040 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1504 vs. $2(248) = 496$.
time = 0.90, size = 3254, normalized size = 12.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/48*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 +
 (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^4 - (2*c^4*d - 17*c
^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3 - (c^5 - 5*c^4*d -
54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^
4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e) + (2*c^5 -
14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 - (c^3*d^2 - 9*c^2*
d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c^3*d^2 - 30*c^2*d^3
 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2
*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*l
```

og(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*
d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x
 + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin
(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^
2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7
*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 +
3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*c
os(f*x + e) - 4)) - 8*(3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 -
3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^
4*d + 39*c^3*d^2 - 29*c^2*d^3 - 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 +
 c^4*d + 12*c^3*d^2 + 4*c^2*d^3 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 -
 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^
3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*
c*d^4 - 2*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*
d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d
 - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2
*d^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 -
 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*
d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^
4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 +
6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3
 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(
f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3
*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2
*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x +
 e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)
*f)*sin(f*x + e)), -1/24*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 -
86*c*d^4 - 22*d^5 + (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^
4 - (2*c^4*d - 17*c^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3
 - (c^5 - 5*c^4*d - 54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x +
 e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f
*x + e) + (2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5
 - (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c
^3*d^2 - 30*c^2*d^3 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d -
38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sq
rt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)
*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^
2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) + 4*(3*c^5 -
3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3
 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^4*d + 39*c^3*d^2 - 29*c^2*d^3 -
 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 + c^4*d + 12*c^3*d^2 + 4*c^2*d^3
 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^
3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x

```
+ e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*c*d^4 - 2*d^5)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^6*d^
2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7
 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3
 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^
3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4
 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)
^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*
cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 +
 a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 -
 a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2
*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 -
a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^...
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.600 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=260

$$-\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{\sqrt{c-d}\,(3c^2+14cd+43d^2)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{a+}}\right.}{16\sqrt{2}\,a^{5/2}f}$$

```
[Out] -2*d^(5/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/a^(5/2)/f-1/4*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a
+a*sin(f*x+e))^(5/2)-1/32*(3*c^2+14*c*d+43*d^2)*arctanh(1/2*cos(f*x+e)*a^(1
/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*(c-d
)^(1/2)/a^(5/2)/f*2^(1/2)-1/16*(c-d)*(3*c+11*d)*cos(f*x+e)*(c+d*sin(f*x+e))
^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.57, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2844, 3056, 3061, 2861, 214, 2854, 211}

$$-\frac{2d^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{\sqrt{c-d}\,(3c^2+14cd+43d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(5/2)*f) - (Sqrt[c - d]*(3*c^2 + 14*c*d + 4
3*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*(3*c
 + 11*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(a + a*Sin[e + f*x]
)^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\,\left(-\frac{1}{2}a(3c-d\right.}{(a+a\sin(e+fx))}}{4a^2}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c}{4f(a+a\sin(e}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c}{4f(a+a\sin(e}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c}{4f(a+a\sin(e}$$

$$= -\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{\sqrt{c-d}\,(3c^2+}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 17.76, size = 1845, normalized size = 7.10

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-1/4*(c - d)^2/(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^3 - (3*(c - d)*(c + 5*d))/(16*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])) + (3*(c^2*Sin[(e + f*x)/2] + 4*c*d*Sin[(e + f*x)/2] - 5*d^2*S
in[(e + f*x)/2]))/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (c^2*Sin[(e
 + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(2*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^4))*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e +
f*x]))^(5/2)) + ((Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[1 + Ta
n[(e + f*x)/2]] - Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[c - d
+ 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-
c + d)*Tan[(e + f*x)/2]] - (32*I)*Sqrt[c - d]*d^(5/2)*(Log[(c - I*(d + (1 +
 I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]])
 + ((-I)*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(I + Tan[(e + f*x)/2]))] - Lo
g[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
```

```
 d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(-I + Tan[(e + f
*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((3*c^3)/(32*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (11*c^2*d)/(32*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (29*c*d^2)/(3
2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (11*d^3
)/(32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (d^
3*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]])))/(f*(a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^3 + 11*c^2*d + 29*c*d^2 - 43
*d^3)*Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(3*c^
3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[
c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x
]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Si
n[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - (32*I)*Sqrt[c - d]*d^(5/2)*
((16*d^(7/2)*(I + Tan[(e + f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 -
 I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*S
qrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)
*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(16*d^(7/2)*(I + Tan[(e +
 f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(
1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f
*x)/2]))/(32*d^(7/2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c
 + d)*Tan[(e + f*x)/2]) - (16*d^(7/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*
Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x
])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e
 + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(16*d^
(7/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqr
t[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c
+ d)*Tan[(e + f*x)/2]))/(32*d^(7/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d +
 (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f
*x]] + (I*c + d)*Tan[(e + f*x)/2]))))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 10731 vs. $2(219) = 438$.
time = 0.30, size = 10732, normalized size = 41.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(231) = 462$.
time = 0.94, size = 4041, normalized size = 15.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2
 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2
- 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 1
72*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14
*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(
1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos
(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*
x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*
d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^
2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*s
qrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4
+ c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13
*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e
)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17
*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2
 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^
3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2
 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^
2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 +
 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 3
2*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15
*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) +
1)) + 2*(3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (
7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*
c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*si
n(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
```

```
3*f)*sin(f*x + e)), 1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*
x + e)^3 - 12*a*c^2 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d
^2)*cos(f*x + e)^2 - 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a
*c^2 + 56*a*c*d + 172*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^
2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c -
 d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*s
qrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2
 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x +
e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) + 16*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*c
os(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*
d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(3*(c^2 + 4*c*d
 - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)
*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f
*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
 (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), -1/
16*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 -
56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2
*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*
a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*
c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt
(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((
c - d)*cos(f*x + e))) - 4*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 -
2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x +
e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2
*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 -
32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d
^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c
*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
 + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*
c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e
))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/
a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 ...
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{5/2}}{(a+a\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.601 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=184

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}\sqrt{c-d}\,f}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -3/32*(c+d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)/(c-d)^(1/2)-1/4*(c-d
)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c+7*d)
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]

time = 0.35, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2844, 3057, 12, 2861, 214}

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f\sqrt{c-d}}-\frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c + d)^2*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*Sqrt[c - d]
*f) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c + 7*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(
a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
```

```
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c^2+6cd-d^2)-ad(c+3d)\sin(e+\ldots}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+\ldots}}}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{16af(a+a\sin(e+fx))^{3\ldots}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{16af(a+a\sin(e+fx))^{3\ldots}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin\ldots}}{16af(a+a\sin(e+fx))^{3\ldots}}$$

$$= -\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}\sqrt{c-d}\,f} - \frac{(c-d\ldots}{\ldots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 396 vs. 2(184) = 368.
time = 6.94, size = 396, normalized size = 2.15

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))(7c+3d+(3c+7d)\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{3(c+d)^2\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{32f(a(1+\sin(e+fx)))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x])*(7*c + 3*d + (3*c + 7*d)*Sin[e + f*x]))/(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3*(c + d)^2*(Log[1 + Tan[(e + f*x)/2]
] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[
e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e
+ f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e
+ f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e
 + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*S
in[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*f*(a*(1 + Sin[e + f*x]))^(
5/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. 2(155) = 310.
time = 11.34, size = 3050, normalized size = 16.58

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3050 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/f*(6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)
-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c^2-
12*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*c^2+28*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*d^2+12*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*cos(f*x+e)^3*c^2-28*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)^3*d^2-16*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*c*d+16*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*c*d-12*2^(1/2)*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(
f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*(2*c-2*d)^(1/2)*c^2-12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
```

(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*d^2-28*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c^2+12*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2-3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^3*c^2-3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^3*d^2+9*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*c^2+9*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*d^2-12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*c^2-12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*d^2+6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*c^2+6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*d^2-24*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*c*d+16*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d+6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2+18*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*c*d-24*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*c*d+12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*c*d+3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*c^2+3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos

```
(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)
^(1/2)*sin(f*x+e)*cos(f*x+e)^2*d^2-6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)
*cos(f*x+e)^3*c*d+6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f
*x+e)^2*c*d+12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)
*c*d)*(c+d*sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/...
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(164) = 328$.
time = 0.58, size = 1364, normalized size = 7.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/128*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^2 + 2*c*d + d^2)*cos(f
*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e) + ((
c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 14*a*c
*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 2
2*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3*c - d)
*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*c + 4
*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) -
 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d + 4*a*d
^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
(cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) +
 8*((3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2
```

```
 - 4*c*d - 3*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d -
7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
+ e) + c))/((a^3*c - a^3*d)*f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x
+ e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c -
 a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^
3*d)*f)*sin(f*x + e)), -1/64*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^
2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d +
d^2)*cos(f*x + e) + ((c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4
*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*
d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(
f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*
sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 + 4*c*d - 7*d^2)*
cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 4*c*d - 3*d^2)*cos(f*x +
e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c - a^3*d)*
f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f
*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2
*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin(e + fx))^{\frac{3}{2}}}{(a(\sin(e + fx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,\sin(e + f\,x))^{3/2}}{(a + a\,\sin(e + f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.602 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{3/2}f}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] -1/32*(3*c-5*d)*(c+d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a
+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(3/2)/f*2^(1/2)-
1/4*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c-d)
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/(a+a*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2843, 3057, 12, 2861, 214}

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{3/2}}-\frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/16*((3*c - 5*d)*(c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt
[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*(
c - d)^(3/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e
 + f*x])^(5/2)) - ((3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*(
c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(3c+d)+ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,d}{4a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^{3/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 412 vs. $2(191) = 382$.
time = 6.68, size = 412, normalized size = 2.16

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(7c-5d+(3c-d)\sin(e+fx))(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{(3c^2-2cd-5d^2)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{32(c-d)f(a(1+\sin(e+fx)))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 5*d + (3*c - d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 2*c*d - 5*d^2)*(Log[1 + Tan[(e
 + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[
c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 +
 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((
1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c
 + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*(c - d)*f*(a*(1
+ Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. $2(162) = 324$.
time = 11.13, size = 3050, normalized size = 15.97

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3050 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(-6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c^
2+12*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*c^2+4*((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*d^2-12*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*cos(f*x+e)^3*c^2-4*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)^3*d^2-16*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*c*d+16*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*c*d+12*2^(1/2)*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(
f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*(2*c-2*d)^(1/2)*c^2-20*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
```

(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*d^2+28*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c^2+20*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2+3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^3*c^2-5*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^3*d^2-9*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*c^2+15*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*d^2+12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*c^2-20*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*d^2-6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*c^2+10*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*d^2-8*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*c*d-48*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d+10*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*d^2+6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)^2*c*d-8*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*c*d+4*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*cos(f*x+e)*c*d-3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*c^2+5*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(

```
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^
(1/2)*sin(f*x+e)*cos(f*x+e)^2*d^2-2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*
cos(f*x+e)^3*c*d+2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*
x+e)^2*c*d+4*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c
*d)*(c+d*sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c...
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(171) = 342$.
time = 0.63, size = 1534, normalized size = 8.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/128*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d^2)
*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f
*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2
 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d
)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4
*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(
f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*
sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a
*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2
*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) -
2*cos(f*x + e) - 4)) + 8*((3*c^2 - 4*c*d + d^2)*cos(f*x + e)^2 + 4*c^2 - 8*
```

```
c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^
2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x +
e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*
a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f + (
(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d +
 a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f)*sin(f*x + e
)), -1/64*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d
^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*co
s(f*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*
d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2
*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*s
in(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 - 4*c*d + d^2)
*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x
+ e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3
*c*d + a^3*d^2)*f*cos(f*x + e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(
f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2
- 2*a^3*c*d + a^3*d^2)*f + ((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^
2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c
*d + a^3*d^2)*f)*sin(f*x + e))]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sin{(e + fx)}}}{(a(\sin{(e + fx)} + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.603 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=201

$$-\frac{(3c^2-10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^{5/2}f}-\frac{\cos(e+fx)\sqrt{c+d\sin(e-}}{4(c-d)f(a+a\sin(e+f}$$

```
[Out] -1/32*(3*c^2-10*c*d+19*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(5/2)/f*2^
(1/2)-1/4*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-
3/16*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/f/(a+a*sin(f*x+e))
^(3/2)
```

**Rubi** [A]

time = 0.34, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2845, 3057, 12, 2861, 214}

$$-\frac{(3c^2-10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^{5/2}}-\frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -1/16*((3*c^2 - 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])
/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[2]*a^(
5/2)*(c - d)^(5/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*(c - d)*
f*(a + a*Sin[e + f*x])^(5/2)) - (3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-7c}{(a+a\sin(e+fx))^{3/2}\sqrt{}}}{4a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+}{16a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+}{16a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+}{16a(c-d)^2f(a}$$

$$= -\frac{(3c^2-10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right.}{16\sqrt{2}a^{5/2}(c-d)^{5/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 411 vs. 2(201) = 402.

time = 7.35, size = 411, normalized size = 2.04

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(7c-13d+3(c-3d)\sin(e+fx))(c+d\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{(3c^2-10cd+19d^2)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{32(c-d)^2f(a(1+\sin(e+fx)))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 13*d + 3*(c - 3*d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 10*c*d + 19*d^2)*(Log[1 + T
an[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*
Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2
/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c -
d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/S
qrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1
)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*(c - d)^2*f
*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. 2(172) = 344.

time = 11.34, size = 2805, normalized size = 13.96

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2805 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/32/f*(-10*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*c*d+6*(2*c-2*d)^
(1/2)*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c-18*(2*c-2*d)^(
1/2)*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-6*(2*c-2*d)^(1/
2)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+18*(2*c-2*d)^(1/2)*
cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-3*ln(2*((2*c-2*d)^(1/2
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d
*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f
*x+e)^3*2^(1/2)*c^2-19*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)^3*2^(1/2)*d^2+9*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(
f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*cos(f*x+e)^2*2^(1/2)*c^2+57*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+
e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*d^2-12
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*c^2-76*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)
*d^2-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-76*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin
(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+6*ln(2*((2
*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+co
s(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*
x+e)))*cos(f*x+e)*2^(1/2)*c^2+38*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+
e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)*d^2+40*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*2^(1/2)*c*d+3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
```

```
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*
c^2+19*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*d^2+10*ln(2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+
e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))
*cos(f*x+e)^3*2^(1/2)*c*d+6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)*2^(1/2)*c^
2+38*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(
f*x+e)+sin(f*x+e)))*sin(f*x+e)*cos(f*x+e)*2^(1/2)*d^2-30*ln(2*((2*c-2*d)^(1
/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c
-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos
(f*x+e)^2*2^(1/2)*c*d+40*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*c*d-20*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos
(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*cos(f*x+e)*2^(1/2)*c*d-14*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+26*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+
e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-20*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*c
os(f*x+e)*2^(1/2)*c*d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))
^(5/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/(2*c-2*d)^(1/2)/(c-d)^2
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(181) = 362$.
time = 0.78, size = 1704, normalized size = 8.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/128*(((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*
d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2
)*cos(f*x + e) + ((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*
d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2
*a*c - 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 -
 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c
 - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) +
 4*c - 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x
 + e) - (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*
x + e)^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(co
s(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin
(f*x + e) - 2*cos(f*x + e) - 4)) + 8*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^
2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) - (4*c^2
 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2*d + 3*
a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*
d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(((3*c^2 -
10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^
2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e) + (
(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*
c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arc
tan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x +
e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*
x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) -
 (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2
*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3
*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c
*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a
^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)
^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(
a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

**Sympy** [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(c + d*sin(e + f*x))), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.604 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=270

$$-\frac{3(c^2-6cd+25d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^{7/2}f}-\frac{\cos(e+}{4(c-d)f(a+a\sin(e+fx))}$$

```
[Out] -3/32*(c^2-6*c*d+25*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)
/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(7/2)/f*2^(1/
2)-1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2)-1/1
6*(3*c-13*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))
^(1/2)-1/16*(c-7*d)*d*(3*c+7*d)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(a+a*sin(f*x
+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.59, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2845, 3057, 3063, 12, 2861, 214}

$$-\frac{3(c^2-6cd+25d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^{7/2}}-\frac{d(c-7d)(3c+7d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{(3c-13d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-3*(c^2 - 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt
[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2
)*(c - d)^(7/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*S
qrt[c + d*Sin[e + f*x]]) - ((3*c - 13*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a
 + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]) - ((c - 7*d)*d*(3*c + 7*
d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} -$$

$$= -\frac{3(c^2-6cd+25d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\ c}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\ \cdots}\right.}{16\sqrt{2}\,a^{5/2}(c-d)^{7/2}f}$$

**Mathematica** [A]
time = 8.94, size = 462, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-14c^3+25c^2d+56cd^2+113d^3+d(3c^2-14cd-49d^2)\cos(2(e+fx))+(-6c^3+14c^2d+62cd^2+170d^3)\sin(e+fx)\right)}{(c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{3(c^2-6cd+25d^2)\left(\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}}\right)}{32(c-d)^3f(a(1+\sin(e+fx)))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(((Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(-14*c^3 + 25*c^2*d + 56*c*d^2 + 113*d^3 + d*(3*c^2 - 14*c*d - 49*d^2
)*Cos[2*(e + f*x)] + (-6*c^3 + 14*c^2*d + 62*c*d^2 + 170*d^3)*Sin[e + f*x])
)/((c + d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (3*(c^2 - 6*c*d + 25*
d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^
2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*S
in[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/
(32*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 4261 vs. $2(235) = 470$.

time = 11.62, size = 4262, normalized size = 15.79

| method | result | size |
|---|---|---|
| default | Expression too large to display | 4262 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/32/f*(150*2^(1/2)*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*d^3-3*sin(f*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*c
os(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3-162*(2*c-2*d)^(1/2)*cos(f*x+e)
*d^3+98*(2*c-2*d)^(1/2)*cos(f*x+e)^3*d^3+14*(2*c-2*d)^(1/2)*cos(f*x+e)*c^3+
12*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos
(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+300*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-170*(2*c-
2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*d^3-22*(2*c-2*d)^(1/2)*cos(f*x+e)*c^2*d-70
*(2*c-2*d)^(1/2)*cos(f*x+e)*c*d^2-14*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*
c^2*d-3*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*c^3-75*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d
*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*d^3-9*2^(1/2)*cos(f*x+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-
cos(f*x+e)+sin(f*x+e)))*c^3+150*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+
c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3-6*(2*c-2*d)^(1/2)*cos(f*x+e)^3*c^2*d+28*
(2*c-2*d)^(1/2)*cos(f*x+e)^3*c*d^2+6*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*
c^3-75*sin(f*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)
-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(co
```

```
s(f*x+e)+1))^(1/2)*d^3+6*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3+15*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d
*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*d-57*cos(f*x+e)^3*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(
f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d^2-225*2^(1/2)*cos(
f*x+e)^2*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3+12*ln(2
*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+si
n(f*x+e)))*c^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+3
00*2^(1/2)*sin(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*
d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x
+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e))
)*d^3+6*2^(1/2)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2
*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+co
s(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*
x+e)))*c^3-60*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d
)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)+228*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)-62*(2*c-2*d)^(1/2)*sin(f*x+e)*cos(f*x+e)*c*d^2-30*ln(2*((2*c-2*d)^(1/
2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-
d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*c^2*d+114
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)+c*sin(f*x+e)-d...
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(247) = 494$.
time = 1.02, size = 3056, normalized size = 11.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/128*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^4 + 4*c^4
- 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^4 - 3*c^3*d + 9*c^2*d^2
+ 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 32*c^2*d^2 + 170*
c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3
+ 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^3 - (c^4 - 2*c^3*
d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c
^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)
*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*
a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f
*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*s
in(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(
d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*
c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*
(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
+ 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2
*cos(f*x + e) - 4)) + 8*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*
c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2
 - 19*c*d^3 + 36*d^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*
c*d^3 + 81*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*
d - 17*c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24
*c^2*d^2 - 54*c*d^3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d
^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a
^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^
3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3
*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^
3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d
^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a
^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c
^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x +
 e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*
d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2
+ 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(
a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c
```

```
*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 +
 25*d^4)*cos(f*x + e)^4 + 4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^4 - 3*c^3*d + 9*c^2*d^2 + 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^
4 - 10*c^3*d + 32*c^2*d^2 + 170*c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 -
4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d
+ 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4
)*cos(f*x + e)^3 - (c^4 - 2*c^3*d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x
+ e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*si
n(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)
/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))
) + 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 35*c*d^3
 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2 - 19*c*d^3 + 36*d
^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*c*d^3 + 81*d^4)*co
s(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 3
5*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24*c^2*d^2 - 54*c*d^
3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4
 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c
^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x +
 e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2
*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*
d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*co
s(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c
^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3
*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 + (a^3*c^6 -
 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f
*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 -
a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^
5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)
*sin(f*x + e))]
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.605 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=355

$$-\frac{(3c^2-26cd+163d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{9/2}f}-\frac{\cos(e}{4(c-d)f(a+a\sin(e+fx}$$

```
[Out] -1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2)-1/16*
(3*c-17*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(
3/2)-1/32*(3*c^2-26*c*d+163*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(9/2)
/f*2^(1/2)-1/48*d*(9*c^2-54*c*d-95*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d
*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-1/48*d*(9*c^3-57*c^2*d-493*c*d^2-
299*d^3)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f
*x+e))^(1/2)
```

**Rubi** [A]

time = 0.84, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2845, 3057, 3063, 12, 2861, 214}

$$-\frac{(3c^2-26cd+163d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{9/2}}-\frac{d(9c^2-54cd-95d^2)\cos(e+fx)}{48a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{d(9c^3-57c^2d-493cd^2-299d^3)\cos(e+fx)}{48a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{(3c-17d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/16*((3*c^2 - 26*c*d + 163*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x]
)/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[2]*a^
(5/2)*(c - d)^(9/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/
2)*(c + d*Sin[e + f*x])^(3/2)) - ((3*c - 17*d)*Cos[e + f*x])/(16*a*(c - d)^
2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^2 - 54
*c*d - 95*d^2)*Cos[e + f*x])/(48*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^3 - 57*c^2*d - 493*c*d^2 - 299
*d^3)*Cos[e + f*x])/(48*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}$$

$$= -\frac{(3c^2-26cd+163d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-}}{\sqrt{2}\sqrt{a+a\sin(e+fx}}\right.}{16\sqrt{2}\,a^{5/2}(c-d)^{9/2}f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 717 vs. $2(355) = 710$.
time = 10.19, size = 717, normalized size = 2.02

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^5\sqrt{c+d\sin(e+fx)}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{2(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}-\frac{1}{4(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{-3c+25d}{16(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{3c\sin\left(\frac{1}{2}(e+fx)\right)-25d\sin\left(\frac{1}{2}(e+fx)\right)}{8(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{2\left(d^3\cos\left(\frac{1}{2}(e+fx)\right)-d^3\sin\left(\frac{1}{2}(e+fx)\right)\right)}{3(c-d)^3(c+d)(c+d\sin(e+fx))^2}+\frac{2\left(11cd^3\cos\left(\frac{1}{2}(e+fx)\right)+7d^4\cos\left(\frac{1}{2}(e+fx)\right)-11cd^3\sin\left(\frac{1}{2}(e+fx)\right)-7d^4\sin\left(\frac{1}{2}(e+fx)\right)\right)}{3(c-d)^4(c+d)^2(c+d\sin(e+fx))}\right)}{f(a(1+\sin(e+fx)))^{5/2}}+\frac{(3c^2-26cd+163d^2)\left(\log\left(1+\tan\left(\tfrac{1}{2}(e+fx)\right)\right)-\log\left(c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\tfrac{1}{2}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4}{32(c-d)^4f(a(1+\sin(e+fx)))^{5/2}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2+2\tan\left(\frac{1}{2}(e+fx)\right)}-\frac{-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{c-d}\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}(d+d\cos(e+fx)+c\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{c+d\sin(e+fx)}+(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c + d*Sin[e + f*x]]*(Sin[(e +
 f*x)/2]/(2*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - 1/(4*(c -
d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (-3*c + 25*d)/(16*(c - d)^4
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (3*c*Sin[(e + f*x)/2] - 25*d*Sin[
(e + f*x)/2])/(8*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(d
^3*Cos[(e + f*x)/2] - d^3*Sin[(e + f*x)/2]))/(3*(c - d)^3*(c + d)*(c + d*Si
n[e + f*x])^2) + (2*(11*c*d^3*Cos[(e + f*x)/2] + 7*d^4*Cos[(e + f*x)/2] - 1
1*c*d^3*Sin[(e + f*x)/2] - 7*d^4*Sin[(e + f*x)/2]))/(3*(c - d)^4*(c + d)^2*
```

```
(c + d*Sin[e + f*x]))))/(f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((3*c^2 - 26*c*d
 + 163*d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1
+ Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(32*(c - d)^4*f*(a*(1 + Sin[e +
f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f
*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f
*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[
e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 8034 vs. $2(314) = 628$.
time = 11.40, size = 8035, normalized size = 22.63

| method | result | size |
|---|---|---|
| default | Expression too large to display | 8035 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 2348 vs. $2(329) = 658$.
time = 2.01, size = 4942, normalized size = 13.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/384*(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 +
2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 +
```

163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 942*c^2*d^4
 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c^4*d^2 + 6
96*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3 - (9*c^6 -
 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^5 + 1141*d
^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c
^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d + 308*c^4*d
^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c
^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3*c^5*d - 1
7*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 + 463*c*d^5 + 163*d^6)*cos(f*x + e)^3
 - (3*c^6 - 2*c^5*d + 9*c^4*d^2 + 884*c^3*d^3 + 2533*c^2*d^4 + 2478*c*d^5 +
815*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 +
877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c -
 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c
*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d
)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c -
 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e)
 - (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)
^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x
 + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x +
 e) - 2*cos(f*x + e) - 4)) - 8*(12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3
 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 - (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 +
 194*c*d^5 + 299*d^6)*cos(f*x + e)^4 - (18*c^5*d - 111*c^4*d^2 - 618*c^3*d^
3 - 520*c^2*d^4 + 728*c*d^5 + 503*d^6)*cos(f*x + e)^3 + 3*(3*c^6 - 14*c^5*d
 - 29*c^4*d^2 - 144*c^3*d^3 - 59*c^2*d^4 + 158*c*d^5 + 85*d^6)*cos(f*x + e)
^2 + 3*(7*c^6 - 16*c^5*d - 73*c^4*d^2 - 312*c^3*d^3 - 91*c^2*d^4 + 328*c*d^
5 + 157*d^6)*cos(f*x + e) - (12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 -
12*c^2*d^4 - 24*c*d^5 + 12*d^6 + (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 19
4*c*d^5 + 299*d^6)*cos(f*x + e)^3 - 6*(3*c^5*d - 20*c^4*d^2 - 92*c^3*d^3 -
14*c^2*d^4 + 89*c*d^5 + 34*d^6)*cos(f*x + e)^2 - 3*(3*c^6 - 8*c^5*d - 69*c^
4*d^2 - 328*c^3*d^3 - 87*c^2*d^4 + 336*c*d^5 + 153*d^6)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^7*d^2 -
 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7
 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 -
7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2
*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8
*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*
d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d -
20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c
^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a
^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*
c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x +
e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4
 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f +
 ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^

```
6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d -
2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d
^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*
a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d
^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^
9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d
^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) +
4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*
a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f
*x + e)), -1/192*(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*
c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 30
0*c*d^5 + 163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 9
42*c^2*d^4 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c
^4*d^2 + 696*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3
- (9*c^6 - 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^
5 + 1141*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d
^3 + 877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d +
 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*
d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3
*c^5*d - 17*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 ...
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.606 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=129

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos
(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+d*sin(
f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
+ Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e
 + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c -
d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+f$$

$$\overline{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+}}{\sqrt{\frac{1}{2}-}}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,(c+d\sin(e+fx))^n\,\left(\right.\right.}{\sqrt{2}\,f(a-a\sin}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e-}{c-d}\right.}{f(1+}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(129) = 258.

time = 1.57, size = 373, normalized size = 2.89

$$\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^n\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f\left(-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(4dnF_1\left(\frac{3}{2};\frac{1}{2}-m,1-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (6*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x
])^n*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3*(c + d)*AppellF1[1/2,
1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/
4]^2)/(c + d)] + (4*d*n*AppellF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c+d\sin(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m (c + d\sin(e + fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**n, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

# 3.607 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=320

$$-\frac{d(d^2(4+m)-cd(5-3m-2m^2)+2c^2(8+6m+m^2))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)(3+m)}-\frac{2^{\frac{1}{2}+m}(d^3m(5+$$

```
[Out] -d*(d^2*(4+m)-c*d*(-2*m^2-3*m+5)+2*c^2*(m^2+6*m+8))*cos(f*x+e)*(a+a*sin(f*x
+e))^m/f/(3+m)/(m^2+3*m+2)-2^(1/2+m)*(d^3*m*(m^2+3*m+5)+3*c^2*d*m*(m^2+5*m+
6)+3*c*d^2*(m^3+4*m^2+4*m+3)+c^3*(m^3+6*m^2+11*m+6))*cos(f*x+e)*hypergeom([
1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+
e))^m/f/(3+m)/(m^2+3*m+2)-d^2*(d*m+c*(5+m))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+
m)/a/f/(2+m)/(3+m)-d*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+
m)
```

**Rubi** [A]
time = 0.46, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2862, 3047, 3102, 2830, 2731, 2730}

$$-\frac{d(2c^2(m^2+6m+8)-cd(-2m^2-3m+5)+d^2(m+4))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)(m+2)(m+3)}-\frac{2^{m+\frac{1}{2}}(c^3(m^3+6m^2+11m+6)+3c^2dm(m^2+5m+6)+3cd^2(m^3+4m^2+4m+3)+d^3m(m^2+3m+5))\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)(m+2)(m+3)}-\frac{d^2(c(m+5)+dm)\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)(m+3)}-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((d*(d^2*(4 + m) - c*d*(5 - 3*m - 2*m^2) + 2*c^2*(8 + 6*m + m^2))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m))) - (2^(1/2 + m)*(d
^3*m*(5 + 3*m + m^2) + 3*c^2*d*m*(6 + 5*m + m^2) + 3*c*d^2*(3 + 4*m + 4*m^2
 + m^3) + c^3*(6 + 11*m + 6*m^2 + m^3))*Cos[e + f*x]*Hypergeometric2F1[1/2,
 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*S
in[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d^2*(d*m + c*(5 + m))*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (d*Cos[e + f*x
]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2862

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + n))), x] + Dist[1/(b*(m + n)), Int
[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n
 - 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} +$$
$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} +$$
$$= -\frac{d^2(dm+c(5+m))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)} -$$
$$= -\frac{d(d^2(4+m)-cd(5-3m-2m^2)+2c^2(8+6m+m^2))}{f(1+m)(2+m)(3+m}$$
$$= -\frac{d(d^2(4+m)-cd(5-3m-2m^2)+2c^2(8+6m+m^2))}{f(1+m)(2+m)(3+m}$$
$$= -\frac{d(d^2(4+m)-cd(5-3m-2m^2)+2c^2(8+6m+m^2))}{f(1+m)(2+m)(3+m}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 3599 vs. $2(320) = 640$.
time = 63.06, size = 3599, normalized size = 11.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-22*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2
)^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^3*(945*(c + d)^3*Gamm
a[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]
 - 1890*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin
[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 142*(c + d)^3*Gamma[3
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2 + 60*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{
3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
*x)/2]^2 + 8*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 2
268*d^2*(c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e
 + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 564*d*(c + d)^2*Gamma[3/2
 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin
[(-e + Pi/2 - f*x)/2]^4 - 312*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 -
f*x)/2]^4 - 48*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4
 - 1080*d^3*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + P
```

i/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 744*d^2*(c + d)*Gamma[3/2 - m
]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
 + Pi/2 - f*x)/2]^6 + 528*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)
/2]^6 + 96*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 - 3
68*d^3*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2
- f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 288*d^3*Gamma[3/2 - m]*Hypergeome
tricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e +
 Pi/2 - f*x)/2]^8 - 64*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8
)*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(3*f*(3465*(c + d)^3*Gam
ma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2
] - 20790*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, S
in[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 - 385*(c + d)^3*(-1 +
 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 1562*(c + d)^3*Gamma[3/2 - m]*Hype
rgeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2 + 660*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2
 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 +
88*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1,
11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 41580*d^2*(
c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 + 770*d*(c + d)^2*(-1 + 2*m)*Gamma[1
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^4 - 10340*d*(c + d)^2*Gamma[3/2 - m]*Hypergeometric
2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2
]^4 - 142*(c + d)^3*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 -
m, 13/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 5720*d*(c
 + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 120*(c + d)^3*(-3 + 2*
m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e +
 Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 880*d*(c + d)^2*Gamma[3/2 -
 m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 32*(c + d)^3*(-3 + 2*m)*Gamma[3/2 -
 m]*HypergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 27720*d^3*Gamma[1/2 - m]*Hypergeome
tric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x
)/2]^6 - 924*d^2*(c + d)*(-1 + 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3
/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 1909
6*d^2*(c + d)*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e
+ Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 564*d*(c + d)^2*(-3 + 2*m)
*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e + Pi/2 - f*x)
/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 13552*d^2*(c + d)*Gamma[3/2 - m]*Hyperg
eometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(

```
-e + Pi/2 - f*x)/2]^6 + 624*d*(c + d)^2*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeom
etricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
+ Pi/2 - f*x)/2]^6 + 2464*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 ...
```

**Maple** [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx+e))^m (c + d\sin(fx+e))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3
*c^2*d - d^3)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (c + d\sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\,\sin(e + f\,x))^m\,(c + d\,\sin(e + f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3, x)
```

# 3.608 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=193

$$\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{2^{\frac{1}{2}+m}(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+}{}$$

```
[Out] d*(d-2*c*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)*(2*c*
d*m*(2+m)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))*cos(f*x+e)*hypergeom([1/2, 1/2-m],
[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(m^2
+3*m+2)-d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

**Rubi** [A]
time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2840, 2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}\left(c^2(m^2+3m+2)+2cdm(m+2)+d^2(m^2+m+1)\right)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)(m+2)}+\frac{d(d-2c(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)(m+2)}-\frac{d^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (d*(d - 2*c*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m
)) - (2^(1/2 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2
))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*
(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) -
 (d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx &= -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}+\frac{\int (a+a\sin(e+f\\ &= \frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos\\ &= \frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos\\ &= \frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{2^{\frac{1}{2}+m}(\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1774 vs. $2(193) = 386$.
time = 71.86, size = 1774, normalized size = 9.19

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)
^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2*(4*Gamma[3/2 - m]*Hy
pergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Si
n[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Ga
mma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^
2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2)
 + d^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) +
 7*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x
```

```
)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*
Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)))*(a + a*Sin[e
+ f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(f*(4*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Gamma[3/2 - m]*Hyp
ergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Si
n[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 7*Gamma[1/2 - m
]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x)/2]^2]*(15*c^2
+ 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*Sin[(-e + Pi/2 -
 f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)) + (2*Sin[(-e + Pi/2 - f*x)/2]^
2*(-48*d*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + P
i/2 - f*x)/2]^2) + 12*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {
1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2)^2 - 4*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2,
 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d
*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 48*d*Gamma[3/2 - m]*Hypergeometric2F1[3/2,
 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]*(-3*c*S
in[(-e + Pi/2 - f*x)/2] + d*Sin[(-e + Pi/2 - f*x)/2]*(-3 + 4*Sin[(-e + Pi/2
 - f*x)/2]^2)) + 84*d*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, S
in[(-e + Pi/2 - f*x)/2]^2]*(-5*c + d*(-5 + 6*Sin[(-e + Pi/2 - f*x)/2]^2)) +
 48*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*
x)/2]^2]*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Sin[(-e
 + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) - 8*(-3 + 2*m)*Gamma[3
/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d
^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 3*(
1/2 - m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2
 - f*x)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15
 - 20*Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4))))/3))
```

**Maple** [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (a + a \sin (fx + e))^m (c + d \sin (fx + e))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(a*sin(f*x
+ e) + a)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m (c+d\sin (e+fx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (a+a\,\sin \left (e+f\,x\right )\right )}^m\,{\left (c+d\,\sin \left (e+f\,x\right )\right )}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

# 3.609 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=117

$$-\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(c+cm+dm)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1+m)}$$

```
[Out] -d*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(c*m+d*m+c)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

**Rubi** [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}(cm+c+dm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -((d*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(c +
c*m + d*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{(c+cm+dm)\int(a+\ldots}{1+\ldots}$$
$$=-\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{((c+cm+dm)(1+\ldots}{\ldots}$$
$$=-\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(c+cm+dm)\ldots}{\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 2.10, size = 275, normalized size = 2.35

$$-\frac{(a(1+\sin(e+fx)))^m\left(\frac{\sqrt[4]{-1}\,2^{-1-2m}de^{-\frac{3}{2}i(e+fx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\right)^{1+2m}\left(e^{2i(e+fx)}(-1+m)\,{}_2F_1\left(1,m;-m;-ie^{-i(e+fx)}\right)-(1+m)\,{}_2F_1\left(1,2+m;2-m;-ie^{-i(e+fx)}\right)\right)}{-1+m^2}+\frac{2\sqrt{2}\,c\cos^{1+2m}\left(\frac{1}{4}(2e-\pi+2fx)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{(1+2m)\sqrt{1-\sin(e+fx)}}\right)\sin^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*d*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*c*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (c+d\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

# 3.610 $\int (a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=74

$$-\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+s
in(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^m\,dx &= \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx\\ &= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}}{f}\end{aligned}$$

**Mathematica** [A]
time = 0.17, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m}{(f+2fm)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

**Maple** [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
[Out] int((a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\,\sin\left(e + f\,x\right))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m, x)
```

# 3.611 $\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=100

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2867, 142, 141}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)*f*(
1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)}\,dx, x, \sin(e+\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a-}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 363 vs. 2(100) = 200.
time = 1.22, size = 363, normalized size = 3.63

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,1;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f(c+d\sin(e+fx))\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,1;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,1;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2
 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(c + d)] + (4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1
[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m}{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin\left(e + f\,x\right))^m}{c + d\,\sin\left(e + f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x)), x)
```

# 3.612 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=100

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m}{(c-d)^2 f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2867, 142, 141}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^2}\,dx, x, \sin(e-\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a-}{(c-d)^2f(1+2m)\sqrt{1-\sin(e+fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 363 vs. $2(100) = 200$.
time = 1.35, size = 363, normalized size = 3.63

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f(c+d\sin(e+fx))^2\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(8dF_1\left(\frac{3}{2};\frac{1}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^2*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4
]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appell
F1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m}{(c + d\sin(fx + e))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^2, x)
```

# 3.613 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=100

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m}{(c-d)^3 f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,3,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^3/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2867, 142, 141}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^3\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^3*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^3}\,dx, x, \sin(e\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a}{(c-d)^3 f(1+2m)\sqrt{1-\sin(e+fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 363 vs. 2(100) = 200.
time = 1.50, size = 363, normalized size = 3.63

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,3;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f(c+d\sin(e+fx))^3\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,3;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(12dF_1\left(\frac{3}{2};\frac{1}{2}-m,4;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^3*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (12*d*AppellF1[3/2, 1/2 - m, 4, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appel
lF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m}{(c + d\sin(fx + e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a \sin(e+f x))^m}{(c+d \sin(e+f x))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^3, x)
```

# 3.614 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=138

$$\frac{\sqrt{2}\,(c-d)^2F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{\phantom{x}}}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (c-d)^2*AppellF1(1/2+m,-5/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f
*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*
m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(c-d)^2\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*(c - d)^2*AppellF1[1/2 + m, 1/2, -5/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*
Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*S
in[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{5/2}\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{5/2}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}}}{\sqrt{\frac{1}{2}}}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(}}$$

$$= \frac{\left((ac-ad)^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{c+d\sin(}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+}}$$

$$= \frac{\sqrt{2}\,(c-d)^2 F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),\right.}{f(1+2m)\sqrt{1-\sin}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(138) = 276.
time = 1.91, size = 365, normalized size = 2.64

$$-\frac{3\sqrt{2}\,(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{5}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sqrt{1+\sin(e+fx)}\,(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{5/2}\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f\sqrt{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}\left(-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{5}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -5/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(5/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (10*d*AppellF1[3/2, 1/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF
1[3/2, 3/2 - m, -5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
+ 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c+d\sin(fx+e))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\,\sin\left(e + f\,x\right))^m\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.615 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=136

$$\frac{\sqrt{2}\,(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{}}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)
/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

**Rubi** [A]

time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\,dx,x,\sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$=\frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}}}{\sqrt{\frac{1}{2}}}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(}}$$

$$=\frac{\left(a(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{c+d\sin(}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+}}$$

$$=\frac{\sqrt{2}\,(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\right.}{f(1+2m)\sqrt{1-\sin}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(136) = 272.
time = 1.36, size = 365, normalized size = 2.68

$$-\frac{3\sqrt{2}\,(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sqrt{1+\sin(e+fx)}\,(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{3/2}\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f\sqrt{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}\left(-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(3/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (6*d*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1
[3/2, 3/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin\left(e+f\,x\right))^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.616 $\int (a+a\sin(e+fx))^m \sqrt{c+d\sin(e+fx)}\, dx$

**Optimal**. Leaf size=131

$$\frac{\sqrt{2}\, F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*c
os(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)/(1-si
n(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\, F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(
1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d
*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x
])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx,x,\sin(e-\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=\frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+}}{\sqrt{\frac{1}{2}}}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(}}$$

$$=\frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+}}$$

$$=\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e}{c-d}\right.}{f(1+2m)\sqrt{1-\sin(e+}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(131) = 262.

time = 1.17, size = 365, normalized size = 2.79

$$-\frac{3\sqrt{2}\,(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sqrt{1+\sin(e+fx)}\,(a(1+\sin(e+fx)))^m\sqrt{c+d\sin(e+fx)}\,\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f\sqrt{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}\left(-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*Tan[(2*e - Pi + 2*f*x)/4])
/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m, -1
/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)] + (2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/
2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m \sqrt{c+d\sin (e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin \left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin \left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.617 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=131

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{\frac{c+d\sin}{c-}}}{f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/(1+2*m
)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
```

```
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{c+dx}}\,dx, x, \sin\ldots\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\ldots}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \ldots\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\ldots}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a\ldots}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+\ldots}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(131) = 262.
time = 1.23, size = 373, normalized size = 2.85

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f\sqrt{c+d\sin(e+fx)}\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*Sqrt[c + d*Sin[e + f*x]]*(3*(c + d)*AppellF1[1/
2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c+d\sin\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.618 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c-d)/f/
(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^{3/2}}\,dx, x, \sin(\ldots\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{\frac{1}{2}\ldots}}\right.}{\sqrt{2}\,(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\ldots}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a\ldots}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin\ldots}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(138) = 276.
time = 1.42, size = 373, normalized size = 2.70

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f(c+d\sin(e+fx))^{3/2}\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(3/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5/2, Cos[(2*e + Pi
+ 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2
*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple** [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{(c+d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin{(e+fx)}+1))^m}{(c+d\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.619 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=138

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{\frac{c+d\sin\cdots}{c-\cdots}}}{(c-d)^2f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,5/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c-d)^2/
f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2867, 145, 144, 143}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 5/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)^2*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^{5/2}}\,dx, x, \sin(\ldots\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^4\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \ldots\right.}{\sqrt{2}\,(ac-ad)^2 f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\ldots}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a\ldots}{(c-d)^2 f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin\ldots}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(138) = 276.
time = 1.64, size = 373, normalized size = 2.70

$$-\frac{6(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{5}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f(c+d\sin(e+fx))^{5/2}\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{5}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{7}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(5/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (10*d*AppellF1[3/2, 1/2 - m, 7/2, 5/2, Cos[(2*e + Pi
 + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 +
2*m)*AppellF1[3/2, 3/2 - m, 5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m}{(c+d\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin (e+fx)+1))^m}{(c+d\sin (e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin (e+f\,x))^m}{(c+d\,\sin (e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.620 $\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=62

$$-\frac{4^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{1-\sin(e+fx)}{4(1+\sin(e+fx))}\right)}{f(1+\sin(e+fx))}$$

```
[Out] -4^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/4*(1-sin(f*x+e))/(1+sin(f
*x+e)))/f/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$-\frac{2^{-2m-1}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{5\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(5\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{5\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1 - 2*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -((1 - Si
n[e + f*x])/(3 + 5*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e
+ f*x])/(3 + 5*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 5*Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+5x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+f$$
$$\overline{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2^{-1-2m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{3+5\sin(e+fx)}\right)(1-}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.48, size = 238, normalized size = 3.84

$$\frac{4^m\,{}_2F_1\left(1+m,1+2m;2(1+m);\frac{4\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(1+\sin(e+fx))^m(1+i\cos(e+fx)+\sin(e+fx))(3+5\sin(e+fx))^{-m}\left(-\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((2-i)-(1-2i)\cos(e+fx)+(2+i)\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(1 + Sin[e
+ f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f*x)/
4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - P
i + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2 - I)
 - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (1+\sin(fx+e))^m(3+5\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="fricas")
[Out] integral((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+5*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\sin\left(e+f\,x\right)+1\right)^m}{\left(5\,\sin\left(e+f\,x\right)+3\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(5*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(5*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.621 $\int (1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=64

$$-\frac{\left(\frac{7}{2}\right)^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{1-\sin(e+fx)}{7(1+\sin(e+fx))}\right)}{f(1+\sin(e+fx))}$$

```
[Out] -(7/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/7*(1-sin(f*x+e))/(1+s
in(f*x+e)))/f/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$-\frac{2^{m+\frac{1}{2}}7^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{4\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(4\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{2(4\sin(e+fx)+3)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*7^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, -1/2*(1 - Sin[e + f*x])/(3 + 4*Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 + m
)*((1 + Sin[e + f*x])/(3 + 4*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 4*Sin[e + f*
x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+4x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}7^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+f}{2(3+4\sin(e+}\right.}{}$$

**Mathematica** [A]
time = 0.53, size = 88, normalized size = 1.38

$$-\frac{2\,7^{-1-m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(1+\sin(e+fx))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(1 + Sin[e + f*x])^m)/(f*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^m)
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (1+\sin(fx+e))^m (3+4\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
[Out] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+4*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(\sin(e+f\,x)+1)^m}{(4\,\sin(e+f\,x)+3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(4*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] int((sin(e + f*x) + 1)^m/(4*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.622 $\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=28

$$-\frac{3^{-1-m}\cos(e+fx)}{f(1+\sin(e+fx))}$$

```
[Out] -3^(-1-m)*cos(f*x+e)/f/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {22, 2727}

$$-\frac{3^{-m-1}\cos(e+fx)}{f(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((3^(-1 - m)*Cos[e + f*x])/(f*(1 + Sin[e + f*x])))
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx &= 3^{-m}\int \frac{1}{3+3\sin(e+fx)}\,dx\\ &= -\frac{3^{-1-m}\cos(e+fx)}{f(1+\sin(e+fx))}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 45, normalized size = 1.61

$$\frac{2\,3^{-1-m}\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
[Out] (2*3^(-1 - m)*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(1+\sin \left(fx+e\right)\right)^{m} \left(3+3\sin \left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
[Out] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
```

**Maxima [A]**
time = 0.51, size = 37, normalized size = 1.32

$$-\frac{2}{\left(3^{m+1}+\frac{3^{m+1}\sin(fx+e)}{\cos(fx+e)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] -2/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

**Fricas [A]**
time = 0.33, size = 58, normalized size = 2.07

$$-\frac{3^{-m-1}(\cos \left(fx+e\right)+1)-3^{-m-1}\sin \left(fx+e\right)}{f\cos \left(fx+e\right)+f\sin \left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="fricas")
[Out] -(3^(-m - 1)*(cos(f*x + e) + 1) - 3^(-m - 1)*sin(f*x + e))/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1}\int \frac{1}{\sin \left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+3*sin(f*x+e))**(-1-m),x)
```

```
[Out] 3**(-m - 1)*Integral(1/(sin(e + f*x) + 1), x)
```

**Giac [A]**
time = 0.64, size = 45, normalized size = 1.61

$$\frac{3^{-m-1}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3^{-m-1}}{f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] (3^(-m - 1)*tan(1/2*f*x + 1/2*e) - 3^(-m - 1))/(f*tan(1/2*f*x + 1/2*e) + f)
```

**Mupad [B]**
time = 0.43, size = 41, normalized size = 1.46

$$\frac{\frac{1}{3^{m+1}}\left(-\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,1\mathrm{i}+1\mathrm{i}\right)}{f\,\left(\sin\left(e+f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] (1/3^(m + 1)*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(sin(e + f*x) + 1))
```

# 3.623 $\int (1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=122

$$-\frac{2^{\frac{1}{2}+m}5^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(3+2\sin(e+fx))}\right)(1+\sin(e+fx))^{-1+m}\left(\frac{1+\sin(e+fx)}{3+2\sin(e+fx)}\right)^{\frac{1}{2}-m}(3+2\sin(e+fx))^{-m}}{f}$$

```
[Out] -2^(1/2+m)*5^(-1/2-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(1-sin(f*
x+e))/(3+2*sin(f*x+e)))*(1+sin(f*x+e))^(-1+m)*((1+sin(f*x+e))/(3+2*sin(f*x+
e)))^(1/2-m)/f/((3+2*sin(f*x+e))^m)
```

**Rubi** [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$-\frac{2^{m+\frac{1}{2}}5^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{2\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(2\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(2\sin(e+fx)+3)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*5^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, (1 - Sin[e + f*x])/(2*(3 + 2*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)
*((1 + Sin[e + f*x])/(3 + 2*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 2*Sin[e + f*x
])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m (3+2\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+2x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}5^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(3+2\sin(e+fx}\right.}{}$$

**Mathematica [A]**
time = 0.61, size = 131, normalized size = 1.07

$$\frac{2\,5^{-1-m}\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{1}{5}\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(1+\sin(e+fx))^m(3+2\sin(e+fx))^{-m}\left(\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(3+2\sin(e+fx))\right)^m\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2*5^(-1 - m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/5*(Cos[(2*e + Pi + 2*f*
x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(1 + Sin[e + f*x])^m*(Sec[(2*e - Pi +
 2*f*x)/4]^2*(3 + 2*Sin[e + f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(3 + 2*S
in[e + f*x])^m)
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int (1+\sin(fx+e))^m (3+2\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="fricas")
[Out] integral((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+2*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin(e+f\,x)+1)^m}{(2\,\sin(e+f\,x)+3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(2*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(2*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.624 $\int (1+\sin(e+fx))^m(3+\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=106

$$-\frac{2^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{3+\sin(e+fx)}\right)(1+\sin(e+fx))^{-1+m}\left(\frac{1+\sin(e+fx)}{3+\sin(e+fx)}\right)^{\frac{1}{2}-m}(3+\sin(e+fx)}{f}$$

```
[Out] -2^(-1/2-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],(1-sin(f*x+e))/(3+sin(f
*x+e)))*(1+sin(f*x+e))^(-1+m)*((1+sin(f*x+e))/(3+sin(f*x+e)))^(1/2-m)/f/((3
+sin(f*x+e))^m)
```

**Rubi** [A]

time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2867, 134}

$$-\frac{2^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[
e + f*x])/(3 + Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e + f*x
])/(3 + Sin[e + f*x]))^(1/2 - m))/(f*(3 + Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1))))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{3+\sin(e+fx)}\right)(1+\sin(}{f}$$

**Mathematica** [A]
time = 0.61, size = 167, normalized size = 1.58

$$\frac{2^{-1-2m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{1}{2}\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(1+\sin(e+fx))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+\sin(e+fx)}\right)^m\left(\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(3+\sin(e+fx))\right)^m\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(-1 - 2*m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/2*(Cos[(2*e + Pi + 2*f*
x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi +
 2*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(3 + Sin[e
+ f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(Cos[(2*e - Pi + 2*f*x)/4]^2)^m)
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (1+\sin(fx+e))^m (3+\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin(e+f\,x)+1)^m}{(\sin(e+f\,x)+3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] int((sin(e + f*x) + 1)^m/(sin(e + f*x) + 3)^(m + 1), x)
```

# 3.625 $\int 3^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=65

$$-\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*3^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x
+e))/f/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {12, 2730}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int 3^{-1-m}(1+\sin(e+fx))^m\,dx &= 3^{-1-m}\int(1+\sin(e+fx))^m\,dx\\ &= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{1+\sin(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.15, size = 95, normalized size = 1.46

$$\frac{\sqrt{2}\,3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(1+\sin(e+fx))^m}{(f+2fm)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(1 + Sin[e + f*x])^m)/((f +
2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

**Maple** [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int 3^{-1-m}(1+\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] 3^(-m - 1)*integrate((sin(f*x + e) + 1)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1}\int(\sin(e+fx)+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((sin(e + f*x) + 1)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{3^{m+1}}\,(\sin\left(e+f\,x\right)+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3^(m + 1)*(sin(e + f*x) + 1)^m,x)
```

```
[Out] int(1/3^(m + 1)*(sin(e + f*x) + 1)^m, x)
```

# 3.626 $\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=94

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-\sin(e+fx))^{-1-m}\left(\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)^{1+m}(1+\sin(e+fx))^m}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-2*(1-sin(f*x+e))/(1+sin(f*x+e)))*(3
-sin(f*x+e))^(-1-m)*((3-sin(f*x+e))/(1+sin(f*x+e)))^(1+m)*(1+sin(f*x+e))^m/
f
```

**Rubi** [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$-\frac{\cos(e+fx)(3-\sin(e+fx))^{-m-1}\left(\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)^{m+1}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-2*(1 - Sin[e + f*x]))/
(1 + Sin[e + f*x])]*(3 - Sin[e + f*x])^(-1 - m)*((3 - Sin[e + f*x])/(1 + Si
n[e + f*x]))^(1 + m)*(1 + Sin[e + f*x])^m)/f)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(3-x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, 1+m; \frac{3}{2}; -\frac{2(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-\sin(e+}{f}$$

**Mathematica [A]**
time = 1.11, size = 182, normalized size = 1.94

$$-\frac{2^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{4\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+\sin(e+fx)}\right)(3-\sin(e+fx))^{-m}\left(-\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+\sin(e+fx)}\right)^{\frac{1}{2}-m}(1+\sin(e+fx))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(1 + Sin[e + f*x])^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f
*(3 - Sin[e + f*x])^m))
```

**Maple [F]**
time = 0.16, size = 0, normalized size = 0.00

$$\int (3-\sin(fx+e))^{-1-m}(1+\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin(e+f\,x)+1)^m}{(3-\sin(e+f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - sin(e + f*x))^(m + 1), x)
```

# 3.627 $\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=114

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin(e+fx))^{-m}\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(1+\sin(e+fx))^m}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

```
[Out] 1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2*(3-2*sin(f*x+e))/(1+sin(f*x+e)))
*(1+sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-2*sin(f*x+
e))^m)/(1-sin(f*x+e))*5^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(1 + Sin[e
 + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1))))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(3-2x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin}{\sqrt{5}\,fm(1-\text{s}}$$

**Mathematica** [A]
time = 0.92, size = 177, normalized size = 1.55

$$-\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{5\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3-2\sin(e+fx)}\right)(3-2\sin(e+fx))^{-m}(1+\sin(e+fx))^m\left(-\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+2\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(1 + Sin[e + f*x])^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2*Sin
[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 2*S
in[e + f*x])^m)
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (3-2\sin(fx+e))^{-1-m}(1+\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin (e+f\,x)+1)^m}{(3-2\,\sin (e+f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 2*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 2*sin(e + f*x))^(m + 1), x)
```

# 3.628 $\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=43

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m}{f(1+2m)}$$

```
[Out] cos(f*x+e)*(3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m/f/(1+2*m)
```

**Rubi** [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2821}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(\sin(e+fx)+1)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m)/(f*(1 + 2
*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx=\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m}{f(1+2m)}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.
time = 0.51, size = 97, normalized size = 2.26

$$\frac{\cos^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2m}(6-6\sin(e+fx))^{-m}(1+\sin(e+fx))^m\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{3(f+2fm)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(1 + Sin[e + f*x])^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(6 -
6*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(3-3\sin \left(fx+e\right)\right)^{-1-m} \left(1+\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [A]
time = 0.37, size = 44, normalized size = 1.02

$$\frac{\left(\sin \left(fx+e\right)+1\right)^{m} \left(-3\, \sin \left(fx+e\right)+3\right)^{-m-1} \cos \left(fx+e\right)}{2\, fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m + f
)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

[Out] Timed out

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [B]
time = 7.85, size = 43, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(\sin\left(e+f\,x\right)+1\right)}^m}{f\,\left(2\,m+1\right)\,{\left(3-3\,\sin\left(e+f\,x\right)\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 3*sin(e + f*x))^(m + 1),x)
```

```
[Out] (cos(e + f*x)*(sin(e + f*x) + 1)^m)/(f*(2*m + 1)*(3 - 3*sin(e + f*x))^(m +
1))
```

# 3.629 $\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=83

$$\frac{2^{1+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{7(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}(-3+4\sin(e+fx))^m}{f(1+\sin(e+fx))}$$

```
[Out] 2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],7*(1-sin(f*x+e))/(1+sin(f*x+e
)))*(-3+4*sin(f*x+e))^m/f/((3-4*sin(f*x+e))^m)/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e +
 f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(3-4x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4}{\sqrt{7}\,fm(1-\mathrm{s}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(83) = 166.
time = 0.91, size = 176, normalized size = 2.12

$$\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{7\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3-4\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}(1+\sin(e+fx))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+4\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Sin[e
+ f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*Sin[e
 + f*x])^m)
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (3-4\sin(fx+e))^{-1-m}(1+\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin(e+f\,x)+1)^m}{(3-4\,\sin(e+f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 4*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 4*sin(e + f*x))^(m + 1), x)
```

# 3.630 $\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=78

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{4(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-5\sin(e+fx))^{-m}(-3+5\sin(e+fx))^m}{f(1+\sin(e+fx))}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],4*(1-sin(f*x+e))/(1+sin(f*x+e)))*(-3+
5*sin(f*x+e))^m/f/((3-5*sin(f*x+e))^m)/(1+sin(f*x+e))
```

**Rubi** [A]

time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e + f
*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(3-5x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+f\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; -\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(}{4fm(1-\sin(e}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.61, size = 246, normalized size = 3.15

$$-\frac{2^{-1+2m}\,{}_2F_1\left(1+m,1+2m;2(1+m);\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(3-5\sin(e+fx))^{-m}(1+\sin(e+fx))^m(1+i\cos(e+fx)+\sin(e+fx))\left(\frac{-\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((1-2i)-(2-i)\cos(e+fx)+(1+2i)\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(1 + Sin[e + f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi +
 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Si
n[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)
*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e
 + f*x])))
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (3-5\sin(fx+e))^{-1-m}(1+\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\sin(e+f\,x)+1)^m}{(3-5\,\sin(e+f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 5*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 5*sin(e + f*x))^(m + 1), x)
```

# 3.631 $\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=81

$$-\frac{4^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{a-a\sin(e+fx)}{4(a+a\sin(e+fx))}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -4^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/4*(a-a*sin(f*x+e))/(a+a*s
in(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/4*(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(
4*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*S
in[e + f*x])^m)/(f*m*(1 - Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; \frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)\sqrt{\frac{1-}{1+}}}{4fm(1-}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.61, size = 240, normalized size = 2.96

$$\frac{4^m\,{}_2F_1\left(1+m, 1+2m; 2(1+m); \frac{4\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(a(1+\sin(e+fx)))^m(1+i\cos(e+fx)+\sin(e+fx))(3+5\sin(e+fx))^{-m}\left(-\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((2-i)-(1-2i)\cos(e+fx)+(2+i)\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + Sin
[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f
*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e
 - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2
- I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^
m)
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (3+5\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^m}{(5\,\sin(e + f\,x) + 3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.632 $\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=83

$$-\frac{\left(\frac{7}{2}\right)^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{a-a\sin(e+fx)}{7(a+a\sin(e+fx))}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -(7/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/7*(a-a*sin(f*x+e))/(a
+a*sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/
(7*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*
Sin[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(3 + 4*Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))]])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)\sqrt{\frac{1-}{1+}}}{\sqrt{7}\,fm(1-}$$

**Mathematica** [A]
time = 0.53, size = 90, normalized size = 1.08

$$-\frac{2\,7^{-1-m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{-m}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(a*(1 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi
 + 2*f*x)/4]^2)^m)
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (3+4\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{{\left(4\,\sin\left(e+f\,x\right)+3\right)}^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.633 $\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=39

$$-\frac{\cos(e+fx)(3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -cos(f*x+e)*(3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {23, 2727}

$$-\frac{\cos(e+fx)(3\sin(e+fx)+3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int \frac{}{a+}$$
$$= -\frac{\cos(e+fx)(3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(39) = 78.

time = 5.22, size = 104, normalized size = 2.67

$$-\frac{2^{-m}3^{-1-m}\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2(1+m)}(1+\sin(e+fx))^{-1-m}(a(1+\sin(e+fx)))^m\sin^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*
e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

**Maple [F]**
time = 0.14, size = 0, normalized size = 0.00

$$\int (3+3\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima [A]**
time = 0.51, size = 40, normalized size = 1.03

$$-\frac{2\,a^m}{\left(3^{m+1}+\frac{3^{m+1}\sin(fx+e)}{\cos(fx+e)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -2*a^m/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

**Fricas [A]**
time = 0.35, size = 47, normalized size = 1.21

$$-\frac{\left(\frac{1}{3}\,a\right)^m(\cos(fx+e)-\sin(fx+e)+1)}{3\,(f\cos(fx+e)+f\sin(fx+e)+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -1/3*(1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1} \int \frac{(a\sin(e+fx)+a)^m}{(\sin(e+fx)+1)^m \sin(e+fx)+(\sin(e+fx)+1)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((a*sin(e + f*x) + a)**m/((sin(e + f*x) + 1)**m*sin(e +
 f*x) + (sin(e + f*x) + 1)**m), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(42) = 84.
time = 0.72, size = 815, normalized size = 20.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi + pi*m*floor(-1/4*sgn(a)
 + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2
*e)^3 + 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi + pi*m*floor(-1
/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*
f*x + 1/2*e)^2 + 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi + pi*m
*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)*t
an(1/2*f*x + 1/2*e)^3 - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi
 + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1
/2*e)^2*tan(1/2*f*x + 1/2*e) - 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan
(3/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*
f*x - 1/2*e)*tan(1/2*f*x + 1/2*e)^2 - e^(-m*log(3) + m*log(abs(a)) - log(3)
)*tan(1/2*f*x + 1/2*e)^3 - e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*p
i + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x -
1/2*e)^2 - 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi + pi*m*floor
(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)*tan(1/2
*f*x + 1/2*e) - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*
e)^2 + 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(3/4*pi + pi*m*floor(-1/
4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e) + 3*e^(-m*l
og(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*e) + e^(-m*log(3) + m*log
(abs(a)) - log(3)))/(f*tan(3/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*
m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2*e)^3 + f*tan(3/4
*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x
- 1/2*e)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(3/4*pi + pi*m*floor(-1/4*sgn(a) +
 1/2) + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2*e
```

```
) + f*tan(1/2*f*x + 1/2*e)^3 + f*tan(3/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2)
 + 1/4*pi*m*sgn(a) + 3/4*pi*m - 1/2*f*x - 1/2*e)^2 + f*tan(1/2*f*x + 1/2*e)
^2 + f*tan(1/2*f*x + 1/2*e) + f)
```

**Mupad** [B]
time = 0.43, size = 52, normalized size = 1.33

$$\frac{(a (\sin (e + f x) + 1))^m (-\cos (e + f x) + \sin (e + f x)\ 1\mathrm{i} + 1\mathrm{i})}{f (3 \sin (e + f x) + 3)^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(3*sin(
e + f*x) + 3)^(m + 1))
```

# 3.634 $\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m \, dx$

**Optimal**. Leaf size=83

$$-\frac{\left(\frac{5}{2}\right)^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{a-a\sin(e+fx)}{5(a+a\sin(e+fx))}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -(5/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/5*(-a+a*sin(f*x+e))/(
a+a*sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/(5
*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a
*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(3 + 2*Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; \frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)\sqrt{-\frac{1}{1}}}{\sqrt{5}\,fm(1}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(83) = 166.
time = 0.61, size = 179, normalized size = 2.16

$$-\frac{2\,5^{-1-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{1}{5}\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m(3+2\sin(e+fx))^{-m}\left(\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(3+2\sin(e+fx))\right)^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*5^(-1 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/5*(Cos[(2*e + Pi + 2*f*x)/4]^2
*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(a*(1 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*
f*x)/4]^2*(3 + 2*Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/
(f*(3 + 2*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (3+2\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^m}{(2\,\sin(e+f\,x)+3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) + 3)^(m + 1), x)
```

# 3.635 $\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=81

$$-\frac{2^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{a-a\sin(e+fx)}{2(a+a\sin(e+fx))}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/2*(-a+a*sin(f*x+e))/(a+a*
sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e
 + f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 + Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)\sqrt{-\frac{1-\sin}{1+\sin}}}{2\sqrt{2}\,fm(1-\sin}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(81) = 162.
time = 0.69, size = 166, normalized size = 2.05

$$-\frac{2^{-1-2m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right){}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{1}{2}\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+\sin(e+fx)}\right)^m\left(\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(3+\sin(e+fx))\right)^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 - 2*m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2
, -1/2*(Cos[(2*e + Pi + 2*f*x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(a*(1 + S
in[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*
e - Pi + 2*f*x)/4]^2*(3 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi + 2*f*x)/4]^2
)^m))
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (3+\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m}{(\sin\left(e+f\,x\right)+3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(sin(e + f*x) + 3)^(m + 1), x)
```

# 3.636 $\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=81

$$-\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(1/2+m)*3^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x
+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {12, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx = 3^{-1-m}\int (a+a\sin(e+fx))^m\,dx$$
$$= \left(3^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m$$
$$= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin}{f}$$

**Mathematica** [A]
time = 0.18, size = 97, normalized size = 1.20

$$\frac{\sqrt{2}\,3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m}{(f+2fm)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((
f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int 3^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] 3^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1}\int(a\sin(e+fx)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{3^{m+1}}\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3^(m + 1)*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(1/3^(m + 1)*(a + a*sin(e + f*x))^m, x)
```

# 3.637 $\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=72

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{2(a-a\sin(e+fx))}{a+a\sin(e+fx)}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-2*(a-a*sin(f*x+e))/(a+a*sin(f*x+e))
)*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Sin
[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e + f
*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 - Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(3-\sin(e+}{2\sqrt{2}\,fm(1-\sin}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 184 vs. 2(72) = 144.
time = 0.85, size = 184, normalized size = 2.56

$$-\frac{2^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{4\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+\sin(e+fx)}\right)(3-\sin(e+fx))^{-m}\left(-\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+\sin(e+fx)}\right)^{\frac{1}{2}-m}(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m)
)/(f*(3 - Sin[e + f*x])^m))
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (3-\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m}{(3-\sin\left(e+f\,x\right))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - sin(e + f*x))^(m + 1), x)
```

# 3.638 $\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=77

$$-\frac{2^{1+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{5(a-a\sin(e+fx))}{a+a\sin(e+fx)}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-5*(a-a*sin(f*x+e))/(a+a*sin
(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = $\{2867, 134\}$

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin
[e + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2}{\sqrt{5}\,fm(1}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(77) = 154.
time = 0.67, size = 179, normalized size = 2.32

$$-\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{5\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3-2\sin(e+fx)}\right)(3-2\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m\left(-\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+2\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2
*Sin[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 -
 2*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (3-2\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m}{(3-2\,\sin\left(e+f\,x\right))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 2*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 2*sin(e + f*x))^(m + 1), x)
```

# 3.639 $\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=45

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

```
[Out] cos(f*x+e)*(3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f/(1+2*m)
```

**Rubi** [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2821}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 +
 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(45) = 90.
time = 0.60, size = 99, normalized size = 2.20

$$\frac{\cos^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2m}(6-6\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{3(f+2fm)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(
6 - 6*Sin[e + f*x])^m)
```

**Maple [F]**
time = 0.17, size = 0, normalized size = 0.00

$$\int (3-3\sin(fx+e))^{-1-m}\,(a+a\sin(fx+e))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas [A]**
time = 0.36, size = 46, normalized size = 1.02

$$\frac{(a\sin(fx+e)+a)^m(-3\,\sin(fx+e)+3)^{-m-1}\cos(fx+e)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [B]
time = 0.37, size = 45, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{f\,\left(2\,m+1\right)\,{\left(3-3\,\sin\left(e+f\,x\right)\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 3*sin(e + f*x))^(m + 1),x)
[Out] (cos(e + f*x)*(a*(sin(e + f*x) + 1))^m)/(f*(2*m + 1)*(3 - 3*sin(e + f*x))^(
m + 1))
```

# 3.640 $\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=115

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

```
[Out] 1/7*cos(f*x+e)*hypergeom([1/2, -m],[1-m],-2*(3-4*sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-4*sin(f*
x+e))^m)/(1-sin(f*x+e))*7^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-}{\sqrt{7}\,fm(1}$$

**Mathematica** [A]
time = 0.68, size = 178, normalized size = 1.55

$$\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{7\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3-4\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{-3+4\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Si
n[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*S
in[e + f*x])^m)
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (3-4\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m}{(3-4\,\sin\left(e+f\,x\right))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 4*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 4*sin(e + f*x))^(m + 1), x)
```

# 3.641 $\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=113

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(e+fx))^{-m}\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx))^m}{4fm(1-\sin(e+fx))}$$

```
[Out] 1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],(-3+5*sin(f*x+e))/(1+sin(f*x+e)))*
(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-5*sin(f*x+
e))^m)/(1-sin(f*x+e))
```

**Rubi** [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e +
 f*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \text{si}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\,\text{si}}{4fm(1-\text{si}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.60, size = 248, normalized size = 2.19

$$-\frac{2^{-1+2m}\,{}_2F_1\left(1+m,1+2m;2(1+m);\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(3-5\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m(1+i\cos(e+fx)+\sin(e+fx))\left(\frac{-\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((1-2i)-(2-i)\cos(e+fx)+(1+2i)\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(a*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e -
Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] +
2*Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 +
2*m)*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*S
in[e + f*x])))
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (3-5\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin\left(e + f\,x\right))^m}{(3 - 5\,\sin\left(e + f\,x\right))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 5*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 5*sin(e + f*x))^(m + 1), x)
```

# 3.642 $\int (-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=72

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{4(a-a\sin(e+fx))}{a+a\sin(e+fx)}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],4*(a-a*sin(f*x+e))/(a+a*sin(f*x+e)))
*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/4*(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])
/(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Si
n[e + f*x])^m)/(f*m*(1 - Sin[e + f*x])*(-3 + 5*Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(-3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)\sqrt{\frac{1}{1}}}{4fm(}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.65, size = 247, normalized size = 3.43

$$\frac{2^{-1+2m}\,{}_2F_1\left(1+m,1+2m;2(1+m);\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(a(1+\sin(e+fx)))^m(1+i\cos(e+fx)+\sin(e+fx))(-3+5\sin(e+fx))^{-m}\left(\frac{-\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((1-2i)-(2-i)\cos(e+fx)+(1+2i)\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e - Pi
 + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*(a
*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi
 + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*
Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*
m)*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e + f*x])*(-3 + 5*Sin[
e + f*x])^m)
```

**Maple** [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(-3+5\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m}{(5\,\sin\left(e+f\,x\right)-3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.643 $\int (-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=77

$$-\frac{2^{1+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};\frac{7(a-a\sin(e+fx))}{a+a\sin(e+fx)}\right)(1+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],7*(a-a*sin(f*x+e))/(a+a*sin(
f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))
/(1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin
[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(-3 + 4*Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(-3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{}}{\sqrt{7}\,fm}$$

**Mathematica [A]**
time = 0.66, size = 154, normalized size = 2.00

$$-\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};7\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m(-3+4\sin(e+fx))^{-m}\left(\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(-3+4\sin(e+fx))\right)^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1 + m, 3/2, 7*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 4*Sin[e + f*x]))^m*(Sin[(2*e
 + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 4*Sin[e + f*x])^m)
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int (-3+4\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^m}{(4 \sin(e + f x) - 3)^{m+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.644 $\int (-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=45

$$\frac{\cos(e+fx)(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

```
[Out] cos(f*x+e)*(-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f/(1+2*m)
```

**Rubi** [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2821}

$$\frac{\cos(e+fx)(3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1
+ 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+f$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. 2(45) = 90.
time = 0.60, size = 110, normalized size = 2.44

$$\frac{2^{-m}3^{-1-m}\cos^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2(1+m)}(-1+\sin(e+fx))^{-1-m}(a(1+\sin(e+fx)))^m\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f+2fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^(2*(1 + m))*(-1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))
^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*(f + 2*f*m))
```

**Maple** [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(-3+3 \sin \left(f x+e\right)\right)^{-1-m} \left(a+a \sin \left(f x+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [**A**]
time = 0.35, size = 46, normalized size = 1.02

$$\frac{{\left(a \sin \left(f x+e\right)+a\right)}^{m} {\left(3 \, \sin \left(f x+e\right)-3\right)}^{-m-1} \cos \left(f x+e\right)}{2 \, f m+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] (a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad [B]**
time = 7.83, size = 45, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(\frac{a\,\left(\sin\left(e+f\,x\right)+1\right)}{3}\right)}^m}{3\,f\,\left(2\,m+1\right)\,{\left(\sin\left(e+f\,x\right)-1\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] (cos(e + f*x)*((a*(sin(e + f*x) + 1))/3)^m)/(3*f*(2*m + 1)*(sin(e + f*x) -
1)^(m + 1))
```

# 3.645 $\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=117

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(-3+2\sin(e+fx))^{-m}(a+a\sin(e+fx))^m}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2*(3-2*sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/(1-sin(f*x+
e))/((-3+2*sin(f*x+e))^m)*5^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/
(1 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*S
in[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(-3 + 2*Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(-3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{-}}{\sqrt{5}\,fm}$$

**Mathematica** [A]
time = 0.68, size = 155, normalized size = 1.32

$$\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-5\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m(-3+2\sin(e+fx))^{-m}\left(-\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(-3+2\sin(e+fx))\right)^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1 + m, 3/2, -5*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(-(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 2*Sin[e + f*x])))^m*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 2*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(-3+2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^m}{(2\,\sin(e+f\,x)-3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.646 $\int (-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=116

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(-3+\sin(e+fx))^{-m}\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx))^m}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],(3-sin(f*x+e))/(1+sin(f*x+e)))*(a
+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/(1-sin(f*x+e))/
((-3+sin(f*x+e))^m)*2^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2867, 134}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/2*(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1
+ Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[
e + f*x])^m)/(Sqrt[2]*f*m*(1 - Sin[e + f*x])*(-3 + Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(-3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x,\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(-3+\text{s}}{2\sqrt{2}\,fm(1}$$

**Mathematica [A]**
time = 0.77, size = 155, normalized size = 1.34

$$\frac{2^{-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-2\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(-3+\sin(e+fx))^{-m}\left(-\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)(-3+\sin(e+fx))\right)^m(a(1+\sin(e+fx)))^m\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyperge
ometric2F1[1/2, 1 + m, 3/2, -2*Tan[(2*e - Pi + 2*f*x)/4]^2]*(-(Sec[(2*e - P
i + 2*f*x)/4]^2*(-3 + Sin[e + f*x])))^m*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e
+ Pi + 2*f*x)/4]^2)^(1/2 - m))/(2^m*f*(-3 + Sin[e + f*x])^m)
```

**Maple [F]**
time = 0.13, size = 0, normalized size = 0.00

$$\int (-3+\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^m}{(\sin(e+f\,x)-3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(sin(e + f*x) - 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(sin(e + f*x) - 3)^(m + 1), x)
```

# 3.647 $\int (-3)^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=81

$$-\frac{(-3)^{-1-m}2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx$$

```
[Out] -(-3)^(-1-m)*2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

**Rubi** [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {12, 2731, 2730}

$$-\frac{(-3)^{-m-1}2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(((-3)^(-1 - m)*2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3
/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x]
)^m)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-3)^{-1-m}(a+a\sin(e+fx))^m\,dx = (-3)^{-1-m}\int (a+a\sin(e+fx))^m\,dx$$
$$= \left((-3)^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+$$
$$= -\frac{(-3)^{-1-m}2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1}{f}$$

**Mathematica** [A]
time = 0.17, size = 97, normalized size = 1.20

$$\frac{(-3)^{-1-m}\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m}{(f+2fm)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((-3)^(-1 - m)*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m
, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)
/((f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (-3)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (-3)^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$(-3)^{-m-1}\int(a\sin(e+fx)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] (-3)**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{(-3)^{m+1}}\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3)^(m + 1)*(a + a*sin(e + f*x))^m,x)
[Out] int(1/(-3)^(m + 1)*(a + a*sin(e + f*x))^m, x)
```

# 3.648 $\int (-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=119

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)(-3-\sin(e+fx))^{-m}\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],1/2*(3+sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-sin(f*
x+e))^m)/(1-sin(f*x+e))*2^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/2*(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*
(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*
Sin[e + f*x])^m)/(Sqrt[2]*f*m*(-3 - Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(-3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)(-3-}{2\sqrt{2}\,fm(}$$

**Mathematica** [A]
time = 0.95, size = 131, normalized size = 1.10

$$\frac{4^{-m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{2\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+\sin(e+fx)}\right)(-3-\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m(3+\sin(e+fx))^{-\frac{1}{2}+m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (2*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(3 + Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(3 + Sin[
e + f*x])^(-1/2 + m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(4^m*f*(-3 -
Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (-3-\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^m}{(-\sin(e + f\,x) - 3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- sin(e + f*x) - 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(- sin(e + f*x) - 3)^(m + 1), x)
```

# 3.649 $\int (-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=119

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)(-3-2\sin(e+fx))^{-m}\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+\ldots}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2/5*(3+2*sin(f*x+e))/(1+sin(f*x+e
)))*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-2*si
n(f*x+e))^m)/(1-sin(f*x+e))*5^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/
(5*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a +
 a*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(-3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x])
))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\operatorname{Subst}\left(\int \frac{(-3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x$$
$$f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)(-3}{\sqrt{5}\,fm}$$

**Mathematica** [A]
time = 1.23, size = 186, normalized size = 1.56

$$\frac{2\,5^{-\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+2\sin(e+fx)}\right)(-3-2\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+2\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*5^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, Sin[(2*e - Pi + 2*f*x)/4]^2/(3
 + 2*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(
3 + 2*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*
(-3 - 2*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(-3-2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^m}{(-2 \sin(e + f x) - 3)^{m+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 2*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 2*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.650 $\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=39

$$-\frac{\cos(e+fx)(-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

[Out] -cos(f*x+e)*(-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f

**Rubi** [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {23, 2727}

$$-\frac{\cos(e+fx)(-3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]

[Out] -((Cos[e + f*x]*(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int$$

$$= -\frac{\cos(e+fx)(-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+}{f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(39) = 78.
time = 0.50, size = 106, normalized size = 2.72

$$-\frac{2^{-m}3^{-1-m}\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2(1+m)}(-1-\sin(e+fx))^{-1-m}(a(1+\sin(e+fx)))^m\sin^{-1-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(-1 - Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2
*e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(-3-3\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [A]
time = 0.56, size = 47, normalized size = 1.21

$$\frac{2\,a^{m}}{\left(3^{m+1}\left(-1\right)^{m}+\frac{3^{m+1}(-1)^{m}\sin(fx+e)}{\cos(fx+e)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] 2*a^m/((3^(m + 1)*(-1)^m + 3^(m + 1)*(-1)^m*sin(f*x + e)/(cos(f*x + e) + 1)
)*f)
```

**Fricas** [A]
time = 0.35, size = 47, normalized size = 1.21

$$\frac{\left(-\frac{1}{3}\,a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] 1/3*(-1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m (-3\sin (e+fx)-3)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-3*sin(e + f*x) - 3)**(-m - 1), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(42) = 84.
time = 0.76, size = 815, normalized size = 20.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi + pi*m*floor(-1/4*sgn(a)
 + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2
*e)^3 + 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi + pi*m*floor(-1
/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*
f*x + 1/2*e)^2 + 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi + pi*m
*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)*t
an(1/2*f*x + 1/2*e)^3 - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi
 + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1
/2*e)^2*tan(1/2*f*x + 1/2*e) - 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan
(5/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*
f*x - 1/2*e)*tan(1/2*f*x + 1/2*e)^2 - e^(-m*log(3) + m*log(abs(a)) - log(3)
)*tan(1/2*f*x + 1/2*e)^3 - e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*p
i + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x -
1/2*e)^2 - 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi + pi*m*floor
(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)*tan(1/2
*f*x + 1/2*e) - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*
e)^2 + 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(5/4*pi + pi*m*floor(-1/
4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e) + 3*e^(-m*l
og(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*e) + e^(-m*log(3) + m*log
(abs(a)) - log(3)))/(f*tan(5/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*
m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2*e)^3 + f*tan(5/4
*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x
- 1/2*e)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(5/4*pi + pi*m*floor(-1/4*sgn(a) +
```

```
 1/2) + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)^2*tan(1/2*f*x + 1/2*e
) + f*tan(1/2*f*x + 1/2*e)^3 + f*tan(5/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2)
 + 1/4*pi*m*sgn(a) + 5/4*pi*m - 1/2*f*x - 1/2*e)^2 + f*tan(1/2*f*x + 1/2*e)
^2 + f*tan(1/2*f*x + 1/2*e) + f)
```

**Mupad** [B]
time = 0.39, size = 52, normalized size = 1.33

$$\frac{(a (\sin (e+f x)+1))^m (-\cos (e+f x)+\sin (e+f x) \, 1\mathrm{i}+1\mathrm{i})}{f (-3 \sin (e+f x)-3)^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 3*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(- 3*si
n(e + f*x) - 3)^(m + 1))
```

# 3.651 $\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=117

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)(-3-4\sin(e+fx))^{-m}\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx))^m}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

```
[Out] 1/7*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2/7*(3+4*sin(f*x+e))/(1+sin(f*x+e)
))*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-4*sin(
f*x+e))^m)/(1-sin(f*x+e))*7^(1/2)
```

**Rubi** [A]

time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/(7
*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Si
n[e + f*x])^m)/(Sqrt[7]*f*m*(-3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

`!IntegerQ[m]`

Rubi steps

$$\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(-3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+f}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)(-3}{\sqrt{7}\,fm(}$$

**Mathematica** [A]
time = 1.24, size = 187, normalized size = 1.60

$$\frac{2\,7^{-\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+4\sin(e+fx)}\right)(-3-4\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{3+4\sin(e+fx)}\right)^{\frac{1}{2}-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*7^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -(Sin[(2*e - Pi + 2*f*x)/4]^2/
(3 + 4*Sin[e + f*x]))]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^
2/(3 + 4*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/
(f*(-3 - 4*Sin[e + f*x])^m)
```

**Maple** [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(-3-4\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^m}{(-4\,\sin(e + f\,x) - 3)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 4*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 4*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.652 $\int (-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=115

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)(-3-5\sin(e+fx))^{-m}\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(a+a\sin(e+fx)}{4fm(1-\sin(e+fx))}$$

```
[Out] 1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],1/4*(3+5*sin(f*x+e))/(1+sin(f*x+e)
))*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-5*sin(
f*x+e))^m)/(1-sin(f*x+e))
```

**Rubi** [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(-3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

`!IntegerQ[m]`

Rubi steps

$$\int (-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(-3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)(-3-5}{4fm(1-}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.60, size = 241, normalized size = 2.10

$$-\frac{4^m\,{}_2F_1\left(1+m,1+2m;2(1+m);\frac{4\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)(-3-5\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m(1+i\cos(e+fx)+\sin(e+fx))\left(-\frac{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)}{2\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)+\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}\right)^m(\cosh(m\log(4))-\sinh(m\log(4)))}{f(1+2m)((2-i)-(1-2i)\cos(e+fx)+(2+i)\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*
x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + S
in[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2
*f*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2
*e - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*(-
3 - 5*Sin[e + f*x])^m*((2 - I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f
*x])))
```

**Maple** [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (-3-5\sin(fx+e))^{-1-m}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^m}{(-5 \sin(e + f x) - 3)^{m+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 5*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 5*sin(e + f*x) - 3)^(m + 1), x)
```

# 3.653 $\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=116

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)(d\sin(e+fx))^{-m}\left(\frac{1+\sin(e+fx)}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a+a\sin(e+fx))^m}{dfm(1+\sin(e+fx))}$$

```
[Out] -cos(f*x+e)*hypergeom([-m, 1/2-m],[1-m],-2*sin(f*x+e)/(1-sin(f*x+e)))*((1+s
in(f*x+e))/(1-sin(f*x+e)))^(1/2-m)*(a+a*sin(f*x+e))^m/d/f/m/((d*sin(f*x+e))
^m)/(1+sin(f*x+e))
```

**Rubi** [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2866, 2865, 2864, 134}

$$-\frac{\cos(e+fx)\left(\frac{\sin(e+fx)+1}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)}{dfm(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2 - m, -m, 1 - m, (-2*Sin[e + f*x])/(1
- Sin[e + f*x])]*((1 + Sin[e + f*x])/(1 - Sin[e + f*x]))^(1/2 - m)*(a + a*S
in[e + f*x])^m)/(d*f*m*(d*Sin[e + f*x])^m*(1 + Sin[e + f*x])))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+f$$

$$= \frac{(\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e+fx))^{-m}(a+a}{d}$$

$$= -\frac{\left(\cos(e+fx)\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e-\right.}{}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)(d\sin}{dfm(1+\sin(}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.61, size = 194, normalized size = 1.67

$$\frac{(1-i)2^m\,{}_2F_1\left(1+m,1+2m;2(1+m);\sqrt{2}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\right)((1-i)(1+\cos(e+fx)-i\sin(e+fx)))^m((1+i)(1-\cos(e+fx)+i\sin(e+fx)))^{-m}(d\sin(e+fx))^{-m}(a(1+\sin(e+fx)))^m(\cos(e+fx)-i(1+\sin(e+fx)))(\cosh(m\log(2))-\sinh(m\log(2)))}{df(1+2m)(-1+\cos(e+fx)-i\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((1 - I)*2^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), Sqrt[2]*Cos[(2*e
- Pi + 2*f*x)/4]*Csc[(e + f*x)/2]]*((1 - I)*(1 + Cos[e + f*x] - I*Sin[e + f
*x]))^m*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] - I*(1 + Sin[e + f*x]))*(Cos
h[m*Log[2]] - Sinh[m*Log[2]]))/(d*f*(1 + 2*m)*(-1 + Cos[e + f*x] - I*Sin[e
+ f*x])*((1 + I)*(1 - Cos[e + f*x] + I*Sin[e + f*x]))^m*(d*Sin[e + f*x])^m)
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^{-1-m} (a+a\sin(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (d\sin(e+fx))^{-m-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**(-m - 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(d\,\sin\left(e+f\,x\right)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(d*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(d*sin(e + f*x))^(m + 1), x)
```

# 3.654 $\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$

**Optimal**. Leaf size=129

$$-\frac{2^{\frac{1}{2}+m} a \cos(e+fx)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx))^{-1+m}\left(\frac{(c+d)(1+\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+}{(c+d)f}$$

```
[Out] -2^(1/2+m)*a*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-sin(f*x+e
))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/(c+d*sin
(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)
```

**Rubi** [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2867, 134}

$$-\frac{a 2^{m+\frac{1}{2}} \cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)
*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])^(-1 + m
)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((c + d)*f
*(c + d*Sin[e + f*x])^m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{a-ax}}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right.}{}$$

**Mathematica [A]**
time = 1.47, size = 187, normalized size = 1.45

$$-\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)(a(1+\sin(e+fx)))^m\left(\frac{(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)^{-\frac{1}{2}-m}(c+d\sin(e+fx))^{-1-m}\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(((c + d)*Cos[(2*e - Pi + 2*f*x)/
4]^2)/(c + d*Sin[e + f*x]))^(-1/2 - m)*(c + d*Sin[e + f*x])^(-1 - m)*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/f
```

**Maple [F]**
time = 0.14, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m\,(c+d\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(e + f\,x))^m}{(c + d\,\sin(e + f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(m + 1), x)
```

# 3.655 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=107

$$-\frac{8\sqrt{2}\,a^3F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -8*a^3*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2863, 144, 143}

$$-\frac{8\sqrt{2}\,a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^3*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{(a^3\cos(e+fx))\,\text{Subst}\left(\int \frac{(1+x)^{5/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Su}}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+}}$$

$$= -\frac{8\sqrt{2}\,a^3F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{1+\sin(e+}}$$

**Mathematica** [F]
time = 28.06, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

**Maple** [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^3\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin (e+f\,x))^3\,(c+d\,\sin (e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n, x)
```

# 3.656 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=107

$$-\frac{4\sqrt{2}\,a^2F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -4*a^2*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2863, 144, 143}

$$-\frac{4\sqrt{2}\,a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a^2*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{S}}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1}}$$

$$= -\frac{4\sqrt{2}\,a^2F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right.}{f\sqrt{1+\sin(e}}$$

**Mathematica [F]**
time = 12.47, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

**Maple [F]**
time = 0.31, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^2\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*sin(f*x + e)
 + c)^n, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\,\sin\left(e + f\,x\right))^2\,\left(c + d\,\sin\left(e + f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

# 3.657 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=105

$$-\frac{2\sqrt{2}\, aF_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-}}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2*a*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f
*x+e))^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2834, 144, 143}

$$-\frac{2\sqrt{2}\, a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}\,(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subs}}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\text{c}}{f\sqrt{1+\sin(e+}}$$

**Mathematica** [**F**]
time = 5.70, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

**Maple** [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))(c+d\sin(fx+e))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

# 3.658 $\int (c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=104

$$-\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x
+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e)
)^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2744, 144, 143}

$$-\frac{\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e
+ f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e +
f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)}{\sqrt{1-x}\sqrt{1+}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica [A]**
time = 0.30, size = 120, normalized size = 1.15

$$\frac{F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\sec(e+fx)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{\frac{d(1+\sin(e+fx))}{-c+d}}(c+d\sin(e+fx))^{1+n}}{df(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[
e + f*x])/(c + d)]*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sq
rt[(d*(1 + Sin[e + f*x]))/(-c + d)]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 +
 n))
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int (c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n,x)
```

```
[Out] int((c+d*sin(f*x+e))^n,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + d \sin (e + fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n,x)
[Out] Integral((c + d*sin(e + f*x))**n, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c + d \sin (e + f x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n,x)
[Out] int((c + d*sin(e + f*x))^n, x)
```

# 3.659 $\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=107

$$-\frac{F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{\sqrt{2}\,af\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+sin(
f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2863, 144, 143}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}\,af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,(1+x)^{3/2}} \cdots\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+f \cdots}{\sqrt{2}\,af\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [**F**]
time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

**Maple** [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin(e+f\,x))^n}{a+a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x)), x)
```

# 3.660 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=109

$$-\frac{F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{2\sqrt{2}\,a^2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/4*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+si
n(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2863, 144, 143}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*(AppellF1[1/2, 5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + S
in[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)}{\sqrt{1-x}\,(1+x)^{5}}\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+}{2\sqrt{2}\,a^2 f\sqrt{1+\sin(e+fx)}}$$

**Mathematica** [F]
time = 5.06, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

**Maple** [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) -
 2*a^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^n}{(a+a\,\sin\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^2, x)
```

# 3.661 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=109

$$-\frac{F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{4\sqrt{2}\,a^3f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/8*AppellF1(1/2,-n,7/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+si
n(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2863, 144, 143}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{4\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/4*(AppellF1[1/2, 7/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^3*f*Sqrt[1 + S
in[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,(1+x)^{7/2}}\right.}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{4\sqrt{2}\,a^3 f\sqrt{1+\sin(e+fx)}}$$

**Mathematica [F]**
time = 8.81, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3, x]
```

**Maple [F]**
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f
*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^n}{(a+a\,\sin\left(e+f\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^3, x)
```

# 3.662 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=257

$$\frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+$$

```
[Out] 2*a^3*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(3+2*n)/(5+2
*n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(3*c^2-2*c*d*(7+4*n)+d^2*(16*n^2+56*n+43))
*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+
e))^n/d^2/f/(3+2*n)/(5+2*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(
1/2)-2*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+
2*n)
```

**Rubi** [A]

time = 0.33, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2842, 3060, 2855, 72, 71}

$$-\frac{2a^3(3c^2-2cd(4n+7)+d^2(16n^2+56n+43))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{n+1}}{df(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(5 + 2*n)) - (2*a^3
*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*Hypergeo
metric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x]
)^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{1-}}{df(5+2n)}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} -$$

**Mathematica [A]**
time = 28.21, size = 190, normalized size = 0.74

$$\frac{a^2\sec(e+fx)(-1+\sin(e+fx))\sqrt{a(1+\sin(e+fx))}\,(c+d\sin(e+fx))^n\left(-((3c-d(11+4n))(c+d\sin(e+fx)))+d(3+2n)(1+\sin(e+fx))(c+d\sin(e+fx))+(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(-1+\sin(e+fx))}{c+d}\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\right)}{d^2f\left(\frac{5}{2}+n\right)(3+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*Sec[e + f*x]*(-1 + Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin
[e + f*x])^n*(-((3*c - d*(11 + 4*n))*(c + d*Sin[e + f*x])) + d*(3 + 2*n)*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x]) + ((3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43
 + 56*n + 16*n^2))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x])
)/(c + d))])/((c + d*Sin[e + f*x])/(c + d))^n))/(d^2*f*(5/2 + n)*(3 + 2*n))
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^n, x)
```

# 3.663 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=160

$$-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a^2(c-d(5+4n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\,\mathrm{s}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
+2*a^2*(c-d*(5+4*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/
(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin
(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2842, 21, 2855, 72, 71}

$$\frac{2a^2(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a^2*(c - d*(5 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[
1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(d*f*
(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+f}{}}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-d(5+4}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{(a^3(c-d(5+}{df(3}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{\left(a^3(c-d(5+\right.}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(c-d(5+}{}$$

**Mathematica [A]**
time = 4.29, size = 133, normalized size = 0.83

$$-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left((-c+d(5+4n))\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(-1+\sin(e+fx))}{c+d}\right)+(c+d)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{1+n}\right)}{df(3+2n)\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*((-c + d*(5 + 4*n))*Hypergeomet
ric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + (c + d)*((c + d*
Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*
((c + d*Sin[e + f*x])/(c + d))^n)
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{3}{2}}(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x))^{3/2} (c + d \sin(e + f x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n, x)
```

# 3.664 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=85

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(
f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2855, 72, 71}

$$-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(
c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e +
 f*x])/(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
```

```
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$
$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+}}$$
$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e}{f\sqrt{a+a\sin(e+fx)}}$$

**Mathematica [F]**
time = 4.34, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x]
```

**Maple [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(fx+e)}\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**n,x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**n, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n, x)
```

# 3.665 $\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=99

$$-\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e
)*(c+d*sin(f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2867, 142, 141}

$$-\frac{\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a + a*Si
n[e + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+a}}}\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+f}{c+d}}}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(99) = 198.
time = 3.71, size = 236, normalized size = 2.38

$$\frac{\cos(e+fx)\sqrt{a(1+\sin(e+fx))}\,(c+d\sin(e+fx))^n\left(-F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),\frac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}+\frac{4F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}\left(1+\frac{c-d}{d+d\sin(e+fx)}\right)^{-n}}{1+2n}\right)}{4af(-1+\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-((AppellF
1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqr
t[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*AppellF1[-1/2
 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)*(1 + (c - d)/(d
 + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^n}{\sqrt{a+a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2), x)
```

# 3.666 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{2af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^
(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2867, 142, 141}

$$\frac{d\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}-\frac{ad}{ac+\cdots}}}\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{dF_1\left(1+n;\frac{1}{2},2;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+\cdots}{c+d}}}{(c-d)^2f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+f\cdots}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(104) = 208.
time = 4.59, size = 319, normalized size = 3.07

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^2F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),\frac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\,(1+\sin(e+fx))^2\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}-\frac{4a(1+\sin(e+fx))\sqrt{1-\frac{2}{1+\sin(e+fx)}}\left(2a(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)(1+\sin(e+fx))\right)\left(1+\frac{c-d}{d+d\sin(e+fx)}\right)^{-n}}{-1+4n^2}\right)}{8a^3f\sqrt{a(1+\sin(e+fx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^2)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a*(1 + Sin[e + f*x
])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n
, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-1 + 2
*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
 + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c - d)/(d + d*
Sin[e + f*x]))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^{n}}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)
^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^{n}}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2), x)
```

# 3.667 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{F_1\left(\frac{1}{2};-n,3;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{4a^2f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/4*AppellF1(1/2,-n,3,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e)
)^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2867, 142, 141}

$$-\frac{d^2\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},3;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^3\sqrt{a\sin(e+fx)+a}\,(a^2-a^2\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((d^2*AppellF1[1 + n, 1/2, 3, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*
Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c
 + d*Sin[e + f*x])^(1 + n))/((c - d)^3*f*(1 + n)*Sqrt[a + a*Sin[e + f*x]]*(
a^2 - a^2*Sin[e + f*x])))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^3}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\mathrm{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^3\sqrt{\frac{ad}{ac+ad}-\frac{ad}{ac+\cdots}}}\cdots\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{d^2F_1\left(1+n;\frac{1}{2},3;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e\cdots}{c+d\cdots}}}{(c-d)^3f(1+n)\sqrt{a+a\sin(e+fx)}\,(a^2-a^2\sin(e\cdots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 414 vs. 2(104) = 208.

time = 7.53, size = 414, normalized size = 3.98

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^3F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),\frac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\,(1+\sin(e+fx))^3\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}-\frac{4a^2(1+\sin(e+fx))\sqrt{1-\frac{2}{1+\sin(e+fx)}}\left(1+\frac{c-d}{d+d\sin(e+fx)}\right)^{-n}\left(a(3-8n+4n^2)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)(1+\sin(e+fx))^2+2(1+2n)\left(2a(-1+2n)F_1\left(\frac{3}{2}-n;-\frac{1}{2},-n;\frac{5}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)+a(-3+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)(1+\sin(e+fx))\right)\right)}{(-3+2n)(-1+2n)(1+2n)}\right)}{16a^4f(a(1+\sin(e+fx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^3*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^3)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a^2*(1 + Sin[e + f
*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(a*(3 - 8*n + 4*n^2)*AppellF1[-1/2 - n,
 -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1
 + Sin[e + f*x])^2 + 2*(1 + 2*n)*(2*a*(-1 + 2*n)*AppellF1[3/2 - n, -1/2, -n
, 5/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-3 + 2
*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*(1 + Sin[e + f*x]))))/((-3 + 2*n)*(-1 + 2*n)*(1 + 2*n)*(
```

```
1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(16*a^4*f*(a*(1 + Sin[e + f*x]))^(3/
2))
```

**Maple** [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x +
e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(5/2),x)
[Out] Integral((c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(5/2), x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(5/2), x)
```

# 3.668 $\int (a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=107

$$-\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}}{f\sqrt{1+\sin(e+fx)}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2*a*AppellF1(1/2,-1/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)*2^(1/2)/f/((c+d*sin(f*x+e))/(c+d))^(1/3)/
(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2834, 144, 143}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int (a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx = \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}\,\sqrt[3]{c+dx}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}\,\sqrt[3]{-\cdots}}{\sqrt{\cdots}}\cdots\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{\cdots}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cdots}{f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{\frac{c+d\sin(e\cdots}{c+d\cdots}}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1736 vs. $2(107) = 214$.
time = 6.44, size = 1736, normalized size = 16.22

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
```

- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e])^(2/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e])^(2/3)))/(4*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Se
c[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[e]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^
2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - Ar
cTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Cs
c[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
+ Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
+ Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2]
- d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(2/3
))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(2/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f
*x])*(c + d*Sin[e + f*x])^(1/3)*((-3*Cos[e]*Cos[f*x])/(4*f) + (3*Sin[e]*Sin
[f*x])/(4*f) + (3*(c + 4*d)*Tan[e])/(4*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2
+ (f*x)/2])^2 + (3*AppellF1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*
Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan
[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sq
rt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*S
qrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])
/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(4*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (3*c*Appell
F1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sq
rt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]
^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^
2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[
e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2]
- d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Ta
n[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + A
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right) \left(c+d\sin \left(fx+e\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
```

```
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt[3]{c+d\sin \left(e+fx\right)}\,\sin \left(e+fx\right) dx+\int \sqrt[3]{c+d\sin \left(e+fx\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral((c + d*sin(e + f*x))**(1/3)*sin(e + f*x), x) + Integral((c + d*
sin(e + f*x))**(1/3), x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x)) (c + d \sin(e + f x))^{1/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/3),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/3), x)
```

# 3.669 $\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] -2*a*AppellF1(1/2,1/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/f/(c+d*sin(f*x+e))^(1/3)/(
1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2834, 144, 143}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx = \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,\sqrt[3]{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,\sqrt[3]{-\frac{c}{-c-d}-\frac{d}{-c}}}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d}{}}}{f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 886 vs. 2(107) = 214.
time = 6.29, size = 886, normalized size = 8.28

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
```

```
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(1/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(2*(d^2*Cos[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot
[e]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e])^(1/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*Tan[e])/(2*d*f*(Cos[e/2 + (f*x)
/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/2, 1/2, 5/3, -((Sec[e]*(c
 + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[
e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c
*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + S
in[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[
1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2
] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1
 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^
(2/3))/(2*d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2
]))
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a+a\sin\left(fx+e\right)}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)
[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{\sqrt[3]{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt[3]{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c + d*sin(e + f*x))**(1/3), x) + Integral((c + d*
sin(e + f*x))**(-1/3), x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/3),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/3), x)
```

# 3.670 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx$

**Optimal**. Leaf size=112

$$-\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{(c+d)f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] -2*a*AppellF1(1/2,4/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(c+d)/f/(c+d*sin(f*x+e))^(
1/3)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2834, 144, 143}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(c+d)\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
(c + d)*f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx = \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,(c+dx)^{4/3}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^{4/3}}\,d\right.}{(c+d)f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+}{}}}{(c+d)f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 942 vs. 2(112) = 224.
time = 6.49, size = 942, normalized size = 8.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*((-3*Csc[e]*Sec[e])/(d*(c
 + d)*f) + (3*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f
*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (2*Sec[e]*(1 + Sin[e
+ f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]*(c + d*Cos[f*x - ArcT
an[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e
```

```
])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sq
rt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1
+ Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt
[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d
*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3))) - ((3*d*Sin[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(2*(d^2*Cos
[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]
^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3)))/((
c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/
2, 1/2, 5/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan
[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -
((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sq
rt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x
 + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*
Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
)/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[
e]]]*Sqrt[1 + Tan[e]^2])^(2/3))/(2*d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

**Maple [F]**
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a + a\sin(fx+e)}{(c + d\sin(fx+e))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)
[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(2/3)/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(4/3), x)
```

# 3.671 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=171

$$\frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)x-\frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\cos(e+fx)}{6f}-\frac{d(6bc^2+20acd+9bd^2)\,\text{c}}{24}$$

```
[Out] 1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*x-1/6*(4*a*d*(4*c^2+d^2)+3*b*(c
^3+4*c*d^2))*cos(f*x+e)/f-1/24*d*(20*a*c*d+6*b*c^2+9*b*d^2)*cos(f*x+e)*sin(
f*x+e)/f-1/12*(4*a*d+3*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f-1/4*b*cos(f*x+e
)*(c+d*sin(f*x+e))^3/f
```

**Rubi** [A]
time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{d(20acd+6bc^2+9bd^2)\sin(e+fx)\cos(e+fx)}{24f}-\frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\cos(e+fx)}{6f}+\frac{1}{8}x(8ac^3+12acd^2+12bc^2d+3bd^3)-\frac{(4ad+3bc)\cos(e+fx)(c+d\sin(e+fx))^2}{12f}-\frac{b\cos(e+fx)(c+d\sin(e+fx))^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*x)/8 - ((4*a*d*(4*c^2 + d^2)
 + 3*b*(c^3 + 4*c*d^2))*Cos[e + f*x])/(6*f) - (d*(6*b*c^2 + 20*a*c*d + 9*b*
d^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((3*b*c + 4*a*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^3}{4f}+\frac{1}{4}\int (c+d\sin(e+fx)$$

$$= -\frac{(3bc+4ad)\cos(e+fx)(c+d\sin(e+fx))^2}{12f}-\frac{b\cos(e+f}{}$$

$$= \frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)x-\frac{(4ad(4c^2+d^2)+3b(}{}$$

**Mathematica [A]**
time = 0.80, size = 143, normalized size = 0.84

$$\frac{-24(3ad(4c^2+d^2)+b(4c^3+9cd^2))\cos(e+fx)+8d^2(3bc+ad)\cos(3(e+fx))+3(4(8ac^3+12bc^2d+12acd^2+3bd^3)(e+fx)-8d(3acd+b(3c^2+d^2))\sin(2(e+fx))+bd^3\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-24*(3*a*d*(4*c^2 + d^2) + b*(4*c^3 + 9*c*d^2))*Cos[e + f*x] + 8*d^2*(3*b*
c + a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d
^3)*(e + f*x) - 8*d*(3*a*c*d + b*(3*c^2 + d^2))*Sin[2*(e + f*x)] + b*d^3*Si
n[4*(e + f*x)]))/(96*f)
```

**Maple [A]**
time = 0.32, size = 182, normalized size = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{a\,c^3(fx+e)-3c^2da\cos(fx+e)+3ac\,d^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-c^3b\cos(fx+e)+3}{}$ |
| default | $\frac{a\,c^3(fx+e)-3c^2da\cos(fx+e)+3ac\,d^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-c^3b\cos(fx+e)+3}{}$ |
| risch | $a\,c^3x+\frac{3xac\,d^2}{2}+\frac{3xb\,c^2d}{2}+\frac{3xb\,d^3}{8}-\frac{3\cos(fx+e)c^2da}{f}-\frac{3\cos(fx+e)d^3a}{4f}-\frac{\cos(fx+e)c^3b}{f}-\frac{9\cos(fx+e)}{4f}$ |
| norman | $\frac{(a\,c^3+\frac{3}{2}ac\,d^2+\frac{3}{2}b\,c^2d+\frac{3}{8}b\,d^3)x+(a\,c^3+\frac{3}{2}ac\,d^2+\frac{3}{2}b\,c^2d+\frac{3}{8}b\,d^3)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a\,c^3+6ac\,d^2+6b\,c^2d+\frac{3}{2}b\,d^3)x\left(\tan\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^3*(f*x+e)-3*c^2*d*a*cos(f*x+e)+3*a*c*d^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-1/3*d^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)-c^3*b*cos(f*x+e)+3*
b*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-c*d^2*b*(2+sin(f*x+e)^2)
```

```
*cos(f*x+e)+b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

**Maxima [A]**
time = 0.31, size = 189, normalized size = 1.11

$$\frac{96 (fx+e)ac^3 + 72 (2 fx + 2 e - \sin (2 fx + 2 e))bc^2d + 72 (2 fx + 2 e - \sin (2 fx + 2 e))acd^2 + 96 (\cos (fx+e)^3 - 3 \cos (fx+e))bcd^2 + 32 (\cos (fx+e)^3 - 3 \cos (fx+e))ad^3 + 3 (12 fx + 12 e + \sin (4 fx + 4 e) - 8 \sin (2 fx + 2 e))bd^3 - 96 bc^3 \cos (fx+e) - 288 ac^2d \cos (fx+e)}{96 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c^2*d + 72
*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d^2 + 96*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*b*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b*d^3 - 96*b*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.37, size = 150, normalized size = 0.88

$$\frac{8 (3 bcd^2 + ad^3) \cos (fx+e)^3 + 3 (8 ac^3 + 12 bc^2d + 12 acd^2 + 3 bd^3)fx - 24 (bc^3 + 3 ac^2d + 3 bcd^2 + ad^3) \cos (fx+e) + 3 (2 bd^3 \cos (fx+e)^3 - (12 bc^2d + 12 acd^2 + 5 bd^3) \cos (fx+e)) \sin (fx+e)}{24 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*b*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*b*c^2*d + 12*a
*c*d^2 + 3*b*d^3)*f*x - 24*(b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*b*d^3*cos(f*x + e)^3 - (12*b*c^2*d + 12*a*c*d^2 + 5*b*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(165) = 330$.
time = 0.26, size = 386, normalized size = 2.26

$$\begin{cases} ac^3x - \frac{3ac^2d\cos(e+fx)}{f} + \frac{3acd^2x\sin^2(e+fx)}{2} + \frac{3acd^2x\cos^2(e+fx)}{2} - \frac{3acd^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{ad^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ad^3\cos^3(e+fx)}{3f} - \frac{bc^3\cos(e+fx)}{f} + \frac{3bc^2dx\sin^2(e+fx)}{2} + \frac{3bc^2dx\cos^2(e+fx)}{2} - \frac{3bc^2d\sin(e+fx)\cos(e+fx)}{2f} - \frac{3bcd^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2bcd^2\cos^3(e+fx)}{f} + \frac{3bd^3x\sin^4(e+fx)}{8} + \frac{3bd^3x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3bd^3x\cos^4(e+fx)}{8} - \frac{5bd^3\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3bd^3\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for}\ f \neq 0 \\ x(a+b\sin(e))(c+d\sin(e))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)
**2/2 + 3*a*c*d**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*d**3*cos(e + f*x)**3
/(3*f) - b*c**3*cos(e + f*x)/f + 3*b*c**2*d*x*sin(e + f*x)**2/2 + 3*b*c**2*
d*x*cos(e + f*x)**2/2 - 3*b*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b*c*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*c*d**2*cos(e + f*x)**3/f + 3*b*d*
*3*x*sin(e + f*x)**4/8 + 3*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
```

```
*d**3*x*cos(e + f*x)**4/8 - 5*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*(c
 + d*sin(e))**3, True))
```

**Giac [A]**
time = 0.48, size = 152, normalized size = 0.89

$$\frac{bd^3\sin(4fx+4e)}{32f}+\frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)x+\frac{(3bcd^2+ad^3)\cos(3fx+3e)}{12f}-\frac{(4bc^3+12ac^2d+9bcd^2+3ad^3)\cos(fx+e)}{4f}-\frac{(3bc^2d+3acd^2+bd^3)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/32*b*d^3*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*
b*d^3)*x + 1/12*(3*b*c*d^2 + a*d^3)*cos(3*f*x + 3*e)/f - 1/4*(4*b*c^3 + 12*
a*c^2*d + 9*b*c*d^2 + 3*a*d^3)*cos(f*x + e)/f - 1/4*(3*b*c^2*d + 3*a*c*d^2
+ b*d^3)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.08, size = 183, normalized size = 1.07

$$\frac{2ad^3\cos(3e+3fx)-6bd^3\sin(2e+2fx)+\frac{3bd^3\sin(4e+4fx)}{4}-18ad^3\cos(e+fx)-24bc^3\cos(e+fx)-72ac^2d\cos(e+fx)-54bcd^2\cos(e+fx)+24ac^3fx+9bd^3fx+6bcd^2\cos(3e+3fx)-18acd^2\sin(2e+2fx)-18bc^2d\sin(2e+2fx)+36acd^2fx+36bc^2dfx}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (2*a*d^3*cos(3*e + 3*f*x) - 6*b*d^3*sin(2*e + 2*f*x) + (3*b*d^3*sin(4*e + 4
*f*x))/4 - 18*a*d^3*cos(e + f*x) - 24*b*c^3*cos(e + f*x) - 72*a*c^2*d*cos(e
 + f*x) - 54*b*c*d^2*cos(e + f*x) + 24*a*c^3*f*x + 9*b*d^3*f*x + 6*b*c*d^2*
cos(3*e + 3*f*x) - 18*a*c*d^2*sin(2*e + 2*f*x) - 18*b*c^2*d*sin(2*e + 2*f*x
) + 36*a*c*d^2*f*x + 36*b*c^2*d*f*x)/(24*f)
```

# 3.672 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=106

$$\frac{1}{2}\left(2bcd+a\left(2c^2+d^2\right)\right)x-\frac{2(3acd+b(c^2+d^2))\cos(e+fx)}{3f}-\frac{d(2bc+3ad)\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(e-$$

```
[Out] 1/2*(2*b*c*d+a*(2*c^2+d^2))*x-2/3*(3*a*c*d+b*(c^2+d^2))*cos(f*x+e)/f-1/6*d*
(3*a*d+2*b*c)*cos(f*x+e)*sin(f*x+e)/f-1/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^2/f
```

**Rubi** [A]

time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{2(3acd+b(c^2+d^2))\cos(e+fx)}{3f}+\frac{1}{2}x\left(a\left(2c^2+d^2\right)+2bcd\right)-\frac{d(3ad+2bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(c+d\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*x)/2 - (2*(3*a*c*d + b*(c^2 + d^2))*Cos[e + f*
x])/(3*f) - (d*(2*b*c + 3*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(3*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx=-\frac{b\cos(e+fx)(c+d\sin(e+fx))^2}{3f}+\frac{1}{3}\int (c+d\sin(e+fx))$$

$$=\frac{1}{2}\left(2bcd+a\left(2c^2+d^2\right)\right)x-\frac{2(3acd+b(c^2+d^2))\cos(e+fx)}{3f}$$

**Mathematica [A]**
time = 0.36, size = 90, normalized size = 0.85

$$\frac{6(2bcd + a(2c^2 + d^2))(e + fx) - 3(4bc^2 + 8acd + 3bd^2)\cos(e + fx) + bd^2\cos(3(e + fx)) - 3d(2bc + ad)\sin(2(e + fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (6*(2*b*c*d + a*(2*c^2 + d^2))*(e + f*x) - 3*(4*b*c^2 + 8*a*c*d + 3*b*d^2)*
Cos[e + f*x] + b*d^2*Cos[3*(e + f*x)] - 3*d*(2*b*c + a*d)*Sin[2*(e + f*x)])
/(12*f)
```

**Maple [A]**
time = 0.15, size = 115, normalized size = 1.08

| method | result |
|---|---|
| derivativedivides | $\frac{c^2 a(fx+e)-2acd\cos(fx+e)+d^2 a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-b\,c^2\cos(fx+e)+2bcd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| default | $\frac{c^2 a(fx+e)-2acd\cos(fx+e)+d^2 a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-b\,c^2\cos(fx+e)+2bcd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| risch | $a\,c^2 x+\frac{x\,d^2 a}{2}+xbcd-\frac{2\cos(fx+e)acd}{f}-\frac{\cos(fx+e)b\,c^2}{f}-\frac{3\cos(fx+e)b\,d^2}{4f}+\frac{b\,d^2\cos(3fx+3e)}{12f}-\frac{\sin(2f}{}$ |
| norman | $\frac{(c^2a+\frac{1}{2}d^2a+bcd)x+(c^2a+\frac{1}{2}d^2a+bcd)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3c^2a+\frac{3}{2}d^2a+3bcd)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3c^2a+\frac{3}{2}d^2a+3bcd)}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^2*a*(f*x+e)-2*a*c*d*cos(f*x+e)+d^2*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)-b*c^2*cos(f*x+e)+2*b*c*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/
2*e)-1/3*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima [A]**
time = 0.30, size = 121, normalized size = 1.14

$$\frac{12\,(fx+e)ac^2+6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))bcd+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ad^2+4\,(\cos(fx+e)^3-3\,\cos(fx+e))bd^2-12\,bc^2\cos(fx+e)-24\,acd\cos(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c*d + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*d^2 + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
*d^2 - 12*b*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.36, size = 94, normalized size = 0.89

$$\frac{2\,bd^{2}\cos\left(fx+e\right)^{3}+3\,\left(2\,ac^{2}+2\,bcd+ad^{2}\right)fx-3\,\left(2\,bcd+ad^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\,\left(bc^{2}+2\,acd+bd^{2}\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*b*c*d + a*d^2)*f*x - 3*(2*b*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(b*c^2 + 2*a*c*d + b*d^2)*cos(f*x
+ e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(97) = 194.
time = 0.18, size = 199, normalized size = 1.88

$$\begin{cases} ac^{2}x-\frac{2acd\cos\left(e+fx\right)}{f}+\frac{ad^{2}x\sin^{2}\left(e+fx\right)}{2}+\frac{ad^{2}x\cos^{2}\left(e+fx\right)}{2}-\frac{ad^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{bc^{2}\cos\left(e+fx\right)}{f}+bcdx\sin^{2}\left(e+fx\right)+bcdx\cos^{2}\left(e+fx\right)-\frac{bcd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{bd^{2}\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2bd^{2}\cos^{3}\left(e+fx\right)}{3f} & \text{for}\: f\neq 0 \\ x\left(a+b\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - 2*a*c*d*cos(e + f*x)/f + a*d**2*x*sin(e + f*x)**2/2 +
 a*d**2*x*cos(e + f*x)**2/2 - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - b*c*
*2*cos(e + f*x)/f + b*c*d*x*sin(e + f*x)**2 + b*c*d*x*cos(e + f*x)**2 - b*c
*d*sin(e + f*x)*cos(e + f*x)/f - b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e))**
2, True))
```

**Giac** [A]
time = 0.46, size = 96, normalized size = 0.91

$$\frac{bd^{2}\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\,\left(2\,ac^{2}+2\,bcd+ad^{2}\right)x-\frac{\left(4\,bc^{2}+8\,acd+3\,bd^{2}\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(2\,bcd+ad^{2}\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/12*b*d^2*cos(3*f*x + 3*e)/f + 1/2*(2*a*c^2 + 2*b*c*d + a*d^2)*x - 1/4*(4*
b*c^2 + 8*a*c*d + 3*b*d^2)*cos(f*x + e)/f - 1/4*(2*b*c*d + a*d^2)*sin(2*f*x
 + 2*e)/f
```

**Mupad** [B]
time = 7.89, size = 108, normalized size = 1.02

$$-\frac{\frac{3\,a\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{b\,d^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+6\,b\,c^2\,\cos\left(e+f\,x\right)+\frac{9\,b\,d^2\,\cos\left(e+f\,x\right)}{2}+3\,b\,c\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a\,c^2\,f\,x-3\,a\,d^2\,f\,x+12\,a\,c\,d\,\cos\left(e+f\,x\right)-6\,b\,c\,d\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -((3*a*d^2*sin(2*e + 2*f*x))/2 - (b*d^2*cos(3*e + 3*f*x))/2 + 6*b*c^2*cos(e
 + f*x) + (9*b*d^2*cos(e + f*x))/2 + 3*b*c*d*sin(2*e + 2*f*x) - 6*a*c^2*f*x
 - 3*a*d^2*f*x + 12*a*c*d*cos(e + f*x) - 6*b*c*d*f*x)/(6*f)
```

# 3.673 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=53

$$\frac{1}{2}(2ac+bd)x-\frac{(bc+ad)\cos(e+fx)}{f}-\frac{bd\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*(2*a*c+b*d)*x-(a*d+b*c)*cos(f*x+e)/f-1/2*b*d*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2813}

$$-\frac{(ad+bc)\cos(e+fx)}{f}+\frac{1}{2}x(2ac+bd)-\frac{bd\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*x)/2 - ((b*c + a*d)*Cos[e + f*x])/f - (b*d*Cos[e + f*x]*Sin[
e + f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}(2ac+bd)x-\frac{(bc+ad)\cos(e+fx)}{f}-\frac{bd\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.12, size = 52, normalized size = 0.98

$$\frac{2bde+4acfx+2bdfx-4(bc+ad)\cos(e+fx)-bd\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*d*e + 4*a*c*f*x + 2*b*d*f*x - 4*(b*c + a*d)*Cos[e + f*x] - b*d*Sin[2*(
e + f*x)])/(4*f)
```

**Maple [A]**
time = 0.09, size = 59, normalized size = 1.11

| method | result |
|---|---|
| risch | $acx+\frac{xbd}{2}-\frac{\cos(fx+e)ad}{f}-\frac{\cos(fx+e)bc}{f}-\frac{bd\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{bd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ad\cos(fx+e)-bc\cos(fx+e)+ac(fx+e)}{f}$ |
| default | $\frac{bd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ad\cos(fx+e)-bc\cos(fx+e)+ac(fx+e)}{f}$ |
| norman | $\frac{\left(ac+\frac{bd}{2}\right)x+\left(ac+\frac{bd}{2}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2ac+bd)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(2ad+2bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{bd\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a*d*cos(f*x+e)-b*c*cos(
f*x+e)+a*c*(f*x+e))
```

**Maxima [A]**
time = 0.32, size = 62, normalized size = 1.17

$$\frac{4(fx+e)ac+(2fx+2e-\sin(2fx+2e))bd-4bc\cos(fx+e)-4ad\cos(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*b*d - 4*b*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.35, size = 51, normalized size = 0.96

$$-\frac{bd\cos(fx+e)\sin(fx+e)-(2ac+bd)fx+2(bc+ad)\cos(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + b*d)*f*x + 2*(b*c + a*d)*cos
(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(44) = 88$.
time = 0.09, size = 94, normalized size = 1.77

$$\begin{cases} acx - \frac{ad\cos(e+fx)}{f} - \frac{bc\cos(e+fx)}{f} + \frac{bdx\sin^2(e+fx)}{2} + \frac{bdx\cos^2(e+fx)}{2} - \frac{bd\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\sin(e))(c+d\sin(e)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*d*cos(e + f*x)/f - b*c*cos(e + f*x)/f + b*d*x*sin(e +
f*x)**2/2 + b*d*x*cos(e + f*x)**2/2 - b*d*sin(e + f*x)*cos(e + f*x)/(2*f),
Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e)), True))
```

**Giac** [A]
time = 0.45, size = 48, normalized size = 0.91

$$\frac{1}{2}(2\,ac + bd)x - \frac{bd\sin(2\,fx + 2\,e)}{4\,f} - \frac{(bc + ad)\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c + b*d)*x - 1/4*b*d*sin(2*f*x + 2*e)/f - (b*c + a*d)*cos(f*x + e)
/f
```

**Mupad** [B]
time = 7.72, size = 52, normalized size = 0.98

$$a\,c\,x + \frac{b\,d\,x}{2} - \frac{a\,d\,\cos(e + f\,x)}{f} - \frac{b\,c\,\cos(e + f\,x)}{f} - \frac{b\,d\,\sin(2\,e + 2\,f\,x)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] a*c*x + (b*d*x)/2 - (a*d*cos(e + f*x))/f - (b*c*cos(e + f*x))/f - (b*d*sin(
2*e + 2*f*x))/(4*f)
```

# 3.674 $\int (a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=16

$$ax - \frac{b\cos(e+fx)}{f}$$

[Out] a*x-b*cos(f*x+e)/f

**Rubi** [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2718}

$$ax - \frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e + f*x])/f

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax + b\int \sin(e+fx)\,dx\\ &= ax - \frac{b\cos(e+fx)}{f}\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 27, normalized size = 1.69

$$ax - \frac{b\cos(e)\cos(fx)}{f} + \frac{b\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f

**Maple** [A]
time = 0.04, size = 17, normalized size = 1.06

| method | result | size |
|---|---|---|
| default | $ax - \frac{b\cos(fx+e)}{f}$ | 17 |
| risch | $ax - \frac{b\cos(fx+e)}{f}$ | 17 |
| derivativedivides | $\frac{(fx+e)a-b\cos(fx+e)}{f}$ | 22 |
| norman | $\frac{ax+ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{2b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$ | 52 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] a*x-b*cos(f*x+e)/f
```

**Maxima** [A]
time = 0.29, size = 17, normalized size = 1.06

$$ax - \frac{b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")
[Out] a*x - b*cos(f*x + e)/f
```

**Fricas** [A]
time = 0.36, size = 19, normalized size = 1.19

$$\frac{afx - b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")
[Out] (a*f*x - b*cos(f*x + e))/f
```

**Sympy** [A]
time = 0.05, size = 19, normalized size = 1.19

$$ax + b\left(\begin{cases} -\frac{\cos(e+fx)}{f} & \text{for}\ f \neq 0 \\ x\sin(e) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)
```

```
[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

**Giac [A]**
time = 0.44, size = 17, normalized size = 1.06

$$ax - \frac{b\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

**Mupad [B]**
time = 7.64, size = 25, normalized size = 1.56

$$a\,x - \frac{2\,b}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x),x)
```

```
[Out] a*x - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

# 3.675 $\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=65

$$\frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}$$

```
[Out] b*x/d-2*(-a*d+b*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d/f/(c^
2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2814, 2739, 632, 210}

$$\frac{bx}{d}-\frac{2(bc-ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (b*x)/d - (2*(b*c - a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
(d*Sqrt[c^2 - d^2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}\\
&= \frac{bx}{d}-\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\
&= \frac{bx}{d}+\frac{(4(bc-ad))\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\
&= \frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}
\end{aligned}$$

**Mathematica [A]**
time = 0.16, size = 67, normalized size = 1.03

$$\frac{b(e+fx)+\frac{(-2bc+2ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
[Out] (b*(e + f*x) + ((-2*b*c + 2*a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/Sqrt[c^2 - d^2])/(d*f)
```

**Maple [A]**
time = 0.16, size = 76, normalized size = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d}+\frac{2(ad-bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}}}{f}$ |

| | |
|---|---|
| default | $\frac{\frac{2b \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d}+\frac{2(ad-bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}}}{f}$ |
| risch | $\frac{bx}{d}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)a}{\sqrt{-c^2+d^2}\,f}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)bc}{\sqrt{-c^2+d^2}\,fd}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{}}{\sqrt{}}\right.}{\sqrt{}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b/d*arctan(tan(1/2*f*x+1/2*e))+2*(a*d-b*c)/d/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.38, size = 264, normalized size = 4.06

$$\left[\frac{2\,(bc^2-bd^2)fx+(bc-ad)\sqrt{-c^2+d^2}\,\log\left(\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2+2\,(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)}{2\,(c^2d-d^3)f},\ \frac{(bc^2-bd^2)fx+(bc-ad)\sqrt{c^2-d^2}\,\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)}{(c^2d-d^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - b*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^
2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(
f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2)))/((c^2*d - d^3)*f), ((b*c^2 - b*d^2)*f*x + (b*c -
a*d)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
+ e))))/((c^2*d - d^3)*f)]
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(53) = 106$.

time = 41.06, size = 537, normalized size = 8.26

$$\begin{cases} \frac{\tilde{\infty} x(a+b\sin(e))}{\sin(e)} & \text{for } c=0 \wedge d=0 \wedge f=0 \\ \frac{x(a+b\sin(e))}{c+d\sin(e)} & \text{for } f=0 \\ \frac{ax-\frac{b\cos(e+fx)}{f}}{c} & \text{for } d=0 \\ \frac{\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+bx}{d} & \text{for } c=0 \\ \frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{bd^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{2bd^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}-\frac{bdfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}} & \text{for } c=-\sqrt{d^2} \\ -\frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{bd^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{2bd^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{bdfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}} & \text{for } c=\sqrt{d^2} \\ \frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}-\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}-\frac{bc\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{df\sqrt{-c^2+d^2}}+\frac{bc\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{df\sqrt{-c^2+d^2}}+\frac{bx}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*sin(e))/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (x
*(a + b*sin(e))/(c + d*sin(e)), Eq(f, 0)), ((a*x - b*cos(e + f*x)/f)/c, Eq(
d, 0)), ((a*log(tan(e/2 + f*x/2))/f + b*x)/d, Eq(c, 0)), (2*a*d*sqrt(d**2)/
(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + b*d**2*f*x*tan(e/2 + f*x/2)/(
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + 2*b*d**2/(d**3*f*tan(e/2 + f*x
/2) - f*(d**2)**(3/2)) - b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) - f*(d
**2)**(3/2)), Eq(c, -sqrt(d**2))), (-2*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x
/2) + f*(d**2)**(3/2)) + b*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/
2) + f*(d**2)**(3/2)) + 2*b*d**2/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)
) + b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)), Eq(c, s
qrt(d**2))), (a*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(f*sqrt(
-c**2 + d**2)) - a*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(f*sq
rt(-c**2 + d**2)) - b*c*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/
(d*f*sqrt(-c**2 + d**2)) + b*c*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d*
*2)/c)/(d*f*sqrt(-c**2 + d**2)) + b*x/d, True))
```

**Giac [A]**
time = 0.45, size = 86, normalized size = 1.32

$$\frac{\frac{(fx+e)b}{d}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(bc-ad)}{\sqrt{c^2-d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*b/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(b*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

**Mupad [B]**
time = 9.68, size = 342, normalized size = 5.26

$$\frac{2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f}+\frac{c\,\left(b\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(c+d\right)\,\left(c-d\right)}-b\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{d^2-c^2}\right)-a\,d\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(c+d\right)\,\left(c-d\right)}+a\,d\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{d^2-c^2}}{d\,f\,\left(c^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f) + (c*(b*log((d*cos(
e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2
))/cos(e/2 + (f*x)/2))*(-(c + d)*(c - d))^(1/2) - b*log((d*cos(e/2 + (f*x)/
2) + c*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 +
 (f*x)/2))*(d^2 - c^2)^(1/2)) - a*d*log((d*cos(e/2 + (f*x)/2) + c*sin(e/2 +
 (f*x)/2) - cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 + (f*x)/2))*(-(c
+ d)*(c - d))^(1/2) + a*d*log((d*cos(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)
+ cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 + (f*x)/2))*(d^2 - c^2)^(1/
2))/(d*f*(c^2 - d^2))
```

# 3.676 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=98

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}f}-\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(3/2
)/f-(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

**Rubi** [A]

time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{2(ac-bd)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^
2)^(3/2)*f) - ((b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f(c+d\sin(e+fx))}+\frac{\int \frac{-ac+bd}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f(c+d\sin(e+fx))}+\frac{(ac-bd)\int \frac{1}{c+d\sin(e+fx)}\,dx}{c^2-d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f(c+d\sin(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e\right.\right.}{(c^2-d^2)\,f}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f(c+d\sin(e+fx))}-\frac{(4(ac-bd))\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+\right.}{(c^2-d^2)\,f}$$

$$= \frac{2(ac-bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}\,f}-\frac{(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f(c+d\sin(e+fx))}$$

**Mathematica** [A]
time = 0.44, size = 96, normalized size = 0.98

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}+\frac{(-bc+ad)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^
2)^(3/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*
x])))/f
```

## Maple [A]

time = 0.22, size = 142, normalized size = 1.45

| method | result |
|---|---|
| derivativedivides | $\dfrac{\dfrac{\frac{2d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{2(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\dfrac{2(ac-bd)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{\frac{3}{2}}}}{f}$ |
| default | $\dfrac{\dfrac{\frac{2d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{2(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\dfrac{2(ac-bd)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{\frac{3}{2}}}}{f}$ |
| risch | $\frac{2i(-ad+bc)\left(id+c\,e^{i(fx+e)}\right)}{d(c^2-d^2)f\left(-ie^{2i(fx+e)}d+id+2c\,e^{i(fx+e)}\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)ac}{\sqrt{-c^2+d^2}\,(c+d)(c-d)f}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c}}{\sqrt{-}}\right.}{\sqrt{-c^2+d^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(d*(a*d-b*c)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+(a*d-b*c)/(c^2-d^2))/(c*
tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+2*(a*c-b*d)/(c^2-d^2)^(3/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

## Fricas [A]

time = 0.36, size = 409, normalized size = 4.17

$$\left[-\frac{(ac^2-bcd+(acd-bd^2)\sin(fx+e))\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2(bc^3-ac^2d-bcd^2+ad^3)\cos(fx+e)}{2((c^4d-2c^2d^3+d^5)f\sin(fx+e)+(c^5-2c^3d^2+cd^4)f)},-\frac{(ac^2-bcd+(acd-bd^2)\sin(fx+e))\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+(bc^3-ac^2d-bcd^2+ad^3)\cos(fx+e)}{(c^4d-2c^2d^3+d^5)f\sin(fx+e)+(c^5-2c^3d^2+cd^4)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(
((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f
```

```
*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3
)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin(f*x + e) + (c^5 - 2*c^3*d^
2 + c*d^4)*f), -((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(c^2 -
d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^3
- a*c^2*d - b*c*d^2 + a*d^3)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin
(f*x + e) + (c^5 - 2*c^3*d^2 + c*d^4)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.49, size = 158, normalized size = 1.61

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-bd)}{(c^2-d^2)^{\frac{3}{2}}}-\frac{bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+bc^2-acd}{(c^3-cd^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(a*c - b*d)/(c^2 - d^2)^(3/2) - (b*c*d*tan(1/2*f*x
 + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e) + b*c^2 - a*c*d)/((c^3 - c*d^2)*(c*t
an(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

**Mupad** [B]
time = 7.92, size = 214, normalized size = 2.18

$$\frac{\frac{2\,(a\,d-b\,c)}{c^2-d^2}+\frac{2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,d-b\,c)}{c\,(c^2-d^2)}}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{2\,\left(c^2\,d-d^3\right)(a\,c-b\,d)}{(c+d)^{3/2}\,\left(c^2-d^2\right)(c-d)^{3/2}}+\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,c-b\,d)}{(c+d)^{3/2}\,(c-d)^{3/2}}\right)(c^2-d^2)}{2\,(a\,c-b\,d)}\right)(a\,c-b\,d)}{f\,(c+d)^{3/2}\,(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a*d - b*c))/(c^2 - d^2) + (2*d*tan(e/2 + (f*x)/2)*(a*d - b*c))/(c*(c^2
 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (2*at
an((((2*(c^2*d - d^3)*(a*c - b*d))/((c + d)^(3/2)*(c^2 - d^2)*(c - d)^(3/2)
) + (2*c*tan(e/2 + (f*x)/2)*(a*c - b*d))/((c + d)^(3/2)*(c - d)^(3/2)))*(c^
2 - d^2))/(2*(a*c - b*d)))*(a*c - b*d))/(f*(c + d)^(3/2)*(c - d)^(3/2))
```

# 3.677 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=164

$$-\frac{(3bcd-a(2c^2+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}f}-\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2}+\frac{(3acd-b(c^2+2d^2))\cos(}{2(c^2-d^2)^2f(c+d\sin(e}$$

```
[Out] -(3*b*c*d-a*(2*c^2+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(
c^2-d^2)^(5/2)/f-1/2*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+1
/2*(3*a*c*d-b*(c^2+2*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$-\frac{(3bcd-a(2c^2+d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\,(c^2-d^2)^{5/2}}+\frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2f\,(c^2-d^2)^2\,(c+d\sin(e+fx))}-\frac{(bc-ad)\cos(e+fx)}{2f\,(c^2-d^2)\,(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c^2 - d^2)^(5/2)*f)) - ((b*c - a*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^2) + ((3*a*c*d - b*(c^2 + 2*d^2))*Cos[e + f*x])/(2*(c^
2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx = -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)-(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2(c^2-d^2)}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} + \frac{\int \frac{-3}{c}\cdots}{2\cdots}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} - \frac{(3bc\cdots}{\cdots}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} - \frac{(3bc\cdots}{\cdots}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} + \frac{(2(3\cdots}{\cdots}$$

$$= -\frac{(3bcd-a(2c^2+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}f} - \frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+f\cdots}$$

## **Mathematica** [A]

time = 0.72, size = 157, normalized size = 0.96

$$\frac{\frac{2(-3bcd+a(2c^2+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} + \frac{(-bc+ad)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))^2} - \frac{(-3acd+b(c^2+2d^2))\cos(e+fx)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(5/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*
(c + d*Sin[e + f*x])^2) - ((-3*a*c*d + b*(c^2 + 2*d^2))*Cos[e + f*x])/((c -
 d)^2*(c + d)^2*(c + d*Sin[e + f*x])))/(2*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(155) = 310$.

time = 0.37, size = 351, normalized size = 2.14

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\frac{d\left(5c^2da-2d^3a-3c^3b\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c\left(c^4-2c^2d^2+d^4\right)}+\frac{\left(4a\,c^4d+7a\,c^2d^3-2a\,d^5-2b\,c^5-5b\,c^3d^2-2bc\,d^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(c^4-2c^2d^2+d^4\right)c^2}+\frac{d\left(11c^2da-2d^3a-5c^3b-\right.}{c\left(c^4-2c^2d\right.}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}}{f}$ |
| default | $\frac{\frac{\frac{d\left(5c^2da-2d^3a-3c^3b\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c\left(c^4-2c^2d^2+d^4\right)}+\frac{\left(4a\,c^4d+7a\,c^2d^3-2a\,d^5-2b\,c^5-5b\,c^3d^2-2bc\,d^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(c^4-2c^2d^2+d^4\right)c^2}+\frac{d\left(11c^2da-2d^3a-5c^3b-\right.}{c\left(c^4-2c^2d\right.}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right)^2}}{f}$ |
| risch | $\frac{i\left(2id^2a\,c^2\mathrm{e}^{3i(fx+e)}+id^4a\,\mathrm{e}^{3i(fx+e)}-3id^3bc\,\mathrm{e}^{3i(fx+e)}-10ia\,c^2d^2\mathrm{e}^{i(fx+e)}+ia\,d^4\mathrm{e}^{i(fx+e)}+4ib\,c^3d\,\mathrm{e}^{i(fx+e)}+5ibc\,d^3\mathrm{e}^{i(fx+\right.}{\left(-i\mathrm{e}^{2i(fx+e)}d+id+2c\,\mathrm{e}^{i(fx\right.}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*d*(5*a*c^2*d-2*a*d^3-3*b*c^3)/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x
+1/2*e)^3+1/2*(4*a*c^4*d+7*a*c^2*d^3-2*a*d^5-2*b*c^5-5*b*c^3*d^2-2*b*c*d^4)
/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d*(11*a*c^2*d-2*a*d^3-5*b
*c^3-4*b*c*d^2)/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)+1/2*(4*a*c^2*d-a*d
^3-2*b*c^3-b*c*d^2)/(c^4-2*c^2*d^2+d^4))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/
2*f*x+1/2*e)+c)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(160) = 320$.
time = 0.40, size = 816, normalized size = 4.98

$$\left[\frac{2\left(bc^4d-3ac^3d^2+bc^2d^3+3acd^4-2bd^5\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2ac^4-3bc^3d+3ac^2d^2-3bcd^3+ad^4-\left(2ac^2d^2-3bcd^3+ad^4\right)\cos\left(fx+e\right)^2+2\left(2ac^3d-3bc^2d^2+acd^3\right)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2\left(2bc^5-4ac^4d-bc^3d^2+5ac^2d^3-bcd^4-ad^5\right)\cos\left(fx+e\right)}{4\left(\left(c^6d^2-3c^4d^4+3c^2d^6-d^8\right)f\cos\left(fx+e\right)^2-2\left(c^7d-3c^5d^3+3c^3d^5-cd^7\right)f\sin\left(fx+e\right)-\left(c^8-2c^6d^2+2c^2d^6-d^8\right)f\right)}, \frac{\left(bc^4d-3ac^3d^2+bc^2d^3+3acd^4-2bd^5\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2ac^4-3bc^3d+3ac^2d^2-3bcd^3+ad^4-\left(2ac^2d^2-3bcd^3+ad^4\right)\cos\left(fx+e\right)^2+2\left(2ac^3d-3bc^2d^2+acd^3\right)\sin\left(fx+e\right)\right)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+\left(2bc^5-4ac^4d-bc^3d^2+5ac^2d^3-bcd^4-ad^5\right)\cos\left(fx+e\right)}{2\left(\left(c^6d^2-3c^4d^4+3c^2d^6-d^8\right)f\cos\left(fx+e\right)^2-2\left(c^7d-3c^5d^3+3c^3d^5-cd^7\right)f\sin\left(fx+e\right)-\left(c^8-2c^6d^2+2c^2d^6-d^8\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5)*cos(f*x +
 e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 -
 (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*
d^2 + a*c*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)) + 2*(2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 -
a*d^5)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e
)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c
^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a
*c*d^4 - 2*b*d^5)*cos(f*x + e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^
2*d^2 - 3*b*c*d^3 + a*d^4 - (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^
2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b*c^5 - 4*a*
c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))/((c^6*d^2
- 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*
c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(160) = 320$.
time = 0.45, size = 428, normalized size = 2.61

$$\frac{\frac{\left(2ac^2-3bcd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^4-2c^2d^2+d^4\right)\sqrt{c^2-d^2}}-\frac{3bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2bc^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4ac^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+5bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-7ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2bcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+5bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-11ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4bc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2bc^5-4ac^4d+bc^3d^2+ac^2d^3}{\left(c^6-2c^4d^2+c^2d^4\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 - 2*c^2*d^2 + d^4)
*sqrt(c^2 - d^2)) - (3*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 2*a*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*b*c^5*tan(1/2*f*x +
1/2*e)^2 - 4*a*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^3*d^2*tan(1/2*f*x + 1/2
*e)^2 - 7*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*d^4*tan(1/2*f*x + 1/2*e)
^2 + 2*a*d^5*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^4*d*tan(1/2*f*x + 1/2*e) - 11*a
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 4*b*c^2*d^3*tan(1/2*f*x + 1/2*e) + 2*a*c*d^
4*tan(1/2*f*x + 1/2*e) + 2*b*c^5 - 4*a*c^4*d + b*c^3*d^2 + a*c^2*d^3)/((c^6
 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e
) + c)^2))/f
```

**Mupad [B]**
time = 9.61, size = 477, normalized size = 2.91

$$\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,c^4\,d-4\,c^2\,d^3+2\,d^5\right)\,\left(2\,a\,c^2-3\,b\,c\,d+a\,d^2\right)}{2\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{5/2}\,\left(c^4-2\,c^2\,d^2+d^4\right)}+\frac{c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a\,c^2-3\,b\,c\,d+a\,d^2\right)}{{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{5/2}}\right)\,\left(c^4-2\,c^2\,d^2+d^4\right)}{2\,a\,c^2-3\,b\,c\,d+a\,d^2}\right)\,\left(2\,a\,c^2-3\,b\,c\,d+a\,d^2\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{5/2}}-\frac{\frac{2\,b\,c^3-4\,a\,c^2\,d+b\,c\,d^2+a\,d^3}{c^4-2\,c^2\,d^2+d^4}+\frac{d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(3\,b\,c^3-5\,a\,c^2\,d+2\,a\,d^3\right)}{c\,\left(c^4-2\,c^2\,d^2+d^4\right)}+\frac{d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,b\,c^3-11\,a\,c^2\,d+4\,b\,c\,d^2+2\,a\,d^3\right)}{c\,\left(c^4-2\,c^2\,d^2+d^4\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(c^2+2\,d^2\right)\,\left(2\,b\,c^3-4\,a\,c^2\,d+b\,c\,d^2+a\,d^3\right)}{c^2\,\left(c^4-2\,c^2\,d^2+d^4\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2+4\,d^2\right)+c^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+c^2+4\,c\,d\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*c^4*d + 2*d^5 - 4*c^2*d^3)*(2*a*c^2 + a*d^2 - 3*b*c*d))/(2*(c +
 d)^(5/2)*(c - d)^(5/2)*(c^4 + d^4 - 2*c^2*d^2)) + (c*tan(e/2 + (f*x)/2)*(2
*a*c^2 + a*d^2 - 3*b*c*d))/((c + d)^(5/2)*(c - d)^(5/2)))*(c^4 + d^4 - 2*c^
2*d^2))/(2*a*c^2 + a*d^2 - 3*b*c*d))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d
)^(5/2)*(c - d)^(5/2)) - ((a*d^3 + 2*b*c^3 - 4*a*c^2*d + b*c*d^2)/(c^4 + d^
4 - 2*c^2*d^2) + (d*tan(e/2 + (f*x)/2)^3*(2*a*d^3 + 3*b*c^3 - 5*a*c^2*d))/(
c*(c^4 + d^4 - 2*c^2*d^2)) + (d*tan(e/2 + (f*x)/2)*(2*a*d^3 + 5*b*c^3 - 11*
a*c^2*d + 4*b*c*d^2))/(c*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(
c^2 + 2*d^2)*(a*d^3 + 2*b*c^3 - 4*a*c^2*d + b*c*d^2))/(c^2*(c^4 + d^4 - 2*c
^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)
^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

# 3.678 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=314

$$\frac{1}{8}\left(6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)+4a^2\left(2c^3+3cd^2\right)\right)x-\frac{\left(20a^2d^2\left(4c^2+d^2\right)+30abcd\left(c^2+4d^2\right)-b^2\left(3c^4-\right.\right.}{30df}$$

```
[Out] 1/8*(6*a*b*d*(4*c^2+d^2)+b^2*c*(4*c^2+9*d^2)+4*a^2*(2*c^3+3*c*d^2))*x-1/30*
(20*a^2*d^2*(4*c^2+d^2)+30*a*b*c*d*(c^2+4*d^2)-b^2*(3*c^4-52*c^2*d^2-16*d^4
))*cos(f*x+e)/d/f-1/120*(100*a^2*c*d^2+30*a*b*d*(2*c^2+3*d^2)-b^2*(6*c^3-71
*c*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/60*(4*(5*a^2+4*b^2)*d^2-3*b*c*(-10*a*d+b
*c))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*b*(-10*a*d+b*c)*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d/f-1/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f
```

**Rubi** [A]
time = 0.40, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2870, 2832, 2813}

$$-\frac{(20a^2d^2(4c^2+d^2)+30abcd(c^2+4d^2)-(b^2(3c^4-52c^2d^2-16d^4)))\cos(e+fx)}{30df}-\frac{(100a^2cd^2+30abd(2c^2+3d^2)-(b^2(6c^3-71cd^2)))\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}x\left(4a^2\left(2c^3+3cd^2\right)+6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)\right)-\frac{(4d^2(5a^2+4b^2)-3bc(bc-10ad))\cos(e+fx)(c+d\sin(e+fx))^2}{60df}+\frac{b(bc-10ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20df}-\frac{b^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((6*a*b*d*(4*c^2 + d^2) + b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*
x)/8 - ((20*a^2*d^2*(4*c^2 + d^2) + 30*a*b*c*d*(c^2 + 4*d^2) - b^2*(3*c^4 -
 52*c^2*d^2 - 16*d^4))*Cos[e + f*x])/(30*d*f) - ((100*a^2*c*d^2 + 30*a*b*d*
(2*c^2 + 3*d^2) - b^2*(6*c^3 - 71*c*d^2))*Cos[e + f*x]*Sin[e + f*x])/(120*f
) - ((4*(5*a^2 + 4*b^2)*d^2 - 3*b*c*(b*c - 10*a*d))*Cos[e + f*x]*(c + d*Sin
[e + f*x])^2)/(60*d*f) + (b*(b*c - 10*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(20*d*f) - (b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df}+\frac{\int (c+d\sin(e+fx))^3 \cdots}{\cdots}$$
$$= \frac{b(bc-10ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20df}-\frac{b^2\cos(e+f \cdots}{\cdots}$$
$$= -\frac{(4(5a^2+4b^2)\,d^2-3bc(bc-10ad))\cos(e+fx)(c+d\sin(e \cdots}{60df}$$
$$= \frac{1}{8}\left(6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)+4a^2\left(2c^3+3cd^2\right)\right)x - \cdots$$

**Mathematica [A]**
time = 1.46, size = 249, normalized size = 0.79

$$\frac{-60(b^2d(18c^2+5d^2)+4abc(4c^2+9d^2)+6a^2(4c^2d+d^3))\cos(e+fx)+10d(24abcd+4a^2d^2+b^2(12c^2+5d^2))\cos(3(e+fx))-6b^2d^3\cos(5(e+fx))+15(4(6abd(4c^2+d^2)+b^2c(4c^2+9d^2)+4a^2(2c^3+3cd^2))\,(e+fx)-8(3a^2cd^2+2abd(3c^2+d^2)+b^2(c^3+3cd^2))\sin(2(e+fx))+bd^2(3bc+2ad)\sin(4(e+fx)))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-60*(b^2*d*(18*c^2 + 5*d^2) + 4*a*b*c*(4*c^2 + 9*d^2) + 6*a^2*(4*c^2*d + d
^3))*Cos[e + f*x] + 10*d*(24*a*b*c*d + 4*a^2*d^2 + b^2*(12*c^2 + 5*d^2))*Co
s[3*(e + f*x)] - 6*b^2*d^3*Cos[5*(e + f*x)] + 15*(4*(6*a*b*d*(4*c^2 + d^2)
+ b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*(e + f*x) - 8*(3*a^2*c*d
^2 + 2*a*b*d*(3*c^2 + d^2) + b^2*(c^3 + 3*c*d^2))*Sin[2*(e + f*x)] + b*d^2*
(3*b*c + 2*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

**Maple [A]**
time = 0.43, size = 325, normalized size = 1.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x+e)+3*a^2*c*d^2*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a*b*c^3*c
os(f*x+e)+6*a*b*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a*b*c*d^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))
*cos(f*x+e)+3/8*f*x+3/8*e)+b^2*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*
e)-b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*b^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^2*d^3*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))

**Maxima** [A]
time = 0.31, size = 338, normalized size = 1.08

$$\frac{480\,(fx+e)a^2c^3 + 120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2c^3 + 720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))abc^2d + 480\,(\cos(fx+e)^3-3\,\cos(fx+e))b^2c^2d + 360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2cd^2 + 960\,(\cos(fx+e)^3-3\,\cos(fx+e))abcd^2 + 45\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))b^2cd^2 + 160\,(\cos(fx+e)^3-3\,\cos(fx+e))a^2d^3 + 30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))abd^3 - 32\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))b^2d^3 - 960\,abc^3\cos(fx+e) - 1440\,a^2c^2d\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 1/480*(480*(f*x + e)*a^2*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3
 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*b^2*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b*c*d^2 + 45*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2*c*d^2 + 160*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*
e))*a*b*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^2*d^3 - 960*a*b*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f

**Fricas** [A]
time = 0.37, size = 253, normalized size = 0.81

$$-\frac{24\,b^2d^3\cos(fx+e)^5 - 40\,(3\,b^2c^2d+6\,abcd^2+(a^2+2\,b^2)d^3)\cos(fx+e)^3 - 15\,(24\,abc^2d+6\,abd^3+4\,(2\,a^2+b^2)c^3+3\,(4\,a^2+3\,b^2)cd^2)fx + 120\,(2\,abc^3+6\,abcd^2+3\,(a^2+b^2)c^2d+(a^2+b^2)d^3)\cos(fx+e) - 15\,(2\,(3\,b^2cd^2+2\,abd^3)\cos(fx+e)^3 - (4\,b^2c^3+24\,abc^2d+10\,abd^3+3\,(4\,a^2+5\,b^2)cd^2)\cos(fx+e))\sin(fx+e)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/120*(24*b^2*d^3*cos(f*x + e)^5 - 40*(3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 +
2*b^2)*d^3)*cos(f*x + e)^3 - 15*(24*a*b*c^2*d + 6*a*b*d^3 + 4*(2*a^2 + b^2)
*c^3 + 3*(4*a^2 + 3*b^2)*c*d^2)*f*x + 120*(2*a*b*c^3 + 6*a*b*c*d^2 + 3*(a^2
 + b^2)*c^2*d + (a^2 + b^2)*d^3)*cos(f*x + e) - 15*(2*(3*b^2*c*d^2 + 2*a*b*
d^3)*cos(f*x + e)^3 - (4*b^2*c^3 + 24*a*b*c^2*d + 10*a*b*d^3 + 3*(4*a^2 + 5
*b^2)*c*d^2)*cos(f*x + e))*sin(f*x + e))/f

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(292) = 584$.
time = 0.48, size = 729, normalized size = 2.32

$$\begin{cases} a^2c^3x - \frac{3a^2c^2d\cos(e+fx)}{f} + \frac{3a^2cd^2x\sin^2(e+fx)}{2} + \frac{3a^2cd^2x\cos^2(e+fx)}{2} - \frac{3a^2cd^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{a^2d^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2a^2d^3\cos^3(e+fx)}{3f} - \frac{2abc^3\cos(e+fx)}{f} + 3abc^2dx\sin^2(e+fx) + 3abc^2dx\cos^2(e+fx) - \frac{3abc^2d\sin(e+fx)\cos(e+fx)}{f} - \frac{6abcd^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{4abcd^2\cos^3(e+fx)}{f} + \frac{3abd^3x\sin^4(e+fx)}{4} + \frac{3abd^3x\sin^2(e+fx)\cos^2(e+fx)}{2} + \frac{3abd^3x\cos^4(e+fx)}{4} - \frac{5abd^3\sin^3(e+fx)\cos(e+fx)}{4f} - \frac{3abd^3\sin(e+fx)\cos^3(e+fx)}{4f} + \frac{b^2c^3x\sin^2(e+fx)}{2} + \frac{b^2c^3x\cos^2(e+fx)}{2} - \frac{b^2c^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{3b^2c^2d\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2b^2c^2d\cos^3(e+fx)}{f} + \frac{9b^2cd^2x\sin^4(e+fx)}{8} + \frac{9b^2cd^2x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{9b^2cd^2x\cos^4(e+fx)}{8} - \frac{15b^2cd^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{9b^2cd^2\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{b^2d^3\sin^4(e+fx)\cos(e+fx)}{f} - \frac{4b^2d^3\sin^2(e+fx)\cos^3(e+fx)}{3f} - \frac{8b^2d^3\cos^5(e+fx)}{15f} & \text{for}\ f \neq 0 \\ x(a+b\sin(e))^2(c+d\sin(e))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x - 3*a**2*c**2*d*cos(e + f*x)/f + 3*a**2*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 3*a**2*c*d**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2
*d**3*cos(e + f*x)**3/(3*f) - 2*a*b*c**3*cos(e + f*x)/f + 3*a*b*c**2*d*x*si
n(e + f*x)**2 + 3*a*b*c**2*d*x*cos(e + f*x)**2 - 3*a*b*c**2*d*sin(e + f*x)*
cos(e + f*x)/f - 6*a*b*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c*d**2
*cos(e + f*x)**3/f + 3*a*b*d**3*x*sin(e + f*x)**4/4 + 3*a*b*d**3*x*sin(e +
f*x)**2*cos(e + f*x)**2/2 + 3*a*b*d**3*x*cos(e + f*x)**4/4 - 5*a*b*d**3*sin
(e + f*x)**3*cos(e + f*x)/(4*f) - 3*a*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(
4*f) + b**2*c**3*x*sin(e + f*x)**2/2 + b**2*c**3*x*cos(e + f*x)**2/2 - b**2
*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b**2*c**2*d*sin(e + f*x)**2*cos(e
 + f*x)/f - 2*b**2*c**2*d*cos(e + f*x)**3/f + 9*b**2*c*d**2*x*sin(e + f*x)*
*4/8 + 9*b**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**2*c*d**2*x*
cos(e + f*x)**4/8 - 15*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b
**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**2*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*
d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))
**3, True))
```

**Giac** [**A**]
time = 0.49, size = 274, normalized size = 0.87

$$-\frac{b^{2} d^{3} \cos\left(5 \, f x + 5 \, e\right)}{80 \, f} + \frac{1}{8} \, {\left(8 \, a^{2} c^{3} + 4 \, b^{2} c^{3} + 24 \, a b c^{2} d + 12 \, a^{2} c d^{2} + 9 \, b^{2} c d^{2} + 6 \, a b d^{3}\right)} x + \frac{{\left(12 \, b^{2} c^{2} d + 24 \, a b c d^{2} + 4 \, a^{2} d^{3} + 5 \, b^{2} d^{3}\right)} \cos\left(3 \, f x + 3 \, e\right)}{48 \, f} - \frac{{\left(16 \, a b c^{3} + 24 \, a^{2} c^{2} d + 18 \, b^{2} c^{2} d + 36 \, a b c d^{2} + 6 \, a^{2} d^{3} + 5 \, b^{2} d^{3}\right)} \cos\left(f x + e\right)}{8 \, f} + \frac{{\left(3 \, b^{2} c d^{2} + 2 \, a b d^{3}\right)} \sin\left(4 \, f x + 4 \, e\right)}{32 \, f} - \frac{{\left(b^{2} c^{3} + 6 \, a b c^{2} d + 3 \, a^{2} c d^{2} + 3 \, b^{2} c d^{2} + 2 \, a b d^{3}\right)} \sin\left(2 \, f x + 2 \, e\right)}{4 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^2*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*a^2*c^3 + 4*b^2*c^3 + 24*a*b*c^2*
d + 12*a^2*c*d^2 + 9*b^2*c*d^2 + 6*a*b*d^3)*x + 1/48*(12*b^2*c^2*d + 24*a*b
*c*d^2 + 4*a^2*d^3 + 5*b^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a*b*c^3 + 24*a
^2*c^2*d + 18*b^2*c^2*d + 36*a*b*c*d^2 + 6*a^2*d^3 + 5*b^2*d^3)*cos(f*x + e
)/f + 1/32*(3*b^2*c*d^2 + 2*a*b*d^3)*sin(4*f*x + 4*e)/f - 1/4*(b^2*c^3 + 6*
a*b*c^2*d + 3*a^2*c*d^2 + 3*b^2*c*d^2 + 2*a*b*d^3)*sin(2*f*x + 2*e)/f
```

**Mupad** [**B**]
time = 8.44, size = 358, normalized size = 1.14

$$-\frac{90\,a^2\,d^3\,\cos\left(e+f\,x\right)+75\,b^2\,d^3\,\cos\left(e+f\,x\right)-10\,a^2\,d^3\,\cos\left(3\,e+3\,f\,x\right)-\frac{25\,b^2\,d^3\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{3\,b^2\,d^3\,\cos\left(5\,e+5\,f\,x\right)}{2}+30\,b^2\,c^3\,\sin\left(2\,e+2\,f\,x\right)-30\,b^2\,c^2\,d\,\cos\left(3\,e+3\,f\,x\right)+90\,a^2\,c\,d^2\,\sin\left(2\,e+2\,f\,x\right)+90\,b^2\,c\,d^2\,\sin\left(2\,e+2\,f\,x\right)-\frac{45\,b^2\,c\,d^2\,\sin\left(4\,e+4\,f\,x\right)}{4}+240\,a\,b\,c^3\,\cos\left(e+f\,x\right)+360\,a^2\,c^2\,d\,\cos\left(e+f\,x\right)+270\,b^2\,c^2\,d\,\cos\left(e+f\,x\right)+60\,a\,b\,d^3\,\sin\left(2\,e+2\,f\,x\right)-\frac{15\,a\,b\,d^3\,\sin\left(4\,e+4\,f\,x\right)}{2}-120\,a^2\,c^3\,f\,x-60\,b^2\,c^3\,f\,x-60\,a\,b\,c\,d^2\,\cos\left(3\,e+3\,f\,x\right)+180\,a\,b\,c^2\,d\,\sin\left(2\,e+2\,f\,x\right)-180\,a^2\,c\,d^2\,f\,x-135\,b^2\,c\,d^2\,f\,x+540\,a\,b\,c\,d^2\,\cos\left(e+f\,x\right)-90\,a\,b\,d^3\,f\,x-360\,a\,b\,c^2\,d\,f\,x}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] -(90*a^2*d^3*cos(e + f*x) + 75*b^2*d^3*cos(e + f*x) - 10*a^2*d^3*cos(3*e +
3*f*x) - (25*b^2*d^3*cos(3*e + 3*f*x))/2 + (3*b^2*d^3*cos(5*e + 5*f*x))/2 +
 30*b^2*c^3*sin(2*e + 2*f*x) - 30*b^2*c^2*d*cos(3*e + 3*f*x) + 90*a^2*c*d^2
*sin(2*e + 2*f*x) + 90*b^2*c*d^2*sin(2*e + 2*f*x) - (45*b^2*c*d^2*sin(4*e +
 4*f*x))/4 + 240*a*b*c^3*cos(e + f*x) + 360*a^2*c^2*d*cos(e + f*x) + 270*b^
2*c^2*d*cos(e + f*x) + 60*a*b*d^3*sin(2*e + 2*f*x) - (15*a*b*d^3*sin(4*e +
4*f*x))/2 - 120*a^2*c^3*f*x - 60*b^2*c^3*f*x - 60*a*b*c*d^2*cos(3*e + 3*f*x
) + 180*a*b*c^2*d*sin(2*e + 2*f*x) - 180*a^2*c*d^2*f*x - 135*b^2*c*d^2*f*x
+ 540*a*b*c*d^2*cos(e + f*x) - 90*a*b*d^3*f*x - 360*a*b*c^2*d*f*x)/(120*f)
```

# 3.679 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=217

$$\frac{1}{8}\left(16abcd+4a^2\left(2c^2+d^2\right)+b^2\left(4c^2+3d^2\right)\right)x-\frac{\left(8a^2bcd+8b^3cd-a^3d^2+4ab^2\left(3c^2+2d^2\right)\right)\cos(e+fx)}{6bf}-\frac{(2ac$$

```
[Out] 1/8*(16*a*b*c*d+4*a^2*(2*c^2+d^2)+b^2*(4*c^2+3*d^2))*x-1/6*(8*a^2*b*c*d+8*b
^3*c*d-a^3*d^2+4*a*b^2*(3*c^2+2*d^2))*cos(f*x+e)/b/f-1/24*(2*a*d*(-a*d+8*b*
c)+3*b^2*(4*c^2+3*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/12*d*(-a*d+8*b*c)*cos(f*x
+e)*(a+b*sin(f*x+e))^2/b/f-1/4*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^3/b/f
```

**Rubi** [A]
time = 0.21, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2870, 2832, 2813}

$$\frac{1}{8}x\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-\frac{\left(a^3\left(-d^2\right)+8a^2bcd+4ab^2\left(3c^2+2d^2\right)+8b^3cd\right)\cos(e+fx)}{6bf}-\frac{\left(2ad(8bc-ad)+3b^2\left(4c^2+3d^2\right)\right)\sin(e+fx)\cos(e+fx)}{24f}-\frac{d(8bc-ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12bf}-\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^3}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(4*c^2 + 3*d^2))*x)/8 - ((8*a^2*b*
c*d + 8*b^3*c*d - a^3*d^2 + 4*a*b^2*(3*c^2 + 2*d^2))*Cos[e + f*x])/(6*b*f)
- ((2*a*d*(8*b*c - a*d) + 3*b^2*(4*c^2 + 3*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(24*f) - (d*(8*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*b*f) -
(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*b*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^3}{4bf} + \frac{\int(a+b\sin(e+fx)}{}$$

$$= -\frac{d(8bc-ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12bf} - \frac{d^2\cos(e+}{}$$

$$= \frac{1}{8}\left(16abcd+4a^2\left(2c^2+d^2\right)+b^2\left(4c^2+3d^2\right)\right)x - \frac{(8a^2bcd+}{}$$

## Mathematica [A]

time = 0.83, size = 160, normalized size = 0.74

$$\frac{-48(4a^2cd+3b^2cd+ab(4c^2+3d^2))\cos(e+fx)+16bd(bc+ad)\cos(3(e+fx))+3(4(16abcd+4a^2(2c^2+d^2)+b^2(4c^2+3d^2))(e+fx)-8(4abcd+a^2d^2+b^2(c^2+d^2))\sin(2(e+fx))+b^2d^2\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-48*(4*a^2*c*d + 3*b^2*c*d + a*b*(4*c^2 + 3*d^2))*Cos[e + f*x] + 16*b*d*(b
*c + a*d)*Cos[3*(e + f*x)] + 3*(4*(16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(
4*c^2 + 3*d^2))*(e + f*x) - 8*(4*a*b*c*d + a^2*d^2 + b^2*(c^2 + d^2))*Sin[2
*(e + f*x)] + b^2*d^2*Sin[4*(e + f*x)]))/(96*f)
```

## Maple [A]

time = 0.28, size = 216, normalized size = 1.00

| method | result |
|---|---|
| derivativedivides | $\frac{a^2c^2(fx+e)-2a^2dc\cos(fx+e)+a^2d^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\,c^2\cos(fx+e)+4abcd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}\right.}{}$ |
| default | $\frac{a^2c^2(fx+e)-2a^2dc\cos(fx+e)+a^2d^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\,c^2\cos(fx+e)+4abcd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}\right.}{}$ |
| risch | $a^2c^2x+\frac{x\,a^2d^2}{2}+2xabcd+\frac{x\,b^2c^2}{2}+\frac{3x\,d^2b^2}{8}-\frac{2\cos(fx+e)a^2dc}{f}-\frac{2\cos(fx+e)ab\,c^2}{f}-\frac{3\cos(fx+e)ab\,d}{2f}$ |

| | |
|---|---|
| norman | $\frac{\left(a^2c^2+\frac{1}{2}a^2d^2+2abcd+\frac{1}{2}b^2c^2+\frac{3}{8}d^2b^2\right)x+\left(a^2c^2+\frac{1}{2}a^2d^2+2abcd+\frac{1}{2}b^2c^2+\frac{3}{8}d^2b^2\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(4a^2c^2+2a^2d^2+8abc\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c^2*(f*x+e)-2*a^2*d*c*cos(f*x+e)+a^2*d^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-2*a*b*c^2*cos(f*x+e)+4*a*b*c*d*(-1/2*cos(f*x+e)*sin(f*x+e
)+1/2*f*x+1/2*e)-2/3*a*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+b^2*c^2*(-1/2*cos(
f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/3*b^2*d*c*(2+sin(f*x+e)^2)*cos(f*x+e)+d^
2*b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

**Maxima** [A]
time = 0.29, size = 224, normalized size = 1.03

$$\frac{96\,(fx+e)a^2c^2+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2c^2+96\,(2\,fx+2\,e-\sin(2\,fx+2\,e))abcd+64\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)b^2cd+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2d^2+64\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)abd^2+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))b^2d^2-192\,abc^2\cos(fx+e)-192\,a^2cd\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^2*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 +
96*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*b^2*c*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^2 + 64*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*a*b*d^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*b^2*d^2 - 192*a*b*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

**Fricas** [A]
time = 0.35, size = 168, normalized size = 0.77

$$\frac{16\,(b^2cd+abd^2)\cos(fx+e)^3+3\,\left(16\,abcd+4\,(2\,a^2+b^2)c^2+(4\,a^2+3\,b^2)d^2\right)fx-48\,\left(abc^2+abd^2+(a^2+b^2)cd\right)\cos(fx+e)+3\,\left(2\,b^2d^2\cos(fx+e)^3-\left(4\,b^2c^2+16\,abcd+(4\,a^2+5\,b^2)d^2\right)\cos(fx+e)\right)\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(b^2*c*d + a*b*d^2)*cos(f*x + e)^3 + 3*(16*a*b*c*d + 4*(2*a^2 + b^
2)*c^2 + (4*a^2 + 3*b^2)*d^2)*f*x - 48*(a*b*c^2 + a*b*d^2 + (a^2 + b^2)*c*d
)*cos(f*x + e) + 3*(2*b^2*d^2*cos(f*x + e)^3 - (4*b^2*c^2 + 16*a*b*c*d + (4
*a^2 + 5*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(202) = 404$.
time = 0.26, size = 459, normalized size = 2.12

$$\begin{cases} a^2c^2x-\frac{2a^2cd\cos(e+fx)}{f}+\frac{a^2d^2x\sin^2(e+fx)}{2}+\frac{a^2d^2x\cos^2(e+fx)}{2}-\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2abc^2\cos(e+fx)}{f}+2abcdx\sin^2(e+fx)+2abcdx\cos^2(e+fx)-\frac{2abcd\sin(e+fx)\cos(e+fx)}{f}-\frac{2abd^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{4abd^2\cos^3(e+fx)}{3f}+\frac{b^2c^2x\sin^2(e+fx)}{2}+\frac{b^2c^2x\cos^2(e+fx)}{2}-\frac{b^2c^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2b^2cd\sin^2(e+fx)\cos(e+fx)}{f}-\frac{4b^2cd\cos^3(e+fx)}{3f}+\frac{3b^2d^2x\sin^4(e+fx)}{8}+\frac{3b^2d^2x\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{3b^2d^2x\cos^4(e+fx)}{8}-\frac{5b^2d^2\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{3b^2d^2\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for}\ f\neq 0 \\ x(a+b\sin(e))^2\,(c+d\sin(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x - 2*a**2*c*d*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*
x)**2/2 + a**2*d**2*x*cos(e + f*x)**2/2 - a**2*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 2*a*b*c**2*cos(e + f*x)/f + 2*a*b*c*d*x*sin(e + f*x)**2 + 2*a*b*
c*d*x*cos(e + f*x)**2 - 2*a*b*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*b*d**2*
sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*d**2*cos(e + f*x)**3/(3*f) + b**2*c*
*2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e + f*x)**2/2 - b**2*c**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - 2*b**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*b**
2*c*d*cos(e + f*x)**3/(3*f) + 3*b**2*d**2*x*sin(e + f*x)**4/8 + 3*b**2*d**2
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**2*d**2*x*cos(e + f*x)**4/8 - 5*
b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**2*d**2*sin(e + f*x)*cos
(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))**2, True
))
```

**Giac [A]**
time = 0.52, size = 176, normalized size = 0.81

$$\frac{b^2d^2\sin(4\,fx+4\,e)}{32\,f}+\frac{1}{8}\left(8\,a^2c^2+4\,b^2c^2+16\,abcd+4\,a^2d^2+3\,b^2d^2\right)x+\frac{(b^2cd+abd^2)\cos(3\,fx+3\,e)}{6\,f}-\frac{(4\,abc^2+4\,a^2cd+3\,b^2cd+3\,abd^2)\cos(fx+e)}{2\,f}-\frac{(b^2c^2+4\,abcd+a^2d^2+b^2d^2)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*a^2*c^2 + 4*b^2*c^2 + 16*a*b*c*d +
 4*a^2*d^2 + 3*b^2*d^2)*x + 1/6*(b^2*c*d + a*b*d^2)*cos(3*f*x + 3*e)/f - 1/
2*(4*a*b*c^2 + 4*a^2*c*d + 3*b^2*c*d + 3*a*b*d^2)*cos(f*x + e)/f - 1/4*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2 + b^2*d^2)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.14, size = 221, normalized size = 1.02

$$-\frac{6\,a^2\,d^2\,\sin(2\,e+2\,f\,x)+6\,b^2\,c^2\,\sin(2\,e+2\,f\,x)+6\,b^2\,d^2\,\sin(2\,e+2\,f\,x)-\frac{3\,b^2\,d^2\,\sin(4\,e+4\,f\,x)}{4}+48\,a\,b\,c^2\,\cos(e+f\,x)+36\,a\,b\,d^2\,\cos(e+f\,x)+48\,a^2\,c\,d\,\cos(e+f\,x)+36\,b^2\,c\,d\,\cos(e+f\,x)-4\,a\,b\,d^2\,\cos(3\,e+3\,f\,x)-4\,b^2\,c\,d\,\cos(3\,e+3\,f\,x)-24\,a^2\,c^2\,f\,x-12\,a^2\,d^2\,f\,x-12\,b^2\,c^2\,f\,x-9\,b^2\,d^2\,f\,x+24\,a\,b\,c\,d\,\sin(2\,e+2\,f\,x)-48\,a\,b\,c\,d\,f\,x}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -(6*a^2*d^2*sin(2*e + 2*f*x) + 6*b^2*c^2*sin(2*e + 2*f*x) + 6*b^2*d^2*sin(2
*e + 2*f*x) - (3*b^2*d^2*sin(4*e + 4*f*x))/4 + 48*a*b*c^2*cos(e + f*x) + 36
*a*b*d^2*cos(e + f*x) + 48*a^2*c*d*cos(e + f*x) + 36*b^2*c*d*cos(e + f*x) -
 4*a*b*d^2*cos(3*e + 3*f*x) - 4*b^2*c*d*cos(3*e + 3*f*x) - 24*a^2*c^2*f*x -
 12*a^2*d^2*f*x - 12*b^2*c^2*f*x - 9*b^2*d^2*f*x + 24*a*b*c*d*sin(2*e + 2*f
*x) - 48*a*b*c*d*f*x)/(24*f)
```

# 3.680 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=107

$$\frac{1}{2}\left(2a^2c+b^2c+2abd\right)x-\frac{2(3abc+a^2d+b^2d)\cos(e+fx)}{3f}-\frac{b(3bc+2ad)\cos(e+fx)\sin(e+fx)}{6f}-\frac{d\cos(e+\ldots}{}$$

```
[Out] 1/2*(2*a^2*c+2*a*b*d+b^2*c)*x-2/3*(a^2*d+3*a*b*c+b^2*d)*cos(f*x+e)/f-1/6*b*
(2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+e)/f-1/3*d*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

**Rubi** [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{2(a^2d+3abc+b^2d)\cos(e+fx)}{3f}+\frac{1}{2}x\left(2a^2c+2abd+b^2c\right)-\frac{b(2ad+3bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{d\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a^2*c + b^2*c + 2*a*b*d)*x)/2 - (2*(3*a*b*c + a^2*d + b^2*d)*Cos[e + f*
x])/(3*f) - (b*(3*b*c + 2*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx &= -\frac{d\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\ldots\\ &= \frac{1}{2}\left(2a^2c+b^2c+2abd\right)x-\frac{2(3abc+a^2d+b^2d)\cos(e+fx)}{3f}-\ldots\end{aligned}$$

**Mathematica [A]**
time = 0.34, size = 90, normalized size = 0.84

$$\frac{6(2a^2c+b^2c+2abd)(e+fx)-3(8abc+4a^2d+3b^2d)\cos(e+fx)+b^2d\cos(3(e+fx))-3b(bc+2ad)\sin(2(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (6*(2*a^2*c + b^2*c + 2*a*b*d)*(e + f*x) - 3*(8*a*b*c + 4*a^2*d + 3*b^2*d)*
Cos[e + f*x] + b^2*d*Cos[3*(e + f*x)] - 3*b*(b*c + 2*a*d)*Sin[2*(e + f*x)])
/(12*f)
```

**Maple [A]**
time = 0.14, size = 115, normalized size = 1.07

| method | result |
|---|---|
| derivativedivides | $\frac{a^2c(fx+e)-a^2d\cos(fx+e)-2abc\cos(fx+e)+2abd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| default | $\frac{a^2c(fx+e)-a^2d\cos(fx+e)-2abc\cos(fx+e)+2abd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right.}{f}$ |
| risch | $a^2cx+xabd+\frac{x\,b^2c}{2}-\frac{\cos(fx+e)a^2d}{f}-\frac{2\cos(fx+e)abc}{f}-\frac{3\cos(fx+e)b^2d}{4f}+\frac{b^2d\cos(3fx+3e)}{12f}-\frac{\sin(2fx}{2}$ |
| norman | $\frac{(a^2c+abd+\frac{1}{2}b^2c)x+(a^2c+abd+\frac{1}{2}b^2c)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^2c+3abd+\frac{3}{2}b^2c)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^2c+3abd+\frac{3}{2}b^2c)x}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(f*x+e)-a^2*d*cos(f*x+e)-2*a*b*c*cos(f*x+e)+2*a*b*d*(-1/2*cos(f*
x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+b^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/
2*e)-1/3*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e))
```

**Maxima [A]**
time = 0.29, size = 121, normalized size = 1.13

$$\frac{12\,(fx+e)a^2c+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2c+6\,(2\,fx+2\,e-\sin(2\,fx+2\,e))abd+4\,(\cos(fx+e)^3-3\,\cos(fx+e))b^2d-24\,abc\cos(fx+e)-12\,a^2d\cos(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a^2*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 6*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*b*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
^2*d - 24*a*b*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

**Fricas [A]**
time = 0.35, size = 93, normalized size = 0.87

$$\frac{2\,b^2d\cos\left(fx+e\right)^3+3\,\left(2\,abd+\left(2\,a^2+b^2\right)c\right)fx-3\,\left(b^2c+2\,abd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\,\left(2\,abc+\left(a^2+b^2\right)d\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d*cos(f*x + e)^3 + 3*(2*a*b*d + (2*a^2 + b^2)*c)*f*x - 3*(b^2*c
+ 2*a*b*d)*cos(f*x + e)*sin(f*x + e) - 6*(2*a*b*c + (a^2 + b^2)*d)*cos(f*x
+ e))/f
```

**Sympy [A]**
time = 0.15, size = 199, normalized size = 1.86

$$\begin{cases} a^2cx-\frac{a^2d\cos\left(e+fx\right)}{f}-\frac{2abc\cos\left(e+fx\right)}{f}+abdx\sin^2\left(e+fx\right)+abdx\cos^2\left(e+fx\right)-\frac{abd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{b^2cx\sin^2\left(e+fx\right)}{2}+\frac{b^2cx\cos^2\left(e+fx\right)}{2}-\frac{b^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^2d\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^2d\cos^3\left(e+fx\right)}{3f} & \text{for}\ f\neq 0\\ x\left(a+b\sin\left(e\right)\right)^2\left(c+d\sin\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d*cos(e + f*x)/f - 2*a*b*c*cos(e + f*x)/f + a*b*
d*x*sin(e + f*x)**2 + a*b*d*x*cos(e + f*x)**2 - a*b*d*sin(e + f*x)*cos(e +
f*x)/f + b**2*c*x*sin(e + f*x)**2/2 + b**2*c*x*cos(e + f*x)**2/2 - b**2*c*s
in(e + f*x)*cos(e + f*x)/(2*f) - b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e)
), True))
```

**Giac [A]**
time = 0.43, size = 96, normalized size = 0.90

$$\frac{b^2d\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\,\left(2\,a^2c+b^2c+2\,abd\right)x-\frac{\left(8\,abc+4\,a^2d+3\,b^2d\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(b^2c+2\,abd\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*b^2*d*cos(3*f*x + 3*e)/f + 1/2*(2*a^2*c + b^2*c + 2*a*b*d)*x - 1/4*(8*
a*b*c + 4*a^2*d + 3*b^2*d)*cos(f*x + e)/f - 1/4*(b^2*c + 2*a*b*d)*sin(2*f*x
 + 2*e)/f
```

**Mupad [B]**
time = 7.77, size = 108, normalized size = 1.01

$$-\frac{\frac{3\,b^2\,c\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{b^2\,d\,\cos\left(3\,e+3\,f\,x\right)}{2}+6\,a^2\,d\,\cos\left(e+f\,x\right)+\frac{9\,b^2\,d\,\cos\left(e+f\,x\right)}{2}+3\,a\,b\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a^2\,c\,f\,x-3\,b^2\,c\,f\,x+12\,a\,b\,c\,\cos\left(e+f\,x\right)-6\,a\,b\,d\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*b^2*c*sin(2*e + 2*f*x))/2 - (b^2*d*cos(3*e + 3*f*x))/2 + 6*a^2*d*cos(e
 + f*x) + (9*b^2*d*cos(e + f*x))/2 + 3*a*b*d*sin(2*e + 2*f*x) - 6*a^2*c*f*x
 - 3*b^2*c*f*x + 12*a*b*c*cos(e + f*x) - 6*a*b*d*f*x)/(6*f)
```

# 3.681 $\int (a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=50

$$\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*(2*a^2+b^2)*x-2*a*b*cos(f*x+e)/f-1/2*b^2*cos(f*x+e)*sin(f*x+e)/f
```

**Rubi** [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2723\}$

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

**Mathematica** [A]
time = 0.12, size = 46, normalized size = 0.92

$$-\frac{-2(2a^2+b^2)(e+fx)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)
])/f
```

**Maple [A]**
time = 0.09, size = 51, normalized size = 1.02

| method | result |
|---|---|
| risch | $a^2x+\frac{b^2x}{2}-\frac{2ab\cos(fx+e)}{f}-\frac{b^2\sin(2fx+2e)}{4f}$ |
| derivativedivides | $\frac{b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\cos(fx+e)+a^2(fx+e)}{f}$ |
| default | $\frac{b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\cos(fx+e)+a^2(fx+e)}{f}$ |
| norman | $\frac{\left(a^2+\frac{b^2}{2}\right)x+\frac{b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\left(a^2+\frac{b^2}{2}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2a^2+b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{4ab\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{b^2\tan(}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

**Maxima [A]**
time = 0.29, size = 49, normalized size = 0.98

$$a^2x+\frac{(2\,fx+2\,e-\sin(2\,fx+2\,e))b^2}{4\,f}-\frac{2\,ab\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

**Fricas [A]**
time = 0.35, size = 48, normalized size = 0.96

$$-\frac{b^2\cos(fx+e)\sin(fx+e)-(2\,a^2+b^2)fx+4\,ab\cos(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

**Sympy** [A]
time = 0.08, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos(e+fx)}{f} + \frac{b^2x\sin^2(e+fx)}{2} + \frac{b^2x\cos^2(e+fx)}{2} - \frac{b^2\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\sin(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

**Giac** [A]
time = 0.46, size = 45, normalized size = 0.90

$$\frac{1}{2}\left(2a^2+b^2\right)x - \frac{2ab\cos(fx+e)}{f} - \frac{b^2\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

**Mupad** [B]
time = 7.61, size = 44, normalized size = 0.88

$$-\frac{\frac{b^2\sin(2e+2fx)}{2} + 4ab\cos(e+fx) - 2a^2fx - b^2fx}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2,x)
```

```
[Out] -((b^2*sin(2*e + 2*f*x))/2 + 4*a*b*cos(e + f*x) - 2*a^2*f*x - b^2*f*x)/(2*f
)
```

# 3.682 $\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=93

$$-\frac{b(bc-2ad)x}{d^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{b^2\cos(e+fx)}{df}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2-b^2*cos(f*x+e)/d/f+2*(-a*d+b*c)^2*arctan((d+c*tan(1/2
*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2825, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)^2\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)/2
])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (b^2*Cos[e + f*x])/(d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx &= -\frac{b^2\cos(e+fx)}{df}+\frac{\int \frac{a^2d-b(bc-2ad)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}\\
&= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}+\frac{(bc-ad)^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2}\\
&= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}+\frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\tfrac{1}{2}\right.\right.}{d^2f}\\
&= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}-\frac{(4(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d\right.}{d^2f}\\
&= -\frac{b(bc-2ad)x}{d^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{b^2\cos(e+fx)}{df}
\end{aligned}$$

**Mathematica [A]**
time = 0.25, size = 89, normalized size = 0.96

$$-\frac{b(bc-2ad)(e+fx)-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b^2d\cos(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*(e + f*x) - (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)
/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + b^2*d*Cos[e + f*x])/(d^2*f))
```

**Maple [A]**
time = 0.21, size = 119, normalized size = 1.28

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b\left(-\frac{bd}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2ad-bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^{2}}+\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{d^{2}\sqrt{c^{2}-d^{2}}}}{f}$ |
| default | $\frac{\frac{2b\left(-\frac{bd}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2ad-bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^{2}}+\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{d^{2}\sqrt{c^{2}-d^{2}}}}{f}$ |
| risch | $\frac{2bxa}{d}-\frac{b^{2}xc}{d^{2}}-\frac{b^{2}e^{i(fx+e)}}{2df}-\frac{b^{2}e^{-i(fx+e)}}{2df}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^{2}+d^{2}}\,c-c^{2}+d^{2}}{\sqrt{-c^{2}+d^{2}}\,d}\right)a^{2}}{\sqrt{-c^{2}+d^{2}}\,f}+\frac{2\ln\left(e^{i(fx+e)}+\frac{i\sqrt{}}{}\right.}{\sqrt{-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b/d^2*(-b*d/(1+tan(1/2*f*x+1/2*e)^2)+(2*a*d-b*c)*arctan(tan(1/2*f*x+
1/2*e)))+2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 386, normalized size = 4.15

$$\left[-\frac{2\left(b^{2}c^{3}-2abc^{2}d-b^{2}cd^{2}+2abd^{3}\right)fx+\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\sqrt{-c^{2}+d^{2}}\log\left(\frac{(2c^{2}-d^{2})\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^{2}+d^{2}}}{d^{2}\cos(fx+e)^{2}-2cd\sin(fx+e)-c^{2}-d^{2}}\right)+2\left(b^{2}c^{2}d-b^{2}d^{3}\right)\cos(fx+e)}{2\left(c^{2}d^{2}-d^{4}\right)f},\ -\frac{\left(b^{2}c^{3}-2abc^{2}d-b^{2}cd^{2}+2abd^{3}\right)fx+\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\sqrt{c^{2}-d^{2}}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^{2}-d^{2}}\cos(fx+e)}\right)+\left(b^{2}c^{2}d-b^{2}d^{3}\right)\cos(fx+e)}{\left(c^{2}d^{2}-d^{4}\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c^3 - 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
```

```
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + 2*(b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f), -((b^2*c^3
- 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e
))) + (b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 4032 vs. $2(78) = 156$.
time = 178.53, size = 4032, normalized size = 43.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*sin(e))**2/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (2*a**2*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**
3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2
)) + 2*a**2*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x
/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*a*b*d**2*f
*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2
) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*a*b*d**2*f*x
*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f
*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 4*a*b*d**2*tan(e/2
+ f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2
)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 4*a*b*d**2/(d**3*f*tan(e/
2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)*
*2 - f*(d**2)**(3/2)) - 2*a*b*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*
tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f
*x/2)**2 - f*(d**2)**(3/2)) - 2*a*b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/
2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(
d**2)**(3/2)) - b**2*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)*
*3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) - b**2*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f
*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - 2*b**2*d**
2*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) -
f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + b**2*d*f*x*sqrt(d*
*2)*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/
2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + b**2*d*f*x*sq
rt(d**2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*
x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*b**2*d*sq
rt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 +
 f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 4*b**2*d
*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2
)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)), Eq(c, -sqrt(d**2))), (-2*a
```

```
**2*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*t
an(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) -
2*a**2*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) +
 f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*a*b*d**2*f*x*ta
n(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f
*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*a*b*d**2*f*x*tan(
e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**
2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 4*a*b*d**2*tan(e/2 + f*x
/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3
/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 4*a*b*d**2/(d**3*f*tan(e/2 + f
*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 +
f*(d**2)**(3/2)) + 2*a*b*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e
/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)
**2 + f*(d**2)**(3/2)) + 2*a*b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3
 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)
**(3/2)) - b**2*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(
3/2)) - b**2*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2)
 + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 2*b**2*d**2*tan
(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d*
*2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - b**2*d*f*x*sqrt(d**2)*t
an(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) +
f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - b**2*d*f*x*sqrt(d*
*2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2)
+ f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 2*b**2*d*sqrt(d*
*2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/
2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 4*b**2*d*sqrt
(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3
/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)), Eq(c, sqrt(d**2))), ((a**2*x -
2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*x*cos(e + f*x)**2/2
- b**2*sin(e + f*x)*cos(e + f*x)/(2*f))/c, Eq(d, 0)), (x*(a + b*sin(e))**2/
(c + d*sin(e)), Eq(f, 0)), ((a**2*log(tan(e/2 + f*x/2))*tan(e/2 + f*x/2)**2
/(f*tan(e/2 + f*x/2)**2 + f) + a**2*log(tan(e/2 + f*x/2))/(f*tan(e/2 + f*x/
2)**2 + f) + 2*a*b*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + 2*
a*b*f*x/(f*tan(e/2 + f*x/2)**2 + f) - 2*b**2/(f*tan(e/2 + f*x/2)**2 + f))/d
, Eq(c, 0)), (a**2*d**2*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)*
tan(e/2 + f*x/2)**2/(d**2*f*sqrt(-c**2 + d**2)*...
```

**Giac** [A]

time = 0.56, size = 134, normalized size = 1.44

$$-\frac{\frac{(b^2c-2\,abd)(fx+e)}{d^2}+\frac{2\,b^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((b^2*c - 2*a*b*d)*(f*x + e)/d^2 + 2*b^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

**Mupad [B]**
time = 12.43, size = 2629, normalized size = 28.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b*atan((64*b^6*c^4*tan(e/2 + (f*x)/2))/(64*b^6*c^4 + 128*a^2*b^4*c^4 - 5
12*a^3*b^3*c*d^3 - 512*a^3*b^3*c^3*d + 768*a^2*b^4*c^2*d^2 + 576*a^4*b^2*c^
2*d^2 - 384*a*b^5*c^3*d - 128*a^5*b*c*d^3) + (384*a*b^5*c^3*tan(e/2 + (f*x)
/2))/(384*a*b^5*c^3 + 512*a^3*b^3*c^3 - (64*b^6*c^4)/d - 768*a^2*b^4*c^2*d
+ 512*a^3*b^3*c*d^2 - 576*a^4*b^2*c^2*d - (128*a^2*b^4*c^4)/d + 128*a^5*b*c
*d^2) + (768*a^2*b^4*c^2*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2
*c^2 + (64*b^6*c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*
c^3)/d + (128*a^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d) + (576*a^4*b^2*c^2*tan(e/
2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2*c^2 + (64*b^6*c^4)/d^2 - (384*
a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*c^3)/d + (128*a^2*b^4*c^4)/d^2
- 512*a^3*b^3*c*d) + (512*a^3*b^3*c^3*tan(e/2 + (f*x)/2))/(384*a*b^5*c^3 +
512*a^3*b^3*c^3 - (64*b^6*c^4)/d - 768*a^2*b^4*c^2*d + 512*a^3*b^3*c*d^2 -
576*a^4*b^2*c^2*d - (128*a^2*b^4*c^4)/d + 128*a^5*b*c*d^2) + (128*a^2*b^4*c
^4*tan(e/2 + (f*x)/2))/(64*b^6*c^4 + 128*a^2*b^4*c^4 - 512*a^3*b^3*c*d^3 -
512*a^3*b^3*c^3*d + 768*a^2*b^4*c^2*d^2 + 576*a^4*b^2*c^2*d^2 - 384*a*b^5*c
^3*d - 128*a^5*b*c*d^3) - (128*a^5*b*c*d*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c
^2 + 576*a^4*b^2*c^2 + (64*b^6*c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d
 - (512*a^3*b^3*c^3)/d + (128*a^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d) - (512*a^
3*b^3*c*d*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2*c^2 + (64*b^6*
c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*c^3)/d + (128*a
^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d))*(2*a*d - b*c))/(d^2*f) - (2*b^2)/(d*f*(
tan(e/2 + (f*x)/2)^2 + 1)) - (atan((((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2
*((32*(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 - (32*tan(e/2
+ (f*x)/2)*(a^4*c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a
*b^3*c^4*d^2 - 8*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3
 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4
- 2*a*b*c*d^5))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 -
 4*a*b*c^2*d^5))/d^3 + ((32*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c
^3*d^5))/d^3)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^
4 - c^2*d^2))*1i)/(d^4 - c^2*d^2) - ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2
*((32*tan(e/2 + (f*x)/2)*(a^4*c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3
```

*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2
*c^3*d^3))/d^3 - (32*(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2
 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4
 - 2*a*b*c*d^5))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 -
 4*a*b*c^2*d^5))/d^3 - ((32*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c
^3*d^5))/d^3)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^
4 - c^2*d^2))*1i)/(d^4 - c^2*d^2))/((64*tan(e/2 + (f*x)/2)*(2*b^6*c^5 + 8*a
^4*b^2*c*d^4 + 26*a^2*b^4*c^3*d^2 - 24*a^3*b^3*c^2*d^3 - 12*a*b^5*c^4*d))/d
^3 - (64*(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 5*a^4*b^2*c^2*d^2 - 2*a^5*b*c*d^3
))/d^2 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 - (32*tan(e/2 + (f*x)/2)*(a^4*c*d^5 + 2*b
^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8*a^2*b^2*c*
d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2
)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5))/d^2 + (32*t
an(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 - 4*a*b*c^2*d^5))/d^3 + ((32
*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(-(c + d)*(c
 - d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)))/(d^4 - c^2*d
^2) + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*tan(e/2 + (f*x)/2)*(a^4*
c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8
*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3 - (32*(b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 + ((-(c + d)*(c - d))^(1/2)*(
a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5))/d^2 + (32*tan(
e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 - 4*a*b*c^2*d^5))/d^3 - ((32*c^
2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(-(c + d)*(c - d
))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)
))*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*2i)/(f*(d^4 - c^2*d^2))

# 3.683 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=129

$$\frac{b^2x}{d^2}-\frac{2(bc-ad)\left(acd+b(c^2-2d^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

```
[Out] b^2*x/d^2-2*(-a*d+b*c)*(a*c*d+b*(c^2-2*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e)
)/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2
)/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2869, 2814, 2739, 632, 210}

$$-\frac{2(bc-ad)\left(acd+b(c^2-2d^2)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*x)/d^2 - (2*(b*c - a*d)*(a*c*d + b*(c^2 - 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(d*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{d\left(\left(a^2+b^2\right)c-2abd\right)+b^2\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c(c^2 - \cdots\right.}{d^2\left(c^2-d^2\right)}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(2\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c(c^2 \cdots\right.\right.}{d \cdots}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(4\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c(c^2 \cdots\right.\right.}{\cdots}$$

$$= \frac{b^2x}{d^2} - \frac{2(bc-ad)\left(bc^2+acd-2bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f} + \frac{(bc-ad)^2 \cdots}{d\left(c^2-d^2\right)f(c \cdots}$$

**Mathematica [A]**
time = 0.65, size = 134, normalized size = 1.04

$$\frac{b^2(e+fx) - \frac{2\left(-a^2cd^2+2abd^3+b^2\left(c^3-2cd^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d(bc-ad)^2\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*(e + f*x) - (2*(-(a^2*c*d^2) + 2*a*b*d^3 + b^2*(c^3 - 2*c*d^2))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (d*(b*c - a
*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])))/(d^2*f)
```

**Maple [A]**
time = 0.31, size = 218, normalized size = 1.69

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\frac{2\left(\frac{d^2\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{c^2-d^2}\right)}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{2\left(a^2c\,d^2-2ab\,d^3-b^2c^3+2b^2c\,d^2\right)\mathrm{ar}}{\left(c^2-d^2\right)}}{d^2}}{f}$ |
| default | $\frac{\frac{2b^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\frac{2\left(\frac{d^2\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{c^2-d^2}\right)}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{2\left(a^2c\,d^2-2ab\,d^3-b^2c^3+2b^2c\,d^2\right)\mathrm{ar}}{\left(c^2-d^2\right)}}{d^2}}{f}$ |
| risch | $\frac{b^2x}{d^2}-\frac{2i\left(a^2d^2-2abcd+b^2c^2\right)\left(id+c\,\mathrm{e}^{i(fx+e)}\right)}{d^2\left(c^2-d^2\right)f\left(-i\mathrm{e}^{2i(fx+e)}d+id+2c\,\mathrm{e}^{i(fx+e)}\right)}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)a^2c}{\sqrt{-c^2+d^2}\,(c+d)(c-d)f}+\frac{2d\ln\left(\mathrm{e}^{i(fx+}\right.}{\sqrt{-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))+2/d^2*((d^2*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2-d^
2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(a^2*c*d^2-2*a*b*d^3-
b^2*c^3+2*b^2*c*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(127) = 254$.

time = 0.40, size = 694, normalized size = 5.38

$$\left[\frac{2\,(b^2c^4d-2\,b^2c^2d^3+b^2d^5)fx\sin(fx+e)+2\,(b^2c^5-2\,b^2c^3d^2+b^2cd^4)fx-(b^2c^4+2\,abcd^3-(a^2+2\,b^2)c^2d^2+(b^2c^3d+2\,abd^4-(a^2+2\,b^2)cd^3)\sin(fx+e))\sqrt{-c^2+d^2}\log\left(-\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2-2\,(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)+2\,(b^2c^4d-2\,abc^3d^2+2\,abcd^4-a^2d^5+(a^2-b^2)c^2d^3)\cos(fx+e)}{2\,((c^4d^3-2\,c^2d^5+d^7)f\sin(fx+e)+(c^5d^2-2\,c^3d^4+cd^6)f)}, \frac{(b^2c^4d-2\,b^2c^2d^3+b^2d^5)fx\sin(fx+e)+(b^2c^5-2\,b^2c^3d^2+b^2cd^4)fx+(b^2c^4+2\,abcd^3-(a^2+2\,b^2)c^2d^2+(b^2c^3d+2\,abd^4-(a^2+2\,b^2)cd^3)\sin(fx+e))\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+(b^2c^4d-2\,abc^3d^2+2\,abcd^4-a^2d^5+(a^2-b^2)c^2d^3)\cos(fx+e)}{(c^4d^3-2\,c^2d^5+d^7)f\sin(fx+e)+(c^5d^2-2\,c^3d^4+cd^6)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + 2*(b^2*c^5
 - 2*b^2*c^3*d^2 + b^2*c*d^4)*f*x - (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*
c^2*d^2 + (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(
-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^2*c^4*d - 2*a*
b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x + e))/((c^
4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*d^6)*f),
 ((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + (b^2*c^5 - 2*b^2
*c^3*d^2 + b^2*c*d^4)*f*x + (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*c^2*d^2
+ (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^
2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b^2*c^4*
d - 2*a*b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x +
e))/((c^4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*
d^6)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.50, size = 249, normalized size = 1.93

$$\frac{\frac{(fx+e)b^2}{d^2}-\frac{2\,(b^2c^3-a^2cd^2-2\,b^2cd^2+2\,abd^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^2-d^4)\sqrt{c^2-d^2}}+\frac{2\,(b^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,abcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b^2c^3-2\,abc^2d+a^2cd^2)}{(c^3d-cd^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^2/d^2 - 2*(b^2*c^3 - a^2*c*d^2 - 2*b^2*c*d^2 + 2*a*b*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c^2*d^2 - d^4)*sqrt(c^2 - d^2)) + 2*(b^2*c^2*d*tan(1/2*
```

```
f*x + 1/2*e) - 2*a*b*c*d^2*tan(1/2*f*x + 1/2*e) + a^2*d^3*tan(1/2*f*x + 1/2
*e) + b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/((c^3*d - c*d^3)*(c*tan(1/2*f*x +
1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

**Mupad [B]**
time = 15.47, size = 2500, normalized size = 19.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2
)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (
f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) - (2*b^2*atan(((b^2*((b^2*((32*(b^2*c*d^
8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5
))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*
a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 +
 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) - (b^2*((32*(c^2*d^9 - 2*c^4*d
^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^
11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d
^2)*1i)/d^2 - (32*(b^4*c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d
^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3
*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 +
4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))
/(d^7 - 2*c^2*d^5 + c^4*d^3)))/d^2 - (b^2*((32*(b^4*c^6*d + b^4*c^2*d^5 - 2
*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^2*c*d^8 + a^2*c^3
*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2
*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6
 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*
d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (b^2*((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^
5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*
d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2)*1i)/d^2
 - (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*
c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d
^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2
*d^5 + c^4*d^3)))/d^2)/((b^2*((b^2*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*
d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*
d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^
8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*
c^2*d^5 + c^4*d^3) - (b^2*((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^
2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7
 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2)*1i)/d^2 - (32*(b^4*c^6
*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/
2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b
```

^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2 - (64*(2*b^6*c^3*d^2 - a^2*b^4*c^5 - b^6*c^5 - 6*a*b^5*c^2*d^3 + 4*a^2*b^4*c*d^4 + 3*a^2*b^4*c^3*d^2 - 4*a^3*b^3*c^2*d^3 + a^4*b^2*c^3*d^2 + 2*a*b^5*c^4*d))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^4*c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (b^2*((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2)*1i)/d^2 - (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2 + (64*tan(e/2 + (f*x)/2)*(2*b^6*c^6 + 4*b^6*c^2*d^4 - 6*b^6*c^4*d^2 + 4*a*b^5*c^3*d^3 + 2*a^2*b^4*c^2*d^4 - 2*a^2*b^4*c^4*d^2 - 4*a*b^5*c*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))))/(d^2*f) + (atan((((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^4*c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) - (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b*d^2 + a*c*d)*1i)/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2) - ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)...

# 3.684 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=196

$$-\frac{\left(6abcd-a^2(2c^2+d^2)-b^2(c^2+2d^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}\,f}+\frac{(bc-ad)^2\cos(e+fx)}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2}-\frac{(bc-a\cdots}{2d\cdots}$$

```
[Out] -(6*a*b*c*d-a^2*(2*c^2+d^2)-b^2*(c^2+2*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e)
)/(c^2-d^2)^(1/2))/(c^2-d^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2
)/f/(c+d*sin(f*x+e))^2-1/2*(-a*d+b*c)*(3*a*c*d+b*(c^2-4*d^2))*cos(f*x+e)/d/
(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.22, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2869, 2833, 12, 2739, 632, 210}

$$-\frac{\left(-\left(a^2(2c^2+d^2)\right)+6abcd-b^2(c^2+2d^2)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\,(c^2-d^2)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2df\,(c^2-d^2)\,(c+d\sin(e+fx))^2}-\frac{(3acd+b(c^2-4d^2))\,(bc-ad)\cos(e+fx)}{2df\,(c^2-d^2)^2\,(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(3*a*c*d
 + b*(c^2 - 4*d^2))*Cos[e + f*x])/(2*d*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2d(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{\int \frac{2d((a^2+b^2)c-2abd)+(b^2c^2+2abcd-(a^2+2b^2)d^2)\sin(e}{(c+d\sin(e+fx))^2}}{2d(c^2-d^2)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)(3acd+b(c^2-4d^2))\cos(e+fx)}{2d(c^2-d^2)^2 f(c+d\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)(3acd+b(c^2-4d^2))\cos(e+fx)}{2d(c^2-d^2)^2 f(c+d\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)(3acd+b(c^2-4d^2))\cos(e+fx)}{2d(c^2-d^2)^2 f(c+d\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)(3acd+b(c^2-4d^2))\cos(e+fx)}{2d(c^2-d^2)^2 f(c+d\sin(e+fx))}$$

$$= -\frac{(6abcd-a^2(2c^2+d^2)-b^2(c^2+2d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}f} + \frac{(bc-a}{2d(c^2-d^2)}$$

**Mathematica** [**A**]
time = 1.00, size = 202, normalized size = 1.03

$$\frac{\frac{2(-6abcd+a^2(2c^2+d^2)+b^2(c^2+2d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} + \frac{(bc-ad)^2\cos(e+fx)}{(c-d)d(c+d)(c+d\sin(e+fx))^2} - \frac{(-3a^2cd^2+2abd(c^2+2d^2)+b^2(c^3-4cd^2))\cos(e+fx)}{(c-d)^2d(c+d)^2(c+d\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((c - d)*d*(c + d)*(c + d*Sin[e + f*x])^2) - ((-3*a^2*c*d^2 + 2*a*b*d
*(c^2 + 2*d^2) + b^2*(c^3 - 4*c*d^2))*Cos[e + f*x])/((c - d)^2*d*(c + d)^2*
(c + d*Sin[e + f*x])))/(2*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(187) = 374$.
time = 0.51, size = 467, normalized size = 2.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*(5*a^2*c^2*d^2-2*a^2*d^4-6*a*b*c^3*d+b^2*c^4+2*b^2*c^2*d^2)/(c^
4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3+1/2*(4*a^2*c^4*d+7*a^2*c^2*d^3-2*a^
```

```
2*d^5-4*a*b*c^5-10*a*b*c^3*d^2-4*a*b*c*d^4+3*b^2*c^4*d+6*b^2*c^2*d^3)/(c^4-
2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*(11*a^2*c^2*d^2-2*a^2*d^4-10*a*
b*c^3*d-8*a*b*c*d^3-b^2*c^4+10*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f
*x+1/2*e)+1/2*(4*a^2*c^2*d-a^2*d^3-4*a*b*c^3-2*a*b*c*d^2+3*b^2*c^2*d)/(c^4-
2*c^2*d^2+d^4))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+(2*a^2*
c^2+a^2*d^2-6*a*b*c*d+b^2*c^2+2*b^2*d^2)/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(192) = 384$.
time = 0.41, size = 1050, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^5 + 2*a*b*c^4*d + 2*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2
)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d
 + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*
d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e
)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 + b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a*b
*c^5 - 2*a*b*c^3*d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a
^2 + 3*b^2)*c^2*d^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e
) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b^2*c^5 + 2*a*b*c^4*d + 2
*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)
*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4
 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)
*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 +
```

```
b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*
sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (4*a*b*c^5 - 2*a*b*c^3*
d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a^2 + 3*b^2)*c^2*d
^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2
 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*
d^2 + 2*c^2*d^6 - d^8)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(192) = 384.
time = 0.49, size = 609, normalized size = 3.11

$$\frac{\frac{(2a^2c^2+b^2c^2-6abcd+a^2d^2+2b^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^4-2c^2d^2+d^4)\sqrt{c^2-d^2}}+\frac{b^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6abc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2b^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4abc^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4a^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3b^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-10abc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6b^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4abcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-10abc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+11a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+10b^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-8abc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4abc^5+4a^2c^4d+3b^2c^4d-2abc^3d^2-a^2c^2d^3}{(c^6-2c^4d^2+c^2d^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2
)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(c^2 - d^2)) + (b^2*c^5*tan(1/2*f*x + 1/2*
e)^3 - 6*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 5*a^2*c^3*d^2*tan(1/2*f*x + 1/2
*e)^3 + 2*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^4*tan(1/2*f*x + 1/
2*e)^3 - 4*a*b*c^5*tan(1/2*f*x + 1/2*e)^2 + 4*a^2*c^4*d*tan(1/2*f*x + 1/2*e
)^2 + 3*b^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b*c^3*d^2*tan(1/2*f*x + 1/2
*e)^2 + 7*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 6*b^2*c^2*d^3*tan(1/2*f*x +
1/2*e)^2 - 4*a*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^5*tan(1/2*f*x + 1/2
*e)^2 - b^2*c^5*tan(1/2*f*x + 1/2*e) - 10*a*b*c^4*d*tan(1/2*f*x + 1/2*e) +
11*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 10*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e) -
 8*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a^2*c*d^4*tan(1/2*f*x + 1/2*e) - 4*
a*b*c^5 + 4*a^2*c^4*d + 3*b^2*c^4*d - 2*a*b*c^3*d^2 - a^2*c^2*d^3)/((c^6 -
2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

**Mupad** [**B**]
time = 10.26, size = 641, normalized size = 3.27

$$\frac{\mathrm{atan}\left(\frac{\left(\frac{(2c^4d-4c^2d^3+2d^5)(2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2)}{2(c+d)^{5/2}(c-d)^{5/2}(c^4-2c^2d^2+d^4)}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2)}{(c+d)^{5/2}(c-d)^{5/2}}\right)(c^4-2c^2d^2+d^4)}{2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2}\right)(2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2)}{f(c+d)^{5/2}(c-d)^{5/2}}-\frac{\frac{-4a^2c^2d+a^2d^3+4abc^3+2abcd^2-3b^2c^2d}{c^4-2c^2d^2+d^4}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(-11a^2c^2d^2+2a^2d^4+10abc^3d+8abcd^3+b^2c^4-10b^2c^2d^2)}{c(c^4-2c^2d^2+d^4)}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(5a^2c^2d^2-2a^2d^4-6abc^3d+b^2c^4+2b^2c^2d^2)}{c(c^4-2c^2d^2+d^4)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(c^2+2d^2)(-4a^2c^2d+a^2d^3+4abc^3+2abcd^2-3b^2c^2d)}{c^2(c^4-2c^2d^2+d^4)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(2c^2+4d^2)+c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+c^2+4cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*c^4*d + 2*d^5 - 4*c^2*d^3)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b
^2*d^2 - 6*a*b*c*d))/(2*(c + d)^(5/2)*(c - d)^(5/2)*(c^4 + d^4 - 2*c^2*d^2)
) + (c*tan(e/2 + (f*x)/2)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*
b*c*d))/((c + d)^(5/2)*(c - d)^(5/2)))*(c^4 + d^4 - 2*c^2*d^2))/(2*a^2*c^2
+ a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*b*c*d))*(2*a^2*c^2 + a^2*d^2 + b^2*c^
2 + 2*b^2*d^2 - 6*a*b*c*d))/(f*(c + d)^(5/2)*(c - d)^(5/2)) - ((a^2*d^3 - 4
*a^2*c^2*d - 3*b^2*c^2*d + 4*a*b*c^3 + 2*a*b*c*d^2)/(c^4 + d^4 - 2*c^2*d^2)
 + (tan(e/2 + (f*x)/2)*(2*a^2*d^4 + b^2*c^4 - 11*a^2*c^2*d^2 - 10*b^2*c^2*d
^2 + 8*a*b*c*d^3 + 10*a*b*c^3*d))/(c*(c^4 + d^4 - 2*c^2*d^2)) - (tan(e/2 +
(f*x)/2)^3*(b^2*c^4 - 2*a^2*d^4 + 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 - 6*a*b*c^3
*d))/(c*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(a^2
*d^3 - 4*a^2*c^2*d - 3*b^2*c^2*d + 4*a*b*c^3 + 2*a*b*c*d^2))/(c^2*(c^4 + d^
4 - 2*c^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (
f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

# 3.685 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=305

$$-\frac{(2abd(4c^2+d^2)-b^2c(c^2+4d^2)-a^2(2c^3+3cd^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{7/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{3d(c^2-d^2)f(c+d\sin(e+fx}$$

```
[Out] -(2*a*b*d*(4*c^2+d^2)-b^2*c*(c^2+4*d^2)-a^2*(2*c^3+3*c*d^2))*arctan((d+c*ta
n(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(7/2)/f+1/3*(-a*d+b*c)^2*cos(f
*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^3-1/6*(-a*d+b*c)*(5*a*c*d+b*(c^2-6*d^2
))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^2+1/6*(a^2*d^2*(11*c^2+4*d^2
)-a*b*(4*c^3*d+26*c*d^3)-b^2*(c^4-10*c^2*d^2-6*d^4))*cos(f*x+e)/d/(c^2-d^2)
^3/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.41, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2869, 2833, 12, 2739, 632, 210}

$$-\frac{(-(a^2(2c^3+3cd^2))+2abd(4c^2+d^2)-b^2c(c^2+4d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\,(c^2-d^2)^{7/2}}+\frac{(a^2d^2(11c^2+4d^2)-ab(4c^3d+26cd^3)-(b^2(c^4-10c^2d^2-6d^4)))\cos(e+fx)}{6df\,(c^2-d^2)^3\,(c+d\sin(e+fx))}+\frac{(bc-ad)^2\cos(e+fx)}{3df\,(c^2-d^2)\,(c+d\sin(e+fx))^3}-\frac{(5acd+b(c^2-6d^2))\,(bc-ad)\cos(e+fx)}{6df\,(c^2-d^2)^2\,(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((2*a*b*d*(4*c^2 + d^2) - b^2*c*(c^2 + 4*d^2) - a^2*(2*c^3 + 3*c*d^2))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((
b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^3) - ((b
*c - a*d)*(5*a*c*d + b*(c^2 - 6*d^2))*Cos[e + f*x])/(6*d*(c^2 - d^2)^2*f*(c
 + d*Sin[e + f*x])^2) + ((a^2*d^2*(11*c^2 + 4*d^2) - a*b*(4*c^3*d + 26*c*d^
3) - b^2*(c^4 - 10*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(6*d*(c^2 - d^2)^3*f*(c
+ d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{\int \frac{3d\left(\left(a^2+b^2\right)c-2abd\right)+\left(4abcd-2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sin(}{(c+d\sin(e+fx))^3}}{3d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b(c^2-6d^2)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b(c^2-6d^2)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b(c^2-6d^2)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b(c^2-6d^2)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b(c^2-6d^2)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2}$$

$$= -\frac{\left(2abd(4c^2+d^2)-b^2c(c^2+4d^2)-a^2(2c^3+3cd^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}f} +$$

**Mathematica** [**A**]
time = 1.49, size = 346, normalized size = 1.13

$$\frac{\frac{12\left(-2abd\left(4c^2+d^2\right)+b^2c\left(c^2+4d^2\right)+a^2\left(2c^3+3cd^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}} + \frac{\cos(e+fx)\left(-24abc^5+36a^2c^4d+25b^2c^4d-44abc^3d^2+a^2c^2d^3+14b^2c^2d^3-22abcd^4+8a^2d^5+6b^2d^5+d\left(-a^2d^2\left(11c^2+4d^2\right)+ab\left(4c^3d+26cd^3\right)+b^2\left(c^4-10c^2d^2-6d^4\right)\right)\cos(2(e+fx))-6\left(-a^2cd^2\left(9c^2+d^2\right)-2abd\left(-2c^4-9c^2d^2+d^4\right)+b^2\left(c^5-9c^3d^2-2cd^4\right)\right)\sin(e+fx)\right)}{\left(c^2-d^2\right)^3(c+d\sin(e+fx))^3}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((12*(-2*a*b*d*(4*c^2 + d^2) + b^2*c*(c^2 + 4*d^2) + a^2*(2*c^3 + 3*c*d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(7/2) + (Cos
[e + f*x]*(-24*a*b*c^5 + 36*a^2*c^4*d + 25*b^2*c^4*d - 44*a*b*c^3*d^2 + a^2
*c^2*d^3 + 14*b^2*c^2*d^3 - 22*a*b*c*d^4 + 8*a^2*d^5 + 6*b^2*d^5 + d*(-(a^2
*d^2*(11*c^2 + 4*d^2)) + a*b*(4*c^3*d + 26*c*d^3) + b^2*(c^4 - 10*c^2*d^2 -
 6*d^4))*Cos[2*(e + f*x)] - 6*(-(a^2*c*d^2*(9*c^2 + d^2)) - 2*a*b*d*(-2*c^4
 - 9*c^2*d^2 + d^4) + b^2*(c^5 - 9*c^3*d^2 - 2*c*d^4))*Sin[e + f*x]))/((c^2
 - d^2)^3*(c + d*Sin[e + f*x])^3))/(12*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(294) = 588.
time = 0.91, size = 933, normalized size = 3.06

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\left(9a^2c^4d^2-6a^2c^2d^4+2a^2d^6-8abc^5d-2abc^3d^3+b^2c^6+4b^2c^4d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c\left(c^6-3c^4d^2+3c^2d^4-d^6\right)}+\frac{\left(6a^2c^6d+27a^2c^4d^3-12a^2c^2d^5+4a^2d^7-4abc^7\right.}{\left(c^6\right.}}{}$ |
| default | $\frac{\frac{\left(9a^2c^4d^2-6a^2c^2d^4+2a^2d^6-8abc^5d-2abc^3d^3+b^2c^6+4b^2c^4d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c\left(c^6-3c^4d^2+3c^2d^4-d^6\right)}+\frac{\left(6a^2c^6d+27a^2c^4d^3-12a^2c^2d^5+4a^2d^7-4abc^7\right.}{\left(c^6\right.}}{}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*(9*a^2*c^4*d^2-6*a^2*c^2*d^4+2*a^2*d^6-8*a*b*c^5*d-2*a*b*c^3*d^
3+b^2*c^6+4*b^2*c^4*d^2)/c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)
^5+1/2*(6*a^2*c^6*d+27*a^2*c^4*d^3-12*a^2*c^2*d^5+4*a^2*d^7-4*a*b*c^7-28*a*
b*c^5*d^2-22*a*b*c^3*d^4+4*a*b*c*d^6+5*b^2*c^6*d+20*b^2*c^4*d^3)/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)/c^2*tan(1/2*f*x+1/2*e)^4+1/3/c^3*d*(54*a^2*c^6*d+21*a^2
*c^4*d^3-4*a^2*c^2*d^5+4*a^2*d^7-36*a*b*c^7-84*a*b*c^5*d^2-34*a*b*c^3*d^4+4
*a*b*c*d^6+39*b^2*c^6*d+32*b^2*c^4*d^3+4*b^2*c^2*d^5)/(c^6-3*c^4*d^2+3*c^2*
d^4-d^6)*tan(1/2*f*x+1/2*e)^3+1/c^2*(6*a^2*c^6*d+20*a^2*c^4*d^3-3*a^2*c^2*d
^5+2*a^2*d^7-4*a*b*c^7-20*a*b*c^5*d^2-28*a*b*c^3*d^4+2*a*b*c*d^6+4*b^2*c^6*
d+17*b^2*c^4*d^3+4*b^2*c^2*d^5)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1
/2*e)^2+1/2*(27*a^2*c^4*d^2-4*a^2*c^2*d^4+2*a^2*d^6-16*a*b*c^5*d-38*a*b*c^3
*d^3+4*a*b*c*d^5-b^2*c^6+22*b^2*c^4*d^2+4*b^2*c^2*d^4)/c/(c^6-3*c^4*d^2+3*c
^2*d^4-d^6)*tan(1/2*f*x+1/2*e)+1/6*(18*a^2*c^4*d-5*a^2*c^2*d^3+2*a^2*d^5-12
*a*b*c^5-20*a*b*c^3*d^2+2*a*b*c*d^4+13*b^2*c^4*d+2*b^2*c^2*d^3)/(c^6-3*c^4*
d^2+3*c^2*d^4-d^6))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^3+(2*
a^2*c^3+3*a^2*c*d^2-8*a*b*c^2*d-2*a*b*d^3+b^2*c^3+4*b^2*c*d^2)/(c^6-3*c^4*d
^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(298) = 596$.
time = 0.44, size = 1753, normalized size = 5.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 11*(
a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*cos(f
*x + e)^3 - 6*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 + 2*
a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*b^2
)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6*a*
b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2)*c
^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2 +
4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*b*d
^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*c*
d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*
c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) - 12*(2*a*b*c^7 + 2*a*b*c^5*d^2 + 2*a^2*c^4*d^3 + b^2*c
^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 + b^2)*d^7)*cos(f*x + e
))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)
^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f +
((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3
*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x
+ e)), -1/6*((b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 1
1*(a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*co
s(f*x + e)^3 - 3*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 +
 2*a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*
b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6
*a*b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2
)*c^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2
 + 4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*
b*d^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)
*c*d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^
2)*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
+ e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a*b*c^7 + 2*a*b*c^5*d^2 +
2*a^2*c^4*d^3 + b^2*c^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 +
b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c
```

```
*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3
*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*
f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^
9 + d^11)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(298) = 596$.
time = 0.58, size = 1316, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^2*c^3 + b^2*c^3 - 8*a*b*c^2*d + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a
*b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x +
1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2
 - d^2)) + (3*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 24*a*b*c^7*d*tan(1/2*f*x + 1
/2*e)^5 + 27*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 12*b^2*c^6*d^2*tan(1/2*f*
x + 1/2*e)^5 - 6*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^4*d^4*tan(1/
2*f*x + 1/2*e)^5 + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^8*tan(1/
2*f*x + 1/2*e)^4 + 18*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 + 15*b^2*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 84*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 81*a^2*c^5*d^3*
tan(1/2*f*x + 1/2*e)^4 + 60*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 - 66*a*b*c^4
*d^4*tan(1/2*f*x + 1/2*e)^4 - 36*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 12*a*
b*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 + 12*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 72
*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
+ 78*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 168*a*b*c^5*d^3*tan(1/2*f*x + 1/2
*e)^3 + 42*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 64*b^2*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^3 - 68*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 8*a^2*c^2*d^6*tan(1/2*
f*x + 1/2*e)^3 + 8*b^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*b*c*d^7*tan(1/2
*f*x + 1/2*e)^3 + 8*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 - 24*a*b*c^8*tan(1/2*f*x
 + 1/2*e)^2 + 36*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*c^7*d*tan(1/2*f*
x + 1/2*e)^2 - 120*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^2*c^5*d^3*tan
(1/2*f*x + 1/2*e)^2 + 102*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 168*a*b*c^4*
d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 24*b^2
```

```
*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 1
2*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 3*b^2*c^8*tan(1/2*f*x + 1/2*e) - 48*a*
b*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 66*b^2
*c^6*d^2*tan(1/2*f*x + 1/2*e) - 114*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a
^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*
a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) - 12*
a*b*c^8 + 18*a^2*c^7*d + 13*b^2*c^7*d - 20*a*b*c^6*d^2 - 5*a^2*c^5*d^3 + 2*
b^2*c^5*d^3 + 2*a*b*c^4*d^4 + 2*a^2*c^3*d^5)/((c^9 - 3*c^7*d^2 + 3*c^5*d^4
- c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f
```

**Mupad [B]**
time = 11.16, size = 1220, normalized size = 4.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^4,x)
```

```
[Out] ((2*a^2*d^5 + 18*a^2*c^4*d + 13*b^2*c^4*d - 5*a^2*c^2*d^3 + 2*b^2*c^2*d^3 -
 12*a*b*c^5 + 2*a*b*c*d^4 - 20*a*b*c^3*d^2)/(3*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)) + (tan(e/2 + (f*x)/2)^4*(4*a^2*d^7 + 6*a^2*c^6*d + 5*b^2*c^6*d - 1
2*a^2*c^2*d^5 + 27*a^2*c^4*d^3 + 20*b^2*c^4*d^3 - 4*a*b*c^7 + 4*a*b*c*d^6 -
 22*a*b*c^3*d^4 - 28*a*b*c^5*d^2))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)
) + (tan(e/2 + (f*x)/2)*(2*a^2*d^6 - b^2*c^6 - 4*a^2*c^2*d^4 + 27*a^2*c^4*d
^2 + 4*b^2*c^2*d^4 + 22*b^2*c^4*d^2 + 4*a*b*c*d^5 - 16*a*b*c^5*d - 38*a*b*c
^3*d^3))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (2*tan(e/2 + (f*x)/2)^2*
(2*a^2*d^7 + 6*a^2*c^6*d + 4*b^2*c^6*d - 3*a^2*c^2*d^5 + 20*a^2*c^4*d^3 + 4
*b^2*c^2*d^5 + 17*b^2*c^4*d^3 - 4*a*b*c^7 + 2*a*b*c*d^6 - 28*a*b*c^3*d^4 -
20*a*b*c^5*d^2))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (tan(e/2 + (f*
x)/2)^5*(2*a^2*d^6 + b^2*c^6 - 6*a^2*c^2*d^4 + 9*a^2*c^4*d^2 + 4*b^2*c^4*d^
2 - 8*a*b*c^5*d - 2*a*b*c^3*d^3))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) +
 (2*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(2*a^2*d^5 + 18*a^2*c^4*d + 13*b
^2*c^4*d - 5*a^2*c^2*d^3 + 2*b^2*c^2*d^3 - 12*a*b*c^5 + 2*a*b*c*d^4 - 20*a*
b*c^3*d^2))/(3*c^3*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)))/(f*(c^3*tan(e/2 +
(f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4
*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2
*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (f*x)/2)^5)) + (atan((((c*tan(e/2
 + (f*x)/2)*(2*a^2*c^3 + b^2*c^3 + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 -
8*a*b*c^2*d))/((c + d)^(7/2)*(c - d)^(7/2)) + ((2*c^6*d - 2*d^7 + 6*c^2*d^5
 - 6*c^4*d^3)*(2*a^2*c^3 + b^2*c^3 + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3
- 8*a*b*c^2*d))/(2*(c + d)^(7/2)*(c - d)^(7/2)*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)))*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2))/(2*a^2*c^3 + b^2*c^3 + 3*a^2
*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 - 8*a*b*c^2*d))*(2*a^2*c^3 + b^2*c^3 + 3*a
^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 - 8*a*b*c^2*d))/(f*(c + d)^(7/2)*(c - d)
^(7/2))
```

# 3.686 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=400

$$\frac{1}{16}\left(18a^2bd\left(4c^2+d^2\right)+b^3d\left(18c^2+5d^2\right)+6ab^2c\left(4c^2+9d^2\right)+8a^3\left(2c^3+3cd^2\right)\right)x-\frac{\left(3ab^2d\left(3c^2+d^2\right)+3a^2b\right.}{}$$

```
[Out] 1/16*(18*a^2*b*d*(4*c^2+d^2)+b^3*d*(18*c^2+5*d^2)+6*a*b^2*c*(4*c^2+9*d^2)+8
*a^3*(2*c^3+3*c*d^2))*x-(3*a*b^2*d*(3*c^2+d^2)+3*a^2*b*c*(c^2+3*d^2)+b^3*c*
(c^2+3*d^2)+a^3*(3*c^2*d+d^3))*cos(f*x+e)/f+1/3*(a*d+b*c)*(8*a*b*c*d+a^2*d^
2+b^2*(c^2+6*d^2))*cos(f*x+e)^3/f-3/5*b^2*d^2*(a*d+b*c)*cos(f*x+e)^5/f-1/16
*(24*a^3*c*d^2+18*a^2*b*d*(4*c^2+d^2)+b^3*d*(18*c^2+5*d^2)+6*a*b^2*c*(4*c^2
+9*d^2))*cos(f*x+e)*sin(f*x+e)/f-5/24*b^3*d^3*cos(f*x+e)*sin(f*x+e)^3/f-3/4
*b*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*b^3*d^3*cos(
f*x+e)*sin(f*x+e)^5/f
```

**Rubi** [A]
time = 0.66, antiderivative size = 493, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2872, 3102, 2832, 2813}

$$\frac{b\left(-90a^2d^2+18abcd-\left(b^2\left(2c^2+25d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}+\frac{1}{16}x\left(8a^3\left(2c^3+3cd^2\right)+18a^2bd\left(4c^2+d^2\right)+6ab^2c\left(4c^2+9d^2\right)+b^3d\left(18c^2+5d^2\right)\right)-\frac{\left(40a^3d^3+90a^2bcd^2-ab^2\left(18c^2d-96d^3\right)+b^3\left(2c^3+21cd^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}-\frac{\left(200a^3cd^3+90a^2bd^2\left(2c^2+3d^2\right)-6ab^2d\left(6c^3-71cd^2\right)+b^3\left(4c^4+36c^2d^2+75d^4\right)\right)\sin(e+fx)\cos(e+fx)}{240df}-\frac{\left(40a^3d^3\left(4c^2+d^2\right)+90a^2bcd^2\left(c^2+4d^2\right)-6ab^2d\left(3c^4-52c^2d^2-16d^4\right)+b^3\left(2c^5+17c^3d^2+96cd^4\right)\right)\cos(e+fx)}{60d^2f}+\frac{b^2(2bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^4}{6df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((18*a^2*b*d*(4*c^2 + d^2) + b^3*d*(18*c^2 + 5*d^2) + 6*a*b^2*c*(4*c^2 + 9*
d^2) + 8*a^3*(2*c^3 + 3*c*d^2))*x)/16 - ((40*a^3*d^3*(4*c^2 + d^2) + 90*a^2
*b*c*d^2*(c^2 + 4*d^2) - 6*a*b^2*d*(3*c^4 - 52*c^2*d^2 - 16*d^4) + b^3*(2*c
^5 + 17*c^3*d^2 + 96*c*d^4))*Cos[e + f*x])/(60*d^2*f) - ((200*a^3*c*d^3 + 9
0*a^2*b*d^2*(2*c^2 + 3*d^2) - 6*a*b^2*d*(6*c^3 - 71*c*d^2) + b^3*(4*c^4 + 3
6*c^2*d^2 + 75*d^4))*Cos[e + f*x]*Sin[e + f*x])/(240*d*f) - ((90*a^2*b*c*d^
2 + 40*a^3*d^3 + b^3*(2*c^3 + 21*c*d^2) - a*b^2*(18*c^2*d - 96*d^3))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) + (b*(18*a*b*c*d - 90*a^2*d^2 -
b^2*(2*c^2 + 25*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (b
^2*(2*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (b^2*
Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^4}{6df} + \frac{\int(}{}$$

$$= \frac{b^2(2bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} - \frac{b^2\cos(e+}{}$$

$$= \frac{b(18abcd-90a^2d^2-b^2(2c^2+25d^2))\cos(e+fx)(c+d\sin(e}{120d^2f}$$

$$= -\frac{(90a^2bcd^2+40a^3d^3+b^3(2c^3+21cd^2)-ab^2(18c^2d-96d^3)}{120d^2f}$$

$$= \frac{1}{16}\left(18a^2bd\left(4c^2+d^2\right)+b^3d\left(18c^2+5d^2\right)+6ab^2c\left(4c^2+9d^2\right)\right.$$

**Mathematica [A]**
time = 1.20, size = 552, normalized size = 1.38

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (960*a^3*c^3*e + 1440*a*b^2*c^3*e + 4320*a^2*b*c^2*d*e + 1080*b^3*c^2*d*e +
 1440*a^3*c*d^2*e + 3240*a*b^2*c*d^2*e + 1080*a^2*b*d^3*e + 300*b^3*d^3*e +
 960*a^3*c^3*f*x + 1440*a*b^2*c^3*f*x + 4320*a^2*b*c^2*d*f*x + 1080*b^3*c^2
*d*f*x + 1440*a^3*c*d^2*f*x + 3240*a*b^2*c*d^2*f*x + 1080*a^2*b*d^3*f*x + 3
00*b^3*d^3*f*x - 360*(b^3*c*(2*c^2 + 5*d^2) + a*b^2*d*(18*c^2 + 5*d^2) + 2*
a^2*b*c*(4*c^2 + 9*d^2) + 2*a^3*(4*c^2*d + d^3))*Cos[e + f*x] + 20*(36*a^2*
b*c*d^2 + 4*a^3*d^3 + 3*a*b^2*d*(12*c^2 + 5*d^2) + b^3*(4*c^3 + 15*c*d^2))*
Cos[3*(e + f*x)] - 36*b^3*c*d^2*Cos[5*(e + f*x)] - 36*a*b^2*d^3*Cos[5*(e +
f*x)] - 720*a*b^2*c^3*Sin[2*(e + f*x)] - 2160*a^2*b*c^2*d*Sin[2*(e + f*x)]
- 720*b^3*c^2*d*Sin[2*(e + f*x)] - 720*a^3*c*d^2*Sin[2*(e + f*x)] - 2160*a*
b^2*c*d^2*Sin[2*(e + f*x)] - 720*a^2*b*d^3*Sin[2*(e + f*x)] - 225*b^3*d^3*S
in[2*(e + f*x)] + 90*b^3*c^2*d*Sin[4*(e + f*x)] + 270*a*b^2*c*d^2*Sin[4*(e
+ f*x)] + 90*a^2*b*d^3*Sin[4*(e + f*x)] + 45*b^3*d^3*Sin[4*(e + f*x)] - 5*b
^3*d^3*Sin[6*(e + f*x)])/(960*f)
```

**Maple [A]**
time = 0.70, size = 489, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*c^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-1/3*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^2*b*c^3
*cos(f*x+e)+9*a^2*b*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3*a^2*
b*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin
(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a*b^2*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+
1/2*f*x+1/2*e)-3*a*b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a*b^2*c*d^2*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a*b^2*d^3*(8/
3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c^3*(2+sin(f*x+e)^2)*co
s(f*x+e)+3*b^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-3/5*b^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b^3*d^
3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e))
```

**Maxima [A]**
time = 0.32, size = 513, normalized size = 1.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(960*(f*x + e)*a^3*c^3 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^3 + 2160*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^2*b*c^2*d + 2880*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*
c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c^2*
d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d^2 + 2880*(cos(f*x + e)^3 -
 3*cos(f*x + e))*a^2*b*c*d^2 + 270*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*a*b^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*b^3*c*d^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 +
 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b*d^3 - 192
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*b^2*d^3 + 5*(4*
sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*
e))*b^3*d^3 - 2880*a^2*b*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.39, size = 382, normalized size = 0.96

$$-\frac{144\,(b^3cd^2+ab^2d^3)\cos(fx+e)^5-80\,(b^3c^3+9\,ab^2c^2d+3\,(3\,a^2b+2\,b^3)cd^2+(a^3+6\,ab^2)d^3)\cos(fx+e)^3-15\,(8\,(2\,a^3+3\,ab^2)c^3+18\,(4\,a^2b+b^3)c^2d+6\,(4\,a^3+9\,ab^2)cd^2+(18\,a^2b+5\,b^3)d^3)fx+240\,((3\,a^2b+b^3)c^3+3\,(a^3+3\,ab^2)c^2d+3\,(3\,a^2b+b^3)cd^2+(a^3+3\,ab^2)d^3)\cos(fx+e)+5\,(8\,b^3d^3\cos(fx+e)^5-2\,(18\,b^3c^2d+54\,ab^2cd^2+(18\,a^2b+13\,b^3)d^3)\cos(fx+e)^3+3\,(24\,ab^2c^3+6\,(12\,a^2b+5\,b^3)c^2d+6\,(4\,a^3+15\,ab^2)cd^2+(30\,a^2b+11\,b^3)d^3)\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(b^3*c*d^2 + a*b^2*d^3)*cos(f*x + e)^5 - 80*(b^3*c^3 + 9*a*b^2*
c^2*d + 3*(3*a^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^3 - 1
5*(8*(2*a^3 + 3*a*b^2)*c^3 + 18*(4*a^2*b + b^3)*c^2*d + 6*(4*a^3 + 9*a*b^2)
*c*d^2 + (18*a^2*b + 5*b^3)*d^3)*f*x + 240*((3*a^2*b + b^3)*c^3 + 3*(a^3 +
3*a*b^2)*c^2*d + 3*(3*a^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3)*cos(f*x + e
) + 5*(8*b^3*d^3*cos(f*x + e)^5 - 2*(18*b^3*c^2*d + 54*a*b^2*c*d^2 + (18*a^
2*b + 13*b^3)*d^3)*cos(f*x + e)^3 + 3*(24*a*b^2*c^3 + 6*(12*a^2*b + 5*b^3)*
c^2*d + 6*(4*a^3 + 15*a*b^2)*c*d^2 + (30*a^2*b + 11*b^3)*d^3)*cos(f*x + e))
*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(400) = 800$.
time = 0.63, size = 1217, normalized size = 3.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*c**3*x - 3*a**3*c**2*d*cos(e + f*x)/f + 3*a**3*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*a**3*c*d**2*sin(e +
```

```
f*x)*cos(e + f*x)/(2*f) - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3
*d**3*cos(e + f*x)**3/(3*f) - 3*a**2*b*c**3*cos(e + f*x)/f + 9*a**2*b*c**2*
d*x*sin(e + f*x)**2/2 + 9*a**2*b*c**2*d*x*cos(e + f*x)**2/2 - 9*a**2*b*c**2
*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 9*a**2*b*c*d**2*sin(e + f*x)**2*cos(e
+ f*x)/f - 6*a**2*b*c*d**2*cos(e + f*x)**3/f + 9*a**2*b*d**3*x*sin(e + f*x)
**4/8 + 9*a**2*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**2*b*d**3*x
*cos(e + f*x)**4/8 - 15*a**2*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*
a**2*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a*b**2*c**3*x*sin(e + f*
x)**2/2 + 3*a*b**2*c**3*x*cos(e + f*x)**2/2 - 3*a*b**2*c**3*sin(e + f*x)*co
s(e + f*x)/(2*f) - 9*a*b**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 6*a*b**
2*c**2*d*cos(e + f*x)**3/f + 27*a*b**2*c*d**2*x*sin(e + f*x)**4/8 + 27*a*b*
*2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 27*a*b**2*c*d**2*x*cos(e +
f*x)**4/8 - 45*a*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 27*a*b**2
*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*d**3*sin(e + f*x)**4*
cos(e + f*x)/f - 4*a*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b**2
*d**3*cos(e + f*x)**5/(5*f) - b**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**3*c**3*cos(e + f*x)**3/(3*f) + 9*b**3*c**2*d*x*sin(e + f*x)**4/8 + 9*b**
3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**3*c**2*d*x*cos(e + f*x)
**4/8 - 15*b**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b**3*c**2*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*b**3*c*d**2*sin(e + f*x)**4*cos(e + f
*x)/f - 4*b**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*b**3*c*d**2*cos
(e + f*x)**5/(5*f) + 5*b**3*d**3*x*sin(e + f*x)**6/16 + 15*b**3*d**3*x*sin(
e + f*x)**4*cos(e + f*x)**2/16 + 15*b**3*d**3*x*sin(e + f*x)**2*cos(e + f*x
)**4/16 + 5*b**3*d**3*x*cos(e + f*x)**6/16 - 11*b**3*d**3*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - 5*b**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*
b**3*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))
**3*(c + d*sin(e))**3, True))
```

**Giac [A]**
time = 0.72, size = 416, normalized size = 1.04

$$-\frac{b^3d^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(16\,a^3c^3+24\,ab^2c^3+72\,a^2bc^2d+18\,b^3c^2d+24\,a^3cd^2+54\,ab^2cd^2+18\,a^2bd^3+5\,b^3d^3\right)x-\frac{3\left(b^3cd^2+ab^2d^3\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(4\,b^3c^3+36\,ab^2c^2d+36\,a^2bcd^2+15\,b^3cd^2+4\,a^3d^3+15\,ab^2d^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{3\left(8\,a^2bc^3+2\,b^3c^3+8\,a^3c^2d+18\,ab^2c^2d+18\,a^2bcd^2+5\,b^3cd^2+2\,a^3d^3+5\,ab^2d^3\right)\cos\left(fx+e\right)}{8\,f}+\frac{3\left(2\,b^3c^2d+6\,ab^2cd^2+2\,a^2bd^3+b^3d^3\right)\sin\left(4\,fx+4\,e\right)}{64\,f}-\frac{3\left(16\,ab^2c^3+48\,a^2bc^2d+16\,b^3c^2d+16\,a^3cd^2+48\,ab^2cd^2+16\,a^2bd^3+5\,b^3d^3\right)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*d^3*sin(6*f*x + 6*e)/f + 1/16*(16*a^3*c^3 + 24*a*b^2*c^3 + 72*a^
2*b*c^2*d + 18*b^3*c^2*d + 24*a^3*c*d^2 + 54*a*b^2*c*d^2 + 18*a^2*b*d^3 + 5
*b^3*d^3)*x - 3/80*(b^3*c*d^2 + a*b^2*d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*b^3
*c^3 + 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 + 15*b^3*c*d^2 + 4*a^3*d^3 + 15*a*b^
2*d^3)*cos(3*f*x + 3*e)/f - 3/8*(8*a^2*b*c^3 + 2*b^3*c^3 + 8*a^3*c^2*d + 18
*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 5*b^3*c*d^2 + 2*a^3*d^3 + 5*a*b^2*d^3)*cos(
f*x + e)/f + 3/64*(2*b^3*c^2*d + 6*a*b^2*c*d^2 + 2*a^2*b*d^3 + b^3*d^3)*sin
(4*f*x + 4*e)/f - 3/64*(16*a*b^2*c^3 + 48*a^2*b*c^2*d + 16*b^3*c^2*d + 16*a
^3*c*d^2 + 48*a*b^2*c*d^2 + 16*a^2*b*d^3 + 5*b^3*d^3)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**

time = 8.92, size = 574, normalized size = 1.44

$$-\frac{180\,a^3\,d^3\,\cos\left(e+f\,x\right)+180\,b^3\,c^3\,\cos\left(e+f\,x\right)-20\,a^3\,d^3\,\cos\left(3\,e+3\,f\,x\right)-20\,b^3\,c^3\,\cos\left(3\,e+3\,f\,x\right)+\frac{225\,b^3\,d^3\,\sin\left(2\,e+2\,f\,x\right)}{4}-\frac{45\,b^3\,d^3\,\sin\left(4\,e+4\,f\,x\right)}{4}+\frac{5\,b^3\,d^3\,\sin\left(6\,e+6\,f\,x\right)}{4}-75\,a\,b^2\,d^3\,\cos\left(3\,e+3\,f\,x\right)+9\,a\,b^2\,d^3\,\cos\left(5\,e+5\,f\,x\right)-75\,b^3\,c\,d^2\,\cos\left(3\,e+3\,f\,x\right)+9\,b^3\,c\,d^2\,\cos\left(5\,e+5\,f\,x\right)+180\,a\,b^2\,c^3\,\sin\left(2\,e+2\,f\,x\right)+180\,a^2\,b\,d^3\,\sin\left(2\,e+2\,f\,x\right)-\frac{45\,a^2\,b\,d^3\,\sin\left(4\,e+4\,f\,x\right)}{2}+180\,a^3\,c\,d^2\,\sin\left(2\,e+2\,f\,x\right)+180\,b^3\,c^2\,d\,\sin\left(2\,e+2\,f\,x\right)-\frac{45\,b^3\,c^2\,d\,\sin\left(4\,e+4\,f\,x\right)}{2}+720\,a^2\,b\,c^3\,\cos\left(e+f\,x\right)+450\,a\,b^2\,d^3\,\cos\left(e+f\,x\right)+720\,a^3\,c^2\,d\,\cos\left(e+f\,x\right)+450\,b^3\,c\,d^2\,\cos\left(e+f\,x\right)-240\,a^3\,c^3\,f\,x-75\,b^3\,d^3\,f\,x+1620\,a\,b^2\,c^2\,d\,\cos\left(e+f\,x\right)+1620\,a^2\,b\,c\,d^2\,\cos\left(e+f\,x\right)-360\,a\,b^2\,c^3\,f\,x-270\,a^2\,b\,d^3\,f\,x-360\,a^3\,c\,d^2\,f\,x-270\,b^3\,c^2\,d\,f\,x-180\,a\,b^2\,c^2\,d\,\cos\left(3\,e+3\,f\,x\right)-180\,a^2\,b\,c\,d^2\,\cos\left(3\,e+3\,f\,x\right)+540\,a\,b^2\,c\,d^2\,\sin\left(2\,e+2\,f\,x\right)+540\,a^2\,b\,c^2\,d\,\sin\left(2\,e+2\,f\,x\right)-\frac{135\,a\,b^2\,c\,d^2\,\sin\left(4\,e+4\,f\,x\right)}{2}-810\,a\,b^2\,c\,d^2\,f\,x-1080\,a^2\,b\,c^2\,d\,f\,x}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] -(180*a^3*d^3*cos(e + f*x) + 180*b^3*c^3*cos(e + f*x) - 20*a^3*d^3*cos(3*e
+ 3*f*x) - 20*b^3*c^3*cos(3*e + 3*f*x) + (225*b^3*d^3*sin(2*e + 2*f*x))/4 -
 (45*b^3*d^3*sin(4*e + 4*f*x))/4 + (5*b^3*d^3*sin(6*e + 6*f*x))/4 - 75*a*b^
2*d^3*cos(3*e + 3*f*x) + 9*a*b^2*d^3*cos(5*e + 5*f*x) - 75*b^3*c*d^2*cos(3*
e + 3*f*x) + 9*b^3*c*d^2*cos(5*e + 5*f*x) + 180*a*b^2*c^3*sin(2*e + 2*f*x)
+ 180*a^2*b*d^3*sin(2*e + 2*f*x) - (45*a^2*b*d^3*sin(4*e + 4*f*x))/2 + 180*
a^3*c*d^2*sin(2*e + 2*f*x) + 180*b^3*c^2*d*sin(2*e + 2*f*x) - (45*b^3*c^2*d
*sin(4*e + 4*f*x))/2 + 720*a^2*b*c^3*cos(e + f*x) + 450*a*b^2*d^3*cos(e + f
*x) + 720*a^3*c^2*d*cos(e + f*x) + 450*b^3*c*d^2*cos(e + f*x) - 240*a^3*c^3
*f*x - 75*b^3*d^3*f*x + 1620*a*b^2*c^2*d*cos(e + f*x) + 1620*a^2*b*c*d^2*co
s(e + f*x) - 360*a*b^2*c^3*f*x - 270*a^2*b*d^3*f*x - 360*a^3*c*d^2*f*x - 27
0*b^3*c^2*d*f*x - 180*a*b^2*c^2*d*cos(3*e + 3*f*x) - 180*a^2*b*c*d^2*cos(3*
e + 3*f*x) + 540*a*b^2*c*d^2*sin(2*e + 2*f*x) + 540*a^2*b*c^2*d*sin(2*e + 2
*f*x) - (135*a*b^2*c*d^2*sin(4*e + 4*f*x))/2 - 810*a*b^2*c*d^2*f*x - 1080*a
^2*b*c^2*d*f*x)/(240*f)
```

# 3.687 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=315

$$\frac{1}{8}\left(24a^2bcd+6b^3cd+4a^3\left(2c^2+d^2\right)+3ab^2\left(4c^2+3d^2\right)\right)x-\frac{\left(30a^3bcd+120ab^3cd-3a^4d^2+4b^4(5c^2+4d^2)+\right.}{30bf}$$

```
[Out] 1/8*(24*a^2*b*c*d+6*b^3*c*d+4*a^3*(2*c^2+d^2)+3*a*b^2*(4*c^2+3*d^2))*x-1/30
*(30*a^3*b*c*d+120*a*b^3*c*d-3*a^4*d^2+4*b^4*(5*c^2+4*d^2)+4*a^2*b^2*(20*c^
2+13*d^2))*cos(f*x+e)/b/f-1/120*(60*a^2*b*c*d+90*b^3*c*d-6*a^3*d^2+a*b^2*(1
00*c^2+71*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/60*(3*a*d*(-a*d+10*b*c)+4*b^2*(5*
c^2+4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^2/b/f-1/20*d*(-a*d+10*b*c)*cos(f*x+
e)*(a+b*sin(f*x+e))^3/b/f-1/5*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^4/b/f
```

**Rubi** [A]
time = 0.33, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2870, 2832, 2813}

$$-\frac{(-6a^3d^2+60a^2bcd+ab^2(100c^2+71d^2)+90b^3cd)\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}x\left(4a^3\left(2c^2+d^2\right)+24a^2bcd+3ab^2\left(4c^2+3d^2\right)+6b^3cd\right)-\frac{(-3a^4d^2+30a^3bcd+4a^2b^2(20c^2+13d^2)+120ab^3cd+4b^4(5c^2+4d^2))\cos(e+fx)}{30bf}-\frac{(3ad(10bc-ad)+4b^2(5c^2+4d^2))\cos(e+fx)(a+b\sin(e+fx))^2}{60bf}-\frac{d(10bc-ad)\cos(e+fx)(a+b\sin(e+fx))^3}{20bf}-\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^4}{5bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((24*a^2*b*c*d + 6*b^3*c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))
*x)/8 - ((30*a^3*b*c*d + 120*a*b^3*c*d - 3*a^4*d^2 + 4*b^4*(5*c^2 + 4*d^2)
+ 4*a^2*b^2*(20*c^2 + 13*d^2))*Cos[e + f*x])/(30*b*f) - ((60*a^2*b*c*d + 90
*b^3*c*d - 6*a^3*d^2 + a*b^2*(100*c^2 + 71*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(120*f) - ((3*a*d*(10*b*c - a*d) + 4*b^2*(5*c^2 + 4*d^2))*Cos[e + f*x]*(a
+ b*Sin[e + f*x])^2)/(60*b*f) - (d*(10*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^3)/(20*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int (a+b\sin(e+fx))^3 \cdots}{\cdots}$$

$$= -\frac{d(10bc-ad)\cos(e+fx)(a+b\sin(e+fx))^3}{20bf} - \frac{d^2\cos(e+\cdots}{\cdots}$$

$$= -\frac{(3ad(10bc-ad)+4b^2(5c^2+4d^2))\cos(e+fx)(a+b\sin(e\cdots}{60bf}$$

$$= \frac{1}{8}\left(24a^2bcd+6b^3cd+4a^3\left(2c^2+d^2\right)+3ab^2\left(4c^2+3d^2\right)\right)x - \cdots$$

**Mathematica** [A]
time = 1.60, size = 246, normalized size = 0.78

$$\frac{-60(16a^3cd+36ab^2cd+6a^2b(4c^2+3d^2)+b^3(6c^2+5d^2))\cos(e+fx)+10b(24abcd+12a^2d^2+b^2(4c^2+5d^2))\cos(3(e+fx))-6b^3d^2\cos(5(e+fx))+15(4(24a^2bcd+6b^3cd+4a^3(2c^2+d^2)+3ab^2(4c^2+3d^2))(e+fx)-8(6a^2bcd+2b^3cd+a^3d^2+3ab^2(c^2+d^2))\sin(2(e+fx))+b^2d(2bc+3ad)\sin(4(e+fx)))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-60*(16*a^3*c*d + 36*a*b^2*c*d + 6*a^2*b*(4*c^2 + 3*d^2) + b^3*(6*c^2 + 5*
d^2))*Cos[e + f*x] + 10*b*(24*a*b*c*d + 12*a^2*d^2 + b^2*(4*c^2 + 5*d^2))*C
os[3*(e + f*x)] - 6*b^3*d^2*Cos[5*(e + f*x)] + 15*(4*(24*a^2*b*c*d + 6*b^3*
c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))*(e + f*x) - 8*(6*a^2*b
*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*(c^2 + d^2))*Sin[2*(e + f*x)] + b^2*d*
(2*b*c + 3*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

**Maple** [A]
time = 0.42, size = 325, normalized size = 1.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*c^2*(f*x+e)-2*a^3*d*c*cos(f*x+e)+a^3*d^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*c^2*cos(f*x+e)+6*a^2*b*c*d*(-1/2*cos(f*x+e)*sin(f
*x+e)+1/2*f*x+1/2*e)-a^2*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*c^2*(-1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a*b^2*d*c*(2+sin(f*x+e)^2)*cos(f*x
+e)+3*a*b^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-1/3*b^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*b^3*c*d*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^3*d^2*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))
```

**Maxima** [**A**]
time = 0.31, size = 338, normalized size = 1.07

$$\frac{480\,(fx+e)a^3c^2+360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ab^2c^2+160\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)b^3c^2+720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2bcd+960\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)ab^2cd+30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))b^3cd+120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^3d^2+480\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)a^2bd^2+45\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))ab^2d^2-32\,\left(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e)\right)b^3d^2-1440\,a^2bc^2\cos(fx+e)-960\,a^3cd\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a^3*c^2 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^2 + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*b*c*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*c*d
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c*d + 120*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*a^2*b*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*a*b^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^3*d^2 - 1440*a^2*b*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

**Fricas** [**A**]
time = 0.37, size = 259, normalized size = 0.82

$$-\frac{24\,b^3d^2\cos(fx+e)^5-40\,(b^3c^2+6\,ab^2cd+(3\,a^2b+2\,b^3)d^2)\cos(fx+e)^3-15\,(4\,(2\,a^3+3\,ab^2)c^2+6\,(4\,a^2b+b^3)cd+(4\,a^3+9\,ab^2)d^2)fx+120\,((3\,a^2b+b^3)c^2+2\,(a^3+3\,ab^2)cd+(3\,a^2b+b^3)d^2)\cos(fx+e)-15\,\left(2\,(2\,b^3cd+3\,ab^2d^2)\cos(fx+e)^3-(12\,ab^2c^2+2\,(12\,a^2b+5\,b^3)cd+(4\,a^3+15\,ab^2)d^2)\cos(fx+e)\right)\sin(fx+e)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*d^2*cos(f*x + e)^5 - 40*(b^3*c^2 + 6*a*b^2*c*d + (3*a^2*b +
2*b^3)*d^2)*cos(f*x + e)^3 - 15*(4*(2*a^3 + 3*a*b^2)*c^2 + 6*(4*a^2*b + b^3
)*c*d + (4*a^3 + 9*a*b^2)*d^2)*f*x + 120*((3*a^2*b + b^3)*c^2 + 2*(a^3 + 3*
a*b^2)*c*d + (3*a^2*b + b^3)*d^2)*cos(f*x + e) - 15*(2*(2*b^3*c*d + 3*a*b^2
*d^2)*cos(f*x + e)^3 - (12*a*b^2*c^2 + 2*(12*a^2*b + 5*b^3)*c*d + (4*a^3 +
15*a*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(301) = 602$.
time = 0.46, size = 729, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**3*c**2*x - 2*a**3*c*d*cos(e + f*x)/f + a**3*d**2*x*sin(e + f*
x)**2/2 + a**3*d**2*x*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 3*a**2*b*c**2*cos(e + f*x)/f + 3*a**2*b*c*d*x*sin(e + f*x)**2 +
3*a**2*b*c*d*x*cos(e + f*x)**2 - 3*a**2*b*c*d*sin(e + f*x)*cos(e + f*x)/f -
 3*a**2*b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*b*d**2*cos(e + f*x)*
*3/f + 3*a*b**2*c**2*x*sin(e + f*x)**2/2 + 3*a*b**2*c**2*x*cos(e + f*x)**2/
2 - 3*a*b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a*b**2*c*d*sin(e + f*
x)**2*cos(e + f*x)/f - 4*a*b**2*c*d*cos(e + f*x)**3/f + 9*a*b**2*d**2*x*sin
(e + f*x)**4/8 + 9*a*b**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a*b*
*2*d**2*x*cos(e + f*x)**4/8 - 15*a*b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 9*a*b**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**3*c**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 2*b**3*c**2*cos(e + f*x)**3/(3*f) + 3*b**3*c*d*x*
sin(e + f*x)**4/4 + 3*b**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*b**3
*c*d*x*cos(e + f*x)**4/4 - 5*b**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) -
3*b**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**3*d**2*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**3*
d**2*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*(c + d*sin(e))
**2, True))
```

**Giac [A]**
time = 0.48, size = 274, normalized size = 0.87

$$-\frac{b^3d^2\cos(5\,fx+5\,e)}{80\,f}+\frac{1}{8}\,(8\,a^3c^2+12\,ab^2c^2+24\,a^2bcd+6\,b^3cd+4\,a^3d^2+9\,ab^2d^2)x+\frac{(4\,b^3c^2+24\,ab^2cd+12\,a^2bd^2+5\,b^3d^2)\cos(3\,fx+3\,e)}{48\,f}-\frac{(24\,a^2bc^2+6\,b^3c^2+16\,a^3cd+36\,ab^2cd+18\,a^2bd^2+5\,b^3d^2)\cos(fx+e)}{8\,f}+\frac{(2\,b^3cd+3\,ab^2d^2)\sin(4\,fx+4\,e)}{32\,f}-\frac{(3\,ab^2c^2+6\,a^2bcd+2\,b^3cd+a^3d^2+3\,ab^2d^2)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*b^3*d^2*cos(5*f*x + 5*e)/f + 1/8*(8*a^3*c^2 + 12*a*b^2*c^2 + 24*a^2*b
*c*d + 6*b^3*c*d + 4*a^3*d^2 + 9*a*b^2*d^2)*x + 1/48*(4*b^3*c^2 + 24*a*b^2*
c*d + 12*a^2*b*d^2 + 5*b^3*d^2)*cos(3*f*x + 3*e)/f - 1/8*(24*a^2*b*c^2 + 6*
b^3*c^2 + 16*a^3*c*d + 36*a*b^2*c*d + 18*a^2*b*d^2 + 5*b^3*d^2)*cos(f*x + e
)/f + 1/32*(2*b^3*c*d + 3*a*b^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a*b^2*c^2
+ 6*a^2*b*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*d^2)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 8.52, size = 358, normalized size = 1.14

$$-\frac{90\,b^3\,c^2\,\cos(e+f\,x)+75\,b^3\,d^2\,\cos(e+f\,x)-10\,b^3\,c^2\,\cos(3\,e+3\,f\,x)-\frac{25\,b^3\,d^2\,\cos(3\,e+3\,f\,x)}{2}+\frac{3\,b^3\,d^2\,\cos(5\,e+5\,f\,x)}{2}+30\,a^3\,d^2\,\sin(2\,e+2\,f\,x)-30\,a^2\,b\,d^2\,\cos(3\,e+3\,f\,x)+90\,a\,b^2\,c^2\,\sin(2\,e+2\,f\,x)+90\,a\,b^2\,d^2\,\sin(2\,e+2\,f\,x)-\frac{45\,a\,b^2\,d^2\,\sin(4\,e+4\,f\,x)}{4}+240\,a^3\,c\,d\,\cos(e+f\,x)+360\,a^2\,b\,c^2\,\cos(e+f\,x)+270\,a^2\,b\,d^2\,\cos(e+f\,x)+60\,b^3\,c\,d\,\sin(2\,e+2\,f\,x)-\frac{15\,b^3\,c\,d\,\sin(4\,e+4\,f\,x)}{2}-120\,a^3\,c^2\,f\,x-60\,a^3\,d^2\,f\,x-60\,a\,b^2\,c\,d\,\cos(3\,e+3\,f\,x)+180\,a^2\,b\,c\,d\,\sin(2\,e+2\,f\,x)-180\,a\,b^2\,c^2\,f\,x-135\,a\,b^2\,d^2\,f\,x+540\,a\,b^2\,c\,d\,\cos(e+f\,x)-90\,b^3\,c\,d\,f\,x-360\,a^2\,b\,c\,d\,f\,x}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -(90*b^3*c^2*cos(e + f*x) + 75*b^3*d^2*cos(e + f*x) - 10*b^3*c^2*cos(3*e +
3*f*x) - (25*b^3*d^2*cos(3*e + 3*f*x))/2 + (3*b^3*d^2*cos(5*e + 5*f*x))/2 +
 30*a^3*d^2*sin(2*e + 2*f*x) - 30*a^2*b*d^2*cos(3*e + 3*f*x) + 90*a*b^2*c^2
*sin(2*e + 2*f*x) + 90*a*b^2*d^2*sin(2*e + 2*f*x) - (45*a*b^2*d^2*sin(4*e +
 4*f*x))/4 + 240*a^3*c*d*cos(e + f*x) + 360*a^2*b*c^2*cos(e + f*x) + 270*a^
2*b*d^2*cos(e + f*x) + 60*b^3*c*d*sin(2*e + 2*f*x) - (15*b^3*c*d*sin(4*e +
4*f*x))/2 - 120*a^3*c^2*f*x - 60*a^3*d^2*f*x - 60*a*b^2*c*d*cos(3*e + 3*f*x
) + 180*a^2*b*c*d*sin(2*e + 2*f*x) - 180*a*b^2*c^2*f*x - 135*a*b^2*d^2*f*x
+ 540*a*b^2*c*d*cos(e + f*x) - 90*b^3*c*d*f*x - 360*a^2*b*c*d*f*x)/(120*f)
```

# 3.688 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=171

$$\frac{1}{8}\left(8a^3c+12ab^2c+12a^2bd+3b^3d\right)x-\frac{(16a^2bc+4b^3c+3a^3d+12ab^2d)\cos(e+fx)}{6f}-\frac{b(20abc+6a^2d+9b^2d)}{2}$$

```
[Out] 1/8*(8*a^3*c+12*a^2*b*d+12*a*b^2*c+3*b^3*d)*x-1/6*(3*a^3*d+16*a^2*b*c+12*a*
b^2*d+4*b^3*c)*cos(f*x+e)/f-1/24*b*(6*a^2*d+20*a*b*c+9*b^2*d)*cos(f*x+e)*si
n(f*x+e)/f-1/12*(3*a*d+4*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^2/f-1/4*d*cos(f*x
+e)*(a+b*sin(f*x+e))^3/f
```

**Rubi** [A]
time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2832, 2813}

$$-\frac{b(6a^2d+20abc+9b^2d)\sin(e+fx)\cos(e+fx)}{24f}-\frac{(3a^3d+16a^2bc+12ab^2d+4b^3c)\cos(e+fx)}{6f}+\frac{1}{8}x(8a^3c+12a^2bd+12ab^2c+3b^3d)-\frac{(3ad+4bc)\cos(e+fx)(a+b\sin(e+fx))^2}{12f}-\frac{d\cos(e+fx)(a+b\sin(e+fx))^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^3*d)*x)/8 - ((16*a^2*b*c + 4*b^3*
c + 3*a^3*d + 12*a*b^2*d)*Cos[e + f*x])/(6*f) - (b*(20*a*b*c + 6*a^2*d + 9*
b^2*d)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((4*b*c + 3*a*d)*Cos[e + f*x]*(a
 + b*Sin[e + f*x])^2)/(12*f) - (d*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f
)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{4}\int (a+b\sin(e+fx$$

$$= -\frac{(4bc+3ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12f} - \frac{d\cos(e+f$$

$$= \frac{1}{8}\left(8a^3c+12ab^2c+12a^2bd+3b^3d\right)x - \frac{(16a^2bc+4b^3c+3a^3$$

**Mathematica** [A]
time = 0.72, size = 142, normalized size = 0.83

$$\frac{-24(12a^2bc+3b^3c+4a^3d+9ab^2d)\cos(e+fx)+8b^2(bc+3ad)\cos(3(e+fx))+3(4(8a^3c+12ab^2c+12a^2bd+3b^3d)(e+fx)-8b(3abc+3a^2d+b^2d)\sin(2(e+fx))+b^3d\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-24*(12*a^2*b*c + 3*b^3*c + 4*a^3*d + 9*a*b^2*d)*Cos[e + f*x] + 8*b^2*(b*c
 + 3*a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^
3*d)*(e + f*x) - 8*b*(3*a*b*c + 3*a^2*d + b^2*d)*Sin[2*(e + f*x)] + b^3*d*S
in[4*(e + f*x)]))/(96*f)
```

**Maple** [A]
time = 0.30, size = 182, normalized size = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{a^3c(fx+e)-a^3d\cos(fx+e)-3a^2bc\cos(fx+e)+3a^2bd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+3a\,b^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{}\right.}{}$ |
| default | $\frac{a^3c(fx+e)-a^3d\cos(fx+e)-3a^2bc\cos(fx+e)+3a^2bd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+3a\,b^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{}\right.}{}$ |
| risch | $a^3cx+\frac{3x\,a^2bd}{2}+\frac{3xa\,b^2c}{2}+\frac{3x\,b^3d}{8}-\frac{\cos(fx+e)a^3d}{f}-\frac{3\cos(fx+e)a^2bc}{f}-\frac{9\cos(fx+e)a\,b^2d}{4f}-\frac{3\cos(fx+e}{4f}$ |
| norman | $\frac{(a^3c+\frac{3}{2}a^2bd+\frac{3}{2}a\,b^2c+\frac{3}{8}b^3d)x+(a^3c+\frac{3}{2}a^2bd+\frac{3}{2}a\,b^2c+\frac{3}{8}b^3d)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a^3c+6a^2bd+6a\,b^2c+\frac{3}{2}b^3d)x\left(\tan^2\left(\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*c*(f*x+e)-a^3*d*cos(f*x+e)-3*a^2*b*c*cos(f*x+e)+3*a^2*b*d*(-1/2*co
s(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3*a*b^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/
2*f*x+1/2*e)-a*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c*(2+sin(f*x+e)^2)
```

```
*cos(f*x+e)+b^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

**Maxima** [A]
time = 0.30, size = 189, normalized size = 1.11

$$\frac{96(fx+e)a^3c+72(2fx+2e-\sin(2fx+2e))ab^2c+32(\cos(fx+e)^3-3\cos(fx+e))b^3c+72(2fx+2e-\sin(2fx+2e))a^2bd+96(\cos(fx+e)^3-3\cos(fx+e))ab^2d+3(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e))b^3d-288a^2bc\cos(fx+e)-96a^3d\cos(fx+e)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c + 32
*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*
e))*a^2*b*d + 96*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*d + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*d - 288*a^2*b*c*cos(f*x + e
) - 96*a^3*d*cos(f*x + e))/f
```

**Fricas** [A]
time = 0.38, size = 153, normalized size = 0.89

$$\frac{8(b^3c+3ab^2d)\cos(fx+e)^3+3(4(2a^3+3ab^2)c+3(4a^2b+b^3)d)fx-24((3a^2b+b^3)c+(a^3+3ab^2)d)\cos(fx+e)+3(2b^3d\cos(fx+e)^3-(12ab^2c+(12a^2b+5b^3)d)\cos(fx+e))\sin(fx+e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(b^3*c + 3*a*b^2*d)*cos(f*x + e)^3 + 3*(4*(2*a^3 + 3*a*b^2)*c + 3*(
4*a^2*b + b^3)*d)*f*x - 24*((3*a^2*b + b^3)*c + (a^3 + 3*a*b^2)*d)*cos(f*x
+ e) + 3*(2*b^3*d*cos(f*x + e)^3 - (12*a*b^2*c + (12*a^2*b + 5*b^3)*d)*cos(
f*x + e))*sin(f*x + e))/f
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(170) = 340$.
time = 0.25, size = 386, normalized size = 2.26

$$\begin{cases} a^3cx - \frac{a^3d\cos(e+fx)}{f} - \frac{3a^2bc\cos(e+fx)}{f} + \frac{3a^2bdx\sin^2(e+fx)}{2} + \frac{3a^2bdx\cos^2(e+fx)}{2} - \frac{3a^2bd\sin(e+fx)\cos(e+fx)}{2f} + \frac{3ab^2cx\sin^2(e+fx)}{2} + \frac{3ab^2cx\cos^2(e+fx)}{2} - \frac{3ab^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{3ab^2d\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ab^2d\cos^3(e+fx)}{f} - \frac{b^3c\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2b^3c\cos^3(e+fx)}{3f} + \frac{3b^3dx\sin^4(e+fx)}{8} + \frac{3b^3dx\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3b^3dx\cos^4(e+fx)}{8} - \frac{5b^3d\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3b^3d\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for } f \neq 0 \\ x(a+b\sin(e))^3(c+d\sin(e)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d*cos(e + f*x)/f - 3*a**2*b*c*cos(e + f*x)/f + 3
*a**2*b*d*x*sin(e + f*x)**2/2 + 3*a**2*b*d*x*cos(e + f*x)**2/2 - 3*a**2*b*d
*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*a*b**2*c*x*sin(e + f*x)**2/2 + 3*a*b**
2*c*x*cos(e + f*x)**2/2 - 3*a*b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*
b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*d*cos(e + f*x)**3/f - b**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*c*cos(e + f*x)**3/(3*f) + 3*b**3
*d*x*sin(e + f*x)**4/8 + 3*b**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
```

```
**3*d*x*cos(e + f*x)**4/8 - 5*b**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3
*(c + d*sin(e)), True))
```

**Giac [A]**
time = 0.45, size = 152, normalized size = 0.89

$$\frac{b^3 d \sin(4fx+4e)}{32f} + \frac{1}{8}\left(8a^3c + 12ab^2c + 12a^2bd + 3b^3d\right)x + \frac{\left(b^3c + 3ab^2d\right)\cos(3fx+3e)}{12f} - \frac{\left(12a^2bc + 3b^3c + 4a^3d + 9ab^2d\right)\cos(fx+e)}{4f} - \frac{\left(3ab^2c + 3a^2bd + b^3d\right)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*b^3*d*sin(4*f*x + 4*e)/f + 1/8*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*
b^3*d)*x + 1/12*(b^3*c + 3*a*b^2*d)*cos(3*f*x + 3*e)/f - 1/4*(12*a^2*b*c +
3*b^3*c + 4*a^3*d + 9*a*b^2*d)*cos(f*x + e)/f - 1/4*(3*a*b^2*c + 3*a^2*b*d
+ b^3*d)*sin(2*f*x + 2*e)/f
```

**Mupad [B]**
time = 7.95, size = 183, normalized size = 1.07

$$\frac{2b^3c\cos(3e+3fx) - 6b^3d\sin(2e+2fx) + \frac{3b^3d\sin(4e+4fx)}{4} - 24a^3d\cos(e+fx) - 18b^3c\cos(e+fx) - 72a^2bc\cos(e+fx) - 54ab^2d\cos(e+fx) + 24a^3cfx + 9b^3dfx + 6ab^2d\cos(3e+3fx) - 18ab^2c\sin(2e+2fx) - 18a^2bd\sin(2e+2fx) + 36ab^2cfx + 36a^2bdfx}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b^3*c*cos(3*e + 3*f*x) - 6*b^3*d*sin(2*e + 2*f*x) + (3*b^3*d*sin(4*e + 4
*f*x))/4 - 24*a^3*d*cos(e + f*x) - 18*b^3*c*cos(e + f*x) - 72*a^2*b*c*cos(e
 + f*x) - 54*a*b^2*d*cos(e + f*x) + 24*a^3*c*f*x + 9*b^3*d*f*x + 6*a*b^2*d*
cos(3*e + 3*f*x) - 18*a*b^2*c*sin(2*e + 2*f*x) - 18*a^2*b*d*sin(2*e + 2*f*x
) + 36*a*b^2*c*f*x + 36*a^2*b*d*f*x)/(24*f)
```

# 3.689 $\int (a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=90

$$\frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*x-2/3*b*(4*a^2+b^2)*cos(f*x+e)/f-5/6*a*b^2*cos(f*x+e)*s
in(f*x+e)/f-1/3*b*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

**Rubi** [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2735, 2813}

$$-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))^3\,dx &= -\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\sin\right. \\ &= \frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b}{}\end{aligned}$$

**Mathematica [A]**
time = 0.20, size = 71, normalized size = 0.79

$$\frac{6a(2a^2+3b^2)(e+fx)-9b(4a^2+b^2)\cos(e+fx)+b^3\cos(3(e+fx))-9ab^2\sin(2(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

**Maple [A]**
time = 0.14, size = 76, normalized size = 0.84

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a\,b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos(fx+e)+a^3(fx+e)}{f}$ |
| default | $\frac{-\frac{b^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+3a\,b^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos(fx+e)+a^3(fx+e)}{f}$ |
| risch | $a^3x+\frac{3a\,b^2x}{2}-\frac{3b\cos(fx+e)a^2}{f}-\frac{3b^3\cos(fx+e)}{4f}+\frac{b^3\cos(3fx+3e)}{12f}-\frac{3a\,b^2\sin(2fx+2e)}{4f}$ |
| norman | $\frac{(a^3+\frac{3}{2}a\,b^2)x+(a^3+\frac{3}{2}a\,b^2)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{18a^2b+}{3f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+a^3*(f*x+e))
```

**Maxima [A]**
time = 0.33, size = 79, normalized size = 0.88

$$a^3x+\frac{3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))ab^2}{4\,f}+\frac{\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)b^3}{3\,f}-\frac{3\,a^2b\cos(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

**Fricas [A]**
time = 0.36, size = 75, normalized size = 0.83

$$\frac{2\,b^3\cos\left(fx+e\right)^3-9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,(2\,a^3+3\,ab^2)fx-6\,(3\,a^2b+b^3)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

**Sympy [A]**
time = 0.13, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+fx\right)}{3f} & \text{for } f\neq 0 \\ x(a+b\sin\left(e\right))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

**Giac [A]**
time = 0.44, size = 75, normalized size = 0.83

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x-\frac{3\,(4\,a^2b+b^3)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

**Mupad [B]**
time = 7.69, size = 127, normalized size = 1.41

$$a^3\,x-\frac{4\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f}+\frac{8\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3\,f}+\frac{3\,a\,b^2\,x}{2}-\frac{6\,a^2\,b\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f}-\frac{6\,a\,b^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}+\frac{3\,a\,b^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3,x)
```

```
[Out] a^3*x - (4*b^3*cos(e/2 + (f*x)/2)^4)/f + (8*b^3*cos(e/2 + (f*x)/2)^6)/(3*f)
 + (3*a*b^2*x)/2 - (6*a^2*b*cos(e/2 + (f*x)/2)^2)/f - (6*a*b^2*cos(e/2 + (f
*x)/2)^3*sin(e/2 + (f*x)/2))/f + (3*a*b^2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x
)/2))/f
```

# 3.690 $\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=156

$$-\frac{b(6abcd-6a^2d^2-b^2(2c^2+d^2))\,x}{2d^3}-\frac{2(bc-ad)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\,c\cdots}{}$$

```
[Out] -1/2*b*(6*a*b*c*d-6*a^2*d^2-b^2*(2*c^2+d^2))*x/d^3+1/2*b^2*(-5*a*d+2*b*c)*c
os(f*x+e)/d^2/f-1/2*b^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/f-2*(-a*d+b*c)^3*arct
an((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.28, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2872, 3102, 2814, 2739, 632, 210}

$$-\frac{bx(-6a^2d^2+6abcd-(b^2(2c^2+d^2)))}{2d^3}-\frac{2(bc-ad)^3\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/2*(b*(6*a*b*c*d - 6*a^2*d^2 - b^2*(2*c^2 + d^2))*x)/d^3 - (2*(b*c - a*d)
^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f
) + (b^2*(2*b*c - 5*a*d)*Cos[e + f*x])/(2*d^2*f) - (b^2*Cos[e + f*x]*(a + b
*Sin[e + f*x]))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

a^2 - b^2, 0]

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \frac{\int \frac{b^3c+2a^3d-b(abc-6a^2d-b^2d)\sin(e+fx)-b^2(2bc-5}{c+d\sin(e+fx)}}{2d}$$

$$= \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \int \frac{d(b^3c+2a^3d)-}{}$$

$$= -\frac{b(6abcd-6a^2d^2-b^2(2c^2+d^2))\,x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+}{}$$

$$= -\frac{b(6abcd-6a^2d^2-b^2(2c^2+d^2))\,x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+}{}$$

$$= -\frac{b(6abcd-6a^2d^2-b^2(2c^2+d^2))\,x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+}{}$$

$$= -\frac{b(6abcd-6a^2d^2-b^2(2c^2+d^2))\,x}{2d^3} - \frac{2(bc-ad)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} +$$

**Mathematica [A]**
time = 0.39, size = 137, normalized size = 0.88

$$\frac{2b(-6abcd+6a^2d^2+b^2(2c^2+d^2))(e+fx) - \frac{8(bc-ad)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 4b^2d(bc-3ad)\cos(e+fx) - b^3d^2\sin(2(e+fx))}{4d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*(-6*a*b*c*d + 6*a^2*d^2 + b^2*(2*c^2 + d^2))*(e + f*x) - (8*(b*c - a*d
)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + 4*b
^2*d*(b*c - 3*a*d)*Cos[e + f*x] - b^3*d^2*Sin[2*(e + f*x)])/(4*d^3*f)
```

**Maple [A]**
time = 0.26, size = 229, normalized size = 1.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-3ab\,d^2+b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-3ab\,d^2+b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(6a^2d^2-6abcd+2b^2c^2+d^2b^2\right)\text{arc}}{2}\right.}{d^3}}{f}$ |

| | |
|---|---|
| default | $\frac{\frac{2b\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-3ab\,d^2+b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-3ab\,d^2+b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(6a^2d^2-6abcd+2b^2c^2+d^2b^2\right)\arctan}{2}\right.}{d^3}}{f}$ |
| risch | $\frac{3bx\,a^2}{d}-\frac{3b^2xac}{d^2}+\frac{b^3x\,c^2}{d^3}+\frac{b^3x}{2d}-\frac{3b^2e^{i(fx+e)}a}{2df}+\frac{b^3e^{i(fx+e)}c}{2d^2f}-\frac{3b^2e^{-i(fx+e)}a}{2df}+\frac{b^3e^{-i(fx+e)}c}{2d^2f}-\frac{\ln\left(e^{i(fx\right.}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b/d^3*((1/2*d^2*b^2*tan(1/2*f*x+1/2*e)^3+(-3*a*b*d^2+b^2*c*d)*tan(1/
2*f*x+1/2*e)^2-1/2*d^2*b^2*tan(1/2*f*x+1/2*e)-3*a*b*d^2+b^2*c*d)/(1+tan(1/2
*f*x+1/2*e)^2)^2+1/2*(6*a^2*d^2-6*a*b*c*d+2*b^2*c^2+b^2*d^2)*arctan(tan(1/2
*f*x+1/2*e)))+2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.41, size = 593, normalized size = 3.80

$$\left[\frac{\left(2b^3c^4-6ab^2c^3d+6ab^2cd^3+\left(6a^2b-b^3\right)c^2d^2-\left(6a^2b+b^3\right)d^4\right)fx-\left(b^3c^2d^2-b^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2\left(b^3c^3d-3ab^2c^2d^2-b^3cd^3+3ab^2d^4\right)\cos\left(fx+e\right)}{2\left(c^2d^3-d^5\right)f},\frac{\left(2b^3c^4-6ab^2c^3d+6ab^2cd^3+\left(6a^2b-b^3\right)c^2d^2-\left(6a^2b+b^3\right)d^4\right)fx-\left(b^3c^2d^2-b^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+2\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+2\left(b^3c^3d-3ab^2c^2d^2-b^3cd^3+3ab^2d^4\right)\cos\left(fx+e\right)}{2\left(c^2d^3-d^5\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((2*b^3*c^4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2
- (6*a^2*b + b^3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x +
 e) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)*
log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*c
os(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 -
```

```
b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f), 1/2*((2*b^3*c^
4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2 - (6*a^2*b + b^
3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x + e) + 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)*arctan(-(c*sin
(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 - b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.44, size = 252, normalized size = 1.62

$$\frac{\frac{(2b^3c^2-6ab^2cd+6a^2bd^2+b^3d^2)(fx+e)}{d^3}-\frac{4(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(b^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2b^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6ab^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2b^3c-6ab^2d\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2d^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*b^3*c^2 - 6*a*b^2*c*d + 6*a^2*b*d^2 + b^3*d^2)*(f*x + e)/d^3 - 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^3) + 2*(b^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c*tan(1/2*f*x
+ 1/2*e)^2 - 6*a*b^2*d*tan(1/2*f*x + 1/2*e)^2 - b^3*d*tan(1/2*f*x + 1/2*e)
+ 2*b^3*c - 6*a*b^2*d)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

**Mupad** [**B**]
time = 14.56, size = 2500, normalized size = 16.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x)),x)
```

```
[Out] ((2*(b^3*c - 3*a*b^2*d))/d^2 + (b^3*tan(e/2 + (f*x)/2)^3)/d + (2*tan(e/2 +
(f*x)/2)^2*(b^3*c - 3*a*b^2*d))/d^2 - (b^3*tan(e/2 + (f*x)/2))/d)/(f*(2*tan
(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (atan((((b^3*c^2*1i + (b*d
^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4
*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 6
```

0*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(
e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 -
8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 2
4*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6
 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4
*b^2*c^3*d^6))/d^6 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*
3i)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7
- 24*a^2*b*c^2*d^8))/d^6 - (8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6
- 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*
x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/
2 - a*b^2*c*d*3i))/d^3))/d^3)*1i)/d^3 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)
*1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 1
2*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^
4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(
2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 -
 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8
+ 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c
^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d
^6 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(2*b^3*c
*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/
d^5 - (8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7
 - 24*a^2*b*c^2*d^8))/d^6 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10
 - 8*c^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i
))/d^3))/d^3)*1i)/d^3)/((16*(2*b^9*c^7 + b^9*c^5*d^2 - 3*a*b^8*c^4*d^3 + 4*
a^3*b^6*c^6*d - 2*a^6*b^3*c*d^6 + 3*a^2*b^7*c^3*d^4 + 30*a^2*b^7*c^5*d^2 -
a^3*b^6*c^2*d^5 - 36*a^3*b^6*c^4*d^3 + 18*a^4*b^5*c^3*d^4 - 24*a^4*b^5*c^5*
d^2 + 60*a^5*b^4*c^4*d^3 - 76*a^6*b^3*c^3*d^4 + 48*a^7*b^2*c^2*d^5 - 12*a*b
^8*c^6*d - 12*a^8*b*c*d^6))/d^5 - ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2
 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^
5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72
*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*
c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*
b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a
^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4
 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + (
(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*tan(e/2 + (f*x
)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d
^6 - (8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 1
2*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c
^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i))/d^3
))/d^3))/d^3 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((
8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^
5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 +
36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8
 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*

```
b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a
^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*
d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + ((b^3*c^2*1i + (b*d^
2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*
b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 - (8*tan(e/2 + (f*x)/
2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d^6
 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(b^3*
c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i))/d^3))/d^3))/d^3 + (16*
tan(e/2 + (f*x)/2)*(8*b^9*c^8 + 2*b^9*c^4*d^4 + 8*b^9*c^6*d^2 - 6*a*b^8*c^3
*d^5 - 48*a*b^8*c^5*d^3 - 2*a^3*b^6*c*d^7 - 24*a^5*b^4*c*d^7 - 72*a^7*b^2*c
*d^7 + 6*a^2*b^7*c^2*d^6 + 120*a^2*b^7*c^4*d^4 + 288*a^2*b^7*c^6*d^2 - 152*
a^3*b^6*c^3*d^5 - 656*a^3*b^6*c^5*d^3 + 96*a^4*b^5*c^2*d^6 + 912*a^4*b^5*c^
4*d^4 - 768*a^5*b^4*c^3*d^5 + 360*a^6*b^3*c^2*d^6 - 72*a*b^8*c^7*d))/d^6))*
(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*2i)/(d^3*f) + (ata
n((((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4
 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6
 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36...
```

## 3.691 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=208

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{2(bc-ad)^2\left(2bc^2+acd-3bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f}+\frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)}{d^2\left(c^2-d^2\right)f}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+2*(-a*d+b*c)^2*(a*c*d+2*b*c^2-3*b*d^2)*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c^2-d^2)^(3/2)/f+b*(2*a*b*c*d-a
^2*d^2-b^2*(2*c^2-d^2))*cos(f*x+e)/d^2/(c^2-d^2)/f+(-a*d+b*c)^2*cos(f*x+e)*
(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.36, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2871, 3102, 2814, 2739, 632, 210}

$$\frac{b\left(-a^2d^2+2abcd-\left(b^2\left(2c^2-d^2\right)\right)\right)\cos(e+fx)}{d^2f\left(c^2-d^2\right)}+\frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b*d^
2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)
*f) + (b*(2*a*b*c*d - a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x])/(d^2*(c^2
- d^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2
)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\,(c^2-d^2)\,f(c+d\sin(e+fx))} - \frac{\int \frac{b^3c^2-a^3cd-3ab^2cd+3a^2bd^2-b(abc^2+(a^2+b}{}}{}$$

$$= \frac{b(2abcd-a^2d^2-b^2(2c^2-d^2))\cos(e+fx)}{d^2\,(c^2-d^2)\,f} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e}{d\,(c^2-d^2)\,f(c+d\sin(e+f}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b(2abcd-a^2d^2-b^2(2c^2-d^2))\cos(e+fx)}{d^2\,(c^2-d^2)\,f} + \frac{(bc-ad)^2\,\mathrm{co}}{d\,(c^2-}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b(2abcd-a^2d^2-b^2(2c^2-d^2))\cos(e+fx)}{d^2\,(c^2-d^2)\,f} + \frac{(bc-ad)^2\,\mathrm{co}}{d\,(c^2-}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b(2abcd-a^2d^2-b^2(2c^2-d^2))\cos(e+fx)}{d^2\,(c^2-d^2)\,f} + \frac{(bc-ad)^2\,\mathrm{co}}{d\,(c^2-}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{2(bc-ad)^2\,(2bc^2+acd-3bd^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\,(c^2-d^2)^{3/2}\,f} +$$

**Mathematica** [A]
time = 1.16, size = 152, normalized size = 0.73

$$\frac{-b^2(2bc-3ad)(e+fx) + \frac{2(bc-ad)^2(2bc^2+acd-3bd^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} - b^3d\cos(e+fx) + \frac{d(-bc+ad)^3\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-(b^2*(2*b*c - 3*a*d)*(e + f*x)) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b
*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) -
 b^3*d*Cos[e + f*x] + (d*(-(b*c) + a*d)^3*Cos[e + f*x])/((c - d)*(c + d)*(c
 + d*Sin[e + f*x])))/(d^3*f)
```

**Maple** [A]
time = 0.42, size = 302, normalized size = 1.45

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b^2\left(-\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3ad-2bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(a^3d^3-3a^2b\right.}{}\right.}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}}{f}$ |

| | |
|---|---|
| default | $\frac{\frac{2b^2\left(-\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3ad-2bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{\frac{d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(a^3d^3-3a^2 \cdots\right.}{\cdots}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}\cdots}{\cdots}}{f}$ |
| risch | $\frac{3b^2xa}{d^2}-\frac{2b^3xc}{d^3}-\frac{b^3e^{i(fx+e)}}{2d^2f}-\frac{b^3e^{-i(fx+e)}}{2d^2f}+\frac{2i\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(id+c\,e^{i(fx+e)}\right)}{d^3\left(c^2-d^2\right)f\left(-ie^{2i(fx+e)}d+id+2c\,e^{i(fx+e)}\right)}-\frac{\ln\left(e^{i(f \cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/d^3*(-b*d/(1+tan(1/2*f*x+1/2*e)^2)+(3*a*d-2*b*c)*arctan(tan(1/2*
f*x+1/2*e)))+2/d^3*((d^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(c^2
-d^2)/c*tan(1/2*f*x+1/2*e)+d*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
(c^2-d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(a^3*c*d^3-3*a
^2*b*d^4-3*a*b^2*c^3*d+6*a*b^2*c*d^3+2*b^3*c^4-3*b^3*c^2*d^2)/(c^2-d^2)^(3/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(208) = 416.
time = 0.42, size = 1081, normalized size = 5.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*b^3*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b
^3*c^2*d^4 - 3*a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^
2 - 3*a^2*b*c*d^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^
2 - 3*b^3*c^2*d^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt
```

```
(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*b^3*c^5*d - 3*
a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d^4
 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + 2*((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*b^3
*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b^3*c^2*d^4 - 3*
a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^2 - 3*a^2*b*c*d
^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^2 - 3*b^3*c^2*d
^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b^3*c^5*d -
3*a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d
^4 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + ((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(208) = 416.
time = 0.49, size = 585, normalized size = 2.81

$$\frac{\frac{2\left(2b^3c^4-3ab^2c^3d-3b^3c^2d^2+a^3cd^3+6ab^2cd^3-3a^2bd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^3-d^5\right)\sqrt{c^2-d^2}}-\frac{2\left(b^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3ab^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^2bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-a^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2b^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3ab^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3a^2bc^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b^3c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3b^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3ab^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3a^2bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2b^3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-a^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2b^3c^4-3ab^2c^3d+3a^2bc^2d^2-b^3c^2d^2-a^3cd^3\right)}{\left(c^3d^2-cd^4\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}-\frac{\left(2b^3c-3ab^2d\right)(fx+e)}{d^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*b^3*c^4 - 3*a*b^2*c^3*d - 3*b^3*c^2*d^2 + a^3*c*d^3 + 6*a*b^2*c*d^3 -
 3*a^2*b*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*
f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(
b^3*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 +
 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - a^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*
b^3*c^4*tan(1/2*f*x + 1/2*e)^2 - 3*a*b^2*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a
^2*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 -
a^3*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*b^3*c^3*d*tan(1/2*f*x + 1/2*e) - 3*a*b
```

```
^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*b^
3*c*d^3*tan(1/2*f*x + 1/2*e) - a^3*d^4*tan(1/2*f*x + 1/2*e) + 2*b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - b^3*c^2*d^2 - a^3*c*d^3)/((c^3*d^2 - c*d^4
)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*b^3*c - 3*a*b^2*d)*(f*x +
e)/d^3)/f
```

**Mupad [B]**
time = 17.64, size = 2500, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^3*d^3 - 2*b^3*c^3 + b^3*c*d^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(d^2
*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2*(a^3*d^3 - 2*b^3*c^3 + b^3*c*d^2 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(
a^3*d^3 - 3*b^3*c^3 + 2*b^3*c*d^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(c*d*(c
^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(
e/2 + (f*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (2
*b^2*atan(((b^2*(3*a*d - 2*b*c)*((32*(4*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6
*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b
^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^
4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d
^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 +
 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10
 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c
^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a
^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3
*a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b
^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2
*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*
c^2*d^8 + c^4*d^6) - (32*(a^3*c^5*d^7 - a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^
3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*
d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (b^2*((32*(c^2*d^12 -
2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)
*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*
d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3))/d^3 + (b^2*(3*a*d - 2*b*c)*((32*(4
*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*
c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2
*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^
3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 2
4*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 -
```

```
18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8
 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^
3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d
^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*(a^3*c^5*d^7 - a^3*c
^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3
*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4
*d^5) - (32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*
d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^
9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8
 + c^4*d^6) + (b^2*((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7
 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 -
2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3)
)/d^3)/((64*(6*b^9*c^6*d^2 - 4*b^9*c^8 - 39*a*b^8*c^5*d^3 + 4*a^3*b^6*c^7*d
 - 27*a^5*b^4*c*d^7 + 105*a^2*b^7*c^4*d^4 - 57*a^2*b^7*c^6*d^2 - 144*a^3*b^
6*c^3*d^5 + 55*a^3*b^6*c^5*d^3 + 99*a^4*b^5*c^2*d^6 + 3*a^4*b^5*c^4*d^4 - 1
2*a^4*b^5*c^6*d^2 - 39*a^5*b^4*c^3*d^5 + 9*a^5*b^4*c^5*d^3 + 18*a^6*b^3*c^2
*d^6 + 2*a^6*b^3*c^4*d^4 - 3*a^7*b^2*c^3*d^5 + 24*a*b^8*c^7*d))/(d^9 - 2*c^
2*d^7 + c^4*d^5) + (64*tan(e/2 + (f*x)/2)*(40*b^9*c^7*d^2 - 24*b^9*c^5*d^4
- 16*b^9*c^9 + 120*a*b^8*c^4*d^5 - 192*a*b^8*c^6*d^3 - 54*a^4*b^5*c*d^8 - 2
22*a^2*b^7*c^3*d^6 + 330*a^2*b^7*c^5*d^4 - 108*a^2*b^7*c^7*d^2 + 180*a^3*b^
6*c^2*d^7 - 226*a^3*b^6*c^4*d^5 + 46*a^3*b^6*c^6*d^3 + 30*a^4*b^5*c^3*d^6 +
 24*a^4*b^5*c^5*d^4 + 18*a^5*b^4*c^2*d^7 - 18*a^5*b^4*c^4*d^5 + 72*a*b^8*c^
8*d))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*(4*b^6*c^4*d
^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 -
12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d
^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*
b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^
2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4
*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*
b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7
+ 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^
2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*
d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12
*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6
*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(a^3*c^5*d^7 - a^3*c^3*d
^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2
*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11)...
```

# 3.692 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=255

$$\frac{b^3x}{d^3}-\frac{\left(9a^2bcd^4-a^3d^3(2c^2+d^2)-3ab^2d^3(c^2+2d^2)+b^3(2c^5-5c^3d^2+6cd^4)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{5/2}f}+\frac{(bc$$

```
[Out] b^3*x/d^3-(9*a^2*b*c*d^4-a^3*d^3*(2*c^2+d^2)-3*a*b^2*d^3*(c^2+2*d^2)+b^3*(2
*c^5-5*c^3*d^2+6*c*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d
^3/(c^2-d^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^
2)/f/(c+d*sin(f*x+e))^2+1/2*(-a*d+b*c)^2*(3*a*c*d+2*b*c^2-5*b*d^2)*cos(f*x+
e)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]

time = 0.46, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2871, 3100, 2814, 2739, 632, 210}

$$-\frac{\left(-a^3d^3(2c^2+d^2)+9a^2bcd^4-3ab^2d^3(c^2+2d^2)+b^3(2c^5-5c^3d^2+6cd^4)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\left(3acd+2bc^2-5bd^2\right)\cos(e+fx)}{2d^2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}+\frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2df\left(c^2-d^2\right)(c+d\sin(e+fx))^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (b^3*x)/d^3 - ((9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 +
2*d^2) + b^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(5/2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*
(a + b*Sin[e + f*x]))/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((b*c -
a*d)^2*(2*b*c^2 + 3*a*c*d - 5*b*d^2)*Cos[e + f*x])/(2*d^2*(c^2 - d^2)^2*f*(
c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} - \frac{\int \frac{b^3c^2-2a^3cd-4ab^2cd+5a^2bd^2-(4a^2bcd+2b\cdots}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\,(2bc^2+3acd-5bd^2)\,\mathrm{c}\cdots}{2d^2\,(c^2-d^2)^2\,f(c+d\sin(e+\cdots}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\,(2bc^2+3acd-\cdots}{2d^2\,(c^2-d^2)^2\,f(c+d\cdots}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\,(2bc^2+3acd-\cdots}{2d^2\,(c^2-d^2)^2\,f(c+d\cdots}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\,(2bc^2+3acd-\cdots}{2d^2\,(c^2-d^2)^2\,f(c+d\cdots}$$

$$= \frac{b^3x}{d^3} - \frac{(9a^2bcd^4-a^3d^3(2c^2+d^2)-3ab^2d^3(c^2+2d^2)+b^3(2c^5-5c^3d^2+6cd^4))\cdots}{d^3\,(c^2-d^2)^{5/2}\,f}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 521 vs. 2(255) = 510.
time = 2.50, size = 521, normalized size = 2.04

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 + 2*d^2) + b
^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(c^2 - d^2)^(5/2) + (4*b^3*c^6*e - 6*b^3*c^4*d^2*e + 2*b^3*d^6*e +
 4*b^3*c^6*f*x - 6*b^3*c^4*d^2*f*x + 2*b^3*d^6*f*x - 2*d*(b*c - a*d)^2*(-2*
b*c^3 - 4*a*c^2*d + 5*b*c*d^2 + a*d^3)*Cos[e + f*x] - 2*b^3*(-(c^2*d) + d^3
)^2*(e + f*x)*Cos[2*(e + f*x)] + 8*b^3*c^5*d*e*Sin[e + f*x] - 16*b^3*c^3*d^
3*e*Sin[e + f*x] + 8*b^3*c*d^5*e*Sin[e + f*x] + 8*b^3*c^5*d*f*x*Sin[e + f*x
] - 16*b^3*c^3*d^3*f*x*Sin[e + f*x] + 8*b^3*c*d^5*f*x*Sin[e + f*x] + 3*b^3*
c^4*d^2*Sin[2*(e + f*x)] - 3*a*b^2*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*b*c^2*d
^4*Sin[2*(e + f*x)] - 6*b^3*c^2*d^4*Sin[2*(e + f*x)] + 3*a^3*c*d^5*Sin[2*(e
 + f*x)] + 12*a*b^2*c*d^5*Sin[2*(e + f*x)] - 6*a^2*b*d^6*Sin[2*(e + f*x)])/
((c^2 - d^2)^2*(c + d*Sin[e + f*x])^2))/(4*d^3*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(246) = 492.
time = 0.76, size = 670, normalized size = 2.63

| method | result |
|---|---|
| derivativedivides | $\frac{2\left(\frac{d^2\left(5a^3c^2d^3-2a^3d^5-9a^2b\,c^3d^2+3a\,b^2c^4d+6a\,b^2c^2d^3+b^3c^5-4b^3c^3d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^4-2c^2d^2+d^4\right)c}+\frac{d\left(4a^3c^4d^3+7a^3c^2d^5-2a^3d^7-6a^2b\,c^5d^2\right.}{}\right.}{}$ |
| default | $\frac{2\left(\frac{d^2\left(5a^3c^2d^3-2a^3d^5-9a^2b\,c^3d^2+3a\,b^2c^4d+6a\,b^2c^2d^3+b^3c^5-4b^3c^3d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^4-2c^2d^2+d^4\right)c}+\frac{d\left(4a^3c^4d^3+7a^3c^2d^5-2a^3d^7-6a^2b\,c^5d^2\right.}{}\right.}{}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/d^3*((1/2*d^2*(5*a^3*c^2*d^3-2*a^3*d^5-9*a^2*b*c^3*d^2+3*a*b^2*c^4*d
+6*a*b^2*c^2*d^3+b^3*c^5-4*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1
/2*e)^3+1/2*d*(4*a^3*c^4*d^3+7*a^3*c^2*d^5-2*a^3*d^7-6*a^2*b*c^5*d^2-15*a^2
*b*c^3*d^4-6*a^2*b*c*d^6+9*a*b^2*c^4*d^3+18*a*b^2*c^2*d^5+2*b^3*c^7-b^3*c^5
*d^2-10*b^3*c^3*d^4)/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(
11*a^3*c^2*d^3-2*a^3*d^5-15*a^2*b*c^3*d^2-12*a^2*b*c*d^4-3*a*b^2*c^4*d+30*a
*b^2*c^2*d^3+7*b^3*c^5-16*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/
2*e)+1/2*d*(4*a^3*c^2*d^3-a^3*d^5-6*a^2*b*c^3*d^2-3*a^2*b*c*d^4+9*a*b^2*c^2
*d^3+2*b^3*c^5-5*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4))/(c*tan(1/2*f*x+1/2*e)^2+
2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(2*a^3*c^2*d^3+a^3*d^5-9*a^2*b*c*d^4+3*a*b^
2*c^2*d^3+6*a*b^2*d^5-2*b^3*c^5+5*b^3*c^3*d^2-6*b^3*c*d^4)/(c^4-2*c^2*d^2+d
^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
))+2*b^3/d^3*arctan(tan(1/2*f*x+1/2*e)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(252) = 504$.

time = 0.48, size = 1732, normalized size = 6.79

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
[Out] [1/4*(4*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 4*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos
(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2)) - 2*(2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*d^8 - (6*a^2*b +
7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*b^3)*c^3*d^5 - (5
*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - 2*(4*(b^3*c^7*d - 3*b^3*c^5*d^3 + 3
*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d^3 + 2*a^2*b*d^
8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2*b - 2*b^3)*c^2
*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((c^6*d^5 - 3*c^4
*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c^5*d^6 + 3*c^3*
d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*d^9 - d^11)*f),
 1/2*(2*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 2*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e)
 + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*
d^8 - (6*a^2*b + 7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*
b^3)*c^3*d^5 - (5*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - (4*(b^3*c^7*d - 3*
b^3*c^5*d^3 + 3*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d
^3 + 2*a^2*b*d^8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2
*b - 2*b^3)*c^2*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((
c^6*d^5 - 3*c^4*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c
^5*d^6 + 3*c^3*d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*
d^9 - d^11)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(252) = 504.
time = 0.54, size = 889, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^3/d^3 - (2*b^3*c^5 - 5*b^3*c^3*d^2 - 2*a^3*c^2*d^3 - 3*a*b^2*c
^2*d^3 + 9*a^2*b*c*d^4 + 6*b^3*c*d^4 - a^3*d^5 - 6*a*b^2*d^5)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^4*d^3 - 2*c^2*d^5 + d^7)*sqrt(c^2 - d^2)) + (b^3*c^6*d*tan(1/2
*f*x + 1/2*e)^3 + 3*a*b^2*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 - 9*a^2*b*c^4*d^3*
tan(1/2*f*x + 1/2*e)^3 - 4*b^3*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d
^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*b^2*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*
c*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c^7*tan(1/2*f*x + 1/2*e)^2 - 6*a^2*b*c
^5*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*a^3*
c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*a*b^2*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 - 1
5*a^2*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 10*b^3*c^3*d^4*tan(1/2*f*x + 1/2*e
)^2 + 7*a^3*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a*b^2*c^2*d^5*tan(1/2*f*x +
 1/2*e)^2 - 6*a^2*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^7*tan(1/2*f*x +
1/2*e)^2 + 7*b^3*c^6*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^5*d^2*tan(1/2*f*x +
 1/2*e) - 15*a^2*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 16*b^3*c^4*d^3*tan(1/2*f*
x + 1/2*e) + 11*a^3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 30*a*b^2*c^3*d^4*tan(1/2
*f*x + 1/2*e) - 12*a^2*b*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*a^3*c*d^6*tan(1/2
*f*x + 1/2*e) + 2*b^3*c^7 - 6*a^2*b*c^5*d^2 - 5*b^3*c^5*d^2 + 4*a^3*c^4*d^3
 + 9*a*b^2*c^4*d^3 - 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)/((c^6*d^2 - 2*c^4*d^4 +
 c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

**Mupad** [B]
time = 20.46, size = 2500, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((a^3*d^5 - 2*b^3*c^5 - 4*a^3*c^2*d^3 + 5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 +
 6*a^2*b*c^3*d^2 + 3*a^2*b*c*d^4)/(d^2*(c^4 + d^4 - 2*c^2*d^2)) - (tan(e/2
```

+ (f*x)/2)^3*(b^3*c^5 - 2*a^3*d^5 + 5*a^3*c^2*d^3 - 4*b^3*c^3*d^2 + 6*a*b^2
*c^2*d^3 - 9*a^2*b*c^3*d^2 + 3*a*b^2*c^4*d))/(c*d*(c^4 + d^4 - 2*c^2*d^2))
+ (tan(e/2 + (f*x)/2)*(2*a^3*d^5 - 7*b^3*c^5 - 11*a^3*c^2*d^3 + 16*b^3*c^3*
d^2 - 30*a*b^2*c^2*d^3 + 15*a^2*b*c^3*d^2 + 3*a*b^2*c^4*d + 12*a^2*b*c*d^4)
)/(c*d*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(a^3*
d^5 - 2*b^3*c^5 - 4*a^3*c^2*d^3 + 5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 + 6*a^2*b
*c^3*d^2 + 3*a^2*b*c*d^4))/(c^2*d^2*(c^4 + d^4 - 2*c^2*d^2)))/(f*(tan(e/2 +
(f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/
2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (2*b^3*atan(((b^3*((8*(4*b^6*
c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^
2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f
*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6
*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11
*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^
8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*
b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6
- 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^
3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12
+ 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (b^3*((b^3*((8*(4*c^2*d^16 - 16*c^4*d
^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d
^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16
+ 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^1
2 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3 - (8*(4*b^3*c*d^14 - 2*a^3*c
^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 -
4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*
a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/
(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/
2)*(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b
^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*
c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^
2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^
8 + c^8*d^6))*1i)/d^3))/d^3 + (b^3*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 2
4*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^
4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*
d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 +
105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d^11 + 24
*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*d^12 + 1
2*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 144*a^2*b^4*c^3*d
^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a
^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d
^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^
8*d^6) + (b^3*((8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^
8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 1
2*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12
- 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4

*c^6*d^7 + c^8*d^5) + (b^3*((8*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16
*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d
^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^
7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6
*d^8 + c^8*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*c^5
*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d^10
 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 -
 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*d^1
5))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3))/d^3)/
((16*(24*b^9*c^3*d^6 - 2*b^9*c^9 - 26*b^9*c^5*d^4 + 13*b^9*c^7*d^2 - 60*a*b
^8*c^2*d^7 + 6*a*b^8*c^4*d^5 + 6*a*b^8*c^6*d^3 + 36*a^2*b^7*c*d^8 - 4*a^3*b
^6*c^8*d + 12*a^4*b^5*c*d^8 + a^6*b^3*c*d^8 + 126*a^2*b^7*c^3*d^6 - 45*a^2*
b^7*c^5*d^4 + 18*a^2*b^7*c^7*d^2 - 118*a^3*b^6*c^2*d^7 - 68*a^3*b^6*c^4*d^5
 + 10*a^3*b^6*c^6*d^3 + 111*a^4*b^5*c^3*d^6 + 12*a^4*b^5*c^5*d^4 - 18*a^5*b
^4*c^2*d^7 - 36*a^5*b^4*c^4*d^5 + 4*a^6*b^3*c^3*d^6 + 4*a^6*b^3*c^5*d^4 - 6
*a*b^8*c^8*d))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (b^3
*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4
*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*
tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*
d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b...

# 3.693 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

**Optimal**. Leaf size=325

$$-\frac{(ac-bd)\left(10abcd-b^2(3c^2+2d^2)-a^2(2c^2+3d^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}f}+\frac{(bc-ad)^2\cos(e+fx)(a+}{3d\left(c^2-d^2\right)f(c+d\sin(}$$

```
[Out] -(a*c-b*d)*(10*a*b*c*d-b^2*(3*c^2+2*d^2)-a^2*(2*c^2+3*d^2))*arctan((d+c*tan
(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(7/2)/f+1/3*(-a*d+b*c)^2*cos(f*
x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^3+1/6*(-a*d+b*c)^2*(5*
a*c*d+2*b*c^2-7*b*d^2)*cos(f*x+e)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^2-1/6*
(-a*d+b*c)*(5*a*b*c*d*(c^2-7*d^2)+a^2*d^2*(11*c^2+4*d^2)+b^2*(2*c^4-5*c^2*d
^2+18*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^3/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 0.53, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3100, 2833, 12, 2739, 632, 210}

$$-\frac{(ac-bd)\left(-\left(a^2(2c^2+3d^2)\right)+10abcd-b^2(3c^2+2d^2)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}-\frac{\left(a^2d^2(11c^2+4d^2)+5abcd(c^2-7d^2)+b^2(2c^4-5c^2d^2+18d^4)\right)(bc-ad)\cos(e+fx)}{6d^2f\left(c^2-d^2\right)^3(c+d\sin(e+fx))}+\frac{(5acd+2bc^2-7bd^2)(bc-ad)^2\cos(e+fx)}{6d^2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))^2}+\frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((a*c - b*d)*(10*a*b*c*d - b^2*(3*c^2 + 2*d^2) - a^2*(2*c^2 + 3*d^2))*Arc
Tan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((b
*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c + d*Si
n[e + f*x])^3) + ((b*c - a*d)^2*(2*b*c^2 + 5*a*c*d - 7*b*d^2)*Cos[e + f*x])
/(6*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(5*a*b*c*d*(
c^2 - 7*d^2) + a^2*d^2*(11*c^2 + 4*d^2) + b^2*(2*c^4 - 5*c^2*d^2 + 18*d^4))
*Cos[e + f*x])/(6*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} - \frac{\int \frac{b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2-(7a^2bcd+3b}{}}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\,(2bc^2+5acd-7bd^2)\,\mathrm{c}}{6d^2\,(c^2-d^2)^2\,f(c+d\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\,(2bc^2+5acd-7bd^2)\,\mathrm{c}}{6d^2\,(c^2-d^2)^2\,f(c+d\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\,(2bc^2+5acd-7bd^2)\,\mathrm{c}}{6d^2\,(c^2-d^2)^2\,f(c+d\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\,(2bc^2+5acd-7bd^2)\,\mathrm{c}}{6d^2\,(c^2-d^2)^2\,f(c+d\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\,(2bc^2+5acd-7bd^2)\,\mathrm{c}}{6d^2\,(c^2-d^2)^2\,f(c+d\sin(e+}$$

$$= -\frac{(ac-bd)\,(10abcd-b^2(3c^2+2d^2)-a^2(2c^2+3d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{7/2}\,f}$$

**Mathematica** [**A**]
time = 5.33, size = 345, normalized size = 1.06

$$\frac{\frac{6\left(-3a^2bd(4c^2+d^2)-b^3d(3c^2+2d^2)+3ab^2c(c^2+4d^2)+a^3(2c^3+3cd^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{7/2}} + \frac{2(bc-ad)^3\cos(e+fx)}{d^2(-c^2+d^2)(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(4bc^2+5acd-9bd^2)\cos(e+fx)}{d^2(c^2-d^2)^2(c+d\sin(e+fx))^2} + \frac{\left(-a^3d^3(11c^2+4d^2)+3a^2bcd^2(2c^2+13d^2)+3ab^2d(c^4-10c^2d^2-6d^4)+b^3(2c^5-5c^3d^2+18cd^4)\right)\cos(e+fx)}{d^2(-c^2+d^2)^3(c+d\sin(e+fx))}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((6*(-3*a^2*b*d*(4*c^2 + d^2) - b^3*d*(3*c^2 + 2*d^2) + 3*a*b^2*c*(c^2 + 4*
d^2) + a^3*(2*c^3 + 3*c*d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^
2]])/(c^2 - d^2)^(7/2) + (2*(b*c - a*d)^3*Cos[e + f*x])/(d^2*(-c^2 + d^2)*(
c + d*Sin[e + f*x])^3) + ((b*c - a*d)^2*(4*b*c^2 + 5*a*c*d - 9*b*d^2)*Cos[e
 + f*x])/(d^2*(c^2 - d^2)^2*(c + d*Sin[e + f*x])^2) + ((-(a^3*d^3*(11*c^2 +
 4*d^2)) + 3*a^2*b*c*d^2*(2*c^2 + 13*d^2) + 3*a*b^2*d*(c^4 - 10*c^2*d^2 - 6
*d^4) + b^3*(2*c^5 - 5*c^3*d^2 + 18*c*d^4))*Cos[e + f*x])/(d^2*(-c^2 + d^2)
^3*(c + d*Sin[e + f*x])))/(6*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(314) = 628$.
time = 1.26, size = 1154, normalized size = 3.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*(9*a^3*c^4*d^2-6*a^3*c^2*d^4+2*a^3*d^6-12*a^2*b*c^5*d-3*a^2*b*c
^3*d^3+3*a*b^2*c^6+12*a*b^2*c^4*d^2-3*b^3*c^5*d-2*b^3*c^3*d^3)/c/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5+1/2*(6*a^3*c^6*d+27*a^3*c^4*d^3-12
*a^3*c^2*d^5+4*a^3*d^7-6*a^2*b*c^7-42*a^2*b*c^5*d^2-33*a^2*b*c^3*d^4+6*a^2*
b*c*d^6+15*a*b^2*c^6*d+60*a*b^2*c^4*d^3-15*b^3*c^5*d^2-10*b^3*c^3*d^4)/(c^6
-3*c^4*d^2+3*c^2*d^4-d^6)/c^2*tan(1/2*f*x+1/2*e)^4+1/3/c^3*d*(54*a^3*c^6*d+
21*a^3*c^4*d^3-4*a^3*c^2*d^5+4*a^3*d^7-54*a^2*b*c^7-126*a^2*b*c^5*d^2-51*a^
2*b*c^3*d^4+6*a^2*b*c*d^6+117*a*b^2*c^6*d+96*a*b^2*c^4*d^3+12*a*b^2*c^2*d^5
-12*b^3*c^7-41*b^3*c^5*d^2-22*b^3*c^3*d^4)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*ta
n(1/2*f*x+1/2*e)^3+(6*a^3*c^6*d+20*a^3*c^4*d^3-3*a^3*c^2*d^5+2*a^3*d^7-6*a^
2*b*c^7-30*a^2*b*c^5*d^2-42*a^2*b*c^3*d^4+3*a^2*b*c*d^6+12*a*b^2*c^6*d+51*a
*b^2*c^4*d^3+12*a*b^2*c^2*d^5-2*b^3*c^7-6*b^3*c^5*d^2-17*b^3*c^3*d^4)/(c^6-
3*c^4*d^2+3*c^2*d^4-d^6)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*(27*a^3*c^4*d^2-4*a^3
*c^2*d^4+2*a^3*d^6-24*a^2*b*c^5*d-57*a^2*b*c^3*d^3+6*a^2*b*c*d^5-3*a*b^2*c^
6+66*a*b^2*c^4*d^2+12*a*b^2*c^2*d^4-5*b^3*c^5*d-20*b^3*c^3*d^3)/c/(c^6-3*c^
4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)+1/6*(18*a^3*c^4*d-5*a^3*c^2*d^3+2*a
^3*d^5-18*a^2*b*c^5-30*a^2*b*c^3*d^2+3*a^2*b*c*d^4+39*a*b^2*c^4*d+6*a*b^2*c
^2*d^3-4*b^3*c^5-11*b^3*c^3*d^2)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6))/(c*tan(1/2*
f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^3+(2*a^3*c^3+3*a^3*c*d^2-12*a^2*b*c^
2*d-3*a^2*b*d^3+3*a*b^2*c^3+12*a*b^2*c*d^2-3*b^3*c^2*d-2*b^3*d^3)/(c^6-3*c^
4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(322) = 644$.
time = 0.49, size = 2165, normalized size = 6.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)*c^5*d^2 - 11*(a^3
+ 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^3 + 12*a*b^2)*c^2*d
^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^7)*cos(f*x + e)^3 -
 6*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d - 3*(3*a^3 + 10*a*b^2
)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b^2)*c^3*d^4 - 3*(10
*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e)
- 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3*(3*a^3 + 7*a*b^2)*
c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2)*c^2*d^4 - 3*(3*a^
2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4*a^2*b + b^3)*c^3*d
^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5)*cos(f*x + e)^2 +
(3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 + (11*a^3 + 39*a*b^2
)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*
b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b + b^3)*c^2*d^4 + 3
*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x + e)^2)*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(3*a^2*b*
c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)
*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*
d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10
)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8
+ 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos
(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d
^11)*f)*sin(f*x + e)), -1/6*((2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)
*c^5*d^2 - 11*(a^3 + 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^
3 + 12*a*b^2)*c^2*d^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^
7)*cos(f*x + e)^3 - 3*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d -
3*(3*a^3 + 10*a*b^2)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b
^2)*c^3*d^4 - 3*(10*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x
 + e)*sin(f*x + e) - 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3
*(3*a^3 + 7*a*b^2)*c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2
)*c^2*d^4 - 3*(3*a^2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4
*a^2*b + b^3)*c^3*d^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5
)*cos(f*x + e)^2 + (3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 +
 (11*a^3 + 39*a*b^2)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b
^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b
 + b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x
+ e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^
2 - d^2)*cos(f*x + e))) - 6*(3*a^2*b*c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^
4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*
a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7
*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 -
 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5
+ 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 +
 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x + e))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1677 vs. 2(322) = 644.
time = 0.54, size = 1677, normalized size = 5.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3*c^3 + 3*a*b^2*c^3 - 12*a^2*b*c^2*d - 3*b^3*c^2*d + 3*a^3*c*d^
2 + 12*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*b^3*d^3)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 -
 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2 - d^2)) + (9*a*b^2*c^8*tan(1/2*f*x +
 1/2*e)^5 - 36*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 9*b^3*c^7*d*tan(1/2*f*x
 + 1/2*e)^5 + 27*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(
1/2*f*x + 1/2*e)^5 - 9*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^5*d^3
*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*c^2
*d^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^4 + 18*a^3*
c^7*d*tan(1/2*f*x + 1/2*e)^4 + 45*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 - 126*
a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^
4 + 81*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 + 180*a*b^2*c^5*d^3*tan(1/2*f*x +
 1/2*e)^4 - 99*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 - 30*b^3*c^4*d^4*tan(1/
2*f*x + 1/2*e)^4 - 36*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 18*a^2*b*c^2*d^6
*tan(1/2*f*x + 1/2*e)^4 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 108*a^2*b*c
^7*d*tan(1/2*f*x + 1/2*e)^3 - 24*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^3
*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 + 234*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
- 252*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 82*b^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 42*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 192*a*b^2*c^4*d^4*tan(1/2
*f*x + 1/2*e)^3 - 102*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 44*b^3*c^3*d^5
*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 24*a*b^2*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*b*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*a
^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^2 - 12*b^
3*c^8*tan(1/2*f*x + 1/2*e)^2 + 36*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 72*a*b
^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 - 180*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2
- 36*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^3*c^5*d^3*tan(1/2*f*x + 1/2
```

```
*e)^2 + 306*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 252*a^2*b*c^4*d^4*tan(1/
2*f*x + 1/2*e)^2 - 102*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^3*c^3*d^5*
tan(1/2*f*x + 1/2*e)^2 + 72*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a^2*b
*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 9*a
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 72*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e) - 15*b^
3*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 198*a*
b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 171*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e) -
 60*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)
+ 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/
2*e) + 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^8 - 4*b^3*c^8 + 18*a
^3*c^7*d + 39*a*b^2*c^7*d - 30*a^2*b*c^6*d^2 - 11*b^3*c^6*d^2 - 5*a^3*c^5*d
^3 + 6*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 + 2*a^3*c^3*d^5)/((c^9 - 3*c^7*d^2 +
 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)
+ c)^3))/f
```

**Mupad [B]**
time = 11.80, size = 1423, normalized size = 4.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^4,x)
```

```
[Out] ((2*a^3*d^5 - 4*b^3*c^5 - 18*a^2*b*c^5 + 18*a^3*c^4*d - 5*a^3*c^2*d^3 - 11*
b^3*c^3*d^2 + 6*a*b^2*c^2*d^3 - 30*a^2*b*c^3*d^2 + 39*a*b^2*c^4*d + 3*a^2*b
*c*d^4)/(3*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) - (tan(e/2 + (f*x)/2)^5*(3*
b^3*c^5*d - 3*a*b^2*c^6 - 2*a^3*d^6 + 6*a^3*c^2*d^4 - 9*a^3*c^4*d^2 + 2*b^3
*c^3*d^3 - 12*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + 12*a^2*b*c^5*d))/(c*(c^6 -
d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (2*tan(e/2 + (f*x)/2)^2*(2*a^3*d^7 - 2*b^3*
c^7 - 6*a^2*b*c^7 + 6*a^3*c^6*d - 3*a^3*c^2*d^5 + 20*a^3*c^4*d^3 - 17*b^3*c
^3*d^4 - 6*b^3*c^5*d^2 + 12*a*b^2*c^2*d^5 + 51*a*b^2*c^4*d^3 - 42*a^2*b*c^3
*d^4 - 30*a^2*b*c^5*d^2 + 12*a*b^2*c^6*d + 3*a^2*b*c*d^6))/(c^2*(c^6 - d^6
+ 3*c^2*d^4 - 3*c^4*d^2)) - (tan(e/2 + (f*x)/2)*(3*a*b^2*c^6 - 2*a^3*d^6 +
5*b^3*c^5*d + 4*a^3*c^2*d^4 - 27*a^3*c^4*d^2 + 20*b^3*c^3*d^3 - 12*a*b^2*c^
2*d^4 - 66*a*b^2*c^4*d^2 + 57*a^2*b*c^3*d^3 - 6*a^2*b*c*d^5 + 24*a^2*b*c^5*
d))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (tan(e/2 + (f*x)/2)^4*(4*a^3*
d^7 - 6*a^2*b*c^7 + 6*a^3*c^6*d - 12*a^3*c^2*d^5 + 27*a^3*c^4*d^3 - 10*b^3*
c^3*d^4 - 15*b^3*c^5*d^2 + 60*a*b^2*c^4*d^3 - 33*a^2*b*c^3*d^4 - 42*a^2*b*c
^5*d^2 + 15*a*b^2*c^6*d + 6*a^2*b*c*d^6))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)) + (2*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(2*a^3*d^5 - 4*b^3*c^5
 - 18*a^2*b*c^5 + 18*a^3*c^4*d - 5*a^3*c^2*d^3 - 11*b^3*c^3*d^2 + 6*a*b^2*c
^2*d^3 - 30*a^2*b*c^3*d^2 + 39*a*b^2*c^4*d + 3*a^2*b*c*d^4))/(3*c^3*(c^6 -
d^6 + 3*c^2*d^4 - 3*c^4*d^2)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*
x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(
e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*
```

```
c^2*d*tan(e/2 + (f*x)/2)^5)) + (atan((((c*tan(e/2 + (f*x)/2)*(a*c - b*d)*(2
*a^2*c^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/((c + d)^(7/2)*
(c - d)^(7/2)) + ((a*c - b*d)*(2*c^6*d - 2*d^7 + 6*c^2*d^5 - 6*c^4*d^3)*(2*
a^2*c^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/(2*(c + d)^(7/2)
*(c - d)^(7/2)*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)))*(c^6 - d^6 + 3*c^2*d^4
 - 3*c^4*d^2))/(2*a^3*c^3 - 2*b^3*d^3 + 3*a*b^2*c^3 - 3*a^2*b*d^3 + 3*a^3*c
*d^2 - 3*b^3*c^2*d + 12*a*b^2*c*d^2 - 12*a^2*b*c^2*d))*(a*c - b*d)*(2*a^2*c
^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/(f*(c + d)^(7/2)*(c -
 d)^(7/2))
```

# 3.694 $\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=54

$$\frac{Bx}{b}-\frac{2\sqrt{a^2-b^2}\,B\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}$$

```
[Out] B*x/b-2*B*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a/b
```

**Rubi** [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2814, 2739, 632, 210}

$$\frac{Bx}{b}-\frac{2B\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a^2 - b^2]*B*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a
*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx &= \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\sin(x)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{\left(2\left(aB-\frac{b^2B}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{\left(4\left(aB-\frac{b^2B}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}-\frac{2\sqrt{a^2-b^2}\,B\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}\end{aligned}$$

**Mathematica [A]**
time = 0.08, size = 52, normalized size = 0.96

$$\frac{B\left(ax-2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
[Out] (B*(a*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]))/(a*b
)
```

**Maple [A]**
time = 0.12, size = 67, normalized size = 1.24

| method | result | size |
|---|---|---|
| default | $\frac{2B\left(\frac{a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b}+\frac{\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}\right)}{a}$ | 67 |
| risch | $\frac{Bx}{b}+\frac{\sqrt{-a^2+b^2}\,B\ln\left(e^{ix}-\frac{-ia+\sqrt{-a^2+b^2}}{b}\right)}{ba}-\frac{\sqrt{-a^2+b^2}\,B\ln\left(e^{ix}+\frac{ia+\sqrt{-a^2+b^2}}{b}\right)}{ba}$ | 102 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sin(x))/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/a*(a/b*arctan(tan(1/2*x))+(-a^2+b^2)/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.36, size = 163, normalized size = 3.02

$$\left[\frac{2\,Bax+\sqrt{-a^2+b^2}\,B\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\,ab},\ \frac{Bax+\sqrt{a^2-b^2}\,B\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\,\cos(x)}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*x + sqrt(-a^2 + b^2)*B*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(
x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(
x)^2 - 2*a*b*sin(x) - a^2 - b^2)))/(a*b), (B*a*x + sqrt(a^2 - b^2)*B*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))))/(a*b)]
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(41) = 82$.
time = 25.28, size = 180, normalized size = 3.33

$$\begin{cases} \text{NaN} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{B \cos{\left(x \right)}}{a} & \text{for}\: b = 0 \\ \frac{B x}{b} & \text{for}\: a = - \sqrt{b^{2}} \vee a = \sqrt{b^{2}} \\ - \frac{B a \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} - \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{b \sqrt{- a^{2} + b^{2}}} + \frac{B a \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} + \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{b \sqrt{- a^{2} + b^{2}}} + \frac{B x}{b} + \frac{B b \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} - \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{a \sqrt{- a^{2} + b^{2}}} - \frac{B b \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} + \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{a \sqrt{- a^{2} + b^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (-B*cos(x)/a, Eq(b, 0)), (B*x/b, Eq(a
, sqrt(b**2)) | Eq(a, -sqrt(b**2))), (-B*a*log(tan(x/2) + b/a - sqrt(-a**2
+ b**2)/a)/(b*sqrt(-a**2 + b**2)) + B*a*log(tan(x/2) + b/a + sqrt(-a**2 + b
**2)/a)/(b*sqrt(-a**2 + b**2)) + B*x/b + B*b*log(tan(x/2) + b/a - sqrt(-a**
2 + b**2)/a)/(a*sqrt(-a**2 + b**2)) - B*b*log(tan(x/2) + b/a + sqrt(-a**2 +
 b**2)/a)/(a*sqrt(-a**2 + b**2)), True))
```

**Giac** [A]
time = 0.49, size = 73, normalized size = 1.35

$$\frac{Bx}{b} - \frac{2\left(Ba^2 - Bb^2\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a^2 - B*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*x) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b)
```

**Mupad** [B]
time = 8.12, size = 94, normalized size = 1.74

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b} + \frac{2\,B\,\mathrm{atanh}\left(\frac{-\sin\left(\frac{x}{2}\right)\,a^2+\cos\left(\frac{x}{2}\right)\,a\,b+2\,\sin\left(\frac{x}{2}\right)\,b^2}{\sqrt{b^2-a^2}\,\left(2\,b\,\sin\left(\frac{x}{2}\right)+a\,\cos\left(\frac{x}{2}\right)\right)}\right)\,\sqrt{b^2-a^2}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sin(x) + (B*b)/a)/(a + b*sin(x)),x)
```

```
[Out] (2*B*atan(sin(x/2)/cos(x/2)))/b + (2*B*atanh((2*b^2*sin(x/2) - a^2*sin(x/2)
 + a*b*cos(x/2))/((b^2 - a^2)^(1/2)*(2*b*sin(x/2) + a*cos(x/2))))*(b^2 - a^
2)^(1/2))/(a*b)
```

# 3.695 $\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx$

**Optimal**. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

**Rubi** [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

**Mathematica** [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

**Maple** [**A**]
time = 0.05, size = 7, normalized size = 1.17

| method | result | size |
|---|---|---|
| default | $\frac{Bx}{b}$ | 7 |
| risch | $\frac{Bx}{b}$ | 7 |
| norman | $\frac{\frac{Bx}{b}+\frac{Bx\left(\tan^2\left(\frac{x}{2}\right)\right)}{b}}{1+\tan^2\left(\frac{x}{2}\right)}$ | 31 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sin(x))/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.32, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

**Sympy** [**A**]
time = 0.14, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] B*x/b
```

**Giac** [A]
time = 0.44, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

**Mupad** [B]
time = 7.70, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sin(x) + (B*a)/b)/(a + b*sin(x)),x)
```

```
[Out] (B*x)/b
```

# 3.696 $\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx$

**Optimal**. Leaf size=12

$$-\frac{\cos(x)}{b+a\sin(x)}$$

```
[Out] -cos(x)/(b+a*sin(x))
```

**Rubi** [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2833, 8}

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])/(b + a*Sin[x])^2,x]
```

```
[Out] -(Cos[x]/(b + a*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{b+a\sin(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= -\frac{\cos(x)}{b+a\sin(x)}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 12, normalized size = 1.00

$$-\frac{\cos(x)}{b+a\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])/(b + a*Sin[x])^2,x]
```

```
[Out] -(Cos[x]/(b + a*Sin[x]))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 0.12, size = 36, normalized size = 3.00

| method | result | size |
|---|---|---|
| default | $\frac{-\frac{a\tan\left(\frac{x}{2}\right)}{b}-1}{\frac{b\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+a\tan\left(\frac{x}{2}\right)+\frac{b}{2}}$ | 36 |
| risch | $-\frac{2\left(ia+b\,e^{ix}\right)}{a\left(a\,e^{2ix}-a+2ib\,e^{ix}\right)}$ | 40 |
| norman | $\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{2a\tan\left(\frac{x}{2}\right)}{b}-\frac{2a\left(\tan^3\left(\frac{x}{2}\right)\right)}{b}-2}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b\right)}$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x))/(b+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2*a/b*tan(1/2*x)-1/2)/(1/2*b*tan(1/2*x)^2+a*tan(1/2*x)+1/2*b)
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

**Fricas** [A]
time = 0.34, size = 12, normalized size = 1.00

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/(a*sin(x) + b)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))**2,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.
time = 0.43, size = 32, normalized size = 2.67

$$-\frac{2\left(a\tan\left(\frac{1}{2}x\right)+b\right)}{\left(b\tan\left(\frac{1}{2}x\right)^2+2\,a\tan\left(\frac{1}{2}x\right)+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*x) + b)/((b*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b)*b)
```

**Mupad** [**B**]
time = 7.87, size = 24, normalized size = 2.00

$$-\frac{a\,\sin\left(x\right)+b\left(\cos\left(x\right)+1\right)}{b\,\left(b+a\,\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x))/(b + a*sin(x))^2,x)
```

```
[Out] -(a*sin(x) + b*(cos(x) + 1))/(b*(b + a*sin(x)))
```

# 3.697 $\int \frac{2-\sin(x)}{2+\sin(x)}\,dx$

**Optimal**. Leaf size=34

$$-x+\frac{4x}{\sqrt{3}}+\frac{8\tan^{-1}\left(\frac{\cos(x)}{2+\sqrt{3}+\sin(x)}\right)}{\sqrt{3}}$$

```
[Out] -x+4/3*x*3^(1/2)+8/3*arctan(cos(x)/(2+sin(x)+3^(1/2)))*3^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2814, 2736}

$$\frac{8\text{ArcTan}\left(\frac{\cos(x)}{\sin(x)+\sqrt{3}+2}\right)}{\sqrt{3}}+\frac{4x}{\sqrt{3}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[x])/(2 + Sin[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[3] + (8*ArcTan[Cos[x]/(2 + Sqrt[3] + Sin[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sin(x)}{2+\sin(x)}\,dx &= -x+4\int \frac{1}{2+\sin(x)}\,dx\\ &= -x+\frac{4x}{\sqrt{3}}+\frac{8\tan^{-1}\left(\frac{\cos(x)}{2+\sqrt{3}+\sin(x)}\right)}{\sqrt{3}}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 28, normalized size = 0.82

$$-x+\frac{8\tan^{-1}\left(\frac{1+2\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[x])/(2 + Sin[x]),x]
[Out] -x + (8*ArcTan[(1 + 2*Tan[x/2])/Sqrt[3]])/Sqrt[3]
```

**Maple** [A]
time = 0.07, size = 28, normalized size = 0.82

| method | result | size |
|---|---|---|
| default | $-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{8\sqrt{3}\,\arctan\left(\frac{\left(2\tan\left(\frac{x}{2}\right)+1\right)\sqrt{3}}{3}\right)}{3}$ | 28 |
| risch | $-x+\frac{4i\sqrt{3}\,\ln\left(e^{ix}+2i+i\sqrt{3}\right)}{3}-\frac{4i\sqrt{3}\,\ln\left(e^{ix}+2i-i\sqrt{3}\right)}{3}$ | 47 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sin(x))/(2+sin(x)),x,method=_RETURNVERBOSE)
[Out] -2*arctan(tan(1/2*x))+8/3*3^(1/2)*arctan(1/3*(2*tan(1/2*x)+1)*3^(1/2))
```

**Maxima** [A]
time = 0.53, size = 36, normalized size = 1.06

$$\frac{8}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)\right)-2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="maxima")
[Out] 8/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sin(x)/(cos(x) + 1) + 1)) - 2*arctan(sin(
x)/(cos(x) + 1))
```

**Fricas** [A]
time = 0.34, size = 27, normalized size = 0.79

$$\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\sin\left(x\right)+\sqrt{3}}{3\,\cos\left(x\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*sin(x) + sqrt(3))/cos(x)) - x
```

**Sympy** [A]
time = 0.28, size = 42, normalized size = 1.24

$$-x+\frac{8\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x)
```

```
[Out] -x + 8*sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 + sqrt(3)/3) + pi*floor((x/2 - pi
/2)/pi))/3
```

**Giac** [A]
time = 0.54, size = 51, normalized size = 1.50

$$\frac{4}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin\left(x\right)-\cos\left(x\right)-2\sin\left(x\right)-1}{\sqrt{3}\cos\left(x\right)+\sqrt{3}-2\cos\left(x\right)+\sin\left(x\right)+2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - cos(x) - 2*sin(x) - 1)/(sqrt(3
)*cos(x) + sqrt(3) - 2*cos(x) + sin(x) + 2))) - x
```

**Mupad** [B]
time = 7.83, size = 36, normalized size = 1.06

$$-x-\frac{8\sqrt{3}\operatorname{atan}\left(-\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)-\sqrt{3}}{3\tan\left(\frac{x}{2}\right)+3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 2)/(sin(x) + 2),x)
```

```
[Out] - x - (8*3^(1/2)*atan(-(3^(1/2)*tan(x/2) - 3^(1/2))/(3*tan(x/2) + 3)))/3
```

# 3.698 $\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=235

$$\frac{d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\,x}{2b^4}+\frac{2(bc-ad)^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,f}+\frac{d^2(12abcd-$$

```
[Out] 1/2*d*(8*a^2*b*c*d^2-2*a^3*d^3+4*b^3*c*(2*c^2+d^2)-a*b^2*d*(12*c^2+d^2))*x/
b^4+1/3*d^2*(12*a*b*c*d-3*a^2*d^2-b^2*(17*c^2+2*d^2))*cos(f*x+e)/b^3/f-1/6*
d^3*(-3*a*d+8*b*c)*cos(f*x+e)*sin(f*x+e)/b^2/f-1/3*d^2*cos(f*x+e)*(c+d*sin(
f*x+e))^2/b/f+2*(-a*d+b*c)^4*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2
))/b^4/f/(a^2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.48, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2872, 3112, 3102, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4f\sqrt{a^2-b^2}}+\frac{d^2(-3a^2d^2+12abcd-(b^2(17c^2+2d^2)))\cos(e+fx)}{3b^3f}+\frac{dx(-2a^3d^3+8a^2bcd^2-ab^2d(12c^2+d^2)+4b^3c(2c^2+d^2))}{2b^4}-\frac{d^3(8bc-3ad)\sin(e+fx)\cos(e+fx)}{6b^2f}-\frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2 + d
^2))*x)/(2*b^4) + (2*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*Sqrt[a^2 - b^2]*f) + (d^2*(12*a*b*c*d - 3*a^2*d^2 - b^2*(17*
c^2 + 2*d^2))*Cos[e + f*x])/(3*b^3*f) - (d^3*(8*b*c - 3*a*d)*Cos[e + f*x]*S
in[e + f*x])/(6*b^2*f) - (d^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*b*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} + \frac{\int \frac{(c+d\sin(e+fx))(3bc^3+2ad^3+d(9bc^2-acd+2bd^2)\sin}{a+b\sin(e+fx)}}{3b}$$

$$= -\frac{d^3(8bc-3ad)\cos(e+fx)\sin(e+fx)}{6b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} +$$

$$= \frac{d^2(12abcd-3a^2d^2-b^2(17c^2+2d^2))\cos(e+fx)}{3b^3f} - \frac{d^3(8bc-3ad)\cos(e+fx)\,\mathrm{s}}{6b^2f}$$

$$= \frac{d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\,x}{2b^4} + \frac{d^2(12abcd-3a^2d^2}{}$$

$$= \frac{d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\,x}{2b^4} + \frac{d^2(12abcd-3a^2d^2}{}$$

$$= \frac{d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\,x}{2b^4} + \frac{d^2(12abcd-3a^2d^2}{}$$

$$= \frac{d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\,x}{2b^4} + \frac{2(bc-ad)^4\tan^{-1}\Big(}{b^4\sqrt{a^2-}}$$

## Mathematica [A]

time = 0.66, size = 203, normalized size = 0.86

$$\frac{-6d(-8a^2bcd^2+2a^3d^3-4b^3c(2c^2+d^2)+ab^2d(12c^2+d^2))(e+fx)+\frac{24(bc-ad)^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-3bd^2(-16abcd+4a^2d^2+3b^2(8c^2+d^2))\cos(e+fx)+b^3d^4\cos(3(e+fx))-3b^2d^3(4bc-ad)\sin(2(e+fx))}{12b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-6*d*(-8*a^2*b*c*d^2 + 2*a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2
 + d^2))*(e + f*x) + (24*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt
[a^2 - b^2]])/Sqrt[a^2 - b^2] - 3*b*d^2*(-16*a*b*c*d + 4*a^2*d^2 + 3*b^2*(8
*c^2 + d^2))*Cos[e + f*x] + b^3*d^4*Cos[3*(e + f*x)] - 3*b^2*d^3*(4*b*c - a
*d)*Sin[2*(e + f*x)])/(12*b^4*f)
```

## Maple [A]

time = 0.30, size = 375, normalized size = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{2\left(a^4d^4-4a^3bc\,d^3+6d^2a^2b^2c^2-4da\,b^3c^3+b^4c^4\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}} - \frac{2d\left(\left(\frac{1}{2}a\,b^2d^3-2b^3c\,d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(a^2b\,d^3-\right.\right.}{}$ |

| | |
|---|---|
| default | $\frac{\frac{2\left(a^4d^4-4a^3bc\,d^3+6d^2a^2b^2c^2-4da\,b^3c^3+b^4c^4\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{2d\left(\frac{\left(\frac{1}{2}a\,b^2d^3-2b^3c\,d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(a^2b\,d^3\right.}{}\right.}{}}{}$ |
| risch | $\frac{4d^3x\,a^2c}{b^3}-\frac{6d^2xa\,c^2}{b^2}-\frac{d^4\mathrm{e}^{i(fx+e)}a^2}{2b^3f}-\frac{3d^2\mathrm{e}^{i(fx+e)}c^2}{bf}-\frac{d^4\mathrm{e}^{-i(fx+e)}a^2}{2b^3f}-\frac{3d^2\mathrm{e}^{-i(fx+e)}c^2}{bf}-\frac{3d^4\mathrm{e}^{i(fx+e)}}{8bf}-$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^4/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))-2*
d/b^4*(((1/2*a*b^2*d^3-2*b^3*c*d^2)*tan(1/2*f*x+1/2*e)^5+(a^2*b*d^3-4*a*b^2
*c*d^2+6*b^3*c^2*d)*tan(1/2*f*x+1/2*e)^4+(2*a^2*b*d^3-8*a*b^2*c*d^2+12*b^3*
c^2*d+2*b^3*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*a*b^2*d^3+2*b^3*c*d^2)*tan(1/2*
f*x+1/2*e)+a^2*b*d^3-4*a*b^2*c*d^2+6*b^3*c^2*d+2/3*b^3*d^3)/(1+tan(1/2*f*x+
1/2*e)^2)^3+1/2*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2*c^2*d+a*b^2*d^3-8*b^3*c^3
-4*b^3*c*d^2)*arctan(tan(1/2*f*x+1/2*e))))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

## Fricas [A]

time = 0.41, size = 812, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^2*b^3 - b^5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 1
2*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 -
a^3*b^2 - a*b^4)*d^4)*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*
d^4)*cos(f*x + e)*sin(f*x + e) - 3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
```

```
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2)) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 +
 (a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f), 1/6*(2*(a^2*b^3 - b^
5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c
^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)
*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)*sin
(f*x + e) - 6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*co
s(f*x + e))) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 + (
a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(228) = 456.
time = 0.48, size = 465, normalized size = 1.98

$$\frac{\frac{3\left(8b^3c^3d-12ab^2c^2d^2+8a^2bcd^3+4b^3cd^3-2a^3d^4-ab^2d^4\right)(fx+e)}{b^4}+\frac{12\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(12b^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-3abd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-36b^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+24abcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-6a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-72b^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+48abcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12b^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12b^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3abd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-36b^2c^2d^2+24abcd^3-6a^2d^4-4b^2d^4\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^3b^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*(f*x + e)/b^4 + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^4) + 2*(12*b^2*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*a*b*d^4*tan(1/2*f*x + 1/2*
e)^5 - 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a*b*c*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^2 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*d^4*tan(1/2*f*x + 1/
2*e)^2 - 12*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*b^2*c*d^3*tan(1/2*f*x + 1/2
*e) + 3*a*b*d^4*tan(1/2*f*x + 1/2*e) - 36*b^2*c^2*d^2 + 24*a*b*c*d^3 - 6*a^
2*d^4 - 4*b^2*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*b^3))/f
```

**Mupad [B]**
time = 16.49, size = 2500, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x)),x)
```

```
[Out] - ((2*(3*a^2*d^4 + 2*b^2*d^4 + 18*b^2*c^2*d^2 - 12*a*b*c*d^3))/(3*b^3) + (t
an(e/2 + (f*x)/2)^5*(a*d^4 - 4*b*c*d^3))/b^2 + (4*tan(e/2 + (f*x)/2)^2*(a^2
*d^4 + b^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/b^3 + (2*tan(e/2 + (f*x)/2)^
4*(a^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/b^3 - (tan(e/2 + (f*x)/2)*(a*d^4
 - 4*b*c*d^3))/b^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 + t
an(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^
8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*
b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5
 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5
*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (((8*(4*a^2*b^10*c^4 + 2*a^2*b^1
0*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*
d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b
^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 +
 ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d
^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2
 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (
b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2
)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3
*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2
*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*
a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c
^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 +
416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b
^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*
c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i)*1i)/b^4 + (
((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5
*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 6
4*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*
c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))
/b^8 - (((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*
d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan
(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*
b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 - ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2
)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*
1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i
+ (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*
b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*
a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^1
1*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56
*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5
 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a
```

^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i)*1i)/b^4)/((16*(2*a^10*d^12 + a^8*b^2*d^12 + 8*a*b^9*c^9*d^3 - 12*a^7*b^3*c*d^11 + 16*a^2*b^8*c^6*d^6 - 2*a^2*b^8*c^8*d^4 - 88*a^2*b^8*c^10*d^2 - 72*a^3*b^7*c^5*d^7 - 128*a^3*b^7*c^7*d^5 + 208*a^3*b^7*c^9*d^3 + 129*a^4*b^6*c^4*d^8 + 416*a^4*b^6*c^6*d^6 - 276*a^4*b^6*c^8*d^4 - 116*a^5*b^5*c^3*d^9 - 640*a^5*b^5*c^5*d^7 + 224*a^5*b^5*c^7*d^5 + 54*a^6*b^4*c^2*d^10 + 584*a^6*b^4*c^4*d^8 - 112*a^6*b^4*c^6*d^6 - 336*a^7*b^3*c^3*d^9 + 32*a^7*b^3*c^5*d^7 + 120*a^8*b^2*c^2*d^10 - 4*a^8*b^2*c^4*d^8 + 16*a*b^9*c^11*d - 24*a^9*b*c*d^11))/b^8 + (((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*...

# 3.699 $\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=156

$$-\frac{d(6abcd-2a^2d^2-b^2(6c^2+d^2))\,x}{2b^3}+\frac{2(bc-ad)^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,f}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2$$

```
[Out] -1/2*d*(6*a*b*c*d-2*a^2*d^2-b^2*(6*c^2+d^2))*x/b^3-1/2*d^2*(-2*a*d+5*b*c)*c
os(f*x+e)/b^2/f-1/2*d^2*cos(f*x+e)*(c+d*sin(f*x+e))/b/f+2*(-a*d+b*c)^3*arct
an((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/f/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2872, 3102, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\sqrt{a^2-b^2}}-\frac{dx(-2a^2d^2+6abcd-(b^2(6c^2+d^2)))}{2b^3}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/2*(d*(6*a*b*c*d - 2*a^2*d^2 - b^2*(6*c^2 + d^2))*x)/b^3 + (2*(b*c - a*d)
^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]*f
) - (d^2*(5*b*c - 2*a*d)*Cos[e + f*x])/(2*b^2*f) - (d^2*Cos[e + f*x]*(c + d
*Sin[e + f*x]))/(2*b*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \frac{\int \frac{2bc^3+ad^3-d(acd-b(6c^2+d^2))\sin(e+fx)+d^2(5bc-}{a+b\sin(e+fx)}}{2b}$$

$$= -\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \frac{\int \frac{b(2bc^3+ad}{}}{}$$

$$= -\frac{d(6abcd-2a^2d^2-b^2(6c^2+d^2))\,x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+}{}$$

$$= -\frac{d(6abcd-2a^2d^2-b^2(6c^2+d^2))\,x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+}{}$$

$$= -\frac{d(6abcd-2a^2d^2-b^2(6c^2+d^2))\,x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+}{}$$

$$= -\frac{d(6abcd-2a^2d^2-b^2(6c^2+d^2))\,x}{2b^3} + \frac{2(bc-ad)^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,f} -$$

**Mathematica** [A]
time = 0.39, size = 138, normalized size = 0.88

$$\frac{2d(-6abcd+2a^2d^2+b^2(6c^2+d^2))\,(e+fx) + \frac{8(bc-ad)^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4bd^2(3bc-ad)\cos(e+fx) - b^2d^3\sin(2(e+fx))}{4b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*(e + f*x) + (8*(b*c - a*d
)^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*b
*d^2*(3*b*c - a*d)*Cos[e + f*x] - b^2*d^3*Sin[2*(e + f*x)])/(4*b^3*f)
```

**Maple** [A]
time = 0.27, size = 229, normalized size = 1.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2d\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(ab\,d^2-3b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+ab\,d^2-3b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2a^2d^2-6abcd+6b^2c^2+d^2b^2\right)\text{arcta}}{2}\right.}{b^3}}{f}$ |

| | |
|---|---|
| default | $\frac{2d\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(ab\,d^2-3b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+ab\,d^2-3b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2a^2d^2-6abcd+6b^2c^2+d^2b^2\right)\arctan}{2}\right.}{b^3}}{f}$ |
| risch | $\frac{d^3x\,a^2}{b^3}-\frac{3d^2xac}{b^2}+\frac{3dx\,c^2}{b}+\frac{d^3x}{2b}+\frac{d^3\mathrm{e}^{i(fx+e)}a}{2b^2f}-\frac{3d^2\mathrm{e}^{i(fx+e)}c}{2bf}+\frac{d^3\mathrm{e}^{-i(fx+e)}a}{2b^2f}-\frac{3d^2\mathrm{e}^{-i(fx+e)}c}{2bf}-\frac{\ln\left(\mathrm{e}^{i(\right.}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d/b^3*((1/2*d^2*b^2*tan(1/2*f*x+1/2*e)^3+(a*b*d^2-3*b^2*c*d)*tan(1/2
*f*x+1/2*e)^2-1/2*d^2*b^2*tan(1/2*f*x+1/2*e)+a*b*d^2-3*b^2*c*d)/(1+tan(1/2*
f*x+1/2*e)^2)^2+1/2*(2*a^2*d^2-6*a*b*c*d+6*b^2*c^2+b^2*d^2)*arctan(tan(1/2*
f*x+1/2*e)))+2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.40, size = 581, normalized size = 3.72

$$\left[-\frac{(a^2b^2-b^4)d^3\cos(fx+e)\sin(fx+e)-\left(6(a^2b^2-b^4)c^2d-6(a^3b-ab^3)cd^2+(2a^4-a^2b^2-b^4)d^3\right)fx-(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2-2(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2}\right)+2\left(3(a^2b^2-b^4)cd^2-(a^3b-ab^3)d^3\right)\cos(fx+e)}{2(a^2b^3-b^5)f},-\frac{(a^2b^2-b^4)d^3\cos(fx+e)\sin(fx+e)-\left(6(a^2b^2-b^4)c^2d-6(a^3b-ab^3)cd^2+(2a^4-a^2b^2-b^4)d^3\right)fx+2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)+2\left(3(a^2b^2-b^4)cd^2-(a^3b-ab^3)d^3\right)\cos(fx+e)}{2(a^2b^3-b^5)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b^2 - b^4)*d^3*cos(f*x + e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c
^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*f*x - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*log(-((2*a^2
- b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*
sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*
b*sin(f*x + e) - a^2 - b^2)) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)
```

```
*d^3)*cos(f*x + e))/((a^2*b^3 - b^5)*f), -1/2*((a^2*b^2 - b^4)*d^3*cos(f*x
+ e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2
*a^4 - a^2*b^2 - b^4)*d^3)*f*x + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*c
os(f*x + e))) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x +
 e))/((a^2*b^3 - b^5)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.50, size = 252, normalized size = 1.62

$$\frac{\frac{(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3)(fx+e)}{b^3}+\frac{4\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{2\left(bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,bcd^2+2\,ad^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*(f*x + e)/b^3 + 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*b^3) + 2*(b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*
x + 1/2*e)^2 + 2*a*d^3*tan(1/2*f*x + 1/2*e)^2 - b*d^3*tan(1/2*f*x + 1/2*e)
- 6*b*c*d^2 + 2*a*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^2))/f
```

**Mupad** [**B**]
time = 14.77, size = 2500, normalized size = 16.03

$$\text{Too large to display}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x)),x)
```

```
[Out] ((2*(a*d^3 - 3*b*c*d^2))/b^2 + (d^3*tan(e/2 + (f*x)/2)^3)/b + (2*tan(e/2 +
(f*x)/2)^2*(a*d^3 - 3*b*c*d^2))/b^2 - (d^3*tan(e/2 + (f*x)/2))/b)/(f*(2*tan
(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (atan((((a^2*d^3*1i + (b^2
*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4
*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 3
```

6*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(
e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 -
 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 2
4*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3
 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5
*b^4*c^2*d^4))/b^6 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*
3i)*((8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d
+ 24*a^3*b^7*c*d^2))/b^6 - (8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3
- 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 + ((32*a^2*b^3 + (8*tan(e/2 + (f*
x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/
2 - a*b*c*d^2*3i))/b^3))/b^3)*1i)/b^3 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)
*1i)/2 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 1
2*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^
2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(
2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 +
 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d
- 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c
^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b
^6 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(2*a*b^8
*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/
b^5 - (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d
 + 24*a^3*b^7*c*d^2))/b^6 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10
 - 8*a^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i
))/b^3))/b^3)*1i)/b^3)/((16*(2*a^7*d^9 + a^5*b^2*d^9 - 2*a*b^6*c^6*d^3 - 3*
a^4*b^3*c*d^8 + 4*a^6*b*c^3*d^6 - a^2*b^5*c^3*d^6 + 48*a^2*b^5*c^7*d^2 + 3*
a^3*b^4*c^2*d^7 + 18*a^3*b^4*c^4*d^5 - 76*a^3*b^4*c^6*d^3 - 36*a^4*b^3*c^3*
d^6 + 60*a^4*b^3*c^5*d^4 + 30*a^5*b^2*c^2*d^7 - 24*a^5*b^2*c^4*d^5 - 12*a*b
^6*c^8*d - 12*a^6*b*c*d^8))/b^5 - ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2
 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*
b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72
*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^
8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*
b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a
^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4
 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + (
(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*tan(e/2 + (f*x
)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c*d^2))/b
^6 - (8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 1
2*a*b^8*c^2*d))/b^5 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a
^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i))/b^3
))/b^3))/b^3 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((
8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*
b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 +
60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6
 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*

```
b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a
^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d
^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + ((a^2*d^3*1i + (b^2*
d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*
a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 - (8*tan(e/2 + (f*x)/
2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c*d^2))/b^6
 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6)*(a^2*
d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i))/b^3))/b^3))/b^3 + (16*
tan(e/2 + (f*x)/2)*(8*a^8*d^9 + 2*a^4*b^4*d^9 + 8*a^6*b^2*d^9 - 2*a*b^7*c^3
*d^6 - 24*a*b^7*c^5*d^4 - 72*a*b^7*c^7*d^2 - 6*a^3*b^5*c*d^8 - 48*a^5*b^3*c
*d^8 + 6*a^2*b^6*c^2*d^7 + 96*a^2*b^6*c^4*d^5 + 360*a^2*b^6*c^6*d^3 - 152*a
^3*b^5*c^3*d^6 - 768*a^3*b^5*c^5*d^4 + 120*a^4*b^4*c^2*d^7 + 912*a^4*b^4*c^
4*d^5 - 656*a^5*b^3*c^3*d^6 + 288*a^6*b^2*c^2*d^7 - 72*a^7*b*c*d^8))/b^6))*
(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*2i)/(b^3*f) + (ata
n((((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6
 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4
 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 + 60...
```

# 3.700 $\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=93

$$\frac{d(2bc-ad)x}{b^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,f}-\frac{d^2\cos(e+fx)}{bf}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2-d^2*cos(f*x+e)/b/f+2*(-a*d+b*c)^2*arctan((b+a*tan(1/2*
f*x+1/2*e))/(a^2-b^2)^(1/2))/b^2/f/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2825, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\sqrt{a^2-b^2}}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\cos(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b^2]*f) - (d^2*Cos[e + f*x])/(b*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx &= -\frac{d^2\cos(e+fx)}{bf}+\frac{\int \frac{bc^2+d(2bc-ad)\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b}\\
&= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{(bc-ad)^2\int \frac{1}{a+b\sin(e+fx)}\,dx}{b^2}\\
&= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\right.\right.}{b^2f}\\
&= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}-\frac{(4(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b\right.}{b^2f}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,f}-\frac{d^2\cos(e+fx)}{bf}
\end{aligned}
$$

**Mathematica [A]**
time = 0.18, size = 90, normalized size = 0.97

$$
\frac{d(2bc-ad)(e+fx)+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-bd^2\cos(e+fx)}{b^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*(e + f*x) + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - b*d^2*Cos[e + f*x])/(b^2*f)
```

**Maple [A]**
time = 0.20, size = 117, normalized size = 1.26

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2d\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(ad-2bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^2}+\frac{2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{f}$ |
| default | $\frac{-\frac{2d\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(ad-2bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^2}+\frac{2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{f}$ |
| risch | $-\frac{d^2xa}{b^2}+\frac{2dxc}{b}-\frac{d^2e^{i(fx+e)}}{2bf}-\frac{d^2e^{-i(fx+e)}}{2bf}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)a^2d^2}{\sqrt{-a^2+b^2}\,f\,b^2}+\frac{2\ln\left(e^{i(fx+e)}-\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*d/b^2*(b*d/(1+tan(1/2*f*x+1/2*e)^2)+(a*d-2*b*c)*arctan(tan(1/2*f*x+
1/2*e)))+2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 379, normalized size = 4.08

$$\left[-\frac{2\,(a^2b-b^3)d^2\cos(fx+e)-2\,(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)fx+(b^2c^2-2\,abcd+a^2d^2)\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2+2\,(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2}\right)}{2\,(a^2b^2-b^4)f},\ -\frac{(a^2b-b^3)d^2\cos(fx+e)-(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)fx+(b^2c^2-2\,abcd+a^2d^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)}{(a^2b^2-b^4)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d^2*cos(f*x + e) - 2*(2*(a^2*b - b^3)*c*d - (a^3 - a
*b^2)*d^2)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
```

```
e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2
*a*b*sin(f*x + e) - a^2 - b^2)))/((a^2*b^2 - b^4)*f), -((a^2*b - b^3)*d^2*c
os(f*x + e) - (2*(a^2*b - b^3)*c*d - (a^3 - a*b^2)*d^2)*f*x + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 -
 b^2)*cos(f*x + e))))/((a^2*b^2 - b^4)*f)]
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 4032 vs. $2(78) = 156$.
time = 186.31, size = 4032, normalized size = 43.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*sin(e))**2/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)),
 (2*b**2*c*d*f*x*tan(e/2 + f*x/2)**3/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*t
an(e/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) +
2*b**2*c*d*f*x*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/
2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + 4*b**
2*c*d*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*
x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + 4*b**2*c*d/
(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)*tan
(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) - b**2*d**2*f*x*tan(e/2 + f*x/2)**2/(b*
*3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/
2 + f*x/2)**2 - f*(b**2)**(3/2)) - b**2*d**2*f*x/(b**3*f*tan(e/2 + f*x/2)**
3 + b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2
)**(3/2)) - 2*b**2*d**2*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3
*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)
) + 2*b*c**2*sqrt(b**2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b
**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3
/2)) + 2*b*c**2*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f
*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) - 2*b*c*d*f*
x*sqrt(b**2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e
/2 + f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) - 2*b*
c*d*f*x*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) -
f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + b*d**2*f*x*sqrt(b*
*2)*tan(e/2 + f*x/2)**3/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/
2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + b*d**2*f*x*sq
rt(b**2)*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*
x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + 2*b*d**2*sq
rt(b**2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 +
 f*x/2) - f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)) + 4*b*d**2
*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) - f*(b**2
)**(3/2)*tan(e/2 + f*x/2)**2 - f*(b**2)**(3/2)), Eq(a, -sqrt(b**2))), (2*b*
```

```
*2*c*d*f*x*tan(e/2 + f*x/2)**3/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2
 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) + 2*b**2
*c*d*f*x*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*
x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) + 4*b**2*c*d*
tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) +
 f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) + 4*b**2*c*d/(b**3*
f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 +
 f*x/2)**2 + f*(b**2)**(3/2)) - b**2*d**2*f*x*tan(e/2 + f*x/2)**2/(b**3*f*t
an(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*
x/2)**2 + f*(b**2)**(3/2)) - b**2*d**2*f*x/(b**3*f*tan(e/2 + f*x/2)**3 + b*
*3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/
2)) - 2*b**2*d**2*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan
(e/2 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) - 2*
b*c**2*sqrt(b**2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*
tan(e/2 + f*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) -
 2*b*c**2*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2)
+ f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) + 2*b*c*d*f*x*sqrt
(b**2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f
*x/2) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) + 2*b*c*d*f*
x*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) + f*(b**
2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) - b*d**2*f*x*sqrt(b**2)*ta
n(e/2 + f*x/2)**3/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) + f
*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) - b*d**2*f*x*sqrt(b**
2)*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) +
 f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) - 2*b*d**2*sqrt(b**
2)*tan(e/2 + f*x/2)**2/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2
) + f*(b**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)) - 4*b*d**2*sqrt(
b**2)/(b**3*f*tan(e/2 + f*x/2)**3 + b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/
2)*tan(e/2 + f*x/2)**2 + f*(b**2)**(3/2)), Eq(a, sqrt(b**2))), ((c**2*x - 2
*c*d*cos(e + f*x)/f + d**2*x*sin(e + f*x)**2/2 + d**2*x*cos(e + f*x)**2/2 -
 d**2*sin(e + f*x)*cos(e + f*x)/(2*f))/a, Eq(b, 0)), (x*(c + d*sin(e))**2/(
a + b*sin(e)), Eq(f, 0)), ((c**2*log(tan(e/2 + f*x/2))*tan(e/2 + f*x/2)**2/
(f*tan(e/2 + f*x/2)**2 + f) + c**2*log(tan(e/2 + f*x/2))/(f*tan(e/2 + f*x/2
)**2 + f) + 2*c*d*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + 2*c
*d*f*x/(f*tan(e/2 + f*x/2)**2 + f) - 2*d**2/(f*tan(e/2 + f*x/2)**2 + f))/b,
 Eq(a, 0)), (a**2*d**2*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)*t
an(e/2 + f*x/2)**2/(b**2*f*sqrt(-a**2 + b**2)*t...
```

**Giac [A]**

time = 0.46, size = 134, normalized size = 1.44

$$\frac{\frac{(2\,bcd-ad^2)(fx+e)}{b^2}-\frac{2\,d^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)b}+\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*b*c*d - a*d^2)*(f*x + e)/b^2 - 2*d^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*b) +
 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^2))/f
```

**Mupad [B]**
time = 12.62, size = 2628, normalized size = 28.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x)),x)
```

```
[Out] - (2*d^2)/(b*f*(tan(e/2 + (f*x)/2)^2 + 1)) - (atan((((-(a + b)*(a - b))^(1/
2)*(a*d - b*c)^2*((32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^
2 - (32*tan(e/2 + (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b
^5*c^2*d^2 + 8*a^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b
^3*c^2*d^2))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^
2 + a^2*b^4*d^2 - 2*a*b^5*c*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 +
 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c
)^2*(32*a^2*b^3 + (32*tan(e/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4
- a^2*b^2)))/(b^4 - a^2*b^2))*1i)/(b^4 - a^2*b^2) - ((-(a + b)*(a - b))^(1/
2)*(a*d - b*c)^2*((32*tan(e/2 + (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b
^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*
d^3 + 10*a^3*b^3*c^2*d^2))/b^3 - (32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b
^3*c^2*d^2))/b^2 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^
2 + a^2*b^4*d^2 - 2*a*b^5*c*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 +
 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d))/b^3 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c
)^2*(32*a^2*b^3 + (32*tan(e/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4
- a^2*b^2)))/(b^4 - a^2*b^2))*1i)/(b^4 - a^2*b^2))/((64*tan(e/2 + (f*x)/2)*
(2*a^5*d^6 + 8*a*b^4*c^4*d^2 - 24*a^2*b^3*c^3*d^3 + 26*a^3*b^2*c^2*d^4 - 12
*a^4*b*c*d^5))/b^3 - (64*(a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 5*a^2*b^2*c^4*d^2
 - 2*a*b^3*c^5*d))/b^2 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^4*
b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^2 - (32*tan(e/2 + (f*x)/2)*
(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a^2*b^4*c*d^
3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b^3*c^2*d^2))/b^3 + ((-(a +
b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^2 + a^2*b^4*d^2 - 2*a*b^5*c
*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 + 2*a^3*b^4*d^2 - 4*a^2*b^5*
c*d))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(32*a^2*b^3 + (32*tan(e
/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4 - a^2*b^2)))/(b^4 - a^2*b^2
)))/(b^4 - a^2*b^2) + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*tan(e/2
+ (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a
^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b^3*c^2*d^2))/b^3
 - (32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^2 + ((-(a + b)*
```

(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^2 + a^2*b^4*d^2 - 2*a*b^5*c*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 + 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d))/b^3 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(32*a^2*b^3 + (32*tan(e/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4 - a^2*b^2)))/(b^4 - a^2*b^2)))/(b^4 - a^2*b^2)))*(-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*2i)/(f*(b^4 - a^2*b^2)) - (2*d*atan((64*a^4*d^6*tan(e/2 + (f*x)/2))/(64*a^4*d^6 + 128*a^4*c^2*d^4 - 512*a*b^3*c^3*d^3 - 512*a^3*b*c^3*d^3 + 768*a^2*b^2*c^2*d^4 + 576*a^2*b^2*c^4*d^2 - 128*a*b^3*c^5*d - 384*a^3*b*c*d^5) + (384*a^3*c*d^5*tan(e/2 + (f*x)/2))/(384*a^3*c*d^5 - (64*a^4*d^6)/b + 512*a^3*c^3*d^3 + 512*a*b^2*c^3*d^3 - 768*a^2*b*c^2*d^4 - 576*a^2*b*c^4*d^2 - (128*a^4*c^2*d^4)/b + 128*a*b^2*c^5*d) + (768*a^2*c^2*d^4*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3) + (576*a^2*c^4*d^2*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3) + (512*a^3*c^3*d^3*tan(e/2 + (f*x)/2))/(384*a^3*c*d^5 - (64*a^4*d^6)/b + 512*a^3*c^3*d^3 + 512*a*b^2*c^3*d^3 - 768*a^2*b*c^2*d^4 - 576*a^2*b*c^4*d^2 - (128*a^4*c^2*d^4)/b + 128*a*b^2*c^5*d) + (128*a^4*c^2*d^4*tan(e/2 + (f*x)/2))/(64*a^4*d^6 + 128*a^4*c^2*d^4 - 512*a*b^3*c^3*d^3 - 512*a^3*b*c^3*d^3 + 768*a^2*b^2*c^2*d^4 + 576*a^2*b^2*c^4*d^2 - 128*a*b^3*c^5*d - 384*a^3*b*c*d^5) - (128*a*b*c^5*d*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3) - (512*a*b*c^3*d^3*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3))*(a*d - 2*b*c))/(b^2*f)

# 3.701 $\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=65

$$\frac{dx}{b}+\frac{2(bc-ad)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,f}$$

```
[Out] d*x/b+2*(-a*d+b*c)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b/f/(a^
2-b^2)^(1/2)
```

**Rubi** [A]

time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2814, 2739, 632, 210}

$$\frac{2(bc-ad)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bf\sqrt{a^2-b^2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*x)/b + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/
(b*Sqrt[a^2 - b^2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+b\sin(e+fx)}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}\\ &= \frac{dx}{b}-\frac{(4(bc-ad))\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}\\ &= \frac{dx}{b}+\frac{2(bc-ad)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,f}\end{aligned}$$

**Mathematica [A]**
time = 0.12, size = 67, normalized size = 1.03

$$\frac{d(e+fx)+\frac{2(bc-ad)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
[Out] (d*(e + f*x) + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2])/(b*f)
```

**Maple [A]**
time = 0.15, size = 76, normalized size = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b}+\frac{2(-ad+bc)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{f}$ |

| | |
|---|---|
| default | $\frac{\frac{2d \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b}+\frac{2(-ad+bc)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{f}$ |
| risch | $\frac{dx}{b}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)ad}{\sqrt{-a^2+b^2}\,fb}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)c}{\sqrt{-a^2+b^2}\,f}+\frac{\ln\left(e^{i(fx+e)}+\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d/b*arctan(tan(1/2*f*x+1/2*e))+2*(-a*d+b*c)/b/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.40, size = 262, normalized size = 4.03

$$\left[\frac{2\,(a^2-b^2)dfx+\sqrt{-a^2+b^2}\,(bc-ad)\log\left(-\frac{(2\,a^2-b^2)\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2-2\,(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2}\right)}{2\,(a^2b-b^3)f},\ \frac{(a^2-b^2)dfx-\sqrt{a^2-b^2}\,(bc-ad)\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\,\cos(fx+e)}\right)}{(a^2b-b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*f*x + sqrt(-a^2 + b^2)*(b*c - a*d)*log(-((2*a^2 - b^2
)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f
*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin
(f*x + e) - a^2 - b^2)))/((a^2*b - b^3)*f), ((a^2 - b^2)*d*f*x - sqrt(a^2 -
 b^2)*(b*c - a*d)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e
))))/((a^2*b - b^3)*f)]
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(53) = 106$.

time = 41.16, size = 537, normalized size = 8.26

$$\begin{cases} \frac{\tilde{\infty}x(c+d\sin(e))}{\sin(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{cx-\frac{d\cos(e+fx)}{f}}{a} & \text{for } b=0 \\ \frac{x(c+d\sin(e))}{a+b\sin(e)} & \text{for } f=0 \\ \frac{b^2dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}+\frac{2b^2d}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}+\frac{2bc\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}-\frac{bdfx\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}} & \text{for } a=-\sqrt{b^2} \\ \frac{b^2dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}+\frac{2b^2d}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}-\frac{2bc\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}+\frac{bdfx\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}} & \text{for } a=\sqrt{b^2} \\ \frac{\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+dx}{b} & \text{for } a=0 \\ -\frac{ad\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{bf\sqrt{-a^2+b^2}}+\frac{ad\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{bf\sqrt{-a^2+b^2}}+\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}}-\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}}+\frac{dx}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*sin(e))/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((
c*x - d*cos(e + f*x)/f)/a, Eq(b, 0)), (x*(c + d*sin(e))/(a + b*sin(e)), Eq(
f, 0)), (b**2*d*f*x*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(
3/2)) + 2*b**2*d/(b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)) + 2*b*c*sqrt(b
**2)/(b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)) - b*d*f*x*sqrt(b**2)/(b**3
*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)), Eq(a, -sqrt(b**2))), (b**2*d*f*x*ta
n(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)) + 2*b**2*d/(b**3
*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)) - 2*b*c*sqrt(b**2)/(b**3*f*tan(e/2 +
 f*x/2) + f*(b**2)**(3/2)) + b*d*f*x*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2) +
f*(b**2)**(3/2)), Eq(a, sqrt(b**2))), ((c*log(tan(e/2 + f*x/2))/f + d*x)/b,
 Eq(a, 0)), (-a*d*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b*f*s
qrt(-a**2 + b**2)) + a*d*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b**2)/a)
/(b*f*sqrt(-a**2 + b**2)) + c*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**
2)/a)/(f*sqrt(-a**2 + b**2)) - c*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 +
b**2)/a)/(f*sqrt(-a**2 + b**2)) + d*x/b, True))
```

**Giac [A]**
time = 0.48, size = 86, normalized size = 1.32

$$\frac{\frac{(fx+e)d}{b}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(bc-ad)}{\sqrt{a^2-b^2}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/b + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))*(b*c - a*d)/(sqrt(a^2 - b^2)*b))/f
```

**Mupad [B]**
time = 10.01, size = 343, normalized size = 5.28

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f}-\frac{a\,\left(d\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}-d\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{b^2-a^2}\right)-b\,c\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}+b\,c\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{b^2-a^2}}{b\,f\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x)),x)

[Out] (2*d*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f) - (a*(d*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(-(a + b)*(a - b))^(1/2) - d*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(b^2 - a^2)^(1/2)) - b*c*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(-(a + b)*(a - b))^(1/2) + b*c*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(b^2 - a^2)^(1/2))/(b*f*(a^2 - b^2))

# 3.702 $\int \frac{1}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=47

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}$$

```
[Out] 2*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/f/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin(e+fx)}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= -\frac{4\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}$$

**Mathematica [A]**
time = 0.04, size = 47, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-1),x]
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

**Maple [A]**
time = 0.10, size = 47, normalized size = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$ | 47 |
| default | $\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$ | 47 |
| risch | $-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,f}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,f}$ | 133 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/f/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**A**]
time = 0.38, size = 199, normalized size = 4.23

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2+2(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2}\right)}{2(a^2-b^2)f}, -\frac{\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)}{\sqrt{a^2-b^2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x +
e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2
 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2))/((a^2 - b^2
)*f), -arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))/(sqrt(a
^2 - b^2)*f)]
```

**Sympy** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(37) = 74$.
time = 4.85, size = 185, normalized size = 3.94

$$\begin{cases} \frac{\tilde{\infty}x}{\sin(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{bf} & \text{for } a=0 \\ \frac{2\sqrt{b^2}}{b^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{x}{a+b\sin(e)} & \text{for } f=0 \\ \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e/2 + f*
x/2))/(b*f), Eq(a, 0)), (2*sqrt(b**2)/(b**2*f*tan(e/2 + f*x/2) - b*f*sqrt(b
**2)), Eq(a, -sqrt(b**2))), (-2*sqrt(b**2)/(b**2*f*tan(e/2 + f*x/2) + b*f*s
qrt(b**2)), Eq(a, sqrt(b**2))), (x/(a + b*sin(e)), Eq(f, 0)), (log(tan(e/2
+ f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(f*sqrt(-a**2 + b**2)) - log(tan(e/2
 + f*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(f*sqrt(-a**2 + b**2)), True))
```

**Giac** [A]
time = 0.53, size = 62, normalized size = 1.32

$$\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*f)
```

**Mupad** [B]
time = 7.79, size = 42, normalized size = 0.89

$$\frac{2\,\mathrm{atan}\left(\frac{b+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{f\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)),x)
```

```
[Out] (2*atan((b + a*tan(e/2 + (f*x)/2))/(a^2 - b^2)^(1/2)))/(f*(a^2 - b^2)^(1/2)
)
```

# 3.703 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=117

$$\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)f}-\frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)\sqrt{c^2-d^2}\,f}$$

```
[Out] 2*b*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c)/f/(a^2-b^2)
^(1/2)-2*d*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)/f/(c
^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2826, 2739, 632, 210}

$$\frac{2b\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)}-\frac{2d\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(b*
c - a*d)*f) - (2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c
- a*d)*Sqrt[c^2 - d^2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{b\int \frac{1}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{bc-ad}$$
$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\text{Subst}(}{}$$
$$= -\frac{(4b)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} -$$
$$= \frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)f} - \frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)\sqrt{c^2-d^2}\,f}$$

**Mathematica [A]**
time = 0.19, size = 104, normalized size = 0.89

$$\frac{\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{bcf-adf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2])/(b*c
*f - a*d*f)
```

**Maple [A]**
time = 0.45, size = 114, normalized size = 0.97

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(ad-bc)\sqrt{a^2-b^2}}+\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(ad-bc)\sqrt{c^2-d^2}}}{f}$ |
| default | $\frac{-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(ad-bc)\sqrt{a^2-b^2}}+\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(ad-bc)\sqrt{c^2-d^2}}}{f}$ |
| risch | $-\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)}{\sqrt{-c^2+d^2}\,(ad-bc)f}+\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)}{\sqrt{-c^2+d^2}\,(ad-bc)f}-\frac{b\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-}}{}\right.}{\sqrt{-a^2+}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*b/(a*d-b*c)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)
/(a^2-b^2)^(1/2))+2/(a*d-b*c)*d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 1.21, size = 1093, normalized size = 9.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - b^2)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
```

```
2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x
 + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b
^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^
3 - a*b^2)*d^3)*f), 1/2*(2*(a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*
(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f
*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)))/(((a^2*b - b^3)*c^3 - (a^3 -
a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), 1/2*((a^2 - b^2
)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
 d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(b*c^2 -
b*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*
x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 +
 (a^3 - a*b^2)*d^3)*f), ((a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (b*c^2 - b*d^2)*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))))/(((a^2*b - b^
3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.47, size = 145, normalized size = 1.24

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,(bc-ad)}-\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d}{(bc-ad)\sqrt{c^2-d^2}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*(b*c - a*d)) - (pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d
^2)))*d/((b*c - a*d)*sqrt(c^2 - d^2)))/f
```

**Mupad [B]**
time = 9.99, size = 2500, normalized size = 21.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)

[Out] (b*d^2*atan((b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - a^3*b^3*d^2*(b^2 - a^2)^(1/2)*2i - b^6*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*4i + b^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*4i - b^4*c*d*(b^2 - a^2)^(3/2)*1i + b^6*c*d*(b^2 - a^2)^(1/2)*1i + a*b^3*c^2*(b^2 - a^2)^(3/2)*1i - a*b^3*d^2*(b^2 - a^2)^(3/2)*1i + a*b^5*d^2*(b^2 - a^2)^(1/2)*1i + a^5*b*d^2*(b^2 - a^2)^(1/2)*1i - a^2*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*2i + a^2*b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*2i - a^4*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i + a^2*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^2*b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*9i + a^4*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*6i + a^2*b^2*c*d*(b^2 - a^2)^(3/2)*1i - a^2*b^4*c*d*(b^2 - a^2)^(1/2)*2i + a^4*b^2*c*d*(b^2 - a^2)^(1/2)*1i)/(a^7*d^2 - a*b^6*c^2 + 2*a^3*b^4*c^2 - a^5*b^2*c^2 + a^3*b^4*d^2 - 2*a^5*b^2*d^2 - 2*b^7*c^2*tan(e/2 + (f*x)/2) + 2*a^6*b*d^2*tan(e/2 + (f*x)/2) + 4*a^2*b^5*c^2*tan(e/2 + (f*x)/2) - 2*a^4*b^3*c^2*tan(e/2 + (f*x)/2) + 2*a^2*b^5*d^2*tan(e/2 + (f*x)/2) - 4*a^4*b^3*d^2*tan(e/2 + (f*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*c^2*atan((b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - a^3*b^3*d^2*(b^2 - a^2)^(1/2)*2i - b^6*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*4i + b^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*4i - b^4*c*d*(b^2 - a^2)^(3/2)*1i + b^6*c*d*(b^2 - a^2)^(1/2)*1i + a*b^3*c^2*(b^2 - a^2)^(3/2)*1i - a*b^3*d^2*(b^2 - a^2)^(3/2)*1i + a*b^5*d^2*(b^2 - a^2)^(1/2)*1i + a^5*b*d^2*(b^2 - a^2)^(1/2)*1i - a^2*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*2i + a^2*b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*2i - a^4*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i + a^2*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^2*b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*9i + a^4*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*6i + a^2*b^2*c*d*(b^2 - a^2)^(3/2)*1i - a^2*b^4*c*d*(b^2 - a^2)^(1/2)*2i + a^4*b^2*c*d*(b^2 - a^2)^(1/2)*1i)/(a^7*d^2 - a*b^6*c^2 + 2*a^3*b^4*c^2 - a^5*b^2*c^2 + a^3*b^4*d^2 - 2*a^5*b^2*d^2 - 2*b^7*c^2*tan(e/2 + (f*x)/2) + 2*a^6*b*d^2*tan(e/2 + (f*x)/2) + 4*a^2*b^5*c^2*tan(e/2 + (f*x)/2) - 2*a^4*b^3*c^2*tan(e/2 + (f*x)/2) + 2*a^2*b^5*d^2*tan(e/2 + (f*x)/2) - 4*a^4*b^3*d^2*tan(e/2 + (f*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) + (a^2*d*atan((a^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^3*d^3*(d^2 - c^2)^(1/2)*2i - a^2*d^6*tan(e/2

+ (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + b^2*d^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i - a*b*d^4*(d^2 - c^2)^(3/2)*1i + a*b*d^6*(d^2 - c^2)^(1/2)*1i + a^2*c*d^3*(d^2 - c^2)^(3/2)*1i - b^2*c*d^3*(d^2 - c^2)^(3/2)*1i + b^2*c*d^5*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - a^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*9i + b^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + a*b*c^2*d^2*(d^2 - c^2)^(3/2)*1i - a*b*c^2*d^4*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^2*(d^2 - c^2)^(1/2)*1i)/(b^2*c^7 - a^2*c*d^6 + 2*a^2*c^3*d^4 - a^2*c^5*d^2 + b^2*c^3*d^4 - 2*b^2*c^5*d^2 - 2*a^2*d^7*tan(e/2 + (f*x)/2) + 2*b^2*c^6*d*tan(e/2 + (f*x)/2) + 4*a^2*c^2*d^5*tan(e/2 + (f*x)/2) - 2*a^2*c^4*d^3*tan(e/2 + (f*x)/2) + 2*b^2*c^2*d^5*tan(e/2 + (f*x)/2) - 4*b^2*c^4*d^3*tan(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b^2*d*atan((a^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^3*d^3*(d^2 - c^2)^(1/2)*2i - a^2*d^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + b^2*d^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i - a*b*d^4*(d^2 - c^2)^(3/2)*1i + a*b*d^6*(d^2 - c^2)^(1/2)*1i + a^2*c*d^3*(d^2 - c^2)^(3/2)*1i - b^2*c*d^3*(d^2 - c^2)^(3/2)*1i + b^2*c*d^5*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - a^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*9i + b^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + a*b*c^2*d^2*(d^2 - c^2)^(3/2)*1i - a*b*c^2*d^4*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^2*(d^2 - c^2)^(1/2)*1i)/(b^2*c^7 - a^2*c*d^6 + 2*a^2*c^3*d^4 - a^2*c^5*d^2 + b^2*c^3*d^4 - 2*b^2*c^5*d^2 - 2...

# 3.704 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=185

$$\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^2 f}+\frac{2d(acd-b(2c^2-d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\left(c^2-d^2\right)^{3/2} f}-\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right) f(c+d\sin(e+fx))}$$

```
[Out] 2*d*(a*c*d-b*(2*c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/
(-a*d+b*c)^2/(c^2-d^2)^(3/2)/f-d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*s
in(f*x+e))+2*b^2*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c
)^2/f/(a^2-b^2)^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2881, 3080, 2739, 632, 210}

$$\frac{2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^2}+\frac{2d(acd-b(2c^2-d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}(bc-ad)^2}-\frac{d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^2*f) + (2*d*(a*c*d - b*(2*c^2 - d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*(c^2 - d^2)^(3/2)*f) - (d^2*Cos[e +
f*x])/((b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{d^2\cos(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{\int \frac{-acd+b(c^2-d^2)}{(a+b\sin(e+fx))}}{(bc-ad)}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{b^2\int \frac{1}{a+b\sin(e+fx}}{(bc-ad)^2}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{(2b^2)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))} - \frac{(4b^2)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^2 f} + \frac{2d(acd-b(2c^2-d^2))\tan^{-1}}{(bc-ad)^2\,(c^2-}$$

**Mathematica** [A]

time = 1.00, size = 165, normalized size = 0.89

$$\frac{\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{2d(acd+b(-2c^2+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}+\frac{d^2(-bc+ad)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{(bc-ad)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (2*d*(a*c*d + b*(-2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(3/2) + (d^2*(-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c +
 d)*(c + d*Sin[e + f*x])))/((b*c - a*d)^2*f)
```

## Maple [A]

time = 1.18, size = 234, normalized size = 1.26

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{a^2-b^2}}+\frac{2d\left(\frac{\frac{d^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c\left(c^2-d^2\right)}+\frac{d(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(acd-2bc^2+bd^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{\frac{3}{2}}}\right)}{(ad-bc)^2}}{f}$ |
| default | $\frac{\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{a^2-b^2}}+\frac{2d\left(\frac{\frac{d^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c\left(c^2-d^2\right)}+\frac{d(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(acd-2bc^2+bd^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{\frac{3}{2}}}\right)}{(ad-bc)^2}}{f}$ |
| risch | $-\frac{2d\left(id+c\,e^{i(fx+e)}\right)}{(c^2-d^2)(-ad+bc)f\left(2ic\,e^{i(fx+e)}+e^{2i(fx+e)}d-d\right)}-\frac{d^2\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)ac}{\sqrt{-c^2+d^2}\,(ad-bc)^2(c+d)(c-d)f}+\frac{2d\ln\left(e^{i(fx+\right.}}{\sqrt{-c^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))+2*d/(a*d-b*c)^2*((d^2*(a*d-b*c)/c/(c^2
-d^2)*tan(1/2*f*x+1/2*e)+d*(a*d-b*c)/(c^2-d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d
*tan(1/2*f*x+1/2*e)+c)+(a*c*d-2*b*c^2+b*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(179) = 358$.
time = 93.13, size = 2934, normalized size = 15.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d - 2*b^2*c^2*d^3 +
b^2*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 -
 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*
x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 -
b^2)) + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^
3 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin
(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(
f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqr
t(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(
(a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3
- a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5
*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a
^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d
^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4
- 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b
^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f),
1/2*(2*(2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3
 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(
f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos
(f*x + e))) - (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d - 2*b^2*c^2
*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x +
 e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b
*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) -
 a^2 - b^2)) - 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b -
b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(
a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b
^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 +
 (a^4 - a^2*b^2)*d^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*
```

```
b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3
 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^
2*b^2)*c*d^6)*f), -1/2*(2*(b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d
 - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*
x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*(a^2*b - b^3)*c^3*d - (a^3
 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 -
 a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*
c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x +
 e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 -
2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)
*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^
2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*
d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^
3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)
*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3
*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)
*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f), -((b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^
4 + (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arc
tan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - (2*(a^2*b - b^3
)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^
2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(c^2 - d
^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((a^2*b
 - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2
)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 +
(a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*
a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*s
in(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2
*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^
4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.50, size = 308, normalized size = 1.66

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{a^2-b^2}}-\frac{(2bc^2d-acd^2-bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(b^2c^4-2abc^3d+a^2c^2d^2-b^2c^2d^2+2abcd^3-a^2d^4)\sqrt{c^2-d^2}}-\frac{d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+cd^2}{(bc^4-ac^3d-bc^2d^2+acd^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a^2 - b^
2)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^2*c^4 - 2*a*b
*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 + 2*a*b*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)
) - (d^3*tan(1/2*f*x + 1/2*e) + c*d^2)/((b*c^4 - a*c^3*d - b*c^2*d^2 + a*c*
d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

**Mupad [B]**
time = 22.74, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*d^2)/((c^2 - d^2)*(a*d - b*c)) + (2*d^3*tan(e/2 + (f*x)/2))/(c*(c^2 - d
^2)*(a*d - b*c)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2))
 + (b^2*atan(((b^2*(b^2 - a^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 -
 a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*
b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d
^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b
^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a
^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^
2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 +
 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^
3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3
*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 -
a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^
4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3
*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((
32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*
d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5
*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^
2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3
*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7
*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d)
)/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^
5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d
^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10
+ 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^
6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*
```

a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) - (b^2*(b^2 - a^2)^(1/2)*((32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b...

# 3.705 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=284

$$\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^3f}+\frac{d(6abc^3d-a^2d^2(2c^2+d^2)-b^2(6c^4-5c^2d^2+2d^4))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^3\left(c^2-d^2\right)^{5/2}f}-$$

```
[Out] d*(6*a*b*c^3*d-a^2*d^2*(2*c^2+d^2)-b^2*(6*c^4-5*c^2*d^2+2*d^4))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/(c^2-d^2)^(5/2)/f-1/2*d^
2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2-1/2*d^2*(-3*a*c*d+5*
b*c^2-2*b*d^2)*cos(f*x+e)/(-a*d+b*c)^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+2*b^3
*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c)^3/f/(a^2-b^2)^
(1/2)
```

**Rubi** [A]
time = 0.73, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$\frac{d\left(-a^2d^2(2c^2+d^2)+6abc^3d-b^2(6c^4-5c^2d^2+2d^4)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^3}+\frac{2b^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^3}-\frac{d^2(-3acd+5bc^2-2bd^2)\cos(e+fx)}{2f\left(c^2-d^2\right)^2(bc-ad)^2(c+d\sin(e+fx))}-\frac{d^2\cos(e+fx)}{2f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^3*f) + (d*(6*a*b*c^3*d - a^2*d^2*(2*c^2 + d^2) - b^2*(6*c^4 - 5*
c^2*d^2 + 2*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c -
 a*d)^3*(c^2 - d^2)^(5/2)*f) - (d^2*Cos[e + f*x])/(2*(b*c - a*d)*(c^2 - d^2
)*f*(c + d*Sin[e + f*x])^2) - (d^2*(5*b*c^2 - 3*a*c*d - 2*b*d^2)*Cos[e + f*
x])/(2*(b*c - a*d)^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{d^2\cos(e+fx)}{2(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{\int \frac{-2(acd-b(c^2}{(}}{}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} - \frac{d^2(5bc^2 -}{2(bc-ad)^2\,(}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} - \frac{d^2(5bc^2 -}{2(bc-ad)^2\,(}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} - \frac{d^2(5bc^2 -}{2(bc-ad)^2\,(}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} - \frac{d^2(5bc^2 -}{2(bc-ad)^2\,(}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^3 f} + \frac{d(6abc^3d-a^2d^2(2c^2+d^2)-}{(b}$$

**Mathematica** [A]
time = 2.38, size = 263, normalized size = 0.93

$$\frac{\frac{4b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{2d(-6abc^3d+a^2d^2(2c^2+d^2)+b^2(6c^4-5c^2d^2+2d^4))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} - \frac{d^2(bc-ad)^2\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))^2} + \frac{d^2(bc-ad)(-5bc^2+3acd+2bd^2)\cos(e+fx)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))}}{2(bc-ad)^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 (2*d*(-6*a*b*c^3*d + a^2*d^2*(2*c^2 + d^2) + b^2*(6*c^4 - 5*c^2*d^2 + 2*d^
4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) - (
d^2*(b*c - a*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])^2) +
(d^2*(b*c - a*d)*(-5*b*c^2 + 3*a*c*d + 2*b*d^2)*Cos[e + f*x])/((c - d)^2*(c
 + d)^2*(c + d*Sin[e + f*x])))/(2*(b*c - a*d)^3*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(270) = 540$.
time = 3.65, size = 625, normalized size = 2.20

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2d\left(\frac{d^2\left(5a^2c^2d^2-2a^2d^4-12abc^3d+6abcd^3+7b^2c^4-4b^2c^2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^4-2c^2d^2+d^4\right)c}+\frac{d\left(4a^2c^4d^2+7a^2c^2d^4-2a^2d^6-10abc^5d-16abc^3d^3+\cdots\right.}{2\left(c^4-2c^2\cdots\right.}\cdots\right.}{\cdots}$ |
| default | $\frac{2d\left(\frac{d^2\left(5a^2c^2d^2-2a^2d^4-12abc^3d+6abcd^3+7b^2c^4-4b^2c^2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^4-2c^2d^2+d^4\right)c}+\frac{d\left(4a^2c^4d^2+7a^2c^2d^4-2a^2d^6-10abc^5d-16abc^3d^3+\cdots\right.}{2\left(c^4-2c^2\cdots\right.}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d/(a*d-b*c)^3*((1/2*d^2*(5*a^2*c^2*d^2-2*a^2*d^4-12*a*b*c^3*d+6*a*b*
c*d^3+7*b^2*c^4-4*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3+1
/2*d*(4*a^2*c^4*d^2+7*a^2*c^2*d^4-2*a^2*d^6-10*a*b*c^5*d-16*a*b*c^3*d^3+8*a
*b*c*d^5+6*b^2*c^6+9*b^2*c^4*d^2-6*b^2*c^2*d^4)/(c^4-2*c^2*d^2+d^4)/c^2*tan
(1/2*f*x+1/2*e)^2+1/2*d^2*(11*a^2*c^2*d^2-2*a^2*d^4-28*a*b*c^3*d+10*a*b*c*d
^3+17*b^2*c^4-8*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)+1/2*d
*(4*a^2*c^2*d^2-a^2*d^4-10*a*b*c^3*d+4*a*b*c*d^3+6*b^2*c^4-3*b^2*c^2*d^2)/(
c^4-2*c^2*d^2+d^4))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2
*(2*a^2*c^2*d^2+a^2*d^4-6*a*b*c^3*d+6*b^2*c^4-5*b^2*c^2*d^2+2*b^2*d^4)/(c^4
-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2)))-2*b^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(276) = 552.
time = 0.49, size = 786, normalized size = 2.77

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{a^2-b^2}}-\frac{\left(6b^2c^4d-6abc^3d^2+2a^2c^2d^3-5b^2c^2d^3+a^2d^5+2b^2d^5\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(b^3c^7-3ab^2c^6d+3a^2bc^5d^2-2b^3c^5d^2-a^3c^4d^3+6ab^2c^4d^3-6a^2bc^3d^4+b^3c^3d^4+2a^3c^2d^5-3ab^2c^2d^5+3a^2bcd^6-a^3d^7\right)\sqrt{c^2-d^2}}-\frac{7bc^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5ac^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4bc^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2acd^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6bc^5d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4ac^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9bc^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-7ac^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6bcd^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2ad^7\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+17bc^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-11ac^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-8bc^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2acd^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6bc^5d^2-4ac^4d^3-3bc^3d^4+ac^2d^5}{\left(b^2c^8-2abc^7d+a^2c^6d^2-2b^2c^6d^2+4abc^5d^3-2a^2c^4d^4+b^2c^4d^4-2abc^3d^5+a^2c^2d^6\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a^2 - b^2)) - (6*b^2*c^4*d - 6*a*b*c^3*d^2 + 2*a^2*c^2*d^3 - 5*
b^2*c^2*d^3 + a^2*d^5 + 2*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - 2*b^3*c^5*d^2 - a^3*c^4*d^3 + 6*a*b^2*c^4*d^3 -
 6*a^2*b*c^3*d^4 + b^3*c^3*d^4 + 2*a^3*c^2*d^5 - 3*a*b^2*c^2*d^5 + 3*a^2*b*
c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) - (7*b*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 5
*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^2*d^5*tan(1/2*f*x + 1/2*e)^3 + 2*
a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*a*c
^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^
2*d^5*tan(1/2*f*x + 1/2*e)^2 - 6*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^7*t
an(1/2*f*x + 1/2*e)^2 + 17*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^4*ta
n(1/2*f*x + 1/2*e) - 8*b*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*a*c*d^6*tan(1/2*f
*x + 1/2*e) + 6*b*c^5*d^2 - 4*a*c^4*d^3 - 3*b*c^3*d^4 + a*c^2*d^5)/((b^2*c^
8 - 2*a*b*c^7*d + a^2*c^6*d^2 - 2*b^2*c^6*d^2 + 4*a*b*c^5*d^3 - 2*a^2*c^4*d
^4 + b^2*c^4*d^4 - 2*a*b*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

**Mupad** [**B**]
time = 30.30, size = 2500, normalized size = 8.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (b^3*atan(((b^3*(b^2 - a^2)^(1/2)*((8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 +
 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12
 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5
*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b
^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5
 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^
4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d
^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b
^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9
- 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14
 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*
d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 2
4*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11
- 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^
10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2
*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*
d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a
^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10
*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^
13 + a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a
*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 -
8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12
- 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d
^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336
*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6
*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 +
 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b
^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^
6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6
*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^
8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4
*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c
^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^
9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b
*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*
a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^
3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20
*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*
c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5
*b*c*d^13) + (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 -
 6*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36
```

*a*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d ^7 - 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234 *a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7 *c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^ 3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202 *a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^ 5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9* d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 1 38*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6* b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5 *d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^ 2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^ 8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24* a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b ^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^ 7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4 *b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d ^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8* a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9...

# 3.706 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=306

$$-\frac{d^2(16abcd-6a^2d^2-b^2(12c^2+d^2))\,x}{2b^4}+\frac{2(bc-ad)^3\,(abc+3a^2d-4b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\,(a^2-b^2)^{3/2}\,f}+\frac{d(2bc-\cdots}{\cdots}$$

```
[Out] -1/2*d^2*(16*a*b*c*d-6*a^2*d^2-b^2*(12*c^2+d^2))*x/b^4+2*(-a*d+b*c)^3*(3*a^
2*d+a*b*c-4*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^4/(a^
2-b^2)^(3/2)/f+d*(-a*d+2*b*c)*(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2*d^2))*cos(f*x
+e)/b^3/(a^2-b^2)/f+1/2*d^2*(4*a*b*c*d-3*a^2*d^2-b^2*(2*c^2-d^2))*cos(f*x+e
)*sin(f*x+e)/b^2/(a^2-b^2)/f+(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^2/b/(
a^2-b^2)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.66, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3112, 3102, 2814, 2739, 632, 210}

$$\frac{2(3a^2d+abc-4b^2d)\,(bc-ad)^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4f\,(a^2-b^2)^{3/2}}+\frac{d^2(-3a^2d^2+4abcd-(b^2(2c^2-d^2)))\sin(e+fx)\cos(e+fx)}{2b^2f\,(a^2-b^2)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{bf\,(a^2-b^2)\,(a+b\sin(e+fx))}-\frac{d^2x(-6a^2d^2+16abcd-(b^2(12c^2+d^2)))}{2b^4}+\frac{d(2bc-ad)\,(-3a^2d^2+2abcd-(b^2(c^2-2d^2)))\cos(e+fx)}{b^3f\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*(d^2*(16*a*b*c*d - 6*a^2*d^2 - b^2*(12*c^2 + d^2))*x)/b^4 + (2*(b*c -
a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*f) + (d*(2*b*c - a*d)*(2*a*b*c*d - 3*a^2*d
^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(b^3*(a^2 - b^2)*f) + (d^2*(4*a*b*c*d
 - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^2*(a^2 -
b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(b*(a^2 - b^2
)*f*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{\int \frac{(c+d\sin(e+fx))\left(4b^2c^2d+2a^2d^3-abc(c^2+5\right.}{}}{}$$

$$= \frac{d^2(4abcd-3a^2d^2-b^2(2c^2-d^2))\cos(e+fx)\sin(e+fx)}{2b^2\,(a^2-b^2)\,f} + \frac{(bc-ad)^2\cos(e+}{b\,(a^2-b^2)\,f(}$$

$$= \frac{d(2bc-ad)\,(2abcd-3a^2d^2-b^2(c^2-2d^2))\cos(e+fx)}{b^3\,(a^2-b^2)\,f} + \frac{d^2(4abcd-3a^2d^2-}{}$$

$$= -\frac{d^2(16abcd-6a^2d^2-b^2(12c^2+d^2))\,x}{2b^4} + \frac{d(2bc-ad)\,(2abcd-3a^2d^2-b^2(c^2-}{b^3\,(a^2-b^2)\,f}$$

$$= -\frac{d^2(16abcd-6a^2d^2-b^2(12c^2+d^2))\,x}{2b^4} + \frac{d(2bc-ad)\,(2abcd-3a^2d^2-b^2(c^2-}{b^3\,(a^2-b^2)\,f}$$

$$= -\frac{d^2(16abcd-6a^2d^2-b^2(12c^2+d^2))\,x}{2b^4} + \frac{d(2bc-ad)\,(2abcd-3a^2d^2-b^2(c^2-}{b^3\,(a^2-b^2)\,f}$$

$$= -\frac{d^2(16abcd-6a^2d^2-b^2(12c^2+d^2))\,x}{2b^4} + \frac{2(bc-ad)^3\,(abc+3a^2d-4b^2d)\tan^{-}}{b^4\,(a^2-b^2)^{3/2}\,f}$$

**Mathematica** [A]
time = 2.10, size = 199, normalized size = 0.65

$$-\frac{-2d^2(-16abcd+6a^2d^2+b^2(12c^2+d^2))\,(e+fx)+\frac{8(-bc+ad)^3(abc+3a^2d-4b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+8bd^3(2bc-ad)\cos(e+fx)-\frac{4b(bc-ad)^4\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}+b^2d^4\sin(2(e+fx))}{4b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(e + f*x) + (8*
(-(b*c) + a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2]
)/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 8*b*d^3*(2*b*c - a*d)*Cos[e + f*x]
- (4*b*(b*c - a*d)^4*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) +
 b^2*d^4*Sin[2*(e + f*x)])/(b^4*f)
```

**Maple** [A]
time = 0.52, size = 456, normalized size = 1.49

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{2d^2\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(2ab\,d^2-4b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+2ab\,d^2-4b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(6a^2d^2-16abcd+12b^2c^2+d^2b^2\right.}{2}\right.}{b^4}$ |
| default | $\frac{2d^2\left(\frac{\frac{d^2b^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(2ab\,d^2-4b^2cd\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{d^2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+2ab\,d^2-4b^2cd}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(6a^2d^2-16abcd+12b^2c^2+d^2b^2\right.}{2}\right.}{b^4}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d^2/b^4*((1/2*d^2*b^2*tan(1/2*f*x+1/2*e)^3+(2*a*b*d^2-4*b^2*c*d)*tan
(1/2*f*x+1/2*e)^2-1/2*d^2*b^2*tan(1/2*f*x+1/2*e)+2*a*b*d^2-4*b^2*c*d)/(1+ta
n(1/2*f*x+1/2*e)^2)^2+1/2*(6*a^2*d^2-16*a*b*c*d+12*b^2*c^2+b^2*d^2)*arctan(
tan(1/2*f*x+1/2*e)))-2/b^4*((-b^2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)-b*(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*
e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(3*a^5*d^4-8*a^4*b*c*d^3+6*a^3*b^2*c^2*d^2-4
*a^3*b^2*d^4+12*a^2*b^3*c*d^3-a*b^4*c^4-12*a*b^4*c^2*d^2+4*b^5*c^3*d)/(a^2-
b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(304) = 608$.
time = 0.44, size = 1472, normalized size = 4.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d^4)*f*x + (a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*
(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 + 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*
a^4*b^2)*d^4 + (a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4
*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a
^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2
))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + (2*(a^2*b^5 - b
^7)*c^4 - 8*(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2
*a^5*b^2 - 3*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b
^7)*d^4)*cos(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*
b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d
^4)*f*x - (8*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f
*x + e) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*f), 1/2*((a^4*b^3 - 2*a^2*b^5 + b^7
)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*
b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d
^4)*f*x - 2*(a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*(a^4*b^2 - 2*a^2*b^4)*c^2*d^2
+ 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*a^4*b^2)*d^4 + (a*b^5*c^4 - 4*
b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3
 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(
f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*(a^2*b^5 - b^7)*c^4 - 8*
(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2*a^5*b^2 - 3
*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*d^4)*cos
(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*b^2 - 2*a^3*
b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d^4)*f*x - (8
*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f*x + e) + (a
^5*b^4 - 2*a^3*b^6 + a*b^8)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.50, size = 517, normalized size = 1.69

$$\frac{\frac{4\left(ab^4c^4-4\,b^5c^3d-6\,a^3b^2c^2d^2+12\,ab^4c^2d^2+8\,a^4bcd^3-12\,a^2b^3cd^3-3\,a^5d^4+4\,a^3b^2d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{4\left(b^5c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,ab^4c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,a^2b^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,a^3b^2cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^4bd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4\right)}{\left(a^3b^3-ab^5\right)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)}+\frac{\left(12\,b^2c^2d^2-16\,abcd^3+6\,a^2d^4+b^2d^4\right)\left(fx+e\right)}{b^4}+\frac{2\left(bd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,ad^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-bd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,bcd^3+4\,ad^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2b^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8*
a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 -
b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 4*(b^5*c^4*tan(1/2*f*x + 1/2*e)
- 4*a*b^4*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^2*b^3*c^2*d^2*tan(1/2*f*x + 1/2*
e) - 4*a^3*b^2*c*d^3*tan(1/2*f*x + 1/2*e) + a^4*b*d^4*tan(1/2*f*x + 1/2*e)
+ a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4
)/((a^3*b^3 - a*b^5)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)) + (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*(f*x + e)/b^4
+ 2*(b*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*a*
d^4*tan(1/2*f*x + 1/2*e)^2 - b*d^4*tan(1/2*f*x + 1/2*e) - 8*b*c*d^3 + 4*a*d
^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^3))/f
```

**Mupad [B]**
time = 20.45, size = 2500, normalized size = 8.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(3*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^
3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(b^3*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/
2)^4*(3*a^4*d^4 + b^4*c^4 - b^4*d^4 - a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2 + 4*a
*b^3*c*d^3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(b^3*(a^2 - b^2)) + (2*tan(e/2
 + (f*x)/2)^2*(6*a^4*d^4 + 2*b^4*c^4 + b^4*d^4 - 5*a^2*b^2*d^4 + 12*a^2*b^2
*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*d^3))/(b^3*(a^2 - b^2
)) + (tan(e/2 + (f*x)/2)^5*(3*a^4*d^4 + 2*b^4*c^4 - a^2*b^2*d^4 + 12*a^2*b^
2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)) + (tan(e/2
+ (f*x)/2)*(9*a^4*d^4 + 2*b^4*c^4 - 7*a^2*b^2*d^4 + 12*a^2*b^2*c^2*d^2 + 16
*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 24*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)) + (4*ta
n(e/2 + (f*x)/2)^3*(3*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2
 + 4*a*b^3*c*d^3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)))/(f*
(a + 2*b*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x
)/2)^4 + a*tan(e/2 + (f*x)/2)^6 + 4*b*tan(e/2 + (f*x)/2)^3 + 2*b*tan(e/2 +
(f*x)/2)^5)) + (atan(((((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8
- 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7
+ 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^1
1*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^
4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a
^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8
*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*
```

b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i)*1i)/b^4 + (((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11

```
*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*...
```

# 3.707 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=205

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{2(bc-ad)^2\left(abc+2a^2d-3b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f}+\frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\mathrm{c}}{b^2\left(a^2-b^2\right)f}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+2*(-a*d+b*c)^2*(2*a^2*d+a*b*c-3*b^2*d)*arctan((b+a
*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f+d*(2*a*b*c*d-2*
a^2*d^2-b^2*(c^2-d^2))*cos(f*x+e)/b^2/(a^2-b^2)/f+(-a*d+b*c)^2*cos(f*x+e)*(
c+d*sin(f*x+e))/b/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.33, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2871, 3102, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{d\left(-2a^2d^2+2abcd-\left(b^2\left(c^2-d^2\right)\right)\right)\cos(e+fx)}{b^2f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x(3bc-2ad)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*d)*
ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f)
 + (d*(2*a*b*c*d - 2*a^2*d^2 - b^2*(c^2 - d^2))*Cos[e + f*x])/(b^2*(a^2 - b
^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(b*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{\int \frac{3b^2c^2d+a^2d^3-abc(c^2+3d^2)-d(a^2cd-3b^2cd\cdots}{}}{}$$

$$= \frac{d(2abcd-2a^2d^2-b^2(c^2-d^2))\cos(e+fx)}{b^2\,(a^2-b^2)\,f} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(\cdots}{b\,(a^2-b^2)\,f(a+b\sin(e+f\cdots}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d(2abcd-2a^2d^2-b^2(c^2-d^2))\cos(e+fx)}{b^2\,(a^2-b^2)\,f} + \frac{(bc-ad)^2\cos(\cdots}{b\,(a^2-b^2\cdots}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d(2abcd-2a^2d^2-b^2(c^2-d^2))\cos(e+fx)}{b^2\,(a^2-b^2)\,f} + \frac{(bc-ad)^2\cos(\cdots}{b\,(a^2-b^2\cdots}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d(2abcd-2a^2d^2-b^2(c^2-d^2))\cos(e+fx)}{b^2\,(a^2-b^2)\,f} + \frac{(bc-ad)^2\cos(\cdots}{b\,(a^2-b^2\cdots}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{2(bc-ad)^2\,(abc+2a^2d-3b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\,(a^2-b^2)^{3/2}\,f} + \frac{d\cdots}{}$$

**Mathematica [A]**
time = 1.16, size = 151, normalized size = 0.74

$$\frac{d^2(3bc-2ad)(e+fx) + \frac{2(bc-ad)^2(abc+2a^2d-3b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - bd^3\cos(e+fx) + \frac{b(bc-ad)^3\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*(e + f*x) + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*
d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*
d^3*Cos[e + f*x] + (b*(b*c - a*d)^3*Cos[e + f*x])/((a - b)*(a + b)*(a + b*S
in[e + f*x])))/(b^3*f)
```

**Maple [A]**
time = 0.40, size = 303, normalized size = 1.48

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2d^2\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2ad-3bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^3}+\frac{2\left(-\frac{b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{b\left(a^3d^3-3a^2\cdots\right.}{}\right)}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}}{f}$ |

| | |
|---|---|
| default | $\frac{-\frac{2d^2\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(2ad-3bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^3}+\frac{2\left(-\frac{b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{b\left(a^3d^3-3\right.}{}\right.}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}}{f}$ |
| risch | $-\frac{2d^3xa}{b^3}+\frac{3d^2xc}{b^2}-\frac{d^3\mathrm{e}^{i(fx+e)}}{2b^2f}-\frac{d^3\mathrm{e}^{-i(fx+e)}}{2b^2f}+\frac{2i\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(ib+a\,\mathrm{e}^{i(fx+e)}\right)}{b^3\left(a^2-b^2\right)f\left(-ib\,\mathrm{e}^{2i(fx+e)}+ib+2a\,\mathrm{e}^{i(fx+e)}\right)}-\frac{2\ln\left(\mathrm{e}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*d^2/b^3*(b*d/(1+tan(1/2*f*x+1/2*e)^2)+(2*a*d-3*b*c)*arctan(tan(1/2*
f*x+1/2*e)))+2/b^3*((-b^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^
2-b^2)/a*tan(1/2*f*x+1/2*e)-b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(2*a^4*d^3-3*
a^3*b*c*d^2-3*a^2*b^2*d^3+a*b^3*c^3+6*a*b^3*c*d^2-3*b^4*c^2*d)/(a^2-b^2)^(3
/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(205) = 410$.
time = 0.45, size = 1025, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4
)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (
2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c
*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^
```

```
2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e
)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*
a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^
5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)
*cos(f*x + e) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*cos(f*x + e) - (3*(a^4*b
^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*f*x)*sin(f
*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5
 + a*b^7)*f), ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
a^2*b^4)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*
d^2 + (2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a
*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arct
an(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^4 - b^6)
*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b -
 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*co
s(f*x + e) - (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*d^3)*f*x)*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e)
+ (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(205) = 410.
time = 0.46, size = 579, normalized size = 2.82

$$\frac{\frac{2\left(ab^3c^3-3b^4c^2d-3a^3bcd^2+6ab^3cd^2+2a^4d^3-3a^2b^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}+\frac{2\left(b^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3ab^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^2b^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-a^3bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+ab^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3a^2b^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3a^3bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a^2b^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+b^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3ab^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3a^2b^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3a^3bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2ab^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-2a^4d^3+a^2b^2d^3\right)}{\left(a^3b^2-ab^4\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}+\frac{\left(3bcd^2-2ad^3\right)(fx+e)}{b^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 - 3
*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 2*(
b^4*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a
^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - a^3*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*
b^3*c^3*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 3
*a^3*b*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^4*d^3*tan(1/2*f*x + 1/2*e)^2 + a^
2*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 + b^4*c^3*tan(1/2*f*x + 1/2*e) - 3*a*b^3*c
^2*d*tan(1/2*f*x + 1/2*e) + 3*a^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e) - 3*a^3*b*
```

```
d^3*tan(1/2*f*x + 1/2*e) + 2*a*b^3*d^3*tan(1/2*f*x + 1/2*e) + a*b^3*c^3 - 3
*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - 2*a^4*d^3 + a^2*b^2*d^3)/((a^3*b^2 - a*b^4
)*(a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)) + (3*b*c*d^2 - 2*a*d^3)*(f*x +
e)/b^3)/f
```

**Mupad [B]**
time = 17.83, size = 2500, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(b^3*c^3 - 2*a^3*d^3 + a*b^2*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2
*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(b^3*c^3 - 2*a^3*d^3 + a*b^2*d^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(
b^3*c^3 - 3*a^3*d^3 + 2*a*b^2*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(a*b*(a
^2 - b^2)) - (2*tan(e/2 + (f*x)/2)^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(a*b*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*tan(
e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) + (2
*d^2*atan(((d^2*(2*a*d - 3*b*c)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8
*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b
^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^
4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d
^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2
+ 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5
- 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c
^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^
6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2
*a*d - 3*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a
^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^
10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*
a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^
4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^
2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (d^2*((32*(a^2*b^12 -
2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)
*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*
b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3))/b^3 + (d^2*(2*a*d - 3*b*c)*((32*(4
*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^
5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6
*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^
8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 1
8*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d -
96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3
```

+ 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3))/b^3)/((64*(6*a^6*b^2*d^9 - 4*a^8*d^9 - 27*a*b^7*c^5*d^4 - 39*a^5*b^3*c*d^8 + 4*a^7*b*c^3*d^6 + 99*a^2*b^6*c^4*d^5 + 18*a^2*b^6*c^6*d^3 - 144*a^3*b^5*c^3*d^6 - 39*a^3*b^5*c^5*d^4 - 3*a^3*b^5*c^7*d^2 + 105*a^4*b^4*c^2*d^7 + 3*a^4*b^4*c^4*d^5 + 2*a^4*b^4*c^6*d^3 + 55*a^5*b^3*c^3*d^6 + 9*a^5*b^3*c^5*d^4 - 57*a^6*b^2*c^2*d^7 - 12*a^6*b^2*c^4*d^5 + 24*a^7*b*c*d^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(40*a^7*b^2*d^9 - 24*a^5*b^4*d^9 - 16*a^9*d^9 - 54*a*b^8*c^4*d^5 + 120*a^4*b^5*c*d^8 - 192*a^6*b^3*c*d^8 + 180*a^2*b^7*c^3*d^6 + 18*a^2*b^7*c^5*d^4 - 222*a^3*b^6*c^2*d^7 + 30*a^3*b^6*c^4*d^5 - 226*a^4*b^5*c^3*d^6 - 18*a^4*b^5*c^5*d^4 + 330*a^5*b^4*c^2*d^7 + 24*a^5*b^4*c^4*d^5 + 46*a^6*b^3*c^3*d^6 - 108*a^7*b^2*c^2*d^7 + 72*a^8*b*c*d^8))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2)...

# 3.708 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=129

$$\frac{d^2x}{b^2}+\frac{2(bc-ad)\left(abc+a^2d-2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

```
[Out] d^2*x/b^2+2*(-a*d+b*c)*(a^2*d+a*b*c-2*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e)
)/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)^(3/2)/f+(-a*d+b*c)^2*cos(f*x+e)/b/(a^2-b^2
)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2869, 2814, 2739, 632, 210}

$$\frac{2(bc-ad)\left(a^2d+abc-2b^2d\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/b^2 + (2*(b*c - a*d)*(a*b*c + a^2*d - 2*b^2*d)*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{-b\left(2bcd-a\left(c^2+d^2\right)\right)+\left(a^2-b^2\right)d^2\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left((bc-ad)\left(abc+a^2d-2b^2d\right)\right)\int \frac{}{a+b\sin}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left(2(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\operatorname{Subst}}{b^2\left(a^2-\right.}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(4(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\operatorname{Subst}}{b^2}$$

$$= \frac{d^2x}{b^2} + \frac{2(bc-ad)\left(abc+a^2d-2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}f} + \frac{(bc-ad)^2}{b\left(a^2-b^2\right)f(a}$$

**Mathematica [A]**
time = 0.61, size = 133, normalized size = 1.03

$$\frac{d^2(e+fx) - \frac{2\left(2b^3cd+a^3d^2-ab^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b(bc-ad)^2\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{b^2f}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]

[Out] (d^2*(e + f*x) - (2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*(b*c - a*d)^2 *Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/(b^2*f)

**Maple [A]**
time = 0.30, size = 220, normalized size = 1.71

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{2\left(\frac{-\frac{b^2\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{b\left(a^2d^2-2abcd+b^2c^2\right)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(a^3d^2-a\,b^2c^2-2a\,b^2d^2+2b^3cd\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^2}}{f}$ |
| default | $-\frac{\frac{2\left(\frac{-\frac{b^2\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{b\left(a^2d^2-2abcd+b^2c^2\right)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(a^3d^2-a\,b^2c^2-2a\,b^2d^2+2b^3cd\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^2}}{f}$ |
| risch | $\frac{d^2x}{b^2}-\frac{2i\left(a^2d^2-2abcd+b^2c^2\right)\left(ib+a\,e^{i(fx+e)}\right)}{b^2\left(a^2-b^2\right)f\left(-ib\,e^{2i(fx+e)}+ib+2a\,e^{i(fx+e)}\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)a^3d^2}{\sqrt{-a^2+b^2}\,(a+b)(a-b)f\,b^2}+$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-2/b^2*((-b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)-b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(a^3*d^2-a*b^2*c^2-2*a*b^2*d^2+2*b^3*c*d)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))+2*d^2/b^2*arctan(tan(1/2*f*x+1/2*e)))

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(127) = 254$.
time = 0.40, size = 682, normalized size = 5.29

$$\left[\frac{2\,(a^4b-2\,a^2b^3+b^5)d^2fx\sin\left(fx+e\right)+2\,(a^5-2\,a^3b^2+ab^4)d^2fx+(a^2b^2c^2-2\,ab^3cd-(a^4-2\,a^2b^2)d^2+(ab^3c^2-2\,b^4cd-(a^3b-2\,ab^3)d^2)\sin\left(fx+e\right))\sqrt{-a^2+b^2}\log\left(-\frac{(2\,a^2-b^2)\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2-2\,(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2}\right)+2\,((a^2b^3-b^5)c^2-2\,(a^3b^2-ab^4)cd+(a^4b-a^2b^3)d^2)\cos\left(fx+e\right)}{2\,((a^4b^3-2\,a^2b^5+b^7)f\sin\left(fx+e\right)+(a^5b^2-2\,a^3b^4+ab^6)f)}, \frac{(a^4b-2\,a^2b^3+b^5)d^2fx\sin\left(fx+e\right)+(a^5-2\,a^3b^2+ab^4)d^2fx-(a^2b^2c^2-2\,ab^3cd-(a^4-2\,a^2b^2)d^2+(ab^3c^2-2\,b^4cd-(a^3b-2\,ab^3)d^2)\sin\left(fx+e\right))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)+((a^2b^3-b^5)c^2-2\,(a^3b^2-ab^4)cd+(a^4b-a^2b^3)d^2)\cos\left(fx+e\right)}{(a^4b^3-2\,a^2b^5+b^7)f\sin\left(fx+e\right)+(a^5b^2-2\,a^3b^4+ab^6)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + 2*(a^5 - 2*a^3*b^2
 + a*b^4)*d^2*f*x + (a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a
*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(
a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*
x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^3 - b^5)*c^2 - 2*(a
^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^
2*b^5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f), ((a^4*b - 2
*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2*f*x -
(a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a*b^3*c^2 - 2*b^4*c*d
 - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x
+ e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^3 - b^5)*c^2 - 2*(a^3*b
^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^2*b^
5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.61, size = 249, normalized size = 1.93

$$\frac{\frac{(fx+e)d^2}{b^2}+\frac{2\,(ab^2c^2-2\,b^3cd-a^3d^2+2\,ab^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{a^2-b^2}}+\frac{2\,(b^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ab^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+ab^2c^2-2\,a^2bcd+a^3d^2)}{(a^3b-ab^3)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^2/b^2 + 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/
```

```
sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(b^3*c^2*tan(1/2*f*
x + 1/2*e) - 2*a*b^2*c*d*tan(1/2*f*x + 1/2*e) + a^2*b*d^2*tan(1/2*f*x + 1/2
*e) + a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/((a^3*b - a*b^3)*(a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

**Mupad [B]**
time = 15.47, size = 2500, normalized size = 19.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2
)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (
f*x)/2) + a*tan(e/2 + (f*x)/2)^2)) - (2*d^2*atan(((d^2*((32*tan(e/2 + (f*x)
/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^
4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 +
 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (32*
(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d
^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*
d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (d^2*((32*(a^2*b^9 - 2*a^
4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a
*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i
)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d
^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2
*a^2*b^5 + a^4*b^3))*1i)/b^2))/b^2 - (d^2*((32*(a^6*b*d^4 + a^2*b^5*d^4 - 2
*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (32*tan(e/2 + (f*x)/2)*(2*a^7*
b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7
*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*
c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (d^2*((32*(a*b^
8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5
*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b
^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3
*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2 + (32*t
an(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^
6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^
4*b^3))*1i)/b^2))/b^2)/((64*tan(e/2 + (f*x)/2)*(2*a^6*d^6 + 4*a^2*b^4*d^6 -
 6*a^4*b^2*d^6 + 4*a^3*b^3*c*d^5 + 2*a^2*b^4*c^2*d^4 - 2*a^4*b^2*c^2*d^4 -
4*a*b^5*c*d^5))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (64*(2*a^3*b^2*d^6 - a^5*d^6
- a^5*c^2*d^4 + 4*a*b^4*c^2*d^4 - 6*a^2*b^3*c*d^5 - 4*a^2*b^3*c^3*d^3 + 3*a
^3*b^2*c^2*d^4 + a^3*b^2*c^4*d^2 + 2*a^4*b*c*d^5))/(b^6 - 2*a^2*b^4 + a^4*b
^2) + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4
 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^
2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^
```

7 - 2*a^2*b^5 + a^4*b^3) - (32*(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(
b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c
^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b
^2) - (d^2*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2
) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/
(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^
2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9
*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2)*1i)/b^2 + (d^2
*((32*(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2
) - (32*tan(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3
*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^
3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^
2*b^5 + a^4*b^3) + (d^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b
^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2
*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*ta
n(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a
^2*b^5 + a^4*b^3))*1i)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*
b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a
^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2)*1i)/b^2)))/(b^2*f) + (at
an((((a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b*d^4 + a^2*b^5*d^4
 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (32*tan(e/2 + (f*x)/2)*(2*
a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a
*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*
b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + ((a*d - b*c
)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2
- a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2)
 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 -
6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*
b^5 + a^4*b^3) + (((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 +
a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7
*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2
)*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*
d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d - 2*b
^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) - ((a*d - b*c)*(-
(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)...

# 3.709 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=97

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}+\frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2
)/f+(-a*d+b*c)*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{2(ac-bd)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(3/2)*f) + ((b*c - a*d)*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx &= \frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}+\frac{\int \frac{-ac+bd}{a+b\sin(e+fx)}\,dx}{-a^2+b^2}\\
&= \frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}+\frac{(ac-bd)\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2}\\
&= \frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e\right.\right.}{(a^2-b^2)\,f}\\
&= \frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}-\frac{(4(ac-bd))\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2\right.}{(a^2-b^2)\,f}\\
&= \frac{2(ac-bd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f}+\frac{(bc-ad)\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}\end{aligned}$$

**Mathematica** [A]
time = 0.35, size = 96, normalized size = 0.99

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
[Out] ((2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])
))/f
```

**Maple [A]**
time = 0.21, size = 144, normalized size = 1.48

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{-\frac{2b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{2(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{2(ac-bd)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{f}$ |
| default | $\frac{\frac{-\frac{2b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)a}-\frac{2(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{2(ac-bd)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{f}$ |
| risch | $\frac{2i(ad-bc)\left(ib+a\,e^{i(fx+e)}\right)}{b(a^2-b^2)f\left(-ib\,e^{2i(fx+e)}+ib+2a\,e^{i(fx+e)}\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)ac}{\sqrt{-a^2+b^2}\,(a+b)(a-b)f}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a}}{\sqrt{}}\right.}{\sqrt{-a^2+b}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(-b*(a*d-b*c)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)-(a*d-b*c)/(a^2-b^2))/(a
*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+2*(a*c-b*d)/(a^2-b^2)^(3/2)
*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 414, normalized size = 4.27

$$\left[-\frac{(a^2c-abd+(abc-b^2d)\sin(fx+e))\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2+2(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2ab\sin(fx+e)-a^2-b^2}\right)-2((a^2b-b^3)c-(a^3-ab^2)d)\cos(fx+e)}{2((a^4b-2a^2b^3+b^5)f\sin(fx+e)+(a^5-2a^3b^2+ab^4)f)}, -\frac{(a^2c-abd+(abc-b^2d)\sin(fx+e))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)-((a^2b-b^3)c-(a^3-ab^2)d)\cos(fx+e)}{(a^4b-2a^2b^3+b^5)f\sin(fx+e)+(a^5-2a^3b^2+ab^4)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f
```

```
*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^
2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d
)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (a^5 - 2*a^3*b^
2 + a*b^4)*f), -((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(a^2 -
b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((a^2*b
 - b^3)*c - (a^3 - a*b^2)*d)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin
(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.48, size = 157, normalized size = 1.62

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(ac-bd)}{(a^2-b^2)^{\frac{3}{2}}}+\frac{b^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-abd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+abc-a^2d}{(a^3-ab^2)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*(a*c - b*d)/(a^2 - b^2)^(3/2) + (b^2*c*tan(1/2*f*x
 + 1/2*e) - a*b*d*tan(1/2*f*x + 1/2*e) + a*b*c - a^2*d)/((a^3 - a*b^2)*(a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

**Mupad** [B]
time = 8.02, size = 215, normalized size = 2.22

$$\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{2\,\left(a^2\,b-b^3\right)\,\left(a\,c-b\,d\right)}{{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}+\frac{2\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c-b\,d\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)\,\left(a^2-b^2\right)}{2\,\left(a\,c-b\,d\right)}\right)\,\left(a\,c-b\,d\right)}{f\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{\frac{2\,\left(a\,d-b\,c\right)}{a^2-b^2}+\frac{2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d-b\,c\right)}{a\,\left(a^2-b^2\right)}}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x))^2,x)
```

```
[Out] (2*atan((((2*(a^2*b - b^3)*(a*c - b*d))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^
(3/2)) + (2*a*tan(e/2 + (f*x)/2)*(a*c - b*d))/((a + b)^(3/2)*(a - b)^(3/2))
)*(a^2 - b^2))/(2*(a*c - b*d)))*(a*c - b*d))/(f*(a + b)^(3/2)*(a - b)^(3/2)
) - ((2*(a*d - b*c))/(a^2 - b^2) + (2*b*tan(e/2 + (f*x)/2)*(a*d - b*c))/(a*
(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2))
```

# 3.710 $\int \frac{1}{(a+b\sin(e+fx))^2} dx$

**Optimal**. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

```
[Out] 2*a*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/f+b*co
s(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

**Rubi** [A]

time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 12, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*f
) + (b*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(e+fx))^2}\,dx &= \frac{b\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{\int \frac{a}{a+b\sin(e+fx)}\,dx}{-a^2+b^2} \\
&= \frac{b\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{a\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2} \\
&= \frac{b\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(a^2-b^2)\,f} \\
&= \frac{b\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\right.\right.}{(a^2-b^2)\,f} \\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} + \frac{b\cos(e+fx)}{(a^2-b^2)\,f(a+b\sin(e+fx))}
\end{aligned}$$

**Mathematica** [A]
time = 0.21, size = 82, normalized size = 0.99

$$\frac{\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{b\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) +
 (b*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/f
```

**Maple** [A]
time = 0.18, size = 122, normalized size = 1.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\frac{2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{f}$ |
| default | $\frac{\frac{\frac{2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}}{f}$ |
| risch | $\frac{2ib+2a\,\mathrm{e}^{i(fx+e)}}{\left(a^2-b^2\right)f\left(b\,\mathrm{e}^{2i(fx+e)}-b+2ia\,\mathrm{e}^{i(fx+e)}\right)}-\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)f}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{-a^2+}}{\sqrt{-a^2}}\right.}{\sqrt{-a^2+b^2}\,(a+}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(b^2/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)+b/(a^2-b^2))/(a*tan(1/2*f*x+1/2*
e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [A]**
time = 0.38, size = 351, normalized size = 4.23

$$\left[\frac{\left(ab\sin\left(fx+e\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2-2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right)+2\left(a^2b-b^3\right)\cos\left(fx+e\right)}{2\left(\left(a^4b-2a^2b^3+b^5\right)f\sin\left(fx+e\right)+\left(a^5-2a^3b^2+ab^4\right)f\right)},-\frac{\left(ab\sin\left(fx+e\right)+a^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(fx+e\right)+b}{\sqrt{a^2-b^2}\cos\left(fx+e\right)}\right)-\left(a^2b-b^3\right)\cos\left(fx+e\right)}{\left(a^4b-2a^2b^3+b^5\right)f\sin\left(fx+e\right)+\left(a^5-2a^3b^2+ab^4\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(f*x + e) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
```

```
 - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f
*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f), -((a*b*sin(f*x + e) + a^2)*sq
rt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))
- (a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (
a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

**Giac** [**A**]
time = 0.47, size = 127, normalized size = 1.53

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ab}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2) + (b^2*tan(1/2*f*x + 1/2*e) +
a*b)/((a^3 - a*b^2)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
a)))/f
```

**Mupad** [**B**]
time = 8.24, size = 174, normalized size = 2.10

$$\frac{\frac{2b}{a^2-b^2}+\frac{2b^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a\left(a^2-b^2\right)}}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+2b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+a\right)}+\frac{2a\operatorname{atan}\left(\frac{\left(a^2-b^2\right)\left(\frac{2a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{(a+b)^{3/2}(a-b)^{3/2}}+\frac{2a\left(a^2b-b^3\right)}{(a+b)^{3/2}\left(a^2-b^2\right)(a-b)^{3/2}}\right)}{2a}\right)}{f\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^2,x)
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(e/2 + (f*x)/2))/(a*(a^2 - b^2)))/(f*(a + 2*
b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2)) + (2*a*atan(((a^2 - b^2)*((
2*a^2*tan(e/2 + (f*x)/2))/((a + b)^(3/2)*(a - b)^(3/2)) + (2*a*(a^2*b - b^3
))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/(2*a)))/(f*(a + b)^(3/2)*(a
- b)^(3/2))
```

# 3.711 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=181

$$\frac{2b(abc-2a^2d+b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2f}+\frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\sqrt{c^2-d^2}\,f}+\frac{b^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))}$$

```
[Out] 2*b*(-2*a^2*d+a*b*c+b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))
/(a^2-b^2)^(3/2)/(-a*d+b*c)^2/f+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*
sin(f*x+e))+2*d^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*
c)^2/f/(c^2-d^2)^(1/2)
```

**Rubi** [A]
time = 0.31, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2881, 3080, 2739, 632, 210}

$$\frac{2b(-2a^2d+abc+b^2d)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}(bc-ad)^2}+\frac{b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))}+\frac{2d^2\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b
^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e +
f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{-abc+a^2d-b^2d-a\cdots}{(a+b\sin(e+fx))(c+\cdots}}{(a^2-b^2)\,(b\cdots}$$

$$= \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} + \frac{d^2\int \frac{1}{c+d\sin(e+fx)}\,\cdots}{(bc-ad)^2}$$

$$= \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{\cdots}{c+\cdots}\right.}{\cdots}$$

$$= \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} - \frac{(4d^2)\,\text{Subst}\left(\int \frac{\cdots}{-\cdots}\right.}{\cdots}$$

$$= \frac{2b(abc-2a^2d+b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,(bc-ad)^2f} + \frac{2d^2\tan^{-1}\cdots}{(bc-a\cdots}$$

**Mathematica** [A]

time = 0.89, size = 178, normalized size = 0.98

$$\frac{\frac{2b(abc-2a^2d+b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2}+\frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\sqrt{c^2-d^2}}-\frac{b^2\cos(e+fx)}{(a-b)(a+b)(-bc+ad)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2) + (2*d^2*ArcTan[(d + c*Tan[(e + f*
x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x
])/((a - b)*(a + b)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])))/f
```

## Maple [A]

time = 1.23, size = 236, normalized size = 1.30

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2b\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a(a^2-b^2)}+\frac{b(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{(2a^2d-abc-b^2d)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}\right)}{(ad-bc)^2}+\frac{2d^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(a^2d^2-2abcd+b^2c^2)\sqrt{c^2-d^2}}}{f}$ |
| default | $\frac{-\frac{2b\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a(a^2-b^2)}+\frac{b(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{(2a^2d-abc-b^2d)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}\right)}{(ad-bc)^2}+\frac{2d^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(a^2d^2-2abcd+b^2c^2)\sqrt{c^2-d^2}}}{f}$ |
| risch | $-\frac{2b\left(ib+a\,e^{i(fx+e)}\right)}{(a^2-b^2)(ad-bc)f\left(b\,e^{2i(fx+e)}-b+2ia\,e^{i(fx+e)}\right)}-\frac{2b\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)a^2d}{\sqrt{-a^2+b^2}\,(ad-bc)^2(a+b)(a-b)f}+\frac{b^2\ln\left(e^{i(fx+\cdots}\right.}{\sqrt{-a^2\cdots}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*b/(a*d-b*c)^2*((b^2*(a*d-b*c)/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)+b*(a*d
-b*c)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(2*a^2*d
-a*b*c-b^2*d)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2)))+2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(175) = 350$.
time = 77.60, size = 2923, normalized size = 16.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a
*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2
 - b^4)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e
)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*c
os(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a
^2 - b^2)) - ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2
 + a*b^4)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*s
in(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*
sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 +
(a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*
(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2
*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b -
 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2
 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)
*f), -1/2*(2*(a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^
3*b - a*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*
a^2*b^2 - b^4)*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) +
 b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*
x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)
*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*
x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(
f*x + e) - c^2 - d^2)) - 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d -
 (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 -
2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 -
 (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4
```

```
+ a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 -
2*a^5*b^2 + a^3*b^4)*d^4)*f), -1/2*(2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*
x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
+ e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (a^2*b^2*c^3 - a^2*b^2*c*d^2 -
(2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2
- (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x + e))*sqrt(-a^2
+ b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2
- 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*c
os(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b^3 - b^5)*c^3 -
 (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*c
os(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 -
2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e
) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^
3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3
+ a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), -((a^2*b^2*c^3 - a^
2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3
- a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x +
e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
 e))) + ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*
b^4)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos
(f*x + e))) - ((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b
^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b
^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4
- 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*
b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a
^3*b^4)*d^4)*f)]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.49, size = 304, normalized size = 1.68

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{c^2-d^2}}+\frac{(ab^2c-2a^2bd+b^3d)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^2c^2-b^4c^2-2a^3bcd+2ab^3cd+a^4d^2-a^2b^2d^2)\sqrt{a^2-b^2}}+\frac{b^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ab^2}{(a^3bc-ab^3c-a^4d+a^2b^2d)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e ) + d)/sqrt(c^2 - d^2)))*d^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2 - d^ 2)) + (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a ) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^2*c^2 - b ^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d + a^4*d^2 - a^2*b^2*d^2)*sqrt(a^2 - b^2) ) + (b^3*tan(1/2*f*x + 1/2*e) + a*b^2)/((a^3*b*c - a*b^3*c - a^4*d + a^2*b^ 2*d)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f

**Mupad [B]**
time = 22.80, size = 2500, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)

[Out] (d^2*atan(((d^2*(d^2 - c^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^ 8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5 *d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c ^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7* d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3* a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5 *c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^ 4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7 *b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^ 5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2)^(1/2)*((32* (a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^ 9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4 *a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5 *d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^ 7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/( a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d ^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2 *a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8* b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a^3*b^7*c^4*d^3 - 8*a^ 4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*c^2*d^5 + 12*a^7*b^3* c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d) - (d^2*(d^2 - c^2)^(1/2)*((32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*...

# 3.712 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=290

$$\frac{2b^2(abc-3a^2d+2b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^3f}+\frac{2d^2(3bc^2-acd-2bd^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^3(c^2-d^2)^{3/2}f}+\frac{\ldots}{(a^2-b^2\ldots}$$

```
[Out] 2*b^2*(-3*a^2*d+a*b*c+2*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(3/2)/(-a*d+b*c)^3/f+2*d^2*(-a*c*d+3*b*c^2-2*b*d^2)*arctan((
d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/(c^2-d^2)^(3/2)/f+d*(
a^2*d^2+b^2*(c^2-2*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d
*sin(f*x+e))+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))/(c+d*si
n(f*x+e))
```

**Rubi** [A]
time = 0.80, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$\frac{2b^2(-3a^2d+abc+2b^2d)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}(bc-ad)^3}+\frac{d(a^2d^2+b^2(c^2-2d^2))\cos(e+fx)}{f(a^2-b^2)(c^2-d^2)(bc-ad)^2(c+d\sin(e+fx))}+\frac{b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))(c+d\sin(e+fx))}+\frac{2d^2(-acd+3bc^2-2bd^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c^2-d^2)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^3*f) + (2*d^2*(3*b*c^2 - a*c*d - 2
*b*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^3*(c
^2 - d^2)^(3/2)*f) + (d*(a^2*d^2 + b^2*(c^2 - 2*d^2))*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x]
)/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))}$$

$$= \frac{d(a^2d^2+b^2(c^2-2d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{}{(a^2-}$$

$$= \frac{d(a^2d^2+b^2(c^2-2d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{}{(a^2-}$$

$$= \frac{d(a^2d^2+b^2(c^2-2d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{}{(a^2-}$$

$$= \frac{d(a^2d^2+b^2(c^2-2d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{}{(a^2-}$$

$$= \frac{2b^2(abc-3a^2d+2b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,(bc-ad)^3f} + \frac{2d^2(3bc}{}$$

**Mathematica [A]**
time = 3.04, size = 227, normalized size = 0.78

$$\frac{\frac{2b^2(abc-3a^2d+2b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{2d^2(-3bc^2+acd+2bd^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{b^3(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} + \frac{d^3(bc-ad)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{(bc-ad)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(3/2) - (2*d^2*(-3*b*c^2 + a*c*d + 2*b*d^2)*ArcTan[(
d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (b^3*(b*c - a
*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) + (d^3*(b*c - a*d)
*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])))/((b*c - a*d)^3*f)
```

**Maple [A]**
time = 3.31, size = 331, normalized size = 1.14

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2d^2\left(\frac{\frac{d^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c\left(c^2-d^2\right)}+\frac{d(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(acd-3bc^2+2bd^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{\frac{3}{2}}}\right)}{(ad-bc)^3}+\frac{2b^2\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a\left(a^2-b^2\right)}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan}\right.}{}}{f}$ |

| | |
|---|---|
| default | $\frac{\frac{2d^2\left(\frac{\frac{d^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c(c^2-d^2)}+\frac{d(ad-bc)}{c^2-d^2}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(acd-3b\,c^2+2b\,d^2)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{\frac{3}{2}}}\right)}{(ad-bc)^3}+\frac{2b^2\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a(a^2-b^2)}+}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan(}\right.}{}}{f}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d^2/(a*d-b*c)^3*((d^2*(a*d-b*c)/c/(c^2-d^2)*tan(1/2*f*x+1/2*e)+d*(a*
d-b*c)/(c^2-d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(a*c*d-
3*b*c^2+2*b*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
^2-d^2)^(1/2)))+2*b^2/(a*d-b*c)^3*((b^2*(a*d-b*c)/a/(a^2-b^2)*tan(1/2*f*x+1
/2*e)+b*(a*d-b*c)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)
+a)+(3*a^2*d-a*b*c-2*b^2*d)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2
*e)+2*b)/(a^2-b^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(287) = 574$.

time = 0.70, size = 1005, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((a*b^3*c - 3*a^2*b^2*d + 2*b^4*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3*c^3 -
b^5*c^3 - 3*a^3*b^2*c^2*d + 3*a*b^4*c^2*d + 3*a^4*b*c*d^2 - 3*a^2*b^3*c*d^2
 - a^5*d^3 + a^3*b^2*d^3)*sqrt(a^2 - b^2)) + (3*b*c^2*d^2 - a*c*d^3 - 2*b*d
^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - b^3
*c^3*d^2 - a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*sqrt(c^
2 - d^2)) + (b^4*c^4*tan(1/2*f*x + 1/2*e)^3 - b^4*c^2*d^2*tan(1/2*f*x + 1/2
*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - a^2*b^2*d^4*tan(1/2*f*x + 1/2*e)^3
 + a*b^3*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*b^4*c^3*d*tan(1/2*f*x + 1/2*e)^2 -
a*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + a^4*c*d^3*tan(1/2*f*x + 1/2*e)^2 - a
^2*b^2*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^4*c*d^3*tan(1/2*f*x + 1/2*e)^2 +
2*a^3*b*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*a*b^3*d^4*tan(1/2*f*x + 1/2*e)^2 + b
^4*c^4*tan(1/2*f*x + 1/2*e) + 2*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) - b^4*c^2*
d^2*tan(1/2*f*x + 1/2*e) + 2*a^3*b*c*d^3*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c*d
^3*tan(1/2*f*x + 1/2*e) + a^4*d^4*tan(1/2*f*x + 1/2*e) - a^2*b^2*d^4*tan(1/
2*f*x + 1/2*e) + a*b^3*c^4 - a*b^3*c^2*d^2 + a^4*c*d^3 - a^2*b^2*c*d^3)/((a
^3*b^2*c^5 - a*b^4*c^5 - 2*a^4*b*c^4*d + 2*a^2*b^3*c^4*d + a^5*c^3*d^2 - 2*
a^3*b^2*c^3*d^2 + a*b^4*c^3*d^2 + 2*a^4*b*c^2*d^3 - 2*a^2*b^3*c^2*d^3 - a^5
*c*d^4 + a^3*b^2*c*d^4)*(a*c*tan(1/2*f*x + 1/2*e)^4 + 2*b*c*tan(1/2*f*x + 1
/2*e)^3 + 2*a*d*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*tan(1/2*f*x + 1/2*e)^2 + 4*b
*d*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*tan(1/2*f*x + 1/2*e) + 2*a*d*tan(1/2*f*x
+ 1/2*e) + a*c)))/f
```

**Mupad** [B]

time = 30.89, size = 2500, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*(a^3*d^3 + b^3*c^3 - a*b^2*d^3 - b^3*c*d^2))/((a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)*(a^2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^3*(
a^4*d^4 + b^4*c^4 - a^2*b^2*d^4 - b^4*c^2*d^2))/(a*c*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)*(a^2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)*
(a^4*d^4 + b^4*c^4 - a^2*b^2*d^4 - b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 2*a*b^3*c^
3*d + 2*a^3*b*c*d^3))/(a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c^2 - a^2*d
^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^2*(a*c + 2*b*d)*(a^3*d^3 +
 b^3*c^3 - a*b^2*d^3 - b^3*c*d^2))/(a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^
2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)))/(f*(a*c + tan(e/2 + (f*x)/2)^3*(2*a*
d + 2*b*c) + tan(e/2 + (f*x)/2)^2*(2*a*c + 4*b*d) + tan(e/2 + (f*x)/2)*(2*a
*d + 2*b*c) + a*c*tan(e/2 + (f*x)/2)^4)) - (b^2*atan(((b^2*(-(a + b)^3*(a -
 b)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 +
 a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 +
a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4
+ 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c
^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^
5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3
 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*
c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22
*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^1
0*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*
a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d
^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*
d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*
d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3
*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6
 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^
5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 -
60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c
^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*
a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(
a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*
b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*
a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 +
 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4
*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222
*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c
^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 2
22*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6
*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5
- 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b
^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 -
 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b
^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a
^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^
```

7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c
*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8
*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 4
4*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^
6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a
^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d
^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*
b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4
- 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*t
an(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a
^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 1
6*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^1
0 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*
b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^1
0*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*
a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^
8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 8
8*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*
c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 7
4*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5
*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 -
164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*...

# 3.713 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=458

$$\frac{2b^3(abc-4a^2d+3b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^4f}-\frac{d^2(2abcd(4c^2-d^2)-a^2d^2(2c^2+d^2)-3b^2(4c^4-5c^2d^2+2}{(bc-ad)^4(c^2-d^2)^{5/2}f}$$

```
[Out] 2*b^3*(-4*a^2*d+a*b*c+3*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(3/2)/(-a*d+b*c)^4/f-d^2*(2*a*b*c*d*(4*c^2-d^2)-a^2*d^2*(2*c
^2+d^2)-3*b^2*(4*c^4-5*c^2*d^2+2*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2
-d^2)^(1/2))/(-a*d+b*c)^4/(c^2-d^2)^(5/2)/f+1/2*d*(a^2*d^2+b^2*(2*c^2-3*d^2
))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+b^2*cos
(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*(3*a
^3*c*d^4-3*a*b^2*c*d^4-a^2*b*d^3*(7*c^2-4*d^2)-b^3*(2*c^4*d-11*c^2*d^3+6*d^
5))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]
time = 1.58, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$-\frac{d^2(-a^2d^2(2c^2+d^2)+2abcd(4c^2-d^2)-3b^2(4c^4-5c^2d^2+2d^4))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c^2-d^2)^{5/2}(bc-ad)^4}+\frac{2b^3(-4a^2d+abc+3b^2d)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}(bc-ad)^4}+\frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2f(a^2-b^2)(c^2-d^2)(bc-ad)^2(c+d\sin(e+fx))^2}+\frac{b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))(c+d\sin(e+fx))^2}-\frac{(3a^3cd^4-a^2bd^3(7c^2-4d^2)-3ab^2cd^4-b^3(2c^4d-11c^2d^3+6d^5))\cos(e+fx)}{2f(a^2-b^2)(c^2-d^2)^2(bc-ad)^3(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (2*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4*f) - (d^2*(2*a*b*c*d*(4*c^2 - d^
2) - a^2*d^2*(2*c^2 + d^2) - 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d +
 c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2)*f)
+ (d*(a^2*d^2 + b^2*(2*c^2 - 3*d^2))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + (b^2*Cos[e + f*x])/((a^2 - b^2
)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - ((3*a^3*c*d^
4 - 3*a*b^2*c*d^4 - a^2*b*d^3*(7*c^2 - 4*d^2) - b^3*(2*c^4*d - 11*c^2*d^3 +
 6*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
```

```
EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))^2}$$

$$= \frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{}{(a^2}$$

$$= \frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{}{(a^2}$$

$$= \frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{}{(a^2}$$

$$= \frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{}{(a^2}$$

$$= \frac{d(a^2d^2+b^2(2c^2-3d^2))\cos(e+fx)}{2\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^2} + \frac{}{(a^2}$$

$$= \frac{2b^3(abc-4a^2d+3b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,(bc-ad)^4f} - \frac{d^2(2abcd}{}$$

**Mathematica** [A]
time = 6.93, size = 346, normalized size = 0.76

$$\frac{\frac{4b^3(abc-4a^2d+3b^2d)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^4}+\frac{2d^2(2abcd(-4c^2+d^2)+a^2d^2(2c^2+d^2)+3b^2(4c^4-5c^2d^2+2d^4))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^4(c^2-d^2)^{5/2}}-\frac{2b^4\cos(e+fx)}{(a-b)(a+b)(-bc+ad)^3(a+b\sin(e+fx))}+\frac{d^3\cos(e+fx)}{(c-d)(c+d)(bc-ad)^2(c+d\sin(e+fx))^2}+\frac{d^3(7bc^2-3acd-4bd^2)\cos(e+fx)}{(c-d)^2(c+d)^2(bc-ad)^3(c+d\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4) + (2*d^2*(2*a*b*c*d*(-4*c^2 +
d^2) + a^2*d^2*(2*c^2 + d^2) + 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d
 + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2))
- (2*b^4*Cos[e + f*x])/((a - b)*(a + b)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x
])) + (d^3*Cos[e + f*x])/((c - d)*(c + d)*(b*c - a*d)^2*(c + d*Sin[e + f*x]
)^2) + (d^3*(7*b*c^2 - 3*a*c*d - 4*b*d^2)*Cos[e + f*x])/((c - d)^2*(c + d)^
2*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

## Maple [A]

time = 9.09, size = 760, normalized size = 1.66

| method | result |
|---|---|
| derivativedivides | $-\frac{2b^3\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a\left(a^2-b^2\right)}+\frac{b(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(4a^2d-abc-3b^2d\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{(ad-bc)^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2d^2\left(\frac{d^2\left(5a^2c^2d^2-2a^2d^4-1\right.}{}\right.}{}$ |
| default | $-\frac{2b^3\left(\frac{\frac{b^2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a\left(a^2-b^2\right)}+\frac{b(ad-bc)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(4a^2d-abc-3b^2d\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{(ad-bc)^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2d^2\left(\frac{d^2\left(5a^2c^2d^2-2a^2d^4-1\right.}{}\right.}{}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*b^3/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*((b^2*(a*d-b*c)/a/(a^2-
b^2)*tan(1/2*f*x+1/2*e)+b*(a*d-b*c)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*
tan(1/2*f*x+1/2*e)+a)+(4*a^2*d-a*b*c-3*b^2*d)/(a^2-b^2)^(3/2)*arctan(1/2*(2
*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))+2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c
^2)/(a*d-b*c)^2*((1/2*d^2*(5*a^2*c^2*d^2-2*a^2*d^4-14*a*b*c^3*d+8*a*b*c*d^3
+9*b^2*c^4-6*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3+1/2*d*
(4*a^2*c^4*d^2+7*a^2*c^2*d^4-2*a^2*d^6-12*a*b*c^5*d-18*a*b*c^3*d^3+12*a*b*c
*d^5+8*b^2*c^6+11*b^2*c^4*d^2-10*b^2*c^2*d^4)/(c^4-2*c^2*d^2+d^4)/c^2*tan(1
/2*f*x+1/2*e)^2+1/2*d^2*(11*a^2*c^2*d^2-2*a^2*d^4-34*a*b*c^3*d+16*a*b*c*d^3
+23*b^2*c^4-14*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)+1/2*d*
(4*a^2*c^2*d^2-a^2*d^4-12*a*b*c^3*d+6*a*b*c*d^3+8*b^2*c^4-5*b^2*c^2*d^2)/(c
^4-2*c^2*d^2+d^4))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*
(2*a^2*c^2*d^2+a^2*d^4-8*a*b*c^3*d+2*a*b*c*d^3+12*b^2*c^4-15*b^2*c^2*d^2+6*
b^2*d^4)/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. 2(453) = 906.
time = 0.57, size = 1109, normalized size = 2.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(a*b^4*c - 4*a^2*b^3*d + 3*b^5*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^4*c^4 -
b^6*c^4 - 4*a^3*b^3*c^3*d + 4*a*b^5*c^3*d + 6*a^4*b^2*c^2*d^2 - 6*a^2*b^4*c
^2*d^2 - 4*a^5*b*c*d^3 + 4*a^3*b^3*c*d^3 + a^6*d^4 - a^4*b^2*d^4)*sqrt(a^2
- b^2)) + (12*b^2*c^4*d^2 - 8*a*b*c^3*d^3 + 2*a^2*c^2*d^4 - 15*b^2*c^2*d^4
+ 2*a*b*c*d^5 + a^2*d^6 + 6*b^2*d^6)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^4*c^8 - 4*a*
b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 2*b^4*c^6*d^2 - 4*a^3*b*c^5*d^3 + 8*a*b^3*c
^5*d^3 + a^4*c^4*d^4 - 12*a^2*b^2*c^4*d^4 + b^4*c^4*d^4 + 8*a^3*b*c^3*d^5 -
 4*a*b^3*c^3*d^5 - 2*a^4*c^2*d^6 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*
d^8)*sqrt(c^2 - d^2)) + 2*(b^5*tan(1/2*f*x + 1/2*e) + a*b^4)/((a^3*b^3*c^3
- a*b^5*c^3 - 3*a^4*b^2*c^2*d + 3*a^2*b^4*c^2*d + 3*a^5*b*c*d^2 - 3*a^3*b^3
*c*d^2 - a^6*d^3 + a^4*b^2*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e) + a)) + (9*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^5*tan(1/2
```

```
*f*x + 1/2*e)^3 - 6*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*d^7*tan(1/2*f*
x + 1/2*e)^3 + 8*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a*c^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^2*d^6*tan(1/2*f*x
 + 1/2*e)^2 - 10*b*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^8*tan(1/2*f*x + 1/2
*e)^2 + 23*b*c^4*d^4*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^5*tan(1/2*f*x + 1/2*
e) - 14*b*c^2*d^6*tan(1/2*f*x + 1/2*e) + 2*a*c*d^7*tan(1/2*f*x + 1/2*e) + 8
*b*c^5*d^3 - 4*a*c^4*d^4 - 5*b*c^3*d^5 + a*c^2*d^6)/((b^3*c^9 - 3*a*b^2*c^8
*d + 3*a^2*b*c^7*d^2 - 2*b^3*c^7*d^2 - a^3*c^6*d^3 + 6*a*b^2*c^6*d^3 - 6*a^
2*b*c^5*d^4 + b^3*c^5*d^4 + 2*a^3*c^4*d^5 - 3*a*b^2*c^4*d^5 + 3*a^2*b*c^3*d
^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)
^2))/f
```

**Mupad [B]**
time = 45.34, size = 2500, normalized size = 5.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d^2*atan(((d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^3*
b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^
4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 -
 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^
10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 +
7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^1
2 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4
470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252
*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*
a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*
a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a
^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a
^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5
*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*
b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*
c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^6*b^8*c
^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^1
4 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8
- 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^13 + 50
64*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 - 688*a^8
*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a^9*b^5*
c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b^4*c^3*
d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^
7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d^10 + 6
28*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 - 220*a^1
```

2*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 10...

# 3.714 $\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=534

$$-\frac{d^3(30abcd-12a^2d^2-b^2(20c^2+d^2))\,x}{2b^5}+\frac{(bc-ad)^3\left(6a^3bcd-12ab^3cd+12a^4d^2+a^2b^2(2c^2-29d^2)+b^4(c\right.}{b^5\left(a^2-b^2\right)^{5/2}f}$$

```
[Out] -1/2*d^3*(30*a*b*c*d-12*a^2*d^2-b^2*(20*c^2+d^2))*x/b^5+(-a*d+b*c)^3*(6*a^3
*b*c*d-12*a*b^3*c*d+12*a^4*d^2+a^2*b^2*(2*c^2-29*d^2)+b^4*(c^2+20*d^2))*arc
tan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(5/2)/f-1/2*d*(
30*a^4*b*c*d^3-12*a^5*d^4-a^3*b^2*d^2*(16*c^2-21*d^2)-b^5*c*d*(17*c^2-10*d^
2)-a^2*b^3*c*d*(4*c^2+55*d^2)+a*b^4*(6*c^4+43*c^2*d^2-6*d^4))*cos(f*x+e)/b^
4/(a^2-b^2)^2/f+1/2*d^2*(7*a^3*b*c*d^2-6*a^4*d^3+b^4*d*(8*c^2-d^2)+a^2*b^2*
d*(c^2+10*d^2)-a*b^3*c*(3*c^2+16*d^2))*cos(f*x+e)*sin(f*x+e)/b^3/(a^2-b^2)^
2/f+1/2*(-a*d+b*c)^2*(4*a^2*d+3*a*b*c-7*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^
2/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f
*x+e))^3/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

**Rubi** [A]
time = 1.44, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2871, 3126, 3112, 3102, 2814, 2739, 632, 210}

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^3}{2bf(a^2-b^2)(a+b\sin(e+fx))^2}+\frac{(bc-ad)^2(4a^2d+3abc-7b^2d)\cos(e+fx)(c+d\sin(e+fx))^2}{2b^2f(a^2-b^2)^2(a+b\sin(e+fx))}-\frac{d^3x(-12a^2d^2+30abcd-(b^2(20c^2+d^2)))}{2b^5}+\frac{(bc-ad)^3(12a^4d^2+6a^3bcd+a^2b^2(2c^2-29d^2)-12ab^3cd+b^4(c^2+20d^2))\,\mathrm{ArcTan}\left(\frac{a\tan(\frac{1}{2}(e+fx))+b}{\sqrt{a^2-b^2}}\right)}{b^5f(a^2-b^2)^{5/2}}+\frac{d^2(-6a^4d^3+7a^3bcd^2+a^2b^2d(c^2+10d^2)-ab^3c(3c^2+16d^2)+b^4d(8c^2-d^2))\sin(e+fx)\cos(e+fx)}{2b^3f(a^2-b^2)^2}-\frac{d(-12a^5d^4+30a^4bcd^3-a^3b^2d^2(16c^2-21d^2)-a^2b^3cd(4c^2+55d^2)+ab^4(6c^4+43c^2d^2-6d^4)-b^5cd(17c^2-10d^2))\cos(e+fx)}{2b^4f(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -1/2*(d^3*(30*a*b*c*d - 12*a^2*d^2 - b^2*(20*c^2 + d^2))*x)/b^5 + ((b*c - a
*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2) +
 b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^5
*(a^2 - b^2)^(5/2)*f) - (d*(30*a^4*b*c*d^3 - 12*a^5*d^4 - a^3*b^2*d^2*(16*c
^2 - 21*d^2) - b^5*c*d*(17*c^2 - 10*d^2) - a^2*b^3*c*d*(4*c^2 + 55*d^2) + a
*b^4*(6*c^4 + 43*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(2*b^4*(a^2 - b^2)^2*f) +
(d^2*(7*a^3*b*c*d^2 - 6*a^4*d^3 + b^4*d*(8*c^2 - d^2) + a^2*b^2*d*(c^2 + 10
*d^2) - a*b^3*c*(3*c^2 + 16*d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^3*(a^2 -
b^2)^2*f) + ((b*c - a*d)^2*(3*a*b*c + 4*a^2*d - 7*b^2*d)*Cos[e + f*x]*(c +
d*Sin[e + f*x])^2)/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a
*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e
+ f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^3}{2b\,(a^2-b^2)\,f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2(7b^2c^2d+3a^2d^3-2abc(c^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\,(3abc+4a^2d-7b^2d)\cos(e+fx)(c+d\sin(e+fx))^2}{2b^2\,(a^2-b^2)^2\,f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\,\mathrm{co}\cdots}{2b\,(a^2-\cdots}$$

$$= \frac{d^2(7a^3bcd^2-6a^4d^3+b^4d(8c^2-d^2)+a^2b^2d(c^2+10d^2)-ab^3c(3c^2+16d^2))\cos(\cdots}{2b^3\,(a^2-b^2)^2\,f}$$

$$= -\frac{d(30a^4bcd^3-12a^5d^4-a^3b^2d^2(16c^2-21d^2)-b^5cd(17c^2-10d^2)-a^2b^3cd(4c^2\cdots}{2b^4\,(a^2-b^2)^2\,f}$$

$$= -\frac{d^3(30abcd-12a^2d^2-b^2(20c^2+d^2))\,x}{2b^5} - \frac{d(30a^4bcd^3-12a^5d^4-a^3b^2d^2(16c^2\cdots}{\cdots}$$

$$= -\frac{d^3(30abcd-12a^2d^2-b^2(20c^2+d^2))\,x}{2b^5} - \frac{d(30a^4bcd^3-12a^5d^4-a^3b^2d^2(16c^2\cdots}{\cdots}$$

$$= -\frac{d^3(30abcd-12a^2d^2-b^2(20c^2+d^2))\,x}{2b^5} - \frac{d(30a^4bcd^3-12a^5d^4-a^3b^2d^2(16c^2\cdots}{\cdots}$$

$$= -\frac{d^3(30abcd-12a^2d^2-b^2(20c^2+d^2))\,x}{2b^5} + \frac{(bc-ad)^3\,(6a^3bcd-12ab^3cd+12a\cdots}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 4.06, size = 341, normalized size = 0.64

$$\frac{2d^3(-30abcd+12a^2d^2+b^2(20c^2+d^2))(e+fx)+\frac{4(bc-ad)^3(6a^3bcd-12ab^3cd+12a^4d^2+a^2b^2(2c^2-29d^2)+b^4(c^2+20d^2))\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+2bd^4(-5bc+3ad)(\cos(e+fx)-i\sin(e+fx))+2bd^4(-5bc+3ad)(\cos(e+fx)+i\sin(e+fx))-\frac{2b(bc-ad)^5\cos(e+fx)}{(-a^2+b^2)(a+b\sin(e+fx))^2}+\frac{2b(bc-ad)^4(3abc+7a^2d-10b^2d)\cos(e+fx)}{(a^2-b^2)^2(a+b\sin(e+fx))}-b^2d^5\sin(2(e+fx))}{4b^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (2*d^3*(-30*a*b*c*d + 12*a^2*d^2 + b^2*(20*c^2 + d^2))*(e + f*x) + (4*(b*c
- a*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2
) + b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] - I*Sin[e + f*x])
 + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] + I*Sin[e + f*x]) - (2*b*(b*c - a
*d)^5*Cos[e + f*x])/((-a^2 + b^2)*(a + b*Sin[e + f*x])^2) + (2*b*(b*c - a*d
)^4*(3*a*b*c + 7*a^2*d - 10*b^2*d)*Cos[e + f*x])/((a^2 - b^2)^2*(a + b*Sin[
e + f*x])) - b^2*d^5*Sin[2*(e + f*x)])/(4*b^5*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(519) = 1038$.
time = 1.16, size = 1123, normalized size = 2.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d^3/b^5*((1/2*d^2*b^2*tan(1/2*f*x+1/2*e)^3+(3*a*b*d^2-5*b^2*c*d)*tan
(1/2*f*x+1/2*e)^2-1/2*d^2*b^2*tan(1/2*f*x+1/2*e)+3*a*b*d^2-5*b^2*c*d)/(1+ta
n(1/2*f*x+1/2*e)^2)^2+1/2*(12*a^2*d^2-30*a*b*c*d+20*b^2*c^2+b^2*d^2)*arctan
(tan(1/2*f*x+1/2*e)))-2/b^5*((-1/2*b^2*(5*a^7*d^5-15*a^6*b*c*d^4+10*a^5*b^2
*c^2*d^3-8*a^5*b^2*d^5+10*a^4*b^3*c^3*d^2+30*a^4*b^3*c*d^4-15*a^3*b^4*c^4*d
-40*a^3*b^4*c^2*d^3+5*a^2*b^5*c^5+20*a^2*b^5*c^3*d^2-2*b^7*c^5)/(a^4-2*a^2*
b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3-1/2*b*(6*a^9*d^5-20*a^8*b*c*d^4+20*a^7*b^2*
c^2*d^3+3*a^7*b^2*d^5-5*a^6*b^3*c*d^4-10*a^5*b^4*c^4*d-10*a^5*b^4*c^2*d^3-1
8*a^5*b^4*d^5+4*a^4*b^5*c^5+30*a^4*b^5*c^3*d^2+70*a^4*b^5*c*d^4-25*a^3*b^6*
c^4*d-100*a^3*b^6*c^2*d^3+7*a^2*b^7*c^5+60*a^2*b^7*c^3*d^2-10*a*b^8*c^4*d-2
*b^9*c^5)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2-1/2*b^2*(19*a^7*d^5-
65*a^6*b*c*d^4+70*a^5*b^2*c^2*d^3-28*a^5*b^2*d^5-10*a^4*b^3*c^3*d^2+110*a^4
*b^3*c*d^4-25*a^3*b^4*c^4*d-160*a^3*b^4*c^2*d^3+11*a^2*b^5*c^5+100*a^2*b^5*
c^3*d^2-20*a*b^6*c^4*d-2*b^7*c^5)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)-
1/2*b*(6*a^7*d^5-20*a^6*b*c*d^4+20*a^5*b^2*c^2*d^3-9*a^5*b^2*d^5+35*a^4*b^3
*c*d^4-10*a^3*b^4*c^4*d-50*a^3*b^4*c^2*d^3+4*a^2*b^5*c^5+30*a^2*b^5*c^3*d^2
-5*a*b^6*c^4*d-b^7*c^5)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*ta
n(1/2*f*x+1/2*e)+a)^2+1/2*(12*a^7*d^5-30*a^6*b*c*d^4+20*a^5*b^2*c^2*d^3-29*
a^5*b^2*d^5+75*a^4*b^3*c*d^4-50*a^3*b^4*c^2*d^3+20*a^3*b^4*d^5-2*a^2*b^5*c^
5-10*a^2*b^5*c^3*d^2-60*a^2*b^5*c*d^4+15*a*b^6*c^4*d+60*a*b^6*c^2*d^3-b^7*c
^5-20*b^7*c^3*d^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1556 vs. 2(529) = 1058.
time = 0.53, size = 3203, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(2*(20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 4*(5*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*d^5)*cos(f*x + e)^3 - 2*(20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x - ((2*a^4*b^5 + 3*a^2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*b^7 + 2*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d^3 + 15*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^7*b^2 - 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c^5 - 10*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^2*d^3 - 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 + a*b^8)*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 + 6*a^2*b^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^6 - 5*a^2*b^8 + b^10)*c^5 - 5*(2*a^5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b^8)*c^3*d^2 + 10*(2*a^7*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2 - 15*a^6*b^4 + 7*a^4*b^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - 2*((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*c*d^4 + (12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*x + (3*(a^3*b^7 - a*b^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a^5*b^5 - 5*a^3*b^7 + 4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8)*c^2*d^3 - 5*(9*a^7*b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^13)*f), 1/2*((20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 2*(5*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*d^5)*cos(f*x + e)^3 - (20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x + ((2*a^4*b^5 + 3*a^2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*b^7 + 2*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d^3 + 15*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^

```
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c
^5 - 10*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^
2*d^3 - 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5
*b^4 + 20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 +
 a*b^8)*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 +
6*a^2*b^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8
*b - 29*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a
*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((4*a^4*b^6 - 5*a^2*b^
8 + b^10)*c^5 - 5*(2*a^5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b
^8)*c^3*d^2 + 10*(2*a^7*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2
 - 15*a^6*b^4 + 7*a^4*b^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*
b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - ((a^6*b^4 - 3*a
^4*b^6 + 3*a^2*b^8 - b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^
8)*c*d^4 + (12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*
x + (3*(a^3*b^7 - a*b^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a
^5*b^5 - 5*a^3*b^7 + 4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8
)*c^2*d^3 - 5*(9*a^7*b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a
^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*si
n(f*x + e))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*
a^6*b^7 + 2*a^2*b^11 - b^13)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3162 vs. $2(529) = 1058$.
time = 0.60, size = 3162, normalized size = 5.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(2*a^2*b^5*c^5 + b^7*c^5 - 15*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 + 20*
b^7*c^3*d^2 - 20*a^5*b^2*c^2*d^3 + 50*a^3*b^4*c^2*d^3 - 60*a*b^6*c^2*d^3 +
30*a^6*b*c*d^4 - 75*a^4*b^3*c*d^4 + 60*a^2*b^5*c*d^4 - 12*a^7*d^5 + 29*a^5*
b^2*d^5 - 20*a^3*b^4*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan
```

((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9
)*sqrt(a^2 - b^2)) + 2*(5*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^7 - 2*a*b^8*c^5*
tan(1/2*f*x + 1/2*e)^7 - 15*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 10*a^5*b
^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 20*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)
^7 + 10*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 - 40*a^4*b^5*c^2*d^3*tan(1/2
*f*x + 1/2*e)^7 - 15*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 30*a^5*b^4*c*d^
4*tan(1/2*f*x + 1/2*e)^7 + 6*a^8*b*d^5*tan(1/2*f*x + 1/2*e)^7 - 10*a^6*b^3*
d^5*tan(1/2*f*x + 1/2*e)^7 + a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^7 + 4*a^4*b^5
*c^5*tan(1/2*f*x + 1/2*e)^6 + 7*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*
c^5*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 25*a
^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^6
 + 30*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^6 + 60*a^2*b^7*c^3*d^2*tan(1/2*f
*x + 1/2*e)^6 + 20*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^2*
d^3*tan(1/2*f*x + 1/2*e)^6 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 3
0*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 15*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e
)^6 + 60*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 12*a^9*d^5*tan(1/2*f*x + 1/
2*e)^6 - 5*a^7*b^2*d^5*tan(1/2*f*x + 1/2*e)^6 - 20*a^5*b^4*d^5*tan(1/2*f*x
+ 1/2*e)^6 + 4*a^3*b^6*d^5*tan(1/2*f*x + 1/2*e)^6 + 21*a^3*b^6*c^5*tan(1/2*
f*x + 1/2*e)^5 - 6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^5 - 55*a^4*b^5*c^4*d*tan(
1/2*f*x + 1/2*e)^5 - 20*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 10*a^5*b^4*c
^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 140*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 90*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 240*a^4*b^5*c^2*d^3*tan(1/2*f
*x + 1/2*e)^5 - 135*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 250*a^5*b^4*c*d^
4*tan(1/2*f*x + 1/2*e)^5 - 40*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 54*a^8
*b*d^5*tan(1/2*f*x + 1/2*e)^5 - 90*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 17*
a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^5 + 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^5 +
 12*a^4*b^5*c^5*tan(1/2*f*x + 1/2*e)^4 + 13*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e
)^4 - 4*b^9*c^5*tan(1/2*f*x + 1/2*e)^4 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2
*e)^4 - 55*a^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^4 - 20*a*b^8*c^4*d*tan(1/2*f*
x + 1/2*e)^4 + 90*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 120*a^2*b^7*c^3*
d^2*tan(1/2*f*x + 1/2*e)^4 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 70
*a^5*b^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 200*a^3*b^6*c^2*d^3*tan(1/2*f*x +
 1/2*e)^4 - 90*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 45*a^6*b^3*c*d^4*tan(1/
2*f*x + 1/2*e)^4 + 190*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 40*a^2*b^7*c*
d^4*tan(1/2*f*x + 1/2*e)^4 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^4 - 15*a^7*b^2
*d^5*tan(1/2*f*x + 1/2*e)^4 - 66*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^4 + 24*a^
3*b^6*d^5*tan(1/2*f*x + 1/2*e)^4 + 27*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^3 -
6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^3 - 65*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^
3 - 40*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 10*a^5*b^4*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 220*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 150*a^6*b^3*c^2
*d^3*tan(1/2*f*x + 1/2*e)^3 - 360*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 -
225*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 410*a^5*b^4*c*d^4*tan(1/2*f*x +
1/2*e)^3 - 80*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 90*a^8*b*d^5*tan(1/2*f
*x + 1/2*e)^3 - 162*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^3 + 55*a^4*b^5*d^5*tan
(1/2*f*x + 1/2*e)^3 - 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*b^5*c^5

```
*tan(1/2*f*x + 1/2*e)^2 + 5*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^2 - 2*b^9*c^5*
tan(1/2*f*x + 1/2*e)^2 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 35*a^3*b
^6*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 9
0*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 60*a^2*b^7*c^3*d^2*tan(1/2*f*x +
 1/2*e)^2 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 110*a^5*b^4*c^2*d^3
*tan(1/2*f*x + 1/2*e)^2 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 90*a
^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 85*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e)^2
 + 120*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 40*a^2*b^7*c*d^4*tan(1/2*f*x
+ 1/2*e)^2 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^2 - 31*a^7*b^2*d^5*tan(1/2*f*x
 + 1/2*e)^2 - 40*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 20*a^3*b^6*d^5*tan(1/
2*f*x + 1/2*e)^2 + 11*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e) - 2*a*b^8*c^5*tan(1/
2*f*x + 1/2*e) - 25*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e) - 20*a^2*b^7*c^4*d*t
an(1/2*f*x + 1/2*e) - 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e) + 100*a^3*b^6
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 70*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 1
60*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e) - 105*a^7*b^2*c*d^4*tan(1/2*f*x + 1
/2*e) + 190*a^5*b^4*c*d^4*tan(1/2*f*x + 1/2*e) - 40*a^3*b^6*c*d^4*tan(1/2*f
*x + 1/2*e) + 42*a^8*b*d^5*tan(1/2*f*x + 1/2*e) - 74*a^6*b^3*d^5*tan(1/2*f*
x + 1/2*e) + 23*a^4*b^5*d^5*tan(1/2*f*x + 1/2*e...
```

**Mupad [B]**
time = 25.76, size = 2500, normalized size = 4.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + b*sin(e + f*x))^3,x)
```

```
[Out] - ((b^7*c^5 - 12*a^7*d^5 - 4*a^2*b^5*c^5 - 6*a^3*b^4*d^5 + 21*a^5*b^2*d^5 +
 10*a^2*b^5*c*d^4 + 10*a^3*b^4*c^4*d - 55*a^4*b^3*c*d^4 - 30*a^2*b^5*c^3*d^
2 + 50*a^3*b^4*c^2*d^3 - 20*a^5*b^2*c^2*d^3 + 5*a*b^6*c^4*d + 30*a^6*b*c*d^
4)/(b^4*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 + (f*x)/2)^6*(12*a^9*d^5 - 2*b^
9*c^5 + 7*a^2*b^7*c^5 + 4*a^4*b^5*c^5 + 4*a^3*b^6*d^5 - 20*a^5*b^4*d^5 - 5*
a^7*b^2*d^5 - 25*a^3*b^6*c^4*d + 60*a^4*b^5*c*d^4 - 10*a^5*b^4*c^4*d + 15*a
^6*b^3*c*d^4 + 60*a^2*b^7*c^3*d^2 - 100*a^3*b^6*c^2*d^3 + 30*a^4*b^5*c^3*d^
2 - 10*a^5*b^4*c^2*d^3 + 20*a^7*b^2*c^2*d^3 - 10*a*b^8*c^4*d - 30*a^8*b*c*d
^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(2*b^9*c^5 -
 36*a^9*d^5 - 5*a^2*b^7*c^5 - 12*a^4*b^5*c^5 - 20*a^3*b^6*d^5 + 40*a^5*b^4*
d^5 + 31*a^7*b^2*d^5 + 40*a^2*b^7*c*d^4 + 35*a^3*b^6*c^4*d - 120*a^4*b^5*c*
d^4 + 30*a^5*b^4*c^4*d - 85*a^6*b^3*c*d^4 - 60*a^2*b^7*c^3*d^2 + 100*a^3*b^
6*c^2*d^3 - 90*a^4*b^5*c^3*d^2 + 110*a^5*b^4*c^2*d^3 - 60*a^7*b^2*c^2*d^3 +
 10*a*b^8*c^4*d + 90*a^8*b*c*d^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)) - (tan
(e/2 + (f*x)/2)^5*(54*a^7*d^5 - 6*b^7*c^5 + 4*a*b^6*d^5 + 21*a^2*b^5*c^5 +
17*a^3*b^4*d^5 - 90*a^5*b^2*d^5 - 40*a^2*b^5*c*d^4 - 55*a^3*b^4*c^4*d + 250
*a^4*b^3*c*d^4 + 140*a^2*b^5*c^3*d^2 - 240*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3
*d^2 + 90*a^5*b^2*c^2*d^3 - 20*a*b^6*c^4*d - 135*a^6*b*c*d^4))/(a*b^3*(a^4
```

+ b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^3*(6*b^7*c^5 - 90*a^7*d^5 + 4*a*b^6*d^5 - 27*a^2*b^5*c^5 - 55*a^3*b^4*d^5 + 162*a^5*b^2*d^5 + 80*a^2*b^5*c*d^4 + 65*a^3*b^4*c^4*d - 410*a^4*b^3*c*d^4 - 220*a^2*b^5*c^3*d^2 + 360*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 40*a*b^6*c^4*d + 225*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 + (f*x)/2)^7*(6*a^7*d^5 - 2*b^7*c^5 + 5*a^2*b^5*c^5 + a^3*b^4*d^5 - 10*a^5*b^2*d^5 - 15*a^3*b^4*c^4*d + 30*a^4*b^3*c*d^4 + 20*a^2*b^5*c^3*d^2 - 40*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 15*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)*(2*b^7*c^5 - 42*a^7*d^5 - 11*a^2*b^5*c^5 - 23*a^3*b^4*d^5 + 74*a^5*b^2*d^5 + 40*a^2*b^5*c*d^4 + 25*a^3*b^4*c^4*d - 190*a^4*b^3*c*d^4 - 100*a^2*b^5*c^3*d^2 + 160*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3*d^2 - 70*a^5*b^2*c^2*d^3 + 20*a*b^6*c^4*d + 105*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^4*(3*a^2 + 4*b^2)*(b^7*c^5 - 12*a^7*d^5 - 4*a^2*b^5*c^5 - 6*a^3*b^4*d^5 + 21*a^5*b^2*d^5 + 10*a^2*b^5*c*d^4 + 10*a^3*b^4*c^4*d - 55*a^4*b^3*c*d^4 - 30*a^2*b^5*c^3*d^2 + 50*a^3*b^4*c^2*d^3 - 20*a^5*b^2*c^2*d^3 + 5*a*b^6*c^4*d + 30*a^6*b*c*d^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(4*a^2 + 4*b^2) + tan(e/2 + (f*x)/2)^6*(4*a^2 + 4*b^2) + tan(e/2 + (f*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(e/2 + (f*x)/2)^8 + a^2 + 12*a*b*tan(e/2 + (f*x)/2)^3 + 12*a*b*tan(e/2 + (f*x)/2)^5 + 4*a*b*tan(e/2 + (f*x)/2)^7 + 4*a*b*tan(e/2 + (f*x)/2))) - (atan(((((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 4

```
8*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a
^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 -
 4*a^6*b^14 + a^8*b^12) + ((a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*
b*c*d^4*15i)*((8*tan(e/2 + (f*x)/2)*(4*a*b^21*c...
```

# 3.715 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=318

$$\frac{d^3(4bc-3ad)x}{b^4}+\frac{(bc-ad)^2\left(4a^3bcd-10ab^3cd+6a^4d^2+a^2b^2(2c^2-15d^2)+b^4(c^2+12d^2)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e}{\sqrt{a^2-}}\right.}{b^4\left(a^2-b^2\right)^{5/2}f}$$

```
[Out] d^3*(-3*a*d+4*b*c)*x/b^4+(-a*d+b*c)^2*(4*a^3*b*c*d-10*a*b^3*c*d+6*a^4*d^2+a
^2*b^2*(2*c^2-15*d^2)+b^4*(c^2+12*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^
2-b^2)^(1/2))/b^4/(a^2-b^2)^(5/2)/f+1/2*d^2*(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2
*d^2))*cos(f*x+e)/b^3/(a^2-b^2)/f+3/2*(-a*d+b*c)^3*(a^2*d+a*b*c-2*b^2*d)*co
s(f*x+e)/b^3/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+
d*sin(f*x+e))^2/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

**Rubi** [A]

time = 0.66, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3110, 3102, 2814, 2739, 632, 210}

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2bf(a^2-b^2)(a+b\sin(e+fx))^2}+\frac{d^2(-3a^2d^2+2abcd-(b^2(c^2-2d^2)))\cos(e+fx)}{2b^3f(a^2-b^2)}+\frac{3(bc-ad)^3(a^2d+abc-2b^2d)\cos(e+fx)}{2b^3f(a^2-b^2)^2(a+b\sin(e+fx))}+\frac{(bc-ad)^2(6a^4d^2+4a^3bcd+a^2b^2(2c^2-15d^2)-10ab^3cd+b^4(c^2+12d^2))\text{ArcTan}\left(\frac{a\tan(\frac{1}{2}(e+fx))+b}{\sqrt{a^2-b^2}}\right)}{b^4f(a^2-b^2)^{5/2}}+\frac{d^3x(4bc-3ad)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + ((b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d +
6*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Ta
n[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)*f) + (d^2*(2*a*b*c
*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(2*b^3*(a^2 - b^2)*f) + (
3*(b*c - a*d)^3*(a*b*c + a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*b^3*(a^2 - b^2)^
2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^2)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))(2(3b^2c^2d+a^2d^3-abc(c^2+\cdots}{\cdots}}{\cdots}$$

$$= \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(\cdots}{2b\left(a^2-b^2\right)f(a+b\sin(e+f\cdots}$$

$$= \frac{d^2(2abcd-3a^2d^2-b^2(c^2-2d^2))\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cdots}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2(2abcd-3a^2d^2-b^2(c^2-2d^2))\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\cdots}{2b^3\left(a^2\cdots\right.}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2(2abcd-3a^2d^2-b^2(c^2-2d^2))\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\cdots}{2b^3\left(a^2\cdots\right.}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2(2abcd-3a^2d^2-b^2(c^2-2d^2))\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\cdots}{2b^3\left(a^2\cdots\right.}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{(bc-ad)^2\left(4a^3bcd-10ab^3cd+6a^4d^2+a^2b^2(2c^2-15d^2)+b\cdots\right.}{b^4\left(a^2-b^2\right)^{5/2}f}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 894 vs. 2(318) = 636.
time = 4.31, size = 894, normalized size = 2.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((4*(b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2
- 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (16*a^6*b*c*d^3*e - 24*a^4*b^3*c*d^3*e + 8*b^7*c
*d^3*e - 12*a^7*d^4*e + 18*a^5*b^2*d^4*e - 6*a*b^6*d^4*e + 16*a^6*b*c*d^3*f
*x - 24*a^4*b^3*c*d^3*f*x + 8*b^7*c*d^3*f*x - 12*a^7*d^4*f*x + 18*a^5*b^2*d
^4*f*x - 6*a*b^6*d^4*f*x - b*(8*a*b^5*c^3*d - 16*a^5*b*c*d^3 + 12*a^6*d^4 -
 21*a^4*b^2*d^4 + 8*a^3*b^3*c*d*(2*c^2 + 5*d^2) + b^6*(2*c^4 + d^4) + 2*a^2
```

```
*b^4*(-4*c^4 - 18*c^2*d^2 + d^4))*Cos[e + f*x] + 2*b^2*(a^2 - b^2)^2*d^3*(-
4*b*c + 3*a*d)*(e + f*x)*Cos[2*(e + f*x)] + a^4*b^3*d^4*Cos[3*(e + f*x)] -
2*a^2*b^5*d^4*Cos[3*(e + f*x)] + b^7*d^4*Cos[3*(e + f*x)] + 32*a^5*b^2*c*d^
3*e*Sin[e + f*x] - 64*a^3*b^4*c*d^3*e*Sin[e + f*x] + 32*a*b^6*c*d^3*e*Sin[e
 + f*x] - 24*a^6*b*d^4*e*Sin[e + f*x] + 48*a^4*b^3*d^4*e*Sin[e + f*x] - 24*
a^2*b^5*d^4*e*Sin[e + f*x] + 32*a^5*b^2*c*d^3*f*x*Sin[e + f*x] - 64*a^3*b^4
*c*d^3*f*x*Sin[e + f*x] + 32*a*b^6*c*d^3*f*x*Sin[e + f*x] - 24*a^6*b*d^4*f*
x*Sin[e + f*x] + 48*a^4*b^3*d^4*f*x*Sin[e + f*x] - 24*a^2*b^5*d^4*f*x*Sin[e
 + f*x] + 3*a*b^6*c^4*Sin[2*(e + f*x)] - 4*a^2*b^5*c^3*d*Sin[2*(e + f*x)] -
 8*b^7*c^3*d*Sin[2*(e + f*x)] - 6*a^3*b^4*c^2*d^2*Sin[2*(e + f*x)] + 24*a*b
^6*c^2*d^2*Sin[2*(e + f*x)] + 12*a^4*b^3*c*d^3*Sin[2*(e + f*x)] - 24*a^2*b^
5*c*d^3*Sin[2*(e + f*x)] - 9*a^5*b^2*d^4*Sin[2*(e + f*x)] + 16*a^3*b^4*d^4*
Sin[2*(e + f*x)] - 4*a*b^6*d^4*Sin[2*(e + f*x)])/((a^2 - b^2)^2*(a + b*Sin[
e + f*x])^2))/(4*b^4*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(307) = 614$.

time = 0.97, size = 854, normalized size = 2.69

| method | result |
|---|---|
| derivativedivides | $-\frac{2d^3\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3ad-4bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^4}+\frac{2\left(-\frac{b^2\left(3a^6d^4-4a^5bc\,d^3-6b^2a^4c^2d^2-6a^4b^2d^4+12a^3b^3c^3d+16a^3b\cdots\right)}{2\left(a^4-2a^2b^2+\cdots\right)}\cdots\right)}{\cdots}$ |
| default | $-\frac{2d^3\left(\frac{bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(3ad-4bc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{b^4}+\frac{2\left(-\frac{b^2\left(3a^6d^4-4a^5bc\,d^3-6b^2a^4c^2d^2-6a^4b^2d^4+12a^3b^3c^3d+16a^3b\cdots\right)}{2\left(a^4-2a^2b^2+\cdots\right)}\cdots\right)}{\cdots}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*d^3/b^4*(b*d/(1+tan(1/2*f*x+1/2*e)^2)+(3*a*d-4*b*c)*arctan(tan(1/2*
f*x+1/2*e)))+2/b^4*((-1/2*b^2*(3*a^6*d^4-4*a^5*b*c*d^3-6*a^4*b^2*c^2*d^2-6*
a^4*b^2*d^4+12*a^3*b^3*c^3*d+16*a^3*b^3*c*d^3-5*a^2*b^4*c^4-12*a^2*b^4*c^2*
d^2+2*b^6*c^4)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3-1/2*b*(4*a^8*d^4-
8*a^7*b*c*d^3+a^6*b^2*d^4+8*a^5*b^3*c^3*d+4*a^5*b^3*c*d^3-4*a^4*b^4*c^4-18*
a^4*b^4*c^2*d^2-14*a^4*b^4*d^4+20*a^3*b^5*c^3*d+40*a^3*b^5*c*d^3-7*a^2*b^6*
c^4-36*a^2*b^6*c^2*d^2+8*a*b^7*c^3*d+2*b^8*c^4)/(a^4-2*a^2*b^2+b^4)/a^2*tan
(1/2*f*x+1/2*e)^2-1/2*b^2*(13*a^6*d^4-28*a^5*b*c*d^3+6*a^4*b^2*c^2*d^2-22*a
^4*b^2*d^4+20*a^3*b^3*c^3*d+64*a^3*b^3*c*d^3-11*a^2*b^4*c^4-60*a^2*b^4*c^2*
d^2+16*a*b^5*c^3*d+2*b^6*c^4)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)-1/2*
b*(4*a^6*d^4-8*a^5*b*c*d^3-7*a^4*b^2*d^4+8*a^3*b^3*c^3*d+20*a^3*b^3*c*d^3-4
```

```
*a^2*b^4*c^4-18*a^2*b^4*c^2*d^2+4*a*b^5*c^3*d+b^6*c^4)/(a^4-2*a^2*b^2+b^4))
/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)^2+1/2*(6*a^6*d^4-8*a^5*b
*c*d^3-15*a^4*b^2*d^4+20*a^3*b^3*c*d^3+2*a^2*b^4*c^4+6*a^2*b^4*c^2*d^2+12*a
^2*b^4*d^4-12*a*b^5*c^3*d-24*a*b^5*c*d^3+b^6*c^4+12*b^6*c^2*d^2)/(a^4-2*a^2
*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)
^(1/2))))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(314) = 628$.
time = 0.48, size = 2362, normalized size = 7.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*cos(f*x + e)^3 - 4*(4*
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^7*b^2 - 3*a^5*b^4 + 3*
a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 4*(4*(a^8*b - 2*a^6*b^3 + 2*a^2*
b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*d^4)*f*x - ((2*a
^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*c^3*d + 6*(a^4*b^4 + 3
*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 + a^3*b^5 + 6*a*b^7)*c*d
^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4 + (12*a*b^7*c^3*d - (2
*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4*(2*a^5*b^3 - 5*a^3*b^
5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*d^4)*cos(f*x + e
)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 - 6*(a^3*b^5 + 2*a*b^7)
*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^3 - 3*(2*a^7*b - 5*a^5
*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*co
s(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x +
 e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x
 + e) - a^2 - b^2)) + 2*((4*a^4*b^5 - 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 -
 a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7
*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7
```

```
 - 2*b^9)*d^4)*cos(f*x + e) + 2*(4*(4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3
*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^
3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*
a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*sin(f*x + e) - (a^8*b^4 - 2*a^6*b^6 +
2*a^2*b^10 - b^12)*f), -1/2*(2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*
cos(f*x + e)^3 - 2*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 2*(4*(a^8*
b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a
*b^8)*d^4)*f*x - ((2*a^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*
c^3*d + 6*(a^4*b^4 + 3*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 +
a^3*b^5 + 6*a*b^7)*c*d^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4
+ (12*a*b^7*c^3*d - (2*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4
*(2*a^5*b^3 - 5*a^3*b^5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2
*b^6)*d^4)*cos(f*x + e)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 -
 6*(a^3*b^5 + 2*a*b^7)*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^
3 - 3*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b^5
- 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 - a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^
5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8
*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*d^4)*cos(f*x + e) + (4*(4*
(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*
a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b
^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3
 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*
a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10
- b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*s
in(f*x + e) - (a^8*b^4 - 2*a^6*b^6 + 2*a^2*b^10 - b^12)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(314) = 628$.
time = 0.51, size = 1159, normalized size = 3.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^4*c^4 + b^6*c^4 - 12*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 + 12*b^6*c^2
*d^2 - 8*a^5*b*c*d^3 + 20*a^3*b^3*c*d^3 - 24*a*b^5*c*d^3 + 6*a^6*d^4 - 15*a
^4*b^2*d^4 + 12*a^2*b^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 +
b^8)*sqrt(a^2 - b^2)) - 2*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*b^3) + (5*a^3*b
^5*c^4*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^4
*b^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^
3 + 12*a^3*b^5*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*a^6*b^2*c*d^3*tan(1/2*f*x
 + 1/2*e)^3 - 16*a^4*b^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^7*b*d^4*tan(1/2
*f*x + 1/2*e)^3 + 6*a^5*b^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^4*c^4*tan(
1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c^4*tan(1/2*f*x + 1/2*e)^2 - 2*b^8*c^4*tan(1
/2*f*x + 1/2*e)^2 - 8*a^5*b^3*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 20*a^3*b^5*c^3
*d*tan(1/2*f*x + 1/2*e)^2 - 8*a*b^7*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 18*a^4*b
^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*b^6*c^2*d^2*tan(1/2*f*x + 1/2*e)
^2 + 8*a^7*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*b^3*c*d^3*tan(1/2*f*x + 1
/2*e)^2 - 40*a^3*b^5*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^8*d^4*tan(1/2*f*x +
 1/2*e)^2 - a^6*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 14*a^4*b^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*a^3*b^5*c^4*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c^4*tan(1/2*f*x
 + 1/2*e) - 20*a^4*b^4*c^3*d*tan(1/2*f*x + 1/2*e) - 16*a^2*b^6*c^3*d*tan(1/
2*f*x + 1/2*e) - 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e) + 60*a^3*b^5*c^2*d^
2*tan(1/2*f*x + 1/2*e) + 28*a^6*b^2*c*d^3*tan(1/2*f*x + 1/2*e) - 64*a^4*b^4
*c*d^3*tan(1/2*f*x + 1/2*e) - 13*a^7*b*d^4*tan(1/2*f*x + 1/2*e) + 22*a^5*b^
3*d^4*tan(1/2*f*x + 1/2*e) + 4*a^4*b^4*c^4 - a^2*b^6*c^4 - 8*a^5*b^3*c^3*d
- 4*a^3*b^5*c^3*d + 18*a^4*b^4*c^2*d^2 + 8*a^7*b*c*d^3 - 20*a^5*b^3*c*d^3 -
 4*a^8*d^4 + 7*a^6*b^2*d^4)/((a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*(a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2) + (4*b*c*d^3 - 3*a*d^4)*(f*x
 + e)/b^4)/f
```

**Mupad [B]**
time = 21.74, size = 2500, normalized size = 7.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (2*d^3*atan(((d^3*(3*a*d - 4*b*c)*((8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^
3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*
b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15
*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24
*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c
*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*
```

a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (d^3*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (d^3*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(3*a*d - 4*b*c)*1i)/b^4)*1i)/b^4))/b^4 - (d^3*(3*a*d - 4*b*c)*((8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3

```
*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12
*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^1
0*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*
a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 +
24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17
*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e
/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^1
6*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^
8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4
*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d +
144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*
c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3*((
8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^
16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2
)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a
^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*...
```

# 3.716 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=248

$$\frac{d^3x}{b^3}+\frac{(bc-ad)\left(2a^3bcd-8ab^3cd+2a^4d^2+a^2b^2(2c^2-5d^2)+b^4(c^2+6d^2)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}f}+\frac{(bc-$$

```
[Out] d^3*x/b^3+(-a*d+b*c)*(2*a^3*b*c*d-8*a*b^3*c*d+2*a^4*d^2+a^2*b^2*(2*c^2-5*d^
2)+b^4*(c^2+6*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a
^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)^2*(2*a^2*d+3*a*b*c-5*b^2*d)*cos(f*x+e)/b^2/(
a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))/
b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

**Rubi** [A]

time = 0.54, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2871, 3100, 2814, 2739, 632, 210}

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{(bc-ad)^2\left(2a^2d+3abc-5b^2d\right)\cos(e+fx)}{2b^2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\left(2a^4d^2+2a^3bcd+a^2b^2(2c^2-5d^2)-8ab^3cd+b^4(c^2+6d^2)\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{5/2}}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)*(2*a^3*b*c*d - 8*a*b^3*c*d + 2*a^4*d^2 + a^2*b^2
*(2*c^2 - 5*d^2) + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]])/(b^3*(a^2 - b^2)^(5/2)*f) + ((b*c - a*d)^2*(3*a*b*c + 2*a^2*d
- 5*b^2*d)*Cos[e + f*x])/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b
*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*b*(a^2 - b^2)*f*(a + b*Si
n[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \int \frac{5b^2c^2d+a^2d^3-2abc\left(c^2+2d^2\right)-\left(a^2cd^2+2ab\right.}{}$$

$$= \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin}{2b\left(a^2-b^2\right)f(a+b\sin(e+}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c}{2b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c}{2b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c}{2b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)\left(2a^2b^2c^2+b^4c^2+2a^3bcd-8ab^3cd+2a^4d^2-5a^2b^2d^2+6b^4d^2\right.}{b^3\left(a^2-b^2\right)^{5/2}f}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 524 vs. 2(248) = 496.
time = 2.52, size = 524, normalized size = 2.11

$$\frac{-\frac{4\left(2a^5d^3-5a^3b^2d^3+3ab^4d\left(3c^2+2d^2\right)-a^2b^3c\left(2c^2+3d^2\right)-b^5c\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{4a^6d^3e-6a^4b^2d^3e+2b^6d^3e+4a^6d^3fx-6a^4b^2d^3fx+2b^6d^3fx-2b(bc-ad)^2\left(-4a^2bc+b^3c-2a^3d+5ab^2d\right)\cos(e+fx)-2\left(-a^2b+b^3\right)^2d^3(e+fx)\cos(2(e+fx))+8a^5bd^3e\sin(e+fx)-16a^3b^3d^3e\sin(e+fx)+8ab^5d^3e\sin(e+fx)+8a^5bd^3fx\sin(e+fx)-16a^3b^3d^3fx\sin(e+fx)+8ab^5d^3fx\sin(e+fx)+3ab^5c^3\sin(2(e+fx))-3a^2b^4c^2d\sin(2(e+fx))-6b^6c^2d\sin(2(e+fx))-3a^3b^3cd^2\sin(2(e+fx))+12ab^5cd^2\sin(2(e+fx))+3a^4b^2d^3\sin(2(e+fx))-6a^2b^4d^3\sin(2(e+fx))}{\left(a^2-b^2\right)^2(a+b\sin(e+fx))^2}}{4b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(2*a^5*d^3 - 5*a^3*b^2*d^3 + 3*a*b^4*d*(3*c^2 + 2*d^2) - a^2*b^3*c*(2*
c^2 + 3*d^2) - b^5*c*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(5/2) + (4*a^6*d^3*e - 6*a^4*b^2*d^3*e + 2*b^6*d^3*e
 + 4*a^6*d^3*f*x - 6*a^4*b^2*d^3*f*x + 2*b^6*d^3*f*x - 2*b*(b*c - a*d)^2*(-
4*a^2*b*c + b^3*c - 2*a^3*d + 5*a*b^2*d)*Cos[e + f*x] - 2*(-(a^2*b) + b^3)^
2*d^3*(e + f*x)*Cos[2*(e + f*x)] + 8*a^5*b*d^3*e*Sin[e + f*x] - 16*a^3*b^3*
d^3*e*Sin[e + f*x] + 8*a*b^5*d^3*e*Sin[e + f*x] + 8*a^5*b*d^3*f*x*Sin[e + f
*x] - 16*a^3*b^3*d^3*f*x*Sin[e + f*x] + 8*a*b^5*d^3*f*x*Sin[e + f*x] + 3*a*
b^5*c^3*Sin[2*(e + f*x)] - 3*a^2*b^4*c^2*d*Sin[2*(e + f*x)] - 6*b^6*c^2*d*S
in[2*(e + f*x)] - 3*a^3*b^3*c*d^2*Sin[2*(e + f*x)] + 12*a*b^5*c*d^2*Sin[2*(
e + f*x)] + 3*a^4*b^2*d^3*Sin[2*(e + f*x)] - 6*a^2*b^4*d^3*Sin[2*(e + f*x)]
)/((a^2 - b^2)^2*(a + b*Sin[e + f*x])^2))/(4*b^3*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(239) = 478.
time = 0.68, size = 671, normalized size = 2.71

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2d^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b^3}-\frac{2\left(-\frac{b^2\left(a^5d^3+3a^4bc\,d^2-9a^3b^2c^2d-4a^3b^2d^3+5a^2b^3c^3+6a^2b^3c\,d^2-2b^5c^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}-\frac{b\left(2a^7d^3-\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{2d^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b^3}-\frac{2\left(-\frac{b^2\left(a^5d^3+3a^4bc\,d^2-9a^3b^2c^2d-4a^3b^2d^3+5a^2b^3c^3+6a^2b^3c\,d^2-2b^5c^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}-\frac{b\left(2a^7d^3-\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))-2/b^3*((-1/2*b^2*(a^5*d^3+3*a^4*b
*c*d^2-9*a^3*b^2*c^2*d-4*a^3*b^2*d^3+5*a^2*b^3*c^3+6*a^2*b^3*c*d^2-2*b^5*c^
3)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3-1/2*b*(2*a^7*d^3-6*a^5*b^2*c^
2*d-a^5*b^2*d^3+4*a^4*b^3*c^3+9*a^4*b^3*c*d^2-15*a^3*b^4*c^2*d-10*a^3*b^4*d
^3+7*a^2*b^5*c^3+18*a^2*b^5*c*d^2-6*a*b^6*c^2*d-2*b^7*c^3)/(a^4-2*a^2*b^2+b
^4)/a^2*tan(1/2*f*x+1/2*e)^2-1/2*b^2*(7*a^5*d^3-3*a^4*b*c*d^2-15*a^3*b^2*c^
2*d-16*a^3*b^2*d^3+11*a^2*b^3*c^3+30*a^2*b^3*c*d^2-12*a*b^4*c^2*d-2*b^5*c^3
)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)-1/2*b*(2*a^5*d^3-6*a^3*b^2*c^2*d
-5*a^3*b^2*d^3+4*a^2*b^3*c^3+9*a^2*b^3*c*d^2-3*a*b^4*c^2*d-b^5*c^3)/(a^4-2*
a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)^2+1/2*(2*a^
5*d^3-5*a^3*b^2*d^3-2*a^2*b^3*c^3-3*a^2*b^3*c*d^2+9*a*b^4*c^2*d+6*a*b^4*d^3
-b^5*c^3-6*b^5*c*d^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(245) = 490$.

time = 0.43, size = 1656, normalized size = 6.68

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*cos(f*x + e)^2 - 4*
(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x - ((2*a^4*b^3 + 3*a^2*b^5 + b^7
)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^5 + 2*b^7)*c*d^2 -
 (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c^2*d - (2*a^2*b^5
+ b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*
d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a*b^6)*c^3 - 3*(a^3
*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*d^3)*sin(f*x + e)
)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
- a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^
4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 - a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b
^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*d^3)*cos(f*x + e) -
 2*(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d^3*f*x + 3*((a^3*b^5 - a*b^7
)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d - (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)
*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*d^3)*cos(f*x + e))*sin(f*x + e))
/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*f*cos(f*x + e)^2 - 2*(a^7*b^4 -
3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x + e) - (a^8*b^3 - 2*a^6*b^5 + 2*a
^2*b^9 - b^11)*f), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*c
os(f*x + e)^2 - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x + ((2*a^4*b^3
 + 3*a^2*b^5 + b^7)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^
5 + 2*b^7)*c*d^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c
^2*d - (2*a^2*b^5 + b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a
^3*b^4 + 6*a*b^6)*d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a
*b^6)*c^3 - 3*(a^3*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)
*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2
- b^2)*cos(f*x + e))) - ((4*a^4*b^4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 -
 a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*d^3)*cos(f*x + e) - (4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*d^3*f*x + 3*((a^3*b^5 - a*b^7)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d -
 (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*
d^3)*cos(f*x + e))*sin(f*x + e))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*
f*cos(f*x + e)^2 - 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x +
 e) - (a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11)*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(245) = 490.
time = 0.52, size = 887, normalized size = 3.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^3/b^3 + (2*a^2*b^3*c^3 + b^5*c^3 - 9*a*b^4*c^2*d + 3*a^2*b^3*c
*d^2 + 6*b^5*c*d^2 - 2*a^5*d^3 + 5*a^3*b^2*d^3 - 6*a*b^4*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2
- b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) + (5*a^3*b^4*c^3*tan
(1/2*f*x + 1/2*e)^3 - 2*a*b^6*c^3*tan(1/2*f*x + 1/2*e)^3 - 9*a^4*b^3*c^2*d*
tan(1/2*f*x + 1/2*e)^3 + 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*b^4
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^6*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*b^
3*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c^3*tan(1/2*f*x + 1/2*e)^2 + 7*a^2
*b^5*c^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c^3*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*
b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 - 15*a^3*b^4*c^2*d*tan(1/2*f*x + 1/2*e)^2
- 6*a*b^6*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 9*a^4*b^3*c*d^2*tan(1/2*f*x + 1/2*
e)^2 + 18*a^2*b^5*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*a^7*d^3*tan(1/2*f*x + 1/
2*e)^2 - a^5*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*b^4*d^3*tan(1/2*f*x +
1/2*e)^2 + 11*a^3*b^4*c^3*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c^3*tan(1/2*f*x +
1/2*e) - 15*a^4*b^3*c^2*d*tan(1/2*f*x + 1/2*e) - 12*a^2*b^5*c^2*d*tan(1/2*f
*x + 1/2*e) - 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e) + 30*a^3*b^4*c*d^2*tan(1
/2*f*x + 1/2*e) + 7*a^6*b*d^3*tan(1/2*f*x + 1/2*e) - 16*a^4*b^3*d^3*tan(1/2
*f*x + 1/2*e) + 4*a^4*b^3*c^3 - a^2*b^5*c^3 - 6*a^5*b^2*c^2*d - 3*a^3*b^4*c
^2*d + 9*a^4*b^3*c*d^2 + 2*a^7*d^3 - 5*a^5*b^2*d^3)/((a^6*b^2 - 2*a^4*b^4 +
 a^2*b^6)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

**Mupad** [B]
time = 20.93, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x))^3,x)
```

```
[Out] - ((b^5*c^3 - 2*a^5*d^3 - 4*a^2*b^3*c^3 + 5*a^3*b^2*d^3 - 9*a^2*b^3*c*d^2 +
 6*a^3*b^2*c^2*d + 3*a*b^4*c^2*d)/(b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2
```

+ (f*x)/2)^3*(a^5*d^3 - 2*b^5*c^3 + 5*a^2*b^3*c^3 - 4*a^3*b^2*d^3 + 6*a^2*b
^3*c*d^2 - 9*a^3*b^2*c^2*d + 3*a^4*b*c*d^2))/(a*b*(a^4 + b^4 - 2*a^2*b^2))
+ (tan(e/2 + (f*x)/2)*(2*b^5*c^3 - 7*a^5*d^3 - 11*a^2*b^3*c^3 + 16*a^3*b^2*
d^3 - 30*a^2*b^3*c*d^2 + 15*a^3*b^2*c^2*d + 12*a*b^4*c^2*d + 3*a^4*b*c*d^2)
)/(a*b*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(a^2 + 2*b^2)*(b^5*
c^3 - 2*a^5*d^3 - 4*a^2*b^3*c^3 + 5*a^3*b^2*d^3 - 9*a^2*b^3*c*d^2 + 6*a^3*b
^2*c^2*d + 3*a*b^4*c^2*d))/(a^2*b^2*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 +
(f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^4 + a^2 + 4*a*b*tan(e/
2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2))) - (2*d^3*atan(((d^3*((8*(4*a^2*
b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^
6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*tan(e/
2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*
d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 -
36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^
2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10
- 4*a^6*b^8 + a^8*b^6) - (8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3
- 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b
^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5
*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a
^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6
*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b
^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^
14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^
10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12
*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 12
4*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^
12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24
*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 12
0*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b
^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 +
36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 -
4*a^6*b^8 + a^8*b^6)))/b^3 + (d^3*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 2
4*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^
4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*
a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d
^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*
d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^
2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(4*a*b^15
*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3
- 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 3
6*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2
+ 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6)
+ (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*
b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 +
(f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^1

0 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i )/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b ^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d ^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^ 2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c ^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3* b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d ^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b ^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6)))/b^3)/ ((16*(24*a^3*b^6*d^9 - 2*a^9*d^9 - 26*a^5*b^4*d^9 + 13*a^7*b^2*d^9 + 36*a*b ^8*c^2*d^7 + 12*a*b^8*c^4*d^5 + a*b^8*c^6*d^3 - 60*a^2*b^7*c*d^8 + 6*a^4*b^ 5*c*d^8 + 6*a^6*b^3*c*d^8 - 4*a^8*b*c^3*d^6 - 118*a^2*b^7*c^3*d^6 - 18*a^2* b^7*c^5*d^4 + 126*a^3*b^6*c^2*d^7 + 111*a^3*b^6*c^4*d^5 + 4*a^3*b^6*c^6*d^3 - 68*a^4*b^5*c^3*d^6 - 36*a^4*b^5*c^5*d^4 - 45*a^5*b^4*c^2*d^7 + 12*a^5*b^ 4*c^4*d^5 + 4*a^5*b^4*c^6*d^3 + 10*a^6*b^3*c^3*d^6 + 18*a^7*b^2*c^2*d^7 - 6 *a^8*b*c*d^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (16* tan(e/2 + (f*x)/2)*(8*a^10*d^9 + 24*a^2*b^8*d^9 - 68*a^4*b^6*d^9 + 72*a^6*b ^4*d^9 - 36*a^8*b^2*d^9 - 4*a*b^9*c^3*d^6 + 36*a^3*b^7*c*d^8 - 12*a^7*b^3*c *d^8 + 36*a^2*b^8*c^2*d^7 - 72*a^4*b^6*c^2*d^7 + 12*a^5*b^5*c^3*d^6 + 36*a^ 6*b^4*c^2*d^7 - 8*a^7*b^3*c^3*d^6 - 24*a*b^9*c*...

# 3.717 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=196

$$-\frac{(6abcd-a^2(2c^2+d^2)-b^2(c^2+2d^2))\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2}+\frac{(bc-}{2}$$

```
[Out] -(6*a*b*c*d-a^2*(2*c^2+d^2)-b^2*(c^2+2*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e)
)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)/b/(a^2-b^2
)/f/(a+b*sin(f*x+e))^2+1/2*(-a*d+b*c)*(a^2*d+3*a*b*c-4*b^2*d)*cos(f*x+e)/b/
(a^2-b^2)^2/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.19, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2869, 2833, 12, 2739, 632, 210}

$$-\frac{(-(a^2(2c^2+d^2))+6abcd-b^2(c^2+2d^2))\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2bf(a^2-b^2)(a+b\sin(e+fx))^2}+\frac{(a^2d+3abc-4b^2d)(bc-ad)\cos(e+fx)}{2bf(a^2-b^2)^2(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((b*c - a*d)*(3*a*b*c
 + a^2*d - 4*b^2*d)*Cos[e + f*x])/(2*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{\int \frac{-2b(2bcd-a(c^2+d^2))+(2abcd+a^2d^2-b^2(c^2+2d^2))\,\mathrm{si}}{(a+b\sin(e+fx))^2}}{2b(a^2-b^2)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)(3abc+a^2d-4b^2d)\cos(e+fx}{2b(a^2-b^2)^2f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)(3abc+a^2d-4b^2d)\cos(e+fx}{2b(a^2-b^2)^2f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)(3abc+a^2d-4b^2d)\cos(e+fx}{2b(a^2-b^2)^2f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)(3abc+a^2d-4b^2d)\cos(e+fx}{2b(a^2-b^2)^2f(a+b\sin(e+fx))}$$

$$= -\frac{(6abcd-a^2(2c^2+d^2)-b^2(c^2+2d^2))\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f} + \frac{(bc-}{2b(a^2-b^2}$$

**Mathematica [A]**
time = 1.02, size = 204, normalized size = 1.04

$$\frac{\frac{2(-6abcd+a^2(2c^2+d^2)+b^2(c^2+2d^2))\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(bc-ad)^2\cos(e+fx)}{(a-b)b(a+b)(a+b\sin(e+fx))^2} - \frac{(2a^2bcd+4b^3cd+a^3d^2-ab^2(3c^2+4d^2))\cos(e+fx)}{(a-b)^2b(a+b)^2(a+b\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[
(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((a - b)*b*(a + b)*(a + b*Sin[e + f*x])^2) - ((2*a^2*b*c*d + 4*b^3*c*
d + a^3*d^2 - a*b^2*(3*c^2 + 4*d^2))*Cos[e + f*x])/((a - b)^2*b*(a + b)^2*(
a + b*Sin[e + f*x])))/(2*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(187) = 374$.
time = 0.52, size = 465, normalized size = 2.37

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{\left(a^4d^2-6a^3bcd+5a^2b^2c^2+2a^2b^2d^2-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a}-\frac{\left(4a^5cd-4a^4b\,c^2-3a^4b\,d^2+10a^3b^2cd-7a^2b^3c^2-6a^2b^3d^2+4a\,b^4cd+2b^5\right.}{\left(a^4-2a^2b^2+b^4\right)a^2}}{\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}$ |
| default | $\frac{\frac{\left(a^4d^2-6a^3bcd+5a^2b^2c^2+2a^2b^2d^2-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a}-\frac{\left(4a^5cd-4a^4b\,c^2-3a^4b\,d^2+10a^3b^2cd-7a^2b^3c^2-6a^2b^3d^2+4a\,b^4cd+2b^5\right.}{\left(a^4-2a^2b^2+b^4\right)a^2}}{\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*(a^4*d^2-6*a^3*b*c*d+5*a^2*b^2*c^2+2*a^2*b^2*d^2-2*b^4*c^2)/(a^
4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3-1/2*(4*a^5*c*d-4*a^4*b*c^2-3*a^4*b*
d^2+10*a^3*b^2*c*d-7*a^2*b^3*c^2-6*a^2*b^3*d^2+4*a*b^4*c*d+2*b^5*c^2)/(a^4-
2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2-1/2*(a^4*d^2+10*a^3*b*c*d-11*a^2*b^
2*c^2-10*a^2*b^2*d^2+8*a*b^3*c*d+2*b^4*c^2)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f
*x+1/2*e)-1/2*(4*a^3*c*d-4*a^2*b*c^2-3*a^2*b*d^2+2*a*b^2*c*d+b^3*c^2)/(a^4-
2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)^2+(2*a^2*
c^2+a^2*d^2-6*a*b*c*d+b^2*c^2+2*b^2*d^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(192) = 384$.
time = 0.41, size = 1048, normalized size = 5.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b + a^2*b^3 - 2*b^5)*c*d - (a^5
- 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(f*x + e) - ((2*a^4 + 3*a^2*b^2
 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 + 3*a^2*b^2 + 2*b^4)*d^2 + (6*a*
b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 + 2*b^4)*d^2)*cos(f*x + e)^2 - 2
*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^3*b + 2*a*b^3)*d^2)*sin(f*x +
e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
 b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((4*a^4*b
 - 5*a^2*b^3 + b^5)*c^2 - 2*(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*
b^3)*d^2)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x
+ e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 -
2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b
+ a^2*b^3 - 2*b^5)*c*d - (a^5 - 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(
f*x + e) - ((2*a^4 + 3*a^2*b^2 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 +
3*a^2*b^2 + 2*b^4)*d^2 + (6*a*b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 +
2*b^4)*d^2)*cos(f*x + e)^2 - 2*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^
3*b + 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) +
 b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c^2 - 2*
(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)
*f)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(192) = 384.
time = 0.50, size = 609, normalized size = 3.11

$$\frac{\frac{(2a^2c^2+b^2c^2-6abcd+a^2d^2+2b^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{5a^3b^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ab^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6a^4bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^5d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2a^3b^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^4bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^2b^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2b^5c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4a^5cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-10a^3b^2cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4ab^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3a^4bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6a^2b^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+11a^3b^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2ab^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-10a^4bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-8a^2b^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-a^5d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+10a^3b^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4a^4bc^2-a^2b^3c^2-4a^5cd-2a^3b^2cd+3a^4bd^2}{(a^6-2a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2
)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 2*a*b^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*b*c*d*tan(1/2*f*x +
```

```
1/2*e)^3 + a^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*b^2*d^2*tan(1/2*f*x + 1/
2*e)^3 + 4*a^4*b*c^2*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*c^2*tan(1/2*f*x + 1
/2*e)^2 - 2*b^5*c^2*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*c*d*tan(1/2*f*x + 1/2*e)
^2 - 10*a^3*b^2*c*d*tan(1/2*f*x + 1/2*e)^2 - 4*a*b^4*c*d*tan(1/2*f*x + 1/2*
e)^2 + 3*a^4*b*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*a^2*b^3*d^2*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*c^2*tan(1/2*f*x + 1/2
*e) - 10*a^4*b*c*d*tan(1/2*f*x + 1/2*e) - 8*a^2*b^3*c*d*tan(1/2*f*x + 1/2*e
) - a^5*d^2*tan(1/2*f*x + 1/2*e) + 10*a^3*b^2*d^2*tan(1/2*f*x + 1/2*e) + 4*
a^4*b*c^2 - a^2*b^3*c^2 - 4*a^5*c*d - 2*a^3*b^2*c*d + 3*a^4*b*d^2)/((a^6 -
2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)^2))/f
```

**Mupad [B]**
time = 10.44, size = 641, normalized size = 3.27

$$\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)\,\left(2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2}\right)\,\left(2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2\right)}{f\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}-\frac{\frac{4\,a^3\,c\,d-4\,a^2\,b\,c^2-3\,a^2\,b\,d^2+2\,a\,b^2\,c\,d+b^3\,c^2}{a^4-2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^4\,d^2+10\,a^3\,b\,c\,d-11\,a^2\,b^2\,c^2-10\,a^2\,b^2\,d^2+8\,a\,b^3\,c\,d+2\,b^4\,c^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(a^4\,d^2-6\,a^3\,b\,c\,d+5\,a^2\,b^2\,c^2+2\,a^2\,b^2\,d^2-2\,b^4\,c^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(a^2+2\,b^2\right)\,\left(4\,a^3\,c\,d-4\,a^2\,b\,c^2-3\,a^2\,b\,d^2+2\,a\,b^2\,c\,d+b^3\,c^2\right)}{a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+a^2+4\,a\,b\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*a^4*b + 2*b^5 - 4*a^2*b^3)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b
^2*d^2 - 6*a*b*c*d))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)
) + (a*tan(e/2 + (f*x)/2)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*
b*c*d))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2))/(2*a^2*c^2
+ a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*b*c*d))*(2*a^2*c^2 + a^2*d^2 + b^2*c^
2 + 2*b^2*d^2 - 6*a*b*c*d))/(f*(a + b)^(5/2)*(a - b)^(5/2)) - ((b^3*c^2 - 4
*a^2*b*c^2 - 3*a^2*b*d^2 + 4*a^3*c*d + 2*a*b^2*c*d)/(a^4 + b^4 - 2*a^2*b^2)
 + (tan(e/2 + (f*x)/2)*(a^4*d^2 + 2*b^4*c^2 - 11*a^2*b^2*c^2 - 10*a^2*b^2*d
^2 + 8*a*b^3*c*d + 10*a^3*b*c*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 +
(f*x)/2)^3*(a^4*d^2 - 2*b^4*c^2 + 5*a^2*b^2*c^2 + 2*a^2*b^2*d^2 - 6*a^3*b*c
*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(a^2 + 2*b^2)*(b^3
*c^2 - 4*a^2*b*c^2 - 3*a^2*b*d^2 + 4*a^3*c*d + 2*a*b^2*c*d))/(a^2*(a^4 + b^
4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (
f*x)/2)^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2)))
```

# 3.718 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=162

$$\frac{(2a^2c+b^2c-3abd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f}+\frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2}+\frac{(3abc-a^2d-2b^2d)\cos(e}{2(a^2-b^2)^2f(a+b\sin(e+}$$

```
[Out] (2*a^2*c-3*a*b*d+b^2*c)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a
^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/
2*(-a^2*d+3*a*b*c-2*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2833, 12, 2739, 632, 210}

$$\frac{(2a^2c-3abd+b^2c)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}}+\frac{(a^2(-d)+3abc-2b^2d)\cos(e+fx)}{2f(a^2-b^2)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)}{2f(a^2-b^2)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2
]])/((a^2 - b^2)^(5/2)*f) + ((b*c - a*d)*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a
+ b*Sin[e + f*x])^2) + ((3*a*b*c - a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*(a^2 -
 b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)+(bc-ad)\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(3abc-a^2d-2b^2d)\cos(e+fx)}{2(a^2-b^2)^2f(a+b\sin(e+fx))} + \frac{\int \frac{2a^2c+b^2\ldots}{a+b\sin\ldots}}{2(a^2\ldots}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(3abc-a^2d-2b^2d)\cos(e+fx)}{2(a^2-b^2)^2f(a+b\sin(e+fx))} + \frac{(2a^2c+\ldots}{}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(3abc-a^2d-2b^2d)\cos(e+fx)}{2(a^2-b^2)^2f(a+b\sin(e+fx))} + \frac{(2a^2c+\ldots}{}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(3abc-a^2d-2b^2d)\cos(e+fx)}{2(a^2-b^2)^2f(a+b\sin(e+fx))} - \frac{(2(2a^2c\ldots}{}$$

$$= \frac{(2a^2c+b^2c-3abd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f} + \frac{(bc-ad)\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2\ldots}$$

**Mathematica [A]**
time = 0.68, size = 157, normalized size = 0.97

$$\frac{\frac{2(2a^2c+b^2c-3abd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))^2} - \frac{(-3abc+a^2d+2b^2d)\cos(e+fx)}{(a-b)^2(a+b)^2(a+b\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a +
 b*Sin[e + f*x])^2) - ((-3*a*b*c + a^2*d + 2*b^2*d)*Cos[e + f*x])/((a - b)^
2*(a + b)^2*(a + b*Sin[e + f*x])))/(2*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(153) = 306$.
time = 0.37, size = 349, normalized size = 2.15

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{-\frac{b\left(3a^3d-5a^2bc+2b^3c\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\left(a^4-2a^2b^2+b^4\right)}-\frac{\left(2a^5d-4a^4bc+5a^3b^2d-7a^2b^3c+2a\,b^4d+2b^5c\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a^2}-\frac{b\left(5a^3d-11a^2bc+4a\,b^2}{a\left(a^4-2a\right.}}{\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a\right)^2}}{f}$ |
| default | $\frac{\frac{-\frac{b\left(3a^3d-5a^2bc+2b^3c\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\left(a^4-2a^2b^2+b^4\right)}-\frac{\left(2a^5d-4a^4bc+5a^3b^2d-7a^2b^3c+2a\,b^4d+2b^5c\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a^2}-\frac{b\left(5a^3d-11a^2bc+4a\,b^2}{a\left(a^4-2a\right.}}{\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a\right)^2}}{f}$ |
| risch | $\frac{i\left(2ib^2a^2c\,e^{3i(fx+e)}-3ib^3ad\,e^{3i(fx+e)}+ib^4c\,e^{3i(fx+e)}+4ia^3bd\,e^{i(fx+e)}-10ia^2b^2c\,e^{i(fx+e)}+5ia\,b^3d\,e^{i(fx+e)}+ib^4c\,e^{i(fx-\right.}}{\left(-ib\,e^{2i(fx+e)}+ib+2a\,e^{i(\right.}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(-1/2*b*(3*a^3*d-5*a^2*b*c+2*b^3*c)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*
x+1/2*e)^3-1/2*(2*a^5*d-4*a^4*b*c+5*a^3*b^2*d-7*a^2*b^3*c+2*a*b^4*d+2*b^5*c
)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2-1/2*b*(5*a^3*d-11*a^2*b*c+4*
a*b^2*d+2*b^3*c)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)-1/2*(2*a^3*d-4*a^
2*b*c+a*b^2*d+b^3*c)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1
/2*f*x+1/2*e)+a)^2+(2*a^2*c-3*a*b*d+b^2*c)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(158) = 316$.
time = 0.40, size = 822, normalized size = 5.07

$$\left[-\frac{2\,(3\,(a^3b^2-ab^4)c-(a^4b+a^2b^3-2\,b^5)d)\cos(fx+e)\sin(fx+e)+((3\,ab^3d-(2\,a^2b^2+b^4)c)\cos(fx+e)^2+(2\,a^4+3\,a^2b^2+b^4)c-3\,(a^3b+ab^3)d-2\,(3\,a^2b^2d-(2\,a^3b+ab^3)c)\sin(fx+e))\sqrt{-a^2+b^2}\log\left(-\frac{(2\,a^2-b^2)\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2-2\,(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2}\right)+2\,((4\,a^4b-5\,a^2b^3+b^5)c-(2\,a^5-a^3b^2-ab^4)d)\cos(fx+e)}{4\,((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)f\cos(fx+e)^2-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)f\sin(fx+e)-(a^8-2\,a^6b^2+2\,a^2b^6-b^8)f)},-\frac{(3\,(a^3b^2-ab^4)c-(a^4b+a^2b^3-2\,b^5)d)\cos(fx+e)\sin(fx+e)-((3\,ab^3d-(2\,a^2b^2+b^4)c)\cos(fx+e)^2+(2\,a^4+3\,a^2b^2+b^4)c-3\,(a^3b+ab^3)d-2\,(3\,a^2b^2d-(2\,a^3b+ab^3)c)\sin(fx+e))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)+((4\,a^4b-5\,a^2b^3+b^5)c-(2\,a^5-a^3b^2-ab^4)d)\cos(fx+e)}{2\,((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)f\cos(fx+e)^2-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)f\sin(fx+e)-(a^8-2\,a^6b^2+2\,a^2b^6-b^8)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5)*d)*cos(f*x + e)
*sin(f*x + e) + ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x + e)^2 + (2*a^4
+ 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*d - (2*a^3*b + a*
b^3)*c)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*
x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 -
b^2)) + 2*((4*a^4*b - 5*a^2*b^3 + b^5)*c - (2*a^5 - a^3*b^2 - a*b^4)*d)*cos
(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a
^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2
*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5
)*d)*cos(f*x + e)*sin(f*x + e) - ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x
 + e)^2 + (2*a^4 + 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*
d - (2*a^3*b + a*b^3)*c)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c -
 (2*a^5 - a^3*b^2 - a*b^4)*d)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*si
n(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(158) = 316$.
time = 0.59, size = 429, normalized size = 2.65

$$\frac{\frac{(2\,a^2c+b^2c-3\,abd)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{5\,a^3b^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,ab^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+4\,a^4bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+7\,a^2b^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,b^5c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,a^5d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5\,a^3b^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,ab^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+11\,a^3b^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ab^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-5\,a^4bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,a^2b^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,a^4bc-a^2b^3c-2\,a^5d-a^3b^2d}{(a^6-2\,a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)
*sqrt(a^2 - b^2)) + (5*a^3*b^2*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*c*tan(1/2
*f*x + 1/2*e)^3 - 3*a^4*b*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b*c*tan(1/2*f*x
+ 1/2*e)^2 + 7*a^2*b^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*c*tan(1/2*f*x + 1/2
*e)^2 - 2*a^5*d*tan(1/2*f*x + 1/2*e)^2 - 5*a^3*b^2*d*tan(1/2*f*x + 1/2*e)^2
 - 2*a*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*c*tan(1/2*f*x + 1/2*e) - 2
*a*b^4*c*tan(1/2*f*x + 1/2*e) - 5*a^4*b*d*tan(1/2*f*x + 1/2*e) - 4*a^2*b^3*
d*tan(1/2*f*x + 1/2*e) + 4*a^4*b*c - a^2*b^3*c - 2*a^5*d - a^3*b^2*d)/((a^6
 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
) + a)^2))/f
```

**Mupad [B]**
time = 9.79, size = 477, normalized size = 2.94

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(2a^4b-4a^2b^3+2b^5\right)\left(2ca^2-3dab+cb^2\right)}{2(a+b)^{5/2}(a-b)^{5/2}\left(a^4-2a^2b^2+b^4\right)}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2ca^2-3dab+cb^2\right)}{(a+b)^{5/2}(a-b)^{5/2}}\right)\left(a^4-2a^2b^2+b^4\right)}{2ca^2-3dab+cb^2}\right)\left(2ca^2-3dab+cb^2\right)}{f\,(a+b)^{5/2}(a-b)^{5/2}}-\frac{\frac{2da^3-4ca^2b+dab^2+cb^3}{a^4-2a^2b^2+b^4}+\frac{b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3da^3-5ca^2b+2cb^3\right)}{a\left(a^4-2a^2b^2+b^4\right)}+\frac{b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(5da^3-11ca^2b+4dab^2+2cb^3\right)}{a\left(a^4-2a^2b^2+b^4\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(a^2+2b^2\right)\left(2da^3-4ca^2b+dab^2+cb^3\right)}{a^2\left(a^4-2a^2b^2+b^4\right)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(2a^2+4b^2\right)+a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+a^2+4ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*a^4*b + 2*b^5 - 4*a^2*b^3)*(2*a^2*c + b^2*c - 3*a*b*d))/(2*(a +
 b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(e/2 + (f*x)/2)*(2
*a^2*c + b^2*c - 3*a*b*d))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^
2*b^2))/(2*a^2*c + b^2*c - 3*a*b*d))*(2*a^2*c + b^2*c - 3*a*b*d))/(f*(a + b
)^(5/2)*(a - b)^(5/2)) - ((2*a^3*d + b^3*c - 4*a^2*b*c + a*b^2*d)/(a^4 + b^
4 - 2*a^2*b^2) + (b*tan(e/2 + (f*x)/2)^3*(3*a^3*d + 2*b^3*c - 5*a^2*b*c))/(
a*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)*(5*a^3*d + 2*b^3*c - 11*
a^2*b*c + 4*a*b^2*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(
a^2 + 2*b^2)*(2*a^3*d + b^3*c - 4*a^2*b*c + a*b^2*d))/(a^2*(a^4 + b^4 - 2*a
^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)
^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2)))
```

# 3.719 $\int \frac{1}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=131

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f}+\frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2}+\frac{3ab\cos(e+fx)}{2(a^2-b^2)^2f(a+b\sin(e+fx))}$$

```
[Out] (2*a^2+b^2)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2
)/f+1/2*b*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+3/2*a*b*cos(f*x+e)/(a^2
-b^2)^2/f/(a+b*sin(f*x+e))
```

**Rubi** [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$\frac{(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}}+\frac{3ab\cos(e+fx)}{2f(a^2-b^2)^2(a+b\sin(e+fx))}+\frac{b\cos(e+fx)}{2f(a^2-b^2)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(5/2)*f) + (b*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + (
3*a*b*Cos[e + f*x])/(2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2a+b\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2(a^2-b^2)}$$
$$= \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2(a^2-b^2)^2 f(a+b\sin(e+fx))} + \frac{\int \frac{2a}{a+b\sin}}{2(a^2}$$
$$= \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2(a^2-b^2)^2 f(a+b\sin(e+fx))} + \frac{(2a^2+}{}$$
$$= \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2(a^2-b^2)^2 f(a+b\sin(e+fx))} + \frac{(2a^2+}{}$$
$$= \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2(a^2-b^2)^2 f(a+b\sin(e+fx))} - \frac{(2(2a^2}{}$$
$$= \frac{(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}f} + \frac{b\cos(e+fx)}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{}{2(a^2}$$

**Mathematica** [A]

time = 0.43, size = 114, normalized size = 0.87

$$\frac{\frac{2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b\cos(e+fx)(4a^2-b^2+3ab\sin(e+fx))}{(a-b)^2(a+b)^2(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(5/2) + (b*Cos[e + f*x]*(4*a^2 - b^2 + 3*a*b*Sin[e + f*x]))/((a - b)^2
*(a + b)^2*(a + b*Sin[e + f*x])^2))/(2*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(122) = 244$.
time = 0.29, size = 282, normalized size = 2.15

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\frac{b^2(5a^2-2b^2)(\tan^3(\frac{fx}{2}+\frac{e}{2}))}{(a^4-2a^2b^2+b^4)a}+\frac{b(4a^4+7a^2b^2-2b^4)(\tan^2(\frac{fx}{2}+\frac{e}{2}))}{(a^4-2a^2b^2+b^4)a^2}+\frac{b^2(11a^2-2b^2)\tan(\frac{fx}{2}+\frac{e}{2})}{a(a^4-2a^2b^2+b^4)}+\frac{2b(4a^2-b^2)}{2a^4-4a^2b^2+2b^4}}{\left(a(\tan^2(\frac{fx}{2}+\frac{e}{2}))+2b\tan(\frac{fx}{2}+\frac{e}{2})+a\right)^2}+\frac{(2a^2+b^2)a}{(a^4-2a}}{f}$ |
| default | $\frac{\frac{\frac{b^2(5a^2-2b^2)(\tan^3(\frac{fx}{2}+\frac{e}{2}))}{(a^4-2a^2b^2+b^4)a}+\frac{b(4a^4+7a^2b^2-2b^4)(\tan^2(\frac{fx}{2}+\frac{e}{2}))}{(a^4-2a^2b^2+b^4)a^2}+\frac{b^2(11a^2-2b^2)\tan(\frac{fx}{2}+\frac{e}{2})}{a(a^4-2a^2b^2+b^4)}+\frac{2b(4a^2-b^2)}{2a^4-4a^2b^2+2b^4}}{\left(a(\tan^2(\frac{fx}{2}+\frac{e}{2}))+2b\tan(\frac{fx}{2}+\frac{e}{2})+a\right)^2}+\frac{(2a^2+b^2)a}{(a^4-2a}}{f}$ |
| risch | $-\frac{i(-2iba^2e^{3i(fx+e)}-ie^{3i(fx+e)}b^3+10ia^2be^{i(fx+e)}-ib^3e^{i(fx+e)}+6a^3e^{2i(fx+e)}+3b^2ae^{2i(fx+e)}-3ab^2)}{(-ibe^{2i(fx+e)}+ib+2ae^{i(fx+e)})^2(a^2-b^2)^2f}-\frac{\ln(e^{i(fx+e}}{\sqrt{-}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1/2*b^2*(5*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3+1/
2*b*(4*a^4+7*a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2+1/
2*b^2*(11*a^2-2*b^2)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)+1/2*b*(4*a^2-
b^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)
^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(127) = 254$.
time = 0.37, size = 641, normalized size = 4.89

$$\left[-\frac{6\,(a^3b^2-ab^4)\cos(fx+e)\sin(fx+e)-(2\,a^4+3\,a^2b^2+b^4-(2\,a^2b^2+b^4)\cos(fx+e)^2+2\,(2\,a^3b+ab^3)\sin(fx+e))\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2+2\,(a\cos(fx+e)\sin(fx+e)+b\cos(fx+e))\sqrt{-a^2+b^2}}{b^2\cos(fx+e)^2-2\,ab\sin(fx+e)-a^2-b^2}\right)+2\,(4\,a^4b-5\,a^2b^3+b^5)\cos(fx+e)}{4\,((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)f\cos(fx+e)^2-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)f\sin(fx+e)-(a^8-2\,a^6b^2+2\,a^2b^6-b^8)f)}, -\frac{3\,(a^3b^2-ab^4)\cos(fx+e)\sin(fx+e)-(2\,a^4+3\,a^2b^2+b^4-(2\,a^2b^2+b^4)\cos(fx+e)^2+2\,(2\,a^3b+ab^3)\sin(fx+e))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(fx+e)+b}{\sqrt{a^2-b^2}\cos(fx+e)}\right)+(4\,a^4b-5\,a^2b^3+b^5)\cos(fx+e)}{2\,((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)f\cos(fx+e)^2-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)f\sin(fx+e)-(a^8-2\,a^6b^2+2\,a^2b^6-b^8)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a^3*b^2 - a*b^4)*cos(f*x + e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 +
 b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))
*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) -
a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^
2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(4*a^4*b - 5
*a^2*b^3 + b^5)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*co
s(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (
a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*(3*(a^3*b^2 - a*b^4)*cos(f*x +
e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)
^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(
f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*
a^2*b^6 - b^8)*f)]
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(127) = 254$.
time = 0.46, size = 284, normalized size = 2.17

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2\,a^2+b^2)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{5\,a^3b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,ab^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+7\,a^2b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,b^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+11\,a^3b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ab^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,a^4b-a^2b^3}{(a^6-2\,a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
+ b)/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (5*a^3*b^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*tan(1/2*f*x + 1/2*e)^3 +
4*a^4*b*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*t
an(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*tan(1/2*f
*x + 1/2*e) + 4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

## Mupad [B]

time = 10.15, size = 395, normalized size = 3.02

$$\frac{\frac{4\,a^2\,b-b^3}{a^4-2\,a^2\,b^2+b^4}+\frac{b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(11\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(4\,a^2\,b-b^3\right)\,\left(a^2+2\,b^2\right)}{a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(5\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+a^2+4\,a\,b\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}+\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,a^2+b^2\right)\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a^2+b^2\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,a^2+b^2}\right)\,\left(2\,a^2+b^2\right)}{f\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^3,x)
```

```
[Out] ((4*a^2*b - b^3)/(a^4 + b^4 - 2*a^2*b^2) + (b*tan(e/2 + (f*x)/2)*(11*a^2*b
- 2*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(4*a^2*b - b^
3)*(a^2 + 2*b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)^3*(
5*a^2*b - 2*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*
a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3
+ 4*a*b*tan(e/2 + (f*x)/2))) + (atan(((((2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4*
a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(
e/2 + (f*x)/2)*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2
*a^2*b^2))/(2*a^2 + b^2))*(2*a^2 + b^2))/(f*(a + b)^(5/2)*(a - b)^(5/2))
```

# 3.720 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

**Optimal**. Leaf size=285

$$-\frac{b(6a^3bcd-6a^4d^2-a^2b^2(2c^2-5d^2)-b^4(c^2+2d^2))\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}(bc-ad)^3f}-\frac{2d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^3\sqrt{c^2-d^2}\,f}$$

```
[Out] -b*(6*a^3*b*c*d-6*a^4*d^2-a^2*b^2*(2*c^2-5*d^2)-b^4*(c^2+2*d^2))*arctan((b+
a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-a*d+b*c)^3/f+1/2*b
^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2+1/2*b^2*(-5*a^2*d+3
*a*b*c+2*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))-2*d^
3*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/f/(c^2-d^2)
^(1/2)
```

**Rubi** [A]
time = 0.71, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$\frac{b^2(-5a^2d+3abc+2b^2d)\cos(e+fx)}{2f(a^2-b^2)^2(bc-ad)^2(a+b\sin(e+fx))}+\frac{b^2\cos(e+fx)}{2f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))^2}-\frac{b(-6a^4d^2+6a^3bcd-a^2b^2(2c^2-5d^2)-b^4(c^2+2d^2))\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}(bc-ad)^3}-\frac{2d^3\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((b*(6*a^3*b*c*d - 6*a^4*d^2 - a^2*b^2*(2*c^2 - 5*d^2) - b^4*(c^2 + 2*d^2)
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(b*c
 - a*d)^3*f)) - (2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((
b*c - a*d)^3*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*Cos[e +
 f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2\left(abc-a^2d+b^2 \cdots\right.}{(a+\cdots}}{\cdots}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2(3abc-\cdots}{2\left(a^2-b^2\right)^2(b\cdots}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2(3abc-\cdots}{2\left(a^2-b^2\right)^2(b\cdots}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2(3abc-\cdots}{2\left(a^2-b^2\right)^2(b\cdots}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2(3abc-\cdots}{2\left(a^2-b^2\right)^2(b\cdots}$$

$$= -\frac{b(6a^3bcd-6a^4d^2-a^2b^2(2c^2-5d^2)-b^4(c^2+2d^2))\tan^{-1}\cdots}{\left(a^2-b^2\right)^{5/2}(bc-ad)^3f}$$

**Mathematica** [A]
time = 2.38, size = 275, normalized size = 0.96

$$\frac{-\frac{2b\left(-6a^3bcd+6a^4d^2+a^2b^2\left(2c^2-5d^2\right)+b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(-bc+ad)^3}+\frac{4d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(-bc+ad)^3\sqrt{c^2-d^2}}-\frac{b^2\cos(e+fx)}{(a-b)(a+b)(-bc+ad)(a+b\sin(e+fx))^2}+\frac{b^2\left(3abc-5a^2d+2b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(bc-ad)^2(a+b\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((-2*b*(-6*a^3*b*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2 - 5*d^2) + b^4*(c^2 + 2*d
^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(
-(b*c) + a*d)^3) + (4*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])
/((-(b*c) + a*d)^3*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x])/((a - b)*(a + b)*(
-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*
Cos[e + f*x])/((a - b)^2*(a + b)^2*(b*c - a*d)^2*(a + b*Sin[e + f*x])))/(2*
f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(271) = 542.
time = 3.66, size = 625, normalized size = 2.19

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $-\frac{2b\left(\frac{b^2\left(7a^4d^2-12a^3bcd+5a^2b^2c^2-4a^2b^2d^2+6ab^3cd-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}+\frac{b\left(6a^6d^2-10a^5bcd+4a^4b^2c^2+9a^4b^2d^2-16a^3b^3cd+\ldots\right)}{2\left(a^4-2a\ldots\right.}\ldots\right.}{\ldots}$ |
| default | $-\frac{2b\left(\frac{b^2\left(7a^4d^2-12a^3bcd+5a^2b^2c^2-4a^2b^2d^2+6ab^3cd-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}+\frac{b\left(6a^6d^2-10a^5bcd+4a^4b^2c^2+9a^4b^2d^2-16a^3b^3cd+\ldots\right)}{2\left(a^4-2a\ldots\right.}\ldots\right.}{\ldots}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*b/(a*d-b*c)^3*((1/2*b^2*(7*a^4*d^2-12*a^3*b*c*d+5*a^2*b^2*c^2-4*a^2
*b^2*d^2+6*a*b^3*c*d-2*b^4*c^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3+
1/2*b*(6*a^6*d^2-10*a^5*b*c*d+4*a^4*b^2*c^2+9*a^4*b^2*d^2-16*a^3*b^3*c*d+7*
a^2*b^4*c^2-6*a^2*b^4*d^2+8*a*b^5*c*d-2*b^6*c^2)/(a^4-2*a^2*b^2+b^4)/a^2*ta
n(1/2*f*x+1/2*e)^2+1/2*b^2*(17*a^4*d^2-28*a^3*b*c*d+11*a^2*b^2*c^2-8*a^2*b^
2*d^2+10*a*b^3*c*d-2*b^4*c^2)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)+1/2*
b*(6*a^4*d^2-10*a^3*b*c*d+4*a^2*b^2*c^2-3*a^2*b^2*d^2+4*a*b^3*c*d-b^4*c^2)/
(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)^2+1/
2*(6*a^4*d^2-6*a^3*b*c*d+2*a^2*b^2*c^2-5*a^2*b^2*d^2+b^4*c^2+2*b^4*d^2)/(a^
4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a
^2-b^2)^(1/2)))+2*d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(277) = 554.
time = 0.53, size = 788, normalized size = 2.76

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^3}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{c^2-d^2}}-\frac{\left(2a^2b^3c^2+b^5c^2-6a^3b^2cd+6a^4bd^2-5a^2b^3d^2+2b^5d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3c^3-2a^2b^5c^3+b^7c^3-3a^5b^2c^2d+6a^3b^4c^2d-3ab^6c^2d+3a^6bcd^2-6a^4b^3cd^2+3a^2b^5cd^2-a^7d^3+2a^5b^2d^3-a^3b^4d^3\right)\sqrt{a^2-b^2}}-\frac{5a^3b^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ab^6c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-7a^4b^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^2b^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^4b^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^2b^5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2b^7c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6a^5b^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-9a^3b^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6ab^6d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+11a^3b^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2ab^6c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-17a^4b^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8a^2b^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4a^4b^3c-a^2b^5c-6a^5b^2d+3a^3b^4d}{\left(a^6b^2c^2-2a^4b^4c^2+a^2b^6c^2-2a^7bcd+4a^5b^3cd-2a^3b^5cd+a^8d^2-2a^6b^2d^2+a^4b^4d^2\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(c^2 - d^2)) - (2*a^2*b^3*c^2 + b^5*c^2 - 6*a^3*b^2*c*d + 6*a^4
*b*d^2 - 5*a^2*b^3*d^2 + 2*b^5*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^3*c^3 - 2
*a^2*b^5*c^3 + b^7*c^3 - 3*a^5*b^2*c^2*d + 6*a^3*b^4*c^2*d - 3*a*b^6*c^2*d
+ 3*a^6*b*c*d^2 - 6*a^4*b^3*c*d^2 + 3*a^2*b^5*c*d^2 - a^7*d^3 + 2*a^5*b^2*d
^3 - a^3*b^4*d^3)*sqrt(a^2 - b^2)) - (5*a^3*b^4*c*tan(1/2*f*x + 1/2*e)^3 -
2*a*b^6*c*tan(1/2*f*x + 1/2*e)^3 - 7*a^4*b^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*a
^2*b^5*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^
2*b^5*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*b^2
*d*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 6*a*b^6*d*
tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^4*c*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c*tan(
1/2*f*x + 1/2*e) - 17*a^4*b^3*d*tan(1/2*f*x + 1/2*e) + 8*a^2*b^5*d*tan(1/2*
f*x + 1/2*e) + 4*a^4*b^3*c - a^2*b^5*c - 6*a^5*b^2*d + 3*a^3*b^4*d)/((a^6*b
^2*c^2 - 2*a^4*b^4*c^2 + a^2*b^6*c^2 - 2*a^7*b*c*d + 4*a^5*b^3*c*d - 2*a^3*
b^5*c*d + a^8*d^2 - 2*a^6*b^2*d^2 + a^4*b^4*d^2)*(a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

**Mupad** [**B**]
time = 30.17, size = 2500, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)

[Out] (d^3*atan(((d^3*(d^2 - c^2)^(1/2)*((8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (d^3*(d^2 - c^2)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d -

12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^1...

## 3.721 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=454

$$-\frac{b^2(8a^3bcd-2ab^3cd-12a^4d^2-a^2b^2(2c^2-15d^2)-b^4(c^2+6d^2))\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(e+fx))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}(bc-ad)^4 f}-\frac{2d^3(4bc^2-acd\cdots}{(bc\cdots}$$

```
[Out] -b^2*(8*a^3*b*c*d-2*a*b^3*c*d-12*a^4*d^2-a^2*b^2*(2*c^2-15*d^2)-b^4*(c^2+6*
d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-a*
d+b*c)^4/f-2*d^3*(-a*c*d+4*b*c^2-3*b*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/(-a*d+b*c)^4/(c^2-d^2)^(3/2)/f-1/2*d*(2*a^4*d^3+a^2*b^2*d*(
7*c^2-11*d^2)-2*b^4*d*(2*c^2-3*d^2)-3*a*b^3*c*(c^2-d^2))*cos(f*x+e)/(a^2-b^
2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e))+1/2*b^2*cos(f*x+e)/(a^2-b^2)
/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))+3/2*b^2*(-2*a^2*d+a*b*c+b
^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e
))
```

**Rubi** [A]

time = 1.60, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$\frac{3b^2(-2a^2d+abc+b^2d)\cos(e+fx)}{2f(a^2-b^2)^2(bc-ad)^2(a+b\sin(e+fx))(c+d\sin(e+fx))}+\frac{b^2\cos(e+fx)}{2f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))^2(c+d\sin(e+fx))}-\frac{d(2a^4d^3+a^2b^2d(7c^2-11d^2)-3ab^3c(c^2-d^2)-2b^4d(2c^2-3d^2))\cos(e+fx)}{2f(a^2-b^2)^2(c^2-d^2)(bc-ad)^3(c+d\sin(e+fx))}-\frac{b^2(-12a^4d^2+8a^3bcd-a^2b^2(2c^2-15d^2)-2ab^3cd-b^4(c^2+6d^2))\text{ArcTan}\left(\frac{a\tan(\frac{1}{2}(e+fx))+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}(bc-ad)^4}-\frac{2d^3(-acd+4bc^2-3bd^2)\text{ArcTan}\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{f(c^2-d^2)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((b^2*(8*a^3*b*c*d - 2*a*b^3*c*d - 12*a^4*d^2 - a^2*b^2*(2*c^2 - 15*d^2) -
 b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2
 - b^2)^(5/2)*(b*c - a*d)^4*f)) - (2*d^3*(4*b*c^2 - a*c*d - 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)*f) - (d*(2*a^4*d^3 + a^2*b^2*d*(7*c^2 - 11*d^2) - 2*b^4*d*(2*c^2 - 3*d^2)
 - 3*a*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2
 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (3*b^2*(a*b*c - 2*a^2
*d + b^2*d)*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f
*x])*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
```

```
EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx)}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx)}$$

$$= -\frac{d(2a^4d^3+a^2b^2d(7c^2-11d^2)-2b^4d(2c^2-3d^2)-3ab^3c(c}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e}$$

$$= -\frac{d(2a^4d^3+a^2b^2d(7c^2-11d^2)-2b^4d(2c^2-3d^2)-3ab^3c(c}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e}$$

$$= -\frac{d(2a^4d^3+a^2b^2d(7c^2-11d^2)-2b^4d(2c^2-3d^2)-3ab^3c(c}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e}$$

$$= -\frac{d(2a^4d^3+a^2b^2d(7c^2-11d^2)-2b^4d(2c^2-3d^2)-3ab^3c(c}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e}$$

$$= -\frac{b^2(8a^3bcd-2ab^3cd-12a^4d^2-a^2b^2(2c^2-15d^2)-b^4(c^2+}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4f}$$

## **Mathematica** [A]

time = 5.90, size = 346, normalized size = 0.76

$$\frac{\frac{2b^2\left(-8a^3bcd+2ab^3cd+12a^4d^2+a^2b^2\left(2c^2-15d^2\right)+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4}+\frac{4d^3\left(-4bc^2+acd+3bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^4\left(c^2-d^2\right)^{3/2}}+\frac{b^3\cos(e+fx)}{(a-b)(a+b)(bc-ad)^2(a+b\sin(e+fx))^2}+\frac{b^3\left(-3abc+7a^2d-4b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(-bc+ad)^3(a+b\sin(e+fx))}-\frac{2d^4\cos(e+fx)}{(c-d)(c+d)(bc-ad)^3(c+d\sin(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(-8*a^3*b*c*d + 2*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2)
 + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^4) + (4*d^3*(-4*b*c^2 + a*c*d + 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)) + (b^3*Cos[e + f*x])/((a - b)*(a + b)*(b*c - a*d)^2*(a + b*Sin[e + f*x])
^2) + (b^3*(-3*a*b*c + 7*a^2*d - 4*b^2*d)*Cos[e + f*x])/((a - b)^2*(a + b)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((c - d)*(c
 + d)*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

## Maple [A]

time = 9.08, size = 736, normalized size = 1.62

| method | result |
|---|---|
| derivativedivides | $\frac{2b^2\left(\frac{b^2\left(9a^4d^2-14a^3bcd+5a^2b^2c^2-6a^2b^2d^2+8a\,b^3cd-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}+\frac{b\left(8a^6d^2-12a^5bcd+4a^4b^2c^2+11a^4b^2d^2-18a^3b^3c \ldots\right.}{2\left(a^4- \ldots\right.} \ldots\right.}{\ldots}$ |
| default | $\frac{2b^2\left(\frac{b^2\left(9a^4d^2-14a^3bcd+5a^2b^2c^2-6a^2b^2d^2+8a\,b^3cd-2b^4c^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(a^4-2a^2b^2+b^4\right)a}+\frac{b\left(8a^6d^2-12a^5bcd+4a^4b^2c^2+11a^4b^2d^2-18a^3b^3c \ldots\right.}{2\left(a^4- \ldots\right.} \ldots\right.}{\ldots}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/(a*d-b*c)^4*((1/2*b^2*(9*a^4*d^2-14*a^3*b*c*d+5*a^2*b^2*c^2-6*a^
2*b^2*d^2+8*a*b^3*c*d-2*b^4*c^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3
+1/2*b*(8*a^6*d^2-12*a^5*b*c*d+4*a^4*b^2*c^2+11*a^4*b^2*d^2-18*a^3*b^3*c*d+
7*a^2*b^4*c^2-10*a^2*b^4*d^2+12*a*b^5*c*d-2*b^6*c^2)/(a^4-2*a^2*b^2+b^4)/a^
2*tan(1/2*f*x+1/2*e)^2+1/2*b^2*(23*a^4*d^2-34*a^3*b*c*d+11*a^2*b^2*c^2-14*a
^2*b^2*d^2+16*a*b^3*c*d-2*b^4*c^2)/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)
+1/2*b*(8*a^4*d^2-12*a^3*b*c*d+4*a^2*b^2*c^2-5*a^2*b^2*d^2+6*a*b^3*c*d-b^4*
c^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)
^2+1/2*(12*a^4*d^2-8*a^3*b*c*d+2*a^2*b^2*c^2-15*a^2*b^2*d^2+2*a*b^3*c*d+b^4
*c^2+6*b^4*d^2)/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))+2*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b
*c)^2*((d^2*(a*d-b*c)/c/(c^2-d^2)*tan(1/2*f*x+1/2*e)+d*(a*d-b*c)/(c^2-d^2))
/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(a*c*d-4*b*c^2+3*b*d^2)/
(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

## Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. 2(448) = 896.
time = 0.59, size = 1111, normalized size = 2.45

$$\frac{\frac{(2a^2b^4c^2+b^6c^2-8a^3b^3cd+2ab^5cd+12a^4b^2d^2-15a^2b^4d^2+6b^6d^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^4c^4-2a^2b^6c^4+b^8c^4-4a^5b^3c^3d+8a^3b^5c^3d-4ab^7c^3d+6a^6b^2c^2d^2-12a^4b^4c^2d^2+6a^2b^6c^2d^2-4a^7bcd^3+8a^5b^3cd^3-4a^3b^5cd^3+a^8d^4-2a^6b^2d^4+a^4b^4d^4)\sqrt{a^2-b^2}}-\frac{2(4bc^2d^3-acd^4-3bd^5)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(b^4c^6-4ab^3c^5d+6a^2b^2c^4d^2-b^4c^4d^2-4a^3bc^3d^3+4ab^3c^3d^3+a^4c^2d^4-6a^2b^2c^2d^4+4a^3bcd^5-a^4d^6)\sqrt{c^2-d^2}}-\frac{2\left(d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+cd^4\right)}{(b^3c^6-3ab^2c^5d+3a^2bc^4d^2-b^3c^4d^2-a^3c^3d^3+3ab^2c^3d^3-3a^2bc^2d^4+a^3cd^5)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}+\frac{5a^3b^5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ab^7c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-9a^4b^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6a^2b^6d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^4b^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^2b^6c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2b^8c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-8a^5b^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-11a^3b^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+10ab^7d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+11a^3b^5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2ab^7c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-23a^4b^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+14a^2b^6d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4a^4b^4c-a^2b^6c-8a^5b^3d+5a^3b^5d}{(a^6b^3c^3-2a^4b^5c^3+a^2b^7c^3-3a^7b^2c^2d+6a^5b^4c^2d-3a^3b^6c^2d+3a^8bcd^2-6a^6b^3cd^2+3a^4b^5cd^2-a^9d^3+2a^7b^2d^3-a^5b^4d^3)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^4*c^2 + b^6*c^2 - 8*a^3*b^3*c*d + 2*a*b^5*c*d + 12*a^4*b^2*d^2 -
15*a^2*b^4*d^2 + 6*b^6*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4*c^4 - 2*a^2*b^6
*c^4 + b^8*c^4 - 4*a^5*b^3*c^3*d + 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*
b^2*c^2*d^2 - 12*a^4*b^4*c^2*d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 + 8*a^
5*b^3*c*d^3 - 4*a^3*b^5*c*d^3 + a^8*d^4 - 2*a^6*b^2*d^4 + a^4*b^4*d^4)*sqrt
(a^2 - b^2)) - 2*(4*b*c^2*d^3 - a*c*d^4 - 3*b*d^5)*(pi*floor(1/2*(f*x + e)/
pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((
b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - b^4*c^4*d^2 - 4*a^3*b*c^3*d^3
 + 4*a*b^3*c^3*d^3 + a^4*c^2*d^4 - 6*a^2*b^2*c^2*d^4 + 4*a^3*b*c*d^5 - a^4*
d^6)*sqrt(c^2 - d^2)) - 2*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((b^3*c^6 - 3*
a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - b^3*c^4*d^2 - a^3*c^3*d^3 + 3*a*b^2*c^3*d^3
 - 3*a^2*b*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)) + (5*a^3*b^5*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c*tan(1/2*f
```

```
*x + 1/2*e)^3 - 9*a^4*b^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^2*b^6*d*tan(1/2*f*
x + 1/2*e)^3 + 4*a^4*b^4*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c*tan(1/2*f*x
 + 1/2*e)^2 - 2*b^8*c*tan(1/2*f*x + 1/2*e)^2 - 8*a^5*b^3*d*tan(1/2*f*x + 1/
2*e)^2 - 11*a^3*b^5*d*tan(1/2*f*x + 1/2*e)^2 + 10*a*b^7*d*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^5*c*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c*tan(1/2*f*x + 1/2*e)
- 23*a^4*b^4*d*tan(1/2*f*x + 1/2*e) + 14*a^2*b^6*d*tan(1/2*f*x + 1/2*e) + 4
*a^4*b^4*c - a^2*b^6*c - 8*a^5*b^3*d + 5*a^3*b^5*d)/((a^6*b^3*c^3 - 2*a^4*b
^5*c^3 + a^2*b^7*c^3 - 3*a^7*b^2*c^2*d + 6*a^5*b^4*c^2*d - 3*a^3*b^6*c^2*d
+ 3*a^8*b*c*d^2 - 6*a^6*b^3*c*d^2 + 3*a^4*b^5*c*d^2 - a^9*d^3 + 2*a^7*b^2*d
^3 - a^5*b^4*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)
^2))/f
```

**Mupad [B]**
time = 43.67, size = 2500, normalized size = 5.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 4*a^4*b^2*d^4 - b^6
*c^2*d^2 - 8*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d + 4*a^2*b^4*c^2*d^2 + 5*a*b^5*
c*d^3 - 5*a*b^5*c^3*d)/((a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) +
 (tan(e/2 + (f*x)/2)*(2*a^7*d^5 + 2*b^7*c^5 - 11*a^2*b^5*c^5 + 2*a^3*b^4*d^
5 - 4*a^5*b^2*d^5 - 2*b^7*c^3*d^2 + 12*a*b^6*c^2*d^3 + 18*a^2*b^5*c*d^4 + 1
5*a^3*b^4*c^4*d - 32*a^4*b^3*c*d^4 + a^2*b^5*c^3*d^2 - 15*a^3*b^4*c^2*d^3 +
 16*a^4*b^3*c^3*d^2 - 12*a*b^6*c^4*d + 8*a^6*b*c*d^4))/(a*c*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2
 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(2*a^7*d^5 + 2*b
^7*c^5 - 5*a^2*b^5*c^5 + 2*a^3*b^4*d^5 - 4*a^5*b^2*d^5 - 2*b^7*c^3*d^2 + 6*
a*b^6*c^2*d^3 + 9*a^3*b^4*c^4*d + 5*a^2*b^5*c^3*d^2 - 9*a^3*b^4*c^2*d^3 - 6
*a*b^6*c^4*d))/(a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^
4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (2*
tan(e/2 + (f*x)/2)^2*(b^8*c^5 + 4*a^7*b*d^5 + 2*a^8*c*d^4 - 3*a^2*b^6*c^5 -
 4*a^4*b^4*c^5 + 4*a^3*b^5*d^5 - 8*a^5*b^3*d^5 - b^8*c^3*d^2 + 3*a*b^7*c^2*
d^3 + 18*a^2*b^6*c*d^4 - 8*a^3*b^5*c^4*d - 29*a^4*b^4*c*d^4 + 8*a^5*b^3*c^4
*d - 11*a^2*b^6*c^3*d^2 + 8*a^3*b^5*c^2*d^3 + 27*a^4*b^4*c^3*d^2 - 8*a^5*b^
3*c^2*d^3 - 3*a*b^7*c^4*d))/(a^2*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*
b^2*d^2)) - (tan(e/2 + (f*x)/2)^4*(7*a^2*b^6*c^5 - 8*a^7*b*d^5 - 2*a^8*c*d^
4 - 2*b^8*c^5 + 4*a^4*b^4*c^5 - 8*a^3*b^5*d^5 + 16*a^5*b^3*d^5 + 2*b^8*c^3*
d^2 - 6*a*b^7*c^2*d^3 - 12*a^2*b^6*c*d^4 - a^3*b^5*c^4*d + 16*a^4*b^4*c*d^4
 - 8*a^5*b^3*c^4*d + 4*a^6*b^2*c*d^4 + 5*a^2*b^6*c^3*d^2 + a^3*b^5*c^2*d^3
- 22*a^4*b^4*c^3*d^2 + 8*a^5*b^3*c^2*d^3 + 6*a*b^7*c^4*d))/(a^2*c*(a^3*d^3
```

- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^3*(a^2*d + 2*b^2*d + 2*a*b*c)*(2*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 4*a^4*b^2*d^4 - b^6*c^2*d^2 - 8*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d + 4*a^2*b^4*c^2*d^2 + 5*a*b^5*c*d^3 - 5*a*b^5*c^3*d))/(a^2*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*a^2*c + 4*b^2*c + 8*a*b*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 4*b^2*c + 8*a*b*d) + tan(e/2 + (f*x)/2)^3*(4*a^2*d + 8*b^2*d + 8*a*b*c) + a^2*c + tan(e/2 + (f*x)/2)*(2*a^2*d + 4*a*b*c) + tan(e/2 + (f*x)/2)^5*(2*a^2*d + 4*a*b*c) + a^2*c*tan(e/2 + (f*x)/2)^6)) - (d^3*atan(((d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84...

# 3.722 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=669

$$-\frac{b^3\left(10a^3bcd-4ab^3cd-20a^4d^2-a^2b^2\left(2c^2-29d^2\right)-b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(bc-ad)^5f}-\frac{d^3(a^2d^2(2c^2$$

```
[Out] -b^3*(10*a^3*b*c*d-4*a*b^3*c*d-20*a^4*d^2-a^2*b^2*(2*c^2-29*d^2)-b^4*(c^2+1
2*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-
a*d+b*c)^5/f-d^3*(a^2*d^2*(2*c^2+d^2)-a*b*(10*c^3*d-4*c*d^3)+b^2*(20*c^4-29
*c^2*d^2+12*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*
c)^5/(c^2-d^2)^(5/2)/f-1/2*d*(a^4*d^3-b^4*d*(5*c^2-6*d^2)+2*a^2*b^2*d*(4*c^
2-5*d^2)-3*a*b^3*c*(c^2-d^2))*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)
/f/(c+d*sin(f*x+e))^2+1/2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*
x+e))^2/(c+d*sin(f*x+e))^2+1/2*b^2*(-7*a^2*d+3*a*b*c+4*b^2*d)*cos(f*x+e)/(a
^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2+3/2*d*(a^5*c*d
^4-2*a^3*b^2*c*d^4+a*b^4*c*(c^4-2*c^2*d^2+2*d^4)+b^5*d*(2*c^4-7*c^2*d^2+4*d
^4)-a^2*b^3*d*(3*c^4-12*c^2*d^2+7*d^4)-a^4*b*(3*c^2*d^3-2*d^5))*cos(f*x+e)/
(a^2-b^2)^2/(-a*d+b*c)^4/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

**Rubi** [A]

time = 2.19, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2881, 3134, 3080, 2739, 632, 210}

$$-\frac{d^3\left(a^2d^2\left(2c^2+d^2\right)-ab\left(10c^3d-4cd^3\right)+b^2\left(20c^4-29c^2d^2+12d^4\right)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^5}+\frac{b^2\left(-7a^2d+3abc+4b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))(c+d\sin(e+fx))^2}+\frac{b^2\cos(e+fx)}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}-\frac{d\left(a^4d^3+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)-b^4d\left(5c^2-6d^2\right)\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3(c+d\sin(e+fx))^2}-\frac{b^3\left(-20a^4d^2+10a^3bcd-a^2b^2\left(2c^2-29d^2\right)-4ab^3cd-b^4\left(c^2+12d^2\right)\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^5}+\frac{3d\left(a^5cd^4-a^4b\left(3c^2d^3-2d^5\right)-2a^3b^2cd^4-a^2b^3d\left(3c^4-12c^2d^2+7d^4\right)+ab^4c\left(c^4-2c^2d^2+2d^4\right)+b^5d\left(2c^4-7c^2d^2+4d^4\right)\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2\left(c^2-d^2\right)^2(bc-ad)^4(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((b^3*(10*a^3*b*c*d - 4*a*b^3*c*d - 20*a^4*d^2 - a^2*b^2*(2*c^2 - 29*d^2)
- b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^5*f)) - (d^3*(a^2*d^2*(2*c^2 + d^2) - a*b*(10*c
^3*d - 4*c*d^3) + b^2*(20*c^4 - 29*c^2*d^2 + 12*d^4))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(5/2)*f) - (d*(a^4*
d^3 - b^4*d*(5*c^2 - 6*d^2) + 2*a^2*b^2*d*(4*c^2 - 5*d^2) - 3*a*b^3*c*(c^2
- d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*S
in[e + f*x])^2) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Si
n[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 7*a^2*d + 4*b^2*d)*
Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x])*(c + d*
Sin[e + f*x])^2) + (3*d*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2
*d^2 + 2*d^4) + b^5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c
^2*d^2 + 7*d^4) - a^4*b*(3*c^2*d^3 - 2*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)^2
*(b*c - a*d)^4*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
```

```
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}dx = \frac{b^2\cos(e+fx)}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx$$

$$= \frac{b^2\cos(e+fx)}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx$$

$$= -\frac{d(a^4d^3-b^4d(5c^2-6d^2)+2a^2b^2d(4c^2-5d^2)-3ab^3c(c^2}{2(a^2-b^2)^2(bc-ad)^3(c^2-d^2)f(c+d\sin(e}$$

$$= -\frac{d(a^4d^3-b^4d(5c^2-6d^2)+2a^2b^2d(4c^2-5d^2)-3ab^3c(c^2}{2(a^2-b^2)^2(bc-ad)^3(c^2-d^2)f(c+d\sin(e}$$

$$= -\frac{d(a^4d^3-b^4d(5c^2-6d^2)+2a^2b^2d(4c^2-5d^2)-3ab^3c(c^2}{2(a^2-b^2)^2(bc-ad)^3(c^2-d^2)f(c+d\sin(e}$$

$$= -\frac{d(a^4d^3-b^4d(5c^2-6d^2)+2a^2b^2d(4c^2-5d^2)-3ab^3c(c^2}{2(a^2-b^2)^2(bc-ad)^3(c^2-d^2)f(c+d\sin(e}$$

$$= -\frac{d(a^4d^3-b^4d(5c^2-6d^2)+2a^2b^2d(4c^2-5d^2)-3ab^3c(c^2}{2(a^2-b^2)^2(bc-ad)^3(c^2-d^2)f(c+d\sin(e}$$

$$= -\frac{b^3(10a^3bcd-4ab^3cd-20a^4d^2-a^2b^2(2c^2-29d^2)-b^4(c}{(a^2-b^2)^{5/2}(bc-ad)^5}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1815 vs. $2(669) = 1338$.

time = 8.65, size = 1815, normalized size = 2.71

---

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]

[Out] -((b^3*(2*a^2*b^2*c^2 + b^4*c^2 - 10*a^3*b*c*d + 4*a*b^3*c*d + 20*a^4*d^2 - 29*a^2*b^2*d^2 + 12*b^4*d^2)*ArcTan[(Sec[(e + f*x)/2]*(b*Cos[(e + f*x)/2] + a*Sin[(e + f*x)/2]))/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(-(b*c) + a*d)^5*f)) - (d^3*(20*b^2*c^4 - 10*a*b*c^3*d + 2*a^2*c^2*d^2 - 29*b^2*c^2*d^2 + 4*a*b*c*d^3 + a^2*d^4 + 12*b^2*d^4)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(5/2)*f) + (32*a^2*b^5*c^7*Cos[e + f*x] - 8*b^7*c^7*Cos[e + f*x] - 80*a^3*b^4*c^6*d*Cos[e + f*x] + 68*a*b^6*c^6*d*Cos[e + f*x] - 92*a^2*b^5*c^5*d^2*Cos[e + f*x] + 38*b^7*c^5*d^2*Cos[e + f*x] + 140*a^3*b^4*c^4*d^3*Cos[e + f*x] - 122*a*b^6*c^4*d^3*Cos[e + f*x] - 80*a^6*b*c^3*d^4*Cos[e + f*x] + 140*a^4*b^3*c^3*d^4*Cos[e + f*x] + 48*a^2*b^5*c^3*d^4*Cos[e + f*x] - 72*b^7*c^3*d^4*Cos[e + f*x] + 32*a^7*c^2*d^5*Cos[e + f*x] - 92*a^5*b^2*c^2*d^5*Cos[e + f*x] + 48*a^3*b^4*c^2*d^5*Cos[e + f*x] + 12*a*b^6*c^2*d^5*Cos[e + f*x] + 68*a^6*b*c*d^6*Cos[e + f*x] - 122*a^4*b^3*c*d^6*Cos[e + f*x] + 12*a^2*b^5*c*d^6*Cos[e + f*x] + 36*b^7*c*d^6*Cos[e + f*x] - 8*a^7*d^7*Cos[e + f*x] + 38*a^5*b^2*d^7*Cos[e + f*x] - 72*a^3*b^4*d^7*Cos[e + f*x] + 36*a*b^6*d^7*Cos[e + f*x] - 12*a*b^6*c^6*d*Cos[3*(e + f*x)] + 28*a^2*b^5*c^5*d^2*Cos[3*(e + f*x)] - 22*b^7*c^5*d^2*Cos[3*(e + f*x)] + 20*a^3*b^4*c^4*d^3*Cos[3*(e + f*x)] + 10*a*b^6*c^4*d^3*Cos[3*(e + f*x)] + 20*a^4*b^3*c^3*d^4*Cos[3*(e + f*x)] - 96*a^2*b^5*c^3*d^4*Cos[3*(e + f*x)] + 64*b^7*c^3*d^4*Cos[3*(e + f*x)] + 28*a^5*b^2*c^2*d^5*Cos[3*(e + f*x)] - 96*a^3*b^4*c^2*d^5*Cos[3*(e + f*x)] + 44*a*b^6*c^2*d^5*Cos[3*(e + f*x)] - 12*a^6*b*c*d^6*Cos[3*(e + f*x)] + 10*a^4*b^3*c*d^6*Cos[3*(e + f*x)] + 44*a^2*b^5*c*d^6*Cos[3*(e + f*x)] - 36*b^7*c*d^6*Cos[3*(e + f*x)] - 22*a^5*b^2*d^7*Cos[3*(e + f*x)] + 64*a^3*b^4*d^7*Cos[3*(e + f*x)] - 36*a*b^6*d^7*Cos[3*(e + f*x)] + 12*a*b^6*c^7*Sin[2*(e + f*x)] - 4*a^2*b^5*c^6*d*Sin[2*(e + f*x)] + 16*b^7*c^6*d*Sin[2*(e + f*x)] - 80*a^3*b^4*c^5*d^2*Sin[2*(e + f*x)] + 38*a*b^6*c^5*d^2*Sin[2*(e + f*x)] - 10*a^2*b^5*c^4*d^3*Sin[2*(e + f*x)] - 20*b^7*c^4*d^3*Sin[2*(e + f*x)] - 80*a^5*b^2*c^3*d^4*Sin[2*(e + f*x)] + 320*a^3*b^4*c^3*d^4*Sin[2*(e + f*x)] - 192*a*b^6*c^3*d^4*Sin[2*(e + f*x)] - 4*a^6*b*c^2*d^5*Sin[2*(e + f*x)] - 10*a^4*b^3*c^2*d^5*Sin[2*(e + f*x)] + 64*a^2*b^5*c^2*d^5*Sin[2*(e + f*x)] - 26*b^7*c^2*d^5*Sin[2*(e + f*x)] + 12*a^7*c*d^6*Sin[2*(e + f*x)] + 38*a^5*b^2*c*d^6*Sin[2*(e + f*x)] - 192*a^3*b^4*c*d^6*Sin[2*(e + f*x)] + 124*a*b^6*c*d^6*Sin[2*(e + f*x)] + 16*a^6*b*d^7*Sin[2*(e + f*x)] - 20*a^4*b^3*d^7*Sin[2*(e + f*x)] - 26*a^2*b^5*d^7*Sin[2*(e + f*x)] + 24*b^7*d^7*Sin[2*(e + f*x)] - 3*a*b^6*c^5*d^2*Sin[4*(e + f*x)] + 9*a^2*b^5*c^4*d^3*Sin[4*(e + f*x)] - 6*b^7*c^4*d^3*Sin[4*(e + f*x)] + 6*a*b^6*c^3*d^4*Sin[4*(e + f*x)] + 9*a^4*b^3*c^2*d^5*Sin[4*(e + f*x)] - 36*a^2*b^5*c^2*d^5*Sin[4*(e + f*x)] + 21*b^7*c^2*d^5*Sin[4*(e + f*x)] - 3*a^5*b^2*c*d^6*Sin[4*(e + f*x)] + 6*a^3*b^4*c*d^6*Sin[4*(e + f*x)] - 6*a*b^6*c*d^6*Sin[4*(e + f*x)] - 6*a^4*b^3*d^7*Sin[4*(e + f*x)] + 21*a^2*b^5*d^7*Sin[4*(e + f*x)] - 12*b^7*d^7*Sin[4*(e + f*x)])/(16*(a^2 - b^2)^2*(-(b*c) + a*d)^4*(c^2 - d^2)^2*f*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2)

**Maple** [A]
time = 20.34, size = 1157, normalized size = 1.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*b^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^3*((1/2*b^2*(11*a^4*d^2-1
6*a^3*b*c*d+5*a^2*b^2*c^2-8*a^2*b^2*d^2+10*a*b^3*c*d-2*b^4*c^2)/(a^4-2*a^2*
b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3+1/2*b*(10*a^6*d^2-14*a^5*b*c*d+4*a^4*b^2*c^
2+13*a^4*b^2*d^2-20*a^3*b^3*c*d+7*a^2*b^4*c^2-14*a^2*b^4*d^2+16*a*b^5*c*d-2
*b^6*c^2)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2+1/2*b^2*(29*a^4*d^2-
40*a^3*b*c*d+11*a^2*b^2*c^2-20*a^2*b^2*d^2+22*a*b^3*c*d-2*b^4*c^2)/a/(a^4-2
*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)+1/2*b*(10*a^4*d^2-14*a^3*b*c*d+4*a^2*b^2*c
^2-7*a^2*b^2*d^2+8*a*b^3*c*d-b^4*c^2)/(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*f*x+1
/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)^2+1/2*(20*a^4*d^2-10*a^3*b*c*d+2*a^2*b^2*
c^2-29*a^2*b^2*d^2+4*a*b^3*c*d+b^4*c^2+12*b^4*d^2)/(a^4-2*a^2*b^2+b^4)/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)))+2*d^3
/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a*d-b*c)^2*((1/2*d^2*(5*a^2
*c^2*d^2-2*a^2*d^4-16*a*b*c^3*d+10*a*b*c*d^3+11*b^2*c^4-8*b^2*c^2*d^2)/(c^4
-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3+1/2*d*(4*a^2*c^4*d^2+7*a^2*c^2*d^4-2
*a^2*d^6-14*a*b*c^5*d-20*a*b*c^3*d^3+16*a*b*c*d^5+10*b^2*c^6+13*b^2*c^4*d^2
-14*b^2*c^2*d^4)/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(11*a
^2*c^2*d^2-2*a^2*d^4-40*a*b*c^3*d+22*a*b*c*d^3+29*b^2*c^4-20*b^2*c^2*d^2)/(
c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)+1/2*d*(4*a^2*c^2*d^2-a^2*d^4-14*a*b
*c^3*d+8*a*b*c*d^3+10*b^2*c^4-7*b^2*c^2*d^2)/(c^4-2*c^2*d^2+d^4))/(c*tan(1/
2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(2*a^2*c^2*d^2+a^2*d^4-10*a*
b*c^3*d+4*a*b*c*d^3+20*b^2*c^4-29*b^2*c^2*d^2+12*b^2*d^4)/(c^4-2*c^2*d^2+d^
4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7128 vs. $2(663) = 1326$.
time = 4.51, size = 7128, normalized size = 10.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^5*c^2 + b^7*c^2 - 10*a^3*b^4*c*d + 4*a*b^6*c*d + 20*a^4*b^3*d^2 -
 29*a^2*b^5*d^2 + 12*b^7*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^5*c^5 - 2*a^2*b
^7*c^5 + b^9*c^5 - 5*a^5*b^4*c^4*d + 10*a^3*b^6*c^4*d - 5*a*b^8*c^4*d + 10*
a^6*b^3*c^3*d^2 - 20*a^4*b^5*c^3*d^2 + 10*a^2*b^7*c^3*d^2 - 10*a^7*b^2*c^2*
d^3 + 20*a^5*b^4*c^2*d^3 - 10*a^3*b^6*c^2*d^3 + 5*a^8*b*c*d^4 - 10*a^6*b^3*
c*d^4 + 5*a^4*b^5*c*d^4 - a^9*d^5 + 2*a^7*b^2*d^5 - a^5*b^4*d^5)*sqrt(a^2 -
 b^2)) - (20*b^2*c^4*d^3 - 10*a*b*c^3*d^4 + 2*a^2*c^2*d^5 - 29*b^2*c^2*d^5
+ 4*a*b*c*d^6 + a^2*d^7 + 12*b^2*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn
(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^5*c^9 - 5*a
*b^4*c^8*d + 10*a^2*b^3*c^7*d^2 - 2*b^5*c^7*d^2 - 10*a^3*b^2*c^6*d^3 + 10*a
*b^4*c^6*d^3 + 5*a^4*b*c^5*d^4 - 20*a^2*b^3*c^5*d^4 + b^5*c^5*d^4 - a^5*c^4
*d^5 + 20*a^3*b^2*c^4*d^5 - 5*a*b^4*c^4*d^5 - 10*a^4*b*c^3*d^6 + 10*a^2*b^3
*c^3*d^6 + 2*a^5*c^2*d^7 - 10*a^3*b^2*c^2*d^7 + 5*a^4*b*c*d^8 - a^5*d^9)*sq
rt(c^2 - d^2)) + (5*a^3*b^6*c^9*tan(1/2*f*x + 1/2*e)^7 - 2*a*b^8*c^9*tan(1/
2*f*x + 1/2*e)^7 - 11*a^4*b^5*c^8*d*tan(1/2*f*x + 1/2*e)^7 + 8*a^2*b^7*c^8*
d*tan(1/2*f*x + 1/2*e)^7 - 10*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 4*a*
b^8*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 22*a^4*b^5*c^6*d^3*tan(1/2*f*x + 1/2*e
)^7 - 16*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e)^7 + 5*a^3*b^6*c^5*d^4*tan(1/2
*f*x + 1/2*e)^7 - 2*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e)^7 - 11*a^8*b*c^4*d^5
*tan(1/2*f*x + 1/2*e)^7 + 22*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e)^7 - 22*a^
```

4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e)^7 + 8*a^2*b^7*c^4*d^5*tan(1/2*f*x + 1/2*e)^7 + 5*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 - 10*a^7*b^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 + 5*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 + 8*a^8*b*c^2*d^7*tan(1/2*f*x + 1/2*e)^7 - 16*a^6*b^3*c^2*d^7*tan(1/2*f*x + 1/2*e)^7 + 8*a^4*b^5*c^2*d^7*tan(1/2*f*x + 1/2*e)^7 - 2*a^9*c*d^8*tan(1/2*f*x + 1/2*e)^7 + 4*a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e)^7 - 2*a^5*b^4*c*d^8*tan(1/2*f*x + 1/2*e)^7 + 4*a^4*b^5*c^9*tan(1/2*f*x + 1/2*e)^6 + 7*a^2*b^7*c^9*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*c^9*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^8*d*tan(1/2*f*x + 1/2*e)^6 + 7*a^3*b^6*c^8*d*tan(1/2*f*x + 1/2*e)^6 + 6*a*b^8*c^8*d*tan(1/2*f*x + 1/2*e)^6 - 52*a^4*b^5*c^7*d^2*tan(1/2*f*x + 1/2*e)^6 + 18*a^2*b^7*c^7*d^2*tan(1/2*f*x + 1/2*e)^6 + 4*b^9*c^7*d^2*tan(1/2*f*x + 1/2*e)^6 + 20*a^5*b^4*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 14*a^3*b^6*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 12*a*b^8*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 10*a^8*b*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 20*a^6*b^3*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 82*a^4*b^5*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 - 57*a^2*b^7*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 4*a^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 52*a^7*b^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 82*a^5*b^4*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 37*a^3*b^6*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 6*a*b^8*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 7*a^8*b*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 - 14*a^6*b^3*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 - 37*a^4*b^5*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 + 32*a^2*b^7*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 + 7*a^9*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 18*a^7*b^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 - 57*a^5*b^4*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 32*a^3*b^6*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 6*a^8*b*c*d^8*tan(1/2*f*x + 1/2*e)^6 - 12*a^6*b^3*c*d^8*tan(1/2*f*x + 1/2*e)^6 + 6*a^4*b^5*c*d^8*tan(1/2*f*x + 1/2*e)^6 - 2*a^9*d^9*tan(1/2*f*x + 1/2*e)^6 + 4*a^7*b^2*d^9*tan(1/2*f*x + 1/2*e)^6 - 2*a^5*b^4*d^9*tan(1/2*f*x + 1/2*e)^6 + 21*a^3*b^6*c^9*tan(1/2*f*x + 1/2*e)^5 - 6*a*b^8*c^9*tan(1/2*f*x + 1/2*e)^5 - 35*a^4*b^5*c^8*d*tan(1/2*f*x + 1/2*e)^5 + 64*a^2*b^7*c^8*d*tan(1/2*f*x + 1/2*e)^5 - 8*b^9*c^8*d*tan(1/2*f*x + 1/2*e)^5 - 40*a^5*b^4*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 - 74*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 + 60*a*b^8*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 + 26*a^4*b^5*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 - 96*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 + 16*b^9*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 - 40*a^7*b^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 160*a^5*b^4*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^3*b^6*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 - 102*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 - 35*a^8*b*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 26*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 106*a^4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 - 44*a^2*b^7*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 - 8*b^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 21*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 74*a^7*b^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 45*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 64*a^3*b^6*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 48*a*b^8*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 64*a^8*b*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 96*a^6*b^3*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 44*a^4*b^5*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 + 64*a^2*b^7*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 6*a^9*c*d^8*tan(1/2*f*x + 1/2*e)^5 + 60*a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 102*a^5*b^4*c*d^8*tan(1/2*f*x + 1/2*e)^5 + 48*a^3*b^6*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 8*a^8*b*d^9*tan(1/2*f*x +

1/2*e)^5 + 16*a^6*b^3*d^9*tan(1/2*f*x + 1/2*e)^...

**Mupad** [B]
time = 80.30, size = 2500, normalized size = 3.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)

[Out] (atan((((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*

c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 5538390
4*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*
d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*
b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20
019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^1
4*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 -
 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13
*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 631
24080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^1
3*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 5412
08*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6
*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 322948
08*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^1
6*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040
*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11
*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a
^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^1
8 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^
14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21
 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b
^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263
320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^
18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b
^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 26210
08*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^1
3 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^
6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19
*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 21203
2*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72
240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8
160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 1
76*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 -...

# 3.723 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=298

$$-\frac{2(15bc^2+56acd+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(5bc+7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}$$

```
[Out] -2/35*(7*a*d+5*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/7*b*cos(f*x+e)*(c
+d*sin(f*x+e))^(5/2)/f-2/105*(56*a*c*d+15*b*c^2+25*b*d^2)*cos(f*x+e)*(c+d*s
in(f*x+e))^(1/2)/f-2/105*(161*a*c^2*d+63*a*d^3+15*b*c^3+145*b*c*d^2)*(sin(1
/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e
+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+2/105*(c^2-d^2)*(56*a*c*d+15*b*c^2+25*b*d^2)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d
/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(56acd+15bc^2+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(c^2-d^2)(56acd+15bc^2+25bd^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105df\sqrt{c+d\sin(e+fx)}}+\frac{2(161ac^2d+63ad^3+15bc^3+145bcd^2)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(7ad+5bc)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])
/(105*f) - (2*(5*b*c + 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*
f) - (2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*(15*b*c^3 + 1
61*a*c^2*d + 145*b*c*d^2 + 63*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) - (2*(c^2 - d^2)*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f} + \frac{2}{7}\int (c+d\sin(e+$$

$$= -\frac{2(5bc+7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f} - \frac{2b\cos(e$$

$$= -\frac{2(15bc^2+56acd+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2(15bc^2+56acd+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2(15bc^2+56acd+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

$$= -\frac{2(15bc^2+56acd+25bd^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}$$

**Mathematica** [A]
time = 1.18, size = 275, normalized size = 0.92

$$\frac{-2d(5bd(27c^2+5d^2)+7a(15c^3+17cd^2))F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}} - 2(7ad(23c^2+9d^2)+5b(3c^3+29cd^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right) - cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}} - d\cos(e+fx)(c+d\sin(e+fx))(90bc^2+154acd+65bd^2-15bd^2\cos(2(e+fx))+6d(15bc+7ad)\sin(e+fx))}{105df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d*(5*b*d*(27*c^2 + 5*d^2) + 7*a*(15*c^3 + 17*c*d^2))*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(7*a*d
*(23*c^2 + 9*d^2) + 5*b*(3*c^3 + 29*c*d^2))*((c + d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(c + d*Sin[e + f*x]
)*(90*b*c^2 + 154*a*c*d + 65*b*d^2 - 15*b*d^2*Cos[2*(e + f*x)] + 6*d*(15*b*
c + 7*a*d)*Sin[e + f*x]))/(105*d*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1838 vs. $2(340) = 680$.
time = 7.56, size = 1839, normalized size = 6.17

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1839 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/105*(-25*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^5+63*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^5-15*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^5-63*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^5-25*b*c*d^4-77*a*c^2*d^3-45*b*c^3*d^2+15*b*d^5*sin(f*x+e)^5+21*a*d^5*sin(f*x+e)^4+10*b*d^5*sin(f*x+e)^3-21*a*d^5*sin(f*x+e)^2-25*b*d^5*sin(f*x+e)+90*b*c^2*d^3*sin(f*x+e)^3+77*a*c^2*d^3*sin(f*x+e)^2+45*b*c^3*d^2*sin(f*x+e)^2-35*b*c*d^4*sin(f*x+e)^2-98*a*c*d^4*sin(f*x+e)-90*b*c^2*d^3*sin(f*x+e)+60*b*c*d^4*sin(f*x+e)^4+98*a*c*d^4*sin(f*x+e)^3+105*c^4*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d+56*c^3*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-42*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^2*d^3-56*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^4+15*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^4*d+120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^3*d^2+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^3-120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^4-161*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^4*d+98*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^2*d^3-130*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d

```
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^3*d^2+145*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^4)/d^2/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 592, normalized size = 1.99

$$-\frac{\sqrt{2}\left(30bc^4+7ac^3d-115bc^2d^2-231acd^3-75bd^4\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}\left(30bc^4+7ac^3d-115bc^2d^2-231acd^3-75bd^4\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)+3\sqrt{2}\left(15ibc^3d+161iac^2d^2+145ibcd^3+63iad^4\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3\sqrt{2}\left(-15ibc^3d-161iac^2d^2-145ibcd^3-63iad^4\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)-6\left(15bd^4\cos(fx+e)^3-3(15bcd^3+7ad^4)\cos(fx+e)\sin(fx+e)-(45bc^2d^2+77acd^3+40bd^4)\cos(fx+e)\right)\sqrt{d\sin(fx+e)+c}}{315d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(sqrt(2)*(30*b*c^4 + 7*a*c^3*d - 115*b*c^2*d^2 - 231*a*c*d^3 - 75*b*
d^4)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3
 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) +
 sqrt(2)*(30*b*c^4 + 7*a*c^3*d - 115*b*c^2*d^2 - 231*a*c*d^3 - 75*b*d^4)*sq
rt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*
I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sq
rt(2)*(15*I*b*c^3*d + 161*I*a*c^2*d^2 + 145*I*b*c*d^3 + 63*I*a*d^4)*sqrt(I*
d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^
3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2
)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*
(-15*I*b*c^3*d - 161*I*a*c^2*d^2 - 145*I*b*c*d^3 - 63*I*a*d^4)*sqrt(-I*d)*w
eierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/
d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(15*b*d^4*
cos(f*x + e)^3 - 3*(15*b*c*d^3 + 7*a*d^4)*cos(f*x + e)*sin(f*x + e) - (45*b
*c^2*d^2 + 77*a*c*d^3 + 40*b*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(
d^2*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(5/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.724 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=235

$$-\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}+\frac{2(20acd+3b(c^2+3a$$

```
[Out] -2/5*b*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/15*(5*a*d+3*b*c)*cos(f*x+e)*(c
+d*sin(f*x+e))^(1/2)/f-2/15*(20*a*c*d+3*b*(c^2+3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/(c
+d))^(1/2)+2/15*(5*a*d+3*b*c)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(c^2-d^2)(5ad+3bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2(20acd+3b(c^2+3d^2))\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(5ad+3bc)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(3*b*c + 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*b*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*(20*a*c*d + 3*b*(c^2 + 3*
d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(15*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(3*b*c + 5*a*d)*(c^2 - d
^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f} + \frac{2}{5}\int \sqrt{c+d\sin(e}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - 2b\cos(e$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - 2b\cos(e$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - 2b\cos(e$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - 2b\cos(e$$

**Mathematica** [A]
time = 0.81, size = 218, normalized size = 0.93

$$\frac{-2d(12bcd+5a(3c^2+d^2))\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}} - 2(20acd+3b(c^2+3d^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right) - cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}} - 2d\cos(e+fx)(c+d\sin(e+fx))(6bc+5ad+3bd\sin(e+fx))}{15df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(12*b*c*d + 5*a*(3*c^2 + d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)
/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(20*a*c*d + 3*b*(c^2 + 3*d
^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF
[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- 2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])*(6*b*c + 5*a*d + 3*b*d*Sin[e + f*x]
))/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(281) = 562$.
time = 7.01, size = 1449, normalized size = 6.17

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1449 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(15*a*c^3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*d+5*a*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f
*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-15*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^3-5*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a
*d^4+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))*b*c^3*d+9*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2-3*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^3-9*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^
4-20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1
+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))*a*c^3*d+20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^3-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^4-6*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1
/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2
+9*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*b*d^4+3*b*d^4*sin(f*x+e)^4+5*a*d^4*sin(f*x+e)^3+9*b*c*d^3*sin(f*
x+e)^3+5*a*c*d^3*sin(f*x+e)^2+6*b*c^2*d^2*sin(f*x+e)^2-3*b*d^4*sin(f*x+e)^2
-5*a*d^4*sin(f*x+e)-9*b*c*d^3*sin(f*x+e)-5*a*c*d^3-6*b*c^2*d^2)/d^2/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 521, normalized size = 2.22

$$-\frac{\sqrt{2}\left(6bc^3-5ac^2d-18bcd^2-15ad^3\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}\left(6bc^3-5ac^2d-18bcd^2-15ad^3\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)+3\sqrt{2}\left(3ibc^2d+20iacd^2+9ibd^3\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3\sqrt{2}\left(-3ibc^2d-20iacd^2-9ibd^3\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)+6\left(3bd^3\cos(fx+e)\sin(fx+e)+(6bcd^2+5ad^3)\cos(fx+e)\right)\sqrt{d\sin(fx+e)+c}}{45d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(sqrt(2)*(6*b*c^3 - 5*a*c^2*d - 18*b*c*d^2 - 15*a*d^3)*sqrt(I*d)*weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(6*b*c^3 -
 5*a*c^2*d - 18*b*c*d^2 - 15*a*d^3)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(3*I*b*c^2*d + 20*I*a*c*d^2 + 9
*I*b*d^3)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*
I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)
/d)) + 3*sqrt(2)*(-3*I*b*c^2*d - 20*I*a*c*d^2 - 9*I*b*d^3)*sqrt(-I*d)*weier
strassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(3*b*d^3*cos(f
*x + e)*sin(f*x + e) + (6*b*c*d^2 + 5*a*d^3)*cos(f*x + e))*sqrt(d*sin(f*x +
 e) + c))/(d^2*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))\,(c+d\sin(e+fx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))\,(c+d\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.725 $\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=181

$$-\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2(bc+3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b(c^2-d^2)}{}$$

```
[Out] -2/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f-2/3*(3*a*d+b*c)*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e)
)/(c+d))^(1/2)+2/3*b*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(3ad+bc)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*(b*c + 3*a*d)*Ellip
ticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(c^2 - d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{1}{2}(3ac+bd)+\frac{1}{2}}{\sqrt{c+}}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{(bc+3ad)\int\sqrt{c+d}}{3d}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{\left((bc+3ad)\sqrt{c+d\,\mathrm{s}}\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2(bc+3ad)E\left(\frac{1}{2}(e-\right.}{3df\sqrt{}}$$

**Mathematica** [A]
time = 0.68, size = 152, normalized size = 0.84

$$-\frac{2\left(bd\cos(e+fx)(c+d\sin(e+fx))+(c+d)(bc+3ad)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-b(c^2-d^2)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (c + d)*(b*c + 3*a*d)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- b*(c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]))/(3*d*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(231) = 462.
time = 5.78, size = 862, normalized size = 4.76

| method | result |
|---|---|
| default | $\frac{2ac^2\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(-1+\sin(fx+e))d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)d-2\sqrt{\frac{c+d\sin}{c}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3*a*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*d-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1
+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d-((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^3-3*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*a*c^2*d+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*a*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e)
)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^2+b*d^3*sin(f*x+e)^
3+b*c*d^2*sin(f*x+e)^2-b*d^3*sin(f*x+e)-b*c*d^2)/d^2/cos(f*x+e)/(c+d*sin(f*
x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 453, normalized size = 2.50

$$-\frac{6\sqrt{d\sin(fx+e)+c}\,bd^2\cos(fx+e)+\sqrt{2}(2bc^2-3acd-3bd^2)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)+\sqrt{2}(2bc^2-3acd-3bd^2)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)+3\sqrt{2}(ibcd+3iad^2)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3\sqrt{2}(-ibcd-3iad^2)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)}{9d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*sqrt(d*sin(f*x + e) + c)*b*d^2*cos(f*x + e) + sqrt(2)*(2*b*c^2 - 3*
a*c*d - 3*b*d^2)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) -
 2*I*c)/d) + sqrt(2)*(2*b*c^2 - 3*a*c*d - 3*b*d^2)*sqrt(-I*d)*weierstrassPI
nverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d
```

```
*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(I*b*c*d + 3*I*a
*d^2)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^
3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d))
 + 3*sqrt(2)*(-I*b*c*d - 3*I*a*d^2)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4
*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e)
+ 3*I*d*sin(f*x + e) + 2*I*c)/d)))/(d^2*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\sin(e + f\,x))\,\sqrt{c + d\,\sin(e + f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.726 $\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=140

$$\frac{2bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\mid\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\mid\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/d/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(
1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(b*c - a*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b\int \sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-bc+ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{\left(b\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-bc+ad)\sqrt{\frac{c+\cdots}{}}\right.}{}$$

$$= \frac{2bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right|\cdots}{df\sqrt{c+d\sin\cdots}}$$

**Mathematica [A]**
time = 2.53, size = 101, normalized size = 0.72

$$-\frac{2\left(b(c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(-bc+ad)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(b*c) +
a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [A]
time = 6.93, size = 243, normalized size = 1.74

| method | result |
|---|---|
| default | $-\frac{2(c-d)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\,\sqrt{-\frac{(-1+\sin(fx+e))d}{c+d}}\,\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\left(\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bc+\mathrm{El}\right.}{d^2\cos(fx+e)\sqrt{}}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*b*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*b*d-a*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*d-EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d)/d^2/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 391, normalized size = 2.79

$$\frac{-3i\sqrt{2}\,b\sqrt{id}\,d\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)\right)+3i\sqrt{2}\,b\sqrt{-id}\,d\,\mathrm{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)\right)-\sqrt{2}(2bc-3ad)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)-3id\sin(fx+e)-2ic}{3d}\right)-\sqrt{2}(2bc-3ad)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(-8ic^3+9icd^2)}{27d^3},\frac{3d\cos(fx+e)+3id\sin(fx+e)+2ic}{3d}\right)}{3d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*sqrt(2)*b*sqrt(I*d)*d*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
```

```
2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
 e) - 2*I*c)/d)) + 3*I*sqrt(2)*b*sqrt(-I*d)*d*weierstrassZeta(-4/3*(4*c^2 -
 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
 3*I*d*sin(f*x + e) + 2*I*c)/d)) - sqrt(2)*(2*b*c - 3*a*d)*sqrt(I*d)*weiers
trassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/
3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) - sqrt(2)*(2*b*c - 3*a
*d)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^
3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d))
/(d^2*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin{(e + fx)}}{\sqrt{c + d\sin{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [B]**
time = 8.58, size = 176, normalized size = 1.26

$$\frac{b\left(2\,c\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)-2\,(c+d)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)\right)\sqrt{\cos(e+f\,x)^2}\,\sqrt{\frac{c+d\,\sin(e+f\,x)}{c+d}}}{d\,f\,\cos(e+f\,x)\,\sqrt{c+d\,\sin(e+f\,x)}}-\frac{2\,a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|\frac{2\,d}{c+d}\right)\sqrt{\frac{c+d\,\sin(e+f\,x)}{c+d}}}{f\,\sqrt{c+d\,\sin(e+f\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(1/2),x)
[Out] (b*(2*c*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(c + d)
) - 2*(c + d)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(
c + d)))*(cos(e + f*x)^2)^(1/2)*((c + d*sin(e + f*x))/(c + d))^(1/2))/(d*f*
cos(e + f*x)*(c + d*sin(e + f*x))^(1/2)) - (2*a*ellipticF(pi/4 - e/2 - (f*x
)/2, (2*d)/(c + d))*((c + d*sin(e + f*x))/(c + d))^(1/2))/(f*(c + d*sin(e +
 f*x))^(1/2))
```

# 3.727 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=195

$$-\frac{2(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d\,(c^2-d^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2bF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\right.\right.}{df}$$

```
[Out] -2*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+2*(-a*d+b*c)*(s
in(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1
/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c^2
-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(bc-ad)\cos(e+fx)}{f\,(c^2-d^2)\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\,(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - (2
*(b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(d*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-ac+bd)+\frac{1}{2}(bc-ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{b\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d} - \frac{(bc-ad)}{}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{\left((bc-ad)\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{}{c}}}{d\,(c^2-d^2)\sqrt{\frac{c+d\sin(e}{c+}}}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+}}{d\,(c^2-d^2)\,f\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

**Mathematica [A]**
time = 0.59, size = 159, normalized size = 0.82

$$\frac{2\left(d(-bc+ad)\cos(e+fx)+(c+d)(bc-ad)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-b(c^2-d^2)\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{(c-d)d(c+d)f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*(-(b*c) + a*d)*Cos[e + f*x] + (c + d)*(b*c - a*d)*EllipticE[(-2*e + P
i - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - b*(c^2 -
d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)]))/((c - d)*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(251) = 502$.
time = 15.13, size = 567, normalized size = 2.91

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{2b\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{\frac{d(1-\sin(fx+e))}{c+d}}\sqrt{\frac{(-\sin(fx+e)-1)d}{c-d}}}{d\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\right.}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b/d*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+(a*d-b*c)/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 629, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(b*c*d^2 - a*d^3)*sqrt(d*sin(f*x + e) + c)*cos(f*x + e) - (sqrt(2)*
(2*b*c^2*d + a*c*d^2 - 3*b*d^3)*sin(f*x + e) + sqrt(2)*(2*b*c^3 + a*c^2*d -
 3*b*c*d^2))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*
(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I
*c)/d) - (sqrt(2)*(2*b*c^2*d + a*c*d^2 - 3*b*d^3)*sin(f*x + e) + sqrt(2)*(2
*b*c^3 + a*c^2*d - 3*b*c*d^2))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 -
 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*
d*sin(f*x + e) + 2*I*c)/d) - 3*(sqrt(2)*(I*b*c*d^2 - I*a*d^3)*sin(f*x + e)
+ sqrt(2)*(I*b*c^2*d - I*a*c*d^2))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 -
3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^
2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*
I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(-I*b*c*d^2 + I*a*d^3)*sin(f*x +
 e) + sqrt(2)*(-I*b*c^2*d + I*a*c*d^2))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/
3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x +
 e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)))/((c^2*d^3 - d^5)*f*sin(f*x + e) + (c
^3*d^2 - c*d^4)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.728 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=285

$$-\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}+\frac{2(4acd-b(c^2+3d^2))\,E\left(\frac{1}{2}\left(e-\frac{\pi}{2}-\right.\right.}{3d\left(c^2-d^2\right)^2 f\sqrt{\frac{c}{}}}$$

```
[Out] -2/3*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(4*a*c*d-
b*(c^2+3*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)-2/3*(4*a*c*d
-b*(c^2+3*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*
x+e))^(1/2)/d/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/3*(-a*d+b*c)*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/
2)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.26, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\cos(e+fx)}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(4acd-b(c^2+3d^2))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\left(c^2-d^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
+ (2*(4*a*c*d - b*(c^2 + 3*d^2))*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[c +
d*Sin[e + f*x]]) + (2*(4*a*c*d - b*(c^2 + 3*d^2))*EllipticE[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*(c^2 - d^2)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*(c^2 - d^2)*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(ac-bd)-\frac{1}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3\,(c^2-d^2)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}} + \cdots$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}} + \cdots$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}} + \cdots$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2(4acd-b(c^2+3d^2))\cos(e+fx)}{3\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}} + \cdots$$

**Mathematica** [**A**]
time = 1.67, size = 199, normalized size = 0.70

$$\frac{2\left(\frac{\left((-4acd+b(c^2+3d^2))E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-(c-d)(bc-ad)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}}{(c-d)^2d}-\frac{\cos(e+fx)(ad(-5c^2+d^2)+2bc(c^2+d^2)+d(-4acd+b(c^2+3d^2))\sin(e+fx))}{(c^2-d^2)^2}\right)}{3f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*((((-4*a*c*d + b*(c^2 + 3*d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(
c + d)] - (c - d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)])*((c + d*Sin[e + f*x])/(c + d))^(3/2))/((c - d)^2*d) - (Cos[e + f*x]*(a*
d*(-5*c^2 + d^2) + 2*b*c*(c^2 + d^2) + d*(-4*a*c*d + b*(c^2 + 3*d^2))*Sin[e
 + f*x]))/(c^2 - d^2)^2))/(3*f*(c + d*Sin[e + f*x])^(3/2))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(331) = 662.
time = 24.47, size = 887, normalized size = 3.11

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{b\left(\frac{2d\left(\cos^{2}\left(fx+e\right)\right)}{\left(c^{2}-d^{2}\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}+\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))))+(a*d-b*c)/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*
c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*
d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*d*c/(c^2-d
^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/
(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 1047, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(2*b*c^3*d^2 + a*c^2*d^3 - 6*b*c*d^4 + 3*a*d^5)*cos(f*x + e)^
2 - 2*sqrt(2)*(2*b*c^4*d + a*c^3*d^2 - 6*b*c^2*d^3 + 3*a*c*d^4)*sin(f*x + e
) - sqrt(2)*(2*b*c^5 + a*c^4*d - 4*b*c^3*d^2 + 4*a*c^2*d^3 - 6*b*c*d^4 + 3*
a*d^5))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*
c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d
) + (sqrt(2)*(2*b*c^3*d^2 + a*c^2*d^3 - 6*b*c*d^4 + 3*a*d^5)*cos(f*x + e)^2
 - 2*sqrt(2)*(2*b*c^4*d + a*c^3*d^2 - 6*b*c^2*d^3 + 3*a*c*d^4)*sin(f*x + e)
 - sqrt(2)*(2*b*c^5 + a*c^4*d - 4*b*c^3*d^2 + 4*a*c^2*d^3 - 6*b*c*d^4 + 3*a
*d^5))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I
*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/
d) + 3*(sqrt(2)*(I*b*c^2*d^3 - 4*I*a*c*d^4 + 3*I*b*d^5)*cos(f*x + e)^2 + 2*
sqrt(2)*(-I*b*c^3*d^2 + 4*I*a*c^2*d^3 - 3*I*b*c*d^4)*sin(f*x + e) + sqrt(2)
*(-I*b*c^4*d + 4*I*a*c^3*d^2 - 4*I*b*c^2*d^3 + 4*I*a*c*d^4 - 3*I*b*d^5))*sq
rt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^
2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*
c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sq
rt(2)*(-I*b*c^2*d^3 + 4*I*a*c*d^4 - 3*I*b*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(
I*b*c^3*d^2 - 4*I*a*c^2*d^3 + 3*I*b*c*d^4)*sin(f*x + e) + sqrt(2)*(I*b*c^4*
d - 4*I*a*c^3*d^2 + 4*I*b*c^2*d^3 - 4*I*a*c*d^4 + 3*I*b*d^5))*sqrt(-I*d)*we
ierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, w
eierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d
^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*((b*c^2*d^3
 - 4*a*c*d^4 + 3*b*d^5)*cos(f*x + e)*sin(f*x + e) + (2*b*c^3*d^2 - 5*a*c^2*
d^3 + 2*b*c*d^4 + a*d^5)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c^4*d^4
- 2*c^2*d^6 + d^8)*f*cos(f*x + e)^2 - 2*(c^5*d^3 - 2*c^3*d^5 + c*d^7)*f*sin
(f*x + e) - (c^6*d^2 - c^4*d^4 - c^2*d^6 + d^8)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2), x)
```

## 3.729 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=369

$$-\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}}-\frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}}-\frac{2(3bc^3-23ac^2d+29bcd^2-9ad}{15\,(c^2-d^2)^3\,f\sqrt{c+d\sin(}}$$

```
[Out] -2/5*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(5/2)-2/15*(-8*a*c*
d+3*b*c^2+5*b*d^2)*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(3/2)-2/15*(-2
3*a*c^2*d-9*a*d^3+3*b*c^3+29*b*c*d^2)*cos(f*x+e)/(c^2-d^2)^3/f/(c+d*sin(f*x
+e))^(1/2)+2/15*(-23*a*c^2*d-9*a*d^3+3*b*c^3+29*b*c*d^2)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c^2-d^2)^3/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)-2/15*(-8*a*c*d+3*b*c^2+5*b*d^2)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/(c^2-d^2)^2/
f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.35, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-8acd+3bc^2+5bd^2)\cos(e+fx)}{15f\,(c^2-d^2)^2\,(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\cos(e+fx)}{5f\,(c^2-d^2)\,(c+d\sin(e+fx))^{5/2}}+\frac{2(-8acd+3bc^2+5bd^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{15df\,(c^2-d^2)^2\,\sqrt{c+d\sin(e+fx)}}-\frac{2(-23ac^2d-9ad^3+3bc^3+29bcd^2)\cos(e+fx)}{15f\,(c^2-d^2)^3\,\sqrt{c+d\sin(e+fx)}}-\frac{2(-23ac^2d-9ad^3+3bc^3+29bcd^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{15df\,(c^2-d^2)^3\,\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(5*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2))
 - (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*Cos[e + f*x])/(15*(c^2 - d^2)^2*f*(c + d
*Sin[e + f*x])^(3/2)) - (2*(3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*Co
s[e + f*x])/(15*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (2*(3*b*c^3 - 2
3*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c +
d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d*(c^2 - d^2)^3*f*Sqrt[(c + d*Sin[e + f*x
])/(c + d)]) + (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d*(c^2 - d^2)^2*f
*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}(ac-bd)-\frac{3}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}dx}{5\,(c^2-d^2)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}} -$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}} - \frac{2(3bc^2-8acd+5bd^2)\cos(e+fx)}{15\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}} -$$

**Mathematica** [**A**]
time = 3.29, size = 297, normalized size = 0.80

$$\frac{2\left(\frac{\left((3bc^3-23ac^2d+29bcd^2-9ad^3)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-(c-d)(3bc^2-8acd+5bd^2)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{5/2}}{(c-d)^3d}+\frac{\cos(e+fx)\left(ad(34c^4-5c^2d^2+3d^4)+b(-9c^5-25c^3d^2+2cd^4)+d(-9bc^4+54ac^3d-60bc^2d^2+10acd^3+5bd^4)\sin(e+fx)+d^2(-3bc^3+23ac^2d-29bcd^2+9ad^3)\sin^2(e+fx)\right)}{(c^2-d^2)^3}\right)}{15f(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*((((3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)] - (c - d)*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*((c + d*Sin[e + f*x])/(c + d))^(5/2)
)/((c - d)^3*d) + (Cos[e + f*x]*(a*d*(34*c^4 - 5*c^2*d^2 + 3*d^4) + b*(-9*c
^5 - 25*c^3*d^2 + 2*c*d^4) + d*(-9*b*c^4 + 54*a*c^3*d - 60*b*c^2*d^2 + 10*a
*c*d^3 + 5*b*d^4)*Sin[e + f*x] + d^2*(-3*b*c^3 + 23*a*c^2*d - 29*b*c*d^2 +
9*a*d^3)*Sin[e + f*x]^2))/(c^2 - d^2)^3))/(15*f*(c + d*Sin[e + f*x])^(5/2))
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(411) = 822.
time = 33.28, size = 1049, normalized size = 2.84

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1049 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a*d-b*c)/d*(2/5/(c^2-d^2)/d^2*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^3+16/15*c/(c^2-d^2)
^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+2/15*d*co
s(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(1/d*c-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3
*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/
d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b/d*(2/3/(c^2-d^2
)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*d*cos(
f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2
)/(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 1601, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/45*((3*sqrt(2)*(6*b*c^5*d^2 - a*c^4*d^3 - 23*b*c^3*d^4 + 33*a*c^2*d^5 - 1
5*b*c*d^6)*cos(f*x + e)^2 + (sqrt(2)*(6*b*c^4*d^3 - a*c^3*d^4 - 23*b*c^2*d^
5 + 33*a*c*d^6 - 15*b*d^7)*cos(f*x + e)^2 - sqrt(2)*(18*b*c^6*d - 3*a*c^5*d
^2 - 63*b*c^4*d^3 + 98*a*c^3*d^4 - 68*b*c^2*d^5 + 33*a*c*d^6 - 15*b*d^7))*s
in(f*x + e) - sqrt(2)*(6*b*c^7 - a*c^6*d - 5*b*c^5*d^2 + 30*a*c^4*d^3 - 84*
b*c^3*d^4 + 99*a*c^2*d^5 - 45*b*c*d^6))*sqrt(I*d)*weierstrassPInverse(-4/3*
(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e)
 - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (3*sqrt(2)*(6*b*c^5*d^2 - a*c^4*d^3 - 2
3*b*c^3*d^4 + 33*a*c^2*d^5 - 15*b*c*d^6)*cos(f*x + e)^2 + (sqrt(2)*(6*b*c^4
*d^3 - a*c^3*d^4 - 23*b*c^2*d^5 + 33*a*c*d^6 - 15*b*d^7)*cos(f*x + e)^2 - s
qrt(2)*(18*b*c^6*d - 3*a*c^5*d^2 - 63*b*c^4*d^3 + 98*a*c^3*d^4 - 68*b*c^2*d
^5 + 33*a*c*d^6 - 15*b*d^7))*sin(f*x + e) - sqrt(2)*(6*b*c^7 - a*c^6*d - 5*
b*c^5*d^2 + 30*a*c^4*d^3 - 84*b*c^3*d^4 + 99*a*c^2*d^5 - 45*b*c*d^6))*sqrt(
-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c
*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(3*sq
rt(2)*(3*I*b*c^4*d^3 - 23*I*a*c^3*d^4 + 29*I*b*c^2*d^5 - 9*I*a*c*d^6)*cos(f
*x + e)^2 + (sqrt(2)*(3*I*b*c^3*d^4 - 23*I*a*c^2*d^5 + 29*I*b*c*d^6 - 9*I*a
*d^7)*cos(f*x + e)^2 + sqrt(2)*(-9*I*b*c^5*d^2 + 69*I*a*c^4*d^3 - 90*I*b*c^
3*d^4 + 50*I*a*c^2*d^5 - 29*I*b*c*d^6 + 9*I*a*d^7))*sin(f*x + e) + sqrt(2)*
(-3*I*b*c^6*d + 23*I*a*c^5*d^2 - 38*I*b*c^4*d^3 + 78*I*a*c^3*d^4 - 87*I*b*c
^2*d^5 + 27*I*a*c*d^6))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2,
 -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/
d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x
 + e) - 2*I*c)/d)) + 3*(3*sqrt(2)*(-3*I*b*c^4*d^3 + 23*I*a*c^3*d^4 - 29*I*b
*c^2*d^5 + 9*I*a*c*d^6)*cos(f*x + e)^2 + (sqrt(2)*(-3*I*b*c^3*d^4 + 23*I*a*
c^2*d^5 - 29*I*b*c*d^6 + 9*I*a*d^7)*cos(f*x + e)^2 + sqrt(2)*(9*I*b*c^5*d^2
 - 69*I*a*c^4*d^3 + 90*I*b*c^3*d^4 - 50*I*a*c^2*d^5 + 29*I*b*c*d^6 - 9*I*a*
d^7))*sin(f*x + e) + sqrt(2)*(3*I*b*c^6*d - 23*I*a*c^5*d^2 + 38*I*b*c^4*d^3
 - 78*I*a*c^3*d^4 + 87*I*b*c^2*d^5 - 27*I*a*c*d^6))*sqrt(-I*d)*weierstrassZ
eta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrass
PInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3
*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*((3*b*c^3*d^4 - 23*a*
c^2*d^5 + 29*b*c*d^6 - 9*a*d^7)*cos(f*x + e)^3 - (9*b*c^4*d^3 - 54*a*c^3*d^
4 + 60*b*c^2*d^5 - 10*a*c*d^6 - 5*b*d^7)*cos(f*x + e)*sin(f*x + e) - (9*b*c
^5*d^2 - 34*a*c^4*d^3 + 28*b*c^3*d^4 - 18*a*c^2*d^5 + 27*b*c*d^6 - 12*a*d^7
)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(3*(c^7*d^4 - 3*c^5*d^6 + 3*c^3*d
^8 - c*d^10)*f*cos(f*x + e)^2 - (c^9*d^2 - 6*c^5*d^6 + 8*c^3*d^8 - 3*c*d^10
)*f + ((c^6*d^5 - 3*c^4*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - (3*c^8*d
^3 - 8*c^6*d^5 + 6*c^4*d^7 - d^11)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \sin(e + f x)}{(c + d \sin(e + f x))^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.730 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=451

$$-\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-b^2(5c^3-57cd^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}-\frac{2(7(9a^2+7b^2)\,d^2-10\cdots}{}$$

```
[Out] -2/315*(7*(9*a^2+7*b^2)*d^2-10*b*c*(-9*a*d+b*c))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(3/2)/d/f+4/63*b*(-9*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f-2/9*b
^2*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d/f-4/315*(84*a^2*c*d^2+15*a*b*d*(3*c^
2+5*d^2)-b^2*(5*c^3-57*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f-2/315*
(21*a^2*d^2*(23*c^2+9*d^2)+30*a*b*d*(3*c^3+29*c*d^2)-b^2*(10*c^4-279*c^2*d^
2-147*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e)
)^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/315*(c^2-d^2)*(-84*a^2*c*d^2
-45*a*b*c^2*d-75*a*b*d^3+5*b^2*c^3-57*b^2*c*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))
^(1/2)
```

**Rubi** [A]
time = 0.62, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-(b^2(5c^3-57cd^2)))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4(c^2-d^2)(-84a^2cd^2-45abc^2d-75abd^3+5b^2c^3-57b^2cd^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(21a^2d^2(23c^2+9d^2)+30abd(3c^3+29cd^2)-(b^2(10c^4-279c^2d^2-147d^4)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(7d^2(9a^2+7b^2)-10bc(bc-9ad))\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315df}+\frac{4b(bc-9ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df}-\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-4*(84*a^2*c*d^2 + 15*a*b*d*(3*c^2 + 5*d^2) - b^2*(5*c^3 - 57*c*d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d*f) - (2*(7*(9*a^2 + 7*b^2)*d^2 -
10*b*c*(b*c - 9*a*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(315*d*f) +
(4*b*(b*c - 9*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(63*d*f) - (2*b
^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(9*d*f) + (2*(21*a^2*d^2*(23*c^
2 + 9*d^2) + 30*a*b*d*(3*c^3 + 29*c*d^2) - b^2*(10*c^4 - 279*c^2*d^2 - 147*
d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(315*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(c^2 - d^2)*(5*b^2*c^
3 - 45*a*b*c^2*d - 84*a^2*c*d^2 - 57*b^2*c*d^2 - 75*a*b*d^3)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2
*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df} + \frac{2\int(c+d\sin(e+ \ldots}{\ldots}$$

$$= \frac{4b(bc-9ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df} - \frac{2b^2\cos(e \ldots}{\ldots}$$

$$= -\frac{2(7(9a^2+7b^2)d^2-10bc(bc-9ad))\cos(e+fx)(c+d\sin \ldots}{315df}$$

$$= -\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-b^2(5c^3-57cd^2))\cos(e+ \ldots}{315df}$$

$$= -\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-b^2(5c^3-57cd^2))\cos(e+ \ldots}{315df}$$

$$= -\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-b^2(5c^3-57cd^2))\cos(e+ \ldots}{315df}$$

$$= -\frac{4(84a^2cd^2+15abd(3c^2+5d^2)-b^2(5c^3-57cd^2))\cos(e+ \ldots}{315df}$$

**Mathematica [A]**
time = 1.88, size = 382, normalized size = 0.85

$$\frac{8\left(-d^2(30abd(27c^2+5d^2)+b^2c(155c^2+261d^2)+21a^2(15c^3+17cd^2))F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(-21a^2d^2(23c^2+9d^2)-30abd(3c^3+29cd^2)+b^2(10c^4-279c^2d^2-147d^4))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-d(c+d\sin(e+fx))(2(924a^2cd^2+30abd(36c^2+23d^2)+b^2(20c^3+747cd^2))\cos(e+fx)-10bd^2(19bc+18ad)\cos(3(e+fx))+2d(540abcd+126a^2d^2+b^2(150c^2+133d^2))\sin(2(e+fx))-35b^2d^3\sin(4(e+fx)))}{1260d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (8*(-(d^2*(30*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(155*c^2 + 261*d^2) + 21*a^2*(
15*c^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (-21
*a^2*d^2*(23*c^2 + 9*d^2) - 30*a*b*d*(3*c^3 + 29*c*d^2) + b^2*(10*c^4 - 279
*c^2*d^2 - 147*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e
```

```
+ f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(924*a^2*c*d^2 + 30*a*b*d*(36*
c^2 + 23*d^2) + b^2*(20*c^3 + 747*c*d^2))*Cos[e + f*x] - 10*b*d^2*(19*b*c +
 18*a*d)*Cos[3*(e + f*x)] + 2*d*(540*a*b*c*d + 126*a^2*d^2 + b^2*(150*c^2 +
 133*d^2))*Sin[2*(e + f*x)] - 35*b^2*d^3*Sin[4*(e + f*x)]))/(1260*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(489) = 978$.
time = 33.13, size = 2112, normalized size = 4.68

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2112 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3*b^2*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63/d^2*c*sin(f*x+e)^2*(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21/d^2*c^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(1/d*
c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b*d^3+3*b^2*c*d^2)
*(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35/d^2*c*s
in(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35/d^2*c^2)/d
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35/d^2*c^2+5/21)*(1/d*c-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(
1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*
c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8
/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15/d*c*(1/d*c-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15/d^2*c^2)*(1/d*c-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*Elliptic
```

```
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3
)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+2*(3*a^2*c^2*d+2*a*b*c^3)*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*c^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 803, normalized size = 1.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/945*(sqrt(2)*(20*b^2*c^5 - 180*a*b*c^4*d + 690*a*b*c^2*d^3 + 450*a*b*d^5
- 3*(7*a^2 + 31*b^2)*c^3*d^2 + 3*(231*a^2 + 163*b^2)*c*d^4)*sqrt(I*d)*weier
strassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1
/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(20*b^2*c^5
 - 180*a*b*c^4*d + 690*a*b*c^2*d^3 + 450*a*b*d^5 - 3*(7*a^2 + 31*b^2)*c^3*d
^2 + 3*(231*a^2 + 163*b^2)*c*d^4)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^
2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3
*I*d*sin(f*x + e) + 2*I*c)/d) - 3*sqrt(2)*(-10*I*b^2*c^4*d + 90*I*a*b*c^3*d
^2 + 870*I*a*b*c*d^4 + 3*I*(161*a^2 + 93*b^2)*c^2*d^3 + 21*I*(9*a^2 + 7*b^2
)*d^5)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
```

```
 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c
^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)
) - 3*sqrt(2)*(10*I*b^2*c^4*d - 90*I*a*b*c^3*d^2 - 870*I*a*b*c*d^4 - 3*I*(1
61*a^2 + 93*b^2)*c^2*d^3 - 21*I*(9*a^2 + 7*b^2)*d^5)*sqrt(-I*d)*weierstrass
Zeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstras
sPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(
3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(5*(19*b^2*c*d^4 + 1
8*a*b*d^5)*cos(f*x + e)^3 - (5*b^2*c^3*d^2 + 270*a*b*c^2*d^3 + 240*a*b*d^5
+ 3*(77*a^2 + 86*b^2)*c*d^4)*cos(f*x + e) + (35*b^2*d^5*cos(f*x + e)^3 - 3*
(25*b^2*c^2*d^3 + 90*a*b*c*d^4 + 7*(3*a^2 + 4*b^2)*d^5)*cos(f*x + e))*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c))/(d^3*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^2 (c+d\sin(e+fx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*sin(e + f*x))**(5/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(e+f\,x))^2\,(c+d\,\sin(e+f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.731 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=347

$$-\frac{2(5(7a^2+5b^2)\,d^2-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df}+\frac{4b(bc-7ad)\cos(e+fx)(c+d\sin(e-$$

```
[Out] 4/35*b*(-7*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f-2/7*b^2*cos(f*x+e
)*(c+d*sin(f*x+e))^(5/2)/d/f-2/105*(5*(7*a^2+5*b^2)*d^2-6*b*c*(-7*a*d+b*c))
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f-4/105*(70*a^2*c*d^2+21*a*b*d*(c^2+3*
d^2)-b^2*(3*c^3-41*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/
4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*
(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/105*(c^2-d^2)
*(42*a*b*c*d+35*a^2*d^2-b^2*(6*c^2-25*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2
)
```

**Rubi** [A]
time = 0.41, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(c^2-d^2)\left(35a^2d^2+42abcd-(b^2(6c^2-25d^2))\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4(70a^2cd^2+21abd(c^2+3d^2)-(b^2(3c^3-41cd^2)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(5d^2(7a^2+5b^2)-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df}+\frac{4b(bc-7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df}-\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(5*(7*a^2 + 5*b^2)*d^2 - 6*b*c*(b*c - 7*a*d))*Cos[e + f*x]*Sqrt[c + d*S
in[e + f*x]])/(105*d*f) + (4*b*(b*c - 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(35*d*f) - (2*b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*
f) + (4*(70*a^2*c*d^2 + 21*a*b*d*(c^2 + 3*d^2) - b^2*(3*c^3 - 41*c*d^2))*El
lipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d
^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(42*a*b*c*d + 35*
a^2*d^2 - b^2*(6*c^2 - 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int(c+d\sin(e+$$

$$= \frac{4b(bc-7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2b^2\cos(e$$

$$= -\frac{2(5(7a^2+5b^2)\,d^2-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin}}{105df}$$

$$= -\frac{2(5(7a^2+5b^2)\,d^2-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin}}{105df}$$

$$= -\frac{2(5(7a^2+5b^2)\,d^2-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin}}{105df}$$

$$= -\frac{2(5(7a^2+5b^2)\,d^2-6bc(bc-7ad))\cos(e+fx)\sqrt{c+d\sin}}{105df}$$

**Mathematica** [A]
time = 1.26, size = 292, normalized size = 0.84

$$\frac{4\left(-d^2(168abcd+35a^2(3c^2+d^2)+b^2(51c^2+25d^2))\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-2(70a^2cd^2+21abd(c^2+3d^2)+b^2(-3c^3+41cd^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-d(c+d\sin(e+fx))((336abcd+140a^2d^2+b^2(12c^2+115d^2))\cos(e+fx)+3bd(-5bd\cos(3(e+fx))+4(4bc+7ad)\sin(2(e+fx))))\right)}{210d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*(-(d^2*(168*a*b*c*d + 35*a^2*(3*c^2 + d^2) + b^2*(51*c^2 + 25*d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) - 2*(70*a^2*c*d^2 + 21*a*b*d*(
c^2 + 3*d^2) + b^2*(-3*c^3 + 41*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*((336*a*b*c*d
+ 140*a^2*d^2 + b^2*(12*c^2 + 115*d^2))*Cos[e + f*x] + 3*b*d*(-5*b*d*Cos[3*
(e + f*x)] + 4*(4*b*c + 7*a*d)*Sin[2*(e + f*x)]))))/(210*d^2*f*Sqrt[c + d*Si
n[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(389) = 778$.
time = 25.85, size = 1575, normalized size = 4.54

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1575 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2*b^2*(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35/d^2*c*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35/d^2*c^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35/d^2*c^2+5/21)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b*d^2+2*b^2*c*d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15/d^2*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^2+4*a*b*c*d+b^2*c^2)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(2*a^2*c*d+2*a*b*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*c^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 684, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(12*b^2*c^4 - 84*a*b*c^3*d + 252*a*b*c*d^3 + (35*a^2 - 11*b^
2)*c^2*d^2 + 15*(7*a^2 + 5*b^2)*d^4)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*
c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) -
3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(12*b^2*c^4 - 84*a*b*c^3*d + 252*a
*b*c*d^3 + (35*a^2 - 11*b^2)*c^2*d^2 + 15*(7*a^2 + 5*b^2)*d^4)*sqrt(-I*d)*w
eierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d
^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 6*sqrt(2)*(-3*
I*b^2*c^3*d + 21*I*a*b*c^2*d^2 + 63*I*a*b*d^4 + I*(70*a^2 + 41*b^2)*c*d^3)*
sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*
d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*
I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 6*s
qrt(2)*(3*I*b^2*c^3*d - 21*I*a*b*c^2*d^2 - 63*I*a*b*d^4 - I*(70*a^2 + 41*b^
2)*c*d^3)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*
c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I
*c)/d)) + 6*(15*b^2*d^4*cos(f*x + e)^3 - 6*(4*b^2*c*d^3 + 7*a*b*d^4)*cos(f*
x + e)*sin(f*x + e) - (3*b^2*c^2*d^2 + 84*a*b*c*d^3 + 5*(7*a^2 + 8*b^2)*d^4
)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^3*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin (e + fx))^2 (c + d\sin (e + fx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))^2 (c+d\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.732 $\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=254

$$\frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df}-\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df}+\frac{2(3(5a^2+3b^2)\,d^2-2b$$

```
[Out] -2/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/15*b*(-5*a*d+b*c)*cos(f*x+
e)*(c+d*sin(f*x+e))^(1/2)/d/f-2/15*(3*(5*a^2+3*b^2)*d^2-2*b*c*(-5*a*d+b*c))
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*b*(-5*a*d+b*c)*(c^2-d^2)*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(
c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.27, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\left(3d^2(5a^2+3b^2)-2bc(bc-5ad)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4b(c^2-d^2)\,(bc-5ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df}-\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*b*(b*c - 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*b^2
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) + (2*(3*(5*a^2 + 3*b^2)*d
^2 - 2*b*c*(b*c - 5*a*d))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[c + d*Sin[e + f*x]])/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*b*
(b*c - 5*a*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int \sqrt{c+d\sin(e-\cdots}{\cdots}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+\cdots}{\cdots}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+\cdots}{\cdots}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+\cdots}{\cdots}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+\cdots}{\cdots}$$

**Mathematica** [A]
time = 0.97, size = 214, normalized size = 0.84

$$\frac{2\left(-d^2(15a^2c+7b^2c+10abd)\,F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(-10abcd-15a^2d^2+b^2(2c^2-9d^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-2bd\cos(e+fx)(c+d\sin(e+fx))(bc+10ad+3bd\sin(e+fx))}{15d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*(-(d^2*(15*a^2*c + 7*b^2*c + 10*a*b*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]) + (-10*a*b*c*d - 15*a^2*d^2 + b^2*(2*c^2 - 9*d^2))*((c + d)
*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b*d*Cos[
e + f*x]*(c + d*Sin[e + f*x])*(b*c + 10*a*d + 3*b*d*Sin[e + f*x]))/(15*d^2*
f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. 2(300) = 600.
time = 19.71, size = 1100, normalized size = 4.33

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1100 |

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d*(-2/5/d*sin(f*x+e)*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)+4/15/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(
3/5+8/15/d^2*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
(2*a*b*d+b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(a^2*d+2*a*b*c)*(1/d*c-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*c*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 575, normalized size = 2.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(4*b^2*c^3 - 20*a*b*c^2*d + 30*a*b*d^3 + 3*(5*a^2 + b^2)*c*d^
2)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 -
 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + s
qrt(2)*(4*b^2*c^3 - 20*a*b*c^2*d + 30*a*b*d^3 + 3*(5*a^2 + b^2)*c*d^2)*sqrt
(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*
c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) - 3*sqrt
(2)*(-2*I*b^2*c^2*d + 10*I*a*b*c*d^2 + 3*I*(5*a^2 + 3*b^2)*d^3)*sqrt(I*d)*w
eierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, w
eierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^
3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*sqrt(2)*(2*I
*b^2*c^2*d - 10*I*a*b*c*d^2 - 3*I*(5*a^2 + 3*b^2)*d^3)*sqrt(-I*d)*weierstra
ssZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstr
assPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*(3*b^2*d^3*cos(f*x
 + e)*sin(f*x + e) + (b^2*c*d^2 + 10*a*b*d^3)*cos(f*x + e))*sqrt(d*sin(f*x
+ e) + c))/(d^3*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(e+f\,x))^2\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.733 $\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=203

$$-\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df}-\frac{4b(bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2((3a^2+$$

```
[Out] -2/3*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f+4/3*b*(-3*a*d+b*c)*(sin(1/2*
e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/
4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-2/3*((3*a^2+b^2)*d^2+2*b*c*(-3*a*d+b*c))*(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/
(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2870, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(d^2(3a^2+b^2)+2bc(bc-3ad))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*b*(b*c - 3*a*d)
*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*((3*a^2 + b^2)*d^2 + 2*b*c*(
b*c - 3*a*d))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/(3*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{\frac{1}{2}(3a^2+b^2)d-b(bc-3ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{(2b(bc-3ad))\int \sqrt{c+d\sin(e+fx)}}{3d^2}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2b(bc-3ad)\sqrt{c+d\sin(e+fx)}\right)}{3d^2\sqrt{\frac{c+d\sin(}{c+}}}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4b(bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

**Mathematica** [A]
time = 0.94, size = 173, normalized size = 0.85

$$-\frac{2\left(b^2d\cos(e+fx)(c+d\sin(e+fx))-2b(c+d)(bc-3ad)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+\left(-6abcd+3a^2d^2+b^2(2c^2+d^2)\right)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b^2*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*b*(c + d)*(b*c - 3*a*d)*El
lipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)] + (-6*a*b*c*d + 3*a^2*d^2 + b^2*(2*c^2 + d^2))*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(253) = 506$.
time = 13.16, size = 695, normalized size = 3.42

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(b^2\left(-\frac{2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{3d}+\frac{2\left(\frac{c}{d}\right.}{}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*(-2/3/d*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+4*a*b*(
1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*
c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*(1/d*c-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2
)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 485, normalized size = 2.39

$$-\frac{6\sqrt{d\sin\left(fx+e\right)+c}\,b^2d^2\cos\left(fx+e\right)-\sqrt{2}\left(4b^2c^2-12abcd+3\left(3a^2+b^2\right)d^2\right)\sqrt{id}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)-\sqrt{2}\left(4b^2c^2-12abcd+3\left(3a^2+b^2\right)d^2\right)\sqrt{-id}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)+6\sqrt{2}\left(-ib^2cd+3iabd^2\right)\sqrt{id}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(8ic^3-9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)-3id\sin\left(fx+e\right)-2ic}{3d}\right)\right)+6\sqrt{2}\left(ib^2cd-3iabd^2\right)\sqrt{-id}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4c^2-3d^2\right)}{3d^2},-\frac{8\left(-8ic^3+9icd^2\right)}{27d^3},\frac{3d\cos\left(fx+e\right)+3id\sin\left(fx+e\right)+2ic}{3d}\right)\right)}{9d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*sqrt(d*sin(f*x + e) + c)*b^2*d^2*cos(f*x + e) - sqrt(2)*(4*b^2*c^2
- 12*a*b*c*d + 3*(3*a^2 + b^2)*d^2)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c
^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3
*I*d*sin(f*x + e) - 2*I*c)/d) - sqrt(2)*(4*b^2*c^2 - 12*a*b*c*d + 3*(3*a^2
+ b^2)*d^2)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*
```

```
(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*
I*c)/d) + 6*sqrt(2)*(-I*b^2*c*d + 3*I*a*b*d^2)*sqrt(I*d)*weierstrassZeta(-4
/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInvers
e(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f
*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 6*sqrt(2)*(I*b^2*c*d - 3*I*a*b*
d^2)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 +
 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*
c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d
)))/(d^3*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin{(e+fx)})^2}{\sqrt{c+d\sin{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**2/sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.734 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=228

$$\frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}+\frac{2\left(2b^2c^2-2abcd+\left(a^2-b^2\right)d^2\right)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d^2\left(c^2-d^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] 2*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)-2*(2*b^2*c^2
-2*a*b*c*d+(a^2-b^2)*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(
c+d*sin(f*x+e))^(1/2)/d^2/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4*b*(-
a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.20, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\left(d^2\left(a^2-b^2\right)-2abcd+2b^2c^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) +
 (2*(2*b^2*c^2 - 2*a*b*c*d + (a^2 - b^2)*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) - (4*b*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d^2*f*Sqrt[c + d*Sin[e + f*
x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{d\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}d(a^2c+b^2c-2abd)+\frac{1}{2}(2b^2c^2-2abcd+(a^2-b^2)d^2)}{\sqrt{c+d\sin(e+fx)}}}{d\,(c^2-d^2)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{(2b(bc-ad))\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} -$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{\left((2abcd-a^2d^2-b^2(2c^2-d^2))\sqrt{c+d\mathrm{si}}\right.}{d^2\,(c^2-d^2)\sqrt{\frac{c}{-}}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} - \frac{2(2abcd-a^2d^2-b^2(2c^2-d^2))\,E\left(\frac{1}{2}\left(e-\right.\right.}{d^2\,(c^2-d^2)\,f\sqrt{\frac{c+d}{}}}$$

**Mathematica [A]**
time = 0.93, size = 172, normalized size = 0.75

$$\frac{2\left(\frac{(bc-ad)^2\cos(e+fx)}{c^2-d^2}+\frac{\left((2abcd-a^2d^2+b^2(-2c^2+d^2))E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+2b(c-d)(bc-ad)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(c-d)d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((b*c - a*d)^2*Cos[e + f*x])/(c^2 - d^2) + (((2*a*b*c*d - a^2*d^2 + b^2
*(-2*c^2 + d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + 2*b*(c -
 d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((c - d)*d)))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(284) = 568.
time = 17.31, size = 888, normalized size = 3.89

| method | result |
|---|---|

| | |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(\frac{b\left(\frac{2bd\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{\frac{d(1-\sin(fx+e))}{c+d}}\sqrt{\frac{(-\sin(fx+e}{c-d}}}{\sqrt{-(-d}}\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d^2*(2*b*d*(1/d*c-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+4*a*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))-2*b*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 810, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(d*sin(f*x + e) + c)*cos(f
*x + e) - (sqrt(2)*(4*b^2*c^3*d - 4*a*b*c^2*d^2 + 6*a*b*d^4 - (a^2 + 5*b^2)
*c*d^3)*sin(f*x + e) + sqrt(2)*(4*b^2*c^4 - 4*a*b*c^3*d + 6*a*b*c*d^3 - (a^
2 + 5*b^2)*c^2*d^2))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
e) - 2*I*c)/d) - (sqrt(2)*(4*b^2*c^3*d - 4*a*b*c^2*d^2 + 6*a*b*d^4 - (a^2 +
 5*b^2)*c*d^3)*sin(f*x + e) + sqrt(2)*(4*b^2*c^4 - 4*a*b*c^3*d + 6*a*b*c*d^
3 - (a^2 + 5*b^2)*c^2*d^2))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*
d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*s
in(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(-2*I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*
(a^2 - b^2)*d^4)*sin(f*x + e) + sqrt(2)*(-2*I*b^2*c^3*d + 2*I*a*b*c^2*d^2 -
 I*(a^2 - b^2)*c*d^3))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2,
-8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x
+ e) - 2*I*c)/d)) + 3*(sqrt(2)*(2*I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(a^2 -
b^2)*d^4)*sin(f*x + e) + sqrt(2)*(2*I*b^2*c^3*d - 2*I*a*b*c^2*d^2 + I*(a^2
- b^2)*c*d^3))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e)
+ 2*I*c)/d)))/((c^2*d^4 - d^6)*f*sin(f*x + e) + (c^3*d^3 - c*d^5)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^2}{(c+d\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2), x)
```

## 3.735 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=329

$$\frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}-\frac{4(bc-ad)\left(2acd+b(c^2-3d^2)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}-\frac{4(bc-ad)\left(2acd+b(c^2-\right.}{3d^2\left(c\right.}$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)-4/3*(-a*d+
b*c)*(2*a*c*d+b*(c^2-3*d^2))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1
/2)+4/3*(-a*d+b*c)*(2*a*c*d+b*(c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-2/3*(2*a*b*c*d-a^2*d^2+b^2*(2*c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/(c^2-d^2)/f/
(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2869, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\left(-a^2d^2+2abcd+b^2(2c^2-3d^2)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}-\frac{4\left(2acd+b(c^2-3d^2)\right)(bc-ad)\cos(e+fx)}{3df\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{4\left(2acd+b(c^2-3d^2)\right)(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2
)) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[e + f*x])/(3*d*(c^2 - d
^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^
2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(3*d^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(2*a*b*c*d
- a^2*d^2 + b^2*(2*c^2 - 3*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^2*(c^2 - d^2)*f*Sqrt[c + d*Sin[
e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}d(a^2c+b^2c-2abd)+\frac{1}{2}(2b^2c^2+2abcd-(a^2+3b^2)}{(c+d\sin(e+fx))^{3/2}}}{3d\,(c^2-d^2)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\,(2acd+b(c^2-3d^2))\cos(e+}{3d\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\,(2acd+b(c^2-3d^2))\cos(e+}{3d\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\,(2acd+b(c^2-3d^2))\cos(e+}{3d\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\,(2acd+b(c^2-3d^2))\cos(e+}{3d\,(c^2-d^2)^2\,f\sqrt{c+d\sin(e+fx)}}$$

**Mathematica [A]**
time = 2.66, size = 302, normalized size = 0.92

$$\frac{2\left(\frac{\left(d^2(-8abcd+a^2(3c^2+d^2)+b^2(c^2+3d^2))F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-2(-2a^2cd^2+abd(c^2+3d^2)+b^2(c^3-3cd^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(c-d)^2(c+d)^2}-\frac{d(-bc+ad)\cos(e+fx)\left(-bc^3-5ac^2d+5bcd^2+ad^3-2d(2acd+b(c^2-3d^2))\sin(e+fx)\right)}{(c^2-d^2)^2}\right)}{3d^2f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-8*a*b*c*d + a^2*(3*c^2 + d^2) + b^2*(c^2 + 3*d^2))*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - 2*(-2*a^2*c*d^2 + a*b*d*(c^2 + 3*d^2)
 + b^2*(c^3 - 3*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*Sin[e +
 f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d*(-(b*
c) + a*d)*Cos[e + f*x]*(-(b*c^3) - 5*a*c^2*d + 5*b*c*d^2 + a*d^3 - 2*d*(2*a
*c*d + b*(c^2 - 3*d^2))*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^2*f*(c + d*Sin[
e + f*x])^(3/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. 2(375) = 750.
time = 26.65, size = 1043, normalized size = 3.17

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1043 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^2/d^2*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+2*b/d^2*(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2-d^
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c
^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*
d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c
^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 1422, normalized size = 4.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(4*b^2*c^4*d^2 + 4*a*b*c^3*d^3 - 12*a*b*c*d^5 + (a^2 - 9*b^2)
*c^2*d^4 + 3*(a^2 + 3*b^2)*d^6)*cos(f*x + e)^2 - 2*sqrt(2)*(4*b^2*c^5*d + 4
*a*b*c^4*d^2 - 12*a*b*c^2*d^4 + (a^2 - 9*b^2)*c^3*d^3 + 3*(a^2 + 3*b^2)*c*d
^5)*sin(f*x + e) - sqrt(2)*(4*b^2*c^6 + 4*a*b*c^5*d - 8*a*b*c^3*d^3 + 4*a^2
*c^2*d^4 - 12*a*b*c*d^5 + (a^2 - 5*b^2)*c^4*d^2 + 3*(a^2 + 3*b^2)*d^6))*sqr
t(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c
*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2
)*(4*b^2*c^4*d^2 + 4*a*b*c^3*d^3 - 12*a*b*c*d^5 + (a^2 - 9*b^2)*c^2*d^4 + 3
*(a^2 + 3*b^2)*d^6)*cos(f*x + e)^2 - 2*sqrt(2)*(4*b^2*c^5*d + 4*a*b*c^4*d^2
 - 12*a*b*c^2*d^4 + (a^2 - 9*b^2)*c^3*d^3 + 3*(a^2 + 3*b^2)*c*d^5)*sin(f*x
+ e) - sqrt(2)*(4*b^2*c^6 + 4*a*b*c^5*d - 8*a*b*c^3*d^3 + 4*a^2*c^2*d^4 - 1
2*a*b*c*d^5 + (a^2 - 5*b^2)*c^4*d^2 + 3*(a^2 + 3*b^2)*d^6))*sqrt(-I*d)*weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3,
 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 6*(sqrt(2)*(I*b^2
*c^3*d^3 + I*a*b*c^2*d^4 + 3*I*a*b*d^6 - I*(2*a^2 + 3*b^2)*c*d^5)*cos(f*x +
 e)^2 + 2*sqrt(2)*(-I*b^2*c^4*d^2 - I*a*b*c^3*d^3 - 3*I*a*b*c*d^5 + I*(2*a^
2 + 3*b^2)*c^2*d^4)*sin(f*x + e) + sqrt(2)*(-I*b^2*c^5*d - I*a*b*c^4*d^2 -
4*I*a*b*c^2*d^4 - 3*I*a*b*d^6 + 2*I*(a^2 + b^2)*c^3*d^3 + I*(2*a^2 + 3*b^2)
*c*d^5))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3
 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I
*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/
d)) + 6*(sqrt(2)*(-I*b^2*c^3*d^3 - I*a*b*c^2*d^4 - 3*I*a*b*d^6 + I*(2*a^2 +
 3*b^2)*c*d^5)*cos(f*x + e)^2 + 2*sqrt(2)*(I*b^2*c^4*d^2 + I*a*b*c^3*d^3 +
3*I*a*b*c*d^5 - I*(2*a^2 + 3*b^2)*c^2*d^4)*sin(f*x + e) + sqrt(2)*(I*b^2*c^
5*d + I*a*b*c^4*d^2 + 4*I*a*b*c^2*d^4 + 3*I*a*b*d^6 - 2*I*(a^2 + b^2)*c^3*d
^3 - I*(2*a^2 + 3*b^2)*c*d^5))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d
^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2
- 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I
*d*sin(f*x + e) + 2*I*c)/d)) + 6*(2*(b^2*c^3*d^3 + a*b*c^2*d^4 + 3*a*b*d^6
- (2*a^2 + 3*b^2)*c*d^5)*cos(f*x + e)*sin(f*x + e) + (b^2*c^4*d^2 + 4*a*b*c
^3*d^3 + 4*a*b*c*d^5 + a^2*d^6 - 5*(a^2 + b^2)*c^2*d^4)*cos(f*x + e))*sqrt(
d*sin(f*x + e) + c))/((c^4*d^5 - 2*c^2*d^7 + d^9)*f*cos(f*x + e)^2 - 2*(c^5
*d^4 - 2*c^3*d^6 + c*d^8)*f*sin(f*x + e) - (c^6*d^3 - c^4*d^5 - c^2*d^7 + d
^9)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.736 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=460

$$\frac{2(bc-ad)^2\cos(e+fx)}{5d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}}-\frac{4(bc-ad)\,(4acd+b(c^2-5d^2))\cos(e+fx)}{15d\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}}+\frac{2(a^2d^2(23c^2+9d^2)-ab(}{15d}$$

```
[Out] 2/5*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(5/2)-4/15*(-a*d
+b*c)*(4*a*c*d+b*(c^2-5*d^2))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(
3/2)+2/15*(a^2*d^2*(23*c^2+9*d^2)-a*b*(6*c^3*d+58*c*d^3)-b^2*(2*c^4-19*c^2*
d^2-15*d^4))*cos(f*x+e)/d/(c^2-d^2)^3/f/(c+d*sin(f*x+e))^(1/2)-2/15*(a^2*d^
2*(23*c^2+9*d^2)-a*b*(6*c^3*d+58*c*d^3)-b^2*(2*c^4-19*c^2*d^2-15*d^4))*(sin
(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2
*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c^2
-d^2)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*(-a*d+b*c)*(4*a*c*d+b*(c^2-5*
d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.56, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2869, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(a^2d^2(23c^2+9d^2)-ab(6c^3d+58cd^3)-(b^2(2c^4-19c^2d^2-15d^4)))\cos(e+fx)}{15df\,(c^2-d^2)^3\sqrt{c+d\sin(e+fx)}}+\frac{2(a^2d^2(23c^2+9d^2)-ab(6c^3d+58cd^3)-(b^2(2c^4-19c^2d^2-15d^4)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\,(c^2-d^2)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{5df\,(c^2-d^2)\,(c+d\sin(e+fx))^{5/2}}-\frac{4(4acd+b(c^2-5d^2))\,(bc-ad)\cos(e+fx)}{15df\,(c^2-d^2)^2\,(c+d\sin(e+fx))^{3/2}}+\frac{4(4acd+b(c^2-5d^2))\,(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\,(c^2-d^2)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(5*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2
)) - (4*(b*c - a*d)*(4*a*c*d + b*(c^2 - 5*d^2))*Cos[e + f*x])/(15*d*(c^2 -
d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) + (2*(a^2*d^2*(23*c^2 + 9*d^2) - a*b*(
6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*Cos[e + f*x])/(15*
d*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*d^2*(23*c^2 + 9*d^2)
- a*b*(6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*(c^2 -
d^2)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(b*c - a*d)*(4*a*c*d + b*
(c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(15*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
```

```
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\frac{5}{2}d\left(\left(a^2+b^2\right)c-2abd\right)+\frac{1}{2}\left(6abcd-3a^2d^2+b^2\left(2c^2-\right.\right.}{(c+d\sin(e+fx))^{5/2}}}{5d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b(c^2-5d^2)\right)\cos(e+}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b(c^2-5d^2)\right)\cos(e+}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b(c^2-5d^2)\right)\cos(e+}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b(c^2-5d^2)\right)\cos(e+}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b(c^2-5d^2)\right)\cos(e+}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/}}$$

**Mathematica [A]**
time = 5.36, size = 424, normalized size = 0.92

$$\frac{2\left(-\frac{\left(d^2\left(-2abd\left(27c^2+5d^2\right)+b^2c\left(7c^2+25d^2\right)+a^2\left(15c^3+17cd^2\right)\right)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-\left(-a^2d^2\left(23c^2+9d^2\right)+ab\left(6c^3d+58cd^3\right)+b^2\left(2c^4-19c^2d^2-15d^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{5/2}}{(c-d)^3(c+d)}+\frac{d\cos(e+fx)\left(3(bc-ad)^2\left(c^2-d^2\right)^2-2\left(c^2-d^2\right)\left(-4a^2cd^2+abd\left(3c^2+5d^2\right)+b^2\left(c^3-5cd^2\right)\right)(c+d\sin(e+fx))-\left(-a^2d^2\left(23c^2+9d^2\right)+ab\left(6c^3d+58cd^3\right)+b^2\left(2c^4-19c^2d^2-15d^4\right)\right)(c+d\sin(e+fx))^2\right)}{\left(c^2-d^2\right)^3}\right)}{15d^2f(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(-(((d^2*(-2*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(7*c^2 + 25*d^2) + a^2*(15*c
^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - (-(a^2*d^
2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 58*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 -
15*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*Ellip
ticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))
^(5/2))/((c - d)^3*(c + d))) + (d*Cos[e + f*x]*(3*(b*c - a*d)^2*(c^2 - d^2)
^2 - 2*(c^2 - d^2)*(-4*a^2*c*d^2 + a*b*d*(3*c^2 + 5*d^2) + b^2*(c^3 - 5*c*d
```

```
^2))*(c + d*Sin[e + f*x]) - (-(a^2*d^2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 5
8*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*(c + d*Sin[e + f*x])^2))/(c^2
 - d^2)^3))/(15*d^2*f*(c + d*Sin[e + f*x])^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(502) = 1004$.
time = 42.31, size = 1450, normalized size = 3.15

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1450 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2/d^2*(2*d*cos(f*x+e)^2/(c^2-d^2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2/5/(c^2-
d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^3+16/15
*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)
^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(1
/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)
/(c^2-d^2)^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*b*(
a*d-b*c)/d^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(
f*x+e)+1/d*c)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 2321, normalized size = 5.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/45*((3*sqrt(2)*(4*b^2*c^6*d^2 + 12*a*b*c^5*d^3 - 46*a*b*c^3*d^5 - 30*a*b*
c*d^7 - (a^2 + 17*b^2)*c^4*d^4 + 3*(11*a^2 + 15*b^2)*c^2*d^6)*cos(f*x + e)^
2 + (sqrt(2)*(4*b^2*c^5*d^3 + 12*a*b*c^4*d^4 - 46*a*b*c^2*d^6 - 30*a*b*d^8
- (a^2 + 17*b^2)*c^3*d^5 + 3*(11*a^2 + 15*b^2)*c*d^7)*cos(f*x + e)^2 - sqrt
(2)*(12*b^2*c^7*d + 36*a*b*c^6*d^2 - 126*a*b*c^4*d^4 - 136*a*b*c^2*d^6 - 30
*a*b*d^8 - (3*a^2 + 47*b^2)*c^5*d^3 + 2*(49*a^2 + 59*b^2)*c^3*d^5 + 3*(11*a
^2 + 15*b^2)*c*d^7))*sin(f*x + e) - sqrt(2)*(4*b^2*c^8 + 12*a*b*c^7*d - 10*
a*b*c^5*d^3 - 168*a*b*c^3*d^5 - 90*a*b*c*d^7 - (a^2 + 5*b^2)*c^6*d^2 + 6*(5
*a^2 - b^2)*c^4*d^4 + 9*(11*a^2 + 15*b^2)*c^2*d^6))*sqrt(I*d)*weierstrassPI
nverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*
cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (3*sqrt(2)*(4*b^2*c^6*d^2 +
 12*a*b*c^5*d^3 - 46*a*b*c^3*d^5 - 30*a*b*c*d^7 - (a^2 + 17*b^2)*c^4*d^4 +
3*(11*a^2 + 15*b^2)*c^2*d^6)*cos(f*x + e)^2 + (sqrt(2)*(4*b^2*c^5*d^3 + 12*
a*b*c^4*d^4 - 46*a*b*c^2*d^6 - 30*a*b*d^8 - (a^2 + 17*b^2)*c^3*d^5 + 3*(11*
a^2 + 15*b^2)*c*d^7)*cos(f*x + e)^2 - sqrt(2)*(12*b^2*c^7*d + 36*a*b*c^6*d^
2 - 126*a*b*c^4*d^4 - 136*a*b*c^2*d^6 - 30*a*b*d^8 - (3*a^2 + 47*b^2)*c^5*d
^3 + 2*(49*a^2 + 59*b^2)*c^3*d^5 + 3*(11*a^2 + 15*b^2)*c*d^7))*sin(f*x + e)
 - sqrt(2)*(4*b^2*c^8 + 12*a*b*c^7*d - 10*a*b*c^5*d^3 - 168*a*b*c^3*d^5 - 9
0*a*b*c*d^7 - (a^2 + 5*b^2)*c^6*d^2 + 6*(5*a^2 - b^2)*c^4*d^4 + 9*(11*a^2 +
 15*b^2)*c^2*d^6))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2,
 -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x +
e) + 2*I*c)/d) + 3*(3*sqrt(2)*(2*I*b^2*c^5*d^3 + 6*I*a*b*c^4*d^4 + 58*I*a*b
*c^2*d^6 - I*(23*a^2 + 19*b^2)*c^3*d^5 - 3*I*(3*a^2 + 5*b^2)*c*d^7)*cos(f*x
 + e)^2 + (sqrt(2)*(2*I*b^2*c^4*d^4 + 6*I*a*b*c^3*d^5 + 58*I*a*b*c*d^7 - I*
(23*a^2 + 19*b^2)*c^2*d^6 - 3*I*(3*a^2 + 5*b^2)*d^8)*cos(f*x + e)^2 + sqrt(
2)*(-6*I*b^2*c^6*d^2 - 18*I*a*b*c^5*d^3 - 180*I*a*b*c^3*d^5 - 58*I*a*b*c*d^
7 + I*(69*a^2 + 55*b^2)*c^4*d^4 + 2*I*(25*a^2 + 32*b^2)*c^2*d^6 + 3*I*(3*a^
```

```
2 + 5*b^2)*d^8))*sin(f*x + e) + sqrt(2)*(-2*I*b^2*c^7*d - 6*I*a*b*c^6*d^2 -
 76*I*a*b*c^4*d^4 - 174*I*a*b*c^2*d^6 + I*(23*a^2 + 13*b^2)*c^5*d^3 + 6*I*(
13*a^2 + 12*b^2)*c^3*d^5 + 9*I*(3*a^2 + 5*b^2)*c*d^7))*sqrt(I*d)*weierstras
sZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstras
sPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3
*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(3*sqrt(2)*(-2*I*b^2*
c^5*d^3 - 6*I*a*b*c^4*d^4 - 58*I*a*b*c^2*d^6 + I*(23*a^2 + 19*b^2)*c^3*d^5
+ 3*I*(3*a^2 + 5*b^2)*c*d^7)*cos(f*x + e)^2 + (sqrt(2)*(-2*I*b^2*c^4*d^4 -
6*I*a*b*c^3*d^5 - 58*I*a*b*c*d^7 + I*(23*a^2 + 19*b^2)*c^2*d^6 + 3*I*(3*a^2
 + 5*b^2)*d^8)*cos(f*x + e)^2 + sqrt(2)*(6*I*b^2*c^6*d^2 + 18*I*a*b*c^5*d^3
 + 180*I*a*b*c^3*d^5 + 58*I*a*b*c*d^7 - I*(69*a^2 + 55*b^2)*c^4*d^4 - 2*I*(
25*a^2 + 32*b^2)*c^2*d^6 - 3*I*(3*a^2 + 5*b^2)*d^8))*sin(f*x + e) + sqrt(2)
*(2*I*b^2*c^7*d + 6*I*a*b*c^6*d^2 + 76*I*a*b*c^4*d^4 + 174*I*a*b*c^2*d^6 -
I*(23*a^2 + 13*b^2)*c^5*d^3 - 6*I*(13*a^2 + 12*b^2)*c^3*d^5 - 9*I*(3*a^2 +
5*b^2)*c*d^7))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e)
+ 2*I*c)/d)) - 6*((2*b^2*c^4*d^4 + 6*a*b*c^3*d^5 + 58*a*b*c*d^7 - (23*a^2 +
 19*b^2)*c^2*d^6 - 3*(3*a^2 + 5*b^2)*d^8)*cos(f*x + e)^3 - 2*(3*b^2*c^5*d^3
 + 9*a*b*c^4*d^4 + 60*a*b*c^2*d^6 - 5*a*b*d^8 - (27*a^2 + 25*b^2)*c^3*d^5 -
 5*(a^2 + 2*b^2)*c*d^7)*cos(f*x + e)*sin(f*x + e) - (b^2*c^6*d^2 + 18*a*b*c
^5*d^3 + 56*a*b*c^3*d^5 + 54*a*b*c*d^7 - (34*a^2 + 23*b^2)*c^4*d^4 - 9*(2*a
^2 + 3*b^2)*c^2*d^6 - 3*(4*a^2 + 5*b^2)*d^8)*cos(f*x + e))*sqrt(d*sin(f*x +
 e) + c))/(3*(c^7*d^5 - 3*c^5*d^7 + 3*c^3*d^9 - c*d^11)*f*cos(f*x + e)^2 -
(c^9*d^3 - 6*c^5*d^7 + 8*c^3*d^9 - 3*c*d^11)*f + ((c^6*d^6 - 3*c^4*d^8 + 3*
c^2*d^10 - d^12)*f*cos(f*x + e)^2 - (3*c^8*d^4 - 8*c^6*d^6 + 6*c^4*d^8 - d^
12)*f)*sin(f*x + e))
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \sin(e + f x))^2}{(c + d \sin(e + f x))^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.737 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=642

$$-\frac{2(1848a^3cd^3+495a^2bd^2(3c^2+5d^2)-66ab^2d(5c^3-57cd^2)+5b^3(8c^4+57c^2d^2+135d^4))\cos(e+fx)\sqrt{c\cdot}}{3465d^2f}$$

```
[Out] -2/3465*(1485*a^2*b*c*d^2+693*a^3*d^3-33*a*b^2*d*(10*c^2-49*d^2)+5*b^3*(8*c
^3+67*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f+2/693*b*(66*a*b*c*d-2
97*a^2*d^2-b^2*(8*c^2+81*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f+8/99
*b^2*(-6*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d^2/f-2/11*b^2*cos(f*x+
e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(7/2)/d/f-2/3465*(1848*a^3*c*d^3+495*a
^2*b*d^2*(3*c^2+5*d^2)-66*a*b^2*d*(5*c^3-57*c*d^2)+5*b^3*(8*c^4+57*c^2*d^2+
135*d^4))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/3465*(231*a^3*d^3*(23*c
^2+9*d^2)+495*a^2*b*c*d^2*(3*c^2+29*d^2)-33*a*b^2*d*(10*c^4-279*c^2*d^2-147
*d^4)+5*b^3*(8*c^5+51*c^3*d^2+741*c*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/
2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d
/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+
2/3465*(c^2-d^2)*(1848*a^3*c*d^3+495*a^2*b*d^2*(3*c^2+5*d^2)-66*a*b^2*d*(5*
c^3-57*c*d^2)+5*b^3*(8*c^4+57*c^2*d^2+135*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^
2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1
/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^
(1/2)
```

**Rubi** [A]
time = 0.92, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(1848*a^3*c*d^3 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 5
7*c*d^2) + 5*b^3*(8*c^4 + 57*c^2*d^2 + 135*d^4))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3465*d^2*f) - (2*(1485*a^2*b*c*d^2 + 693*a^3*d^3 - 33*a*b^2*d
*(10*c^2 - 49*d^2) + 5*b^3*(8*c^3 + 67*c*d^2))*Cos[e + f*x]*(c + d*Sin[e +
f*x])^(3/2))/(3465*d^2*f) + (2*b*(66*a*b*c*d - 297*a^2*d^2 - b^2*(8*c^2 + 8
1*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d^2*f) + (8*b^2*(b*c
- 6*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f) - (2*b^2*Cos[e
 + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(11*d*f) + (2*(231
*a^3*d^3*(23*c^2 + 9*d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) - 33*a*b^2*d*(
```

```
10*c^4 - 279*c^2*d^2 - 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3465
*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(1848*a^3*c*d^3
 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 57*c*d^2) + 5*b^3*(8
*c^4 + 57*c^2*d^2 + 135*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3465*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{7/2}}{11df}$$

$$= \frac{8b^2(bc-6ad)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f} - \frac{2b^2\cos(}{}$$

$$= \frac{2b(66abcd-297a^2d^2-b^2(8c^2+81d^2))\cos(e+fx)(c+d\,\mathrm{s}}{693d^2f}$$

$$= -\frac{2(1485a^2bcd^2+693a^3d^3-33ab^2d(10c^2-49d^2)+5b^3(8c}{3465d^2f}$$

$$= -\frac{2(1848a^3cd^3+495a^2bd^2(3c^2+5d^2)-66ab^2d(5c^3-57cd}{3}$$

$$= -\frac{2(1848a^3cd^3+495a^2bd^2(3c^2+5d^2)-66ab^2d(5c^3-57cd}{3}$$

$$= -\frac{2(1848a^3cd^3+495a^2bd^2(3c^2+5d^2)-66ab^2d(5c^3-57cd}{3}$$

$$= -\frac{2(1848a^3cd^3+495a^2bd^2(3c^2+5d^2)-66ab^2d(5c^3-57cd}{3}$$

**<u>Mathematica</u>** **[A]**
time = 2.74, size = 545, normalized size = 0.85

$$\frac{-16\left(d^2(495a^2bd^2(27c^2+5d^2)+231a^3cd(15c^2+17d^2)+33ab^2d(155c^3+261cd^2)+5b^3(2c^4+663c^2d^2+135d^4))F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+(231a^3d^3(23c^2+9d^2)+495a^2bcd^2(3c^2+29d^2)+33ab^2d(-10c^4+279c^2d^2+147d^4)+5b^3(8c^5+51c^3d^2+741cd^4))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d(c+d\sin(e+fx))(2(-20328a^3cd^3-990a^2bd^2(36c^2+23d^2)-66ab^2d(20c^3+747cd^2)+5b^3(32c^4-1866c^2d^2-1305d^4))\cos(e+fx)+5bd^2(2508abcd+1188a^2d^2+b^2(452c^2+513d^2))\cos(3(e+fx))-315b^3d^4\cos(5(e+fx))-4d(8910a^2bcd^2+1386a^3d^3+33ab^2d(150c^2+133d^2)+5b^3(6c^3+619cd^2))\sin(2(e+fx))+70b^2d^3(23bc+33ad)\sin(4(e+fx)))}{27720d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-16*(d^2*(495*a^2*b*d^2*(27*c^2 + 5*d^2) + 231*a^3*c*d*(15*c^2 + 17*d^2) +
 33*a*b^2*d*(155*c^3 + 261*c*d^2) + 5*b^3*(2*c^4 + 663*c^2*d^2 + 135*d^4))*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (231*a^3*d^3*(23*c^2 + 9*
d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) + 33*a*b^2*d*(-10*c^4 + 279*c^2*d^2
 + 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*
```

```
(2*(-20328*a^3*c*d^3 - 990*a^2*b*d^2*(36*c^2 + 23*d^2) - 66*a*b^2*d*(20*c^3
 + 747*c*d^2) + 5*b^3*(32*c^4 - 1866*c^2*d^2 - 1305*d^4))*Cos[e + f*x] + 5*
b*d^2*(2508*a*b*c*d + 1188*a^2*d^2 + b^2*(452*c^2 + 513*d^2))*Cos[3*(e + f*
x)] - 315*b^3*d^4*Cos[5*(e + f*x)] - 4*d*(8910*a^2*b*c*d^2 + 1386*a^3*d^3 +
 33*a*b^2*d*(150*c^2 + 133*d^2) + 5*b^3*(6*c^3 + 619*c*d^2))*Sin[2*(e + f*x
)] + 70*b^2*d^3*(23*b*c + 33*a*d)*Sin[4*(e + f*x)]))/(27720*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2727 vs. $2(676) = 1352$.
time = 43.21, size = 2728, normalized size = 4.25

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2728 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^3*(-2/11/d*sin(f*x+e)^4*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+20/99/d^2*c*sin(f*x+e)^3*(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)-2/7*(9/11+80/99/d^2*c^2)/d*sin(f*x+e)^2*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(-480*c^3-472*c*d^2)/d^4*sin(f*x+e)*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(640*c^4+596*c^2*d^2+675*d^4)
/d^5*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3465*(-320*c^4-348*c^2*d^2+6
75*d^4)/d^4*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/3465*
(-1280*c^5-1032*c^3*d^2-1146*c*d^4)/d^5*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))))+(3*a*b^2*d^3+3*b^3*c*d^2)*(-2/9/d*sin(f*x+e)^3*(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63/d^2*c*sin(f*x+e)^2*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21/d^2*c^2)/d*sin(f*x+e)*(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(1/d*c
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*
EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^3+9*a*b^2*c*d
^2+3*b^3*c^2*d)*(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)+12/35/d^2*c*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+2
```

```
4/35/d^2*c^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35/d^2*c^2+5/
21)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-
44*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^3
*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)+8/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)+4/15/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8
/15/d^2*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^
3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c
*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/
d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(3*a^3*c^2*d+3*
a^2*b*c^3)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^3*c^
3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*
sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.25, size = 1066, normalized size = 1.66

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/10395*(sqrt(2)*(80*b^3*c^6 - 660*a*b^2*c^5*d + 30*(99*a^2*b + 16*b^3)*c^4*d^2 + 33*(7*a^3 + 93*a*b^2)*c^3*d^3 - 15*(759*a^2*b + 169*b^3)*c^2*d^4 - 99*(77*a^3 + 163*a*b^2)*c*d^5 - 675*(11*a^2*b + 3*b^3)*d^6)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(80*b^3*c^6 - 660*a*b^2*c^5*d + 30*(99*a^2*b + 16*b^3)*c^4*d^2 + 33*(7*a^3 + 93*a*b^2)*c^3*d^3 - 15*(759*a^2*b + 169*b^3)*c^2*d^4 - 99*(77*a^3 + 163*a*b^2)*c*d^5 - 675*(11*a^2*b + 3*b^3)*d^6)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(40*I*b^3*c^5*d - 330*I*a*b^2*c^4*d^2 + 15*I*(99*a^2*b + 17*b^3)*c^3*d^3 + 33*I*(161*a^3 + 279*a*b^2)*c^2*d^4 + 15*I*(957*a^2*b + 247*b^3)*c*d^5 + 693*I*(3*a^3 + 7*a*b^2)*d^6)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*(-40*I*b^3*c^5*d + 330*I*a*b^2*c^4*d^2 - 15*I*(99*a^2*b + 17*b^3)*c^3*d^3 - 33*I*(161*a^3 + 279*a*b^2)*c^2*d^4 - 15*I*(957*a^2*b + 247*b^3)*c*d^5 - 693*I*(3*a^3 + 7*a*b^2)*d^6)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) + 6*(315*b^3*d^6*cos(f*x + e)^5 - 5*(113*b^3*c^2*d^4 + 627*a*b^2*c*d^5 + 9*(33*a^2*b + 23*b^3)*d^6)*cos(f*x + e)^3 - (20*b^3*c^4*d^2 - 165*a*b^2*c^3*d^3 - 15*(297*a^2*b + 106*b^3)*c^2*d^4 - 33*(77*a^3 + 258*a*b^2)*c*d^5 - 45*(88*a^2*b + 31*b^3)*d^6)*cos(f*x + e) - (35*(23*b^3*c*d^5 + 33*a*b^2*d^6)*cos(f*x + e)^3 - 3*(5*b^3*c^3*d^3 + 825*a*b^2*c^2*d^4 + 5*(297*a^2*b + 130*b^3)*c*d^5 + 231*(a^3 + 4*a*b^2)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^3 (c + d\sin(e + fx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)

[Out] Integral((a + b*sin(e + f*x))**3*(c + d*sin(e + f*x))**(5/2), x)

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))^3\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.738 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=496

$$-\frac{2(189a^2bcd^2+105a^3d^3-9ab^2d(6c^2-25d^2)+b^3(8c^3+39cd^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}+\frac{2b(54ab$$

```
[Out] 2/315*b*(54*a*b*c*d-189*a^2*d^2-b^2*(8*c^2+49*d^2))*cos(f*x+e)*(c+d*sin(f*x
+e))^(3/2)/d^2/f+8/63*b^2*(-5*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^
2/f-2/9*b^2*cos(f*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2)/d/f-2/315*(1
89*a^2*b*c*d^2+105*a^3*d^3-9*a*b^2*d*(6*c^2-25*d^2)+b^3*(8*c^3+39*c*d^2))*c
os(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/315*(420*a^3*c*d^3+189*a^2*b*d^2*(
c^2+3*d^2)-a*b^2*(54*c^3*d-738*c*d^3)+b^3*(8*c^4+33*c^2*d^2+147*d^4))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)+2/315*(c^2-d^2)*(189*a^2*b*c*d^2+105*a^3*d^3-9*
a*b^2*d*(6*c^2-25*d^2)+b^3*(8*c^3+39*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/
2)
```

**Rubi** [A]
time = 0.67, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2b(-189a^2d^2+54abcd-(b^2(8c^2+49d^2)))\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{2(105a^3d^3+189a^2bcd^2-9ab^2d(6c^2-25d^2)+b^3(8c^3+39cd^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\frac{2(c^2-d^2)(105a^3d^3+189a^2bcd^2-9ab^2d(6c^2-25d^2)+b^3(8c^3+39cd^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^3f\sqrt{c+d\sin(e+fx)}}+\frac{2(420a^3cd^3+189a^2bd^2(c^2+3d^2)-ab^2(54c^3d-738cd^3)+b^3(8c^4+33c^2d^2+147d^4))\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{315d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8b^2(bc-5ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f}-\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(189*a^2*b*c*d^2 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^
3 + 39*c*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d^2*f) + (2*b*(5
4*a*b*c*d - 189*a^2*d^2 - b^2*(8*c^2 + 49*d^2))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^(3/2))/(315*d^2*f) + (8*b^2*(b*c - 5*a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(5/2))/(63*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d
*Sin[e + f*x])^(5/2))/(9*d*f) + (2*(420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*
d^2) - a*b^2*(54*c^3*d - 738*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*
d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(189*a^2*b*c*d^2
 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^3 + 39*c*d^2))*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(315*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
```

```
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}{9df}$$

$$= \frac{8b^2(bc-5ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f} - \frac{2b^2\cos}{}$$

$$= \frac{2b(54abcd-189a^2d^2-b^2(8c^2+49d^2))\cos(e+fx)(c+d}{315d^2f}$$

$$= -\frac{2(189a^2bcd^2+105a^3d^3-9ab^2d(6c^2-25d^2)+b^3(8c^3+}{315d^2f}$$

$$= -\frac{2(189a^2bcd^2+105a^3d^3-9ab^2d(6c^2-25d^2)+b^3(8c^3+}{315d^2f}$$

$$= -\frac{2(189a^2bcd^2+105a^3d^3-9ab^2d(6c^2-25d^2)+b^3(8c^3+}{315d^2f}$$

$$= -\frac{2(189a^2bcd^2+105a^3d^3-9ab^2d(6c^2-25d^2)+b^3(8c^3+}{315d^2f}$$

**Mathematica [A]**
time = 2.51, size = 410, normalized size = 0.83

$$\frac{-8\left(d^2(756a^2bcd^2+105a^3d(3c^2+d^2)+9ab^2d(51c^2+25d^2)+2b^3(c^3+93cd^2))F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+(420a^3cd^3+189a^2bd^2(c^2+3d^2)+ab^2(-54c^3d+738cd^3)+b^3(8c^4+33c^2d^2+147d^4))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+d(c+d\sin(e+fx))\left(-2(1512a^2bcd^2+420a^3d^3+9ab^2d(12c^2+115d^2)+b^3(-16c^3+402cd^2))\cos(e+fx)+bd(10bd(10bc+27ad)\cos(3(e+fx))-2(432abcd+378a^2d^2+b^2(6c^2+133d^2)-35b^2d^2\cos(2(e+fx)))\sin(2(e+fx)))\right)}{1260d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*(d^2*(756*a^2*b*c*d^2 + 105*a^3*d*(3*c^2 + d^2) + 9*a*b^2*d*(51*c^2 + 2
5*d^2) + 2*b^3*(c^3 + 93*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] + (420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*d^2) + a*b^2*(-54*c^3*d + 7
38*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*((c + d)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*(-2*(1
512*a^2*b*c*d^2 + 420*a^3*d^3 + 9*a*b^2*d*(12*c^2 + 115*d^2) + b^3*(-16*c^3
 + 402*c*d^2))*Cos[e + f*x] + b*d*(10*b*d*(10*b*c + 27*a*d)*Cos[3*(e + f*x)
] - 2*(432*a*b*c*d + 378*a^2*d^2 + b^2*(6*c^2 + 133*d^2) - 35*b^2*d^2*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)])))/(1260*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(534) = 1068$.
time = 32.96, size = 2112, normalized size = 4.26

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2112 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^2*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63/d^2*c*sin(f*x+e)^2*(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21/d^2*c^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(1/d*
c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a*b^2*d^2+2*b^3*c*d)
*(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35/d^2*c*s
in(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35/d^2*c^2)/d
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35/d^2*c^2+5/21)*(1/d*c-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
```

```
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(
1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*
c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^2+6*a*b^
2*c*d+b^3*c^2)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8
/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15/d*c*(1/d*c-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15/d^2*c^2)*(1/d*c-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2
)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+2*(2*a^3*c*d+3*a^2*b*c^2)*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^3*c^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 895, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/945*(sqrt(2)*(16*b^3*c^5 - 108*a*b^2*c^4*d + 6*(63*a^2*b + 10*b^3)*c^3*d
^2 - 3*(35*a^3 - 33*a*b^2)*c^2*d^3 - 6*(189*a^2*b + 44*b^3)*c*d^4 - 45*(7*a
^3 + 15*a*b^2)*d^5)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2,
 -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e
) - 2*I*c)/d) + sqrt(2)*(16*b^3*c^5 - 108*a*b^2*c^4*d + 6*(63*a^2*b + 10*b^
3)*c^3*d^2 - 3*(35*a^3 - 33*a*b^2)*c^2*d^3 - 6*(189*a^2*b + 44*b^3)*c*d^4 -
 45*(7*a^3 + 15*a*b^2)*d^5)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*
d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*s
in(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(8*I*b^3*c^4*d - 54*I*a*b^2*c^3*d^2 + 3
*I*(63*a^2*b + 11*b^3)*c^2*d^3 + 6*I*(70*a^3 + 123*a*b^2)*c*d^4 + 21*I*(27*
a^2*b + 7*b^3)*d^5)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/
27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2,
 -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e
) - 2*I*c)/d)) + 3*sqrt(2)*(-8*I*b^3*c^4*d + 54*I*a*b^2*c^3*d^2 - 3*I*(63*a
^2*b + 11*b^3)*c^2*d^3 - 6*I*(70*a^3 + 123*a*b^2)*c*d^4 - 21*I*(27*a^2*b +
7*b^3)*d^5)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*
I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27
*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2
*I*c)/d)) - 6*(5*(10*b^3*c*d^4 + 27*a*b^2*d^5)*cos(f*x + e)^3 + (4*b^3*c^3*
d^2 - 27*a*b^2*c^2*d^3 - 6*(63*a^2*b + 23*b^3)*c*d^4 - 15*(7*a^3 + 24*a*b^2
)*d^5)*cos(f*x + e) + (35*b^3*d^5*cos(f*x + e)^3 - 3*(b^3*c^2*d^3 + 72*a*b^
2*c*d^4 + 7*(9*a^2*b + 4*b^3)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f
*x + e) + c))/(d^4*f)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^3 (c + d\sin(e + fx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))^3\,(c+d\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2), x)
```

## 3.739 $\int (a+b\sin(e+fx))^3 \sqrt{c+d\sin(e+fx)}\, dx$

**Optimal**. Leaf size=375

$$\frac{2b(42abcd-105a^2d^2-b^2(8c^2+25d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8b^2(bc-4ad)\cos(e+fx)(c+d\sin}{35d^2f}$$

```
[Out] 8/35*b^2*(-4*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f-2/7*b^2*cos(f
*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/d/f+2/105*b*(42*a*b*c*d-105*a
^2*d^2-b^2*(8*c^2+25*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/105*(1
05*a^2*b*c*d^2+105*a^3*d^3-21*a*b^2*d*(2*c^2-9*d^2)+b^3*(8*c^3+19*c*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f
/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/105*b*(c^2-d^2)*(42*a*b*c*d-105*a^2*d^2-b
^2*(8*c^2+25*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2
*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*si
n(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.44, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2b(-105a^2d^2+42abcd-(b^2(8c^2+25d^2)))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{2b(c^2-d^2)\left(-105a^2d^2+42abcd-(b^2(8c^2+25d^2))\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\sqrt{c+d\sin(e+fx)}}+\frac{2(105a^3d^3+105a^2bcd^2-21ab^2d(2c^2-9d^2)+b^3(8c^3+19cd^2))\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8b^2(bc-4ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*b*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^2 + 25*d^2))*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(105*d^2*f) + (8*b^2*(b*c - 4*a*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(35*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(
c + d*Sin[e + f*x])^(3/2))/(7*d*f) + (2*(105*a^2*b*c*d^2 + 105*a^3*d^3 - 21
*a*b^2*d*(2*c^2 - 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) + (2*b*(c^2 - d^2)*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^
2 + 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
```

```
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{7df} +$$

$$= \frac{8b^2(bc-4ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f} - \frac{2b^2\cos(e}{}$$

$$= \frac{2b(42abcd-105a^2d^2-b^2(8c^2+25d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{105d^2f}$$

$$= \frac{2b(42abcd-105a^2d^2-b^2(8c^2+25d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{105d^2f}$$

$$= \frac{2b(42abcd-105a^2d^2-b^2(8c^2+25d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{105d^2f}$$

$$= \frac{2b(42abcd-105a^2d^2-b^2(8c^2+25d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{105d^2f}$$

**Mathematica** [A]
time = 1.51, size = 306, normalized size = 0.82

$$\frac{-4\left(d^2(105a^3cd+147ab^2cd+105a^2bd^2+b^3(2c^2+25d^2))F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+(105a^2bcd^2+105a^3d^3+21ab^2d(-2c^2+9d^2)+b^3(8c^3+19cd^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}+bd(c+d\sin(e+fx))\left((-84abcd-420a^2d^2+b^2(16c^2-115d^2))\cos(e+fx)+3bd(5bd\cos(3(e+fx))-2(bc+21ad)\sin(2(e+fx)))\right)}{210d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-4*(d^2*(105*a^3*c*d + 147*a*b^2*c*d + 105*a^2*b*d^2 + b^3*(2*c^2 + 25*d^2
))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (105*a^2*b*c*d^2 + 105
*a^3*d^3 + 21*a*b^2*d*(-2*c^2 + 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*((c + d)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + b*d*(c + d*S
in[e + f*x])*((-84*a*b*c*d - 420*a^2*d^2 + b^2*(16*c^2 - 115*d^2))*Cos[e +
f*x] + 3*b*d*(5*b*d*Cos[3*(e + f*x)] - 2*(b*c + 21*a*d)*Sin[2*(e + f*x)])))
/(210*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1560 vs. $2(417) = 834$.

time = 26.81, size = 1561, normalized size = 4.16

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1561 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d*(-2/7/d*sin(f*x+e)^2*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35/d^2*c*sin(f*x+e)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35/d^2*c^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)+2*(-4/35/d^2*c^2+5/21)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))))+(3*a*b^2*d+b^3*c)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/
5+8/15/d^2*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3
*a^2*b*d+3*a*b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/
d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*Elliptic
```

```
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(a^3*d+3*a^2*b*c)*(1/d*c-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^3*c*(1/d*c-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 743, normalized size = 1.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(sqrt(2)*(16*b^3*c^4 - 84*a*b^2*c^3*d + 2*(105*a^2*b + 16*b^3)*c^2*d
^2 - 21*(5*a^3 + 3*a*b^2)*c*d^3 - 15*(21*a^2*b + 5*b^3)*d^4)*sqrt(I*d)*weie
rstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + sqrt(2)*(16*b^3*c^
4 - 84*a*b^2*c^3*d + 2*(105*a^2*b + 16*b^3)*c^2*d^2 - 21*(5*a^3 + 3*a*b^2)*
c*d^3 - 15*(21*a^2*b + 5*b^3)*d^4)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c
^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) +
3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(8*I*b^3*c^3*d - 42*I*a*b^2*c^2*
d^2 + I*(105*a^2*b + 19*b^3)*c*d^3 + 21*I*(5*a^3 + 9*a*b^2)*d^4)*sqrt(I*d)*
weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3,
weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d
^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*(-8
*I*b^3*c^3*d + 42*I*a*b^2*c^2*d^2 - I*(105*a^2*b + 19*b^3)*c*d^3 - 21*I*(5*
a^3 + 9*a*b^2)*d^4)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8
/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
+ e) + 2*I*c)/d)) - 6*(15*b^3*d^4*cos(f*x + e)^3 - 3*(b^3*c*d^3 + 21*a*b^2*
```

```
d^4)*cos(f*x + e)*sin(f*x + e) + (4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 5*(21*a^
2*b + 8*b^3)*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**3*sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(e+f\,x))^3\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2), x)
```

## 3.740 $\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=302

$$\frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f}-\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{5df}-\frac{2b(}{}$$

```
[Out] 8/15*b^2*(-3*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/5*b^2*cos(f
*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/d/f+2/15*b*(30*a*b*c*d-45*a^2
*d^2-b^2*(8*c^2+9*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+
d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/15*(45*a^2*b*c*d
^2-15*a^3*d^3-15*a*b^2*d*(2*c^2+d^2)+b^3*(8*c^3+7*c*d^2))*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*si
n(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.31, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2872, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2b(-45a^2d^2+30abcd-(b^2(8c^2+9d^2)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(-15a^3d^3+45a^2bcd^2-15ab^2d(2c^2+d^2)+b^3(8c^3+7cd^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f}-\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (8*b^2*(b*c - 3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2
*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) -
(2*b*(30*a*b*c*d - 45*a^2*d^2 - b^2*(8*c^2 + 9*d^2))*EllipticE[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[
e + f*x])/(c + d)]) - (2*(45*a^2*b*c*d^2 - 15*a^3*d^3 - 15*a*b^2*d*(2*c^2 +
 d^2) + b^3*(8*c^3 + 7*c*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{1}{2}(2b^3c+5a^3d+ab^2d)}{}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+}{5df}$$

**Mathematica** [**A**]
time = 1.32, size = 219, normalized size = 0.73

$$\frac{-2\left(d^2(2b^3c+15a^3d+15ab^2d)\,F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+b(-30abcd+45a^2d^2+b^2(8c^2+9d^2))\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}-2b^2d\cos(e+fx)(c+d\sin(e+fx))(-4bc+15ad+3bd\sin(e+fx))}{15d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(d^2*(2*b^3*c + 15*a^3*d + 15*a*b^2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
 (2*d)/(c + d)] + b*(-30*a*b*c*d + 45*a^2*d^2 + b^2*(8*c^2 + 9*d^2))*((c +
d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b^2*d*
Cos[e + f*x]*(c + d*Sin[e + f*x])*(-4*b*c + 15*a*d + 3*b*d*Sin[e + f*x]))/(
15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(348) = 696$.
time = 18.39, size = 1085, normalized size = 3.59

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1085 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/
5+8/15/d^2*c^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*
a*b^2*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))))+6*a^2*b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2)))+2*a^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 605, normalized size = 2.00

$$-\frac{\sqrt{2}\left(16 b^{3} c^{3}-60 a b^{2} c^{2} d+6\left(15 a^{2} b+2 b^{3}\right) c d^{2}-45\left(a^{3}+a b^{2}\right) d^{3}\right) \sqrt{i d}\, \mathrm{weierstrassPInverse}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(8 i c^{3}-9 i c d^{2}\right)}{27 d^{3}}, \frac{3 d \cos(f x+e)-3 i d \sin(f x+e)-2 i c}{3 d}\right)+\sqrt{2}\left(16 b^{3} c^{3}-60 a b^{2} c^{2} d+6\left(15 a^{2} b+2 b^{3}\right) c d^{2}-45\left(a^{3}+a b^{2}\right) d^{3}\right) \sqrt{-i d}\, \mathrm{weierstrassPInverse}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(-8 i c^{3}+9 i c d^{2}\right)}{27 d^{3}}, \frac{3 d \cos(f x+e)+3 i d \sin(f x+e)+2 i c}{3 d}\right)+3 \sqrt{2}\left(8 i b^{3} c^{2} d-30 i a b^{2} c d^{2}+9 i\left(5 a^{2} b+b^{3}\right) d^{3}\right) \sqrt{i d}\, \mathrm{weierstrassZeta}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(8 i c^{3}-9 i c d^{2}\right)}{27 d^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(8 i c^{3}-9 i c d^{2}\right)}{27 d^{3}}, \frac{3 d \cos(f x+e)-3 i d \sin(f x+e)-2 i c}{3 d}\right)\right)+3 \sqrt{2}\left(-8 i b^{3} c^{2} d+30 i a b^{2} c d^{2}-9 i\left(5 a^{2} b+b^{3}\right) d^{3}\right) \sqrt{-i d}\, \mathrm{weierstrassZeta}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(-8 i c^{3}+9 i c d^{2}\right)}{27 d^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(4 c^{2}-3 d^{2}\right)}{3 d^{2}},-\frac{8\left(-8 i c^{3}+9 i c d^{2}\right)}{27 d^{3}}, \frac{3 d \cos(f x+e)+3 i d \sin(f x+e)+2 i c}{3 d}\right)\right)+6\left(3 b^{3} d^{3} \cos(f x+e) \sin(f x+e)-\left(4 b^{3} c d^{2}-15 a b^{2} d^{3}\right) \cos(f x+e)\right) \sqrt{d \sin(f x+e)+c}}{45 d^{4} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/45*(sqrt(2)*(16*b^3*c^3 - 60*a*b^2*c^2*d + 6*(15*a^2*b + 2*b^3)*c*d^2 -
45*(a^3 + a*b^2)*d^3)*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^
2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
 e) - 2*I*c)/d) + sqrt(2)*(16*b^3*c^3 - 60*a*b^2*c^2*d + 6*(15*a^2*b + 2*b^
3)*c*d^2 - 45*(a^3 + a*b^2)*d^3)*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2
 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*
I*d*sin(f*x + e) + 2*I*c)/d) + 3*sqrt(2)*(8*I*b^3*c^2*d - 30*I*a*b^2*c*d^2
+ 9*I*(5*a^2*b + b^3)*d^3)*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^
2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(
f*x + e) - 2*I*c)/d)) + 3*sqrt(2)*(-8*I*b^3*c^2*d + 30*I*a*b^2*c*d^2 - 9*I*
(5*a^2*b + b^3)*d^3)*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -
8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d
^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x
 + e) + 2*I*c)/d)) + 6*(3*b^3*d^3*cos(f*x + e)*sin(f*x + e) - (4*b^3*c*d^2
- 15*a*b^2*d^3)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(d^4*f)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(e+f\,x))^3}{\sqrt{c+d\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.741 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=361

$$\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}+\frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2\left(c^2-d^2\right)f}$$

```
[Out] 2*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(
1/2)+2/3*b*(6*a*b*c*d-3*a^2*d^2-b^2*(4*c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(1/2)/d^2/(c^2-d^2)/f+2/3*(9*a^2*b*c*d^2-3*a^3*d^3-9*a*b^2*d*(2*c^2-d^2)+
b^3*(8*c^3-5*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1
/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*s
in(f*x+e))^(1/2)/d^3/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/3*b*(18*a
*b*c*d-9*a^2*d^2-b^2*(8*c^2+d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1
/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.41, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2871, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2b\left(-3a^2d^2+6abcd-\left(b^2\left(4c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f\left(c^2-d^2\right)}-\frac{2b\left(-9a^2d^2+18abcd-\left(b^2\left(8c^2+d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-3a^3d^3+9a^2bcd^2-9ab^2d\left(2c^2-d^2\right)+b^3\left(8c^3-5cd^2\right)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2)*f*Sqrt[c
 + d*Sin[e + f*x]]) + (2*b*(6*a*b*c*d - 3*a^2*d^2 - b^2*(4*c^2 - d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d^2*(c^2 - d^2)*f) - (2*(9*a^2*b*c*d^
2 - 3*a^3*d^3 - 9*a*b^2*d*(2*c^2 - d^2) + b^3*(8*c^3 - 5*c*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c^2 -
d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(18*a*b*c*d - 9*a^2*d^2 -
 b^2*(8*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-ad\left(\left(a^2+b^2\right)c-2abd\right)\right)-}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right.}{3d^2\left(c^2\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right.}{3d^2\left(c^2\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right.}{3d^2\left(c^2\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right.}{3d^2\left(c^2\right.}$$

**Mathematica** [A]
time = 2.15, size = 311, normalized size = 0.86

$$\frac{2\left(\frac{\left(d^2\left(-3a^3cd-9ab^2cd+9a^2bd^2+b^3\left(2c^2+d^2\right)\right)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+\left(9a^2bcd^2-3a^3d^3+9ab^2d\left(-2c^2+d^2\right)+b^3\left(8c^3-5cd^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(c-d)(c+d)}-\frac{d\cos(e+fx)\left(9ab^2c^2d-9a^2bcd^2+3a^3d^3+b^3\left(-4c^3+cd^2\right)+b^3d\left(-c^2+d^2\right)\sin(e+fx)\right)}{-c^2+d^2}\right)}{3d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((d^2*(-3*a^3*c*d - 9*a*b^2*c*d + 9*a^2*b*d^2 + b^3*(2*c^2 + d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (9*a^2*b*c*d^2 - 3*a^3*d^3 +
9*a*b^2*d*(-2*c^2 + d^2) + b^3*(8*c^3 - 5*c*d^2))*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)*(c + d)) - (d*Cos[e
 + f*x]*(9*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 3*a^3*d^3 + b^3*(-4*c^3 + c*d^2) +
 b^3*d*(-c^2 + d^2)*Sin[e + f*x]))/(-c^2 + d^2)))/(3*d^3*f*Sqrt[c + d*Sin[e
 + f*x]])
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(409) = 818$.

time = 23.59, size = 1398, normalized size = 3.87

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1398 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d^3*(d^2*b^2*(-2/3/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)))+2*(3*a*b*d^2-b^2*c*d)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))+6*a^2*d^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6*
a*b*c*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*b^2*c^2*(
1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 1083, normalized size = 3.00

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/9*((sqrt(2)*(16*b^3*c^4*d - 36*a*b^2*c^3*d^2 + 2*(9*a^2*b - 8*b^3)*c^2*d^3 + 3*(a^3 + 15*a*b^2)*c*d^4 - 3*(9*a^2*b + b^3)*d^5)*sin(f*x + e) + sqrt(2)*(16*b^3*c^5 - 36*a*b^2*c^4*d + 2*(9*a^2*b - 8*b^3)*c^3*d^2 + 3*(a^3 + 15*a*b^2)*c^2*d^3 - 3*(9*a^2*b + b^3)*c*d^4))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d) + (sqrt(2)*(16*b^3*c^4*d - 36*a*b^2*c^3*d^2 + 2*(9*a^2*b - 8*b^3)*c^2*d^3 + 3*(a^3 + 15*a*b^2)*c*d^4 - 3*(9*a^2*b + b^3)*d^5)*sin(f*x + e) + sqrt(2)*(16*b^3*c^5 - 36*a*b^2*c^4*d + 2*(9*a^2*b - 8*b^3)*c^3*d^2 + 3*(a^3 + 15*a*b^2)*c^2*d^3 - 3*(9*a^2*b + b^3)*c*d^4))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(8*I*b^3*c^3*d^2 - 18*I*a*b^2*c^2*d^3 + I*(9*a^2*b - 5*b^3)*c*d^4 - 3*I*(a^3 - 3*a*b^2)*d^5)*sin(f*x + e) + sqrt(2)*(8*I*b^3*c^4*d - 18*I*a*b^2*c^3*d^2 + I*(9*a^2*b - 5*b^3)*c^2*d^3 - 3*I*(a^3 - 3*a*b^2)*c*d^4))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(2)*(-8*I*b^3*c^3*d^2 + 18*I*a*b^2*c^2*d^3 - I*(9*a^2*b - 5*b^3)*c*d^4 + 3*I*(a^3 - 3*a*b^2)*d^5)*sin(f*x + e) + sqrt(2)*(-8*I*b^3*c^4*d + 18*I*a*b^2*c^3*d^2 - I*(9*a^2*b - 5*b^3)*c^2*d^3 + 3*I*(a^3 - 3*a*b^2)*c*d^4))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*((b^3*c^2*d^3 - b^3*d^5)*cos(f*x + e)*sin(f*x + e) + (4*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 - 3*a^3*d^5 + (9*a^2*b - b^3)*c*d^4)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/((c^2*d^5 - d^7)*f*sin(f*x + e) + (c^3*d^4 - c*d^6)*f)

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^3}{(c+d\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2), x)
```

## 3.742 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=391

$$\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{8(bc-ad)^2\left(acd+b(c^2-2d^2)\right)\cos(e+fx)}{3d^2\left(c^2-d^2\right)^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(4a^3cd^3-6ab^2cd$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(3/2)+8/3*(-a*d+b*c)^2*(a*c*d+b*(c^2-2*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2/f/
(c+d*sin(f*x+e))^(1/2)-2/3*(4*a^3*c*d^3-6*a*b^2*c*d*(c^2-3*d^2)-3*a^2*b*d^2
*(c^2+3*d^2)+b^3*(8*c^4-15*c^2*d^2+3*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/3*(-a*d+b*c)*(2*a*b*c*d-a^2*d^2+b^2*(8*c^2-9*d^2))*(sin(1/2*
e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/
4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(
c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.49, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2871, 3100, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(-a^2d^2+2abcd+b^2(8c^2-9d^2))(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(4a^3cd^3-3a^2bd^2(c^2+3d^2)-6ab^2cd(c^2-3d^2)+b^3(8c^4-15c^2d^2+3d^4))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8(acd+b(c^2-2d^2))(bc-ad)^2\cos(e+fx)}{3d^2f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(3/2)) + (8*(b*c - a*d)^2*(a*c*d + b*(c^2 - 2*d^2))*Cos[e
+ f*x])/(3*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(4*a^3*c*d^3
- 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*(8*c^4 - 15*c
^2*d^2 + 3*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (
2*(b*c - a*d)*(2*a*b*c*d - a^2*d^2 + b^2*(8*c^2 - 9*d^2))*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*(c^2
 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
```

```
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-3ad\left(\left(a^2+b^2\right)c-2abd\right)\right)}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e}}$$

**Mathematica [A]**
time = 3.96, size = 357, normalized size = 0.91

$$\frac{2\left(\frac{\left(d^2\left(-12a^2bcd^2+a^3d\left(3c^2+d^2\right)+3ab^2d\left(c^2+3d^2\right)+2b^3\left(c^3-3cd^2\right)\right)F\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)+\left(4a^3cd^3-6ab^2cd\left(c^2-3d^2\right)-3a^2bd^2\left(c^2+3d^2\right)+b^3\left(8c^4-15c^2d^2+3d^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\frac{2d}{c+d}\right)\right)\right)(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(c-d)^2(c+d)^2}-\frac{d(bc-ad)^2\cos(e+fx)\left(-4bc^3-5ac^2d+8bcd^2+ad^3+d\left(-5bc^2-4acd+9bd^2\right)\sin(e+fx)\right)}{\left(c^2-d^2\right)^2}\right)}{3d^3f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-12*a^2*b*c*d^2 + a^3*d*(3*c^2 + d^2) + 3*a*b^2*d*(c^2 + 3*d^2)
+ 2*b^3*(c^3 - 3*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] +
(4*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*
(8*c^4 - 15*c^2*d^2 + 3*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*S
in[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d
```

```
*(b*c - a*d)^2*Cos[e + f*x]*(-4*b*c^3 - 5*a*c^2*d + 8*b*c*d^2 + a*d^3 + d*(
-5*b*c^2 - 4*a*c*d + 9*b*d^2)*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^3*f*(c +
d*Sin[e + f*x])^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(437) = 874$.
time = 31.49, size = 1379, normalized size = 3.53

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1379 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2/d^3*(2*b*d*(1/d*c-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+6*a*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))-4*b*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))+3*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2*d*cos(f*x+e)^2/(c^2-d^2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+
e)+1/d*c)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 1914, normalized size = 4.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*((sqrt(2)*(16*b^3*c^5*d^2 - 12*a*b^2*c^4*d^3 - 6*(a^2*b + 6*b^3)*c^3*d
^4 - (a^3 - 27*a*b^2)*c^2*d^5 + 6*(3*a^2*b + 4*b^3)*c*d^6 - 3*(a^3 + 9*a*b^
2)*d^7)*cos(f*x + e)^2 - 2*sqrt(2)*(16*b^3*c^6*d - 12*a*b^2*c^5*d^2 - 6*(a^
2*b + 6*b^3)*c^4*d^3 - (a^3 - 27*a*b^2)*c^3*d^4 + 6*(3*a^2*b + 4*b^3)*c^2*d
^5 - 3*(a^3 + 9*a*b^2)*c*d^6)*sin(f*x + e) - sqrt(2)*(16*b^3*c^7 - 12*a*b^2
*c^6*d - 4*a^3*c^2*d^5 - 2*(3*a^2*b + 10*b^3)*c^5*d^2 - (a^3 - 15*a*b^2)*c^
4*d^3 + 12*(a^2*b - b^3)*c^3*d^4 + 6*(3*a^2*b + 4*b^3)*c*d^6 - 3*(a^3 + 9*a
*b^2)*d^7))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(
8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*
c)/d) + (sqrt(2)*(16*b^3*c^5*d^2 - 12*a*b^2*c^4*d^3 - 6*(a^2*b + 6*b^3)*c^3
*d^4 - (a^3 - 27*a*b^2)*c^2*d^5 + 6*(3*a^2*b + 4*b^3)*c*d^6 - 3*(a^3 + 9*a*
b^2)*d^7)*cos(f*x + e)^2 - 2*sqrt(2)*(16*b^3*c^6*d - 12*a*b^2*c^5*d^2 - 6*(
a^2*b + 6*b^3)*c^4*d^3 - (a^3 - 27*a*b^2)*c^3*d^4 + 6*(3*a^2*b + 4*b^3)*c^2
*d^5 - 3*(a^3 + 9*a*b^2)*c*d^6)*sin(f*x + e) - sqrt(2)*(16*b^3*c^7 - 12*a*b
^2*c^6*d - 4*a^3*c^2*d^5 - 2*(3*a^2*b + 10*b^3)*c^5*d^2 - (a^3 - 15*a*b^2)*
c^4*d^3 + 12*(a^2*b - b^3)*c^3*d^4 + 6*(3*a^2*b + 4*b^3)*c*d^6 - 3*(a^3 + 9
*a*b^2)*d^7))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/2
7*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) +
2*I*c)/d) - 3*(sqrt(2)*(-8*I*b^3*c^4*d^3 + 6*I*a*b^2*c^3*d^4 + 3*I*(a^2*b +
 5*b^3)*c^2*d^5 - 2*I*(2*a^3 + 9*a*b^2)*c*d^6 + 3*I*(3*a^2*b - b^3)*d^7)*co
s(f*x + e)^2 + 2*sqrt(2)*(8*I*b^3*c^5*d^2 - 6*I*a*b^2*c^4*d^3 - 3*I*(a^2*b
+ 5*b^3)*c^3*d^4 + 2*I*(2*a^3 + 9*a*b^2)*c^2*d^5 - 3*I*(3*a^2*b - b^3)*c*d^
6)*sin(f*x + e) + sqrt(2)*(8*I*b^3*c^6*d - 6*I*a*b^2*c^5*d^2 - I*(3*a^2*b +
 7*b^3)*c^4*d^3 + 4*I*(a^3 + 3*a*b^2)*c^3*d^4 - 12*I*(a^2*b + b^3)*c^2*d^5
+ 2*I*(2*a^3 + 9*a*b^2)*c*d^6 - 3*I*(3*a^2*b - b^3)*d^7))*sqrt(I*d)*weierst
rassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierst
rassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3
*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) - 3*(sqrt(2)*(8*I*b^3*
```

```
c^4*d^3 - 6*I*a*b^2*c^3*d^4 - 3*I*(a^2*b + 5*b^3)*c^2*d^5 + 2*I*(2*a^3 + 9*
a*b^2)*c*d^6 - 3*I*(3*a^2*b - b^3)*d^7)*cos(f*x + e)^2 + 2*sqrt(2)*(-8*I*b^
3*c^5*d^2 + 6*I*a*b^2*c^4*d^3 + 3*I*(a^2*b + 5*b^3)*c^3*d^4 - 2*I*(2*a^3 +
9*a*b^2)*c^2*d^5 + 3*I*(3*a^2*b - b^3)*c*d^6)*sin(f*x + e) + sqrt(2)*(-8*I*
b^3*c^6*d + 6*I*a*b^2*c^5*d^2 + I*(3*a^2*b + 7*b^3)*c^4*d^3 - 4*I*(a^3 + 3*
a*b^2)*c^3*d^4 + 12*I*(a^2*b + b^3)*c^2*d^5 - 2*I*(2*a^3 + 9*a*b^2)*c*d^6 +
 3*I*(3*a^2*b - b^3)*d^7))*sqrt(-I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/
d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*
d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*d*s
in(f*x + e) + 2*I*c)/d)) + 6*((5*b^3*c^4*d^3 - 6*a*b^2*c^3*d^4 - 9*a^2*b*d^
7 - 3*(a^2*b + 3*b^3)*c^2*d^5 + 2*(2*a^3 + 9*a*b^2)*c*d^6)*cos(f*x + e)*sin
(f*x + e) + (4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 - 6*a^2*b*c*d^6 - a^3*d^7 - 2*
(3*a^2*b + 4*b^3)*c^3*d^4 + 5*(a^3 + 3*a*b^2)*c^2*d^5)*cos(f*x + e))*sqrt(d
*sin(f*x + e) + c))/((c^4*d^6 - 2*c^2*d^8 + d^10)*f*cos(f*x + e)^2 - 2*(c^5
*d^5 - 2*c^3*d^7 + c*d^9)*f*sin(f*x + e) - (c^6*d^4 - c^4*d^6 - c^2*d^8 + d
^10)*f)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.743 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$

**Optimal**. Leaf size=532

$$\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{5/2}}+\frac{8(bc-ad)^2\,(2acd+b(c^2-3d^2))\cos(e+fx)}{15d^2\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\,(a^2d^2(\cdots}{\cdots}$$

```
[Out] 2/5*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(5/2)+8/15*(-a*d+b*c)^2*(2*a*c*d+b*(c^2-3*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2
/f/(c+d*sin(f*x+e))^(3/2)-2/15*(-a*d+b*c)*(a^2*d^2*(23*c^2+9*d^2)+2*a*b*d*(
7*c^3-39*c*d^2)+b^2*(8*c^4-21*c^2*d^2+45*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^3/f
/(c+d*sin(f*x+e))^(1/2)+2/15*(-a*d+b*c)*(a^2*d^2*(23*c^2+9*d^2)+2*a*b*d*(7*
c^3-39*c*d^2)+b^2*(8*c^4-21*c^2*d^2+45*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c^2-d^2)^3/f/((c+d*sin(f*x+e)
)/(c+d))^(1/2)+2/15*(8*a^3*c*d^3-6*a*b^2*c*d*(c^2-5*d^2)-3*a^2*b*d^2*(3*c^2
+5*d^2)-b^3*(8*c^4-15*c^2*d^2+15*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/
sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c
+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e
))^(1/2)
```

**Rubi** [A]

time = 0.73, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2871, 3100, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2d^2(23c^2+9d^2)+2abd(7c^3-39cd^2)+b^2(8c^4-21c^2d^2+45d^4))\,(bc-ad)\cos(e+fx)}{15d^2f\,(c^2-d^2)^3\,\sqrt{c+d\sin(e+fx)}}-\frac{2(a^2d^2(23c^2+9d^2)+2abd(7c^3-39cd^2)+b^2(8c^4-21c^2d^2+45d^4))\,(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\,(c^2-d^2)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(8a^3cd^3-3a^2bd^2(3c^2+5d^2)-6ab^2cd(c^2-5d^2)-(b^3(8c^4-15c^2d^2+15d^4)))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\,(c^2-d^2)^2\sqrt{c+d\sin(e+fx)}}+\frac{8(2acd+b(c^2-3d^2))\,(bc-ad)^2\cos(e+fx)}{15d^2f\,(c^2-d^2)^2\,(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5df\,(c^2-d^2)\,(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(5*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(5/2)) + (8*(b*c - a*d)^2*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[
e + f*x])/(15*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2*(b*c - a
*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8*c^4 - 2
1*c^2*d^2 + 45*d^4))*Cos[e + f*x])/(15*d^2*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e
 + f*x]]) - (2*(b*c - a*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*
c*d^2) + b^2*(8*c^4 - 21*c^2*d^2 + 45*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*(c^2 - d^2)^3*f*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]) - (2*(8*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 5*d^2) - 3*
a^2*b*d^2*(3*c^2 + 5*d^2) - b^3*(8*c^4 - 15*c^2*d^2 + 15*d^4))*EllipticF[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^
3*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
```

```
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-5ad\left(\left(a^2+b^2\right)c-2abd\right)\right)}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b(c^2-3d^2)\right)}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b(c^2-3d^2)\right)}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b(c^2-3d^2)\right)}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b(c^2-3d^2)\right)}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b(c^2-3d^2)\right)}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

**Mathematica** [A]
time = 5.55, size = 584, normalized size = 1.10

$$\frac{2\left(\frac{\left(d^2\left(3a^2bd^2\left(27c^2+5d^2\right)-a^3cd\left(15c^2+17d^2\right)-3ab^2d\left(7c^3+25cd^2\right)+b^3\left(2c^4+15c^2d^2+15d^4\right)\right)F\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+\left(-a^3d^3\left(23c^2+9d^2\right)+3a^2bcd^2\left(3c^2+29d^2\right)-3ab^2d\left(-2c^4+19c^2d^2+15d^4\right)+b^3\left(8c^5-21c^3d^2+45cd^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\right)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{5/2}}{(c-d)^3(c+d)}+\frac{d(bc-ad)\cos(e+fx)\left(8b^2c^6+14abc^5d+68a^2c^4d^2-2b^2c^4d^2-146abc^3d^3+13a^2c^2d^4+45b^2c^2d^4-60abcd^5+15a^2d^6+45b^2d^6-d^2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(8c^4-21c^2d^2+45d^4\right)\right)\cos(2(e+fx))+2d\left(2a^2cd^2\left(27c^2+5d^2\right)+abd\left(27c^4-170c^2d^2+15d^4\right)+b^2\left(9c^5-20c^3d^2+75cd^4\right)\right)\sin(e+fx)\right)}{2\left(-c^2+d^2\right)^3}\right)}{15d^3f(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(((d^2*(3*a^2*b*d^2*(27*c^2 + 5*d^2) - a^3*c*d*(15*c^2 + 17*d^2) - 3*a*b
^2*d*(7*c^3 + 25*c*d^2) + b^3*(2*c^4 + 15*c^2*d^2 + 15*d^4))*EllipticF[(-2*
e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(a^3*d^3*(23*c^2 + 9*d^2)) + 3*a^2*b*
c*d^2*(3*c^2 + 29*d^2) - 3*a*b^2*d*(-2*c^4 + 19*c^2*d^2 + 15*d^4) + b^3*(8*
c^5 - 21*c^3*d^2 + 45*c*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*S
in[e + f*x])/(c + d))^(5/2))/((c - d)^3*(c + d)) + (d*(b*c - a*d)*Cos[e + f
*x]*(8*b^2*c^6 + 14*a*b*c^5*d + 68*a^2*c^4*d^2 - 2*b^2*c^4*d^2 - 146*a*b*c^
3*d^3 + 13*a^2*c^2*d^4 + 45*b^2*c^2*d^4 - 60*a*b*c*d^5 + 15*a^2*d^6 + 45*b^
2*d^6 - d^2*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8
*c^4 - 21*c^2*d^2 + 45*d^4))*Cos[2*(e + f*x)] + 2*d*(2*a^2*c*d^2*(27*c^2 +
5*d^2) + a*b*d*(27*c^4 - 170*c^2*d^2 + 15*d^4) + b^2*(9*c^5 - 20*c^3*d^2 +
75*c*d^4))*Sin[e + f*x]))/(2*(-c^2 + d^2)^3)))/(15*d^3*f*(c + d*Sin[e + f*x
])^(5/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(574) = 1148$.
time = 43.82, size = 1621, normalized size = 3.05

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1621 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^3/d^3*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+3/d^3*b^2*(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2-
d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
```

```
2*d-b^3*c^3)*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(si
n(f*x+e)+1/d*c)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)/(sin(f*x+e)+1/d*c)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^
2+45*c^2*d^4-15*d^6)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))+3*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c^2-d^2)/d*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*d*cos(f*x+e)^2
/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4
-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8
/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))
)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 3078, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/45*((3*sqrt(2)*(16*b^3*c^7*d^2 + 12*a*b^2*c^6*d^3 + 6*(3*a^2*b - 8*b^3)*c
^5*d^4 - (a^3 + 51*a*b^2)*c^4*d^5 - 3*(23*a^2*b - 15*b^3)*c^3*d^6 + 3*(11*a
^3 + 45*a*b^2)*c^2*d^7 - 45*(a^2*b + b^3)*c*d^8)*cos(f*x + e)^2 + (sqrt(2)*
(16*b^3*c^6*d^3 + 12*a*b^2*c^5*d^4 + 6*(3*a^2*b - 8*b^3)*c^4*d^5 - (a^3 + 5
1*a*b^2)*c^3*d^6 - 3*(23*a^2*b - 15*b^3)*c^2*d^7 + 3*(11*a^3 + 45*a*b^2)*c*
```

d^8 - 45*(a^2*b + b^3)*d^9)*cos(f*x + e)^2 - sqrt(2)*(48*b^3*c^8*d + 36*a*b
^2*c^7*d^2 + 2*(27*a^2*b - 64*b^3)*c^6*d^3 - 3*(a^3 + 47*a*b^2)*c^5*d^4 - 3
*(63*a^2*b - 29*b^3)*c^4*d^5 + 2*(49*a^3 + 177*a*b^2)*c^3*d^6 - 6*(34*a^2*b
 + 15*b^3)*c^2*d^7 + 3*(11*a^3 + 45*a*b^2)*c*d^8 - 45*(a^2*b + b^3)*d^9))*s
in(f*x + e) - sqrt(2)*(16*b^3*c^9 + 12*a*b^2*c^8*d + 18*a^2*b*c^7*d^2 - (a^
3 + 15*a*b^2)*c^6*d^3 - 3*(5*a^2*b + 33*b^3)*c^5*d^4 + 6*(5*a^3 - 3*a*b^2)*
c^4*d^5 - 18*(14*a^2*b - 5*b^3)*c^3*d^6 + 9*(11*a^3 + 45*a*b^2)*c^2*d^7 - 1
35*(a^2*b + b^3)*c*d^8))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)
/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*
x + e) - 2*I*c)/d) + (3*sqrt(2)*(16*b^3*c^7*d^2 + 12*a*b^2*c^6*d^3 + 6*(3*a
^2*b - 8*b^3)*c^5*d^4 - (a^3 + 51*a*b^2)*c^4*d^5 - 3*(23*a^2*b - 15*b^3)*c^
3*d^6 + 3*(11*a^3 + 45*a*b^2)*c^2*d^7 - 45*(a^2*b + b^3)*c*d^8)*cos(f*x + e
)^2 + (sqrt(2)*(16*b^3*c^6*d^3 + 12*a*b^2*c^5*d^4 + 6*(3*a^2*b - 8*b^3)*c^4
*d^5 - (a^3 + 51*a*b^2)*c^3*d^6 - 3*(23*a^2*b - 15*b^3)*c^2*d^7 + 3*(11*a^3
 + 45*a*b^2)*c*d^8 - 45*(a^2*b + b^3)*d^9)*cos(f*x + e)^2 - sqrt(2)*(48*b^3
*c^8*d + 36*a*b^2*c^7*d^2 + 2*(27*a^2*b - 64*b^3)*c^6*d^3 - 3*(a^3 + 47*a*b
^2)*c^5*d^4 - 3*(63*a^2*b - 29*b^3)*c^4*d^5 + 2*(49*a^3 + 177*a*b^2)*c^3*d^
6 - 6*(34*a^2*b + 15*b^3)*c^2*d^7 + 3*(11*a^3 + 45*a*b^2)*c*d^8 - 45*(a^2*b
 + b^3)*d^9))*sin(f*x + e) - sqrt(2)*(16*b^3*c^9 + 12*a*b^2*c^8*d + 18*a^2*
b*c^7*d^2 - (a^3 + 15*a*b^2)*c^6*d^3 - 3*(5*a^2*b + 33*b^3)*c^5*d^4 + 6*(5*
a^3 - 3*a*b^2)*c^4*d^5 - 18*(14*a^2*b - 5*b^3)*c^3*d^6 + 9*(11*a^3 + 45*a*b
^2)*c^2*d^7 - 135*(a^2*b + b^3)*c*d^8))*sqrt(-I*d)*weierstrassPInverse(-4/3
*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x +
e) + 3*I*d*sin(f*x + e) + 2*I*c)/d) + 3*(3*sqrt(2)*(8*I*b^3*c^6*d^3 + 6*I*a
*b^2*c^5*d^4 + 3*I*(3*a^2*b - 7*b^3)*c^4*d^5 - I*(23*a^3 + 57*a*b^2)*c^3*d^
6 + 3*I*(29*a^2*b + 15*b^3)*c^2*d^7 - 9*I*(a^3 + 5*a*b^2)*c*d^8)*cos(f*x +
e)^2 + (sqrt(2)*(8*I*b^3*c^5*d^4 + 6*I*a*b^2*c^4*d^5 + 3*I*(3*a^2*b - 7*b^3
)*c^3*d^6 - I*(23*a^3 + 57*a*b^2)*c^2*d^7 + 3*I*(29*a^2*b + 15*b^3)*c*d^8 -
 9*I*(a^3 + 5*a*b^2)*d^9)*cos(f*x + e)^2 + sqrt(2)*(-24*I*b^3*c^7*d^2 - 18*
I*a*b^2*c^6*d^3 - I*(27*a^2*b - 55*b^3)*c^5*d^4 + 3*I*(23*a^3 + 55*a*b^2)*c
^4*d^5 - 6*I*(45*a^2*b + 19*b^3)*c^3*d^6 + 2*I*(25*a^3 + 96*a*b^2)*c^2*d^7
 - 3*I*(29*a^2*b + 15*b^3)*c*d^8 + 9*I*(a^3 + 5*a*b^2)*d^9))*sin(f*x + e) +
sqrt(2)*(-8*I*b^3*c^8*d - 6*I*a*b^2*c^7*d^2 - 3*I*(3*a^2*b + b^3)*c^6*d^3 +
 I*(23*a^3 + 39*a*b^2)*c^5*d^4 - 6*I*(19*a^2*b - 3*b^3)*c^4*d^5 + 6*I*(13*a
^3 + 36*a*b^2)*c^3*d^6 - 9*I*(29*a^2*b + 15*b^3)*c^2*d^7 + 27*I*(a^3 + 5*a*
b^2)*c*d^8))*sqrt(I*d)*weierstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I
*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*
(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x + e) - 2*I
*c)/d)) + 3*(3*sqrt(2)*(-8*I*b^3*c^6*d^3 - 6*I*a*b^2*c^5*d^4 - 3*I*(3*a^2*b
 - 7*b^3)*c^4*d^5 + I*(23*a^3 + 57*a*b^2)*c^3*d^6 - 3*I*(29*a^2*b + 15*b^3)
*c^2*d^7 + 9*I*(a^3 + 5*a*b^2)*c*d^8)*cos(f*x + e)^2 + (sqrt(2)*(-8*I*b^3*c
^5*d^4 - 6*I*a*b^2*c^4*d^5 - 3*I*(3*a^2*b - 7*b^3)*c^3*d^6 + I*(23*a^3 + 57
*a*b^2)*c^2*d^7 - 3*I*(29*a^2*b + 15*b^3)*c*d^8 + 9*I*(a^3 + 5*a*b^2)*d^9)*
cos(f*x + e)^2 + sqrt(2)*(24*I*b^3*c^7*d^2 + 18*I*a*b^2*c^6*d^3 + I*(27*a^2
*b - 55*b^3)*c^5*d^4 - 3*I*(23*a^3 + 55*a*b^2)*c^4*d^5 + 6*I*(45*a^2*b + 19

```
*b^3)*c^3*d^6 - 2*I*(25*a^3 + 96*a*b^2)*c^2*d^7 + 3*I*(29*a^2*b + 15*b^3)*c
*d^8 - 9*I*(a^3 + 5*a*b^2)*d^9))*sin(f*x + e) + sqrt(2)*(8*I*b^3*c^8*d + 6*
I*a*b^2*c^7*d^2 + 3*I*(3*a^2*b + b^3)*c^6*d^3 - I*(23*a^3 + 39*a*b^2)*c^5*d
^4 + 6*I*(19*a^2*b - 3*b^3)*c^4*d^5 - 6*I*(13*a^3 + 36*a*b^2)*c^3*d^6 + 9*I
*(29*a^2*b + 15*b^3)*c^2*d^7 - 27*I*(a^3 + 5*a*b^2)*c*d^8))*sqrt(-I*d)*weie
rstrassZeta(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, wei
erstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3
, 1/3*(3*d*cos(f*x + e) + 3*I*d*sin(f*x + e) + 2*I*c)/d)) - 6*((8*b^3*c^5*d
^4 + 6*a*b^2*c^4*d^5 + 3*(3*a^2*b - 7*b^3)*c^3*d^6 - (23*a^3 + 57*a*b^2)*c^
2*d^7 + 3*(29*a^2*b + 15*b^3)*c*d^8 - 9*(a^3 + 5*a*b^2)*d^9)*cos(f*x + e)^3
 - (9*b^3*c^6*d^3 + 18*a*b^2*c^5*d^4 - 15*a^2*b*d^9 + (27*a^2*b - 20*b^3)*c
^4*d^5 - 6*(9*a^3 + 25*a*b^2)*c^3*d^6 + 15*(12*a^2*b + 5*b^3)*c^2*d^7 - 10*
(a^3 + 6*a*b^2)*c*d^8)*cos(f*x + e)*sin(f*x + e) - (4*b^3*c^7*d^2 + 3*a*b^2
*c^6*d^3 + 3*(9*a^2*b + b^3)*c^5*d^4 - (34*a^3 + 69*a*b^2)*c^4*d^5 + 12*(7*
a^2*b + b^3)*c^3*d^6 - 9*(2*a^3 + 9*a*b^2)*c^2*d^7 + 9*(9*a^2*b + 5*b^3)*c*
d^8 - 3*(4*a^3 + 15*a*b^2)*d^9)*cos(f*x + e))*sqrt(d*sin(f*x + e) + c))/(3*
(c^7*d^6 - 3*c^5*d^8 + 3*c^3*d^10 - c*d^12)*f*c...
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2), x)
```

# 3.744 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

**Optimal**. Leaf size=716

$$\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}}+\frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))\cos(e+fx)}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e+fx))^{5/2}}-\frac{2(bc-ad)\,(a^2d^2 [illegible]}{}$$

```
[Out] 2/7*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(7/2)+8/35*(-a*d+b*c)^2*(3*a*c*d+b*(c^2-4*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2
/f/(c+d*sin(f*x+e))^(5/2)-2/105*(-a*d+b*c)*(a^2*d^2*(71*c^2+25*d^2)+a*b*(26
*c^3*d-218*c*d^3)+b^2*(8*c^4-17*c^2*d^2+105*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^
3/f/(c+d*sin(f*x+e))^(3/2)+2/105*(16*a^3*c*d^3*(11*c^2+13*d^2)-6*a*b^2*c*d*
(3*c^4-62*c^2*d^2-133*d^4)-9*a^2*b*d^2*(5*c^4+102*c^2*d^2+21*d^4)-b^3*(8*c^
6-23*c^4*d^2+294*c^2*d^4+105*d^6))*cos(f*x+e)/d^2/(c^2-d^2)^4/f/(c+d*sin(f*
x+e))^(1/2)-2/105*(16*a^3*c*d^3*(11*c^2+13*d^2)-6*a*b^2*c*d*(3*c^4-62*c^2*d
^2-133*d^4)-9*a^2*b*d^2*(5*c^4+102*c^2*d^2+21*d^4)-b^3*(8*c^6-23*c^4*d^2+29
4*c^2*d^4+105*d^6))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/
2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*si
n(f*x+e))^(1/2)/d^3/(c^2-d^2)^4/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/105*(-a*
d+b*c)*(a^2*d^2*(71*c^2+25*d^2)+a*b*(26*c^3*d-218*c*d^3)+b^2*(8*c^4-17*c^2*
d^2+105*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/d^3/(c^2-d^2)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.95, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2871, 3100, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2d^2(71c^2+25d^2)+ab(26c^3d-218cd^3)+b^2(8c^4-17c^2d^2+105d^4))\,(bc-ad)\cos(e+fx)}{105d^2f\,(c^2-d^2)^3\,(c+d\sin(e+fx))^{3/2}}+\frac{2(a^2d^2(71c^2+25d^2)+ab(26c^3d-218cd^3)+b^2(8c^4-17c^2d^2+105d^4))\,(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\,(c^2-d^2)^3\sqrt{c+d\sin(e+fx)}}+\frac{2(16a^3cd^3(11c^2+13d^2)-9a^2bd^2(5c^4+102c^2d^2+21d^4)-6ab^2cd(3c^4-62c^2d^2-133d^4)-(b^3(8c^6-23c^4d^2+294c^2d^4+105d^6)))\cos(e+fx)}{105d^2f\,(c^2-d^2)^4\sqrt{c+d\sin(e+fx)}}+\frac{2(16a^3cd^3(11c^2+13d^2)-9a^2bd^2(5c^4+102c^2d^2+21d^4)-6ab^2cd(3c^4-62c^2d^2-133d^4)-(b^3(8c^6-23c^4d^2+294c^2d^4+105d^6)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f\,(c^2-d^2)^4\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8(3acd+b(c^2-4d^2))\,(bc-ad)^2\cos(e+fx)}{35d^2f\,(c^2-d^2)^2\,(c+d\sin(e+fx))^{5/2}}+\frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7df\,(c^2-d^2)\,(c+d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(7*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(7/2)) + (8*(b*c - a*d)^2*(3*a*c*d + b*(c^2 - 4*d^2))*Cos[
e + f*x])/(35*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(5/2)) - (2*(b*c - a
*d)*(a^2*d^2*(71*c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 -
17*c^2*d^2 + 105*d^4))*Cos[e + f*x])/(105*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e
+ f*x])^(3/2)) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*a*b^2*c*d*(3*c^4 -
62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2 + 21*d^4) - b^3*(8
*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*Cos[e + f*x])/(105*d^2*(c^2 - d
```

```
^2)^4*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*
a*b^2*c*d*(3*c^4 - 62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2
 + 21*d^4) - b^3*(8*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*(c^2 -
d^2)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*(a^2*d^2*(71*
c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 - 17*c^2*d^2 + 105*
d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/(105*d^3*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
```

```
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} - \frac{2\int \frac{\frac{1}{2}(2b(bc-ad)^2-7ad((a^2+b^2)c-2abd))}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\,(c^2-d^2)\,f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\,(3acd+b(c^2-4d^2))}{35d^2\,(c^2-d^2)^2\,f(c+d\sin(e}$$

**Mathematica** [A]
time = 7.11, size = 1127, normalized size = 1.57

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos
[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x]))/(7*d^2*(-c^
2 + d^2)*(c + d*Sin[e + f*x])^4) - (6*(-3*b^3*c^4*Cos[e + f*x] + 2*a*b^2*c^
3*d*Cos[e + f*x] + 5*a^2*b*c^2*d^2*Cos[e + f*x] + 7*b^3*c^2*d^2*Cos[e + f*x
] - 4*a^3*c*d^3*Cos[e + f*x] - 14*a*b^2*c*d^3*Cos[e + f*x] + 7*a^2*b*d^4*Co
s[e + f*x]))/(35*d^2*(-c^2 + d^2)^2*(c + d*Sin[e + f*x])^3) - (2*(-8*b^3*c^
5*Cos[e + f*x] - 18*a*b^2*c^4*d*Cos[e + f*x] - 45*a^2*b*c^3*d^2*Cos[e + f*x
] + 17*b^3*c^3*d^2*Cos[e + f*x] + 71*a^3*c^2*d^3*Cos[e + f*x] + 201*a*b^2*c
```

```
^2*d^3*Cos[e + f*x] - 243*a^2*b*c*d^4*Cos[e + f*x] - 105*b^3*c*d^4*Cos[e +
f*x] + 25*a^3*d^5*Cos[e + f*x] + 105*a*b^2*d^5*Cos[e + f*x]))/(105*d^2*(-c^
2 + d^2)^3*(c + d*Sin[e + f*x])^2) - (2*(8*b^3*c^6*Cos[e + f*x] + 18*a*b^2*
c^5*d*Cos[e + f*x] + 45*a^2*b*c^4*d^2*Cos[e + f*x] - 23*b^3*c^4*d^2*Cos[e +
 f*x] - 176*a^3*c^3*d^3*Cos[e + f*x] - 372*a*b^2*c^3*d^3*Cos[e + f*x] + 918
*a^2*b*c^2*d^4*Cos[e + f*x] + 294*b^3*c^2*d^4*Cos[e + f*x] - 208*a^3*c*d^5*
Cos[e + f*x] - 798*a*b^2*c*d^5*Cos[e + f*x] + 189*a^2*b*d^6*Cos[e + f*x] +
105*b^3*d^6*Cos[e + f*x]))/(105*d^2*(-c^2 + d^2)^4*(c + d*Sin[e + f*x]))))/
f - ((-2*(2*b^3*c^5*d - 105*a^3*c^4*d^2 - 153*a*b^2*c^4*d^2 + 720*a^2*b*c^3
*d^3 + 172*b^3*c^3*d^3 - 254*a^3*c^2*d^4 - 894*a*b^2*c^2*d^4 + 432*a^2*b*c*
d^5 + 210*b^3*c*d^5 - 25*a^3*d^6 - 105*a*b^2*d^6)*EllipticF[(-e + Pi/2 - f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e +
 f*x]] - ((8*b^3*c^6 + 18*a*b^2*c^5*d + 45*a^2*b*c^4*d^2 - 23*b^3*c^4*d^2 -
 176*a^3*c^3*d^3 - 372*a*b^2*c^3*d^3 + 918*a^2*b*c^2*d^4 + 294*b^3*c^2*d^4
- 208*a^3*c*d^5 - 798*a*b^2*c*d^5 + 189*a^2*b*d^6 + 105*b^3*d^6)*((2*(c + d
)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*
d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/
d)/(105*(c - d)^4*d^2*(c + d)^4*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(754) = 1508$.
time = 68.68, size = 2111, normalized size = 2.95

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2111 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/d^3*(2/7/(c^2-d^2)/d^3*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x
+e)+1/d*c)^4+24/35/(c^2-d^2)^2/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)/(sin(f*x+e)+1/d*c)^3+2/105*(71*c^2+25*d^2)/d/(c^2-d^2)^3*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+32/105*d*cos(f*x+e)^2/(c^2-d^2
)^4*c*(11*c^2+13*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(105*c^4+25
4*c^2*d^2+25*d^4)/(105*c^8-420*c^6*d^2+630*c^4*d^4-420*c^2*d^6+105*d^8)*(1/
```

```
d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+32/105*d*c*(11*c^2+13*
d^2)/(c^2-d^2)^4*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3
*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+2/15*d*cos(f*x+e)^2/(c^2-d^
2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*
d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(1/d*c-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*b^2*(a*d-b*c)/d^3*(2/3/(c^2-d^2)/
d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*d*cos(f*
x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/
(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

**Fricas** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 4727, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/315*((sqrt(2)*(16*b^3*c^7*d^4 + 36*a*b^2*c^6*d^5 + 2*(45*a^2*b - 26*b^3)*
c^5*d^6 - (37*a^3 + 285*a*b^2)*c^4*d^7 - 36*(9*a^2*b - 2*b^3)*c^3*d^8 + 2*(
173*a^3 + 543*a*b^2)*c^2*d^9 - 6*(153*a^2*b + 70*b^3)*c*d^10 + 15*(5*a^3 +
21*a*b^2)*d^11)*cos(f*x + e)^4 - 2*sqrt(2)*(48*b^3*c^9*d^2 + 108*a*b^2*c^8*
d^3 + 10*(27*a^2*b - 14*b^3)*c^7*d^4 - 3*(37*a^3 + 273*a*b^2)*c^6*d^5 - 2*(
441*a^2*b - 82*b^3)*c^5*d^6 + (1001*a^3 + 2973*a*b^2)*c^4*d^7 - 54*(57*a^2*
b + 22*b^3)*c^3*d^8 + (571*a^3 + 2031*a*b^2)*c^2*d^9 - 6*(153*a^2*b + 70*b^
3)*c*d^10 + 15*(5*a^3 + 21*a*b^2)*d^11)*cos(f*x + e)^2 - 4*(sqrt(2)*(16*b^3
*c^8*d^3 + 36*a*b^2*c^7*d^4 + 2*(45*a^2*b - 26*b^3)*c^6*d^5 - (37*a^3 + 285
*a*b^2)*c^5*d^6 - 36*(9*a^2*b - 2*b^3)*c^4*d^7 + 2*(173*a^3 + 543*a*b^2)*c^
3*d^8 - 6*(153*a^2*b + 70*b^3)*c^2*d^9 + 15*(5*a^3 + 21*a*b^2)*c*d^10)*cos(
f*x + e)^2 - sqrt(2)*(16*b^3*c^10*d + 36*a*b^2*c^9*d^2 + 18*(5*a^2*b - 2*b^
3)*c^8*d^3 - (37*a^3 + 249*a*b^2)*c^7*d^4 - 2*(117*a^2*b - 10*b^3)*c^6*d^5
+ 3*(103*a^3 + 267*a*b^2)*c^5*d^6 - 6*(207*a^2*b + 58*b^3)*c^4*d^7 + (421*a
^3 + 1401*a*b^2)*c^3*d^8 - 6*(153*a^2*b + 70*b^3)*c^2*d^9 + 15*(5*a^3 + 21*
a*b^2)*c*d^10))*sin(f*x + e) + sqrt(2)*(16*b^3*c^11 + 36*a*b^2*c^10*d + 2*(
45*a^2*b + 22*b^3)*c^9*d^2 - (37*a^3 + 69*a*b^2)*c^8*d^3 + 8*(27*a^2*b - 28
*b^3)*c^7*d^4 + 4*(31*a^3 - 147*a*b^2)*c^6*d^5 - 4*(693*a^2*b + 10*b^3)*c^5
*d^6 + 2*(1057*a^3 + 3273*a*b^2)*c^4*d^7 - 72*(81*a^2*b + 34*b^3)*c^3*d^8 +
 4*(199*a^3 + 744*a*b^2)*c^2*d^9 - 6*(153*a^2*b + 70*b^3)*c*d^10 + 15*(5*a^
3 + 21*a*b^2)*d^11))*sqrt(I*d)*weierstrassPInverse(-4/3*(4*c^2 - 3*d^2)/d^2
, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) - 3*I*d*sin(f*x +
e) - 2*I*c)/d) + (sqrt(2)*(16*b^3*c^7*d^4 + 36*a*b^2*c^6*d^5 + 2*(45*a^2*b
- 26*b^3)*c^5*d^6 - (37*a^3 + 285*a*b^2)*c^4*d^7 - 36*(9*a^2*b - 2*b^3)*c^3
*d^8 + 2*(173*a^3 + 543*a*b^2)*c^2*d^9 - 6*(153*a^2*b + 70*b^3)*c*d^10 + 15
*(5*a^3 + 21*a*b^2)*d^11)*cos(f*x + e)^4 - 2*sqrt(2)*(48*b^3*c^9*d^2 + 108*
a*b^2*c^8*d^3 + 10*(27*a^2*b - 14*b^3)*c^7*d^4 - 3*(37*a^3 + 273*a*b^2)*c^6
*d^5 - 2*(441*a^2*b - 82*b^3)*c^5*d^6 + (1001*a^3 + 2973*a*b^2)*c^4*d^7 - 5
4*(57*a^2*b + 22*b^3)*c^3*d^8 + (571*a^3 + 2031*a*b^2)*c^2*d^9 - 6*(153*a^2
*b + 70*b^3)*c*d^10 + 15*(5*a^3 + 21*a*b^2)*d^11)*cos(f*x + e)^2 - 4*(sqrt(
2)*(16*b^3*c^8*d^3 + 36*a*b^2*c^7*d^4 + 2*(45*a^2*b - 26*b^3)*c^6*d^5 - (37
*a^3 + 285*a*b^2)*c^5*d^6 - 36*(9*a^2*b - 2*b^3)*c^4*d^7 + 2*(173*a^3 + 543
*a*b^2)*c^3*d^8 - 6*(153*a^2*b + 70*b^3)*c^2*d^9 + 15*(5*a^3 + 21*a*b^2)*c*
d^10)*cos(f*x + e)^2 - sqrt(2)*(16*b^3*c^10*d + 36*a*b^2*c^9*d^2 + 18*(5*a^
2*b - 2*b^3)*c^8*d^3 - (37*a^3 + 249*a*b^2)*c^7*d^4 - 2*(117*a^2*b - 10*b^3
)*c^6*d^5 + 3*(103*a^3 + 267*a*b^2)*c^5*d^6 - 6*(207*a^2*b + 58*b^3)*c^4*d^
7 + (421*a^3 + 1401*a*b^2)*c^3*d^8 - 6*(153*a^2*b + 70*b^3)*c^2*d^9 + 15*(5
*a^3 + 21*a*b^2)*c*d^10))*sin(f*x + e) + sqrt(2)*(16*b^3*c^11 + 36*a*b^2*c^
10*d + 2*(45*a^2*b + 22*b^3)*c^9*d^2 - (37*a^3 + 69*a*b^2)*c^8*d^3 + 8*(27*
a^2*b - 28*b^3)*c^7*d^4 + 4*(31*a^3 - 147*a*b^2)*c^6*d^5 - 4*(693*a^2*b + 1
0*b^3)*c^5*d^6 + 2*(1057*a^3 + 3273*a*b^2)*c^4*d^7 - 72*(81*a^2*b + 34*b^3)
*c^3*d^8 + 4*(199*a^3 + 744*a*b^2)*c^2*d^9 - 6*(153*a^2*b + 70*b^3)*c*d^10
+ 15*(5*a^3 + 21*a*b^2)*d^11))*sqrt(-I*d)*weierstrassPInverse(-4/3*(4*c^2 -
```

```
3*d^2)/d^2, -8/27*(-8*I*c^3 + 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x + e) + 3*I*
d*sin(f*x + e) + 2*I*c)/d) + 3*(sqrt(2)*(8*I*b^3*c^6*d^5 + 18*I*a*b^2*c^5*d
^6 + I*(45*a^2*b - 23*b^3)*c^4*d^7 - 4*I*(44*a^3 + 93*a*b^2)*c^3*d^8 + 6*I*
(153*a^2*b + 49*b^3)*c^2*d^9 - 2*I*(104*a^3 + 399*a*b^2)*c*d^10 + 21*I*(9*a
^2*b + 5*b^3)*d^11)*cos(f*x + e)^4 + 2*sqrt(2)*(-24*I*b^3*c^8*d^3 - 54*I*a*
b^2*c^7*d^4 - I*(135*a^2*b - 61*b^3)*c^6*d^5 + 6*I*(88*a^3 + 183*a*b^2)*c^5
*d^6 - I*(2799*a^2*b + 859*b^3)*c^4*d^7 + 2*I*(400*a^3 + 1383*a*b^2)*c^3*d^
8 - 3*I*(495*a^2*b + 203*b^3)*c^2*d^9 + 2*I*(104*a^3 + 399*a*b^2)*c*d^10 -
21*I*(9*a^2*b + 5*b^3)*d^11)*cos(f*x + e)^2 + 4*(sqrt(2)*(-8*I*b^3*c^7*d^4
- 18*I*a*b^2*c^6*d^5 - I*(45*a^2*b - 23*b^3)*c^5*d^6 + 4*I*(44*a^3 + 93*a*b
^2)*c^4*d^7 - 6*I*(153*a^2*b + 49*b^3)*c^3*d^8 + 2*I*(104*a^3 + 399*a*b^2)*
c^2*d^9 - 21*I*(9*a^2*b + 5*b^3)*c*d^10)*cos(f*x + e)^2 + sqrt(2)*(8*I*b^3*
c^9*d^2 + 18*I*a*b^2*c^8*d^3 + 15*I*(3*a^2*b - b^3)*c^7*d^4 - 2*I*(88*a^3 +
 177*a*b^2)*c^6*d^5 + I*(963*a^2*b + 271*b^3)*c^5*d^6 - 6*I*(64*a^3 + 195*a
*b^2)*c^4*d^7 + 3*I*(369*a^2*b + 133*b^3)*c^3*d^8 - 2*I*(104*a^3 + 399*a*b^
2)*c^2*d^9 + 21*I*(9*a^2*b + 5*b^3)*c*d^10))*sin(f*x + e) + sqrt(2)*(8*I*b^
3*c^10*d + 18*I*a*b^2*c^9*d^2 + 5*I*(9*a^2*b + 5*b^3)*c^8*d^3 - 88*I*(2*a^3
 + 3*a*b^2)*c^7*d^4 + 4*I*(297*a^2*b + 41*b^3)*c^6*d^5 - 4*I*(316*a^3 + 753
*a*b^2)*c^5*d^6 + 2*I*(2871*a^2*b + 923*b^3)*c^4*d^7 - 8*I*(178*a^3 + 645*a
*b^2)*c^3*d^8 + 12*I*(171*a^2*b + 77*b^3)*c^2*d^9 - 2*I*(104*a^3 + 399*a*b^
2)*c*d^10 + 21*I*(9*a^2*b + 5*b^3)*d^11))*sqrt(I*d)*weierstrassZeta(-4/3*(4
*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, weierstrassPInverse(-4/
3*(4*c^2 - 3*d^2)/d^2, -8/27*(8*I*c^3 - 9*I*c*d^2)/d^3, 1/3*(3*d*cos(f*x +
e) - 3*I*d*sin(f*x + e) - 2*I*c)/d)) + 3*(sqrt(...
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2), x)
```

## 3.745 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=296

$$-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}+\frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2d(6abc$$

```
[Out] -2/3*d^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/f-2/3*d*(-3*a*d+7*b*c)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^2/f/((c+d
*sin(f*x+e))/(c+d))^(1/2)+2/3*d*(6*a*b*c*d-3*a^2*d^2-b^2*(2*c^2+d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b
^3/f/(c+d*sin(f*x+e))^(1/2)-2*(-a*d+b*c)^3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b
),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^3/(a+b)/f/(c+d*
sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.72, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2872, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{2d(-3a^2d^2+6abcd-(b^2(2c^2+d^2)))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^3f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d(7bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + (2*d*(7*b*c - 3*a*
d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*b^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*d*(6*a*b*c*d - 3*a^2*d^2 -
 b^2*(2*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*b^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*
d)^3*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(b^3*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}+\frac{2\int \frac{\frac{1}{2}(3bc^3+ad^3)-\frac{1}{2}d(2acd-b(9c^2+d^2))\sin(e+}{(a+b\sin(e+fx))\sqrt{c+d\,\mathrm{s}}}}{3b}$$

$$=-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}-\frac{2\int \frac{\frac{1}{2}d(acd(7bc-3ad)-b(3bc^3+ad^3))+\frac{1}{2}d^2(6abc}{(a+b\sin(e+fx))\sqrt{c+d}}}{3b^2d}$$

$$=-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}+\frac{(bc-ad)^3\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(}}}{b^3}$$

$$=-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}+\frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{3b^2f\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

$$=-\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf}+\frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{3b^2f\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 25.58, size = 606, normalized size = 2.05

$$\frac{\frac{4i(-2acd+b(9c^2+d^2))\left((-bc+ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)-ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\sec(e+fx)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{\frac{d(1+\sin(e+fx))}{-c+d}}}{b\sqrt{-\frac{1}{c+d}}(bc-ad)}+\frac{2i(-7bc+3ad)\left(-2b(c-d)(bc-ad)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+d\left(2(a+b)(-bc+ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+(-2a^2+b^2)d\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\right)\sec(e+fx)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{\frac{d(1+\sin(e+fx))}{-c+d}}}{b^2\sqrt{-\frac{1}{c+d}}(bc-ad)}-4d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}-\frac{2(6bc^3+7bcd^2-ad^3)\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)\sqrt{c+d\sin(e+fx)}}}{6bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (((4*I)*(-2*a*c*d + b*(9*c^2 + d^2))*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*Ellipti
cPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
 + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c
 + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c
- a*d)) + ((2*I)*(-7*b*c + 3*a*d)*(-2*b*(c - d)*(b*c - a*d)*EllipticE[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(2
*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^2 + b^2)*d*EllipticPi[(b*(c + d))/
(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d
*(1 + Sin[e + f*x]))/(-c + d)])/(b^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - 4*d
^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]] - (2*(6*b*c^3 + 7*b*c*d^2 - a*d^3)
*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/(6*b*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(375) = 750$.
time = 19.82, size = 1190, normalized size = 4.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1190 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d/b^3*(d^2*b^2*(-2/3/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)))+2*(-a*b*d^2+3*b^2*c*d)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
```

```
(c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2)))+2*a^2*d^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6
*a*b*c*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^
(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+6*b^2*c^2*
(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*(-a^3*d^3+3*a^2*
b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d
))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(
f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a), x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x)), x)
```

# 3.746 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=229

$$\frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2d(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{b^2f\sqrt{c+d\sin(e+fx)}}+2$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/b/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*d*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/f/(c+d*sin(f*x+e
))^(1/2)-2*(-a*d+b*c)^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))
^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2883, 2734, 2732, 2882, 2742, 2740, 2886, 2884}

$$\frac{2d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(b*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*(b*c - a*d)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*f*S
qrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)^2*EllipticPi[(2*b)/(a + b), (e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a +
 b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2883

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{c+d\sin(e+fx)}\,dx}{b} - \frac{(-bc+ad)\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(d(bc-ad))\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin}}}{b^2}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left(d(bc-ad)\sqrt{\frac{c+d\sin(e+}{c+d}}\right.}{b^2\sqrt{c}}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{2d(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+f\right.\right.}{b^2f\sqrt{c+d}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 13.18, size = 242, normalized size = 1.06

$$\frac{2i\left(b(c-d)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)\middle|\frac{c+d}{c-d}\right)+(ad+b(-2c+d))F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)\middle|\frac{c+d}{c-d}\right)+(bc-ad)\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)\middle|\frac{c+d}{c-d}\right)\right)\sec(e+fx)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sin(e+fx))}{c-d}}}{b^2\sqrt{-\frac{1}{c+d}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((2*I)*(b*(c - d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (a*d + b*(-2*c + d))*EllipticF[I*ArcSinh[Sqrt[
-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (b*c - a*d)*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]
))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))])/(b^2*Sqrt[-(c + d)^(-
1)]*f)
```

**Maple** [A]
time = 7.46, size = 391, normalized size = 1.71

| method | result |
|---|---|
| default | $-\frac{2\left(\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bc+\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bd+a\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c+Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d+a*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d-2*b*c*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d-EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/
2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*a*d+EllipticPi(((c+d*sin(f*x+e))
/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*b*c)/b^2*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x)),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x)), x)
```

# 3.747 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=153

$$\frac{2dF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{bf\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{b(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d
))^(1/2)/b/f/(c+d*sin(f*x+e))^(1/2)-2*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*
b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a+b)/f/(
c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2882, 2742, 2740, 2886, 2884}

$$\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/(b*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)*EllipticPi[(2*b)
/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sin(e+fx)}} + \frac{\left((bc-ad)\sqrt{\frac{c+\cdots}{\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{2dF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{bf\sqrt{c+d\sin(e+fx)}} + \frac{2(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+\cdots\right.\right.}{b(a+b)f\sqrt{c\cdots}}$$

**Mathematica [A]**
time = 2.37, size = 114, normalized size = 0.75

$$-\frac{2\left((a+b)dF\left(\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)+(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{b(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*((a + b)*d*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (b*c - a*d
)*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple [A]**
time = 6.86, size = 181, normalized size = 1.18

| method | result |
|---|---|
| default | $\frac{2\left(\text{EllipticF}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)-\text{EllipticPi}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},-\frac{(c-d)b}{ad-bc},\sqrt{\frac{c-d}{c+d}}\right)\right)\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\sqrt{-}}{b\cos(fx+e)\sqrt{c+d\sin\left(fx+e\right)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2)))/b*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x)),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x)), x)
```

# 3.748 $\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=75

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2886, 2884}

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}}{\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

**Mathematica** [A]
time = 0.13, size = 74, normalized size = 0.99

$$-\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e+\pi-2fx)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
[Out] (-2*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c
 + d*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [A]
time = 5.24, size = 151, normalized size = 2.01

| method | result |
|---|---|
| default | $\frac{2(c-d)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(-1+\sin(fx+e))d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},-\frac{(c-d)b}{ad-bc},\sqrt{\frac{c-d}{c+d}}\right)}{(ad-bc)\cos(fx+e)\sqrt{c+d\sin(fx+e)}\,f}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-
d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))/(a*d-b*c)/cos(f*x+e)/(c+d*sin(f*x+e))^(
1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(13),SparseUnivariatePolynomial(InnerPrimeField(13)),?^2+4*?+12)),f
ailed)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sin{(e + fx)})\sqrt{c + d\sin{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,\sin{(e + f\,x)})\,\sqrt{c + d\,\sin{(e + f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.749 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=220

$$-\frac{2d^2\cos(e+fx)}{(bc-ad)(c^2-d^2)f\sqrt{c+d\sin(e+fx)}}-\frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\mid\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{(bc-ad)(c^2-d^2)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\right.\right.}{(a+b)(}$$

```
[Out] -2*d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+2*d*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(-a*d+b*c)/
(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/
(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a+b)/(-a*d+b
*c)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.40, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2881, 3138, 2734, 2732, 12, 2886, 2884}

$$-\frac{2d^2\cos(e+fx)}{f(c^2-d^2)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{f(c^2-d^2)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\mid\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
- (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*El
lipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/((a + b)*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}(-acd+b( \cdots}{(a \cdots}}{\cdots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int -\frac{\cdots}{2(a+b\sin \cdots}}{ba \cdots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\int \frac{\cdots}{(a+b\sin(e+ \cdots}}{\cdots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2} \cdots\right.\right.}{(bc-ad) \cdots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2} \cdots\right.\right.}{(bc-ad) \cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 26.71, size = 617, normalized size = 2.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*((4*d^2*Cos[e + f*x])/((c^2 - d^2)*Sqrt[c + d*Sin[e + f*x]]) + (((4*I)
*(b*c + a*d)*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*EllipticPi[(b*(c + d))/(b*c - a
*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin
[e + f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - ((2*I)*(-2*b*(
c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
 + f*x]]], (c + d)/(c - d)] + d*(2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^
2 + b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)
```

```
]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1
+ Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-
(c + d)^(-1)]*(b*c - a*d)) + (2*(2*b*c^2 - 2*a*c*d - 3*b*d^2)*EllipticPi[(2
*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/((c - d)*(c + d)))/((b*c -
a*d)*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(276) = 552$.
time = 19.35, size = 610, normalized size = 2.77

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{d\left(\frac{2d\left(\cos^{2}\left(fx+e\right)\right)}{\left(c^{2}-d^{2}\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}+\cdots\right.}{\cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d/(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2
-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(a*d-b*c)*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*si
n(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.750 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=399

$$-\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}-\frac{2d^2\left(7bc^2-4acd-3bd^2\right)\cos(e+fx)}{3(bc-ad)^2\left(c^2-d^2\right)^2f\sqrt{c+d\sin(e+fx)}}-\frac{2d(7bc^2-4ac}{3(}$$

```
[Out] -2/3*d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)-2/3*d^2*(
-4*a*c*d+7*b*c^2-3*b*d^2)*cos(f*x+e)/(-a*d+b*c)^2/(c^2-d^2)^2/f/(c+d*sin(f*
x+e))^(1/2)+2/3*d*(-4*a*c*d+7*b*c^2-3*b*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(-a*d+b*c)^2/(c^2-d^2)^2/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e
+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2
))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(
1/2)-2*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d
*sin(f*x+e))/(c+d))^(1/2)/(a+b)/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.07, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2b^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\left(-4acd+7bc^2-3bd^2\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\cos(e+fx)}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3f\left(c^2-d^2\right)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\left(-4acd+7bc^2-3bd^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/
2)) - (2*d^2*(7*b*c^2 - 4*a*c*d - 3*b*d^2)*Cos[e + f*x])/(3*(b*c - a*d)^2*(
c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*d*(7*b*c^2 - 4*a*c*d - 3*b*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*(b*c - a*d)^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*
EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(3*(b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (2*b^2*Ellip
ticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
 f*x])/(c + d)])/((a + b)*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}(acd-}{}}{}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2(7b}{3(bc-ad)}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2(7b}{3(bc-ad)}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2(7b}{3(bc-ad)}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2(7b}{3(bc-ad)}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2(7b}{3(bc-ad)}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 26.97, size = 1079, normalized size = 2.70

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)
*(c + d*Sin[e + f*x])^2) + (2*(-7*b*c^2*d^2*Cos[e + f*x] + 4*a*c*d^3*Cos[e
+ f*x] + 3*b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^2 - d^2)^2*(c + d*Sin[e
 + f*x]))))/f + ((-2*(6*b^2*c^4 - 12*a*b*c^3*d + 6*a^2*c^2*d^2 - 19*b^2*c^2
*d^2 + 8*a*b*c*d^3 + 2*a^2*d^4 + 9*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e +
 Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)
*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-12*b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*
c*d^3 + 4*b^2*c*d^3 + 8*a*b*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcS
```

```
inh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*E
llipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[
-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(
b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
 f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x]
)^2)/d^2)]) - ((2*I)*(7*b^2*c^2*d^2 - 4*a*b*c*d^3 - 3*b^2*d^4)*Cos[e + f*x]
*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c)
 + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqr
t[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*
c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*
(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Si
n[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[
e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(6*(c - d)^2*(c + d)^2*(b*c - a
*d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(474) = 948$.
time = 31.61, size = 1072, normalized size = 2.69

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1072 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-d*b/(a*d-b*c)^2*(2*d*cos(f*x+e)^2
/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*b/(a*d-b*c)^2*(1/d*c-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)
)+d/(a*d-b*c)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin
(f*x+e)+1/d*c)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
```

```
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(19),SparseUnivariatePolynomial(InnerPrimeField(19)),?^2+7)),failed
) cann
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\sin(e+fx))\,(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.751 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=534

$$\frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2(a^2-b^2)f}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx)}{b(a^2-b^2)f(a+b\sin(e+fx))}$$

```
[Out] (-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e
))+1/3*d*(6*a*b*c*d-5*a^2*d^2-b^2*(3*c^2-2*d^2))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(1/2)/b^2/(a^2-b^2)/f-1/3*(29*a^2*b*c*d^2-15*a^3*d^3+b^3*(3*c^3-20*c*d^2)
-a*b^2*(9*c^2*d-12*d^3))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*
Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c
+d*sin(f*x+e))^(1/2)/b^3/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/3*(24
*a^3*b*c*d^3-15*a^4*d^4-12*a*b^3*c*d*(c^2+3*d^2)+2*a^2*b^2*d^2*(c^2+8*d^2)+
b^4*(3*c^4+16*c^2*d^2+2*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e
+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2
))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^4/(a^2-b^2)/f/(c+d*sin(f*x+e))^(1/2)-(-
a*d+b*c)^3*(5*a^2*d+2*a*b*c-7*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1
/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b^4/(a+b)^2/f/(c+
d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.31, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2871, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{d(-5a^2d^2+6abcd-(b^2(3c^2-2d^2)))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2f(a^2-b^2)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{bf(a^2-b^2)(a+b\sin(e+fx))}+\frac{(5a^2d+2abc-7b^2d)(bc-ad)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^4f(a-b)(a+b)^2\sqrt{c+d\sin(e+fx)}}+\frac{(-15a^3d^3+29a^2bcd^2-ab^2(9c^2d-12d^3)+b^3(3c^3-20cd^2))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^3f(a^2-b^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(-15a^4d^4+24a^3bcd^3+2a^2b^2d^2(c^2+8d^2)-12ab^3cd(c^2+3d^2)+b^4(3c^4+16c^2d^2+2d^4))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^4f(a^2-b^2)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d*(6*a*b*c*d - 5*a^2*d^2 - b^2*(3*c^2 - 2*d^2))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3*b^2*(a^2 - b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin
[e + f*x])^(3/2))/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])) + ((29*a^2*b*c*d^2
 - 15*a^3*d^3 + b^3*(3*c^3 - 20*c*d^2) - a*b^2*(9*c^2*d - 12*d^3))*Elliptic
E[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*b^3*(a^2
- b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((24*a^3*b*c*d^3 - 15*a^4*d^
4 - 12*a*b^3*c*d*(c^2 + 3*d^2) + 2*a^2*b^2*d^2*(c^2 + 8*d^2) + b^4*(3*c^4 +
 16*c^2*d^2 + 2*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(3*b^4*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
+ ((b*c - a*d)^3*(2*a*b*c + 5*a^2*d - 7*b^2*d)*EllipticPi[(2*b)/(a + b), (e
```

```
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
b)*b^4*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{\int \sqrt{c+d\sin(e+fx)}\,\left(\frac{1}{2}(7b^2c\right.}{}$$

$$= \frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\,(a^2-b^2)\,f} + \frac{(bc-a}{b}$$

$$= \frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\,(a^2-b^2)\,f} + \frac{(bc-a}{b}$$

$$= \frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\,(a^2-b^2)\,f} + \frac{(bc-a}{b}$$

$$= \frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\,(a^2-b^2)\,f} + \frac{(bc-a}{b}$$

$$= \frac{d(6abcd-5a^2d^2-b^2(3c^2-2d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\,(a^2-b^2)\,f} + \frac{(bc-a}{b}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 27.67, size = 1109, normalized size = 2.08

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^3*Cos[e + f*x])/(3*b^2) + (-(b^3*c^3*Cos[e
 + f*x]) + 3*a*b^2*c^2*d*Cos[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^
3*Cos[e + f*x])/(b^2*(-a^2 + b^2)*(a + b*Sin[e + f*x]))))/f - ((-2*(-12*a*b
^2*c^4 + 39*b^3*c^3*d - 45*a*b^2*c^2*d^2 + a^2*b*c*d^3 + 20*b^3*c*d^3 + 5*a
^3*d^4 - 8*a*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*
x]]) - ((2*I)*(-12*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 72*b^3*c^2*d^2 + 20*a^3
*c*d^3 - 56*a*b^2*c*d^3 + 8*a^2*b*d^4 + 4*b^3*d^4)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
```

```
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(3*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 29*
a^2*b*c*d^3 - 20*b^3*c*d^3 - 15*a^3*d^4 + 12*a*b^2*d^4)*Cos[e + f*x]*Cos[2*
(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]
*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[S
qrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f
*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(12*(a - b)*b^2*(a + b)*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(612) = 1224$.

time = 32.53, size = 1886, normalized size = 3.53

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1886 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^4*(d^2*b^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))))+2*(-2*a*b*d^2+4*b^2*c*d)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))+6*a^2*d^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))-16*a*b*c*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
```

```
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+12*b^
2*c^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/b^4*(a^4*d
^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2
*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1
/d*c+a/b),((c-d)/(c+d))^(1/2)))-8/b^5*d*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/
d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^2, x)
```

# 3.752 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=390

$$\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))}-\frac{(2abcd-3a^2d^2-b^2(c^2-2d^2))\,E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+}}{b^2\,(a^2-b^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e
))+(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/
(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^2/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d
))^(1/2)-(-a*d+b*c)*(2*a*b*c*d+3*a^2*d^2-b^2*(c^2+4*d^2))*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^3/(a^2-b^2)
/f/(c+d*sin(f*x+e))^(1/2)-(-a*d+b*c)^2*(3*a^2*d+2*a*b*c-5*b^2*d)*(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/
4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(
1/2)/(a-b)/b^3/(a+b)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.80, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2871, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(-3a^2d^2+2abcd-(b^2(c^2-2d^2)))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\,(a^2-b^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\,(a^2-b^2)\,(a+b\sin(e+fx))}+\frac{(3a^2d^2+2abcd-(b^2(c^2+4d^2)))\,(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f\,(a^2-b^2)\sqrt{c+d\sin(e+fx)}}+\frac{(3a^2d+2abc-5b^2d)\,(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f(a-b)(a+b)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*(a +
 b*Sin[e + f*x])) - ((2*a*b*c*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(b^2*(a^2 - b^
2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((b*c - a*d)*(2*a*b*c*d + 3*a^2*
d^2 - b^2*(c^2 + 4*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
(c + d*Sin[e + f*x])/(c + d)])/(b^3*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
 + ((b*c - a*d)^2*(2*a*b*c + 3*a^2*d - 5*b^2*d)*EllipticPi[(2*b)/(a + b), (
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
 b)*b^3*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}(5b^2c^2d+a^2d^3-2abc(c^2+2d^2))-d(a^2c}{(a-}}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d(bc-ad)(abc^2+3a^2cd-5b^2cd+abd^2)-}{(a+b\sin(e+fx))}}{b^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{((bc-ad)^2\,(2abc+3a^2d-5b^2d}{2b}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{(2abcd-3a^2d^2-b^2(c^2-2d^2))}{b^2\,(a^2-b^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{(2abcd-3a^2d^2-b^2(c^2-2d^2))}{b^2\,(a^2-b^2}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 27.44, size = 986, normalized size = 2.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x] - a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*(a + b*Sin[e + f*x])) + ((-2*(4
*a*b*c^3 - 9*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3 - 2*b^2*d^3)*EllipticPi[(2*b
)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b*c^2*d + 4*a^2*c*
d^2 - 12*b^2*c*d^2 + 4*a*b*d^3)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
```

```
^2)/d^2)]) - ((2*I)*(-(b^2*c^2*d) + 2*a*b*c*d^2 - 3*a^2*d^3 + 2*b^2*d^3)*Co
s[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a +
b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c -
a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c -
d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b
*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c
*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*b*(a + b)*
f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(476) = 952$.

time = 28.56, size = 1363, normalized size = 3.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1363 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^3*(2*b*d*(1/d*c-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-4*a*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+6*b*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))+1/b^3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-b^2/(a^3
*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(
f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
```

```
+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c
+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))+6/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*
(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*
c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((
c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin(e+f\,x))^{5/2}}{(a+b\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^2, x)
```

# 3.753 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=351

$$\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))}+\frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{b\,(a^2-b^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(2abcd+}{}$$

```
[Out] (-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))-(
-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^
(1/2)/b/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-(2*a*b*c*d+a^2*d^2-b^2*(
c^2+2*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e
))/(c+d))^(1/2)/b^2/(a^2-b^2)/f/(c+d*sin(f*x+e))^(1/2)-(-a*d+b*c)*(a^2*d+2*
a*b*c-3*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x
)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*(
(c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b^2/(a+b)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.67, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2878, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{\left(a^2d^2+2abcd-\left(b^2\left(c^2+2d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{(bc-ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(bc-ad)\left(a^2d+2abc-3b^2d\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a-b)(a+b)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*S
in[e + f*x])) + ((b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) + ((2*a*b*c*d + a^2*d^2 - b^2*(c^2 + 2*d^2))*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a^2 - b^2)*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/((a - b)*b^2*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2878

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d
*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) +
 (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*
d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
 && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}\left(3bcd-a\left(2c^2+d^2\right)\right)-d(ac-bd)\sin(e+f\cdots}{(a+b\sin(e+fx))\sqrt{c+d\sin\cdots}}}{-a^2+b^2\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d\left(a^2cd-3b^2cd+ab\left(c^2+d^2\right)\right)+\frac{1}{2}d\left(2abcd\cdots\right.}{(a+b\sin(e+fx))\sqrt{c+d\cdots}}}{b\left(a^2-b^{2}\cdots\right.}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\left((bc-ad)\left(2abc+a^2d-3b^2d\right)\right)\cdots}{2b^2\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\cdots}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin\cdots}{c\cdots}}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\cdots}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin\cdots}{c\cdots}}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 26.92, size = 891, normalized size = 2.54

$$\frac{(bc\cos(e+fx)-ad\cos(e+fx))\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)f(a+b\sin(e+fx))}+\frac{-\frac{2(4ac^2-5bcd+ad^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)\sqrt{c+d\sin(e+fx)}}-\frac{2i(4acd-4bd^2)\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\sqrt{\frac{d-d\sin(e+fx)}{c+d}}\sqrt{-\frac{d+d\sin(e+fx)}{c-d}}(-bc+ad+b(c+d\sin(e+fx)))}{bd^2\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}\sqrt{-\frac{c^2-d^2-2c(c+d\sin(e+fx))+(c+d\sin(e+fx))^2}{d^2}}}-\frac{2i(-bcd+ad^2)\cos(e+fx)\cos(2(e+fx))\left(2b(c-d)(bc-ad)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+d\left(-2(a+b)(-bc+ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+(2a^2-b^2)d\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\right)\sqrt{\frac{d-d\sin(e+fx)}{c+d}}\sqrt{-\frac{d+d\sin(e+fx)}{c-d}}(-bc+ad+b(c+d\sin(e+fx)))}{b^2d\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}\left(-2c^2+d^2+4c(c+d\sin(e+fx))-2(c+d\sin(e+fx))^2\right)\sqrt{-\frac{c^2-d^2-2c(c+d\sin(e+fx))+(c+d\sin(e+fx))^2}{d^2}}}}{4(a-b)(a+b)f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x])) + ((-2*(4*a*c^2 - 5*b*c*d + a*d^2)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*c*d - 4*b*d^2)*C
os[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d),
 I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])
*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(
-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a
*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b*c*d) + a
*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(
-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c +
d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))
/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d
^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
 + b)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(439) = 878$.
time = 25.03, size = 1027, normalized size = 2.93

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(\frac{2d^2\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{\frac{d(1-\sin(fx+e))}{c+d}}\sqrt{\frac{(-\sin(fx+e)-}{c-d}}}{b^2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos\right.}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^2/b^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+1/b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))-4/b^3*d*(a*d-b*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{3/2}}{(a+b\,\sin\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^2, x)
```

## 3.754 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=307

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))}+\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right.\right.}{b\,(a^2-b^2)\,f\sqrt{c}}$$

```
[Out] b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))-(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/((
c+d*sin(f*x+e))/(c+d))^(1/2)+(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a^2-b^2)/f/(c+d*sin(f*x+e))^
(1/2)-(-a^2*d+2*a*b*c-b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+
1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b/(a+b)^2/f/(c+d*sin(f*x+
e))^(1/2)
```

**Rubi** [A]
time = 0.55, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2875, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\,(a^2-b^2)\,(a+b\sin(e+fx))}-\frac{(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\,(a^2-b^2)\,\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\,(a^2-b^2)\,\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(a^2(-d)+2abc-b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a-b)(a+b)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*Sin[e + f*x
])) + (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*d)*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(b*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - a^2*d - b^2*d)*Ell
ipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/((a - b)*b*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2875

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
 + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}(-2ac+bd)-ad\sin(e+fx)-\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{-a^2+b^2}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2\,(a^2-b^2)} + \frac{\int \frac{\frac{1}{2}bd(ac-bc}{(a+b\sin(e+fx}}{}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} - \frac{(bc-ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2b\,(a^2-b^2)} + \frac{(2c}{}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx}}{(a^2-b^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx}}{(a^2-b^2)\,f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 26.74, size = 846, normalized size = 2.76

$$-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(-a^2+b^2)f(a+b\sin(e+fx))}+\frac{-\frac{2(4ac-bd)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)\sqrt{c+d\sin(e+fx)}}-\frac{8ia\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\sqrt{\frac{d-d\sin(e+fx)}{c+d}}\sqrt{-\frac{d+d\sin(e+fx)}{c-d}}(-bc+ad+b(c+d\sin(e+fx)))}{bd\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}\sqrt{-\frac{c^2-d^2-2c(c+d\sin(e+fx))+(c+d\sin(e+fx))^2}{d^2}}}+\frac{2i\cos(e+fx)\cos(2(e+fx))\left(2b(c-d)(bc-ad)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+d\left(-2(a+b)(-bc+ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+(2a^2-b^2)d\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)\right)\right)\sqrt{\frac{d-d\sin(e+fx)}{c+d}}\sqrt{-\frac{d+d\sin(e+fx)}{c-d}}(-bc+ad+b(c+d\sin(e+fx)))}{b\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}(-2c^2+d^2+4c(c+d\sin(e+fx))-2(c+d\sin(e+fx))^2)\sqrt{-\frac{c^2-d^2-2c(c+d\sin(e+fx))+(c+d\sin(e+fx))^2}{d^2}}}}{4(a-b)(a+b)f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((-a^2 + b^2)*f*(a + b*Sin[e +
f*x]))) + ((-2*(4*a*c - b*d)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2,
 (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin
[e + f*x]]) - ((8*I)*a*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[
(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f
*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*S
in[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d*Sqrt[-
(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqr
t[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])
+ ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I
*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] +
d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c +
d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e +
f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*Sqrt[-(c + d)^(
-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^
2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
- 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
+ b)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(393) = 786$.
time = 24.78, size = 872, normalized size = 2.84

| method | result |
|---|---|
| default | $\frac{\sqrt{-(-d\sin(fx+e)-c)(\cos^2(fx+e))}\left(\frac{(-ad+bc)\left(-\frac{b^2\sqrt{-(-d\sin(fx+e)-c)(\cos^2(fx+e))}}{(a^3d-a^2bc-ab^2d+b^3c)(a+b\sin(fx+e))}\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-a*d+b*c)/b*(-b^2/(a^3*d-a^2*b*c-
a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b
^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c
+a/b),((c-d)/(c+d))^(1/2)))+2*d/b^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-
d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^2, x)
```

# 3.755 $\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=325

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))}+\frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\frac{2}{c}\right.}{(a^2-b^2)f\sqrt{}}$$

```
[Out] b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e
))-b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/(a^2-b^2)/(-a*d+b*c)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a^2-b^2)/f/(c+d
*sin(f*x+e))^(1/2)-(-3*a^2*d+2*a*b*c+b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+
b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^2/(-
a*d+b*c)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.62, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2881, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))}-\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{f(a^2-b^2)\sqrt{c+d\sin(e+fx)}}+\frac{b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{f(a^2-b^2)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(-3a^2d+2abc+b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{f(a-b)(a+b)^2(bc-ad)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*(a +
 b*Sin[e + f*x])) + (b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c
+ d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)]) - (EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
f*x])/(c + d)])/((a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - 3*a^
2*d + b^2*d)*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(a + b)^2*(b*c - a*d)*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}(-2abc+2a^2d-\ldots}{(a+b\sin(e+\ldots}}{(\ldots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} + \frac{b\int \sqrt{c+d\sin(\ldots}}{2\,(a^2-b^2)\,(\ldots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{1}{\sqrt{c+d\sin(e\ldots}}}{2\,(a^2-\ldots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\ldots\right.\right.}{(a^2-b^2)\,(bc\ldots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\ldots\right.\right.}{(a^2-b^2)\,(bc\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 26.89, size = 871, normalized size = 2.68

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(-(b*c) + a*d)*f
*(a + b*Sin[e + f*x]))) + ((-2*(-4*a*b*c + 4*a^2*d - 3*b^2*d)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) + ((8*I)*a*Cos[e + f*x]*((b*c
- a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(
c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e
 + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(
c + d*Sin[e + f*x])))/(d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x
])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
(c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*
```

```
(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a
^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/
(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin
[e + f*x])))/(Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 -
 Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e
+ f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*
x])^2)/d^2)]))/(4*(a - b)*(a + b)*(-(b*c) + a*d)*f)
```

**Maple** [A]
time = 17.10, size = 690, normalized size = 2.12

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{b^{2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{\left(a^{3}d-a^{2}bc-a\,b^{2}d+b^{3}c\right)\left(a+b\sin\left(fx+e\right)\right)}-\frac{ad\left(\frac{c}{d}-1\right)}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*
d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(1/d*c
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)
*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(
c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.756 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=449

$$\frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)(bc-ad)^2(c^2-d^2)f\sqrt{c+d\sin(e+fx)}}+\frac{b^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))\sqrt{c+d\sin(e+}$$

```
[Out] d*(2*a^2*d^2+b^2*(c^2-3*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f
/(c+d*sin(f*x+e))^(1/2)+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+
e))/(c+d*sin(f*x+e))^(1/2)-(2*a^2*d^2+b^2*(c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/(
c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(c+d
*sin(f*x+e))^(1/2)-b*(-5*a^2*d+2*a*b*c+3*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^
2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b
/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^
2/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.06, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{f(a^2-b^2)(c^2-d^2)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{(2a^2d^2+b^2(c^2-3d^2))\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a^2-b^2)(c^2-d^2)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}-\frac{b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a^2-b^2)(bc-ad)\sqrt{c+d\sin(e+fx)}}+\frac{b(-5a^2d+2abc+3b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a-b)(a+b)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(c^2 - 3*d^2))*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)^2
*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/((a^2 - b^2)*
(b*c - a*d)*f*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + ((2*a^2*d^2
+ b^2*(c^2 - 3*d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) - (b*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[c + d*Sin[e +
 f*x]]) + (b*(2*a*b*c - 5*a^2*d + 3*b^2*d)*EllipticPi[(2*b)/(a + b), (e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(
a + b)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))\sqrt{c+d\sin(e+f}}$$

$$= \frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{}{(a}$$

$$= \frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{}{(a}$$

$$= \frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{}{(a}$$

$$= \frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{}{(a}$$

$$= \frac{d(2a^2d^2+b^2(c^2-3d^2))\cos(e+fx)}{(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f\sqrt{c+d\sin(e+fx)}} + \frac{}{(a}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 27.79, size = 1057, normalized size = 2.35

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)^2
*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((b*c - a*d)^2*(c^2 - d^2)*(c
 + d*Sin[e + f*x]))))/f + ((-2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 7*b^3*c^2*d +
 4*a^3*c*d^2 - 8*a*b^2*c*d^2 + 10*a^2*b*d^3 - 9*b^3*d^3)*EllipticPi[(2*b)/(
a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b^2*c^2*d + 4*a^2*b*c
*d^2 - 4*b^3*c*d^2 + 4*a^3*d^3 - 8*a*b^2*d^3)*Cos[e + f*x]*((b*c - a*d)*Ell
ipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
```

```
- d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(
c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[
e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqr
t[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d
*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b^3*c^2*d) - 2*a^2*b*d^3 + 3*b^3*d^3)*
Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
+ b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
- a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
 - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
- d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*
(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4
*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c
*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a + b)*
(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(533) = 1066.

time = 30.92, size = 1266, normalized size = 2.82

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1266 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-b/(a*d-b*c)*(-b^2/(a^3*d-a^2*b*c-
a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b
^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c
+a/b),((c-d)/(c+d))^(1/2)))+d^2/(a*d-b*c)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
```

```
-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))))-2*d/(a*d-b*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x
+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/
(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,\sin\left(e+f\,x\right))^2\,(c+d\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.757 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=661

$$\frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e$$

```
[Out] 1/3*d*(2*a^2*d^2+b^2*(3*c^2-5*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-
d^2)/f/(c+d*sin(f*x+e))^(3/2)+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*si
n(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/3*(8*a^3*c*d^4-8*a*b^2*c*d^4-4*a^2*b*d^3
*(5*c^2-3*d^2)-b^3*(3*c^4*d-26*c^2*d^3+15*d^5))*cos(f*x+e)/(a^2-b^2)/(-a*d+
b*c)^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)+1/3*(8*a^3*c*d^3-8*a*b^2*c*d^3-
4*a^2*b*d^2*(5*c^2-3*d^2)-b^3*(3*c^4-26*c^2*d^2+15*d^4))*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^3
/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/3*(2*a^2*d^2+b^2*(3*c^2-5*d
^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d
))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)-b^2*(-7*
a^2*d+2*a*b*c+5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi
+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^2/(-a*d+b*c)^3/f/(c+d*sin
(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.84, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}-\frac{(2a^2d^2+b^2(3c^2-5d^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{b^2\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}+\frac{b^2(-7a^2d+2abc+5b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a-b)(a+b)^2(bc-ad)^3\sqrt{c+d\sin(e+fx)}}-\frac{(8a^3cd^3-4a^2bd^2(5c^2-3d^2)-8ab^2cd^3-(b^3(3c^4-26c^2d^2+15d^4)))\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)^2(bc-ad)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(8a^3cd^4-4a^2bd^3(5c^2-3d^2)-8ab^2cd^4-b^3(3c^4d-26c^2d^3+15d^5))\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)^2(bc-ad)^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(3*c^2 - 5*d^2))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2)) + (b^2*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - ((8*
a^3*c*d^4 - 8*a*b^2*c*d^4 - 4*a^2*b*d^3*(5*c^2 - 3*d^2) - b^3*(3*c^4*d - 26
*c^2*d^3 + 15*d^5))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^3*c*d^3 - 8*a*b^2*c*d^3 - 4*a^2*b*d^2
*(5*c^2 - 3*d^2) - b^3*(3*c^4 - 26*c^2*d^2 + 15*d^4))*EllipticE[(e - Pi/2 +
 f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d
```

```
)^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((2*a^2*d^2 + b^2
*(3*c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[c +
 d*Sin[e + f*x]]) + (b^2*(2*a*b*c - 7*a^2*d + 5*b^2*d)*EllipticPi[(2*b)/(a
+ b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/((a - b)*(a + b)^2*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = \frac{b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))}$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{d(2a^2d^2+b^2(3c^2-5d^2))\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} +$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 28.48, size = 1319, normalized size = 2.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-((b^4*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)
^3*(a + b*Sin[e + f*x]))) + (2*d^3*Cos[e + f*x])/(3*(b*c - a*d)^2*(c^2 - d^
```

```
2)*(c + d*Sin[e + f*x])^2) - (4*(-5*b*c^2*d^3*Cos[e + f*x] + 2*a*c*d^4*Cos[
e + f*x] + 3*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin
[e + f*x]))))/f + ((-2*(-12*a*b^3*c^5 + 36*a^2*b^2*c^4*d - 33*b^4*c^4*d - 3
6*a^3*b*c^3*d^2 + 60*a*b^3*c^3*d^2 + 12*a^4*c^2*d^3 - 104*a^2*b^2*c^2*d^3 +
 86*b^4*c^2*d^3 + 28*a^3*b*c*d^4 - 40*a*b^3*c*d^4 + 4*a^4*d^5 + 44*a^2*b^2*
d^5 - 45*b^4*d^5)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c +
 d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]])
 - ((2*I)*(-12*a*b^3*c^4*d - 36*a^2*b^2*c^3*d^2 + 36*b^4*c^3*d^2 - 28*a^3*b
*c^2*d^3 + 52*a*b^3*c^2*d^3 + 16*a^4*c*d^4 + 4*a^2*b^2*c*d^4 - 20*b^4*c*d^4
 + 28*a^3*b*d^5 - 40*a*b^3*d^5)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
^2)/d^2)]) - ((2*I)*(3*b^4*c^4*d + 20*a^2*b^2*c^2*d^3 - 26*b^4*c^2*d^3 - 8*
a^3*b*c*d^4 + 8*a*b^3*c*d^4 - 12*a^2*b^2*d^5 + 15*b^4*d^5)*Cos[e + f*x]*Cos
[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a
*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSin
h[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d
 - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) +
 a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a +
 b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e
+ f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e +
f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(12*(a - b)*(a + b)*(c - d)^2*(c +
d)^2*(-(b*c) + a*d)^3*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1730 vs. $2(735) = 1470$.
time = 57.44, size = 1731, normalized size = 2.62

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1731 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2/(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b
*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-
a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
```

```
/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*
d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/
d*c+a/b),((c-d)/(c+d))^(1/2)))-2*d^2/(a*d-b*c)^3*b*(2*d*cos(f*x+e)^2/(c^2-d
^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))))+d^2/(a*d-b*c)^2*(2/3/(c^2-d^2)/d*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+1/d*c)^2+8/3*d*cos(f*x+e)^2/(c
^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*
c^2*d^2+3*d^4)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*
d*c/(c^2-d^2)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+4*
b/(a*d-b*c)^3*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)
/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.758 $\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=816

$$\frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d^2))\cos(e+fx)\sqrt{c+d\sin(e+}{12b^3\left(a^2-b^2\right)^2f}$$

```
[Out] 1/4*(-a*d+b*c)^2*(7*a^2*d+6*a*b*c-13*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/
2)/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(
f*x+e))^(5/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/12*d*(36*a^3*b*c*d^2-35*a^
4*d^3+b^4*d*(45*c^2-8*d^2)-18*a*b^3*c*(c^2+5*d^2)+a^2*b^2*d*(9*c^2+61*d^2))
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b^3/(a^2-b^2)^2/f-1/12*(185*a^4*b*c*d^3-
105*a^5*d^4-b^5*c*d*(51*c^2-104*d^2)-15*a^3*b^2*d^2*(3*c^2-13*d^2)-a^2*b^3*
c*d*(21*c^2+361*d^2)+9*a*b^4*(2*c^4+17*c^2*d^2-8*d^4))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^4/(a^2-b^2)^2/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+1/12*(150*a^5*b*c*d^4-105*a^6*d^5-12*a^3*b^3*c*d^2
*(4*c^2+29*d^2)+a^4*b^2*d^3*(26*c^2+223*d^2)-b^6*d*(57*c^4+136*c^2*d^2+8*d^
4)+6*a*b^5*c*(3*c^4+38*c^2*d^2+48*d^4)-a^2*b^4*d*(33*c^4+70*c^2*d^2+128*d^4
))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))
^(1/2)/b^5/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)^3*(20*a^3*b*
c*d-44*a*b^3*c*d+35*a^4*d^2+2*a^2*b^2*(4*c^2-43*d^2)+b^4*(4*c^2+63*d^2))*(s
in(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(
1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/(a-b)^2/b^5/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 2.05, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2871, 3126, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(35d^2a^4+20bcda^3+2b^2(4c^2-43d^2)a^2-44b^3cda+b^4(4c^2+63d^2))\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}(bc-ad)^3}{4(a-b)^2b^5(a+b)^3f\sqrt{c+d\sin(e+fx)}}+\frac{\cos(e+fx)(c+d\sin(e+fx))^{5/2}(bc-ad)^2}{2b(a^2-b^2)f(a+b\sin(e+fx))^2}+\frac{(7da^2+6bca-13b^2d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}(bc-ad)^2}{4b^2(a^2-b^2)^2f(a+b\sin(e+fx))}+\frac{d(-35d^3a^4+36bcd^2a^3+b^2d(9c^2+61d^2)a^2-18b^3c(c^2+5d^2)a+b^4d(45c^2-8d^2))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{12b^3(a^2-b^2)^2f}-\frac{(-105d^5a^6+150bcd^4a^5+b^2d^3(26c^2+223d^2)a^4-12b^3cd^2(4c^2+29d^2)a^3-b^4d(33c^4+70d^2c^2+128d^4)a^2+6b^5c(3c^4+38d^2c^2+48d^4)a-b^6d(57c^4+136d^2c^2+8d^4))F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{12b^5(a^2-b^2)^2f\sqrt{c+d\sin(e+fx)}}+\frac{(-105d^4a^5+185bcd^3a^4-15b^2d^2(3c^2-13d^2)a^3-b^3cd(21c^2+361d^2)a^2+9b^4(2c^4+17d^2c^2-8d^4)a-b^5cd(51c^2-104d^2))E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{12b^4(a^2-b^2)^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d*(36*a^3*b*c*d^2 - 35*a^4*d^3 + b^4*d*(45*c^2 - 8*d^2) - 18*a*b^3*c*(c^2
+ 5*d^2) + a^2*b^2*d*(9*c^2 + 61*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(12*b^3*(a^2 - b^2)^2*f) + ((b*c - a*d)^2*(6*a*b*c + 7*a^2*d - 13*b^2*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin
[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(2*b*
```

```
(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((185*a^4*b*c*d^3 - 105*a^5*d^4 - b
^5*c*d*(51*c^2 - 104*d^2) - 15*a^3*b^2*d^2*(3*c^2 - 13*d^2) - a^2*b^3*c*d*(
21*c^2 + 361*d^2) + 9*a*b^4*(2*c^4 + 17*c^2*d^2 - 8*d^4))*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(12*b^4*(a^2 - b^2)^2
*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((150*a^5*b*c*d^4 - 105*a^6*d^5 -
12*a^3*b^3*c*d^2*(4*c^2 + 29*d^2) + a^4*b^2*d^3*(26*c^2 + 223*d^2) - b^6*d*
(57*c^4 + 136*c^2*d^2 + 8*d^4) + 6*a*b^5*c*(3*c^4 + 38*c^2*d^2 + 48*d^4) -
a^2*b^4*d*(33*c^4 + 70*c^2*d^2 + 128*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(12*b^5*(a^2 - b^2)^2*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^3*(20*a^3*b*c*d - 44*a*b^3*c*d + 35*
a^4*d^2 + 2*a^2*b^2*(4*c^2 - 43*d^2) + b^4*(4*c^2 + 63*d^2))*EllipticPi[(2*
b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(4*(a - b)^2*b^5*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
```

```
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{2b\,(a^2-b^2)\,f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}\left(5d(bc-ad)^2+4\right.\right.}{}}{}$$

$$= \frac{(bc-ad)^2\,(6abc+7a^2d-13b^2d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4b^2\,(a^2-b^2)^2\,f(a+b\sin(e+fx))} + \frac{(bc-ac}{2b}$$

$$= \frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d}{12b^3\,(a^2-b^2)^2\,f}$$

$$= \frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d}{12b^3\,(a^2-b^2)^2\,f}$$

$$= \frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d}{12b^3\,(a^2-b^2)^2\,f}$$

$$= \frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d}{12b^3\,(a^2-b^2)^2\,f}$$

$$= \frac{d(36a^3bcd^2-35a^4d^3+b^4d(45c^2-8d^2)-18ab^3c(c^2+5d^2)+a^2b^2d(9c^2+61d}{12b^3\,(a^2-b^2)^2\,f}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 28.43, size = 1526, normalized size = 1.87

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^4*Cos[e + f*x])/(3*b^3) + (-(b^4*c^4*Cos[e
 + f*x]) + 4*a*b^3*c^3*d*Cos[e + f*x] - 6*a^2*b^2*c^2*d^2*Cos[e + f*x] + 4*
a^3*b*c*d^3*Cos[e + f*x] - a^4*d^4*Cos[e + f*x])/(2*b^3*(-a^2 + b^2)*(a + b
*Sin[e + f*x])^2) + (6*a*b^4*c^4*Cos[e + f*x] - 7*a^2*b^3*c^3*d*Cos[e + f*x
] - 17*b^5*c^3*d*Cos[e + f*x] - 15*a^3*b^2*c^2*d^2*Cos[e + f*x] + 51*a*b^4*
```

```
c^2*d^2*Cos[e + f*x] + 27*a^4*b*c*d^3*Cos[e + f*x] - 51*a^2*b^3*c*d^3*Cos[e
 + f*x] - 11*a^5*d^4*Cos[e + f*x] + 17*a^3*b^2*d^4*Cos[e + f*x])/(4*b^3*(-a
^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-2*(-48*a^2*b^3*c^5 - 24*b^5*c^5
+ 306*a*b^4*c^4*d - 177*a^2*b^3*c^3*d^2 - 327*b^5*c^3*d^2 - 105*a^3*b^2*c^2
*d^3 + 501*a*b^4*c^2*d^3 + 13*a^4*b*c*d^4 - 53*a^2*b^3*c*d^4 - 104*b^5*c*d^
4 + 35*a^5*d^5 - 73*a^3*b^2*d^5 + 56*a*b^4*d^5)*EllipticPi[(2*b)/(a + b), (
-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a
+ b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-60*a^2*b^3*c^4*d - 12*b^5*c^4*d +
 36*a^3*b^2*c^3*d^2 + 252*a*b^4*c^3*d^2 - 228*a^4*b*c^2*d^3 + 276*a^2*b^3*c
^2*d^3 - 480*b^5*c^2*d^3 + 140*a^5*c*d^4 - 364*a^3*b^2*c*d^4 + 512*a*b^4*c*
d^4 + 56*a^4*b*d^5 - 112*a^2*b^3*d^5 - 16*b^5*d^5)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(18*a*b^4*c^4*d - 21*a^2*b^3*c^3*d^2
 - 51*b^5*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 153*a*b^4*c^2*d^3 + 185*a^4*b*c*d^
4 - 361*a^2*b^3*c*d^4 + 104*b^5*c*d^4 - 105*a^5*d^5 + 195*a^3*b^2*d^5 - 72*
a*b^4*d^5)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE
[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]
+ d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c
+ d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
 (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e
+ f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c
+ d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^
2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2
- d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(48*(a -
 b)^2*b^3*(a + b)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 2774 vs. $2(885) = 1770$.

time = 65.43, size = 2775, normalized size = 3.40

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2775 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^5*(d^2*b^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
```

*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-4/3/d*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))))+2*(-3*a*b*d^2+5*b^2*c*d)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))+12*a^2*d^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))-30*a*b*c*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+20*b
^2*c^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+5/b^5*d*(a^
4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+
e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*
b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/
(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))-20*d^2/b^6*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1
)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2))+1/b^5*(-a^5*d^5+5*a^4*b*c*d^4-10*a^3*b^
2*c^2*d^3+10*a^2*b^3*c^3*d^2-5*a*b^4*c^4*d+b^5*c^5)*(-1/2*b^2/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2
-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b
^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/

```
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4
*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2
+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f
*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+
e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(9/2)/(b*sin(f*x + e) + a)^3, x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(9/2)/(b*sin(f*x + e) + a)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin(e+f\,x))^{9/2}}{(a+b\,\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(9/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(9/2)/(a + b*sin(e + f*x))^3, x)
```

# 3.759 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=605

$$\frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2}$$

```
[Out] 1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/b/(a^2-b^2)/f/(a+b*sin(f
*x+e))^2+1/4*(-a*d+b*c)^2*(5*a^2*d+6*a*b*c-11*b^2*d)*cos(f*x+e)*(c+d*sin(f*
x+e))^(1/2)/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/4*(8*a^3*b*c*d^2-15*a^4*d^
3+b^4*d*(13*c^2-8*d^2)-2*a*b^3*c*(3*c^2+13*d^2)+a^2*b^2*d*(5*c^2+29*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^3/(
a^2-b^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-3/4*(-a*d+b*c)*(4*a^3*b*c*d^2+5
*a^4*d^3+a^2*b^2*d*(c^2-11*d^2)-2*a*b^3*c*(c^2+5*d^2)+b^4*d*(5*c^2+8*d^2))*
(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos
(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1
/2)/b^4/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)^2*(12*a^3*b*c*d
-36*a*b^3*c*d+15*a^4*d^2+2*a^2*b^2*(4*c^2-19*d^2)+b^4*(4*c^2+35*d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2
*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/(a-b)^2/b^4/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.42, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2871, 3126, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(5a^2d+6abc-11b^2d\right)(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{\left(15a^4d^2+12a^3bcd+2a^2b^2(4c^2-19d^2)-36ab^3cd+b^4(4c^2+35d^2)\right)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^4f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}+\frac{3\left(5a^4d^3+4a^3bcd^2+a^2b^2d(c^2-11d^2)-2ab^3c(c^2+5d^2)+b^4d(5c^2+8d^2)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^4f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(-15a^4d^3+8a^3bcd^2+a^2b^2d(5c^2+29d^2)-2ab^3c(3c^2+13d^2)+b^4d(13c^2-8d^2)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^3f\left(a^2-b^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)^2*(6*a*b*c + 5*a^2*d - 11*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos
[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x
])^2) - ((8*a^3*b*c*d^2 - 15*a^4*d^3 + b^4*d*(13*c^2 - 8*d^2) - 2*a*b^3*c*(
3*c^2 + 13*d^2) + a^2*b^2*d*(5*c^2 + 29*d^2))*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^3*(a^2 - b^2)^2*f*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]) + (3*(b*c - a*d)*(4*a^3*b*c*d^2 + 5*a^4*d^3 + a^2
*b^2*d*(c^2 - 11*d^2) - 2*a*b^3*c*(c^2 + 5*d^2) + b^4*d*(5*c^2 + 8*d^2))*El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
```

)])/(4*b^4*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^2*(12*a^3*b*c*d - 36*a*b^3*c*d + 15*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 19*d^2) + b^4*(4*c^2 + 35*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^4*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \sqrt{c+d\sin(e+fx)}\,\left(\frac{1}{2}(3d(bc-\right.}{}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad}{2b\,(}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad}{2b\,(}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad}{2b\,(}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad}{2b\,(}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad}{2b\,(}$$

**Mathematica** [**C**] Result contains complex when optimal does not.
time = 28.42, size = 1323, normalized size = 2.19

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos[e +
 f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x])/(2*b^2*(-a^2 + b
^2)*(a + b*Sin[e + f*x])^2) + (6*a*b^3*c^3*Cos[e + f*x] - 5*a^2*b^2*c^2*d*C
os[e + f*x] - 13*b^4*c^2*d*Cos[e + f*x] - 8*a^3*b*c*d^2*Cos[e + f*x] + 26*a
*b^3*c*d^2*Cos[e + f*x] + 7*a^4*d^3*Cos[e + f*x] - 13*a^2*b^2*d^3*Cos[e + f
*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(16*a^2*b^2*c^4
 + 8*b^4*c^4 - 78*a*b^3*c^3*d + 33*a^2*b^2*c^2*d^2 + 57*b^4*c^2*d^2 + 8*a^3
```

```
*b*c*d^3 - 50*a*b^3*c*d^3 + 5*a^4*d^4 - 7*a^2*b^2*d^4 + 8*b^4*d^4)*Elliptic
Pi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c^3*
d + 4*b^4*c^3*d - 8*a^3*b*c^2*d^2 - 64*a*b^3*c^2*d^2 + 20*a^4*c*d^3 - 12*a^
2*b^2*c*d^3 + 64*b^4*c*d^3 + 8*a^3*b*d^4 - 32*a*b^3*d^4)*Cos[e + f*x]*((b*c
 - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[
e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*
(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
+ f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-6*a*b^3*c^3*d + 5*a^2*b^2*c^
2*d^2 + 13*b^4*c^2*d^2 + 8*a^3*b*c*d^3 - 26*a*b^3*c*d^3 - 15*a^4*d^4 + 29*a
^2*b^2*d^4 - 8*b^4*d^4)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a
*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c +
d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*Elli
pticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Si
n[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-(
(d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^
2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f
*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]))/(16*(a - b)^2*b^2*(a + b)^2*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. $2(678) = 1356$.

time = 52.88, size = 2237, normalized size = 3.70

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2237 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^4*(2*b*d*(1/d*c-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-6*a*d*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+8*b*c*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
```

```
d))^(1/2)))-4/b^4*d*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(-b^2/(a^
3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin
(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b
*c-a*b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*
c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))+12*d^2/b^5*(a^2*d^2-2*a*b*c*d+b^2*c
^2)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-
1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b
),((c-d)/(c+d))^(1/2))+1/b^4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(
a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b
*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2
*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d
*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^
2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d
+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1
/d*c+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^3, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^3, x)
```

## 3.760 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=549

$$\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2}+\frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b\left(a^2-b^2\right)^2f(a+b\sin(e+fx))}$$

```
[Out] 1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f
*x+e))^2+3/4*(-a*d+b*c)*(a^2*d+2*a*b*c-3*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))
^(1/2)/b/(a^2-b^2)^2/f/(a+b*sin(f*x+e))-3/4*(-a*d+b*c)*(a^2*d+2*a*b*c-3*b^2
*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/
b^2/(a^2-b^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/4*(4*a^3*b*c*d^2+3*a^4*d
^3+a^2*b^2*d*(7*c^2-5*d^2)+b^4*d*(11*c^2+8*d^2)-2*a*b^3*c*(3*c^2+11*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/
2)/b^3/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)*(4*a^3*b*c*d-28*
a*b^3*c*d+3*a^4*d^2+2*a^2*b^2*(4*c^2-3*d^2)+b^4*(4*c^2+15*d^2))*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4
*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1
/2)/(a-b)^2/b^3/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.33, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2871, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{3\left(a^2d+2abc-3b^2d\right)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{3\left(a^2d+2abc-3b^2d\right)(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f\left(a^2-b^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{\left(3a^4d^2+4a^3bcd+2a^2b^2\left(4c^2-3d^2\right)-28ab^3cd+b^4\left(4c^2+15d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^3f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}+\frac{\left(3a^4d^3+4a^3bcd^2+a^2b^2d\left(7c^2-5d^2\right)-2ab^3c\left(3c^2+11d^2\right)+b^4d\left(11c^2+8d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^3f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*b*(a^2 - b^2)*f*(a
 + b*Sin[e + f*x])^2) + (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) +
 (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[(c + d*
Sin[e + f*x])/(c + d)]) + ((4*a^3*b*c*d^2 + 3*a^4*d^3 + a^2*b^2*d*(7*c^2 -
5*d^2) + b^4*d*(11*c^2 + 8*d^2) - 2*a*b^3*c*(3*c^2 + 11*d^2))*EllipticF[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^3*
(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(4*a^3*b*c*d - 28*
a*b^3*c*d + 3*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 3*d^2) + b^4*(4*c^2 + 15*d^2))*E
```

llipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^3*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4abc^3+9b^2c^2d-6abcd^2+a^2d^3\right)-(\ldots}{\ldots}}{\ldots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\ldots\right.}{4b\left(a^2-b^2\right)^2 f(\ldots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\ldots\right.}{4b\left(a^2-b^2\right)^2 f(\ldots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\ldots\right.}{4b\left(a^2-b^2\right)^2 f(\ldots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\ldots\right.}{4b\left(a^2-b^2\right)^2 f(\ldots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\ldots\right.}{4b\left(a^2-b^2\right)^2 f(\ldots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 27.78, size = 1149, normalized size = 2.09

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x
] - a^2*d^2*Cos[e + f*x])/(2*b*(-a^2 + b^2)*(a + b*Sin[e + f*x])^2) - (3*(-
2*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x] + 3*b^3*c*d*Cos[e + f*x]
+ a^3*d^2*Cos[e + f*x] - 3*a*b^2*d^2*Cos[e + f*x]))/(4*b*(-a^2 + b^2)^2*(a
+ b*Sin[e + f*x]))))/f - ((-2*(-16*a^2*b*c^3 - 8*b^3*c^3 + 54*a*b^2*c^2*d -
 15*a^2*b*c*d^2 - 21*b^3*c*d^2 + a^3*d^3 + 5*a*b^2*d^3)*EllipticPi[(2*b)/(a
 + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
```

```
)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*a^2*b*c^2*d - 4*b^3*c^
2*d + 4*a^3*c*d^2 + 44*a*b^2*c*d^2 - 8*a^2*b*d^3 - 16*b^3*d^3)*Cos[e + f*x]
*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*
x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e
 + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c^2*d - 3*a^2*b
*c*d^2 - 9*b^3*c*d^2 - 3*a^3*d^3 + 9*a*b^2*d^3)*Cos[e + f*x]*Cos[2*(e + f*x
)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
+ d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Ellipti
cF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)
] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c
+ d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e
+ f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c
 + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e +
f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2
*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
+ d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*b*(a + b)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1887 vs. $2(622) = 1244$.

time = 47.30, size = 1888, normalized size = 3.44

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1888 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^3/b^3*(1/d*c-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+3/b^3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(-b^2/(
a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*s
in(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2
*b*c-a*b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
```

```
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/
d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))-6*d^2/b^4*(a*d-b*c)*(1/d*c-1)*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)
*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*Ellipt
icPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(
1/2))+1/b^3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-1/2*b^2/(a^3*d
-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*
x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*
b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*
b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+
d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2))))/c
os(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^3, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^3, x)
```

# 3.761 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=472

$$\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2}+\frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2f(a+b\sin(e+fx))}+\frac{(6abc-$$

```
[Out] 1/2*(-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e
))^2+1/4*(-a^2*d+6*a*b*c-5*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^
2)^2/f/(a+b*sin(f*x+e))-1/4*(-a^2*d+6*a*b*c-5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*
f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)^2/f/((c+d*sin(
f*x+e))/(c+d))^(1/2)+1/4*(-a*d+b*c)*(a^2*d+6*a*b*c-7*b^2*d)*(sin(1/2*e+1/4*
Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1
/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/(a^2-b^
2)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*(4*a^3*b*c*d+20*a*b^3*c*d+a^4*d^2-b^4*(4*
c^2+3*d^2)-2*a^2*b^2*(4*c^2+5*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin
(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/
2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)^2/b^2/(a+b)^3/f/(c
+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.13, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2878, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{\left(a^2(-d)+6abc-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}-\frac{(bc-ad)\left(a^2d+6abc-7b^2d\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}+\frac{\left(a^2(-d)+6abc-5b^2d\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf\left(a^2-b^2\right)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{\left(a^4d^2+4a^3bcd-2a^2b^2\left(4c^2+5d^2\right)+20ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b
*Sin[e + f*x])^2) + ((6*a*b*c - a^2*d - 5*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(4*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - a^2*d -
 5*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f
*x]])/(4*b*(a^2 - b^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*
d)*(6*a*b*c + a^2*d - 7*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((4*a^3*b*c*d + 20*a*b^3*c*d + a^4*d^2 - b^4*(4*c^2 + 3*d^2) -
 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^2*(a + b)^
3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2878

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d
*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) +
 (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*
d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
 && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(5bcd-a(4c^2+d^2))-(3acd-b(c^2+2d^2)\cdots}{(a+b\sin(e+fx))^2\sqrt{c+\cdots}}}{2(a^2-\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(6abc-a^2d-5b^2d)\cos(e+fx)\cdots}{4(a^2-b^2)^2 f(a+b\sin\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(6abc-a^2d-5b^2d)\cos(e+fx)\cdots}{4(a^2-b^2)^2 f(a+b\sin\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(6abc-a^2d-5b^2d)\cos(e+fx)\cdots}{4(a^2-b^2)^2 f(a+b\sin\cdots}$$

$$=$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(6abc-a^2d-5b^2d)\cos(e+fx)\cdots}{4(a^2-b^2)^2 f(a+b\sin\cdots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2} + \frac{(6abc-a^2d-5b^2d)\cos(e+fx)\cdots}{4(a^2-b^2)^2 f(a+b\sin\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 26.94, size = 1001, normalized size = 2.12

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])/(2*(a^2 -
b^2)*(a + b*Sin[e + f*x])^2) + (6*a*b*c*Cos[e + f*x] - a^2*d*Cos[e + f*x] -
 5*b^2*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(1
6*a^2*c^2 + 8*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2 + b^2*d^2)*EllipticPi[(2*b)/
(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*c*d + 4*b^2*c*d -
 24*a*b*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d)
```

```
)/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c
+ d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2
 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)
*(-6*a*b*c*d + a^2*d^2 + 5*b^2*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c -
 d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 -
 b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*S
qrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c +
 d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e +
 f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1
 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[
e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e +
f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(545) = 1090$.
time = 45.22, size = 1718, normalized size = 3.64

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1718 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2*d*(a*d-b*c)/b^2*(-b^2/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e
))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b
^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(
-1/d*c+a/b),((c-d)/(c+d))^(1/2)))+2*d^2/b^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x
+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2))+(a^2*d^2-2*a
*b*c*d+b^2*c^2)/b^2*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e
```

```
)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a
+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b
^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b
*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*
b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2
*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(
-1/d*c+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^3, x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{3/2}}{(a+b\,\sin\left(e+f\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^3, x)
```

# 3.762 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=487

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)f(a+b\sin(e+fx))^2}+\frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(a^2-b^2)^2(bc-ad)f(a+b\sin(e+fx))}+\frac{(6abc-5a^2d-b}{4(a^2-}$$

```
[Out] 1/2*b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/4*
b*(-5*a^2*d+6*a*b*c-b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-
a*d+b*c)/f/(a+b*sin(f*x+e))-1/4*(-5*a^2*d+6*a*b*c-b^2*d)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+3/4*(-a^2*d+2*a*b*c-b^2*d)*(sin(1/2*e+1/4
*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+
1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a^2-b^2
)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*(12*a^3*b*c*d+12*a*b^3*c*d-3*a^4*d^2-b^4*(
4*c^2-d^2)-2*a^2*b^2*(4*c^2+5*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin
(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/
2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)^2/b/(a+b)^3/(-a*d+
b*c)/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.02, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2875, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b(-5a^2d+6abc-b^2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a^2-b^2)^2(bc-ad)(a+b\sin(e+fx))}+\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a^2-b^2)(a+b\sin(e+fx))^2}-\frac{3(a^2(-d)+2abc-b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf(a^2-b^2)^2\sqrt{c+d\sin(e+fx)}}+\frac{(-5a^2d+6abc-b^2d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a^2-b^2)^2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(-3a^4d^2+12a^3bcd-2a^2b^2(4c^2+5d^2)+12ab^3cd-b^4(4c^2-d^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf(a-b)^2(a+b)^3(bc-ad)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f
*x])^2) + (b*(6*a*b*c - 5*a^2*d - b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - 5*a
^2*d - b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
) - (3*(2*a*b*c - a^2*d - b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((12*a^3*b*c*d + 12*a*b^3*c*d - 3*a^4*d^2 - b^4*(4*c^2 - d^2)
- 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b*(a + b)^3
*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2875

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
 + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(-4ac+bd)+(bc-2ad)\sin(e+fx)+\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b(6abc-5a^2d-b^2d)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 26.97, size = 1038, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*Cos[e + f*x])/(2*(a^2 - b^2)*(a + b*Sin[e + f
*x])^2) - (6*a*b^2*c*Cos[e + f*x] - 5*a^2*b*d*Cos[e + f*x] - b^3*d*Cos[e +
f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)*(a + b*Sin[e + f*x]))))/f + ((-2*(-16
*a^2*b*c^2 - 8*b^3*c^2 + 16*a^3*c*d + 14*a*b^2*c*d - 9*a^2*b*d^2 + 3*b^3*d^
2)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*
a^2*b*c*d - 4*b^3*c*d + 16*a^3*d^2 + 8*a*b^2*d^2)*Cos[e + f*x]*((b*c - a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
```

```
/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)
^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x
])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*
Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])
*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
 + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c*d - 5*a^2*b*d^2 - b^3*d^2)
*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
 + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin
[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
 - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(
c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
 - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]
*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 +
4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*
c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a +
 b)^2*(-(b*c) + a*d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. $2(560) = 1120$.

time = 47.59, size = 1525, normalized size = 3.13

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1525 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-a*d+b*c)/b*(-1/2*b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a
*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c
-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1
/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c
^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin
(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))+d/b*(-b
```

```
^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a
+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d
-a^2*b*c-a*b^2*d+b^3*c)/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),
(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^
(1/2)/f
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^3, x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^3, x)
```

# 3.763 $\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=503

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2}+\frac{3b^2(2abc-3a^2d+b^2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)^2f(a+b\sin(e+fx))}+\frac{3b(2a$$

```
[Out] 1/2*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f
*x+e))^2+3/4*b^2*(-3*a^2*d+2*a*b*c+b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)
/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))-3/4*b*(-3*a^2*d+2*a*b*c+b^2*d)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^
2-b^2)^2/(-a*d+b*c)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/4*(-7*a^2*d+6*a*b*
c+b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(1/2)+1/4*(20*a^3*b*
c*d+4*a*b^3*c*d-15*a^4*d^2-2*a^2*b^2*(4*c^2-3*d^2)-b^4*(4*c^2+3*d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2
*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/(a-b)^2/(a+b)^3/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 1.06, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{3b^2(-3a^2d+2abc+b^2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))}+\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2}-\frac{(-7a^2d+6abc+b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f\left(a^2-b^2\right)^2(bc-ad)\sqrt{c+d\sin(e+fx)}}+\frac{3b(-3a^2d+2abc+b^2d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f\left(a^2-b^2\right)^2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{(-15a^4d^2+20a^3bcd-2a^2b^2(4c^2-3d^2)+4ab^3cd-b^4(4c^2+3d^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a-b)^2(a+b)^3(bc-ad)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a
 + b*Sin[e + f*x])^2) + (3*b^2*(2*a*b*c - 3*a^2*d + b^2*d)*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]
)) + (3*b*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) - ((6*a*b*c - 7*a^2*d + b^2*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a^2 -
b^2)^2*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]]) - ((20*a^3*b*c*d + 4*a*b^3*c
*d - 15*a^4*d^2 - 2*a^2*b^2*(4*c^2 - 3*d^2) - b^4*(4*c^2 + 3*d^2))*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
```

```
x])/(c + d)])/(4*(a - b)^2*(a + b)^3*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x
]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(-4abc+4a^2d \cdots}{(a+b\sin \cdots}}{\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2(2abc-3a \cdots}{4(a^2- \cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2(2abc-3a \cdots}{4(a^2- \cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2(2abc-3a \cdots}{4(a^2- \cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2(2abc-3a \cdots}{4(a^2- \cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2(2abc-3a \cdots}{4(a^2- \cdots}$$

**<u>Mathematica</u>** [**C**] Result contains complex when optimal does not.
time = 26.95, size = 1069, normalized size = 2.13

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-1/2*(b^2*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a
*d)*(a + b*Sin[e + f*x])^2) + (3*(2*a*b^3*c*Cos[e + f*x] - 3*a^2*b^2*d*Cos[
e + f*x] + b^4*d*Cos[e + f*x]))/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^2*(a + b*Si
n[e + f*x]))))/f + ((-2*(16*a^2*b^2*c^2 + 8*b^4*c^2 - 32*a^3*b*c*d + 2*a*b^
3*c*d + 16*a^4*d^2 - 19*a^2*b^2*d^2 + 9*b^4*d^2)*EllipticPi[(2*b)/(a + b),
(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a
 + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c*d + 4*b^4*c*d - 32*a
^3*b*d^2 + 8*a*b^3*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[
```

```
-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi
[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*
Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqr
t[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*
Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)
]) - ((2*I)*(-6*a*b^3*c*d + 9*a^2*b^2*d^2 - 3*b^4*d^2)*Cos[e + f*x]*Cos[2*(
e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*
Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*
EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/
(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d
*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d
 + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*S
in[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*
x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]
) + (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^
2*f)
```

**Maple [A]**
time = 27.93, size = 867, normalized size = 1.72

| method | result |
|---|---|
| default | $\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{b^{2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{2\left(a^{3}d-a^{2}bc-a\,b^{2}d+b^{3}c\right)\left(a+b\sin\left(fx+e\right)\right)^{2}}-\frac{3b^{2}\left(3a\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^
3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*
a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c
)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c
)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((
-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2
-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/
```

```
(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d)
)^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f
*x+e))^(1/2)/f
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)),x)
[Out] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.764 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=682

$$-\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab^3c(c^2-d^2))\cos(e+fx)}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}+\frac{b^2}{2\left(a^2-b^2\right)(bc-ad)f(a+}$$

```
[Out] -1/4*d*(8*a^4*d^3+a^2*b^2*d*(13*c^2-29*d^2)-b^4*d*(7*c^2-15*d^2)-6*a*b^3*c*
(c^2-d^2))*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(1/2)+1/2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2/(c+d*si
n(f*x+e))^(1/2)+1/4*b^2*(-11*a^2*d+6*a*b*c+5*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/
(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2)+1/4*(8*a^4*d^3+a^2*b
^2*d*(13*c^2-29*d^2)-b^4*d*(7*c^2-15*d^2)-6*a*b^3*c*(c^2-d^2))*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*
d+b*c)^3/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/4*b*(-11*a^2*d+6*a*b*
c+5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e)
)/(c+d))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*b*(28*
a^3*b*c*d-4*a*b^3*c*d-35*a^4*d^2-2*a^2*b^2*(4*c^2-19*d^2)-b^4*(4*c^2+15*d^2
))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/(a-b)^2/(a+b)^3/(-a*d+b*c)^3/f/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 1.77, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b^2(-11a^2d+6abc+5b^2d)\cos(e+fx)}{4f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}+\frac{b^2\cos(e+fx)}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}-\frac{b(-11a^2d+6abc+5b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f\left(a^2-b^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-6ab^3c(c^2-d^2)-b^4d(7c^2-15d^2))\cos(e+fx)}{4f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3\sqrt{c+d\sin(e+fx)}}-\frac{(8a^4d^3+a^2b^2d(13c^2-29d^2)-6ab^3c(c^2-d^2)-b^4d(7c^2-15d^2))\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{b(-35a^4d^2+28a^3bcd-2a^2b^2(4c^2-19d^2)-4ab^3cd-b^4(4c^2+15d^2))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a-b)^2(a+b)^3(bc-ad)^3\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/4*(d*(8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*d^2) - b^4*d*(7*c^2 - 15*d^2) -
 6*a*b^3*c*(c^2 - d^2))*Cos[e + f*x])/((a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d
^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) + (b^2*(6*a*b*c - 1
1*a^2*d + 5*b^2*d)*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Si
n[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 2
9*d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a*b^3*c*(c^2 - d^2))*EllipticE[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c
```

```
- a*d)^3*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (b*(6*a*b*c -
11*a^2*d + 5*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e
 + f*x]]) - (b*(28*a^3*b*c*d - 4*a*b^3*c*d - 35*a^4*d^2 - 2*a^2*b^2*(4*c^2
- 19*d^2) - b^4*(4*c^2 + 15*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*(a + b
)^3*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+}}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+}}$$

$$= -\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\,\mathrm{s}}}$$

$$= -\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\,\mathrm{s}}}$$

$$= -\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\,\mathrm{s}}}$$

$$= -\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\,\mathrm{s}}}$$

$$= -\frac{d(8a^4d^3+a^2b^2d(13c^2-29d^2)-b^4d(7c^2-15d^2)-6ab}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\,\mathrm{s}}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 28.79, size = 1318, normalized size = 1.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/(2*(a^2 - b^2)*(-(b*c) + a*d)
^2*(a + b*Sin[e + f*x])^2) - (6*a*b^4*c*Cos[e + f*x] - 13*a^2*b^3*d*Cos[e +
 f*x] + 7*b^5*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^3*(a + b*Sin[
e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d)^3*(c^2 - d^2)*(c + d*Sin[e +
 f*x]))))/f + ((-2*(-16*a^2*b^3*c^4 - 8*b^5*c^4 + 48*a^3*b^2*c^3*d - 18*a*b
^4*c^3*d - 48*a^4*b*c^2*d^2 + 95*a^2*b^3*c^2*d^2 - 29*b^5*c^2*d^2 + 16*a^5*
c*d^3 - 80*a^3*b^2*c*d^3 + 34*a*b^4*c*d^3 + 56*a^4*b*d^4 - 95*a^2*b^3*d^4 +
 45*b^5*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((
2*I)*(-20*a^2*b^3*c^3*d - 4*b^5*c^3*d + 48*a^3*b^2*c^2*d^2 - 24*a*b^4*c^2*d
^2 + 16*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 + 20*b^5*c*d^3 + 16*a^5*d^4 - 80*a^3
*b^2*d^4 + 40*a*b^4*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt
[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticP
i[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
 f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d
*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sq
rt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]) - ((2*I)*(6*a*b^4*c^3*d - 13*a^2*b^3*c^2*d^2 + 7*b^5*c^2*d^2 - 6*a*b^4*
c*d^3 - 8*a^4*b*d^4 + 29*a^2*b^3*d^4 - 15*b^5*d^4)*Cos[e + f*x]*Cos[2*(e +
f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Elli
pticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin
[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b
*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
 + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x])
- 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
(c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(
b*c) + a*d)^3*f)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2098 vs. $2(751) = 1502$.
time = 59.79, size = 2099, normalized size = 3.08

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2099 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/(a*d-b*c)^3*(2*d*cos(f*x+e)^2/
(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(1/d*c-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
```

```
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(1/d*c-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-b/(a*d-b*c)*(-1/2*b^2/(a^3*d-a^2*b
*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^
2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*
b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-
a*b^2*d+b^3*c)^2*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/
4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^
2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(1/d*c-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(
f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c-d)/(c+d))^(1/2)))-2*d^2/(a
*d-b*c)^3*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)/(-1/d*c+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d
*c+a/b),((c-d)/(c+d))^(1/2))-b*d/(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b
^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-1/d*c-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(
1/d*c-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-1/d*c
+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-1/d*c+1)/(-1/d*c+a/b),((c
-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.765 $\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=888

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(14abcd-3a^2d^2+b^2(33c^2+16d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(}{(a+b)(}\right.}{24b^2(bc-a}$$

```
[Out] -1/8*(5*a^2*b*c*d^2-a^3*d^3-a*b^2*d*(15*c^2+4*d^2)-5*b^3*(c^3+4*c*d^2))*Ell
ipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/
2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x
+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*
((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d/f/(a+b)^(1/2
)+1/24*(c-d)*(14*a*b*c*d-3*a^2*d^2+b^2*(33*c^2+16*d^2))*EllipticE((a+b)^(1/
2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(
a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-
a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*
x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/(-a*d+b*c)/f+1/24*(a+b)^(3/2)*(3*a^
2*d^2-6*a*b*d*(2*c+d)+b^2*(33*c^2+26*c*d+16*d^2))*EllipticF((c+d)^(1/2)*(a+
b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(
c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^
(1/2)/b^3/f/(c+d)^(1/2)-1/3*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(3/2)*(c+d*sin(
f*x+e))^(1/2)/b/f-1/24*(14*a*b*c*d-3*a^2*d^2+b^2*(33*c^2+16*d^2))*cos(f*x+e
)*(c+d*sin(f*x+e))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)-1/12*d*(-3*a*d+13*b*c)*
cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

**Rubi** [A]
time = 2.22, antiderivative size = 888, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2872, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d - 3*a^2*d^2 + b^2*(33*c^2 + 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*b^2*(b*c - a*d)*f) - (Sqrt[c + d]*(5*a^2*b*c*d^2 - a
^3*d^3 - a*b^2*d*(15*c^2 + 4*d^2) - 5*b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
```

```
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b^3*Sqrt[a + b]*d*f) - ((14*a*b*c*d - 3*a^2*d^2 +
b^2*(33*c^2 + 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*b*f*Sqrt[
a + b*Sin[e + f*x]]) - (d*(13*b*c - 3*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + ((a + b)^(3/2)*(3*a^2*d^2
- 6*a*b*d*(2*c + d) + b^2*(33*c^2 + 26*c*d + 16*d^2))*EllipticF[ArcSin[(Sqr
t[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - S
in[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[
e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b^3*S
qrt[c + d]*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
```

```
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}{3bf}$$

$$= -\frac{d(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\,\mathrm{s}}}{12bf}$$

$$= -\frac{(14abcd-3a^2d^2+b^2(33c^2+16d^2))\cos(e+fx)\sqrt{c+d}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{(14abcd-3a^2d^2+b^2(33c^2+16d^2))\cos(e+fx)\sqrt{c+d}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\sqrt{c+d}\,(5a^2bcd^2-a^3d^3-ab^2d(15c^2+4d^2)-5b^3(c^3+}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(14abcd-3a^2d^2+b^2(33c^2+16d^2)}{}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 1978 vs. $2(888) = 1776$.

time = 7.21, size = 1978, normalized size = 2.23

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]

[Out] -1/48*((-4*(-(b*c) + a*d)*(-48*a*b*c^3 - 59*b^2*c^2*d - 58*a*b*c*d^2 + a^2*d^3 - 16*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-48*b^2*c^3 - 92*a*b*c^2*d + 4*a^2*c*d^2 - 76*b^2*c*d^2 - 28*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(33*b^2*c^2*d + 14*a*b*c*d^2 - 3*a^2*d^3 + 16*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +

```
d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])))/(b*d)))/(b*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]*(-1/12*(d*(13*b*c + a*d)*Cos[e + f*x])/b - (d^2*Sin[2*(e +
f*x)])/6))/f
```

**Maple** [C] Result contains complex when optimal does not.
time = 30.36, size = 409142, normalized size = 460.75

| method | result | size |
|---|---|---|
| default | Expression too large to display | 409142 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(b*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\sin\left(e+f\,x\right)}\,\left(c+d\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.766 $\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=784

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(5bc+ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-}{(c+}}}{4b(bc-ad)f}$$

```
[Out] 1/4*(6*a*b*c*d-a^2*d^2+b^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f
*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d
)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*
(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-
d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d/f/(a+b)^(1/2)+1/4*(c-d)*(a*d+5*b*c)*Ellipt
icE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),(
(a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+
d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/(-a*d+b*c)/f-1/2*b*cos(f
*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+b*sin(f*x+e))^(1/2)+1/4*(a+b)^(3/2)*(-a*d
+5*b*c+2*b*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d
*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x
+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)+1/2*(-a*d+b*
c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)-1/4*(a*d+5*b*
c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 2.25, antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2900, 3126, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(5*b*c + a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d - a^2*d^2 + b^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
```

```
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b^2*Sqrt[a + b]*d*f) + ((b*c - a*d)*Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*Sqrt[a + b*Sin[e + f*x]]) - ((5*b*c + a*d
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + b*Sin[e + f*x]]) + (
(a + b)^(3/2)*(5*b*c - a*d + 2*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b^2*Sqrt[c + d]*f) - (b*
Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+b\sin(e+fx)}} + \frac{\int \sqrt{c+d\sin(e \ldots}{\ldots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+f \ldots}{2f\sqrt{\ldots}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad) \ldots}{4 \ldots}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad) \ldots}{4 \ldots}$$

$$= \frac{\sqrt{c+d}\,(6abcd-a^2d^2+b^2(3c^2+4d^2))\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(5bc+ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c \ldots}}{\sqrt{c+d}\,\sqrt{a \ldots}}\right.\right.}{\ldots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1879 vs. $2(784) = 1568$.
time = 9.64, size = 1879, normalized size = 2.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/f +
 ((-4*(-(b*c) + a*d)*(8*a*c^2 + 7*b*c*d + 3*a*d^2)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(8*b*c^2 + 12*a*c*d + 4*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*
d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])) + 2*(-5*b*c*d - a*d^2)*((Cos[e + f*x]*Sqrt[c +
d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a +
 b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-
e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))
/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e +
Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a +
b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
- ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elliptic
Pi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(8*f)
```

**Maple** [C] Result contains complex when optimal does not.
time = 15.30, size = 278217, normalized size = 354.87

| method | result | size |
|---|---|---|
| default | Expression too large to display | 278217 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin{(e+fx)}}\,(c+d\sin{(e+fx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.767 $\int \sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=628

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{(bc-ad)f}$$

```
[Out] (a*d+b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*si
n(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)
*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*
x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d/f
/(a+b)^(1/2)+(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)
/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*si
n(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*si
n(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(
-a*d+b*c)/f+(a+b)^(3/2)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^
(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c
+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(
-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/f/(c+d)^(1/2)-b*co
s(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.83, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2900, 3132, 2890, 12, 2880, 2897, 3075}

$$\frac{(a+b)^{3/2}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{bf\sqrt{c+d}}+\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f(bc-ad)}+\frac{\sqrt{c+d}\,(ad+bc)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{bdf\sqrt{a+b}}-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*S
in[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((
a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*
(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) + (Sqrt[c + d]
*(b*c + a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c +
d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*Sqrt[a + b]*d*f) - (b
*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]]) + ((a
+ b)^(3/2)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
```

```
+ b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e
 + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]
*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
```

```
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d(2a^2c-b^2c+abd)+ad}{(a+b\sin(e+f}}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(bc+ad)+\frac{1}{2}b}{(a+b\sin(e+fx))^{3/2}\sqrt{c}}}{b^2}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}\right.\right.}{}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}\right.\right.}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 33.26, size = 228392, normalized size = 363.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
[Out] Result too large to show
```

**Maple** [C] Result contains complex when optimal does not.
time = 13.20, size = 146664, normalized size = 233.54

| method | result | size |
|---|---|---|
| default | Expression too large to display | 146664 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin{(e + fx)}}\,\sqrt{c + d\sin{(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.768 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=198

$$\frac{2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+}{(c+d)(a+b\sin(e+}}}{\sqrt{a+b}\,df}$$

[Out] 2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d/f/(a+b)^(1/2)

**Rubi** [A]

time = 0.07, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2890}

$$\frac{2\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\,\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]
))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+f}{\sqrt{}}$$

**Mathematica** [A]
time = 0.15, size = 197, normalized size = 0.99

$$\frac{2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{\frac{(-bc+ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(c-d)(a+b\sin(e+fx))}}\,(a+b\sin(e+fx))}{\sqrt{a+b}\,df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((-(b*c) + a*d)*(1 - Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

**Maple** [C] Result contains complex when optimal does not.
time = 21.07, size = 248426, normalized size = 1254.68

| method | result | size |
|---|---|---|
| default | Expression too large to display | 248426 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Curve
 not irreducible after change of variable 0 -> infinity
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sin{\left(e + f x \right)}}}{\sqrt{c + d \sin{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.769 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=409

$$-\frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+}{(a+b)(c+d\sin(e+}}}{(c-d)\sqrt{c+d}\,(bc-ad)f}$$

```
[Out] -2*(a-b)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(
f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*
(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)/f/(c+d
)^(1/2)+2*(a-b)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)
/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2874, 2897, 3075}

$$\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}\,(bc-ad)}-\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*
Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) + (
2*(a - b)*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]
])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Si
n[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
 + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
```

```
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{(a-b)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} + \frac{(bc-ad)\int \frac{}{\sqrt{a+b}}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(}{(c-}$$

**Mathematica [A]**

time = 5.48, size = 263, normalized size = 0.64

$$\frac{2\left(-((bc-ad)\cos(e+fx))-\frac{\sqrt{2}\sqrt{\frac{a-b}{a+b}}(a+b)(c+d)\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right)|\frac{2(-bc+ad)}{(a-b)(c+d)}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}{\sqrt{\frac{(a+b)(1+\sin(e+fx))}{a+b\sin(e+fx)}}}\right)}{(c-d)(c+d)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(-((b*c - a*d)*Cos[e + f*x]) - (Sqrt[2]*Sqrt[(a - b)/(a + b)]*(a + b)*(c
 + d)*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Cos
[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a
*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
 + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])/Sqrt[((a + b)*(1 + Sin
[e + f*x]))/(a + b*Sin[e + f*x])]))/((c - d)*(c + d)*f*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 46827 vs. $2(379) = 758$.
time = 11.09, size = 46828, normalized size = 114.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 46828 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sin{(e + fx)}}}{(c + d\sin{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\,\sin\left(e + f\,x\right)}}{{\left(c + d\,\sin\left(e + f\,x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.770 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=489

$$\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{}$$

```
[Out] 2/3*d*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+
2/3*(a-b)*(4*a*c*d-b*(3*c^2+d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*
sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2
)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f+2/3*(a-b)*(3*c+d)*EllipticF((c+d)^(1/2
)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-si
n(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f
```

**Rubi** [A]
time = 0.55, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2875, 3077, 2897, 3075}

$$\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)^2}+\frac{2(a-b)\sqrt{a+b}(3c+d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)}+\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c + d*Sin[e +
 f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(4*a*c*d - b*(3*c^2 + d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^2*f) + (2*(a - b)*Sqrt[a + b]*(3
*c + d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rule 2875

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
 + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)-\frac{1}{2}(3bc-ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}}{3\,(c^2-d^2)}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{((a-b)(3c+d))\int \frac{}{\sqrt{a+b\sin(e+fx)}}}{3(c-d)^2(c+d}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\,(4acd-b(3c^2+d^2))\,E}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2067 vs. $2(489) = 978$.
time = 6.29, size = 2067, normalized size = 4.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*d*Cos[e + f*x])/(3*(
c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (2*(3*b*c^2*d*Cos[e + f*x] - 4*a*c*d^2
*Cos[e + f*x] + b*d^3*Cos[e + f*x]))/(3*(b*c - a*d)*(c^2 - d^2)^2*(c + d*Si
n[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b*c^3 + 3*a^2*c^2*d + b^2*c^2*
d - a*b*c*d^2 + a^2*d^3 - b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-3*b
^2*c^3 + a*b*c^2*d + 4*a^2*c*d^2 - b^2*c*d^2 - a*b*d^3)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
```

```
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 +
b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x
]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[Arc
Sin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*
x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x
]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
+ d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((
(a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])))/(b*d)))/(3*(c - d)^2*(c + d)^2*(-(b*c) + a*d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 196466 vs. $2(449) = 898$.
time = 14.08, size = 196467, normalized size = 401.77

| method | result | size |
|---|---|---|
| default | Expression too large to display | 196467 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*
x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e
)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sin{(e + fx)}}}{(c + d\sin{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\,\sin(e + f\,x)}}{(c + d\,\sin(e + f\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.771 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=1080

$$\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(57a^2bcd^2-9a^3d^3+ab^2d(337c^2+156d^2)+b^3(15c^3+284cd^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{c+d}}\right.\right.$$

```
[Out] -1/64*(20*a^3*b*c*d^3-3*a^4*d^4-60*a*b^3*c*d*(c^2+4*d^2)-6*a^2*b^2*d^2*(15*
c^2+4*d^2)+b^4*(5*c^4-120*c^2*d^2-48*d^4))*EllipticPi((a+b)^(1/2)*(c+d*sin(
f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+
d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)
*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c
-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d^2/f/(a+b)^(1/2)+1/192*(c-d)*(57*a^2*b*c*d
^2-9*a^3*d^3+a*b^2*d*(337*c^2+156*d^2)+b^3*(15*c^3+284*c*d^2))*EllipticE((a
+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*
(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/
2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1
+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d/(-a*d+b*c)/f+1/192*(a+b)^(
3/2)*(9*a^3*d^3-3*a^2*b*d^2*(17*c+6*d)+3*a*b^2*d*(73*c^2+36*c*d+28*d^2)+b^3
*(15*c^3+118*c^2*d+284*c*d^2+72*d^3))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e)
)^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+
e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^3/d/
f/(c+d)^(1/2)-1/24*d*(-3*a*d+17*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(3/2)*(c+d
*sin(f*x+e))^(1/2)/b/f-1/4*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(5/2)*(c+d*sin(f
*x+e))^(1/2)/b/f-1/192*(57*a^2*b*c*d^2-9*a^3*d^3+a*b^2*d*(337*c^2+156*d^2)+
b^3*(15*c^3+284*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/d/f/(a+b*sin(f*
x+e))^(1/2)-1/96*(54*a*b*c*d-9*a^2*d^2+b^2*(59*c^2+36*d^2))*cos(f*x+e)*(a+b
*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

**Rubi** [A]

time = 3.50, antiderivative size = 1080, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2872, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(57*a^2*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337
*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
```

```
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b^2*d*(b*c - a
*d)*f) - (Sqrt[c + d]*(20*a^3*b*c*d^3 - 3*a^4*d^4 - 60*a*b^3*c*d*(c^2 + 4*d
^2) - 6*a^2*b^2*d^2*(15*c^2 + 4*d^2) + b^4*(5*c^4 - 120*c^2*d^2 - 48*d^4))*
EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*
(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a
+ b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*
Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(64*b^3*Sqrt[a + b]*d^2*f) - ((57*a^2
*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^
2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*b*d*f*Sqrt[a + b*Sin[e + f*
x]]) - ((54*a*b*c*d - 9*a^2*d^2 + b^2*(59*c^2 + 36*d^2))*Cos[e + f*x]*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*b*f) - (d*(17*b*c - 3*a*d
)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(24*b*f
) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]])/
(4*b*f) + ((a + b)^(3/2)*(9*a^3*d^3 - 3*a^2*b*d^2*(17*c + 6*d) + 3*a*b^2*d*
(73*c^2 + 36*c*d + 28*d^2) + b^3*(15*c^3 + 118*c^2*d + 284*c*d^2 + 72*d^3))
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(192*b^3*d*Sqrt[c + d]*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
```

```
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+f}}{4bf}$$

$$= -\frac{d(17bc-3ad)\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+}}{24bf}$$

$$= -\frac{(54abcd-9a^2d^2+b^2(59c^2+36d^2))\cos(e+fx)\sqrt{a+}}{96bf}$$

$$= -\frac{(57a^2bcd^2-9a^3d^3+ab^2d(337c^2+156d^2)+b^3(15c^3+}{192bdf\sqrt{a+b\sin(e}}$$

$$= -\frac{(57a^2bcd^2-9a^3d^3+ab^2d(337c^2+156d^2)+b^3(15c^3+}{192bdf\sqrt{a+b\sin(e}}$$

$$= -\frac{\sqrt{c+d}\,(20a^3bcd^3-3a^4d^4-60ab^3cd(c^2+4d^2)-6a^2}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(57a^2bcd^2-9a^3d^3+ab^2d(337c^2}{}$$

**Mathematica [A]**
time = 6.99, size = 2091, normalized size = 1.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/384*((-4*(-(b*c) + a*d)*(-384*a^2*b*c^3 - 133*b^3*c^3 - 971*a*b^2*c^2*d
- 451*a^2*b*c*d^2 - 356*b^3*c*d^2 + 3*a^3*d^3 - 228*a*b^2*d^3)*Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]) - 4*(-(b*c) + a*d)*(-532*a*b^2*c^3 - 664*a^2*b*c^2*d - 644*b^3*c^2*d +
12*a^3*c*d^2 - 1160*a*b^2*c*d^2 - 228*a^2*b*d^3 - 144*b^3*d^3)*((Sqrt[((c +
 d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(15*b^3*c^3 + 337*a*b^
2*c^2*d + 57*a^2*b*c*d^2 + 284*b^3*c*d^2 - 9*a^3*d^3 + 156*a*b^2*d^3)*((Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
 - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
 b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
 (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[
((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*
x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*
d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/
(b*d)))/(b*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/96*(
(59*b^2*c^2 + 122*a*b*c*d + 3*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*x])/b + (b*d^
2*Cos[3*(e + f*x)])/16 - (d*(17*b*c + 9*a*d)*Sin[2*(e + f*x)])/48))/f
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 50.24, size = 576490, normalized size = 533.79

| method | result | size |
|---|---|---|
| default | Expression too large to display | 576490 |

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d
^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e))*sqrt(b*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.772 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=870

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(38abcd+3a^2d^2+b^2(3c^2+16d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c\cdots}{(a+b)(c\cdots}\right.}{24bd(bc-a\cdots}$$

```
[Out] 1/8*(a*d+b*c)*(10*a*b*c*d-a^2*d^2-b^2*(c^2-12*d^2))*EllipticPi((a+b)^(1/2)*
(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,(
(a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(
-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f
*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d^2/f/(a+b)^(1/2)+1/24*(c-d)*(38*a
*b*c*d+3*a^2*d^2+b^2*(3*c^2+16*d^2))*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*
sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x
+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*si
n(f*x+e)))^(1/2)/b/d/(-a*d+b*c)/f-1/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(
a+b*sin(f*x+e))^(1/2)/f-1/24*(a+b)^(3/2)*(3*a^2*d^2-6*a*b*d*(4*c+d)-b^2*(3*
c^2+14*c*d+16*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2
)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*s
in(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/d/f/(c+d)^(1/2)-1/24
*(38*a*b*c*d+3*a^2*d^2+b^2*(3*c^2+16*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2
)/d/f/(a+b*sin(f*x+e))^(1/2)-1/12*(7*a*d+3*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))
^(1/2)*(c+d*sin(f*x+e))^(1/2)/f
```

**Rubi** [A]
time = 2.15, antiderivative size = 870, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2900, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*
d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*
Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*
x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]
*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a +
 b*Sin[e + f*x]))/(24*b*d*(b*c - a*d)*f) + (Sqrt[c + d]*(b*c + a*d)*(10*a*b
*c*d - a^2*d^2 - b^2*(c^2 - 12*d^2))*EllipticPi[(b*(c + d))/((a + b)*d), Ar
```

```
cSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c -
a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)
*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/
(8*b^2*Sqrt[a + b]*d^2*f) - ((38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*d^2)
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d*f*Sqrt[a + b*Sin[e + f*x]])
- ((3*b*c + 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]])/(12*f) - ((a + b)^(3/2)*(3*a^2*d^2 - 6*a*b*d*(4*c + d) - b^2*(3*c^2
 + 14*c*d + 16*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/(24*b^2*d*Sqrt[c + d]*f) - (b*Cos[e + f*x]
*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
```

```
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}{3f}$$

$$= -\frac{(3bc+7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin}}{12f}$$

$$= -\frac{(38abcd+3a^2d^2+b^2(3c^2+16d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{24df\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{(38abcd+3a^2d^2+b^2(3c^2+16d^2))\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{24df\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\,(10abcd-a^2d^2-b^2(c^2-12d^2))\,\Pi\left(\frac{b(c-}{(a+}\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(38abcd+3a^2d^2+b^2(3c^2+16d^2)}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1952 vs. $2(870) = 1740$.
time = 6.25, size = 1952, normalized size = 2.24

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]

[Out] ((-4*(-(b*c) + a*d)*(48*a^2*c^2 + 17*b^2*c^2 + 82*a*b*c*d + 17*a^2*d^2 + 16*b^2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(68*a*b*c^2 + 68*a^2*c*d + 52*b^2*c*d + 52*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-3*b^2*c^2 - 38*a*b*c*d - 3*a^2*d^2 - 16*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d

```
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(48*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-7*(b*c + a*
d)*Cos[e + f*x])/12 - (b*d*Sin[2*(e + f*x)])/6))/f
```

**Maple** [C] Result contains complex when optimal does not.
time = 29.43, size = 408713, normalized size = 469.79

| method | result | size |
|---|---|---|
| default | Expression too large to display | 408713 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(e + fx)})^{\frac{3}{2}} (c + d\sin{(e + fx)})^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin \left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.773 $\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=740

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc+5ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-}{(c+}}}{4d(bc-ad)f}$$

[Out] 1/4*(6*a*b*c*d+3*a^2*d^2-b^2*(c^2-4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d^2/f/(a+b)^(1/2)+1/4*(c-d)*(5*a*d+b*c)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d/(-a*d+b*c)/f+1/4*(a+b)^(3/2)*(3*a*d+b*(c+2*d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/d/f/(c+d)^(1/2)-1/4*b*(5*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+b*sin(f*x+e))^(1/2)-1/2*b*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/f

**Rubi** [A]
time = 1.47, antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2900, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]

[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c + 5*a*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d*(b*c - a*d)*f) + (Sqrt[c + d]*(6*a*b*c*d + 3*a^2*d^2 - b^2*(c^2 - 4*d^2))*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]

```
))]*(a + b*Sin[e + f*x]))/(4*b*Sqrt[a + b]*d^2*f) - (b*(b*c + 5*a*d)*Cos[e
+ f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + b*Sin[e + f*x]]) - (b*Cos[
e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) + ((a + b
)^(3/2)*(3*a*d + b*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[
e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(
c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c +
 d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b*d*Sqrt[c + d]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
```

```
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2f}$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e$$

$$= \frac{\sqrt{c+d}\left(6ac-\frac{bc^2}{d}+\frac{3a^2d}{b}+4bd\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{c+}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc+5ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c}}{\sqrt{c+d}\sqrt{a}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1879 vs. $2(740) = 1480$.

time = 9.01, size = 1879, normalized size = 2.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/2*(b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/f +
 ((-4*(-(b*c) + a*d)*(8*a^2*c + 3*b^2*c + 7*a*b*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(12*a*b*c + 8*a^2*d + 4*b^2*d)*((Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
```

```
*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*
d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^2*c) - 5*a*b*d)*((Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a
 + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[
(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d
))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e
+ Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a
+ b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a
+ b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-
(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d
)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d))/(8*f)
```

**Maple** [C] Result contains complex when optimal does not.
time = 15.30, size = 277278, normalized size = 374.70

| method | result | size |
|---|---|---|
| default | Expression too large to display | 277278 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(e+f\,x))^{3/2}\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2), x)
```

## 3.774 $\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=644

$$-\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}}-\frac{(a-b)b\sqrt{a+b}\,\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b}{(a-b}\right.}{}$$

```
[Out] (b*(c-d)-2*a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/f/(c+d)^(1/2
)-(-3*a*d+b*c)*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f
*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*si
n(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/
d^2/f/(c+d)^(1/2)-(a-b)*b*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b
)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*
(c+d*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(
c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^
(1/2)/d/(-a*d+b*c)/f-b*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f/(c+d*sin(f*x+e))
^(1/2)
```

**Rubi** [A]
time = 0.99, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2900, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[c + d*Sin[e + f*x]])) -
 ((a - b)*b*Sqrt[a + b]*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d*(b*c - a*d)*f) + (Sqrt[a
 + b]*(b*(c - d) - 2*a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*
(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*S
```

```
in[e + f*x]))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f) - (Sqrt[a + b]*(b
*c - 3*a*d)*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a
+ b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((
a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a -
 b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
```

```
 + f*x]))))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x]))))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d\left(2a^2c+b^2c-abd\right)+ad(bc+ad)\sin(e+fx)-\frac{1}{2}bd(b}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+}}{d}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}bc^2d(bc-3ad)+\frac{1}{2}d^3\left(2a^2c+b^2c-abd\right)+d\left(bcd(bc-\right.}{\sqrt{a+b\sin(e+fx)}\,(c+d\,s}}{d^3}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{\sqrt{a+b}\,(bc-3ad)\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{(a-b)b\sqrt{a+b}\,\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.\right.}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 33.89, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 23.92, size = 529026, normalized size = 821.47

| method | result | size |
|---|---|---|
| default | Expression too large to display | 529026 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(e + fx))^{\frac{3}{2}}}{\sqrt{c + d\sin(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2), x)
```

## 3.775 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=600

$$\frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\mid\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{(c-d)d\sqrt{c+d}\,f}$$

```
[Out] 2*(a-b)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f
*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/d/f/(c+d)^(1/2)-2*
(b*(c-2*d)+a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/d^2/f/(c+d
)^(1/2)+2*b*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f
*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2
/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.60, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2877, 2890, 3077, 2897, 3075}

$$-\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\mid\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{d^2f(c-d)\sqrt{c+d}}+\frac{2b\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\mid\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{d^2f\sqrt{c+d}}+\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\mid\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{df(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d*Sqrt[c + d]*f) - (2*Sqrt[a +
b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d^2*Sqrt[c + d]*f) + (2*b*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
```

```
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rule 2877

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^2} - \frac{(bc-ad)\int \frac{bc+ad+2bd\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3}}}{d^2}$$

$$= \frac{2b\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\middle|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\middle|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e}{(c}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 1896 vs. $2(600) = 1200$.
time = 8.92, size = 1896, normalized size = 3.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[a + b*Sin[e + f*x]])/((c^2 -
 d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a^2*c - a*b*d)*Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-
```

```
(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]) - 4*(-(b*c) + a*d)*(a^2*d - b^2*d)*((Sqrt[((c + d)*Cot[(-e + P
i/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((
a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
 + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(b^2*c - a*b*d)*((Cos[e + f*x]*Sqrt[c
 + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(
a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin
[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*
d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e
 + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a
 + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a
 + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
 b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d))/((c - d)
*(c + d)*f)
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 39.34, size = 2620757, normalized size = 4367.93

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2620757 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(e + fx))^{\frac{3}{2}}}{(c + d\sin(e + fx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \sin(e + f x))^{3/2}}{(c + d \sin(e + f x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.776 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=497

$$-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}-\frac{8(a-b)\sqrt{a+b}\,(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{}$$

```
[Out] -2/3*(-a*d+b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(c^2-d^2)/f/(c+d*sin(f*x+
e))^(3/2)-8/3*(a-b)*(a*c-b*d)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x
+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(
f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c
-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f+2/3*(a-b)*(a*(3*c+d)-b*(c+3*d))*EllipticF((c
+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*
c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))
/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f
```

**Rubi** [A]

time = 0.63, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2878, 3077, 2897, 3075}

$$\frac{2(a-b)\sqrt{a+b}\,(a(3c+d)-b(c+3d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)}-\frac{8(a-b)\sqrt{a+b}\,(ac-bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)}-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) - (8*(a - b)*Sqrt[a + b]*(a*c - b*d)*EllipticE[Arc
Sin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d
)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(
3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f) + (2*(a - b)*Sqrt[a + b]*(a*(3*c +
 d) - b*(c + 3*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
 + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e +
f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rule 2878

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
```

```
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d
*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) +
 (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*
d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
 && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3a^2c-b^2c+4abd\right)-\frac{1}{2}(4abc-}{\sqrt{a+b\sin(e+fx)}\,(c}}{3\left(c^2-d^2\right.}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{(4(bc-ad)(ac-bd))\int \frac{}{\sqrt{a}}}{3(c-}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{8(a-b)\sqrt{a+b}\,(ac-bd)E}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2012 vs. $2(497) = 994$.
time = 6.25, size = 2012, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b*c*Cos[e + f*x] -
 a*d*Cos[e + f*x]))/(3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (8*(-(a*c*d*Co
s[e + f*x]) + b*d^2*Cos[e + f*x]))/(3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))
/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^2*d
^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^2*c*d -
 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
```

```
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) +
2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt
[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x
)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt
[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt
[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b
*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]
*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*
(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]))))/(b*d)))/(3*(c - d)^2*(c + d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 190876 vs. $2(457) = 914$.
time = 14.20, size = 190877, normalized size = 384.06

| method | result | size |
|---|---|---|
| default | Expression too large to display | 190877 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(
f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x +
 e)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin{(e + fx)})^{\frac{3}{2}}}{(c + d\sin{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\sin(e + f\,x))^{3/2}}{(c + d\,\sin(e + f\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.777 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=1295

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(360a^3bcd^3-45a^4d^4+2a^2b^2d^2(1877c^2+846d^2)+8ab^3d(45c^3+791cd^2)-b^4(45c^4-$$

[Out] -1/128*(a*d+b*c)*(28*a^3*b*c*d^3-3*a^4*d^4+28*a*b^3*c*d*(c^2-20*d^2)-2*a^2*b^2*d^2*(89*c^2+20*d^2)-b^4*(3*c^4+40*c^2*d^2+240*d^4))*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d^3/f/(a+b)^(1/2)+1/1920*(c-d)*(360*a^3*b*c*d^3-45*a^4*d^4+2*a^2*b^2*d^2*(1877*c^2+846*d^2)+8*a*b^3*d*(45*c^3+791*c*d^2)-b^4*(45*c^4-1692*c^2*d^2-1024*d^4))*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d^2/(-a*d+b*c)/f-1/240*(110*a*b*c*d+93*a^2*d^2-b^2*(15*c^2-64*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+b*sin(f*x+e))^(1/2)/d/f+3/40*b*(-7*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)*(a+b*sin(f*x+e))^(1/2)/d/f-1/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)*(a+b*sin(f*x+e))^(1/2)/d/f+1/1920*(a+b)^(3/2)*(45*a^4*d^4-30*a^3*b*d^3*(11*c+3*d)+30*a^2*b^2*d^2*(64*c^2+23*c*d+22*d^2)+2*a*b^3*d*(165*c^3+917*c^2*d+2392*c*d^2+516*d^3)-b^4*(45*c^4-30*c^3*d-1692*c^2*d^2-1544*c*d^3-1024*d^4))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^3/d^2/f/(c+d)^(1/2)-1/1920*(360*a^3*b*c*d^3-45*a^4*d^4+2*a^2*b^2*d^2*(1877*c^2+846*d^2)+8*a*b^3*d*(45*c^3+791*c*d^2)-b^4*(45*c^4-1692*c^2*d^2-1024*d^4))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/d^2/f/(a+b*sin(f*x+e))^(1/2)-1/960*(917*a^2*b*c*d^2+15*a^3*d^3+a*b^2*d*(345*c^2+772*d^2)-b^3*(45*c^3-516*c*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/d/f

**Rubi** [A]

time = 5.42, antiderivative size = 1295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2872, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(360*a^3*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*
d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*c^3 + 791*c*d^2) - b^4*(45*c^4 - 1
692*c^2*d^2 - 1024*d^4))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(1920*b^2*d^2*(b*c - a*d)*f) - (Sqrt[c
 + d]*(b*c + a*d)*(28*a^3*b*c*d^3 - 3*a^4*d^4 + 28*a*b^3*c*d*(c^2 - 20*d^2)
 - 2*a^2*b^2*d^2*(89*c^2 + 20*d^2) - b^4*(3*c^4 + 40*c^2*d^2 + 240*d^4))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(128*b^3*Sqrt[a + b]*d^3*f) - ((360*a^3
*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*
c^3 + 791*c*d^2) - b^4*(45*c^4 - 1692*c^2*d^2 - 1024*d^4))*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(1920*b*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((917*a^2*
b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(345*c^2 + 772*d^2) - b^3*(45*c^3 - 516*c*d^
2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(960*b*
d*f) + ((a + b)^(3/2)*(45*a^4*d^4 - 30*a^3*b*d^3*(11*c + 3*d) + 30*a^2*b^2*
d^2*(64*c^2 + 23*c*d + 22*d^2) + 2*a*b^3*d*(165*c^3 + 917*c^2*d + 2392*c*d^
2 + 516*d^3) - b^4*(45*c^4 - 30*c^3*d - 1692*c^2*d^2 - 1544*c*d^3 - 1024*d^
4))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sq
rt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]
*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt
[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d
*Sin[e + f*x]))/(1920*b^3*d^2*Sqrt[c + d]*f) - ((110*a*b*c*d + 93*a^2*d^2 -
 b^2*(15*c^2 - 64*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])^(3/2))/(240*d*f) + (3*b*(b*c - 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(5/2))/(40*d*f) - (b^2*Cos[e + f*x]*Sqrt[a +
b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))/(5*d*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + b\sin(e + fx))^{5/2}(c + d\sin(e + fx))^{5/2}\,dx = -\frac{b^2\cos(e + fx)\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e + fx))^{7/}}{5df}$$

$$= \frac{3b(bc - 7ad)\cos(e + fx)\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e}{40df}$$

$$= -\frac{(110abcd + 93a^2d^2 - b^2(15c^2 - 64d^2))\cos(e + fx)\sqrt{a -}}{240df}$$

$$= -\frac{(917a^2bcd^2 + 15a^3d^3 + ab^2d(345c^2 + 772d^2) - b^3(45c^3 -}{}$$

$$= -\frac{(360a^3bcd^3 - 45a^4d^4 + 2a^2b^2d^2(1877c^2 + 846d^2) + 8ab^3}{1}$$

$$= -\frac{(360a^3bcd^3 - 45a^4d^4 + 2a^2b^2d^2(1877c^2 + 846d^2) + 8ab^3}{1}$$

$$= -\frac{\sqrt{c + d}\,(bc + ad)\,(28a^3bcd^3 - 3a^4d^4 + 28ab^3cd(c^2 - 20}{}$$

$$= \frac{\sqrt{a + b}\,(c - d)\sqrt{c + d}\,(360a^3bcd^3 - 45a^4d^4 + 2a^2b^2d^2(18}{}$$

**Mathematica [A]**
time = 7.53, size = 2276, normalized size = 1.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-15*b^4*c^4 + 3840*a^3*b*c^3*d + 4456*a*b^3*c^3*d + 14
702*a^2*b^2*c^2*d^2 + 3236*b^4*c^2*d^2 + 4456*a^3*b*c*d^3 + 10440*a*b^3*c*d
^3 - 15*a^4*d^4 + 3236*a^2*b^2*d^4 + 1024*b^4*d^4)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
```

```
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(-60*a*b^3*c^4 + 6364*a^2*b^2*c^3*d + 2292*b^4*c^3*d + 6364*a^3*
b*c^2*d^2 + 17020*a*b^3*c^2*d^2 - 60*a^4*c*d^3 + 17020*a^2*b^2*c*d^3 + 4624
*b^4*c*d^3 + 2292*a^3*b*d^4 + 4624*a*b^3*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*
Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
+ b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
 b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(45*b^4*c^4 - 360*a*b^3*c^3*d - 3754*a^
2*b^2*c^2*d^2 - 1692*b^4*c^2*d^2 - 360*a^3*b*c*d^3 - 6328*a*b^3*c*d^3 + 45*
a^4*d^4 - 1692*a^2*b^2*d^4 - 1024*b^4*d^4)*((Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[
(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2
 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b
)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f
*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e
+ f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3840*b*d*f) + (Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/960*((15*b^3*c^3 + 1289*
a*b^2*c^2*d + 1289*a^2*b*c*d^2 + 898*b^3*c*d^2 + 15*a^3*d^3 + 898*a*b^2*d^3
)*Cos[e + f*x])/(b*d) + (21*b*d*(b*c + a*d)*Cos[3*(e + f*x)])/160 - ((93*b^
```

```
2*c^2 + 362*a*b*c*d + 93*a^2*d^2 + 88*b^2*d^2)*Sin[2*(e + f*x)])/480 + (b^2
*d^2*Sin[4*(e + f*x)])/40))/f
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 72.56, size = 753715, normalized size = 582.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 753715 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2
)*d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c
^2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*
x + e))*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.778 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=1071

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(337a^2bcd^2+15a^3d^3+ab^2d(57c^2+284d^2)-b^3(9c^3-156cd^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{\cdots}}{\sqrt{c+d}\,\sqrt{\cdots}}\right.\right.}{\cdots}$$

```
[Out] 1/64*(60*a^3*b*c*d^3-5*a^4*d^4-20*a*b^3*c*d*(c^2-12*d^2)+3*b^4*(c^2+4*d^2)^
2+30*a^2*b^2*d^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/
2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c
-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x
+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*si
n(f*x+e)))^(1/2)/b^2/d^3/f/(a+b)^(1/2)+1/192*(c-d)*(337*a^2*b*c*d^2+15*a^3*
d^3+a*b^2*d*(57*c^2+284*d^2)-b^3*(9*c^3-156*c*d^2))*EllipticE((a+b)^(1/2)*(
c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)
/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d^2/(-a*d+b*c)/f+1/24*b*(-17*a*d+3*b*c)*c
os(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+b*sin(f*x+e))^(1/2)/d/f-1/4*b^2*cos(f*x
+e)*(c+d*sin(f*x+e))^(5/2)*(a+b*sin(f*x+e))^(1/2)/d/f-1/192*(a+b)^(3/2)*(15
*a^3*d^3-15*a^2*b*d^2*(11*c+2*d)-a*b^2*d*(51*c^2+172*c*d+212*d^2)+b^3*(9*c^
3-6*c^2*d-156*c*d^2-72*d^3))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(
a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/d^2/f/(c+d)
^(1/2)-1/192*(337*a^2*b*c*d^2+15*a^3*d^3+a*b^2*d*(57*c^2+284*d^2)-b^3*(9*c^
3-156*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*sin(f*x+e))^(1/2
)-1/96*(54*a*b*c*d+59*a^2*d^2-9*b^2*(c^2-4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e)
)^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

**Rubi** [A]
time = 3.33, antiderivative size = 1071, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2872, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(337*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(5
7*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
```

```
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b*d^2*(b*c - a
*d)*f) + (Sqrt[c + d]*(60*a^3*b*c*d^3 - 5*a^4*d^4 - 20*a*b^3*c*d*(c^2 - 12*
d^2) + 3*b^4*(c^2 + 4*d^2)^2 + 30*a^2*b^2*d^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(64*b^2*Sqrt[a + b]*d^3*f) - ((337*a^2*b*c*d^2 +
15*a^3*d^3 + a*b^2*d*(57*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((54*
a*b*c*d + 59*a^2*d^2 - 9*b^2*(c^2 - 4*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*d*f) - ((a + b)^(3/2)*(15*a^3*d^3 - 15
*a^2*b*d^2*(11*c + 2*d) - a*b^2*d*(51*c^2 + 172*c*d + 212*d^2) + b^3*(9*c^3
 - 6*c^2*d - 156*c*d^2 - 72*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*
Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/(
(a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*
(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(192*b^2*d^2*Sqrt[c + d]*f) +
 (b*(3*b*c - 17*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])^(3/2))/(24*d*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(5/2))/(4*d*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
```

```
, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/}}{4df}$$

$$= \frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(}{24df}$$

$$= -\frac{(54abcd+59a^2d^2-9b^2(c^2-4d^2))\cos(e+fx)\sqrt{a+b\,\mathrm{s}}}{96df}$$

$$= -\frac{(337a^2bcd^2+15a^3d^3+ab^2d(57c^2+284d^2)-b^3(9c^3-1}{192d^2f\sqrt{a+b\sin(e}}$$

$$= -\frac{(337a^2bcd^2+15a^3d^3+ab^2d(57c^2+284d^2)-b^3(9c^3-1}{192d^2f\sqrt{a+b\sin(e}}$$

$$= \frac{\sqrt{c+d}\,\left(60a^3bcd^3-5a^4d^4-20ab^3cd(c^2-12d^2)+3b^4(c\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(337a^2bcd^2+15a^3d^3+ab^2d(57c^2}{}$$

**Mathematica** [A]
time = 6.84, size = 2091, normalized size = 1.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-3*b^3*c^3 + 384*a^3*c^2*d + 451*a*b^2*c^2*d + 971*a^2
*b*c*d^2 + 228*b^3*c*d^2 + 133*a^3*d^3 + 356*a*b^2*d^3)*Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4
```

```
*(-(b*c) + a*d)*(-12*a*b^2*c^3 + 664*a^2*b*c^2*d + 228*b^3*c^2*d + 532*a^3*
c*d^2 + 1160*a*b^2*c*d^2 + 644*a^2*b*d^3 + 144*b^3*d^3)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(9*b^3*c^3 - 57*a*b^2*c^2*d -
 337*a^2*b*c*d^2 - 156*b^3*c*d^2 - 15*a^3*d^3 - 284*a*b^2*d^3)*((Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(
a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a
+ b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(
b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b
)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((
c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[
2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f
*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(
b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c +
d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(384*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/96*((3*
b^2*c^2 + 122*a*b*c*d + 59*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*x])/d + (b^2*d*C
os[3*(e + f*x)])/16 - (b*(9*b*c + 17*a*d)*Sin[2*(e + f*x)])/48))/f
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 47.41, size = 576487, normalized size = 538.27

| method | result | size |
|---|---|---|
| default | Expression too large to display | 576487 |

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2
*d*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e))*sqrt(b*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.779 $\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=894

$$\frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(14abcd+33a^2d^2-b^2(3c^2-16d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c-}{(a+b)(c-}\right.}{24d^2(bc-ad}$$

```
[Out] 1/8*(15*a^2*b*c*d^2+5*a^3*d^3-5*a*b^2*d*(c^2-4*d^2)+b^3*(c^3+4*c*d^2))*Elli
pticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2
),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+
e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*(
(-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d^3/f/(a+b)^(1/2)
+1/24*(c-d)*(14*a*b*c*d+33*a^2*d^2-b^2*(3*c^2-16*d^2))*EllipticE((a+b)^(1/2
)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a
+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a
*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x
+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^2/(-a*d+b*c)/f-1/3*b^2*cos(f*x+e)*(c+d
*sin(f*x+e))^(3/2)*(a+b*sin(f*x+e))^(1/2)/d/f+1/24*(a+b)^(3/2)*(15*a^2*d^2+
6*a*b*d*(2*c+3*d)-b^2*(3*c^2-2*c*d-16*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*si
n(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/
b/d^2/f/(c+d)^(1/2)-1/24*b*(14*a*b*c*d+33*a^2*d^2-b^2*(3*c^2-16*d^2))*cos(f
*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*sin(f*x+e))^(1/2)+1/12*b*(-13*a*d+3
*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

**Rubi** [A]
time = 2.15, antiderivative size = 894, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2872, 3128, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d + 33*a^2*d^2 - b^2*(3*c^2 - 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*d^2*(b*c - a*d)*f) + (Sqrt[c + d]*(15*a^2*b*c*d^2 +
5*a^3*d^3 - 5*a*b^2*d*(c^2 - 4*d^2) + b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
```

```
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b*Sqrt[a + b]*d^3*f) - (b*(14*a*b*c*d + 33*a^2*d^2
 - b^2*(3*c^2 - 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d^2*f*S
qrt[a + b*Sin[e + f*x]]) + (b*(3*b*c - 13*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*d*f) + ((a + b)^(3/2)*(15*a^2*d^2 +
 6*a*b*d*(2*c + 3*d) - b^2*(3*c^2 - 2*c*d - 16*d^2))*EllipticF[ArcSin[(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Si
n[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e
 + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b*d^2*
Sqrt[c + d]*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^(3/2))/(3*d*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
```

```
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/}}{3df}$$

$$= \frac{b(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin}}{12df}$$

$$= -\frac{b(14abcd+33a^2d^2-b^2(3c^2-16d^2))\cos(e+fx)\sqrt{c+}}{24d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{b(14abcd+33a^2d^2-b^2(3c^2-16d^2))\cos(e+fx)\sqrt{c+}}{24d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\,(15a^2bcd^2+5a^3d^3-5ab^2d(c^2-4d^2)+b^3(c^3+4}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(14abcd+33a^2d^2-b^2(3c^2-16d^2}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1979 vs. $2(894) = 1788$.

time = 6.68, size = 1979, normalized size = 2.21

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]

[Out] ((-4*(-(b*c) + a*d)*(-(b^3*c^2) + 48*a^3*c*d + 58*a*b^2*c*d + 59*a^2*b*d^2 + 16*b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 92*a^2*b*c*d + 28*b^3*c*d + 48*a^3*d^2 + 76*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 14*a*b^2*c*d - 33*a^2*b*d^2 - 16*b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S

```
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])))/(b*d)))/(48*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]*(-1/12*(b*(b*c + 13*a*d)*Cos[e + f*x])/d - (b^2*Sin[2*(e + f
*x)])/6))/f
```

**Maple** [C] Result contains complex when optimal does not.
time = 27.80, size = 404969, normalized size = 452.99

| method | result | size |
|---|---|---|
| default | Expression too large to display | 404969 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.780 $\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=745

$$-\frac{3b\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-}}{4d^2(bc-ad)f}$$

```
[Out] -1/4*(10*a*b*c*d-15*a^2*d^2-b^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*
sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)
*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^3/f/(a+b)^(1/2)-3/4*b*(c-d)*(-3*a*d+b*c)*
EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(
1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/
2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^2/(-a*d+b*c)/f-1/4
*(a+b)^(3/2)*(-7*a*d+3*b*c-2*b*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/f/(c+d
)^(1/2)+3/4*b^2*(-3*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*s
in(f*x+e))^(1/2)-1/2*b^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))
^(1/2)/d/f
```

**Rubi** [A]

time = 1.42, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2872, 3140, 3132, 2890, 3077, 2897, 3075}

$$-\frac{\sqrt{c+d}\left(-15a^2d^2+10abcd-\left(b^2\left(3c^2+4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{4d^3f\sqrt{a+b}}-\frac{(a+b)^{3/2}(-7ad+3bc-2bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{4d^2f\sqrt{c+d}}-\frac{3b\sqrt{a+b}(c-d)\sqrt{c+d}(bc-3ad)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{4d^2f(bc-ad)}+\frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}}-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-3*b*Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - 3*a*d)*EllipticE[ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d^2*(b*c
 - a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 15*a^2*d^2 - b^2*(3*c^2 + 4*d^2))*E
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
```

```
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*Sqrt[a + b]*d^3*f) + (3*b^2*(b*c -
3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + b*Sin[e + f
*x]]) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)/(2*d*f) - ((a + b)^(3/2)*(3*b*c - 7*a*d - 2*b*d)*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*d^2*Sqrt[
c + d]*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \frac{\frac{1}{2}(b^3c+4a^3d+ab^2d)\cdots}{\sqrt{a+b\cdots}}}{}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e\cdots}}{2d\cdots}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e\cdots}}{2d\cdots}$$

$$= -\frac{\sqrt{c+d}\,(10abcd-15a^2d^2-b^2(3c^2+4d^2))\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+\cdots}}{\sqrt{c+d}\,\sqrt{a+\cdots}}\right.\right.}{}$$

$$= -\frac{3b\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1894 vs. $2(745) = 1490$.

time = 9.42, size = 1894, normalized size = 2.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/2*(b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(
d*f) + ((-4*(-(b*c) + a*d)*(-(b^3*c) + 8*a^3*d + 11*a*b^2*d)*Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-
(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d
)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - 4*(-(b*c) + a*d)*(-4*a*b^2*c + 24*a^2*b*d + 4*b^3*d)*((Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
```

```
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
- (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
 a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c +
d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c - 9*a*b^2*d)*((Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
(2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[(
(a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(
b*d)))/(8*d*f)
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 44.54, size = 729396, normalized size = 979.06

| method | result | size |
|---|---|---|
| default | Expression too large to display | 729396 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(
f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\sin(e + fx))^{5/2}}{\sqrt{c + d\sin(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.781 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=780

$$-\frac{\sqrt{a+b}\,(4abcd-2a^2d^2-b^2(3c^2-d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)}{d^2\sqrt{c+d}\,(bc-ad)f}$$

```
[Out] -(4*a*b*c*d-2*a^2*d^2-b^2*(3*c^2-d^2))*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e
))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2)
)*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)
/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))
^(1/2)/d^2/(-a*d+b*c)/f/(c+d)^(1/2)-b*(-5*a*d+3*b*c)*EllipticPi((a+b)^(1/2)
*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,
((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-
(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(
f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^3/f/(a+b)^(1/2)-(a+b)^(3/2)*(2*a*d-
b*(3*c+d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*si
n(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e)
)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+
sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/(c+d)^(3/2)/f+2*(-a*d+b*c)^2*
cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+b*(4
*a*b*c*d-2*a^2*d^2-b^2*(3*c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/(
c^2-d^2)/f/(a+b*sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 1.64, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2871, 3140, 3132, 2890, 3077, 2897, 3075}

$$-\frac{\sqrt{a+b}\,(-2a^2d^2+4abcd-(b^2(3c^2-d^2)))\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{d^2f\sqrt{c+d}\,(bc-ad)}+\frac{b(-2a^2d^2+4abcd-(b^2(3c^2-d^2)))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f\,(c^2-d^2)\,\sqrt{a+b\sin(e+fx)}}-\frac{b\sqrt{c+d}\,(3bc-5ad)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{d^3f\sqrt{a+b}}-\frac{(a+b)^{3/2}(2ad-b(3c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{d^2f(c+d)^{3/2}}+\frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{df\,(c^2-d^2)\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a + b]*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*EllipticE[ArcSin
[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)
*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1
+ Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(d^2
*Sqrt[c + d]*(b*c - a*d)*f)) - (b*Sqrt[c + d]*(3*b*c - 5*a*d)*EllipticPi[(b
*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[
c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec
```

```
[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x])
)]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d^3*f) + (2*(b*c - a*d)^2*Cos[e + f*x
]*Sqrt[a + b*Sin[e + f*x]])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b
*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((a + b)^(3/2)*(2*a
*d - b*(3*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(
Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))
]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f
*x])))]*(c + d*Sin[e + f*x]))/(d^2*(c + d)^(3/2)*f)
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-a^3cd-3ab^2cd+3a^2bd^2\right)+\frac{1}{2}(}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b(4abcd-2a^2d^2-b^2(3c^2-d^2}{d^2\left(c^2-d^2\right)f\sqrt{}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b(4abcd-2a^2d^2-b^2(3c^2-d^2}{d^2\left(c^2-d^2\right)f\sqrt{}}$$

$$= -\frac{b\sqrt{c+d}\,(3bc-5ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c}{(a+b)(c}\right.}{}$$

$$= -\frac{\sqrt{a+b}\,(4abcd-2a^2d^2-b^2(3c^2-d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+f}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+f}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2006 vs. $2(780) = 1560$.
time = 6.52, size = 2006, normalized size = 2.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[a + b*Sin[e + f*x]])/(d*(-c^2 + d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - ((-
4*(-(b*c) + a*d)*(-(b^3*c^2) - 2*a^3*c*d - 2*a*b^2*c*d + 4*a^2*b*d^2 + b^3*
d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 2*a^2*b*c*d - 2*b^3
*c*d - 2*a^3*d^2 + 6*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
```

```
+ d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 4*a*b^2*c*d + 2*a^2*b*d^2 - b^3*d^2)*((C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[
(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
 - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]
], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqr
t[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c +
a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))
)/(b*d)))/(2*(c - d)*d*(c + d)*f)
```

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 53.38, size = 3432645, normalized size = 4400.83

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3432645 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2), x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.782 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=737

$$\frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(3bc^2+4acd-7bd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+\cdots}}{\sqrt{a+b}\,\sqrt{c+\cdots}}\right.\right.}{\cdots}$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/d/(c^2-d^2)/f/(c+d*sin(f
*x+e))^(3/2)+2/3*(a-b)*(4*a*c*d+3*b*c^2-7*b*d^2)*EllipticE((c+d)^(1/2)*(a+b
*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+
e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*si
n(f*x+e)))^(1/2)/(c-d)^2/d^2/(c+d)^(3/2)/f-2/3*(a^2*d^2*(3*c+d)+a*b*d*(3*c^
2-4*c*d-7*d^2)+b^2*(3*c^3-6*c^2*d-2*c*d^2+9*d^3))*EllipticF((c+d)^(1/2)*(a+
b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x
+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*s
in(f*x+e)))^(1/2)/(c-d)^2/d^3/(c+d)^(3/2)/f+2*b^2*EllipticPi((c+d)^(1/2)*(a
+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*
d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x
+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^3/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 1.18, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2871, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*d*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(3*b*c^2 + 4*a*c*d - 7
*b*d^2)*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*d^2*(c + d)^(3/2)*f) - (2*Sqrt[a + b]*(a^2
*d^2*(3*c + d) + a*b*d*(3*c^2 - 4*c*d - 7*d^2) + b^2*(3*c^3 - 6*c^2*d - 2*c
*d^2 + 9*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
```

```
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*d^3*(c + d)^(3/2)*f) + (2*b^2*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^3*Sqrt[c + d]*f)
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2\right)-}{\sqrt{}}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{b^3\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2b^2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\right.\right.}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\left(3bc^2+4acd\right.}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 2169 vs. $2(737) = 1474$.
time = 6.66, size = 2169, normalized size = 2.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*d*(-c^2 + d^2)*(c +
 d*Sin[e + f*x])^2) - (2*(3*b^2*c^3*Cos[e + f*x] + a*b*c^2*d*Cos[e + f*x] -
 4*a^2*c*d^2*Cos[e + f*x] - 7*b^2*c*d^2*Cos[e + f*x] + 7*a*b*d^3*Cos[e + f*
x]))/(3*d*(-c^2 + d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-
(b^3*c^3) + 3*a^3*c^2*d + 2*a*b^2*c^2*d - 8*a^2*b*c*d^2 + b^3*c*d^2 + a^3*d
^3 + 2*a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 + 3*a^2*
b*c^2*d + b^3*c^2*d + 4*a^3*c*d^2 - 7*a^2*b*d^3 + 3*b^3*d^3)*((Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
```

```
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^3 + a*b^2*c^2*d
 - 4*a^2*b*c*d^2 - 7*b^3*c*d^2 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*C
os[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + P
i/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a
- b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2
- f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin
[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin
[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e
 + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
)/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b
*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-
(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d))/(3*(c - d)^2*d*(
c + d)^2*f)
```

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 79.85, size = 4937517, normalized size = 6699.48

| method | result | size |
|---|---|---|
| default | Expression too large to display | 4937517 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d
^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^{5/2}}{(c+d\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2), x)
```

## 3.783 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=772

$$\frac{3\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,(3bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(\ldots}{(\ldots}}}{4b^2(bc-ad)f}$$

```
[Out] -1/4*(10*a*b*c*d-3*a^2*d^2-b^2*(15*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*
sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)
*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)+3/4*(c-d)*d*(-a*d+3*b*c)*
EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(
1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/
2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/(-a*d+b*c)/f+1/4
*(3*a^2*d^2-a*b*d*(7*c+3*d)+b^2*(8*c^2+9*c*d+2*d^2))*EllipticF((c+d)^(1/2)*
(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b
)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(
f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+
d*sin(f*x+e)))^(1/2)/b^3/f/(c+d)^(1/2)-3/4*d*(-a*d+3*b*c)*cos(f*x+e)*(c+d*s
in(f*x+e))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)-1/2*d^2*cos(f*x+e)*(a+b*sin(f*x
+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

**Rubi** [A]
time = 1.52, antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2872, 3140, 3132, 2890, 3077, 2897, 3075}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (3*Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*(3*b*c - a*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^2*(b*c
- a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 3*a^2*d^2 - b^2*(15*c^2 + 4*d^2))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
```

```
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^3*Sqrt[a + b]*f) - (3*d*(3*b*c - a
*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*f*Sqrt[a + b*Sin[e + f*x]])
 - (d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*
b*f) + (Sqrt[a + b]*(3*a^2*d^2 - a*b*d*(7*c + 3*d) + b^2*(8*c^2 + 9*c*d + 2
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(4*b^3*Sqrt[c + d]*f)
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2bf} + \frac{\int \frac{\frac{1}{2}(ad^3+bc(4c^2+d^2))}{\sqrt{a+}}}{}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e}{2}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e}{2}$$

$$= -\frac{\sqrt{c+d}\,(10abcd-3a^2d^2-b^2(15c^2+4d^2))\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+}}{\sqrt{c+d}\,\sqrt{a+}}\right.\right.}{}$$

$$= \frac{3\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,(3bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1894 vs. $2(772) = 1544$.
time = 9.55, size = 1894, normalized size = 2.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -1/2*(d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(
b*f) + ((-4*(-(b*c) + a*d)*(8*b*c^3 + 11*b*c*d^2 - a*d^3)*Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 4*(-(b*c) + a*d)*(24*b*c^2*d - 4*a*c*d^2 + 4*b*d^3)*((Sqrt[((c + d)*Cot[(-
e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
+ a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((
c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqr
```

```
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d
)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]
*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x
)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-9*b*c*d^2 + 3*a*d^3)*((Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b
)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/
(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*
(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a
+ b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*
Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
 + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d
)))/(8*b*f)
```

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 40.26, size = 731219, normalized size = 947.17

| method | result | size |
|---|---|---|
| default | Expression too large to display | 731219 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,\sin\left(e + f\,x\right))^{5/2}}{\sqrt{a + b\,\sin\left(e + f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(1/2), x)
```

## 3.784 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=644

$$\frac{\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1}{(c+d)(a+}}}{b(bc-ad)f}$$

```
[Out] (-a*d+3*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b
*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x
+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin
(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^
2/f/(a+b)^(1/2)+(c-d)*d*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^
(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a
+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a
+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1
/2)/b/(-a*d+b*c)/f-(a*d-b*(2*c+d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(
1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*se
c(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d
*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/
2)/b^2/f/(c+d)^(1/2)-d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))
^(1/2)
```

**Rubi** [A]
time = 1.01, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2900, 3132, 2890, 3077, 2897, 3075}

$$-\frac{\sqrt{a+b}\,(ad-b(2c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{b^2f\sqrt{c+d}}+\frac{\sqrt{c+d}\,(3bc-ad)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{b^2f\sqrt{a+b}}+\frac{d\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{bf(bc-ad)}-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*(b*c - a*d)*f) + (Sqrt[c
+ d]*(3*b*c - a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)
*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]
))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f
```

```
) - (d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]])
- (Sqrt[a + b]*(a*d - b*(2*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
 + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
 b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
```

```
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}b\left(bcd-a\left(2c^2+d^2\right)\right)+bc(bc+ad)\sin(e+fx)+\ldots}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(\ldots}}}{b}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(3bc-ad)-\frac{1}{2}b^3\left(bcd-a\left(2c^2+d^2\right)\right)+b(-a\ldots}{(a+b\sin(e+fx))^{3/2}\sqrt{c+\ldots}}}{b^3}$$

$$= \frac{\sqrt{c+d}\,(3bc-ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d\ldots}{(a+b)(c-d\ldots}\ldots}{\ldots}$$

$$= \frac{\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\ldots}{\ldots}$$

**Mathematica** [**C**] Result contains complex when optimal does not.
time = 34.97, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 19.26, size = 544053, normalized size = 844.80

| method | result | size |
|---|---|---|
| default | Expression too large to display | 544053 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sin{(e + fx)})^{\frac{3}{2}}}{\sqrt{a + b\sin{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{3/2}}{\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(1/2), x)
```

# 3.785 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=198

$$\frac{2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx}{(a+b)(c+d\sin(e+fx}}}{b\sqrt{c+d}\,f}$$

```
[Out] 2*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e)
)^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*si
n(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/f/(c+d)^(1/
2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2890}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((
a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+$$

**Mathematica** [A]
time = 0.18, size = 197, normalized size = 0.99

$$\frac{2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{\frac{(-bc+ad)(1+\sin(e+fx))}{(a-b)(c+d\sin(e+fx))}}(c+d\sin(e+fx))}{b\sqrt{c+d}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x]))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

**Maple** [C] Result contains complex when optimal does not.
time = 14.83, size = 247463, normalized size = 1249.81

| method | result | size |
|---|---|---|
| default | Expression too large to display | 247463 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Curve
 not irreducible after change of variable 0 -> infinity
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + d\, \sin(e + f\, x)}}{\sqrt{a + b\, \sin(e + f\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(1/2), x)
```

# 3.786 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=192

$$\frac{2\sqrt{a+b}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{\cdots}}{\sqrt{c+d}\,(bc-ad)f}$$

```
[Out] 2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))
^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(
1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(-a*d+b*c)/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2897}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c\cdots}{(a-b)(c\cdots}\right.}{}$$

**Mathematica** [A]
time = 0.15, size = 191, normalized size = 0.99

$$\frac{2\sqrt{a+b}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{\frac{(-bc+ad)(1+\sin(e+fx))}{(a-b)(c+d\sin(e+fx))}}\,(c+d\sin(e+fx))}{\sqrt{c+d}\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x
]))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(177) = 354$.
time = 12.99, size = 1236, normalized size = 6.44

| method | result |
|---|---|
| default | $-\frac{4\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(\sqrt{-c^2+d^2}\,\cos(fx+e)-c\sin(fx+e)-d\cos(fx+e)+\sqrt{-c^2+d^2}-d\right)\left(-a\sqrt{-c^2+d^2}+c\sqrt{-a^2\cdots}\right.}{\left(\sqrt{-c^2+d^2}\,\cos(fx+e)+c\sin(fx+e)+d\cos(fx+e)+\sqrt{-c^2+d^2}+d\right)\left(a\sqrt{-c^2+d^2}+c\sqrt{-a^2\cdots}\right.}}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -4/f*EllipticF((-((-c^2+d^2)^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*cos(f*x+e)+(-c
^2+d^2)^(1/2)-d)*(-a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c)/((-c^2+d^
2)^(1/2)*cos(f*x+e)+c*sin(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)/(a*(-c^2+
d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c))^(1/2),((a*(-c^2+d^2)^(1/2)+c*(-a^2+
b^2)^(1/2)-a*d+b*c)*(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)+a*d-b*c)/(-a*(-c
^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)+a*d-b*c)/(-a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)
^(1/2)-a*d+b*c))^(1/2))*((cos(f*x+e)*(-a^2+b^2)^(1/2)-a*sin(f*x+e)-cos(f*x+
```

```
e)*b+(-a^2+b^2)^(1/2)-b)/((-c^2+d^2)^(1/2)*cos(f*x+e)+c*sin(f*x+e)+d*cos(f*
x+e)+(-c^2+d^2)^(1/2)+d)*(-c^2+d^2)^(1/2)*c/(-a*(-c^2+d^2)^(1/2)+c*(-a^2+b^
2)^(1/2)+a*d-b*c))^(1/2)*((cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+cos(f*x
+e)*b+(-a^2+b^2)^(1/2)+b)/((-c^2+d^2)^(1/2)*cos(f*x+e)+c*sin(f*x+e)+d*cos(f
*x+e)+(-c^2+d^2)^(1/2)+d)*(-c^2+d^2)^(1/2)*c/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^
2)^(1/2)-a*d+b*c))^(1/2)*(-((-c^2+d^2)^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*cos(
f*x+e)+(-c^2+d^2)^(1/2)-d)*(-a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c)
/((-c^2+d^2)^(1/2)*cos(f*x+e)+c*sin(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)
/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c))^(1/2)*(a+b*sin(f*x+e))^(1
/2)*(c+d*sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(cos(f*x+e)*(-
a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)*d-cos(f*x+e)*(-a^2+b^2)^(1/2)*c^2+cos(f*x+e
)*(-a^2+b^2)^(1/2)*d^2-cos(f*x+e)*(-c^2+d^2)^(1/2)*a*c+cos(f*x+e)*(-c^2+d^2
)^(1/2)*b*d-cos(f*x+e)*b*c^2+cos(f*x+e)*b*d^2+c*(-a^2+b^2)^(1/2)*(-c^2+d^2)
^(1/2)*sin(f*x+e)+c*d*(-a^2+b^2)^(1/2)*sin(f*x+e)+b*c*(-c^2+d^2)^(1/2)*sin(
f*x+e)-a*c^2*sin(f*x+e)+b*c*d*sin(f*x+e)+d*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2
)+d^2*(-a^2+b^2)^(1/2)+b*d*(-c^2+d^2)^(1/2)-a*c*d+d^2*b)/sin(f*x+e)^4/(cos(
f*x+e)^2*b*d-sin(f*x+e)*a*d-sin(f*x+e)*b*c-a*c-b*d)/(-c^2+d^2)^(1/2)/(-a*(-
c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d*cos(f*x +
e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin{\left(e + fx \right)}}\,\sqrt{c + d\sin{\left(e + fx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.787 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=405

$$\frac{2(a-b)\sqrt{a+b}\,dE\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+}{(a+b)(c+d\sin(e+}}}{(c-d)\sqrt{c+d}\,(bc-ad)^2f}$$

```
[Out] 2*(a-b)*d*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin
(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))
*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^2/f/(
c+d)^(1/2)+2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)/f/
(c+d)^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2880, 2897, 3075}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}\,(bc-ad)}+\frac{2d(a-b)\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*d*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
```

```
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{}{\sqrt{a}}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,dE\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right.\right.}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 90261 vs. $2(405) = 810$.
time = 33.73, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 41918 vs. $2(375) = 750$.
time = 11.46, size = 41919, normalized size = 103.50

| method | result | size |
|---|---|---|
| default | Expression too large to display | 41919 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(a*c^2 + 2*b*c*d
 + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2
 - 2*a*c*d - b*d^2)*sin(f*x + e)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin{(e + fx)}}\,(c + d\sin{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

## 3.788 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=521

$$-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}-\frac{4(a-b)\sqrt{a+b}\,d\left(2acd-b\left(3c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{}$$

[Out] -2/3*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)-4/3*(a-b)*d*(2*a*c*d-b*(3*c^2-d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f-2/3*(a*d*(3*c+d)-b*(3*c^2+3*c*d-2*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f

**Rubi** [A]
time = 0.66, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2881, 3077, 2897, 3075}

$$-\frac{2\sqrt{a+b}\,(ad(3c+d)-b(3c^2+3cd-2d^2))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)^2}-\frac{4d(a-b)\sqrt{a+b}\,(2acd-b(3c^2-d^2))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(c-d)^2(c+d)^{3/2}(bc-ad)^3}-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]

[Out] (-2*d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2)) - (4*(a - b)*Sqrt[a + b]*d*(2*a*c*d - b*(3*c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f) - (2*Sqrt[a + b]*(a*d*(3*c + d) - b*(3*c^2 + 3*c*d - 2*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^2*f)

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(-\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{(ad(3c\cdots}{\cdots}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}} - \frac{4(a-b\cdots}{\cdots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2102 vs. $2(521) = 1042$.
time = 6.34, size = 2102, normalized size = 4.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(
3*(b*c - a*d)*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (4*(-3*b*c^2*d^2*Cos[e
+ f*x] + 2*a*c*d^3*Cos[e + f*x] + b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^
2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*b^2*c^4 - 6*a
*b*c^3*d + 3*a^2*c^2*d^2 - 5*b^2*c^2*d^2 + 2*a*b*c*d^3 + a^2*d^4 + 2*b^2*d^
4)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-6*b^2*c^3*d - 2*a*b*c^2*d^2 + 4*a^2
*c*d^3 + 2*b^2*c*d^3 + 2*a*b*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-
e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcS
```

```
in[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])) + 2*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 - 2*b^2*d^4)*((Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b
)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/
(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*
(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a
+ b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*
Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
 + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d
)))/(3*(c - d)^2*(c + d)^2*(b*c - a*d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 219145 vs. $2(481) = 962$.
time = 14.42, size = 219146, normalized size = 420.63

| method | result | size |
|---|---|---|
| default | Expression too large to display | 219146 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d^3*cos(f*x +
 e)^4 + a*c^3 + 3*b*c^2*d + 3*a*c*d^2 + b*d^3 - (3*b*c^2*d + 3*a*c*d^2 + 2*
b*d^3)*cos(f*x + e)^2 + (b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3 - (3*b*c*d^2
 + a*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(5/2)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.789 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=822

$$\frac{(c-d)\sqrt{c+d}\,(2b^2c^2-4abcd+3a^2d^2-b^2d^2)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{(a-b)b^2\sqrt{a+b}\,(bc-a}$$

```
[Out] (c-d)*(3*a^2*d^2-4*a*b*c*d+2*b^2*c^2-b^2*d^2)*EllipticE((a+b)^(1/2)*(c+d*si
n(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d)
)^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e)
)/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f
*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+d*(-3*a*d+5*b*c)*EllipticP
i((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(
c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)-(3*a^2
*d^2-2*a*b*d*(c+3*d)-b^2*(2*c^2-6*c*d-d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(
a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x
+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)+2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+
e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)+(4*a*b*c*d-3*a^2*d^2-b^2*(2*
c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^
(1/2)
```

**Rubi** [A]
time = 1.73, antiderivative size = 822, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2871, 3140, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - b^2*d^2)*Elliptic
E[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*
c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c -
a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x
]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (d*Sqrt[c + d]*(5*b*c - 3*a*d
)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e
```

```
+ f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b
)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d
)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[
e + f*x]]) + ((4*a*b*c*d - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[a
+ b]*(3*a^2*d^2 - 2*a*b*d*(c + 3*d) - b^2*(2*c^2 - 6*c*d - d^2))*EllipticF[
ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e
 + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])
)/((a - b)*b^3*Sqrt[c + d]*f)
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
```

```
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)\right)-\frac{1}{2}(}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right.}{b\left(a^2-b^2\right)f\sqrt{}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right.}{b\left(a^2-b^2\right)f\sqrt{}}$$

$$= \frac{d\sqrt{c+d}\,(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+}{(a+b)(c-}\right.}{}$$

$$= -\frac{(c-d)\sqrt{c+d}\,\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin}}{\sqrt{c+d}\,\sqrt{a+b\sin}}\right.\right.}{}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 2005 vs. $2(822) = 1644$.
time = 6.55, size = 2005, normalized size = 2.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-
4*(-(b*c) + a*d)*(2*a*b*c^3 - 4*b^2*c^2*d + 2*a*b*c*d^2 + a^2*d^3 - b^2*d^3
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(2*b^2*c^3 - 2*a*b*c^2*d + 4*a^2*c*d^2
 - 6*b^2*c*d^2 + 2*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
 + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
```

```
n[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]])) + 2*(-2*b^2*c^2*d + 4*a*b*c*d^2 - 3*a^2*d^3 + b^2*d^3)*((Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
(2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[(
(a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(
b*d)))/(2*(a - b)*b*(a + b)*f)
```

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 58.02, size = 3899958, normalized size = 4744.47

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3899958 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(3/2), x)
```

## 3.790 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=600

$$\frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+}{(c+d)(a+b\sin(e+}}}{(a-b)b\sqrt{a+b}\,f}$$

```
[Out] 2*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f
*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/f/(a+b)^(1/2)+2*
d*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e)
)^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*si
n(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(
1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/f/(a+b)^(
1/2)+2*(b*(c-2*d)+a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(
1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+
d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e))
)^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^2
/f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.56, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2877, 2890, 3077, 2897, 3075}

$$\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{b^2f(a-b)\sqrt{c+d}}+\frac{2d\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{b^2f\sqrt{a+b}}+\frac{2(c-d)\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{bf(a-b)\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*f) + (2*d*Sqrt[c
+ d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a
+ b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a
 + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) + (2*Sqrt[a
+ b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
```

```
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^2*Sqrt[c + d]*f)
```

Rule 2877

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{d^2\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)\int \frac{bc+ad+2bd\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b^2}$$

$$= \frac{2d\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{b}$$

$$= \frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{(a-}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1896 vs. $2(600) = 1200$.
time = 8.95, size = 1896, normalized size = 3.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(-(b*c*Cos[e + f*x]) + a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a*c^2 - b*c*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[(
(-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
```

```
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b*c^2 - b*d^2)*((Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/
((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b*c*d) + a*d^2)*((Cos[e + f*x]*
Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a +
 b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b
)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c
) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*C
os[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*S
qrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((
c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
+ f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]
*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d))/((
a - b)*(a + b)*f)
```

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 40.85, size = 2945157, normalized size = 4908.60

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2945157 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)/(b^2*cos(f*x
+ e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sin (e + fx))^{\frac{3}{2}}}{(a + b \sin (e + fx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*x))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(3/2), x)
```

# 3.791 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=409

$$\frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+\ldots}{(c+d)(a+b\sin(e+\ldots}}}{(a-b)\sqrt{a+b}\,(bc-ad)f}$$

```
[Out] 2*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f
*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/f/(a+b)
^(1/2)+2*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+
d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*
x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*
(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/
f/(c+d)^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2874, 2897, 3075}

$$\frac{2\sqrt{a+b}\,(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(a-b)\sqrt{c+d}\,(bc-ad)}+\frac{2(c-d)\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f(a-b)\sqrt{a+b}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)*f) + (2
*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
```

```
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(bc-ad)\int \frac{}{(a+b\sin(e}}{}$$

$$= \frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e}{(a-b)}$$

**Mathematica** [A]

time = 3.35, size = 226, normalized size = 0.55

$$-\frac{2\sqrt{2}\cos\left(\frac{1}{4}(2e-\pi+2fx)\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right)|\frac{2(-bc+ad)}{(a-b)(c+d)}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{c+d\sin(e+fx)}}{\sqrt{\frac{a-b}{a+b}}f\sqrt{\frac{(a+b)(1+\sin(e+fx))}{a+b\sin(e+fx)}}(a+b\sin(e+fx))^{3/2}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b
)]*Cos[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*
c) + a*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[(a - b)/(a + b)]*f*Sqrt[((a + b)*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x])]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[((a + b)*(c + d*Sin[e
 + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 46827 vs. $2(379) = 758$.
time = 12.50, size = 46828, normalized size = 114.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 46828 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x +
e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sin{(e + fx)}}}{(a + b\sin{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(3/2), x)
```

# 3.792 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=405

$$\frac{2b(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+}{(c+d)(a+b\sin(e+}}}{(a-b)\sqrt{a+b}\,(bc-ad)^2f}$$

```
[Out] 2*b*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin
(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))
*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a
*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)^2/f/(
a+b)^(1/2)+2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/f/
(c+d)^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2880, 2897, 3075}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(a-b)\sqrt{c+d}\,(bc-ad)}+\frac{2b(c-d)\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f(a-b)\sqrt{a+b}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*b*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2880

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
```

```
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{b\int \frac{}{(a+}}{}$$

$$= \frac{2b(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right.\right.}{}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 90261 vs. $2(405) = 810$.
time = 36.00, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] Result too large to show
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 40620 vs. $2(375) = 750$.
time = 12.92, size = 40621, normalized size = 100.30

| method | result | size |
|---|---|---|
| default | Expression too large to display | 40621 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*b*d - (b^2*
c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*cos(f*x + e)^2 - 2*a*b
*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sin{(e + fx)})^{\frac{3}{2}}\sqrt{c + d\sin{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.793 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=495

$$\frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}-\frac{2(a^2d^2+b^2(c^2-2d^2))\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{}$$

```
[Out] -2*(a^2*d^2+b^2*(c^2-2*d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(
a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c
)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*(b*(c-2*d)-a*d)*EllipticF((c+d)^(1/2)*(a+b*
sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+
d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1
/2)/(c-d)/(-a*d+b*c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b^2*cos(f*x+e)/(a^2-b^2)
/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

**Rubi** [A]

time = 0.61, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2881, 3077, 2897, 3075}

$$-\frac{2(a^2d^2+b^2(c^2-2d^2))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^3}+\frac{2b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}+\frac{2(b(c-2d)-ad)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - (2*(a^2*d^2 + b^2*(c^2 - 2*d^2))*EllipticE[ArcSin
[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(
1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqr
t[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(b*(c - 2*d) - a*d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}}$$

$$= \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}}$$

$$= \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2082 vs. $2(495) = 990$.
time = 6.62, size = 2082, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/((
a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((
b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a
*b^2*c^3 - 2*a^2*b*c^2*d + 2*b^3*c^2*d + a^3*c*d^2 - 2*a*b^2*c*d^2 + 2*a^2*
b*d^3 - 2*b^3*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elli
pticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b^3*c^3 + a*b^2*c^2*
d + a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*d^3 - 2*a*b^2*d^3)*((Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c
) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
(Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a
*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
```

```
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^3*c^2*d) - a^2*b*d^3 + 2*
b^3*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x
]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[Arc
Sin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*
x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x
]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
+ d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((
(a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 119963 vs. $2(461) = 922$.
time = 12.81, size = 119964, normalized size = 242.35

| method | result | size |
|---|---|---|
| default | Expression too large to display | 119964 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*d^2*cos(f*x
 + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^2 - (b^2*c^2 + 4*a*b*
c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 + a*b*d^2 + (a^2 + b^2
)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sin{(e + fx)})^{\frac{3}{2}}\,(c + d\sin{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f*x))**(3/2)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.794 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=681

$$\frac{2b^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}+\frac{2d(a^2d^2+b^2(3c^2-4d^2))\cos(e+fx)\sqrt{a+}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin}$$

```
[Out] 2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x
+e))^(1/2)+2/3*d*(a^2*d^2+b^2*(3*c^2-4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^(1
/2)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(4*a^3*c*
d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4))*Ell
ipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2
),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*
(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a
-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2)+
2/3*(a^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3))*E
llipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1
/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c
)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/
(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f/(a+b)^(1/2
)
```

**Rubi** [A]

time = 1.81, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2881, 3134, 3077, 2897, 3075}

$$\frac{2(a^2d^2(3c+d)-6abd(c^2-d^2)+b^2(3c^3-9c^2d-6cd^2+8d^3))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f\sqrt{a+b}(c-d)^2(c+d)^{3/2}(bc-ad)^3}+\frac{2d(a^2d^2+b^2(3c^2-4d^2))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f(a^2-b^2)(c^2-d^2)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}+\frac{2b^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}+\frac{2(4a^3cd^3-a^2bd^2(9c^2-5d^2)-4ab^2cd^3-(b^3(3c^4-15c^2d^2+8d^4)))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f\sqrt{a+b}(c-d)^2(c+d)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) + (2*d*(a^2*d^2 + b^2*(3*c^2 - 4*d^2))*Cos[e + f*
x]*Sqrt[a + b*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) + (2*(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c
^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4))*EllipticE[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
 b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
 f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*
x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*
(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) + (2*(a^2*d^2*(3*c + d) - 6*a*b*d*
(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3))*EllipticF[ArcSin[(Sq
```

```
rt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 -
Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin
[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[
a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f)
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+$$

$$= \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+$$

$$= \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+$$

$$= \frac{2b^2\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 2350 vs. 2(681) = 1362.
time = 7.21, size = 2350, normalized size = 3.45

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]

[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/(3*(b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(-9*b*c^2*d^3*Cos[e + f*x] + 4*a*c*d^4*Cos[e + f*x] + 5*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b^3*c^5 + 9*a^2*b^2*c^4*d - 9*b^4*c^4*d - 9*a^3*b*c^3*d^2 + 15*a*b^3*c^3*d^2 + 3*a^4*c^2*d^3 - 20*a^2*b^2*c^2*d^3 + 17*b^4*c^2*d^3 + 5*a^3*b*c*d^4 - 8*a*b^3*c*d^4 + a^4*d^5 + 7*a^2*b^2*d^5 - 8*b^4*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-3*b^4*c^5 - 3*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 15*b^4*c^3*d^2 - 5*a^3*b*c^2*d^3 + 11*a*b^3*c^2*d^3 + 4*a^4*c*d^4 + a^2*b^2*c*d^4 - 8*b^4*c*d^4 + 5*a^3*b*d^5 - 8*a*b^3*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 15*b^4*c^2*d^3 - 4*a^3*b*c*d^4 + 4*a*b^3*c*d^4 - 5*a^2*b^2*d^5 + 8*b^4*d^5)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]

```
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((
a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a
 - b)*(a + b)*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 414383 vs. $2(637) = 1274$.
time = 15.96, size = 414384, normalized size = 608.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 414384 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(6*a*b*c^2*d + 2
*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 + (a^2 + b^2)*c^3 + 3*(
a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3 + 3*(a^2 + 2*b^2)*c*d
^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c^3 + 6*a*b*c*d^2 + 3*
(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 + 2
*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.795 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=736

$$\frac{2(c-d)\sqrt{c+d}\,(4abc+3a^2d-7b^2d)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-}}{3(a-b)^2b^2(a+b)^{3/2}f}$$

```
[Out] 2/3*(c-d)*(3*a^2*d+4*a*b*c-7*b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^
(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*s
ec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a
+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1
/2)/(a-b)^2/b^2/(a+b)^(3/2)/f+2*d^2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b
)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin
(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+
b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)+2/3*(3*a^2*b*(c-2*d)*d+3*a^3*d^2+a*b
^2*(3*c^2-4*c*d-2*d^2)+b^3*(c^2-7*c*d+9*d^2))*EllipticF((c+d)^(1/2)*(a+b*si
n(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*
sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2
)/(a-b)^2/b^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*(-a*d+b*c)^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 1.20, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2871, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c + 3*a^2*d - 7*b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*b^2*(a + b)^(3/2)*f) + (2*d^2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b
)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(
((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*
```

```
c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e +
 f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(3*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a^2*b*(c
- 2*d)*d + 3*a^3*d^2 + a*b^2*(3*c^2 - 4*c*d - 2*d^2) + b^3*(c^2 - 7*c*d + 9
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(3*(a - b)^2*b^3*Sqrt[a + b]*Sqrt[c + d]*f)
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b(a^2-b^2)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3abc^3+7b^2c^2d-5abcd^2+a^2d^3)-}{(a}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b(a^2-b^2)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}a^2(a^2-b^2)d^3+\frac{1}{2}b^2(-3abc^3+7b^2c^2}{}}{}$$

$$= \frac{2d^2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e}{b}$$

$$= \frac{2(c-d)\sqrt{c+d}(4abc+3a^2d-7b^2d)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2172 vs. $2(736) = 1472$.
time = 6.73, size = 2172, normalized size = 2.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*b*(-a^2 + b^2)*(a +
 b*Sin[e + f*x])^2) - (2*(-4*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x
] + 7*b^3*c*d*Cos[e + f*x] + 3*a^3*d^2*Cos[e + f*x] - 7*a*b^2*d^2*Cos[e + f
*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-4*(-(b*c) + a*d)*(
-3*a^2*b*c^3 - b^3*c^3 + 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*
d^3 - a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 - 3*a^2*b
*c^2*d + 7*b^3*c^2*d + 4*a^3*c*d^2 - a^2*b*d^3 - 3*b^3*d^3)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
```

```
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c
) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a
 + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c^2*d - a^2*b*c*
d^2 - 7*b^3*c*d^2 - 3*a^3*d^3 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Co
s[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi
/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a -
 b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 -
 f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[
e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*b*(a
 + b)^2*f)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.
time = 89.33, size = 5973124, normalized size = 8115.66

| method | result | size |
|---|---|---|
| default | Expression too large to display | 5973124 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b
^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin\left(e+f\,x\right))^{5/2}}{(a+b\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(5/2), x)
```

## 3.796 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=497

$$\frac{8(c-d)\sqrt{c+d}\,(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+}}}{3(a-b)^2(a+b)^{3/2}(bc-ad)f}$$

```
[Out] 8/3*(c-d)*(a*c-b*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2
)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)^2/(a+b
)^(3/2)/(-a*d+b*c)/f+2/3*(c-d)*(3*a*c-a*d+b*c-3*b*d)*EllipticF((c+d)^(1/2)*
(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b
)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b
)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)
))^(1/2)/(a-b)^2/(-a*d+b*c)/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*(-a*d+b*c)*cos(f*
x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.64, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2878, 3077, 2897, 3075}

$$\frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)(3ac-ad+bc-3bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(a-b)^2\sqrt{a+b}\sqrt{c+d}(bc-ad)}+\frac{8(c-d)\sqrt{c+d}(ac-bd)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{3f(a-b)^2(a+b)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (8*(c - d)*Sqrt[c + d]*(a*c - b*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2*(a + b)^(3/2)*(
b*c - a*d)*f) + (2*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(a
^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(c - d)*(3*a*c + b*c - a*d - 3
*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*
x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c +
 d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2878

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d
*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) +
 (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*
d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
 && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(4bcd-a\left(3c^2+d^2\right)\right)-\frac{1}{2}\left(4acd-b\left(c^2+\right.\right.}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin}}}{3\left(a^2-b^2\right)}$$

$$= \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} + \frac{((c-d)(3ac+bc-ad-3bd))}{3}$$

$$= \frac{8(c-d)\sqrt{c+d}\,(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right.}{3(a-}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2012 vs. $2(497) = 994$.

time = 6.31, size = 2012, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b*c*Cos[e + f*x]
) + a*d*Cos[e + f*x]))/(3*(a^2 - b^2)*(a + b*Sin[e + f*x])^2) - (8*(-(a*b*c
*Cos[e + f*x]) + b^2*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(a + b*Sin[e + f*x])
)))/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^
2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^2*c*
d - 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
```

```
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))
 + 2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*S
qrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 -
f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/S
qrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*S
qrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a
+ b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a +
b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) -
(2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
 + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 194979 vs. $2(457) = 914$.
time = 13.50, size = 194980, normalized size = 392.31

| method | result | size |
|---|---|---|
| default | Expression too large to display | 194980 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)/(3*a*b^2*cos(
f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x +
 e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin{(e+fx)})^{\frac{3}{2}}}{(a+b\sin{(e+fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*x))**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(5/2), x)
```

# 3.797 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=489

$$\frac{2(c-d)\sqrt{c+d}\,(4abc-3a^2d-b^2d)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-}}{3(a-b)^2(a+b)^{3/2}(bc-ad)^2f}$$

```
[Out] 2/3*(c-d)*(-3*a^2*d+4*a*b*c-b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(
1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*se
c(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+
b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/
2)/(a-b)^2/(a+b)^(3/2)/(-a*d+b*c)^2/f+2/3*(3*a+b)*(c-d)*EllipticF((c+d)^(1/
2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(
a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x
+e)))^(1/2)/(a-b)^2/(-a*d+b*c)/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*b*cos(f*x+e)*(
c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.57, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2875, 3077, 2897, 3075}

$$\frac{2(c-d)\sqrt{c+d}\,(-3a^2d+4abc-b^2d)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{3f(a-b)^2(a+b)^{3/2}(bc-ad)^2}+\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\,(a^2-b^2)\,(a+b\sin(e+fx))^{3/2}}+\frac{2(3a+b)(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(a-b)^2\sqrt{a+b}\,\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c - 3*a^2*d - b^2*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2
*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(3*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a + b)*(c - d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2875

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
 + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(a^2-b^2)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)+\frac{1}{2}(bc-3ad)\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{3(a^2-b^2)}$$

$$= \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(a^2-b^2)f(a+b\sin(e+fx))^{3/2}} + \frac{((3a+b)(c-d))\int \frac{}{\sqrt{a+b\sin(e+fx}}}{3(a-b)^2(a+}$$

$$= \frac{2(c-d)\sqrt{c+d}\,(4abc-3a^2d-b^2d)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2067 vs. $2(489) = 978$.
time = 6.35, size = 2067, normalized size = 4.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b*Cos[e + f*x])/(3*(
a^2 - b^2)*(a + b*Sin[e + f*x])^2) + (2*(-4*a*b^2*c*Cos[e + f*x] + 3*a^2*b*
d*Cos[e + f*x] + b^3*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(-(b*c) + a*d)*(a +
b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b*c^2 - b^3*c^2 + 3*a^3*
c*d + a*b^2*c*d - a^2*b*d^2 + b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/
2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
 d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(
-4*a*b^2*c^2 - a^2*b*c*d + b^3*c*d + 3*a^3*d^2 + a*b^2*d^2)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c
) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
```

```
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a
 + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c*d - 3*a^2*b*d^
2 - b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e +
 f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE
[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e +
 f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])
]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)
*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)
*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]))))/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c) + a*d)*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 197174 vs. $2(449) = 898$.

time = 14.45, size = 197175, normalized size = 403.22

| method | result | size |
|---|---|---|
| default | Expression too large to display | 197175 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*a*b^2*cos(f*
x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e
)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sin{(e + fx)}}}{(a + b\sin{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(5/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c + d\,\sin\left(e + f\,x\right)}}{{\left(a + b\,\sin\left(e + f\,x\right)\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(5/2), x)
```

## 3.798 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=516

$$\frac{4b(c-d)\sqrt{c+d}\,(2abc-3a^2d+b^2d)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(}{(}}}{3(a-b)^2(a+b)^{3/2}(bc-ad)^3f}$$

```
[Out] 4/3*b*(c-d)*(-3*a^2*d+2*a*b*c+b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*
sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(
a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(
1/2)/(a-b)^2/(a+b)^(3/2)/(-a*d+b*c)^3/f+2/3*(3*a*b*(c-d)-3*a^2*d+b^2*(c+2*d
))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e)
)^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d
+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+
e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)^2/(-a*d+b*c)^2/f/(a+b)^(1/2)/(c+d)^
(1/2)+2/3*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b
*sin(f*x+e))^(3/2)
```

**Rubi** [A]
time = 0.67, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2881, 3077, 2897, 3075}

$$\frac{2(-3a^2d+3ab(c-d)+b^2(c+2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f(a-b)^2\sqrt{a+b}\sqrt{c+d}(bc-ad)^2}+\frac{4b(c-d)\sqrt{c+d}(-3a^2d+2abc+b^2d)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{3f(a-b)^2(a+b)^{3/2}(bc-ad)^3}+\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (4*b*(c - d)*Sqrt[c + d]*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*(a + b)^(3/2)*(b*c - a*d)^3*f) + (2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a*b
*(c - d) - 3*a^2*d + b^2*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sq
rt[c + d]*(b*c - a*d)^2*f)
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-2b^2d}{(a+b\sin(}}{}$$

$$= \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{(2b(2abc-}{}$$

$$= \frac{4b(c-d)\sqrt{c+d}\,(2abc-3a^2d+b^2d)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{c+d}}\right.\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2102 vs. $2(516) = 1032$.
time = 6.36, size = 2102, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^2*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (4*(2*a*b^3*c*Cos[e
+ f*x] - 3*a^2*b^2*d*Cos[e + f*x] + b^4*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(
-(b*c) + a*d)^2*(a + b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^2*
c^2 + b^4*c^2 - 6*a^3*b*c*d + 2*a*b^3*c*d + 3*a^4*d^2 - 5*a^2*b^2*d^2 + 2*b
^4*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSi
n[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-
e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b^3*c^2 - 2*a^2*b^2*c*d + 2
*b^4*c*d - 6*a^3*b*d^2 + 2*a*b^3*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c
+ d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[-(b*c) + a*d)/((a + b)*d),
```

```
ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*S
in[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])) + 2*(-4*a*b^3*c*d + 6*a^2*b^2*d^2 - 2*b^4*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(
a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^2*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 242898 vs. $2(476) = 952$.
time = 13.84, size = 242899, normalized size = 470.73

| method | result | size |
|---|---|---|
| default | Expression too large to display | 242899 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d*cos(f*x +
 e)^4 - (3*a*b^2*c + (3*a^2*b + 2*b^3)*d)*cos(f*x + e)^2 + (a^3 + 3*a*b^2)*
c + (3*a^2*b + b^3)*d - ((b^3*c + 3*a*b^2*d)*cos(f*x + e)^2 - (3*a^2*b + b^
3)*c - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin\left(e + fx\right)\right)^{\frac{5}{2}}\sqrt{c + d\sin\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(5/2)*sqrt(c + d*sin(e + f*x))), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)),x)
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

# 3.799 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=688

$$\frac{2b^2\cos(e+fx)}{3(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}+\frac{8b^2(abc-2a^2d+b^2d)\cos(e}{3(a^2-b^2)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] 2/3*(3*a^4*d^3-b^4*d*(5*c^2-8*d^2)+3*a^2*b^2*d*(3*c^2-5*d^2)-4*a*b^3*c*(c^2
-d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(
f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)/(-a*d+b*c)^4/f/(a+b)^
(1/2)/(c+d)^(1/2)-2/3*(3*a^2*b*(2*c-3*d)*d-3*a^3*d^2-3*a*b^2*(c^2-2*d^2)+b^
3*(c^2-6*c*d+8*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*
sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)/(-a*d+b
*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(
a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2)+8/3*b^2*(-2*a^2*d+a*b*c+b^2*d)
*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+
e))^(1/2)
```

**Rubi** [A]

time = 1.23, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2881, 3134, 3077, 2897, 3075}

$$\frac{8b^2(-2a^2d+abc+b^2d)\cos(e+fx)}{3f(a^2-b^2)^2(bc-ad)^2\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\frac{2b^2\cos(e+fx)}{3f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}+\frac{2(3a^4d^3+3a^2b^2d(3c^2-5d^2)-4ab^3c(c^2-d^2)-b^4d(5c^2-8d^2))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f\sqrt{a+b}(a^2-b^2)(c-d)\sqrt{c+d}(bc-ad)^4}-\frac{2(-3a^3d^2+3a^2bd(2c-3d)-3ab^2(c^2-2d^2)+b^3(c^2-6cd+8d^2))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{3f\sqrt{a+b}(a^2-b^2)(c-d)\sqrt{c+d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*Sqrt[c + d*Sin[e + f*x]]) + (8*b^2*(a*b*c - 2*a^2*d + b^2*d)*Cos[e + f*x]
)/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) + (2*(3*a^4*d^3 - b^4*d*(5*c^2 - 8*d^2) + 3*a^2*b^2*d*(3*c^2 - 5
*d^2) - 4*a*b^3*c*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*
(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c
+ d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c -
d)*Sqrt[c + d]*(b*c - a*d)^4*f) - (2*(3*a^2*b*(2*c - 3*d)*d - 3*a^3*d^2 - 3
*a*b^2*(c^2 - 2*d^2) + b^3*(c^2 - 6*c*d + 8*d^2))*EllipticF[ArcSin[(Sqrt[c
```

```
+ d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
 + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b
]*(a^2 - b^2)*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f)
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e}}$$

$$= \frac{2b^2\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e}}$$

$$= \frac{2b^2\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e}}$$

$$= \frac{2b^2\cos(e+fx)}{3\,(a^2-b^2)\,(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2352 vs. 2(688) = 1376.
time = 7.36, size = 2352, normalized size = 3.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/(3
*(a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])^2) - (2*(4*a*b^4*c*Cos[e
 + f*x] - 9*a^2*b^3*d*Cos[e + f*x] + 5*b^5*d*Cos[e + f*x]))/(3*(a^2 - b^2)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d
)^3*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b^3
*c^4 - b^5*c^4 + 9*a^3*b^2*c^3*d - 5*a*b^4*c^3*d - 9*a^4*b*c^2*d^2 + 20*a^2
*b^3*c^2*d^2 - 7*b^5*c^2*d^2 + 3*a^5*c*d^3 - 15*a^3*b^2*c*d^3 + 8*a*b^4*c*d
^3 + 9*a^4*b*d^4 - 17*a^2*b^3*d^4 + 8*b^5*d^4)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c)
+ a*d)*(-4*a*b^4*c^4 + 5*a^2*b^3*c^3*d - 5*b^5*c^3*d + 9*a^3*b^2*c^2*d^2 -
a*b^4*c^2*d^2 + 3*a^4*b*c*d^3 - 11*a^2*b^3*c*d^3 + 8*b^5*c*d^3 + 3*a^5*d^4
- 15*a^3*b^2*d^4 + 8*a*b^4*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
+ d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])) + 2*(4*a*b^4*c^3*d - 9*a^2*b^3*c^2*d^2 + 5*b^5*c^2*d^2 -
4*a*b^4*c*d^3 - 3*a^4*b*d^4 + 15*a^2*b^3*d^4 - 8*b^5*d^4)*((Cos[e + f*x]*Sq
rt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b
)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]
*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c)
+ a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos
[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
(2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
```

```
]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
+ a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*E
llipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(
a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(b*c) + a*d)^3*f)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 438220 vs. $2(642) = 1284$.
time = 18.10, size = 438221, normalized size = 636.95

| method | result | size |
|---|---|---|
| default | Expression too large to display | 438221 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((2*b^3*c*d + 3*
a*b^2*d^2)*cos(f*x + e)^4 + (a^3 + 3*a*b^2)*c^2 + 2*(3*a^2*b + b^3)*c*d + (
a^3 + 3*a*b^2)*d^2 - (3*a*b^2*c^2 + 2*(3*a^2*b + 2*b^3)*c*d + (a^3 + 6*a*b^
2)*d^2)*cos(f*x + e)^2 + (b^3*d^2*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^2 + 2*
(a^3 + 3*a*b^2)*c*d + (3*a^2*b + b^3)*d^2 - (b^3*c^2 + 6*a*b^2*c*d + (3*a^2
*b + 2*b^3)*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

# 3.800 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=941

$$\frac{2b^2\cos(e+fx)}{3(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}+\frac{4b^2(2abc-5a^2d+3b^2d)\cos}{3(a^2-b^2)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx}}$$

```
[Out] 2/3*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f
*x+e))^(3/2)+4/3*b^2*(-5*a^2*d+2*a*b*c+3*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*
d+b*c)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2)-2/3*d*(a^4*d^3+a^2
*b^2*d*(11*c^2-13*d^2)-b^4*d*(7*c^2-8*d^2)-4*a*b^3*c*(c^2-d^2))*cos(f*x+e)*
(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e)
)^(3/2)-8/3*(a^5*c*d^4-2*a^3*b^2*c*d^4+a*b^4*c*(c^4-2*c^2*d^2+2*d^4)+b^5*d*
(2*c^4-7*c^2*d^2+4*d^4)-a^2*b^3*d*(3*c^4-12*c^2*d^2+7*d^4)-a^4*b*(3*c^2*d^3
-2*d^5))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(
f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*
((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+si
n(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)^2/(c+d)^(3/2)/(-a*d
+b*c)^5/f/(a+b)^(1/2)-2/3*(a^4*d^3*(3*c+d)-9*a^3*b*d^2*(c^2-d^2)+a^2*b^2*d*
(9*c^3-18*c^2*d-15*c*d^2+16*d^3)+b^4*(c^4-9*c^3*d+16*c^2*d^2+12*c*d^3-16*d^
4)-3*a*b^3*(c^4-5*c^2*d^2+4*d^4))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/
(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2)
```

**Rubi** [A]
time = 3.05, antiderivative size = 941, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2881, 3134, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*(c + d*Sin[e + f*x])^(3/2)) + (4*b^2*(2*a*b*c - 5*a^2*d + 3*b^2*d)*Cos[e
+ f*x])/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(3/2)) - (2*d*(a^4*d^3 + a^2*b^2*d*(11*c^2 - 13*d^2) - b^4*d*(7
*c^2 - 8*d^2) - 4*a*b^3*c*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]
])/(3*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - (8*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2*d^2 + 2*d^4) + b^
```

```
5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c^2*d^2 + 7*d^4) -
a^4*b*(3*c^2*d^3 - 2*d^5))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
 d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*
Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^5*f) - (2*(a^4*d^3*(3*c + d) - 9*a^3*b*d^2*(c^2
- d^2) + a^2*b^2*d*(9*c^3 - 18*c^2*d - 15*c*d^2 + 16*d^3) + b^4*(c^4 - 9*c^
3*d + 16*c^2*d^2 + 12*c*d^3 - 16*d^4) - 3*a*b^3*(c^4 - 5*c^2*d^2 + 4*d^4))*
EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqr
t[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-((
(b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin
[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^
4*f)
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
```

```
 f*x]))))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 2669 vs. $2(941) = 1882$.
time = 7.94, size = 2669, normalized size = 2.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])^2) + (8*(a*b^5*c*Cos[e
+ f*x] - 3*a^2*b^4*d*Cos[e + f*x] + 2*b^6*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2
*(-(b*c) + a*d)^4*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/(3*(b*c - a*
d)^3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (8*(-3*b*c^2*d^4*Cos[e + f*x] +
a*c*d^5*Cos[e + f*x] + 2*b*d^6*Cos[e + f*x]))/(3*(b*c - a*d)^4*(c^2 - d^2)^
2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^4*c^6 + b^6*c^6
- 12*a^3*b^3*c^5*d + 8*a*b^5*c^5*d + 18*a^4*b^2*c^4*d^2 - 41*a^2*b^4*c^4*d^
2 + 15*b^6*c^4*d^2 - 12*a^5*b*c^3*d^3 + 48*a^3*b^3*c^3*d^3 - 28*a*b^5*c^3*d
^3 + 3*a^6*c^2*d^4 - 41*a^4*b^2*c^2*d^4 + 74*a^2*b^4*c^2*d^4 - 32*b^6*c^2*d
^4 + 8*a^5*b*c*d^5 - 28*a^3*b^3*c*d^5 + 16*a*b^5*c*d^5 + a^6*d^6 + 15*a^4*b
^2*d^6 - 32*a^2*b^4*d^6 + 16*b^6*d^6)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
```

Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4
*a*b^5*c^6 - 8*a^2*b^4*c^5*d + 8*b^6*c^5*d - 12*a^3*b^3*c^4*d^2 - 12*a^4*b^
2*c^3*d^3 + 40*a^2*b^4*c^3*d^3 - 28*b^6*c^3*d^3 - 8*a^5*b*c^2*d^4 + 40*a^3*
b^3*c^2*d^4 - 20*a*b^5*c^2*d^4 + 4*a^6*c*d^5 - 20*a^2*b^4*c*d^5 + 16*b^6*c*
d^5 + 8*a^5*b*d^6 - 28*a^3*b^3*d^6 + 16*a*b^5*d^6)*((Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/
((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-4*a*b^5*c^5*d + 12*a^2*b^4*c^4*d
^2 - 8*b^6*c^4*d^2 + 8*a*b^5*c^3*d^3 + 12*a^4*b^2*c^2*d^4 - 48*a^2*b^4*c^2*
d^4 + 28*b^6*c^2*d^4 - 4*a^5*b*c*d^5 + 8*a^3*b^3*c*d^5 - 8*a*b^5*c*d^5 - 8*
a^4*b^2*d^6 + 28*a^2*b^4*d^6 - 16*b^6*d^6)*((Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[
(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2
 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b
)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f
*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e
+ f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b
)^2*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^4*f)

**Maple** [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 36.37, size = 1120519, normalized size = 1190.77

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1120519 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d^3*cos(f*
x + e)^6 - 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + (a^2*b + b^3)*d^3)*cos(f*x + e)^4
 - (a^3 + 3*a*b^2)*c^3 - 3*(3*a^2*b + b^3)*c^2*d - 3*(a^3 + 3*a*b^2)*c*d^2
- (3*a^2*b + b^3)*d^3 + 3*(a*b^2*c^3 + (3*a^2*b + 2*b^3)*c^2*d + (a^3 + 6*a
*b^2)*c*d^2 + (2*a^2*b + b^3)*d^3)*cos(f*x + e)^2 - (3*(b^3*c*d^2 + a*b^2*d
^3)*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^3 + 3*(a^3 + 3*a*b^2)*c^2*d + 3*(3*a
^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 3*(3*a
^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x
)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,\sin\left(e+f\,x\right))^{5/2}\,(c+d\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

# 3.801 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

**Optimal**. Leaf size=28

$$\text{Int}((a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

**Mathematica** [A]
time = 2.27, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^m\,(c+d\sin(fx+e))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\sin\left(e+f\,x\right))^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

# 3.802 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=311

$$-\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)}+\frac{\sqrt{2}\,(a+b)d(ad-2bc(2+m))F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+ \right.}{b^2f(2+m)\sqrt{\phantom{x}}}$$

```
[Out] -d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1+m)/b/f/(2+m)+(a+b)*d*(a*d-2*b*c*(2+m))*
AppellF1(1/2,-1-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*
x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b^2/f/(2+m)/(((a+b*sin(f*x+e))/(a+b))^m)/(1
+sin(f*x+e))^(1/2)-(a*d*(a*d-2*b*c*(2+m))+b^2*(d^2*(1+m)+c^2*(2+m)))*Appell
F1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(a+
b*sin(f*x+e))^m*2^(1/2)/b^2/f/(2+m)/(((a+b*sin(f*x+e))/(a+b))^m)/(1+sin(f*x
+e))^(1/2)
```

**Rubi** [A]

time = 0.30, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2870, 2835, 2744, 144, 143}

$$-\frac{\sqrt{2}\cos(e+fx)\left(ad(ad-2bc(m+2))+b^2\left(c^2(m+2)+d^2(m+1)\right)\right)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}+\frac{\sqrt{2}\,d(a+b)\cos(e+fx)(ad-2bc(m+2))(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}-\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(2 + m))) + (Sqrt[2]
*(a + b)*d*(a*d - 2*b*c*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e
 + f*x])/2, (b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^
m) - (Sqrt[2]*(a*d*(a*d - 2*b*c*(2 + m)) + b^2*(d^2*(1 + m) + c^2*(2 + m)))
*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]))/(
a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e
+ f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2870

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\int(a+b\sin(e+}{}$$
$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+}{}$$
$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+}{b}$$
$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\left((-a-b)d(ad\right.}{}$$
$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\sqrt{2}\,(a+b)d(ad}{}$$

**Mathematica [F]**
time = 9.40, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2, x]
```

**Maple [F]**
time = 0.40, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^m\,(c+d\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(b*sin(f*x
+ e) + a)^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

# 3.803 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

**Optimal**. Leaf size=229

$$-\frac{\sqrt{2}\,(a+b)dF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin}{a}\right.}{bf\sqrt{1+\sin(e+fx)}}$$

```
[Out] -(a+b)*d*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b/f/(((a+b*sin(f*x+e))/(a+b))^m)/(
1+sin(f*x+e))^(1/2)-(-a*d+b*c)*AppellF1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+
b),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b/f/(((a+b*sin
(f*x+e))/(a+b))^m)/(1+sin(f*x+e))^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2835, 2744, 144, 143}

$$-\frac{\sqrt{2}\,(bc-ad)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}\,d(a+b)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*(a + b)*d*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e + f*x])/2,
(b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*S
qrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)) - (Sqrt[2]*(b*c -
a*d)*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]
))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*Sqrt[1 + Sin[e + f*x]
]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx = \frac{d\int (a+b\sin(e+fx))^{1+m}\,dx}{b} + \frac{(bc-ad)\int (a+b\sin(e+fx$$

$$= \frac{(d\cos(e+fx))\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(e+fx)\right.}{bf\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\left((-a-b)d\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx}{-a-b}\right.\right.}{bf\sqrt{1-\sin(e+fx)}\,\sqrt{}}$$

$$= -\frac{\sqrt{2}\,(a+b)dF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(}{a+}\right.}{bf\sqrt{1+\sin}}$$

**Mathematica** [A]
time = 0.38, size = 200, normalized size = 0.87

$$\frac{\sec(e+fx)\sqrt{-\frac{b(-1+\sin(e+fx))}{a+b}}\,\sqrt{\frac{b(1+\sin(e+fx))}{-a+b}}\,(a+b\sin(e+fx))^{1+m}\left((bc-ad)(2+m)F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{a+b\sin(e+fx)}{a-b},\frac{a+b\sin(e+fx)}{a+b}\right)+d(1+m)F_1\left(2+m;\frac{1}{2},\frac{1}{2};3+m;\frac{a+b\sin(e+fx)}{a-b},\frac{a+b\sin(e+fx)}{a+b}\right)(a+b\sin(e+fx))\right)}{b^2f(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[(b*(1 + Sin[e +
 f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m)*((b*c - a*d)*(2 + m)*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[e + f*x]
)/(a + b)] + d*(1 + m)*AppellF1[2 + m, 1/2, 1/2, 3 + m, (a + b*Sin[e + f*x]
)/(a - b), (a + b*Sin[e + f*x])/(a + b)]*(a + b*Sin[e + f*x])))/(b^2*f*(1 +
 m)*(2 + m))
```

**Maple** [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a + b \sin (fx + e))^m (c + d \sin (fx + e)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin\left(e+f\,x\right))^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

# 3.804 $\int (a+b\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=104

$$-\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -AppellF1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*x
+e)*(a+b*sin(f*x+e))^m*2^(1/2)/f/(((a+b*sin(f*x+e))/(a+b))^m)/(1+sin(f*x+e)
)^(1/2)
```

**Rubi** [A]

time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2744, 144, 143}

$$-\frac{\sqrt{2}\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e
+ f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*Sqrt[1 + Sin[e +
f*x]]*((a + b*Sin[e + f*x])/(a + b))^m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)}{\sqrt{1-x}\sqrt{1+}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e+fx)(a+b\sin(e}{f\sqrt{1+\sin(e+fx)}}$$

**Mathematica [A]**
time = 0.16, size = 120, normalized size = 1.15

$$\frac{F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{a+b\sin(e+fx)}{a-b},\frac{a+b\sin(e+fx)}{a+b}\right)\sec(e+fx)\sqrt{-\frac{b(-1+\sin(e+fx))}{a+b}}\sqrt{\frac{b(1+\sin(e+fx))}{-a+b}}(a+b\sin(e+fx))^{1+m}}{bf(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[
e + f*x])/(a + b)]*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[e + f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(1 +
 m))
```

**Maple [F]**
time = 0.03, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m,x)
```

```
[Out] int((a+b*sin(f*x+e))^m,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(e + fx)})^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m,x)
[Out] Integral((a + b*sin(e + f*x))**m, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m,x)
[Out] int((a + b*sin(e + f*x))^m, x)
```

# 3.805 $\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

**Mathematica** [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

**Maple** [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^m}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x)),x)
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x)), x)
```

# 3.806 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

**Mathematica** [A]
time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

**Maple** [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{(c+d\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^m}{(c+d\,\sin\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^2,x)
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^2, x)
```

# 3.807 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=28

$$\operatorname{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

**Mathematica** [A]
time = 7.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

**Maple** [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{(c+d\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^3, x)
```

# 3.808 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$

**Optimal**. Leaf size=30

$$\text{Int}\left((a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx = \int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

**Mathematica** [A]
time = 31.86, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

**Maple** [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(b*sin(f*x + e) + a)^m, x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(b*sin(f*x + e) + a)^m, x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2), x)
```

# 3.809 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$

**Optimal**. Leaf size=30

$$\mathrm{Int}\left((a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx = \int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

**Mathematica** [A]
time = 8.43, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

**Maple** [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a+b\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2), x)
```

# 3.810 $\int (a+b\sin(e+fx))^m \sqrt{c+d\sin(e+fx)}\,dx$

**Optimal**. Leaf size=30

$$\text{Int}\left((a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)},x\right)$$

[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]

[Out] Defer[Int][(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]

Rubi steps

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx=\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

**Mathematica** [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]

[Out] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^m\sqrt{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(e + fx))^m \sqrt{c + d\sin(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**m*sqrt(c + d*sin(e + f*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)
```

# 3.811 $\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

**Optimal**. Leaf size=30

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

**Rubi** [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

**Mathematica** [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{\sqrt{c+d\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin{(e + fx)})^m}{\sqrt{c + d\sin{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**m/sqrt(c + d*sin(e + f*x)), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

<u>**Mupad**</u> **[A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2), x)
```

# 3.812 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

**Optimal**. Leaf size=30

$$\operatorname{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

**Mathematica** [A]
time = 4.37, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{(c+d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin{(e + fx)})^m}{(c + d\sin{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/(c + d*sin(e + f*x))**(3/2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b \sin(e + f x))^m}{(c + d \sin(e + f x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2), x)
```

# 3.813 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

**Optimal**. Leaf size=30

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

**Mathematica** [A]
time = 6.62, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

**Maple** [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(fx+e))^m}{(c+d\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin{(e + fx)})^m}{(c + d\sin{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/(c + d*sin(e + f*x))**(5/2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2), x)
```

# 3.814 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=272

$$\frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)}+\frac{d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}\left(a^3+a^3\csc(e+fx)\right)}{f(1-n)}+\frac{a^3}{\quad}$$

```
[Out] a^3*d^3*(1-2*n)*cot(f*x+e)*(d*csc(f*x+e))^(-3+n)/f/(n^2-3*n+2)+d^3*cot(f*x+
e)*(d*csc(f*x+e))^(-3+n)*(a^3+a^3*csc(f*x+e))/f/(1-n)+a^3*d^3*(5-4*n)*cos(f
*x+e)*(d*csc(f*x+e))^(-3+n)*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+
e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)^(1/2)+a^3*d^4*(11-4*n)*cos(f*x+e)*(d*csc
(f*x+e))^(-4+n)*hypergeom([1/2, 2-1/2*n],[3-1/2*n],sin(f*x+e)^2)/f/(n^2-6*n
+8)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3899, 4082, 3872, 3857, 2722}

$$\frac{a^3d^4(11-4n)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(5-4n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{n-3}}{f(1-n)(2-n)}+\frac{d^3\cot(e+fx)\left(a^3\csc(e+fx)+a^3\right)(d\csc(e+fx))^{n-3}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2 -
n)) + (d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(a^3 + a^3*Csc[e + f*x]))
/(f*(1 - n)) + (a^3*d^3*(5 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hy
pergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 -
 n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d^4*(11 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x
])^(-4 + n)*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Sin[e + f*x]^2])/(
f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx = d^3\int (d\csc(e+fx))^{-3+n}(a+a\csc(e+fx))^3\,dx$$
$$= \frac{d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}\left(a^3+a^3\csc(e+fx)\right)}{f(1-n)} - \frac{(ad$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)$$

**Mathematica** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 24.93, size = 28213, normalized size = 103.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

**Maple** [**F**]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*csc(f*x + e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int (d\csc{(e+fx)})^{n}\, dx + \int 3(d\csc{(e+fx)})^{n}\sin{(e+fx)}\, dx + \int 3(d\csc{(e+fx)})^{n}\sin^{2}{(e+fx)}\, dx + \int (d\csc{(e+fx)})^{n}\sin^{3}{(e+fx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*csc(e + f*x))**n, x) + Integral(3*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral(3*(d*csc(e + f*x))**n*sin(e + f*x)**2, x) + Integr
al((d*csc(e + f*x))**n*sin(e + f*x)**3, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^3, x)
```

# 3.815 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=203

$$\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a^2*d^2*cot(f*x+e)*(d*csc(f*x+e))^(-2+n)/f/(1-n)+2*a^2*d^2*cos(f*x+e)*(d*cs
c(f*x+e))^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(
cos(f*x+e)^2)^(1/2)+a^2*d^3*(3-2*n)*cos(f*x+e)*(d*csc(f*x+e))^(-3+n)*hyperg
eom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)
^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3873, 3857, 2722, 4131}

$$\frac{a^2d^3(3-2n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{n-2}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a^2*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (a^2*d^3*(3 - 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2
, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx &= d^2\int (d\csc(e+fx))^{-2+n}(a+a\csc(e+fx))^2\,dx\\ &= \left(2a^2d\right)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\left(a^2\right.\\ &= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{\left(a^2d^2(3-2n)\right)\int (d\csc(e\right.}{1-n}\\ &= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e}{}\\ &= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e}{}\end{aligned}$$

**Mathematica** [A]
time = 1.29, size = 346, normalized size = 1.70

$$-\frac{2(d\csc(e+fx))^n\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}(a+a\sin(e+fx))^2\tan\left(\frac{1}{2}(e+fx)\right)\left((-2+n)\,{}_2F_1\left(1-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+4(-2+n)\,{}_2F_1\left(2-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+8\,{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4n\,{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\,{}_2F_1\left(2-n,1-\frac{n}{2};2-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)+4n\,{}_2F_1\left(2-n,1-\frac{n}{2};2-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(-2+n)(-1+n)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)/2]*((-2 + n)*Hy
pergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2] + 4*(-2 +
 n)*Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2] + 8
```

```
*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2] - 4*n*
Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2] - 4*Hyp
ergeometric2F1[2 - n, 1 - n/2, 2 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/
2] + 4*n*Hypergeometric2F1[2 - n, 1 - n/2, 2 - n/2, -Tan[(e + f*x)/2]^2]*Ta
n[(e + f*x)/2]))/(f*(-2 + n)*(-1 + n)*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)
```

**Maple [F]**
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{n} \left(a+a\sin\left(fx+e\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*csc(f*x + e)
)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \left(d \csc{\left(e + f x \right)}\right)^{n}\, dx + \int 2 \left(d \csc{\left(e + f x \right)}\right)^{n} \sin{\left(e + f x \right)}\, dx + \int \left(d \csc{\left(e + f x \right)}\right)^{n} \sin^{2}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*csc(e + f*x))**n, x) + Integral(2*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral((d*csc(e + f*x))**n*sin(e + f*x)**2, x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^2, x)
```

# 3.816 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=149

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{-2+n}\,{}_2F_1\left(\right.}{f(2-n)\sqrt{\cos^2(e+}}$$

```
[Out] a*d*cos(f*x+e)*(d*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)+a*d^2*cos(f*x+e)*(d*csc(f*x+e))
^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(cos(f*x+e
)^2)^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3317, 3872, 3857, 2722}

$$\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (a*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx &= d\int (d\csc(e+fx))^{-1+n}(a+a\csc(e+fx))\,dx\\ &= a\int (d\csc(e+fx))^n\,dx+(ad)\int (d\csc(e+fx))^{-1+n}\,dx\\ &= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}\,dx+\\ &= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.20, size = 278, normalized size = 1.87

$$\frac{2^{-1+n}ae^{-i(e+fnx)}\left(1-e^{2i(e+fx)}\right)^n\left(\frac{ie^{i(e+fx)}}{-1+e^{2i(e+fx)}}\right)^n\csc^{-1-n}(e+fx)(d\csc(e+fx))^n(1+\csc(e+fx))\left(e^{if(-1+n)x}n(1+n)\,{}_2F_1\left(\frac{1}{2}(-1+n),n;\frac{1+n}{2};e^{2i(e+fx)}\right)-e^{ie}(-1+n)\left(2ie^{ifnx}(1+n)\,{}_2F_1\left(\frac{n}{2},n;\frac{2+n}{2};e^{2i(e+fx)}\right)+e^{i(e+f(1+n)x)}n\,{}_2F_1\left(n,\frac{1+n}{2};\frac{3+n}{2};e^{2i(e+fx)}\right)\right)\right)}{f(-1+n)n(1+n)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 + n)*a*(1 - E^((2*I)*(e + f*x)))^n*((I*E^(I*(e + f*x)))/(-1 + E^((2*
I)*(e + f*x))))^n*Csc[e + f*x]^(-1 - n)*(d*Csc[e + f*x])^n*(1 + Csc[e + f*x
])*(E^(I*f*(-1 + n)*x)*n*(1 + n)*Hypergeometric2F1[(-1 + n)/2, n, (1 + n)/2
, E^((2*I)*(e + f*x))] - E^(I*e)*(-1 + n)*((2*I)*E^(I*f*n*x)*(1 + n)*Hyperg
eometric2F1[n/2, n, (2 + n)/2, E^((2*I)*(e + f*x))] + E^(I*(e + f*(1 + n)*x
))*n*Hypergeometric2F1[n, (1 + n)/2, (3 + n)/2, E^((2*I)*(e + f*x))])))/(E^
(I*(e + f*n*x))*f*(-1 + n)*n*(1 + n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2)
```

**Maple** [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (d\csc (e+fx))^n\,dx+\int (d\csc (e+fx))^n\sin (e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((d*csc(e + f*x))**n, x) + Integral((d*csc(e + f*x))**n*sin(e +
f*x), x))
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x)), x)
```

# 3.817 $\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=171

$$-\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))}+\frac{dn\cos(e+fx)(d\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+$$

```
[Out] -cot(f*x+e)*(d*csc(f*x+e))^n/f/(a+a*csc(f*x+e))+d*n*cos(f*x+e)*(d*csc(f*x+e
))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2)/a/f/(1-n)/(c
os(f*x+e)^2)^(1/2)+cos(f*x+e)*(d*csc(f*x+e))^n*hypergeom([1/2, -1/2*n],[1-1
/2*n],sin(f*x+e)^2)/a/f/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3905, 3872, 3857, 2722}

$$\frac{dn\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2(e+fx)\right)}{af\sqrt{\cos^2(e+fx)}}-\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a\csc(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + a*Csc[e + f*x]))) + (d*n*Cos[e
+ f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/
2, Sin[e + f*x]^2])/(a*f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (Cos[e + f*x]*(d*C
sc[e + f*x])^n*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Sin[e + f*x]^2])/(
a*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{(d\csc(e+fx))^{1+n}}{a+a\csc(e+fx)}\,dx}{d}$$

$$= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n(a-a\csc(e+fx))\,dx}{a^2}$$

$$= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n\,dx}{a} - \frac{n\int (d\csc(e+fx))^{1+n}\,\ldots}{ad}$$

$$= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\left(n(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}\ldots}{a}$$

$$= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2\ldots\right)}{af\sqrt{\cos^2(e+fx)}}$$

**Mathematica** [**F**]
time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

**Maple** [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(d \csc(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)
[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\csc(e+fx))^n}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x) + 1), x)/a
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x)),x)
```

```
[Out] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x)), x)
```

# 3.818 $\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=231

$$-\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))}+\frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2}+\frac{2n\cos(e+fx)(d\csc(e+fx))^{2+n}\,{}_2F_1}{3a^2d^2f\sqrt{\cos^2}}$$

```
[Out] -2/3*n*cot(f*x+e)*(d*csc(f*x+e))^(2+n)/a^2/d^2/f/(1+csc(f*x+e))+1/3*cot(f*x
+e)*(d*csc(f*x+e))^(2+n)/d^2/f/(a+a*csc(f*x+e))^2+2/3*n*cos(f*x+e)*(d*csc(f
*x+e))^(2+n)*hypergeom([1/2, -1-1/2*n],[-1/2*n],sin(f*x+e)^2)/a^2/d^2/f/(co
s(f*x+e)^2)^(1/2)-1/3*(1+2*n)*cos(f*x+e)*(d*csc(f*x+e))^(1+n)*hypergeom([1/
2, -1/2-1/2*n],[1/2-1/2*n],sin(f*x+e)^2)/a^2/d/f/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3902, 4105, 3872, 3857, 2722}

$$\frac{2n\cos(e+fx)(d\csc(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\sin^2(e+fx)\right)}{3a^2d^2f\sqrt{\cos^2(e+fx)}}-\frac{2n\cot(e+fx)(d\csc(e+fx))^{n+2}}{3a^2d^2f(\csc(e+fx)+1)}-\frac{(2n+1)\cos(e+fx)(d\csc(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\sin^2(e+fx)\right)}{3a^2df\sqrt{\cos^2(e+fx)}}+\frac{\cot(e+fx)(d\csc(e+fx))^{n+2}}{3d^2f(a\csc(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*n*Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*a^2*d^2*f*(1 + Csc[e + f*x]
)) + (Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*d^2*f*(a + a*Csc[e + f*x])^
2) + (2*n*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Hypergeometric2F1[1/2, (-2
- n)/2, -1/2*n, Sin[e + f*x]^2])/(3*a^2*d^2*f*Sqrt[Cos[e + f*x]^2]) - ((1 +
 2*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1/2, (-1 - n)
/2, (1 - n)/2, Sin[e + f*x]^2])/(3*a^2*d*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(a+a\csc(e+fx))^2}\,dx}{d^2}$$
$$= \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int \frac{(d\csc(e+fx))^{2+n}(a(-1+n)-a(1+n)\csc(e+fx))}{a+a\csc(e+fx)}\,dx}{3a^2d^2}$$
$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int(d\,\mathrm{cs}}{}$$
$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{(2n(2}{}$$
$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\left(2n(2\right.}{}$$
$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} + \frac{2n\cot}{}$$

**Mathematica [F]**
time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

**Maple [F]**
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(fx+e))^n}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(d*csc(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\csc(e+fx))^n}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x))^2,x)
[Out] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x))^2, x)
```

# 3.819 $\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))^m\,dx$

**Optimal**. Leaf size=113

$$-\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-np}(e+fx)\,(c(d\sin(e+fx$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n*p,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
/(sin(f*x+e)^(n*p))
```

**Rubi** [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2905, 2866, 2865, 2864, 138}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\sin^{-np}(e+fx)F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(c(d\sin(e+fx))^p)^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -(n*p), 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x])^(-
1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^(n*p)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))^m \, dx = \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx))$$

$$= \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n (1+\sin(e+fx))^{-}\right.$$

$$= \left(\sin^{-np}(e+fx) (c(d\sin(e+fx))^p)^n (1+\sin(e+fx))^{-m}(a\right.$$

$$= -\frac{\left(\cos(e+fx)\sin^{-np}(e+fx) (c(d\sin(e+fx))^p)^n (1+\sin\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2}; -np, \frac{1}{2}-m; \frac{3}{2}; 1-\sin(e+fx), \frac{1}{2}(1-\sin(e+\right.}{}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2967 vs. 2(113) = 226.
time = 14.00, size = 2967, normalized size = 26.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(n*p)*(c*(d*Sin[e + f*x
])^p)^n*(a + a*Sin[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appell
F1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi
/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2,
Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2
, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*p*AppellF1[1/2, -(n*p), 1 +
m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[
e + f*x]^2*Sin[e + f*x]^(-1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appe
llF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2
, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3
/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -(n*p), 1 + m
 + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e
 + f*x]^(1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p),
 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
 - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2
- f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 +
m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan
[(-e + Pi/2 - f*x)/2]^2)) - (3*m*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e +
 f*x]^(n*p)*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Ap
pellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1
[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin[e + f*x]
^(n*p)*(-1/3*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^
2*Tan[(-e + Pi/2 - f*x)/2]) - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*
AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
e + Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*Appell
F1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + P
i/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -(n*p), 1
 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*C
os[e + f*x]*Sin[e + f*x]^(n*p)*(-2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 +
 m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n
*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)
/2] + 3*(-1/3*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-
```

```
e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]
^2*Tan[(-e + Pi/2 - f*x)/2]) - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2
, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
 f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*((1
+ m + n*p)*((-3*(2 + m + n*p)*AppellF1[5/2, -(n*p), 3 + m + n*p, 7/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2
]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*n*p*AppellF1[5/2, 1 - n*p, 2 + m + n*p
, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + n*p*((-3*(1 + m + n*p)*Appel
lF1[5/2, 1 - n*p, 2 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 +
(3*(1 - n*p)*AppellF1[5/2, 2 - n*p, 1 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)
/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi
/2 - f*x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p)
, 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2
] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 +
 m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Ta
n[(-e + Pi/2 - f*x)/2]^2)^2)))
```

**Maple [F]**
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(c(d\sin\left(fx+e\right))^{p}\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (c(d\sin(e+fx))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c*(d*sin(e + f*x))**p)**n, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,(d\,\sin(e+f\,x))^p)^n\,(a+a\,\sin(e+f\,x))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^m, x)
```

# 3.820 $\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=299

$$-\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)(3+np)}+\frac{a^3(5+4np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+\right.}{f(1+np)(2+n}$$

```
[Out] -a^3*(2*n*p+7)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*p+
3)-cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(a^3+a^3*sin(f*x+e))/f/(n*p
+3)+a^3*(4*n*p+5)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^
2)^(1/2)+a^3*(4*n*p+11)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],s
in(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*p+3)/(cos(f*x
+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.34, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2905, 2842, 3047, 3102, 2827, 2722}

$$\frac{a^3(4np+11)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(4np+5)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+1)(np+2)\sqrt{\cos^2(e+fx)}}-\frac{a^3(2np+7)\sin(e+fx)\cos(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(np+2)(np+3)}-\frac{\sin(e+fx)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c(d\sin(e+fx))^p)^n}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(
2 + n*p)*(3 + n*p))) + (a^3*(5 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2,
 (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])
^p)^n)/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (a^3*(11 + 4*n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*
Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*(3 + n*p)*Sqrt[Cos[e
+ f*x]^2]) - (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(a^3 + a^3
*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^n$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e\right.}{f(3+np)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e\right.}{f(3+np)}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)}$$

**Mathematica** [A]
time = 0.90, size = 297, normalized size = 0.99

$$-\frac{a^3\cos(e+fx)\sqrt{\cos^2(e+fx)}\,\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left((24+26np+9n^2p^2+n^3p^3)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)+\frac{1}{2}(1+np)\sin(e+fx)\left(6(12+7np+n^2p^2)\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)+2(2+np)\sin(e+fx)\left(3(4+np)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3+np);\frac{1}{2}(5+np);\sin^2(e+fx)\right)+(3+np)\,{}_2F_1\left(\frac{1}{2},2+\frac{np}{2};3+\frac{np}{2};\sin^2(e+fx)\right)\sin(e+fx)\right)\right)\right)}{f(1+np)(2+np)(3+np)(4+np)(-1+\sin(e+fx))(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((24 + 26*n*p + 9*n^2*p^2 + n^3*p^3)*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Sin[e + f*x]^2] + ((1 + n*p)*Sin[e + f*x]*(6*(12 + 7*n*p + n
^2*p^2)*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] +
2*(2 + n*p)*Sin[e + f*x]*(3*(4 + n*p)*Hypergeometric2F1[1/2, (3 + n*p)/2, (
5 + n*p)/2, Sin[e + f*x]^2] + (3 + n*p)*Hypergeometric2F1[1/2, 2 + (n*p)/2,
 3 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/2))/(f*(1 + n*p)*(2 + n*p)*(3
 + n*p)*(4 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

**Maple** [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n\left(a+a\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (c(d\sin(e+fx))^p)^n\,dx + \int 3(c(d\sin(e+fx))^p)^n\sin(e+fx)\,dx + \int 3(c(d\sin(e+fx))^p)^n\sin^2(e+fx)\,dx + \int (c(d\sin(e+fx))^p)^n\sin^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**3,x)
[Out] a**3*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral(3*(c*(d*sin(e + f*
x))**p)**n*sin(e + f*x), x) + Integral(3*(c*(d*sin(e + f*x))**p)**n*sin(e +
 f*x)**2, x) + Integral((c*(d*sin(e + f*x))**p)**n*sin(e + f*x)**3, x))
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^3,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^3, x)
```

# 3.821 $\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))^2 \, dx$

**Optimal**. Leaf size=222

$$-\frac{a^2\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)}+\frac{a^2(3+2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(\ldots\right.}{f(1+np)(2+np)\sqrt{\cos^2(\ldots}}$$

```
[Out] -a^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)+a^2*(2*n*p+3)*c
os(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f*x+
e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^2)^(1/2)+2*a^2*cos(
f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c
*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.17, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2905, 2842, 2827, 2722}

$$\frac{2a^2\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2np+3)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+1)(np+2)\sqrt{\cos^2(e+fx)}}-\frac{a^2\sin(e+fx)\cos(e+fx)(c(d\sin(e+fx))^p)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 (a^2*(3 + 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*(2
 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*a^2*Cos[e + f*x]*Hypergeometric2F1[1/2,
(2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x]
)^p)^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))^2 \, dx = \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx)$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)(c(d\sin(e+fx))^p)^n}{f(2+np)} + \frac{((d\sin}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)(c(d\sin(e+fx))^p)^n}{f(2+np)} + \frac{(2a^2(a}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)(c(d\sin(e+fx))^p)^n}{f(2+np)} + \frac{a^2(3+}{}$$

**Mathematica [A]**
time = 0.44, size = 222, normalized size = 1.00

$$-\frac{a^2\cos(e+fx)\sqrt{\cos^2(e+fx)}\,\sin(e+fx)\,(c(d\sin(e+fx))^p)^n\left((6+5np+n^2p^2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)+(1+np)\sin(e+fx)\left(2(3+np)\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)+(2+np)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3+np);\frac{1}{2}(5+np);\sin^2(e+fx)\right)\sin(e+fx)\right)\right)}{f(1+np)(2+np)(3+np)(-1+\sin(e+fx))(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((6 + 5*n*p + n^2*p^2)*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2,
 Sin[e + f*x]^2] + (1 + n*p)*Sin[e + f*x]*(2*(3 + n*p)*Hypergeometric2F1[1/
2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] + (2 + n*p)*Hypergeometric2F1[
1/2, (3 + n*p)/2, (5 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n*p)
*(2 + n*p)*(3 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

**Maple** [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \left(c{\left(d \sin \left(f x +e \right)\right)}^{p}\right)^{n} \left(a +a \sin \left(f x +e \right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*((d*sin(f*x + e
))^p*c)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \left(c \left(d \sin{\left(e + f x \right)}\right)^{p}\right)^{n}\, dx + \int 2 \left(c \left(d \sin{\left(e + f x \right)}\right)^{p}\right)^{n} \sin{\left(e + f x \right)}\, dx + \int \left(c \left(d \sin{\left(e + f x \right)}\right)^{p}\right)^{n} \sin^{2}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**2,x)
[Out] a**2*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral(2*(c*(d*sin(e + f*
x))**p)**n*sin(e + f*x), x) + Integral((c*(d*sin(e + f*x))**p)**n*sin(e + f
*x)**2, x))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(d\sin\left(e+f\,x\right)\right)^p\right)^n\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^2, x)
```

# 3.822 $\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))\,dx$

**Optimal**. Leaf size=163

$$\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(1+np)\sqrt{\cos^2(e+fx)}}+\frac{a\cos(e+fx)\,{}_2F_1\left(\right.}{}$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f
*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(cos(f*x+e)^2)^(1/2)+a*cos(f*x+e)*hy
pergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f
*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2905, 2827, 2722}

$$\frac{a\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
```

```
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c(d\sin(e+fx))^p)^n (a+a\sin(e+fx))\,dx &= \left((d\sin(e+fx))^{-np}(c(d\sin(e+fx))^p)^n\right)\int (d\sin(e+fx))^{\cdots} \\ &= \left(a(d\sin(e+fx))^{-np}(c(d\sin(e+fx))^p)^n\right)\int (d\sin(e+fx)\cdots \\ &= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin\cdots}{f(1+np)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 1.12, size = 307, normalized size = 1.88

$$\frac{2^{-1-np}ae^{-i(e+fx)}\left(1-e^{2i(e+fx)}\right)^{-np}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{np}\left(2ie^{i(e+fx)}(-1+n^2p^2)\,{}_2F_1\left(-np,-\frac{np}{2};1-\frac{np}{2};e^{2i(e+fx)}\right)+np(1-np)\,{}_2F_1\left(-np,\frac{1}{2}(-1-np);\frac{1}{2}(1-np);e^{2i(e+fx)}\right)+e^{2i(e+fx)}np(1+np)\,{}_2F_1\left(-np,\frac{1}{2}(1-np);\frac{1}{2}(3-np);e^{2i(e+fx)}\right)\right)\sin^{-np}(e+fx)\,(c(d\sin(e+fx))^p)^n\,(1+\sin(e+fx))}{fnp(-1+np)(1+np)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 - n*p)*a*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x)))^(n*p)*(
(2*I)*E^(I*(e + f*x))*(-1 + n^2*p^2)*Hypergeometric2F1[-(n*p), -1/2*(n*p),
1 - (n*p)/2, E^((2*I)*(e + f*x))] + n*p*(1 - n*p)*Hypergeometric2F1[-(n*p),
 (-1 - n*p)/2, (1 - n*p)/2, E^((2*I)*(e + f*x))] + E^((2*I)*(e + f*x))*n*p*
(1 + n*p)*Hypergeometric2F1[-(n*p), (1 - n*p)/2, (3 - n*p)/2, E^((2*I)*(e +
 f*x))])*(c*(d*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x]))/(E^(I*(e + f*x))*(1 -
 E^((2*I)*(e + f*x)))^(n*p)*f*n*p*(-1 + n*p)*(1 + n*p)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2*Sin[e + f*x]^(n*p))
```

**Maple** [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+a\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(c(d\sin\left(e+fx\right))^{p}\right)^{n}\,dx+\int\left(c(d\sin\left(e+fx\right))^{p}\right)^{n}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral((c*(d*sin(e + f*x))**
p)**n*sin(e + f*x), x))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x)),x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x)), x)
```

# 3.823 $\int \frac{(c(d\sin(e+fx))^p)^n}{a+a\sin(e+fx)}\,dx$

**Optimal**. Leaf size=189

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(2+np);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+\right.}{af(1+}$$

```
[Out] -cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(a+a*sin(f*x+e))+cos(f*x+e)*hypergeom(
[1/2, 1/2*n*p],[1/2*n*p+1],sin(f*x+e)^2)*(c*(d*sin(f*x+e))^p)^n/a/f/(cos(f*
x+e)^2)^(1/2)-n*p*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a/f/(n*p+1)/(cos(f*x+e)^2)^(1/
2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2905, 2848, 2827, 2722}

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(np+2);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{af(np+1)\sqrt{\cos^2(e+fx)}}-\frac{\cos(e+fx)(c(d\sin(e+fx))^p)^n}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sin[e + f*x]^2]*
(c*(d*Sin[e + f*x])^p)^n)/(a*f*Sqrt[Cos[e + f*x]^2]) - (n*p*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(c*(d*Sin[e + f*x])^p)^n)/(a*f*(1 + n*p)*Sqrt[Cos[e + f*x]^2]) - (Cos[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(a + a*Sin[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2848

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(
```

```
a*f*(a + b*Sin[e + f*x]))), x] + Dist[d*(n/(a*b)), Int[(c + d*Sin[e + f*x])
^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2
*n] || EqQ[c, 0])
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c(d\sin(e+fx))^p)^n}{a+a\sin(e+fx)}\,dx &= \left((d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+a\sin(e+fx)}\,dx \\
&= -\frac{\cos(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(a+a\sin(e+fx))} + \frac{(dnp(d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)}{} \\
&= -\frac{\cos(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(a+a\sin(e+fx))} - \frac{(np(d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n}{a} \\
&= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(2+np);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{af\sqrt{\cos^2(e+fx)}} - \frac{np\cos(e}{}
\end{aligned}$$

**Mathematica [A]**
time = 0.18, size = 157, normalized size = 0.83

$$\frac{\cos(e+fx)\sqrt{\cos^2(e+fx)}\,\sin(e+fx)\,(c(d\sin(e+fx))^p)^n\left(-\left((2+np)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\right)+(1+np)\,{}_2F_1\left(\frac{3}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)\sin(e+fx)\right)}{af(1+np)(2+np)(-1+\sin(e+fx))(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-
((2 + n*p)*Hypergeometric2F1[3/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]
) + (1 + n*p)*Hypergeometric2F1[3/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]
^2]*Sin[e + f*x]))/(a*f*(1 + n*p)*(2 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e
+ f*x]))
```

**Maple [F]**
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(c(d\sin(fx+e))^p)^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c(d\sin{(e+fx)})^p)^n}{\sin{(e+fx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x) + 1), x)/a
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,(d\,\sin\left(e+f\,x\right))^p)^n}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x)),x)
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x)), x)
```

# 3.824 $\int \frac{(c(d\sin(e+fx))^p)^n}{(a+a\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=288

$$-\frac{np(1-2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+np)\sqrt{\cos^2(e+fx)}}+\frac{2(1$$

```
[Out] 2/3*(-n*p+1)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a^2/f/(1+sin(f*x+
e))+1/3*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(a+a*sin(f*x+e))^2-1
/3*n*p*(-2*n*p+1)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a^2/f/(n*p+1)/(cos(f*x+e)^2)^(
1/2)+2/3*(-n^2*p^2+1)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin
(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/a^2/f/(n*p+2)/(cos(f*x+e)^2)
^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2905, 2845, 3057, 2827, 2722}

$$\frac{2(1-n^2p^2)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{np(1-2np)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{2(1-np)\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(\sin(e+fx)+1)}+\frac{\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(n*p*(1 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
 n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(a^2*f*(1 +
 n*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n^2*p^2)*Cos[e + f*x]*Hypergeometric2
F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[
e + f*x])^p)^n)/(3*a^2*f*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n*p)*Cos
[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + Sin[e + f*x]
)) + (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*f*(a + a*Sin[e
 + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c(d\sin(e+fx))^p)^n}{(a+a\sin(e+fx))^2}\,dx &= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3f(a+a\sin(e+fx))^2} + \frac{((d\sin(e+fx))^{-np}\left(c(d\sin(e\right.}{} \\
&= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+}{3f(a+a} \\
&= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+}{3f(a+a} \\
&= -\frac{np(1-2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+}{3a^2f(1+np)\sqrt{\cos^2(e+fx)}}
\end{aligned}$$

**Mathematica [A]**
time = 1.86, size = 195, normalized size = 0.68

$$\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(\frac{np(-1+2np)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)}{(1+np)\sqrt{\cos^2(e+fx)}}+\frac{3-2np+(2-2np)\sin(e+fx)}{(1+\sin(e+fx))^2}-\frac{2(-1+n^2p^2)\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)\sec(e+fx)\tan(e+fx)}{2+np}\right)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*((n*p*(-1 + 2*n*p)*Hype
rgeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*Sq
rt[Cos[e + f*x]^2]) + (3 - 2*n*p + (2 - 2*n*p)*Sin[e + f*x])/(1 + Sin[e + f
*x])^2 - (2*(-1 + n^2*p^2)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 1 +
(n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(2 + n*p))
)/(3*a^2*f)
```

**Maple [F]**
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c(d\sin(e+fx))^p)^n}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
 x)/a**2
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,(d\,\sin(e+f\,x))^p)^n}{(a+a\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x))^2, x)
```

# 3.825 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx$

**Optimal**. Leaf size=298

$$\frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)}+\frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)}+$$

```
[Out] a^2*b*d^3*(1-2*n)*cot(f*x+e)*(d*csc(f*x+e))^(-3+n)/f/(n^2-3*n+2)+a^2*d^3*co
t(f*x+e)*(d*csc(f*x+e))^(-3+n)*(b+a*csc(f*x+e))/f/(1-n)+a*d^3*(3*b^2*(1-n)+
a^2*(2-n))*cos(f*x+e)*(d*csc(f*x+e))^(-3+n)*hypergeom([1/2, 3/2-1/2*n],[5/2
-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)^(1/2)+b*d^4*(b^2*(2-n)+3
*a^2*(3-n))*cos(f*x+e)*(d*csc(f*x+e))^(-4+n)*hypergeom([1/2, 2-1/2*n],[3-1/
2*n],sin(f*x+e)^2)/f/(n^2-6*n+8)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.38, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3927, 4132, 3857, 2722, 4131}

$$\frac{bd^4(3a^2(3-n)+b^2(2-n))\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^3(a^2(2-n)+3b^2(1-n))\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{n-3}}{f(1-n)(2-n)}+\frac{a^2d^3\cot(e+fx)(a\csc(e+fx)+b)(d\csc(e+fx))^{n-3}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*b*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2
- n)) + (a^2*d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(b + a*Csc[e + f*x]
))/(f*(1 - n)) + (a*d^3*(3*b^2*(1 - n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e
 + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]
^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^4*(b^2*(2 - n) + 3*a^2
*(3 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-4 + n)*Hypergeometric2F1[1/2, (4
- n)/2, (6 - n)/2, Sin[e + f*x]^2])/(f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2]
)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx &= d^3\int (d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))^3\,dx \\
&= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int \\
&= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int \\
&= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+ \\
&= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+ \\
&= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+
\end{aligned}
$$

**Mathematica [A]**
time = 0.38, size = 167, normalized size = 0.56

$$
-\frac{d\cos(e+fx)(d\csc(e+fx))^{-1+n}\sin^2(e+fx)^{\frac{1}{2}(-1+n)}\left(3ab^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{3}{2};\cos^2(e+fx)\right)+a^3\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3}{2};\cos^2(e+fx)\right)+b\csc(e+fx)\left(b^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{3}{2};\cos^2(e+fx)\right)+3a^2\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{3}{2};\cos^2(e+fx)\right)\right)\sqrt{\sin^2(e+fx)}\right)}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
3*a*b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a^3*Hyper
geometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*(b^2*Hyp
ergeometric2F1[1/2, (-2 + n)/2, 3/2, Cos[e + f*x]^2] + 3*a^2*Hypergeometric
2F1[1/2, n/2, 3/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2]))/f)
```

**Maple [F]**
time = 0.56, size = 0, normalized size = 0.00

$$
\int \left(d\csc\left(fx+e\right)\right)^{n}\left(a+b\sin\left(fx+e\right)\right)^{3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*csc(f*x + e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\csc(e+fx))^n (a+b\sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{d}{\sin(e+f\,x)}\right)^n (a+b\,\sin(e+f\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^3, x)
```

# 3.826 $\int (d\csc(e+fx))^n (a+b\sin(e+fx))^2\,dx$

**Optimal**. Leaf size=213

$$\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)}+\frac{2abd^2\cos(e+fx)(d\csc(e+fx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a^2*d^2*cot(f*x+e)*(d*csc(f*x+e))^(-2+n)/f/(1-n)+2*a*b*d^2*cos(f*x+e)*(d*cs
c(f*x+e))^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(
cos(f*x+e)^2)^(1/2)+d^3*(b^2*(1-n)+a^2*(2-n))*cos(f*x+e)*(d*csc(f*x+e))^(-3
+n)*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos
(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.18, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3873, 3857, 2722, 4131}

$$\frac{d^3(a^2(2-n)+b^2(1-n))\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{n-2}}{f(1-n)}+\frac{2abd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a*b*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (d^3*(b^2*(1 -
n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[
1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e +
 f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d\csc(e+fx))^n(a+b\sin(e+fx))^2\,dx &= d^2\int (d\csc(e+fx))^{-2+n}(b+a\csc(e+fx))^2\,dx \\
&= (2abd)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\,(b^2 \\
&= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \left(d^2\left(b^2+\frac{a^2(2-n)}{1-n}\right)\right) \\
&= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e \\
&= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e
\end{aligned}$$

**Mathematica [A]**
time = 0.25, size = 135, normalized size = 0.63

$$-\frac{d\cos(e+fx)(d\csc(e+fx))^{-1+n}\sin^2(e+fx)^{\frac{1}{2}(-1+n)}\left(b^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{3}{2};\cos^2(e+fx)\right)+a\left(a\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3}{2};\cos^2(e+fx)\right)+2b\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{3}{2};\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a*(a*Hypergeo
```

```
metric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + 2*b*Csc[e + f*x]*Hypergeom
etric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/f)
```

**Maple [F]**
time = 0.53, size = 0, normalized size = 0.00

$$\int (d\csc(fx+e))^n (a+b\sin(fx+e))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*csc(f*x
+ e))^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\csc(e+fx))^n (a+b\sin(e+fx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**2,x)
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^2, x)
```

# 3.827 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=149

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{-2+n}\,{}_2F_1(}{f(2-n)\sqrt{\cos^2(e+}}$$

```
[Out] a*d*cos(f*x+e)*(d*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)+b*d^2*cos(f*x+e)*(d*csc(f*x+e))
^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(cos(f*x+e
)^2)^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3317, 3872, 3857, 2722}

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx &= d\int (d\csc(e+fx))^{-1+n}(b+a\csc(e+fx))\,dx\\ &= a\int (d\csc(e+fx))^n\,dx+(bd)\int (d\csc(e+fx))^{-1+n}\,dx\\ &= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-n}\,dx+\\ &= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

**Mathematica [A]**
time = 0.14, size = 105, normalized size = 0.70

$$-\frac{d\cos(e+fx)(d\csc(e+fx))^{-1+n}\sin^2(e+fx)^{\frac{1}{2}(-1+n)}\left(a\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3}{2};\cos^2(e+fx)\right)+b\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{3}{2};\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
a*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*H
ypergeometric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/f)
```

**Maple [F]**
time = 0.15, size = 0, normalized size = 0.00

$$\int (d\csc(fx+e))^n\,(a+b\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\csc(e+fx))^n (a+b\sin(e+fx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e)),x)
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\sin(e+f\,x)}\right)^n (a+b\,\sin(e+f\,x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x)), x)
```

# 3.828 $\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=204

$$\frac{bF_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{1+n}\sin(e+fx)\sin^2(e+fx)^{n/2}}{(a^2-b^2)\,df}-\frac{aF_1\left(\right.}{}$$

```
[Out] b*AppellF1(1/2,1/2*n,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*
x+e)*(d*csc(f*x+e))^(1+n)*sin(f*x+e)*(sin(f*x+e)^2)^(1/2*n)/(a^2-b^2)/d/f-a
*AppellF1(1/2,1/2+1/2*n,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos
(f*x+e)*(d*csc(f*x+e))^(1+n)*(sin(f*x+e)^2)^(1/2+1/2*n)/(a^2-b^2)/d/f
```

**Rubi** [A]
time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3954, 2902, 3268, 440}

$$\frac{b\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\,(a^2-b^2)}-\frac{a\cos(e+fx)\sin^2(e+fx)^{\frac{n+1}{2}}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n+1}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*AppellF1[1/2, n/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)
^(n/2))/((a^2 - b^2)*d*f) - (a*AppellF1[1/2, (1 + n)/2, 1, 3/2, Cos[e + f*x
]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1
+ n)*(Sin[e + f*x]^2)^((1 + n)/2))/((a^2 - b^2)*d*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
```

```
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx = \frac{\int \frac{(d\csc(e+fx))^{1+n}}{b+a\csc(e+fx)}\,dx}{d}$$

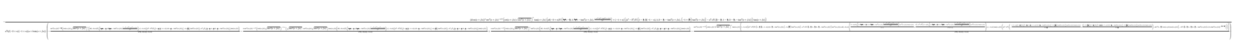
$$= \frac{\left((d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a+b\sin(e+fx)}\,dx}{d}$$

$$= \frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a^2-b^2\sin^2(e+fx)}\,dx}{d} - \frac{(b(d\csc(e+fx))^{1+n}\,\mathrm{s}}{}$$

$$= -\frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+2\left(-\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{\frac{1}{2}+\frac{n}{2}}\right)\mathrm{Subst}\left(\int \frac{(1-x^2}{a^2-b}\right.}{df}$$

$$= \frac{bF_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{1+n}\sin(e+}{(a^2-b^2)\,df}$$

**Mathematica** [**B**] Leaf count is larger than twice the leaf count of optimal. 1665 vs. $2(204) = 408$.
time = 14.99, size = 1665, normalized size = 8.16



Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
[Out] -(((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(a
*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-
1 - n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2
*Hypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f
*x]))/(a^2*b*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x]
)*(-(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(a*b
*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1
- n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*H
ypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*(-2 + n)*(-1 + n))) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1
 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e + f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e +
f*x]*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^
2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 -
n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2
] - a^2*Hypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Ta
n[e + f*x]))/(a^2*b*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)) + (n*(Cot[e +
 f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(a*b*(-2 + n)*AppellF1[(1 - n)
/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[1/2 - n/2
, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(-2 + n)*(-1 +
n)*(Sec[e + f*x]^2)^(n/2)) - ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e +
 f*x]*((-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[1/2 - n
/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Sec[e + f*x]^2 + a*b*(-2 + n)*(((1
- n)*n*AppellF1[1 + (1 - n)/2, 1 - n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2,
((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (
2*(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1/2*n, 2, 1 + (3 - n)/2, -T
an[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(a^2*(3 - n))) + (-1 + n)*Tan[e + f*x]*((a^2 - b^2)*(-(((-1 - n)*(1 - n
/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 1, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (2*(-1 + b
^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 2, 3 - n/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) -
2*a^2*(1 - n/2)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2 - n/2, 1
- n/2, 2 - n/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1/2 + n/2)))))/(a
^2*b*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)))))
```

**Maple** [**F**]

time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(d \csc(fx+e))^n}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\csc{(e+fx)})^n}{a+b\sin{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x)), x)
```

# 3.829 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=321

$$-\frac{b^2 F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{2+n}\sin^3(e+fx)\sin^2(e+}{\left(a^2-b^2\right)^2 d^2 f}$$

```
[Out] -b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2)
)*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*sin(f*x+e)^3*(sin(f*x+e)^2)^(-1/2+1/2*n)/
(a^2-b^2)^2/d^2/f-a^2*AppellF1(1/2,1/2+1/2*n,2,3/2,cos(f*x+e)^2,-b^2*cos(f*
x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*sin(f*x+e)*(sin(f*x+e)^2)
^(1/2+1/2*n)/(a^2-b^2)^2/d^2/f+2*a*b*AppellF1(1/2,1/2*n,2,3/2,cos(f*x+e)^2,
-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*(sin(f*x+e)^2)
^(1+1/2*n)/(a^2-b^2)^2/d^2/f
```

**Rubi** [A]
time = 0.37, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3954, 2903, 3268, 440}

$$-\frac{a^2\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n+1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n+1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2 f\left(a^2-b^2\right)^2}+\frac{2ab\cos(e+fx)\sin^2(e+fx)^{\frac{n+2}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2 f\left(a^2-b^2\right)^2}-\frac{b^2\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n-1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2 f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]^3*(Si
n[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^2*d^2*f)) - (a^2*AppellF1[1/2, (1
+ n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e
+ f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)^((1 + n)/2))/
((a^2 - b^2)^2*d^2*f) + (2*a*b*AppellF1[1/2, n/2, 2, 3/2, Cos[e + f*x]^2, -
((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*(
Sin[e + f*x]^2)^((2 + n)/2))/((a^2 - b^2)^2*d^2*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
```

```
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx &= \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(b+a\csc(e+fx))^2}\,dx}{d^2} \\
&= \frac{((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx))\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{d^2} \\
&= \frac{((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx))\int \left(-\frac{2ab\sin^{1-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^2}+\frac{a^2\sin^{-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^2}+\right.}{d^2} \\
&= \frac{(a^2(d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx))\int \frac{\sin^{-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^2}\,dx}{d^2} - \frac{(2ab(d\csc(e+}{} \\
&= -\frac{\left(b^2(d\csc(e+fx))^{2+n}\sin^{2+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\operatorname{Subst}\left(\int \frac{}{(-}\right.}{d^2f} \\
&= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+f}{(a^2-b^2)^2\,d^2f}
\end{aligned}
$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1872 vs. $2(321) = 642$.
time = 16.42, size = 1872, normalized size = 5.83

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/
2, -1/2*n, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] +
(a^2 - b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*f*(-2
+ n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x])^2*(((Sec[e + f*x]
^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(-(a*(a^2 + b^2)*(-2 +
n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 2, (3 -
n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(-1 + n
)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)) + (n*(
Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e +
 f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2 + b^2)*(-2 + n)*Appell
```

```
F1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 2, (3 - n)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(-1 + n)*AppellF
1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)
^(n/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(-(a*(a^2
 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2,
 (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1
/2*n, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2
- b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-
1 + n)*(Sec[e + f*x]^2)^(n/2)) + ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan
[e + f*x]*(-(a*(a^2 + b^2)*(-2 + n)*(((1 - n)*n*AppellF1[1 + (1 - n)/2, 1 -
 n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(3 - n) + (2*(-1 + b^2/a^2)*(1 - n)*AppellF1[1 +
(1 - n)/2, -1/2*n, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))) + 2*b*((a^2 - b^2)*(-1 + n
)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Sec[e + f*x]^2 + a*b*(-2 + n)*(((1 - n)*n*AppellF1[1 + (1
- n)/2, 1 - n/2, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (4*(-1 + b^2/a^2)*(1 - n)*Ap
pellF1[1 + (1 - n)/2, -1/2*n, 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/
a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n)) + (a^2 - b^2)*(-
1 + n)*Tan[e + f*x]*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2
, 2, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(2 - n/2)) + (4*(-1 + b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2,
(-1 - n)/2, 3, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(2 - n/2)))))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*
(Sec[e + f*x]^2)^(n/2))))
```

**Maple [F]**
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(d \csc (fx+e))^n}{(a+b\sin (fx+e))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \csc{(e + fx)})^n}{(a + b \sin{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{(a + b\,\sin{(e + f\,x)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^2, x)
```

# 3.830 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=432

$$-\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{3+n}\sin^4(e+fx)\sin^2(e+}{(a^2-b^2)^3d^3f}$$

```
[Out] -3*a*b^2*AppellF1(1/2,-1/2+1/2*n,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^4*(sin(f*x+e)^2)^(-1/2+1/2
*n)/(a^2-b^2)^3/d^3/f+b^3*AppellF1(1/2,-1+1/2*n,3,3/2,cos(f*x+e)^2,-b^2*cos
(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^3*(sin(f*x+
e)^2)^(1/2*n)/(a^2-b^2)^3/d^3/f+3*a^2*b*AppellF1(1/2,1/2*n,3,3/2,cos(f*x+e)
^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^
3*(sin(f*x+e)^2)^(1/2*n)/(a^2-b^2)^3/d^3/f-a^3*AppellF1(1/2,1/2+1/2*n,3,3/2
,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*
(sin(f*x+e)^2)^(3/2+1/2*n)/(a^2-b^2)^3/d^3/f
```

**Rubi** [A]

time = 0.48, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3954, 2903, 3268, 440}

$$-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{3a^2b\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{b^3\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-2}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}-\frac{a^3\cos(e+fx)\sin^2(e+fx)^{\frac{n+3}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n+1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^4*
(Sin[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^3*d^3*f) + (b^3*AppellF1[1/2, (
-2 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos
[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/(
(a^2 - b^2)^3*d^3*f) + (3*a^2*b*AppellF1[1/2, n/2, 3, 3/2, Cos[e + f*x]^2,
-((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*
Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/((a^2 - b^2)^3*d^3*f) - (a^3*AppellF
1[1/2, (1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2
))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*(Sin[e + f*x]^2)^((3 + n)/2))/((a
^2 - b^2)^3*d^3*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx = \frac{\int \frac{(d\csc(e+fx))^{3+n}}{(b+a\csc(e+fx))^3}\,dx}{d^3}$$
$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^3}\,dx}{d^3}$$
$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \left(-\frac{3a^2b\sin^{1-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^3}+\frac{3ab^2\sin^{2-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^3}+\right.}{d^3}$$
$$= \frac{\left(a^3(d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a^2-b^2\sin^2(e+fx))^3}\,dx}{d^3} - \frac{(3a^2b(d\csc(e+f}{}$$
$$= -\frac{\left(3ab^2(d\csc(e+fx))^{3+n}\sin^{3+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\operatorname{Subst}\left(\int\right.}{d^3f}$$
$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+}{(a^2-b^2)^3\,d^3f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2406 vs. 2(432) = 864.
time = 16.67, size = 2406, normalized size = 5.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e
+ f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 -
 n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1
- n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e
+ f*x])))/(a^4*(a^2 - b^2)*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a +
b*Sin[e + f*x])^3*(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f
*x]^2])^n*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 -
 n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2 + n)
*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2
, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1
[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f
```

```
*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)) + (n*(Cot[e + f
*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e + f*x]^2*S
qrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 -
 n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF
1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(
-1 + n)*(Sec[e + f*x]^2)^(n/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*
Tan[e + f*x]^2*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2,
 (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2
 + n)*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)
/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*App
ellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[
e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*
x]^2)^(n/2)) + ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a*(a^
2 + 3*b^2)*(-2 + n)*((4*(-1 + b^2/a^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 -
 n/2, 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(3 - n) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 -
 n)/2, -1/2*n, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))) + b*((-1 + n)*((3*a^2 + b^2)*A
ppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Ta
n[e + f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Sec[e + f*x]^2 + 4*a*b*(-2 + n)*((6*(
-1 + b^2/a^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 - n/2, 4, 1 + (3 - n)/2, -
Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])
/(3 - n) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 - n)/2, -1/2*n, 3, 1 + (3
- n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(3 - n)) + (-1 + n)*Tan[e + f*x]*((3*a^2 + b^2)*(-(((-1 - n)*(1 -
 n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 2, 3 - n/2, -Tan[e + f*x]^2, (-1 +
b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (4*(-1 +
 b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 3, 3 - n/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2))
- 4*b^2*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 3, 3 - n/2
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(2 - n/2)) + (6*(-1 + b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2,
4, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(2 - n/2))))))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*
x]^2)^(n/2))))
```

**Maple** [F]

time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(fx+e))^n}{(a+b\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^3, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(d*csc(f*x + e))^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**3,x)
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**3, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^3, x)
```

# 3.831 $\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^m dx$

**Optimal**. Leaf size=56

$$(d\sin(e+fx))^{-np}(c(d\sin(e+fx))^p)^n \operatorname{Int}((d\sin(e+fx))^{np}(a+b\sin(e+fx))^m,x)$$

```
[Out] (c*(d*sin(f*x+e))^p)^n*Unintegrable((d*sin(f*x+e))^(n*p)*(a+b*sin(f*x+e))^m
,x)/((d*sin(f*x+e))^(n*p))
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^m dx$$

Verification is not applicable to the result.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sin[e + f*x])^p)^n*Defer[Int][(d*Sin[e + f*x])^(n*p)*(a + b*Sin[e +
f*x])^m, x])/(d*Sin[e + f*x])^(n*p)
```

Rubi steps

$$\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^m dx = \left((d\sin(e+fx))^{-np}(c(d\sin(e+fx))^p)^n\right)\int (d\sin(e+fx))^{n}$$

**Mathematica** [A]
time = 1.62, size = 0, normalized size = 0.00

$$\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^m dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m, x]
```

**Maple** [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int (c(d\sin(fx+e))^p)^n (a+b\sin(fx+e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c(d\sin{(e+fx)})^{p}\right)^{n}(a+b\sin{(e+fx)})^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**m, x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,(d\,\sin\left(e+f\,x\right))^p)^n\,\left(a+b\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^m,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^m, x)
```

# 3.832 $\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^3 \, dx$

**Optimal**. Leaf size=323

$$-\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)(3+np)}+\frac{a(3b^2(1+np)+a^2(2+np))\cos(e+fx)\,{}_2F_1(\ldots}{f(1+\ldots}$$

```
[Out] -a*b^2*(2*n*p+7)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*
p+3)-b^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))/f/(n
*p+3)+a*(3*b^2*(n*p+1)+a^2*(n*p+2))*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2]
,[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n
*p+2)/(cos(f*x+e)^2)^(1/2)+b*(b^2*(n*p+2)+3*a^2*(n*p+3))*cos(f*x+e)*hyperge
om([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e)
)^p)^n/f/(n*p+2)/(n*p+3)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.38, antiderivative size = 303, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2905, 2872, 3102, 2827, 2722}

$$\frac{b\left(\frac{3a^2}{np+2}+\frac{b^2}{np+3}\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{a\left(\frac{a^2}{np+1}+\frac{3b^2}{np+2}\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f\sqrt{\cos^2(e+fx)}}-\frac{ab^2(2np+7)\sin(e+fx)\cos(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(np+2)(np+3)}-\frac{b^2\sin(e+fx)\cos(e+fx)(a+b\sin(e+fx))\,(c(d\sin(e+fx))^p)^n}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f
*(2 + n*p)*(3 + n*p))) + (a*(a^2/(1 + n*p) + (3*b^2)/(2 + n*p))*Cos[e + f*x
]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[Cos[e + f*x]^2]) + (b*((3*a^2)/(2 +
n*p) + b^2/(3 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 +
 n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[C
os[e + f*x]^2]) - (b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(
a + b*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (c(d\sin(e+fx))^p)^n\,(a+b\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int (d\sin(e+fx))^{n\cdots}$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n\,(a+b\sin(e\cdots}{f(3+np)}$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)(3+np)}\cdots$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)(3+np)}\cdots$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)(3+np)}\cdots$$

**Mathematica** [A]
time = 0.69, size = 230, normalized size = 0.71

$$\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(-\frac{ab^2(7+2np)}{2+np}+\frac{a(3+np)(3b^2(1+np)+a^2(2+np))\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)}{(1+np)(2+np)\sqrt{\cos^2(e+fx)}}+\frac{b(b^2(2+np)+3a^2(3+np))\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)\sin(e+fx)}{(2+np)\sqrt{\cos^2(e+fx)}}-b^2(a+b\sin(e+fx))\right)}{f(3+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-((a*b^2*(7 + 2*n*p))/
(2 + n*p)) + (a*(3 + n*p)*(3*b^2*(1 + n*p) + a^2*(2 + n*p))*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*(2 + n*p)*Sqr
t[Cos[e + f*x]^2]) + (b*(b^2*(2 + n*p) + 3*a^2*(3 + n*p))*Hypergeometric2F1
[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n*p)*Sq
rt[Cos[e + f*x]^2]) - b^2*(a + b*Sin[e + f*x])))/(f*(3 + n*p))
```

**Maple** [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**3,x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**3, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^3,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^3, x)
```

# 3.833 $\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^2 dx$

**Optimal**. Leaf size=231

$$-\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)}+\frac{\left(b^2(1+np)+a^2(2+np)\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\cdots\right.}{f(1+np)(2+np\cdots}$$

```
[Out] -b^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)+(b^2*(n*p+1)+a^
2*(n*p+2))*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)
^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^2)^(1/2
)+2*a*b*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin
(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]

time = 0.16, antiderivative size = 221, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2905, 2868, 2722, 3093}

$$\frac{\left(\frac{a^2}{np+1}+\frac{b^2}{np+2}\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{2ab\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{b^2\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 ((a^2/(1 + n*p) + b^2/(2 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)
/(f*Sqrt[Cos[e + f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 +
 n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)
^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))^2\,dx = \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx))^{n}$$

$$= \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx))^{n}$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)} + \frac{2ab\cos(}{}$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(2+np)} + \frac{\left(a^2+\frac{b^2}{}\right.}{}$$

**Mathematica [A]**
time = 0.23, size = 152, normalized size = 0.66

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-1-np)} (c(d\sin(e+fx))^p)^n \left(b^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-np);\frac{3}{2};\cos^2(e+fx)\right)\sin(e+fx)+a\left(a\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{3}{2};\cos^2(e+fx)\right)\sin(e+fx)+2b\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{3}{2};\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*(b
^2*Hypergeometric2F1[1/2, (-1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] +
 a*(a*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x]
 + 2*b*Hypergeometric2F1[1/2, -1/2*(n*p), 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e +
 f*x]^2])))/f)
```

**Maple [F]**
time = 0.54, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n} \left(a+b\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*((d*sin(f*x
 + e))^p*c)^n, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\sin (e+fx))^p)^n (a+b\sin (e+fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,(d\,\sin{(e+f\,x)})^p)^n\,(a+b\,\sin{(e+f\,x)})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^2,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^2, x)
```

# 3.834 $\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))\,dx$

**Optimal**. Leaf size=163

$$\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\,(c(d\sin(e+fx))^p)^n}{f(1+np)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)\,{}_2F_1}{}$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f
*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(cos(f*x+e)^2)^(1/2)+b*cos(f*x+e)*hy
pergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f
*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2905, 2827, 2722}

$$\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{b\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)(c(d\sin(e+fx))^p)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
```

```
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c(d\sin(e+fx))^p)^n (a+b\sin(e+fx))\,dx &= \left((d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx))^{np} \\ &= \left(a(d\sin(e+fx))^{-np} (c(d\sin(e+fx))^p)^n\right) \int (d\sin(e+fx))^{n} \\ &= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(}{f(1+np)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

**Mathematica** [A]
time = 0.16, size = 129, normalized size = 0.79

$$\frac{\sqrt{\cos^2(e+fx)}\,(c(d\sin(e+fx))^p)^n \left(a(2+np)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)+b(1+np)\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sin^2(e+fx)\right)\sin(e+fx)\right)\tan(e+fx)}{f(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*(c*(d*Sin[e + f*x])^p)^n*(a*(2 + n*p)*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2] + b*(1 + n*p)*Hypergeomet
ric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e +
 f*x])/(f*(1 + n*p)*(2 + n*p))
```

**Maple** [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (c(d\sin(fx+e))^p)^n (a+b\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\sin{(e+fx)})^p)^n (a+b\sin{(e+fx)})\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,\left(a+b\,\sin\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x)),x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x)), x)
```

# 3.835 $\int \frac{(c(d\sin(e+fx))^p)^n}{a+b\sin(e+fx)}\,dx$

**Optimal**. Leaf size=204

$$\frac{bF_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n}{(a^2-b^2)\,f}-\frac{aF_1\left(\frac{1}{2};\frac{1}{2}(1-\right.}{}$$

```
[Out] b*AppellF1(1/2,-1/2*n*p,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos
(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)/f/((sin(f*x+e)^2)^(1/2*n*p))-a*App
ellF1(1/2,-1/2*n*p+1/2,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cot(
f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)/f
```

**Rubi** [A]
time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2905, 2902, 3268, 440}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{a\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-np)}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(1-np),1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*AppellF1[1/2, -1/2*(n*p), 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)
/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f*(Sin[e
 + f*x]^2)^((n*p)/2)) - (a*AppellF1[1/2, (1 - n*p)/2, 1, 3/2, Cos[e + f*x]^
2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 -
 n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
```

```
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(c(d\sin(e+fx))^p)^n}{a+b\sin(e+fx)}\,dx &= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+b\sin(e+fx)}\,dx\\ &= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{(b(d\sin}{}\\ &= \frac{\left(b\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^{\frac{np}{2}}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e+fx)\right.}{f}\\ &= \frac{bF_1\left(\frac{1}{2}; -\frac{np}{2}, 1; \frac{3}{2}; \cos^2(e+fx), -\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(c\right.}{(a^2-b^2)\,f}\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1808 vs. $2(204) = 408$.
time = 15.73, size = 1808, normalized size = 8.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e +
f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/
2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 +
n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*H
ypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Ta
```

```
n[e + f*x])))/(a^2*b*f*(1 + n*p)*(2 + n*p)*(a + b*Sin[e + f*x])*(((Sec[e +
f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2
)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1
[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2] - a*(1 + n*p)*Hypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2
 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) +
(n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1,
 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]
+ a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*
x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*Hypergeometric2F1[1
+ (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*
b*(1 + n*p)*(2 + n*p)) + (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[
e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f
*x]^2/Sqrt[Sec[e + f*x]^2])*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-
1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/
2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*Hyperg
eometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e +
 f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) + ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e +
f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*Appel
lF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + (a^2 - b^2)*(1 + n*p)*Tan[e + f*x]*((2
*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 2, 3 + (n
*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1
+ (-1 + n*p)/2, 1, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x
]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) + a*(-(a*(1 + n*p)*Hyperge
ometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Sec[e +
f*x]^2) + b*(2 + n*p)*((2*(-a^2 + b^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2,
(n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)
/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + n*p)) - (n*p*(1 + n*p)*AppellF
1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)) - 2*a*
(1 + (n*p)/2)*(1 + n*p)*Sec[e + f*x]^2*(-Hypergeometric2F1[1 + (n*p)/2, (1
+ n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^((-1 - n*p)/
2)))))/(a^2*b*(1 + n*p)*(2 + n*p))))
```

**Maple [F]**

time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \sin \left(f x+e \right)\right)^{p}\right)^{n}}{a +b \sin \left(f x+e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(d\sin{(e+fx)})^p)^n}{a+b\sin{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e)),x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x)),x)
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x)), x)
```

# 3.836 $\int \frac{(c(d\sin(e+fx))^p)^n}{(a+b\sin(e+fx))^2}\,dx$

**Optimal**. Leaf size=322

$$\frac{2abF_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n}{\left(a^2-b^2\right)^2 f}-\frac{b^2F_1\left(\frac{1}{2};\right.}{}$$

```
[Out] 2*a*b*AppellF1(1/2,-1/2*n*p,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^2/f/((sin(f*x+e)^2)^(1/2*n*p))
-b^2*AppellF1(1/2,-1/2*n*p-1/2,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p-1/2)*(c*(d*sin(f*x+e))^p
)^n/(a^2-b^2)^2/f-a^2*AppellF1(1/2,-1/2*n*p+1/2,2,3/2,cos(f*x+e)^2,-b^2*cos
(f*x+e)^2/(a^2-b^2))*cot(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*sin(f*x
+e))^p)^n/(a^2-b^2)^2/f
```

**Rubi** [A]

time = 0.35, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2905, 2903, 3268, 440, 16}

$$\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{b^2\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-np-1)}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(-np-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{a^2\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-np)}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(1-np),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*b*AppellF1[1/2, -1/2*(n*p), 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f*
(Sin[e + f*x]^2)^((n*p)/2)) - (b^2*AppellF1[1/2, (-1 - n*p)/2, 2, 3/2, Cos[
e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*Sin[e + f*x]*
(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f)
 - (a^2*AppellF1[1/2, (1 - n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f
*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[
e + f*x])^p)^n)/((a^2 - b^2)^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c(d\sin(e+fx))^p)^n}{(a+b\sin(e+fx))^2}\,dx = \left((d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^2}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int \left(\frac{a^2(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2} - \frac{2ab\sin(}{(a}\right.$$

$$= \left(a^2(d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int \frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx - (2ab$$

$$= \frac{\left(b^2(d\sin(e+fx))^{-np}\,(c(d\sin(e+fx))^p)^n\right)\int \frac{(d\sin(e+fx))^{2+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{(2ab(d\sin(}{}$$

$$= -\frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(1-np),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)}{\left(a^2-b^2\right)^2 f}$$

$$= \frac{2abF_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\,(c(}{\left(a^2-b^2\right)^2 f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 1970 vs. $2(322) = 644$.

time = 16.89, size = 1970, normalized size = 6.12

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1
+ n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[
1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*
Tan[e + f*x]^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*f*(1 + n*p)*(2 + n*p)*(a +
b*Sin[e + f*x])^2*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec
[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)
/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2
*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/
a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-
1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Tan[e + f*x]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p))) - (n*p*(Sec[e + f*x]
^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a
*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (
n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) +
 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)
/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^3*(a^
2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x
]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(-(a*(2 + n*p)*((a^2 + b^2
)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2
/a^2)*Tan[e + f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 +
n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (
-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e +
 f*x]^2/Sqrt[Sec[e + f*x]^2]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Se
c[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n
*p)*(2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (
n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + 2*
b*(a^2 - b^2)*(1 + n*p)*Tan[e + f*x]*((4*(-1 + b^2/a^2)*(1 + (n*p)/2)*Appel
lF1[2 + (n*p)/2, (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p
)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 2, 3 + (n*p)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(
```

```
2 + (n*p)/2)) - a*(2 + n*p)*((a^2 + b^2)*((2*(-1 + b^2/a^2)*(1 + n*p)*Appel
lF1[1 + (1 + n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) - (n*p*(1 + n
*p)*AppellF1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p))
 - 2*b^2*((4*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 3,
 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/(3 + n*p) - (n*p*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, 1
+ (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)))))/(a^3*(a^2 - b^2)*(1 + n*p)*(
2 + n*p)))))
```

**Maple** [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \sin \left(f x +e \right)\right)^{p}\right)^{n}}{\left(a +b \sin \left(f x +e \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \sin \left(e + f x\right)\right)^{p}\right)^{n}}{\left(a + b \sin \left(e + f x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^2, x)
```

# 3.837 $\int \frac{(c(d\sin(e+fx))^p)^n}{(a+b\sin(e+fx))^3}\,dx$

**Optimal**. Leaf size=428

$$\frac{3a^2bF_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n}{\left(a^2-b^2\right)^3f}+\frac{b^3F_1\left(\frac{1}{2};\right.}{}$$

```
[Out] 3*a^2*b*AppellF1(1/2,-1/2*n*p,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*n*p
))+b^3*AppellF1(1/2,-1/2*n*p-1,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*n*
p))-3*a*b^2*AppellF1(1/2,-1/2*n*p-1/2,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/
(a^2-b^2))*cos(f*x+e)*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p-1/2)*(c*(d*sin(f*
x+e))^p)^n/(a^2-b^2)^3/f-a^3*AppellF1(1/2,-1/2*n*p+1/2,3,3/2,cos(f*x+e)^2,-
b^2*cos(f*x+e)^2/(a^2-b^2))*cot(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*
sin(f*x+e))^p)^n/(a^2-b^2)^3/f
```

**Rubi** [A]
time = 0.46, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2905, 2903, 3268, 440, 16}

$$\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^nF_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}-\frac{3ab^2\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-np-1)}\left(c(d\sin(e+fx))^p\right)^nF_1\left(\frac{1}{2};\frac{1}{2}(-np-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}+\frac{b^3\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^nF_1\left(\frac{1}{2};\frac{1}{2}(-np-2),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}-\frac{a^3\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-np)}\left(c(d\sin(e+fx))^p\right)^nF_1\left(\frac{1}{2};\frac{1}{2}(1-np),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*b*AppellF1[1/2, -1/2*(n*p), 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f
*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*
f*(Sin[e + f*x]^2)^((n*p)/2)) + (b^3*AppellF1[1/2, (-2 - n*p)/2, 3, 3/2, Co
s[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e
 + f*x])^p)^n)/((a^2 - b^2)^3*f*(Sin[e + f*x]^2)^((n*p)/2)) - (3*a*b^2*Appe
llF1[1/2, (-1 - n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2
 - b^2))]*Cos[e + f*x]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*S
in[e + f*x])^p)^n)/((a^2 - b^2)^3*f) - (a^3*AppellF1[1/2, (1 - n*p)/2, 3, 3
/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[
e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 2905

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sin[e + f*x
])^p)^FracPart[n]/(d*Sin[e + f*x])^(p*FracPart[n])), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c(d\sin(e+fx))^p)^n}{(a+b\sin(e+fx))^3}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^3}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \left(\frac{a^3(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3} - \frac{3a^2b\sin}{(a}\right.$$

$$= \left(a^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2\right.$$

$$= \frac{\left(b^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{3+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2(d\sin\right.}{}$$

$$= -\frac{a^3F_1\left(\frac{1}{2};\frac{1}{2}(1-np),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)}{\left(a^2-b^2\right)^3 f}$$

$$= \frac{3a^2bF_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\,(c}{\left(a^2-b^2\right)^3 f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2570 vs. $2(428) = 856$.

time = 15.95, size = 2570, normalized size = 6.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(
1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2,
-Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*
p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*
p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*f*(1 + n*p)*(2 + n*p)*(a + b*Sin[e
 + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*
x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)
/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2
*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 +
 b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/
2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
```

x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p))) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + b*(3*a^2 + b^2)*(1 + n*p)*Tan[e + f*x]*((4*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 2, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) - 4*b^3*(1 + n*p)*Tan[e + f*x]*((6*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 4, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) - a*(2 + n*p)*((a^2 + 3*b^2)*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) + (4*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, -1 + (n*p)/2, 3, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)) - 4*b^2*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 3, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) + (6*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, -1 + (n*p)/2, 4, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*S

```
ec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)))))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n
*p)))))
```

**Maple [F]**
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(c(d \sin (fx+e))^p)^n}{(a+b \sin (fx+e))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2
+ (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(d \sin (e+fx))^p)^n}{(a+b \sin (e+fx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**3,x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**3, x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)^n}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^3,x)
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^3, x)
```

# Chapter 4

# Appendix

## Local contents

# 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica_syntax.zip

**Maple and Mupad format** Maple_syntax.zip

**Sympy format** SYMPY_syntax.zip

**Sage math format** SAGE_syntax.zip

# 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

## 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*     is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*     antiderivative*)
(* "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])    #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()):  #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()):  #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1:  #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```